Bose-Einstein condensation of cold dark matter axions

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Axion 2022 Conference

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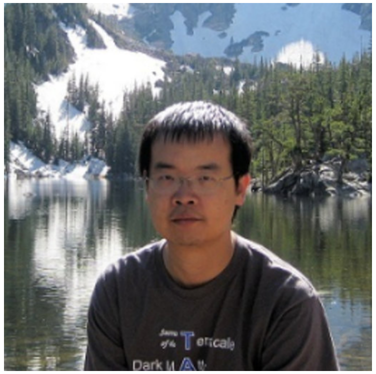
grant DE-SC0022148

based on work done in collaboration with

- Qiaoli Yang 2009
- Ozgur Erken, Heywood Tam and Qiaoli Yang 2012
- Nilanjan Banik 2013
- Elisa Todarello 2017
- Sankha Chakrabarty, Yaqi Han and Anthony Gonzalez 2021



Outline

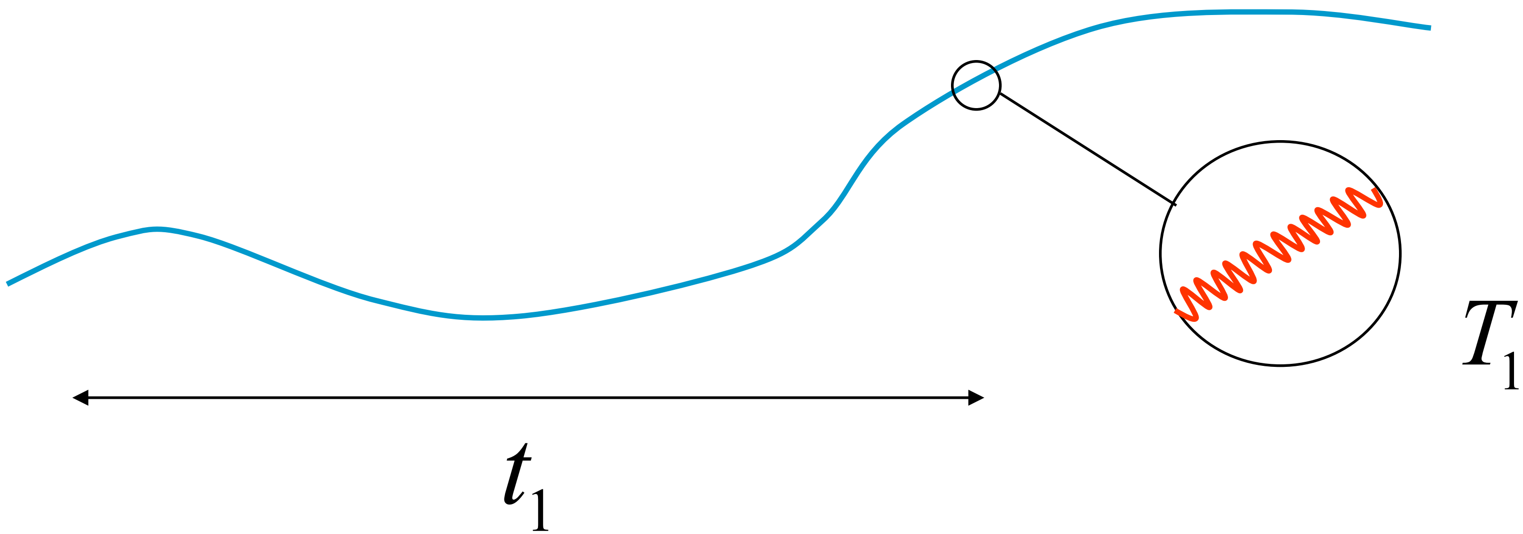
- Cosmic axion Bose-Einstein condensation

  (axions are different)

- Evidence for axion dark matter from the study of caustics (axions are better)

No assumption other than the standard QCD axion

There are two cosmic axion populations: hot and cold.



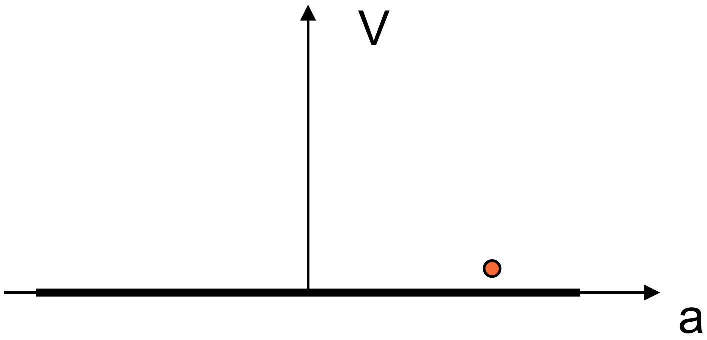
When the axion mass turns on, at QCD time,

$$T_1 \simeq 1 \text{ GeV}$$

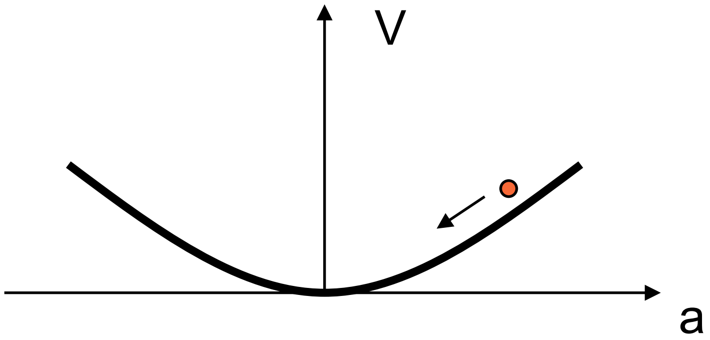
$$t_1 \sim 2 \times 10^{-7} \text{ sec}$$

$$p_a(t_1) \sim \frac{1}{t_1} \sim 3 \times 10^{-9} \text{ eV}$$

Cold axion production by vacuum realignment



$T \geq 1\,\mathrm{GeV}$



$T \leq 1\,\mathrm{GeV}$

$$n_a(t_1) \simeq \frac{1}{2} m_a(t_1) a(t_1)^2 \simeq \frac{1}{2t_1} f_a^2 \alpha(t_1)^2$$

$$\rho_a(t_0) \simeq m_a n_a(t_1) \left(\frac{R_1}{R_0}\right)^3 \propto m_a^{-\frac{7}{6}}$$

initial misalignment angle

J. Preskill, M. Wise & F. Wilczek, L. Abbott & PS, M. Dine & W. Fischler, 1983

Axions today

$$n_a(t_0) = \frac{\rho_{\mathrm{DM}}(t_0)}{m_a} \sim 1.3 \cdot 10^8 \frac{1}{\mathrm{cm}^3}$$

$$\Delta p_a = m_a \Delta v_a \sim \frac{1}{t_1} \frac{10^{-4}\ \mathrm{eV}}{\mathrm{GeV}}$$

$$\Delta v_a \sim 3 \cdot 10^{-17} \sim 10^{-6}\ \frac{\mathrm{cm}}{\mathrm{sec}} \sim \frac{30\ \mathrm{cm}}{\mathrm{year}}$$

$$\Delta x_a \sim \frac{1}{\Delta p_a} \sim 0.7 \cdot 10^{17}\ \mathrm{cm} \simeq 0.02\ \mathrm{pc}$$

$$\mathcal{N}_a = \frac{(2\pi)^3 n_a}{(\Delta p_a)^3} \sim 10^{61}$$ (!)

Axion dark matter is an extremely degenerate Bose gas.

Does it behave the same way as WIMP dark matter in astrophysical contexts?

WIMPs today

$$\rho_{\rm DM} = \Omega_{\rm DM}\ \rho_{\rm crit}$$

$$\Omega_{\rm DM} \simeq 0.23 \qquad\qquad \rho_{\rm crit} \simeq 10^{-29}\ {\rm gr/cc}$$

$$n_W \simeq 0.13\ \frac{1}{\rm m^3}\left(\frac{10\ {\rm GeV}}{m_W}\right)$$

$$\Delta p_W \equiv m_W \Delta v_W \sim \sqrt{2 m_W T_{W{\rm kin}}}\ \frac{T_0}{T_{W{\rm kin}}}$$

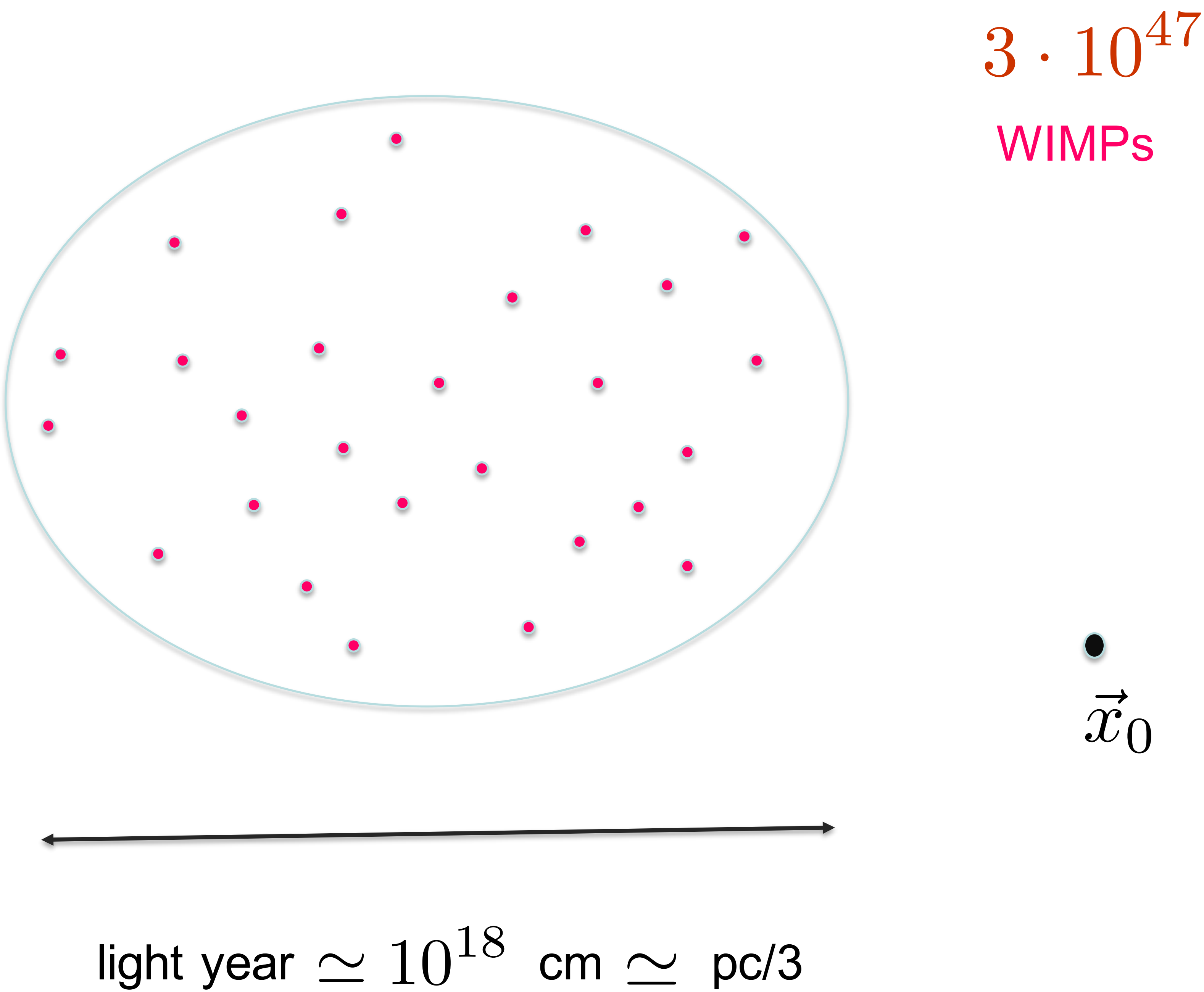
temperature at which WIMPs kinetically decouple

$$T_{W{\rm kin}} \sim {\rm MeV}$$

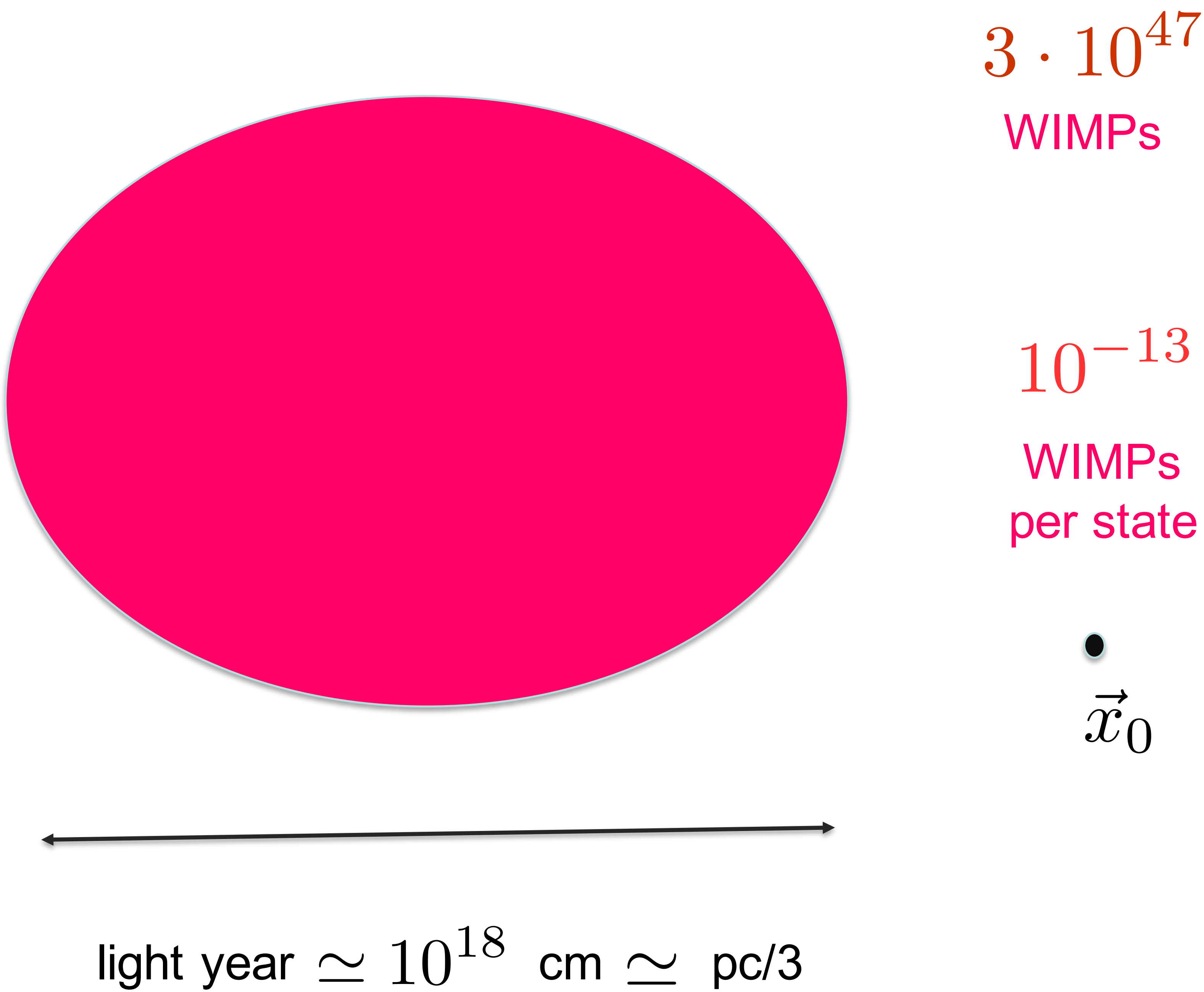
$$\Delta v_W \sim 1.3 \cdot 10^{-12} \sqrt{\frac{10\ \mathrm{GeV}}{m_W}} \qquad (c = 1)$$

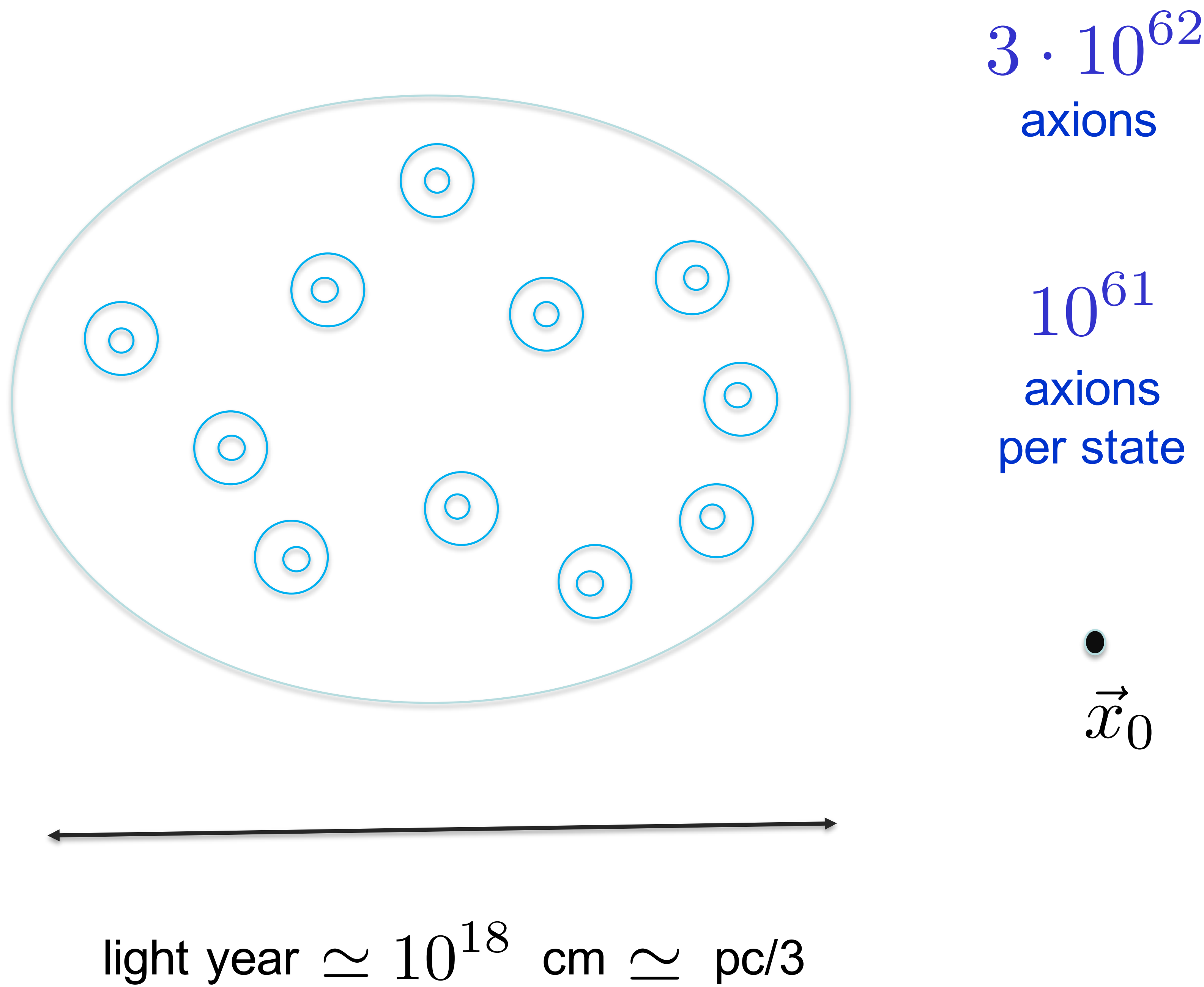
$$\Delta x_W \sim \frac{1}{\Delta p_W} \sim 15\mu\mathrm{m}\ \sqrt{\frac{10\ \mathrm{GeV}}{m_W}} \qquad (\hbar = 1)$$

$$\mathcal{N}_W = \frac{(2\pi)^3 n_W}{(\Delta p_W)^3} \sim 10^{-13} \left(\frac{10\ \mathrm{GeV}}{m_W}\right)^{\frac{5}{2}}$$



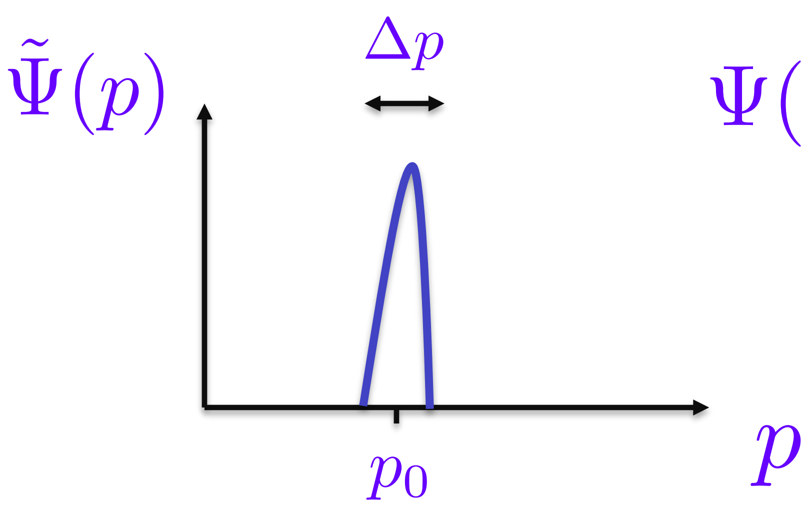
$3 \cdot 10^{47}$
WIMPs
$\vec{x}_0$
light year $\simeq 10^{18}$ cm $\simeq$ pc/3


$3 \cdot 10^{47}$
WIMPs
10^{-13}
WIMPs
per state
$\vec{x}_0$
light year $\simeq 10^{18}$ cm $\simeq$ pc/3

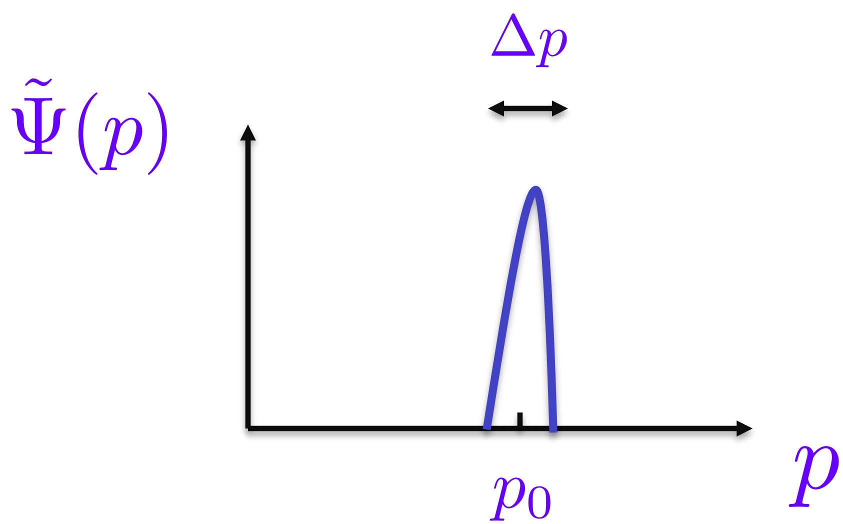

$3 \cdot 10^{62}$
axions
10^{61}
axions
per state
$\vec{x}_0$
light year $\simeq 10^{18}$ cm $\simeq$ pc/3

In the axion case, when the object size becomes of order $\frac{1}{\Delta p_a}$, only a limited number of configurations are possible

$$\ell \equiv \frac{1}{\Delta p_a}$$ = correlation length



$$\Psi(\vec{x}) = \int d^3p \; \tilde{\Psi}(\vec{p}) \; e^{i\vec{p}\cdot\vec{x}}$$



$$\Psi(\vec{x}) = \int d^3p \; \tilde{\Psi}(\vec{p}) \; e^{i\vec{p}\cdot\vec{x}}$$

$$\rho(\vec{x}) = m_a \; |\Psi(\vec{x})|^2$$

$$= \int d^3p \int d^3p' \; \Psi(\vec{p})\Psi(\vec{p}')^* \; e^{i(\vec{p}-\vec{p}')\cdot\vec{x}}$$

cannot change much over a distance $\quad \ell = \dfrac{1}{\Delta p}$

In the axion case, the gravitational fields are necessarily large

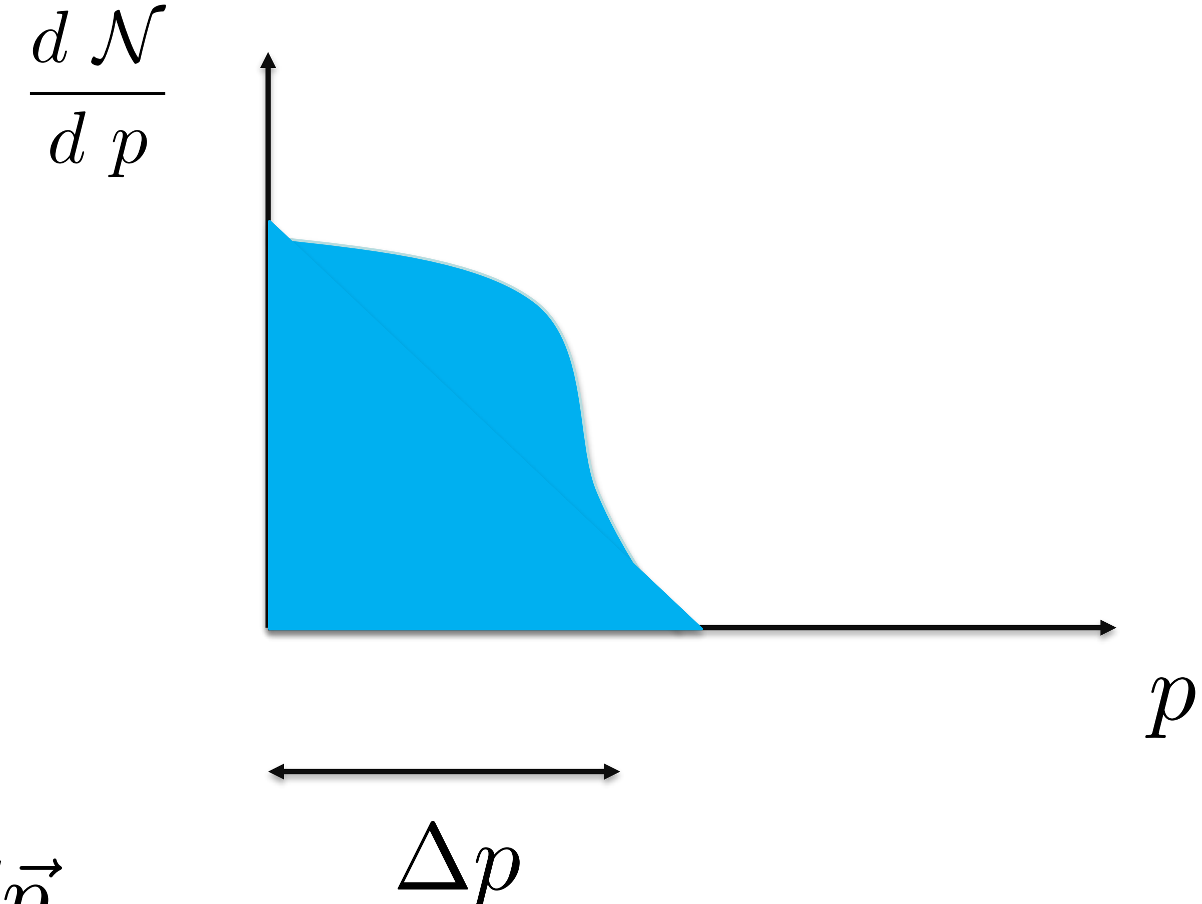
$$\delta g \sim 4\pi G\rho\,\ell$$

regardless of their average value.

For example in a homogeneous universe

$\vec{g} = 0$ in the WIMP case

$\bar{\vec{g}} = 0$ in the axion case, but the typical gravitational field is δg



$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\tau \sim \frac{\Delta p}{m\delta g} \sim 24 \text{ sec}$$

Thermalization occurs due to gravitational interactions

PS + Q. Yang, PRL 103 (2009) 111301

$$\Gamma_g \equiv \frac{1}{\tau} \sim 4\pi G\rho\ell \, m \, \frac{1}{\Delta p} = 4\pi G\rho m\ell^2$$

compare with $$H(t) = \frac{1}{2t}$$

During the QCD phase transition

$$\frac{\Gamma_g(t_1)}{H(t_1)} \sim 4 \cdot 10^{-7} \left(\frac{10^{-5}\ \mathrm{eV}}{m_a} \right)^{\frac{2}{3}}$$

but

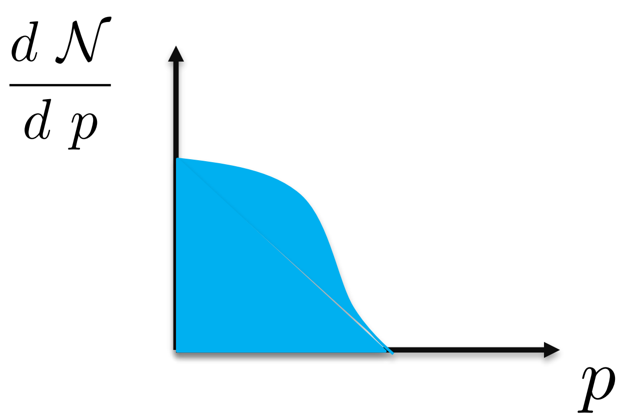
$$\Gamma_g \sim 4\pi G n m^2 \ell^2 \propto n\ \ell^2 \propto a(t)^{-1}$$

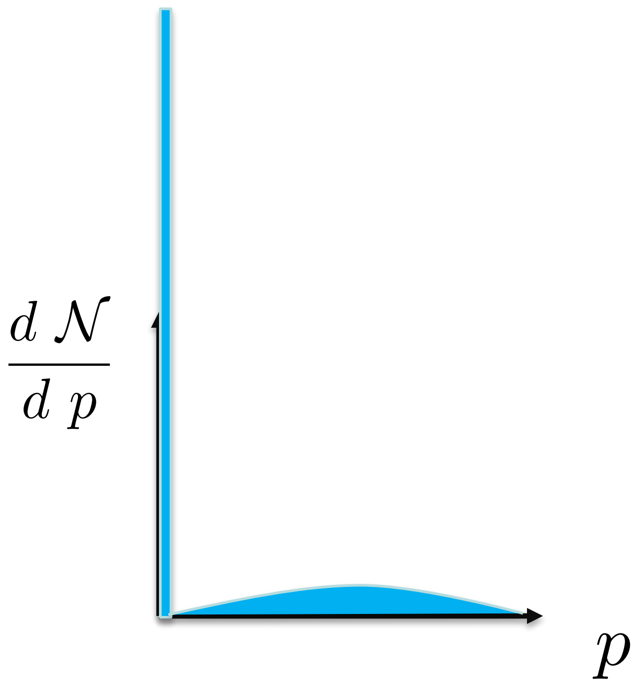
whereas

$$H = \frac{1}{2t} \sim a(t)^{-2}$$

Gravitational interactions thermalize the axions and cause them to form a BEC when the photon temperature

$$T_\gamma \sim 400 \text{ eV} \left(\frac{10^{-5} \text{ eV}}{m_a} \right)^{\frac{1}{2}}$$





Bose-Einstein Condensation

if identical bosonic particles
are highly condensed in phase space
and their total number is conserved
and they thermalize

then most of them go to the lowest energy
available state

On time scales larger than

$$\tau \sim \frac{1}{4\pi G\rho\, m\, \ell^2}$$

the axions thermalize, i.e. they move from one state to another.

Vorticity can be generated and is expected to be generated because the lowest energy state for given total angular momentum is a state of rigid rotation in the angular variables.

$$|l_z = 3\hbar\rangle + |l_z = 5\hbar\rangle \quad \rightarrow \quad |l_z = 2\hbar\rangle + |l_z = 6\hbar\rangle$$

Generation of vorticity is impossible in the case of WIMP dark matter and in the case of dark matter in the form of a classical scalar field.

(A. Natarajan + PS, 2006)

For collisionless dark matter

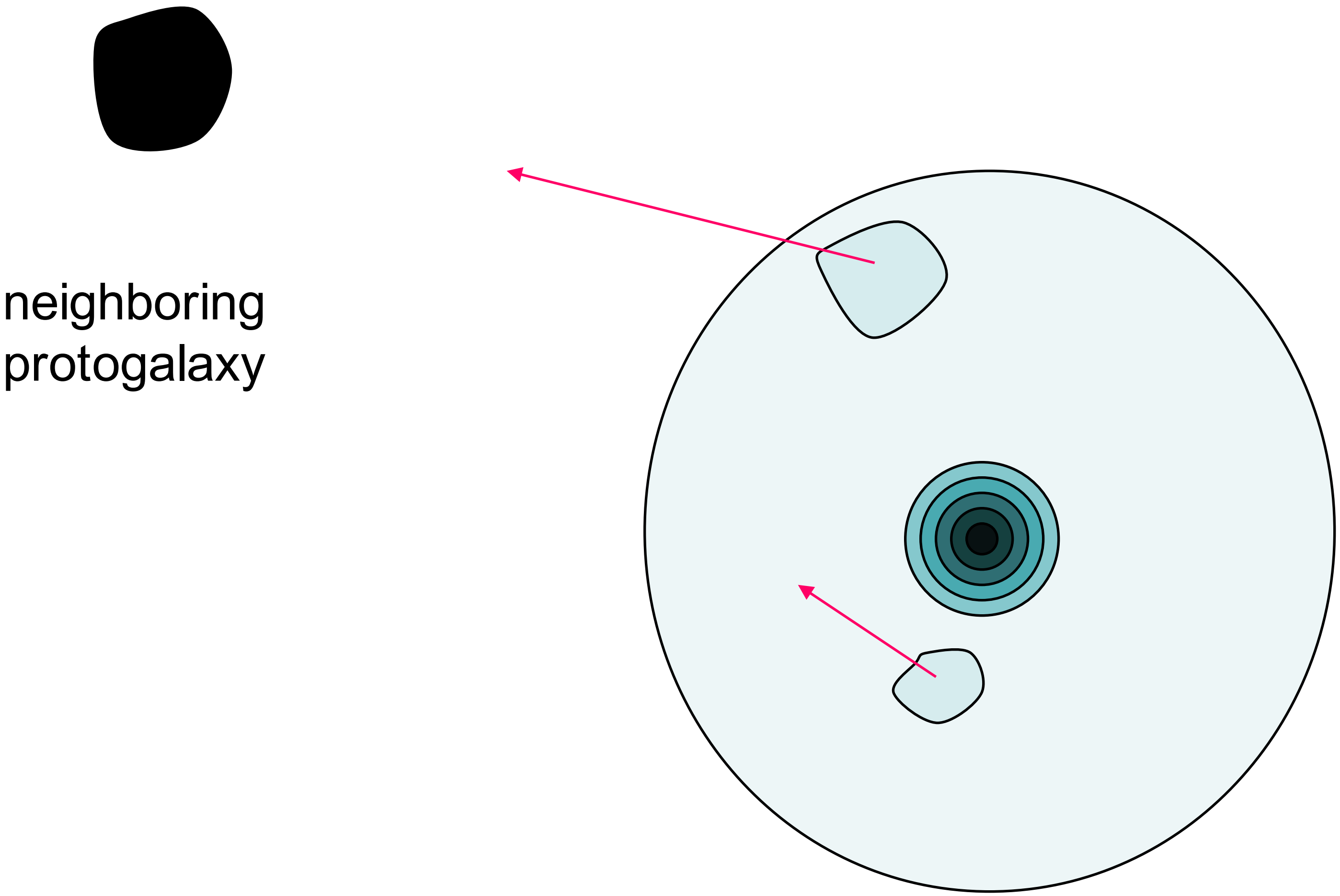
$$\partial_t \vec{v} + \vec{v} \cdot \vec{\nabla} \vec{v} = -\vec{\nabla} \Phi(\vec{x}, t)$$

Newtonian gravitational potential

If $\vec{\nabla} \times \vec{v} = 0$ initially

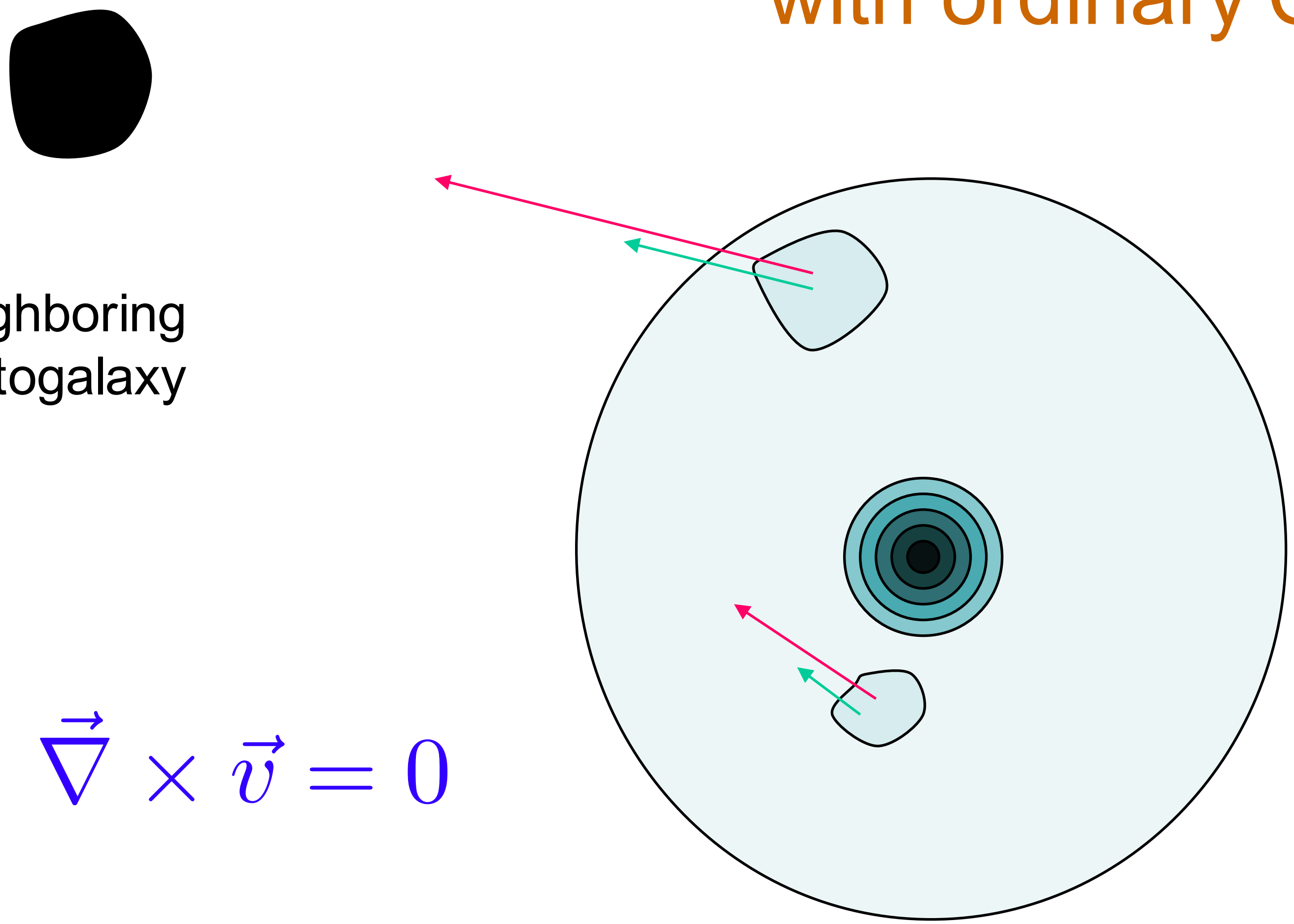
then $\vec{\nabla} \times \vec{v} = 0$ for ever after

Tidal torque theory



Stromberg 1934; Hoyle 1947; Peebles 1969, 1971

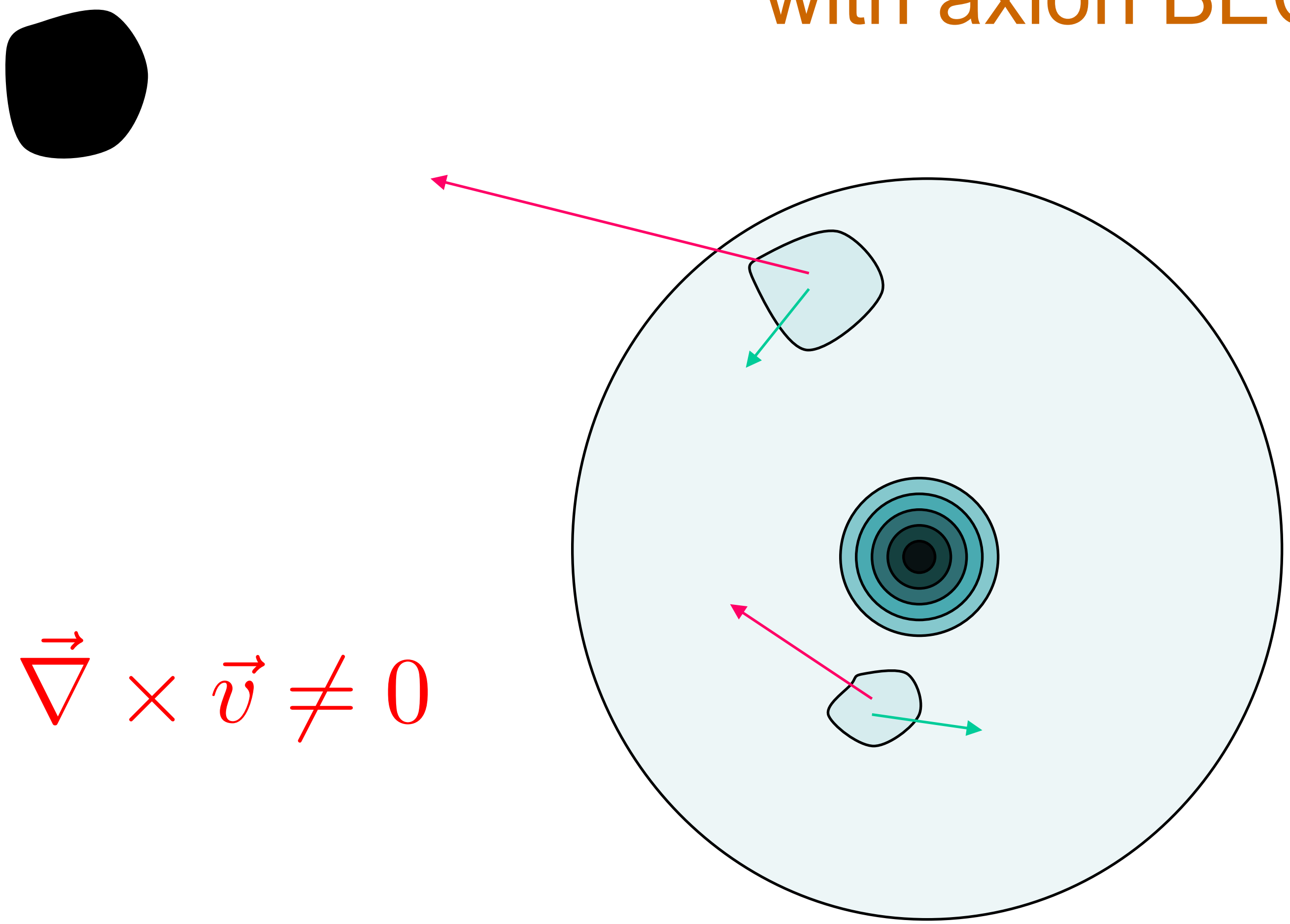
Tidal torque theory with ordinary CDM



$$\vec{\nabla} \times \vec{v} = 0$$

Even though ordinary CDM acquires angular momentum, its velocity field remains irrotational

Tidal torque theory with axion BEC



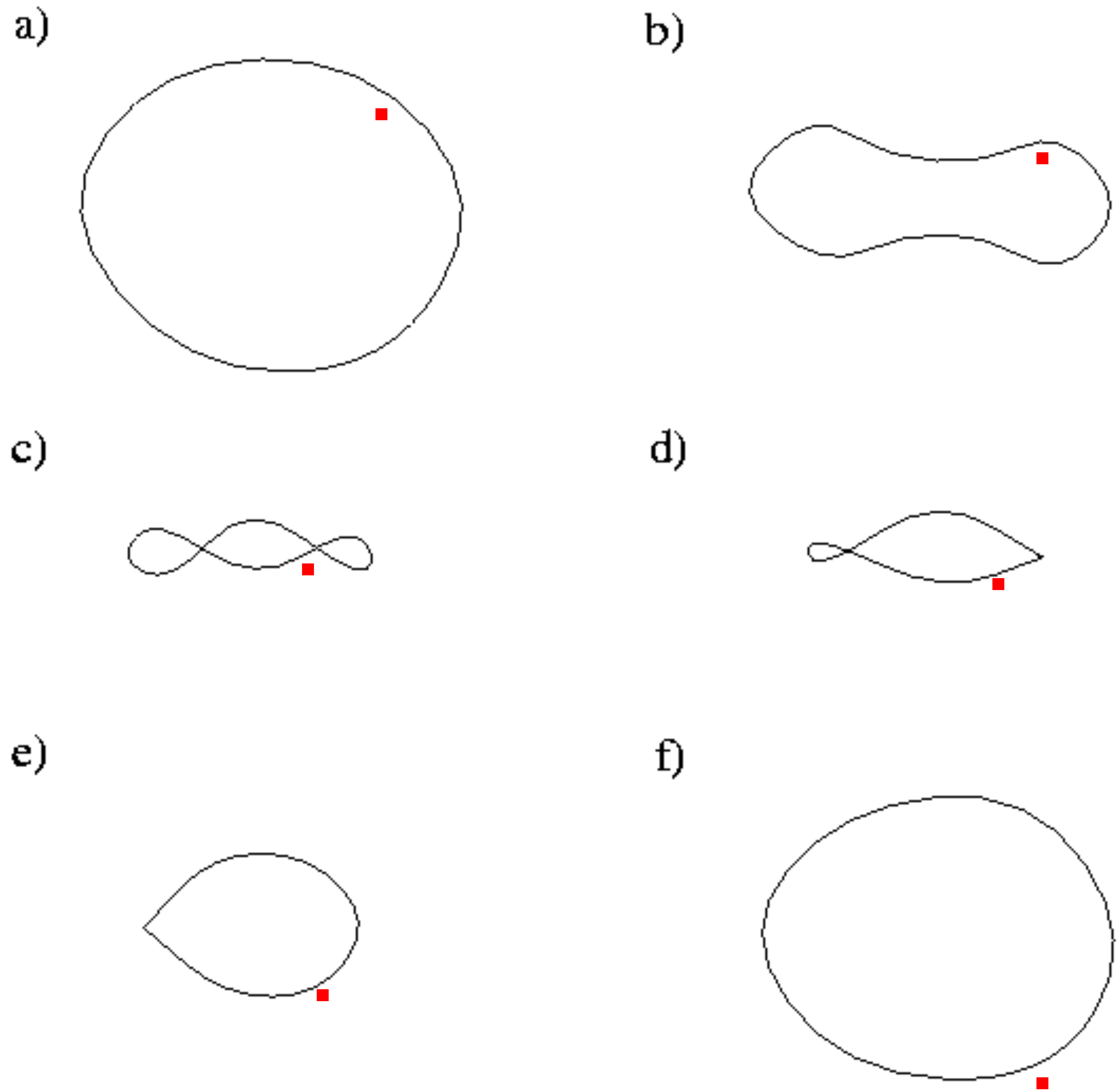
$$\vec{\nabla} \times \vec{v} \neq 0$$

Because most axions move to their lowest energy state, they fall in with net overall rotation.

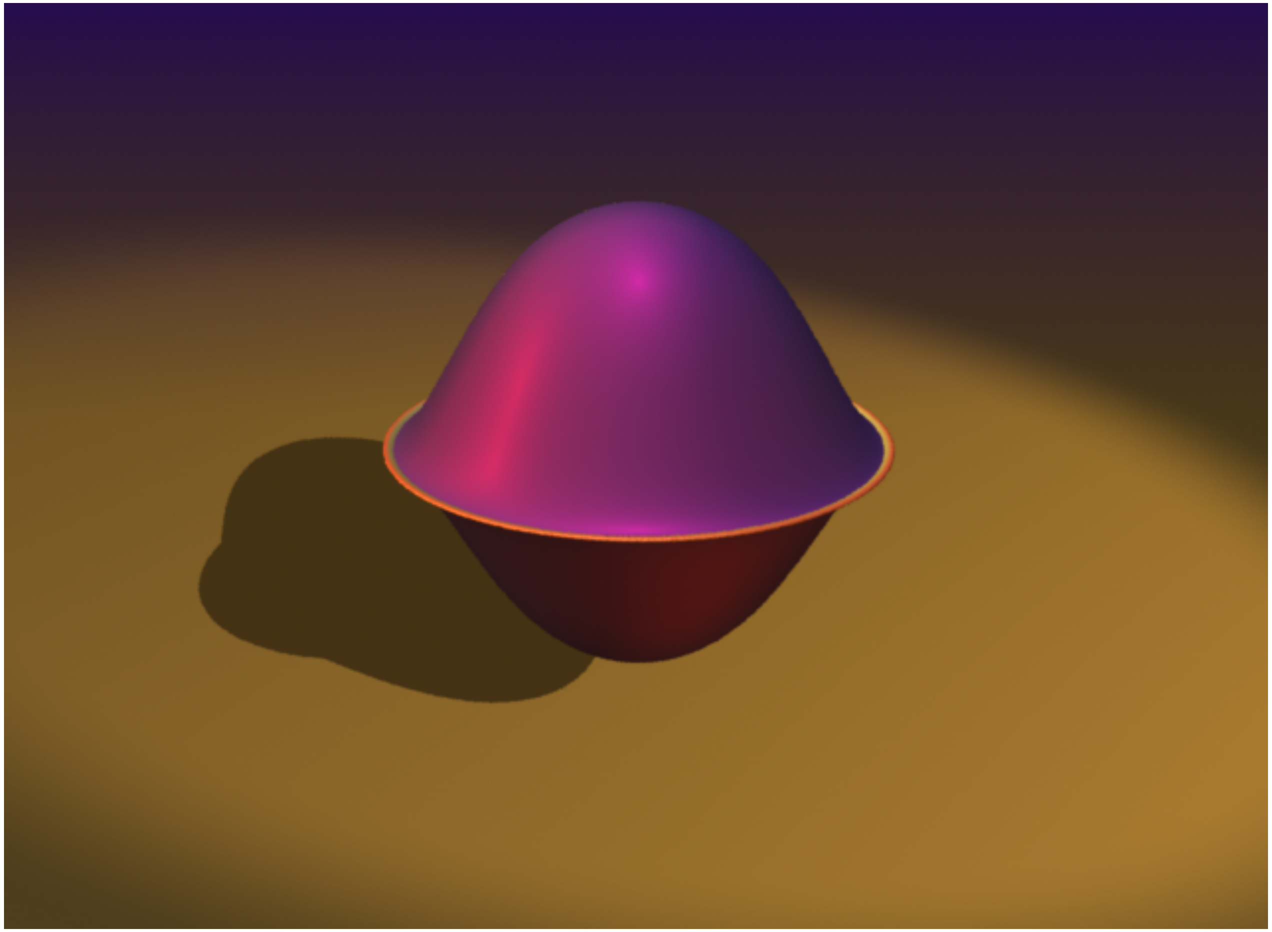
A shell of particles, part of a continuous flow.

The shell has net overall rotation.

As the shell falls in and out of the galaxy, it turns itself inside out.

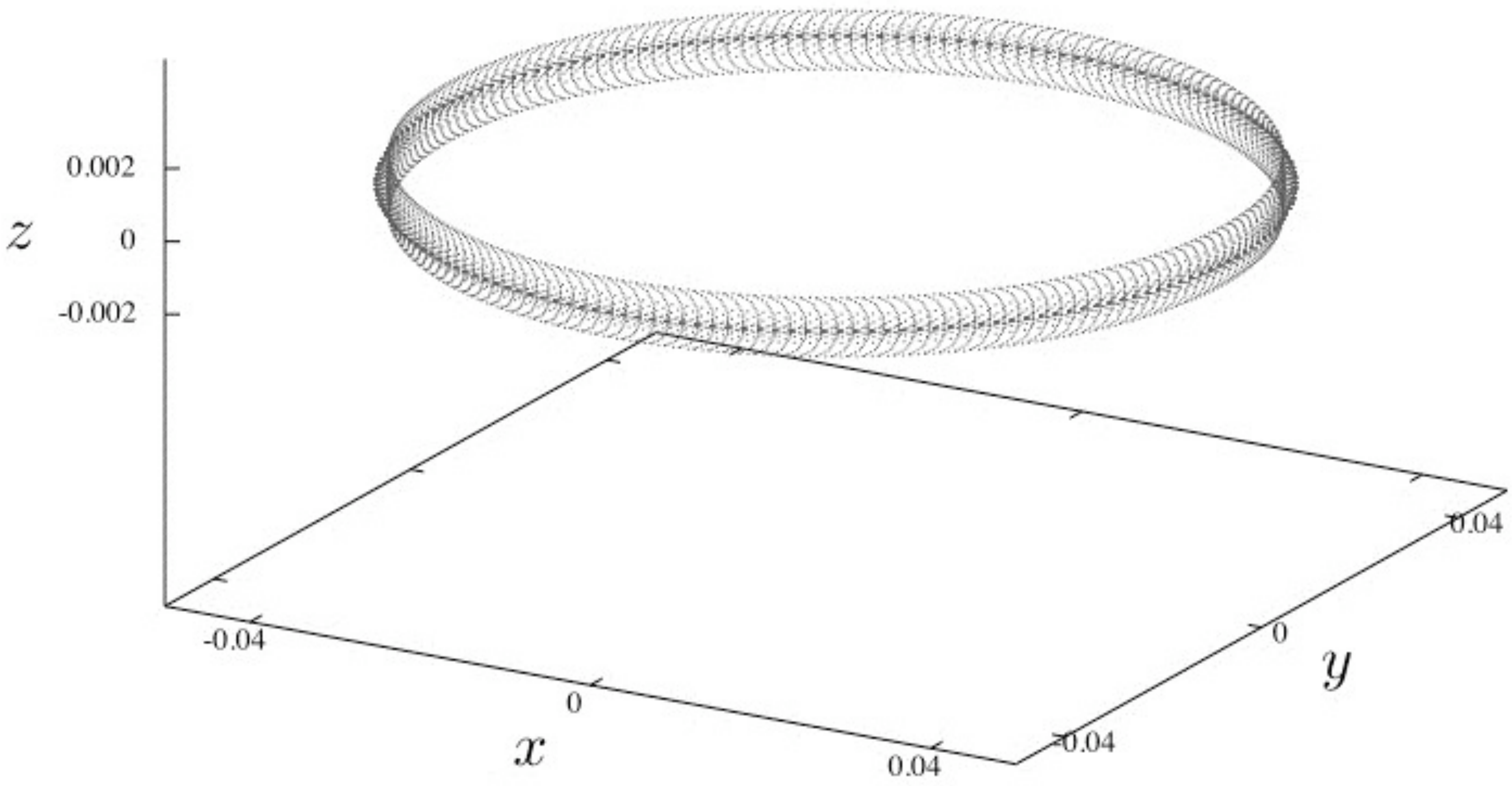


Sphere turning inside out

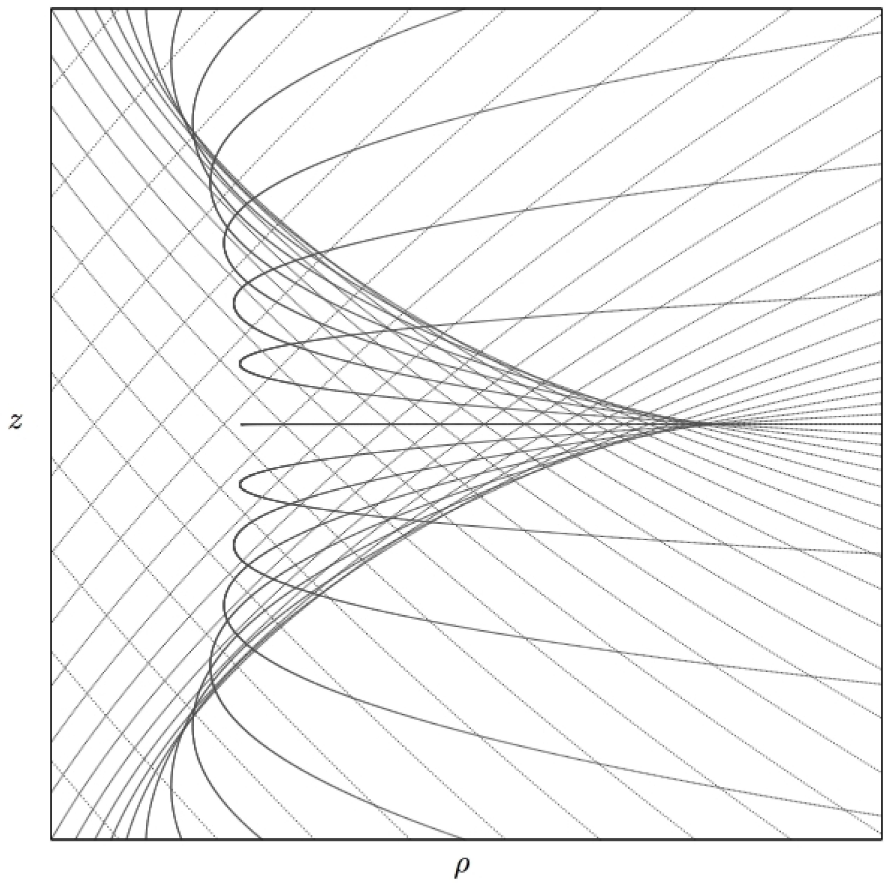


simulations by Arvind Natarajan

in case of net overall rotation

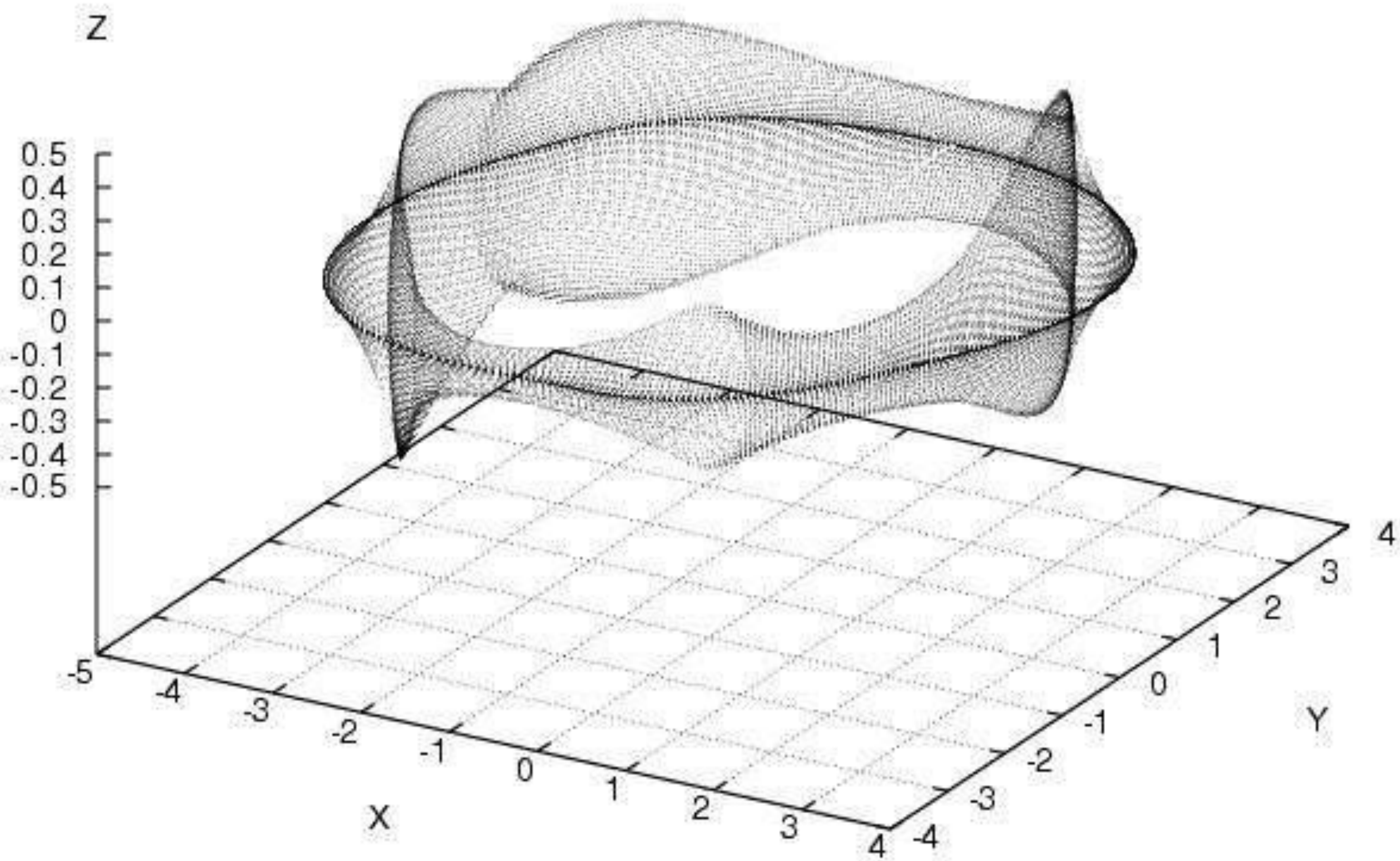


The caustic ring cross-section

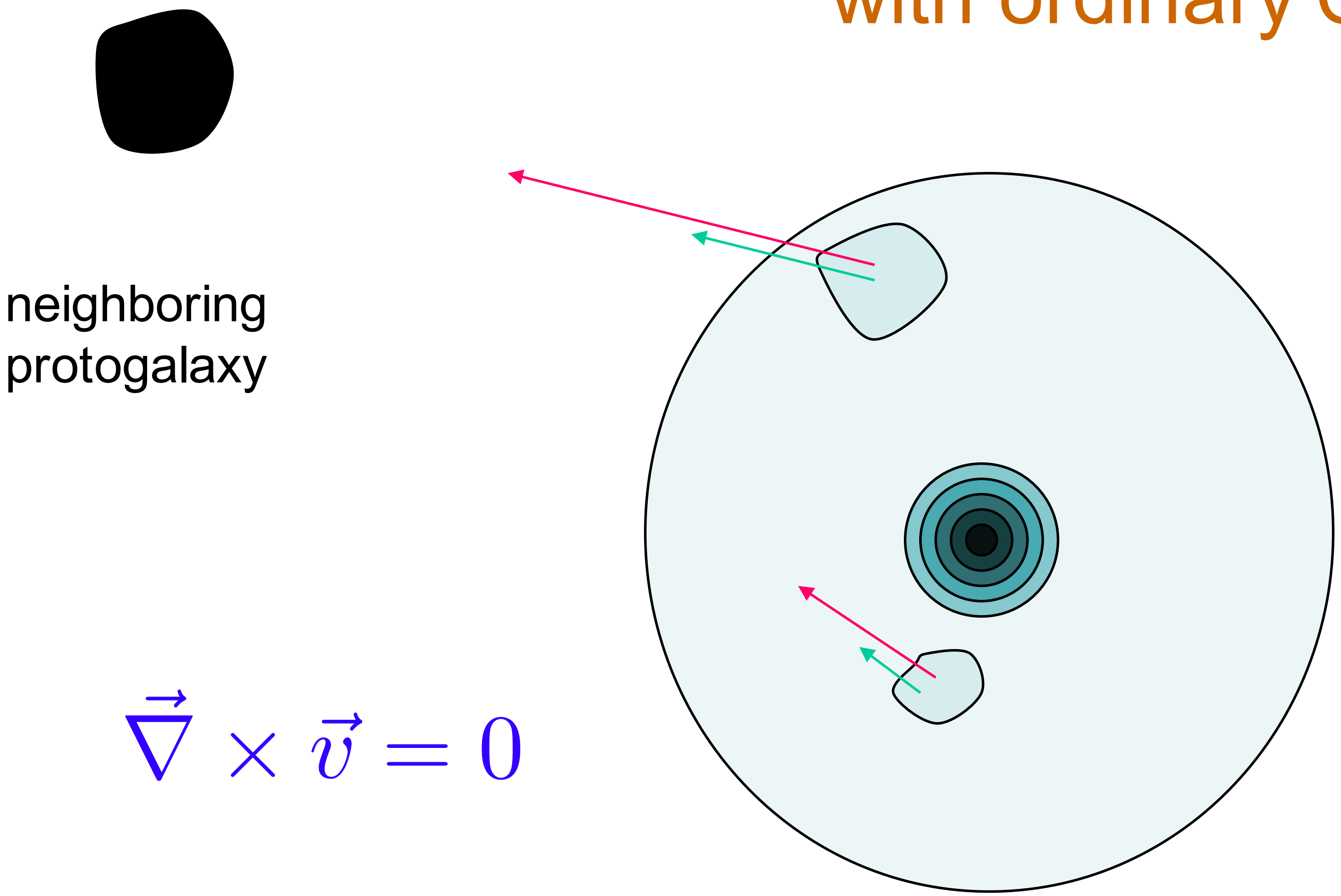


D_{-4}

an elliptic umbilic catastrophe


Z
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-5
-4
-3
-2
-1
0
1
2
3
4
X
-4
-3
-2
-1
0
1
2
3
4
Y

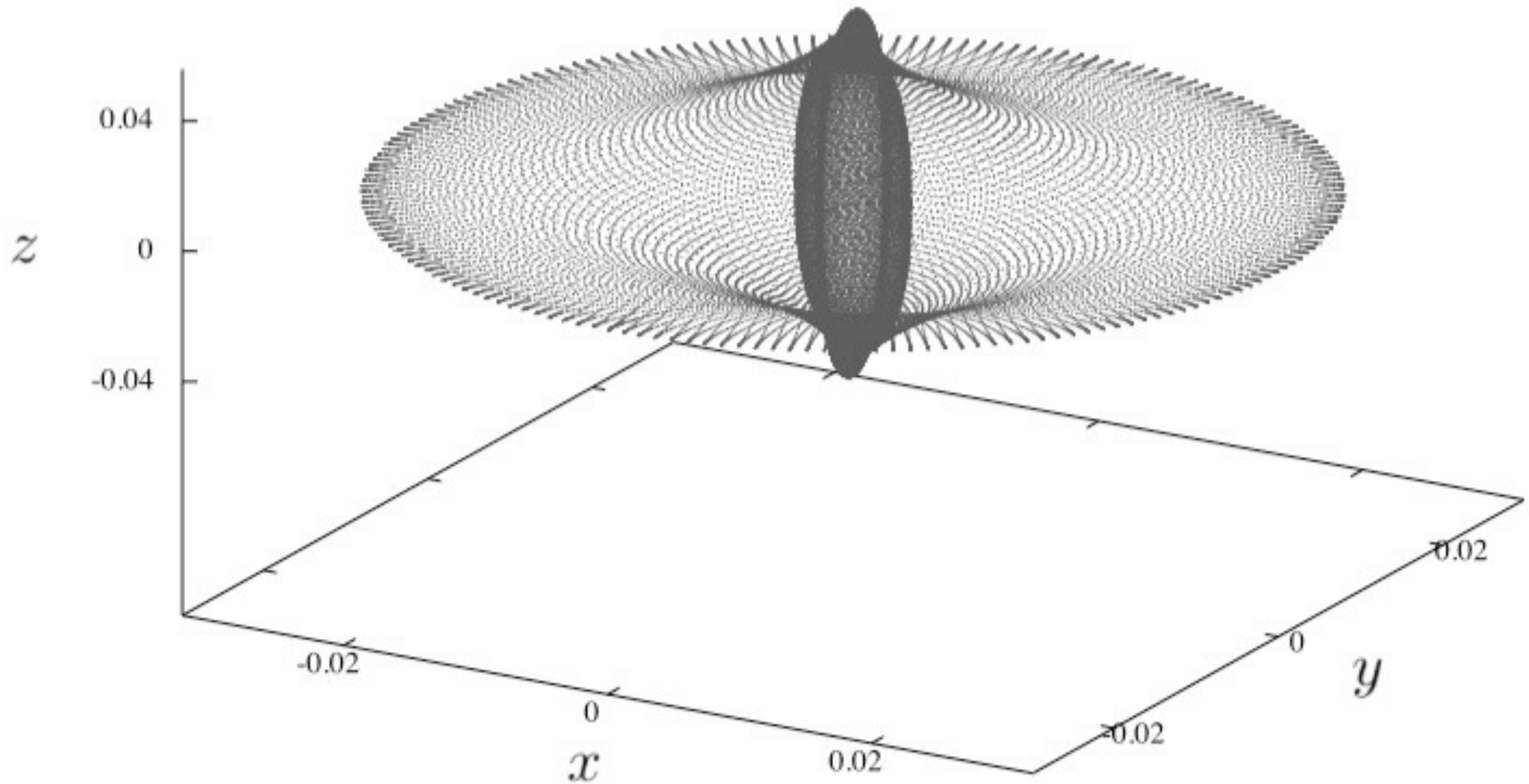
Tidal torque theory with ordinary CDM



$$\vec{\nabla} \times \vec{v} = 0$$

the velocity field remains irrotational

in case of irrotational flow



On the basis of the self-similar infall model (Filmore and Goldreich, Bertschinger) with angular momentum (Tkachev, Wang + PS), the caustic rings were predicted to be

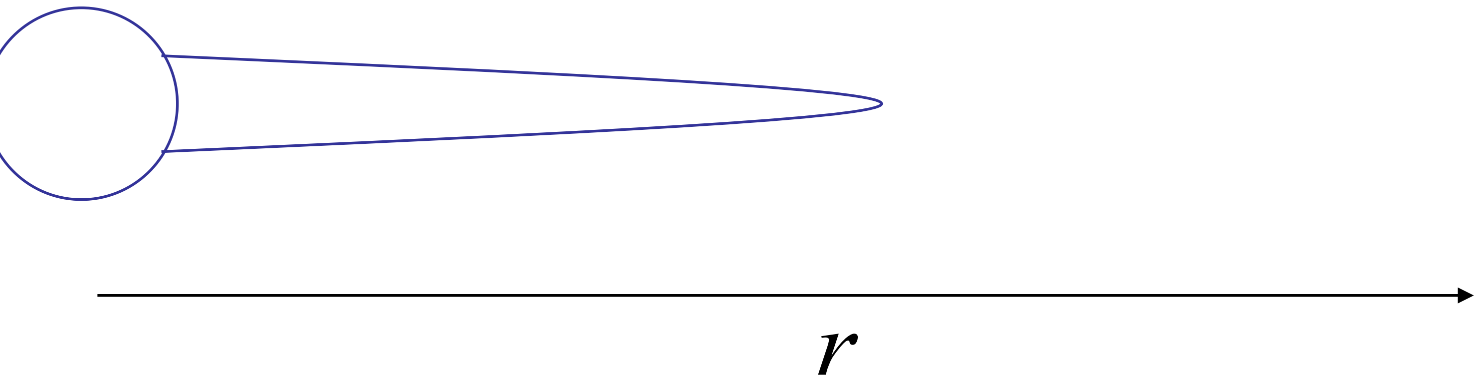
in the galactic plane

with radii $(n = 1, 2, 3 \ldots)$

$$a_n = \frac{40\mathrm{kpc}}{n} \left(\frac{v_{rot}}{220\mathrm{km/s}} \right) \left(\frac{j_{max}}{0.18} \right)$$

$j_{max} \cong 0.18$ was expected for the Milky Way halo from the effect of angular momentum on the inner rotation curve.

Galactic rotation curves

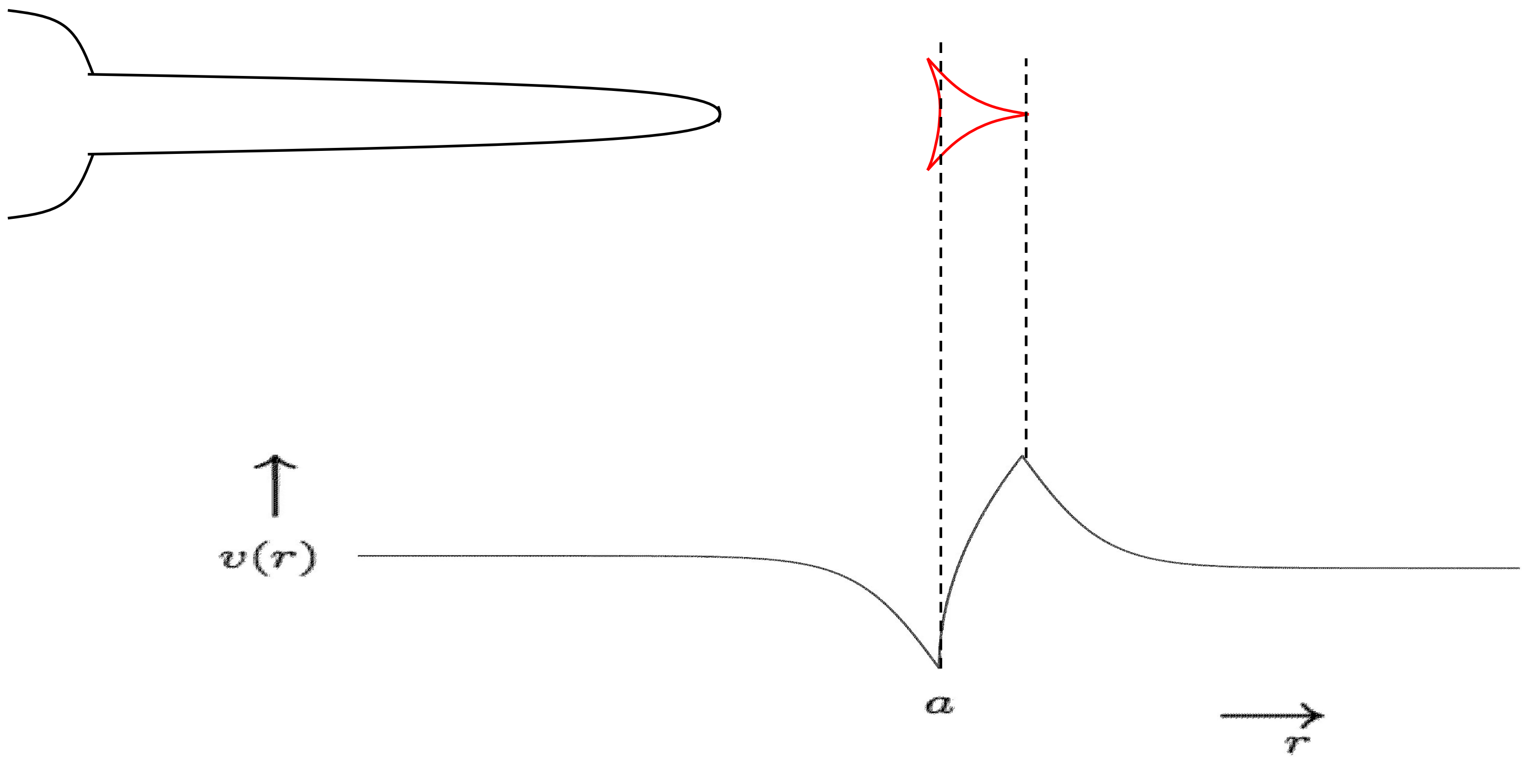


galactic mass

$$v^2(r) = \frac{G\,M(r)}{r}$$

rotation speed

Effect of a caustic ring of dark matter upon the galactic rotation curve



Rotation curve of Andromeda Galaxy

from L. Chemin, C. Carignan & T. Foster, arXiv: 0909.3846

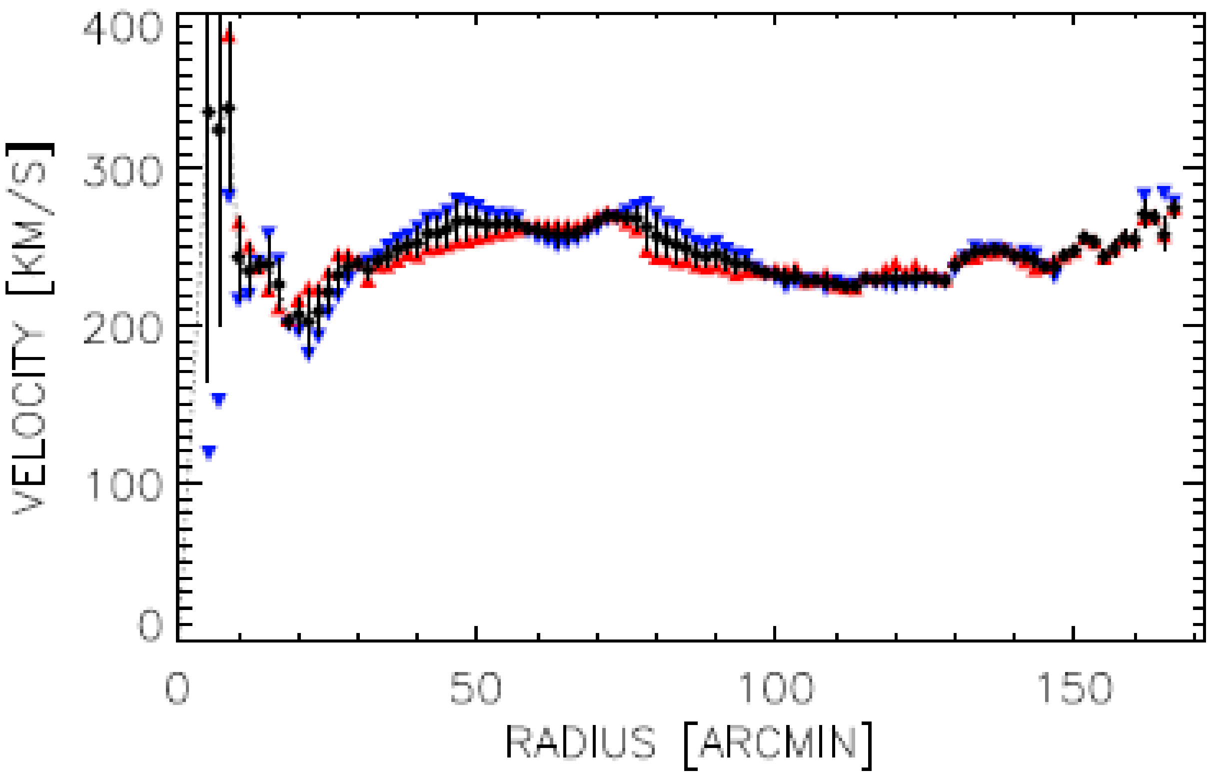
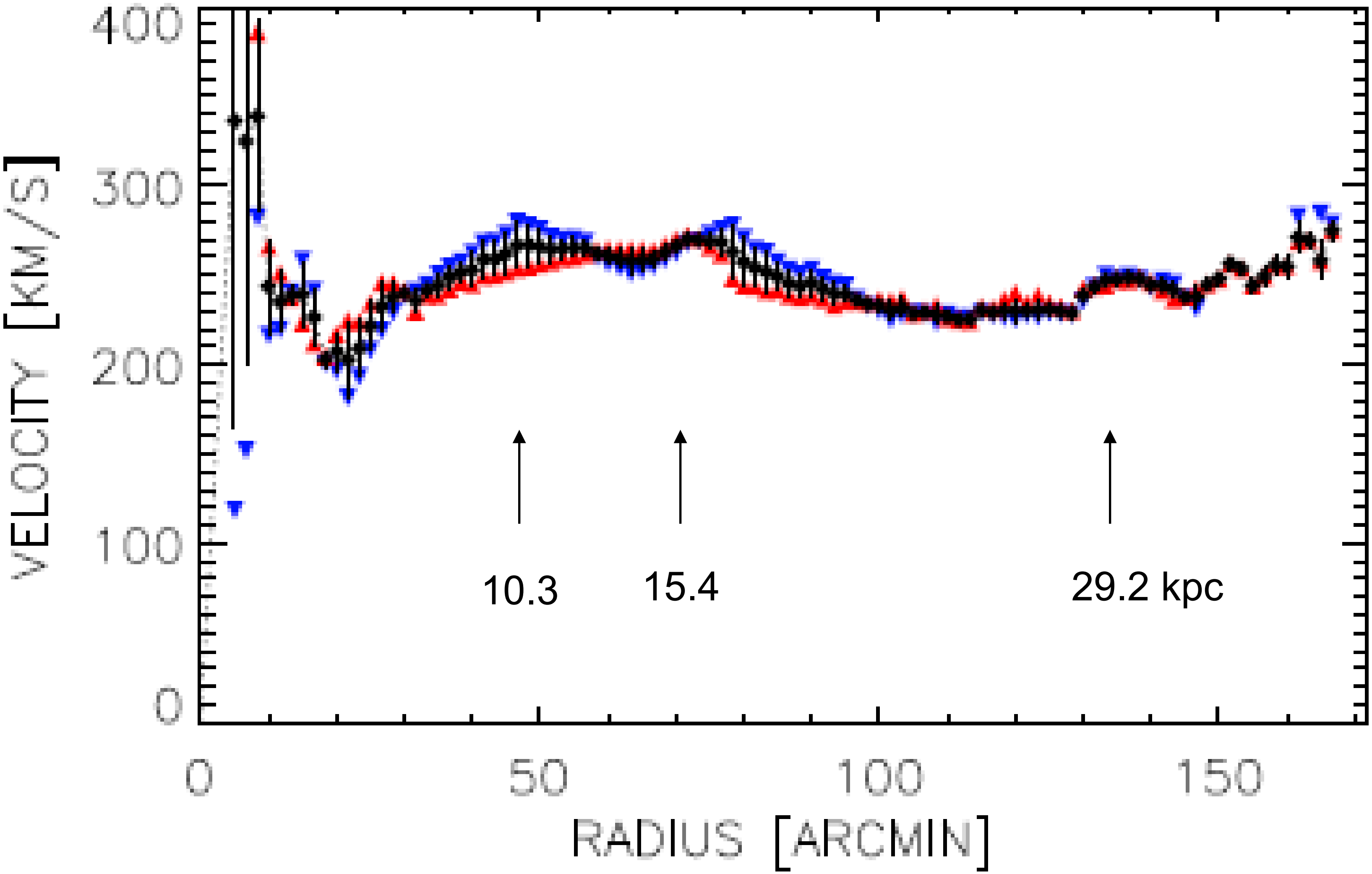


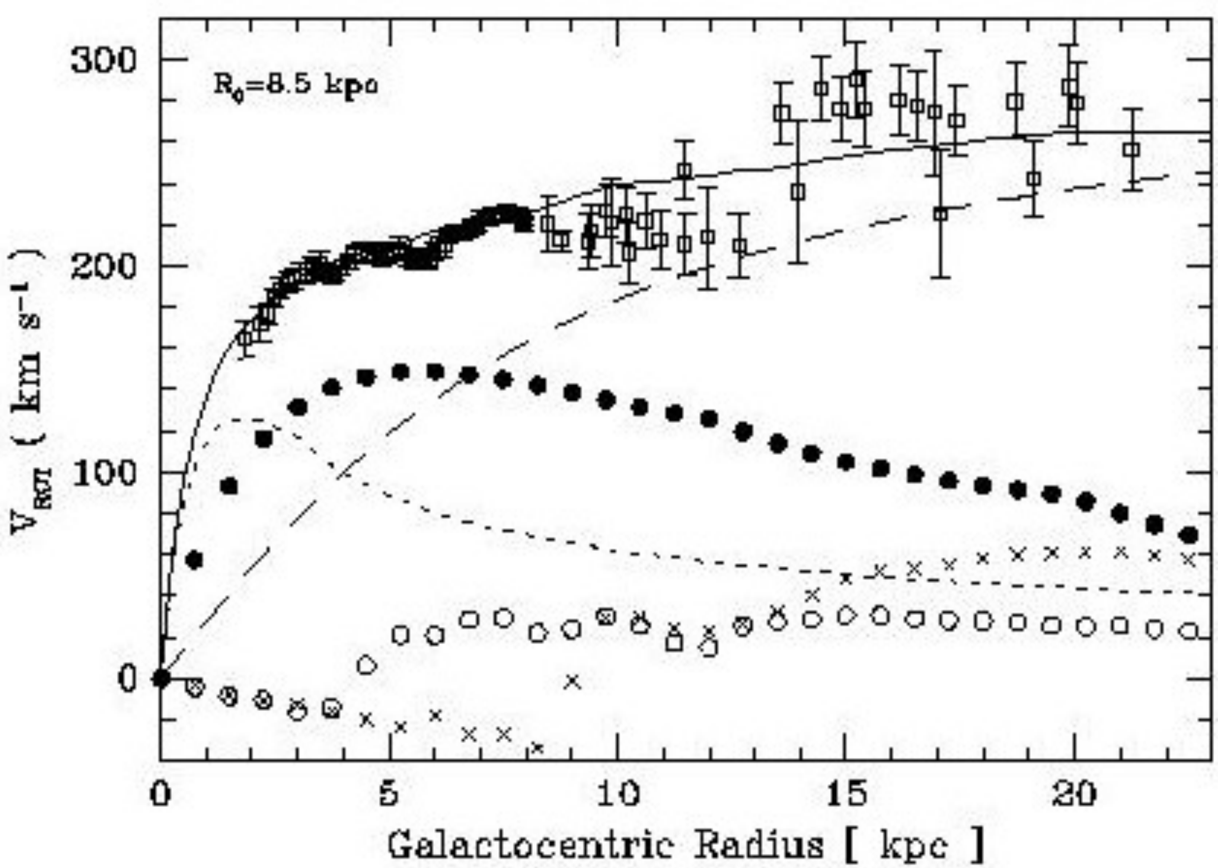
FIG. 10.— HI rotation curve of Messier 31. Filled diamonds are for both halves of the disc fitted simultaneously while blue downward/red upward triangles are for the approaching/receding sides fitted separately (respectively).



10 arcmin = 2.2 kpc

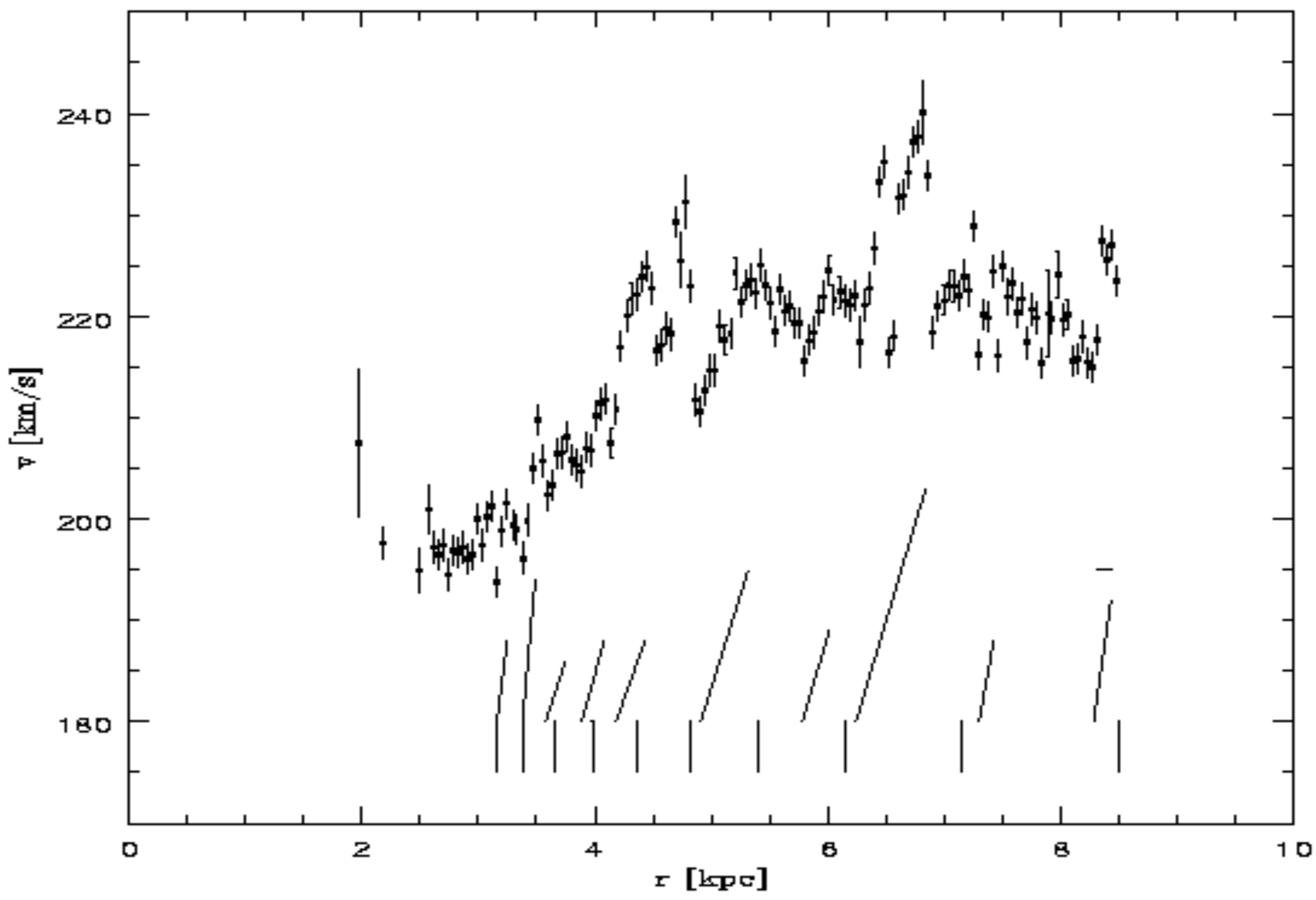
S. Chakrabarty, Y. Han, A. Gonzalez & PS, arXiv:2007.10509

Outer Galactic rotation curve

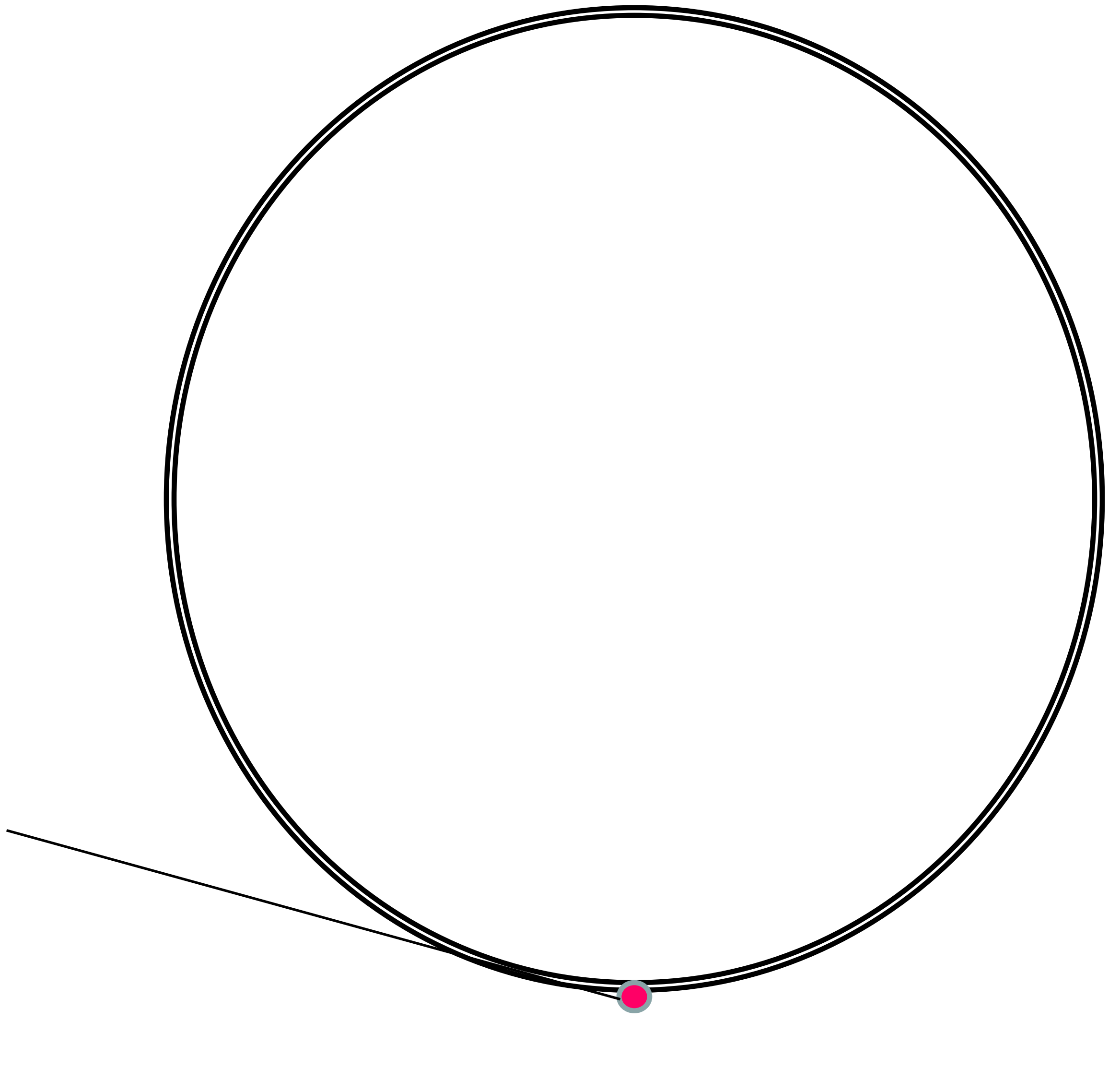


R.P. Olling and M.R. Merrifield, MNRAS 311 (2000) 361

Inner Galactic rotation curve

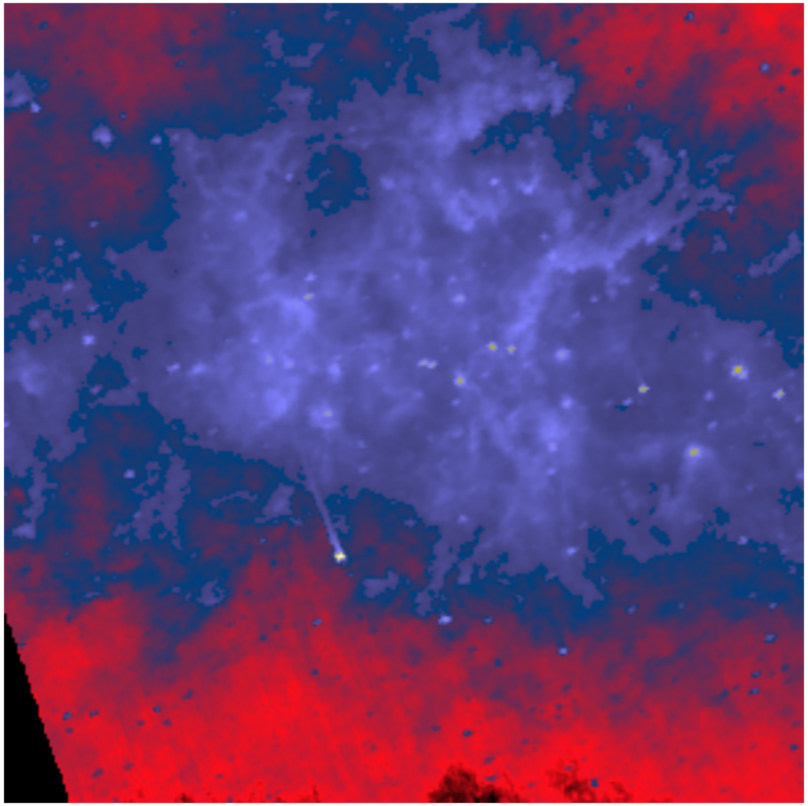


from Massachusetts-Stony Brook North Galactic Pane CO Survey (Clemens, 1985)


Sun

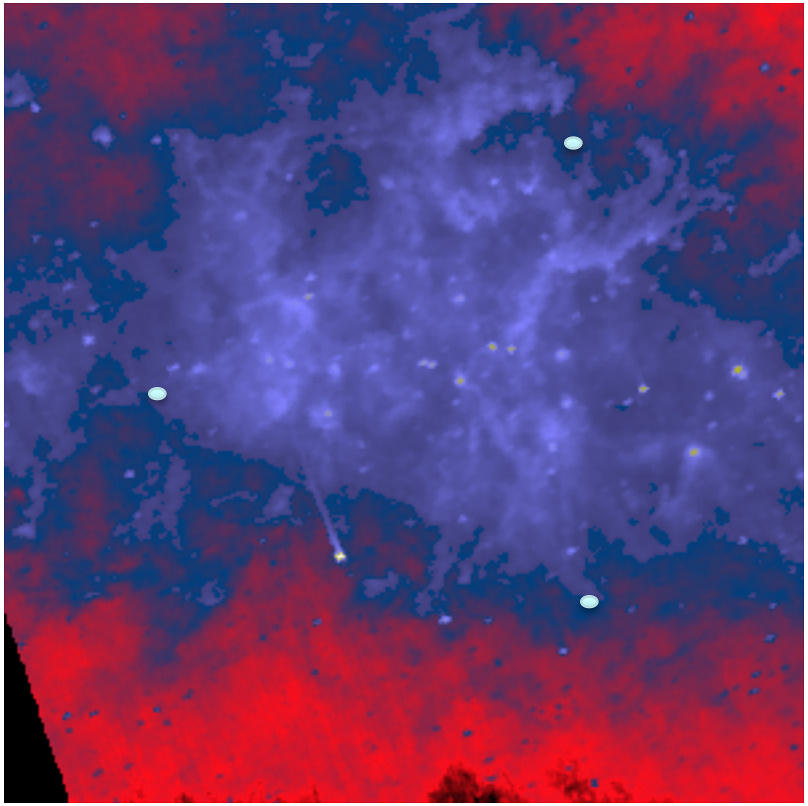
IRAS $\mathbf{12}\ \mu\mathrm{m}$

$(l, b) = (80^{\circ}, 0^{\circ})$ $10^{\circ} \times 10^{\circ}$



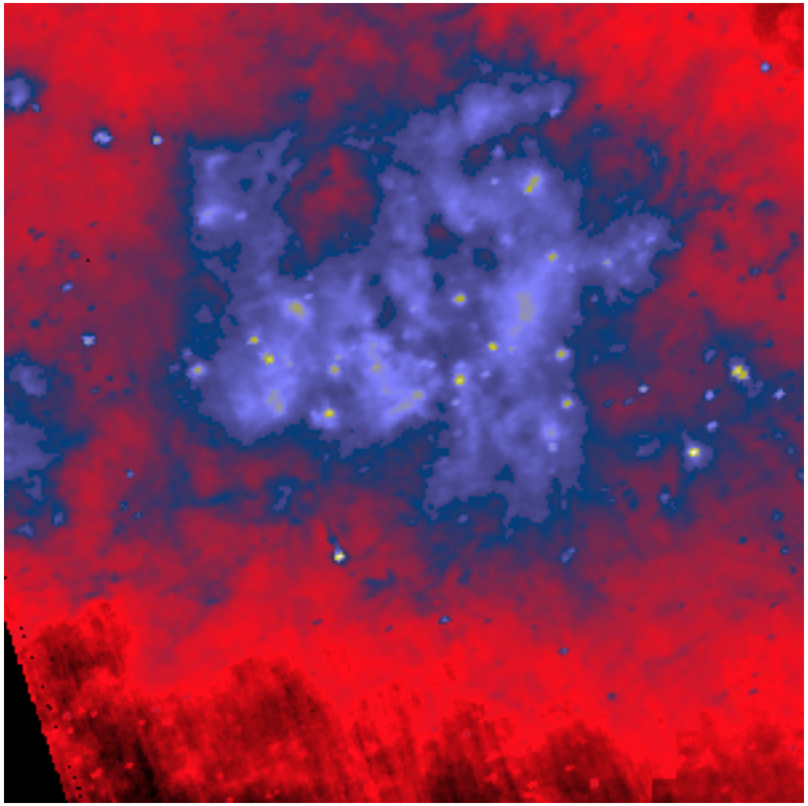
IRAS **12 μm**

$(l, b) = (80^o, 0^o)$ $10^o \times 10^o$

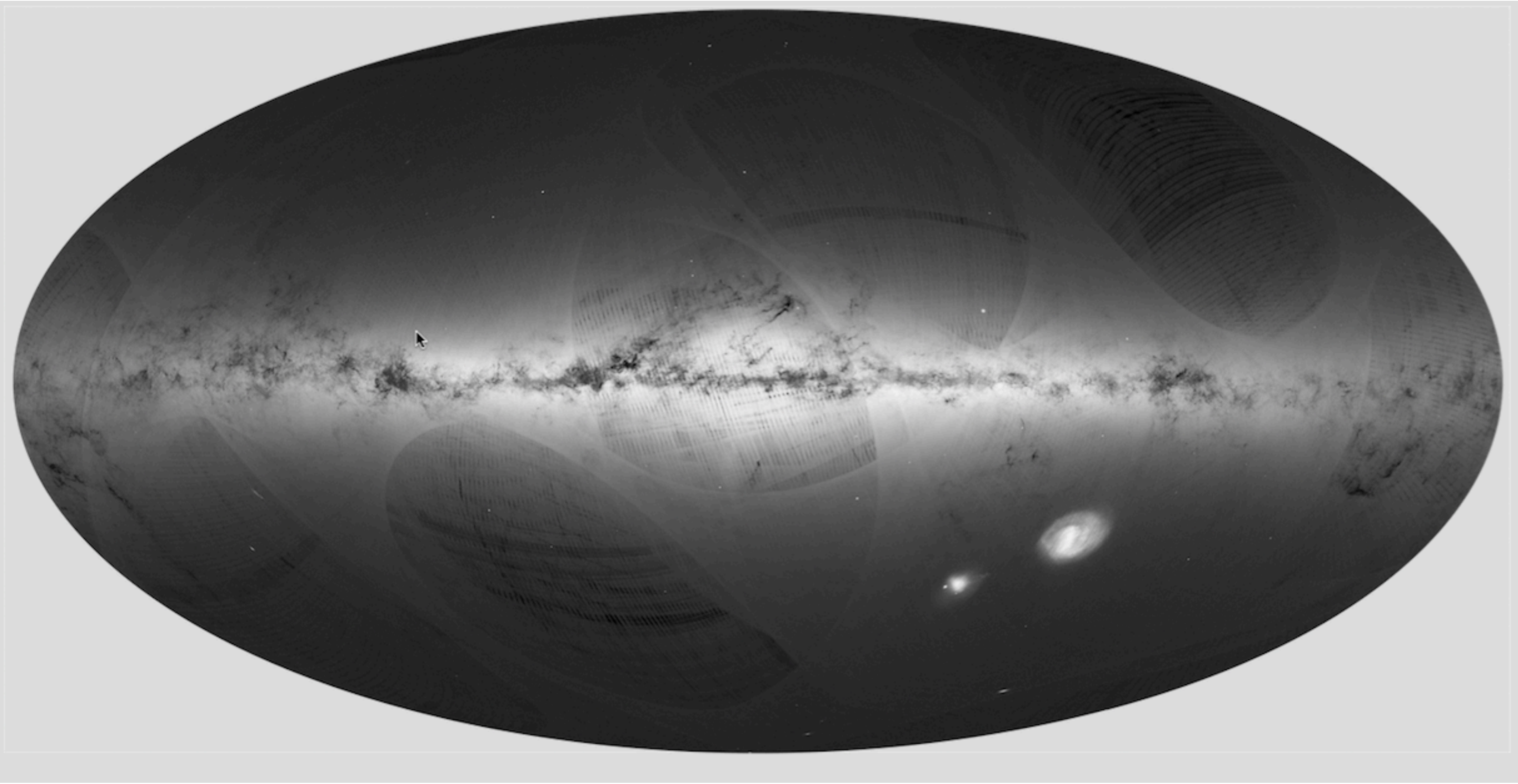


IRAS 25 μm

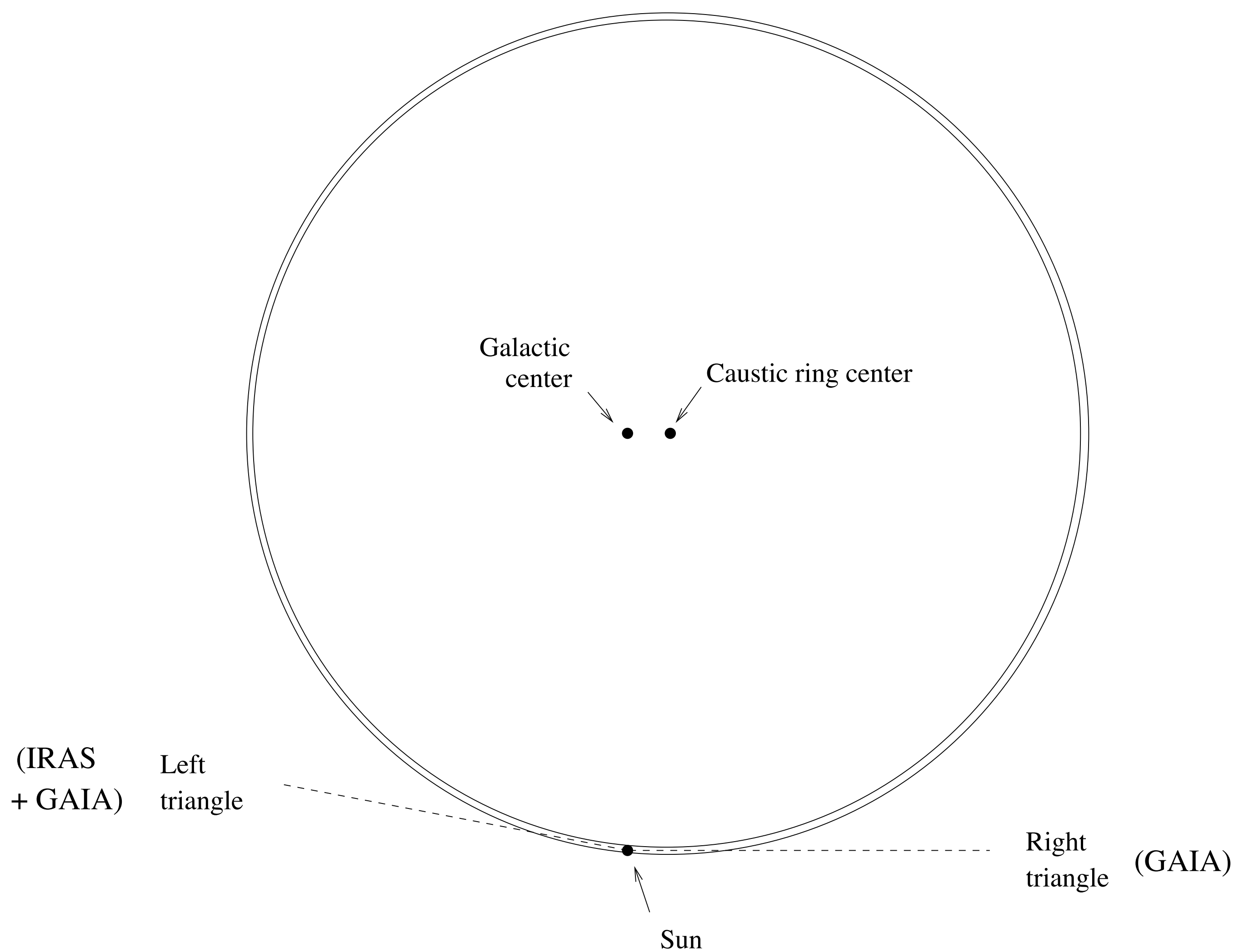
$(l, b) = (80^o, 0^o)$ $10^o \times 10^o$

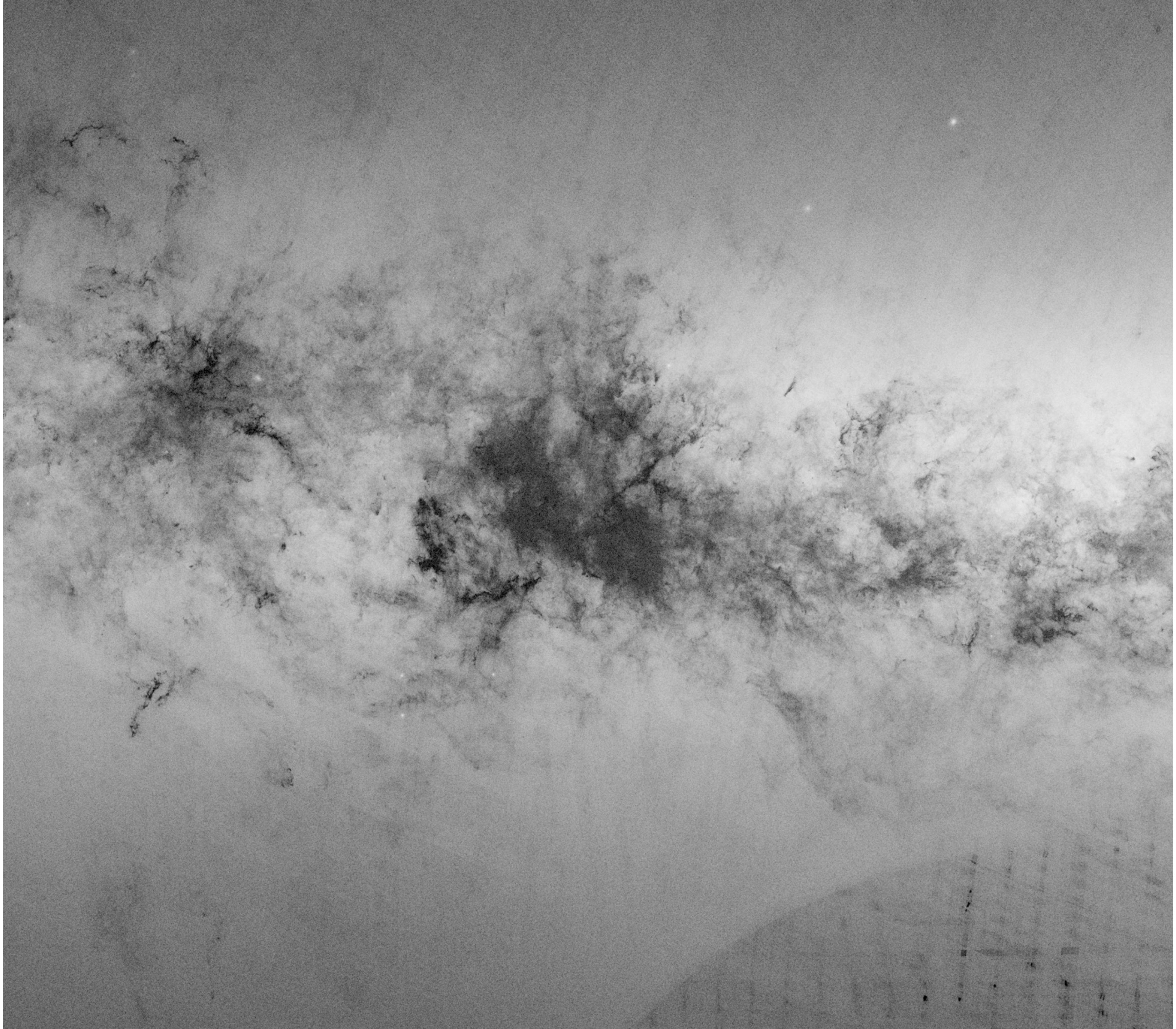


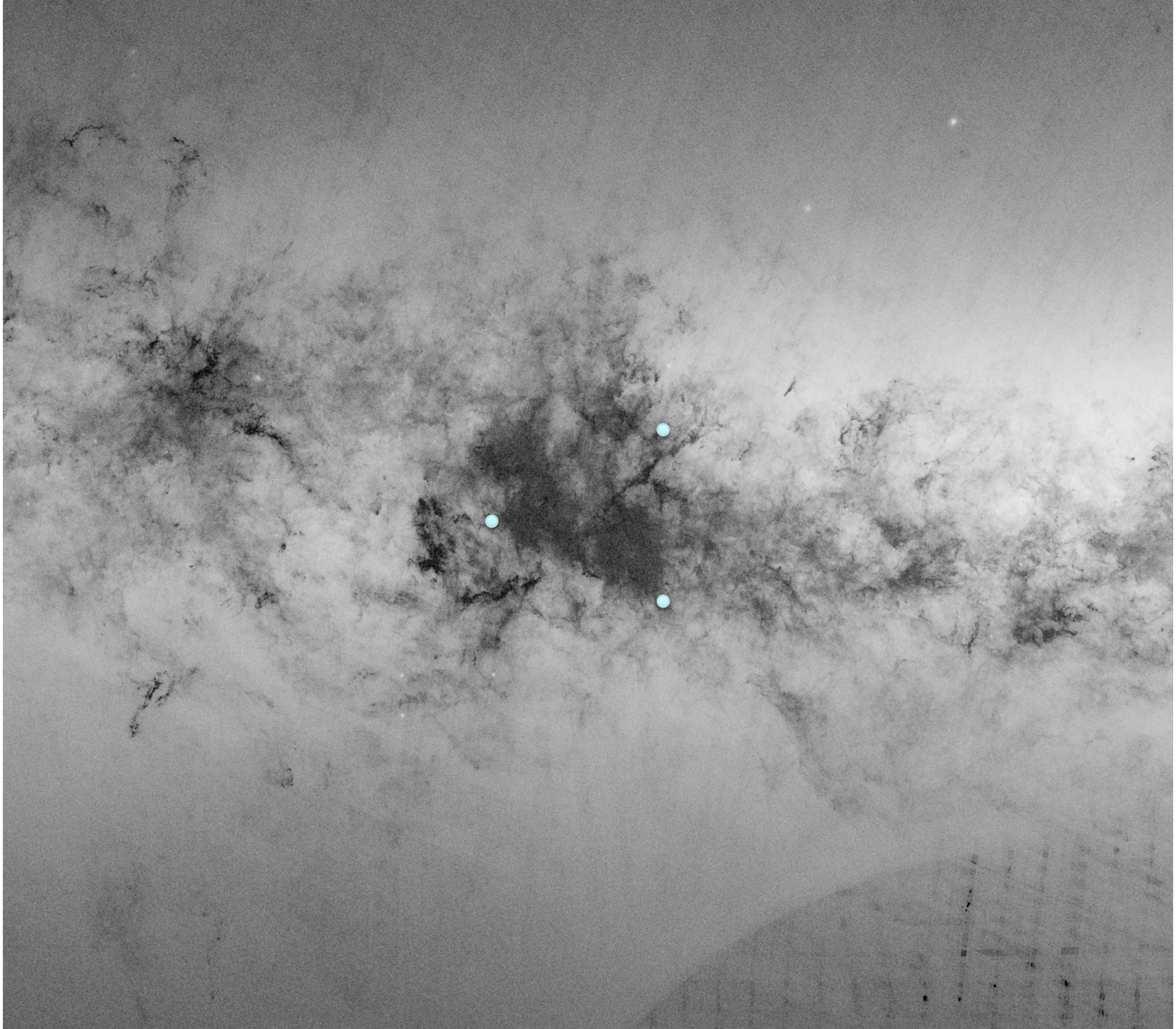
GAIA sky map



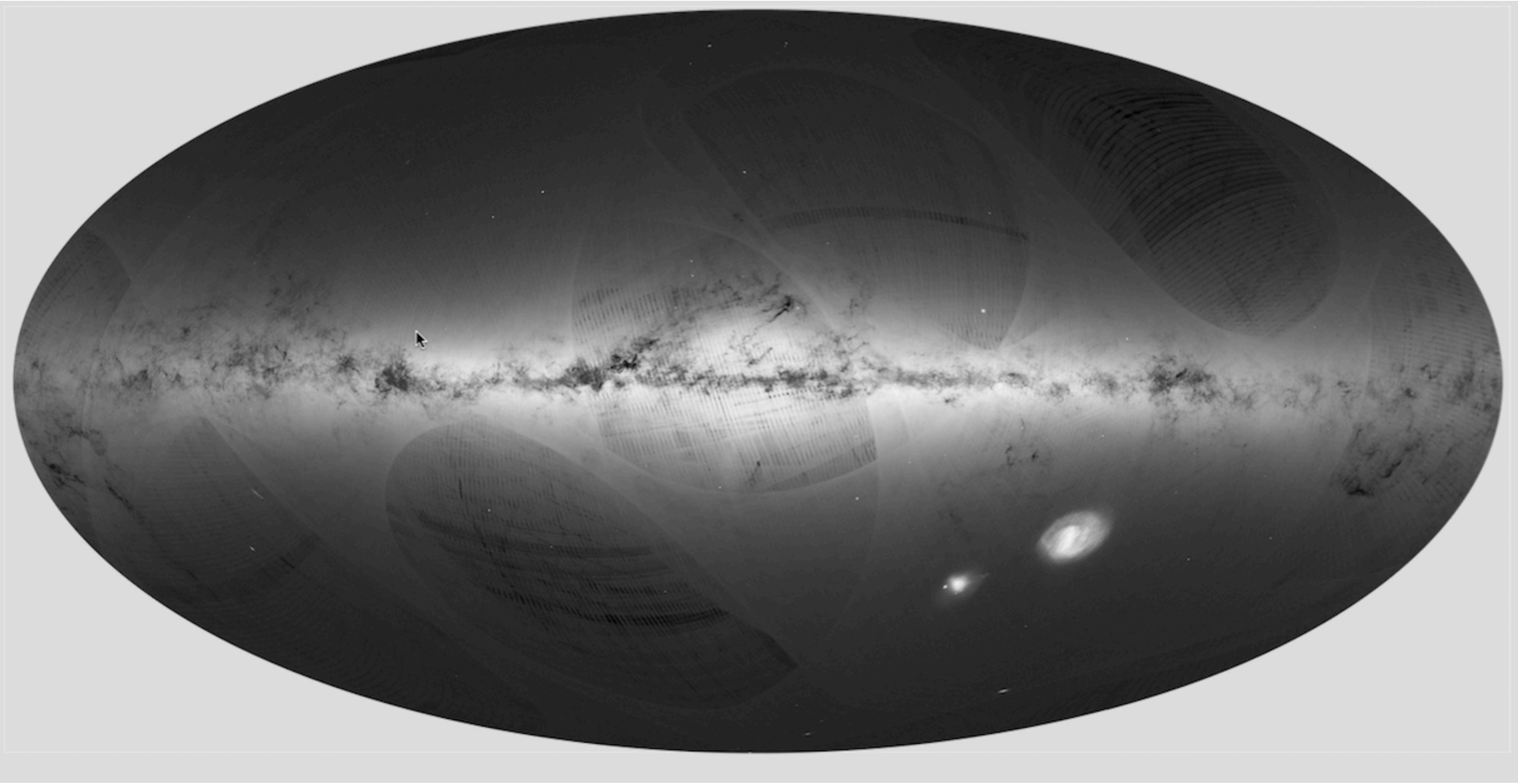
S. Chakrabarty, Y. Han, A. Gonzalez and PS, arXiv: 2007.10509

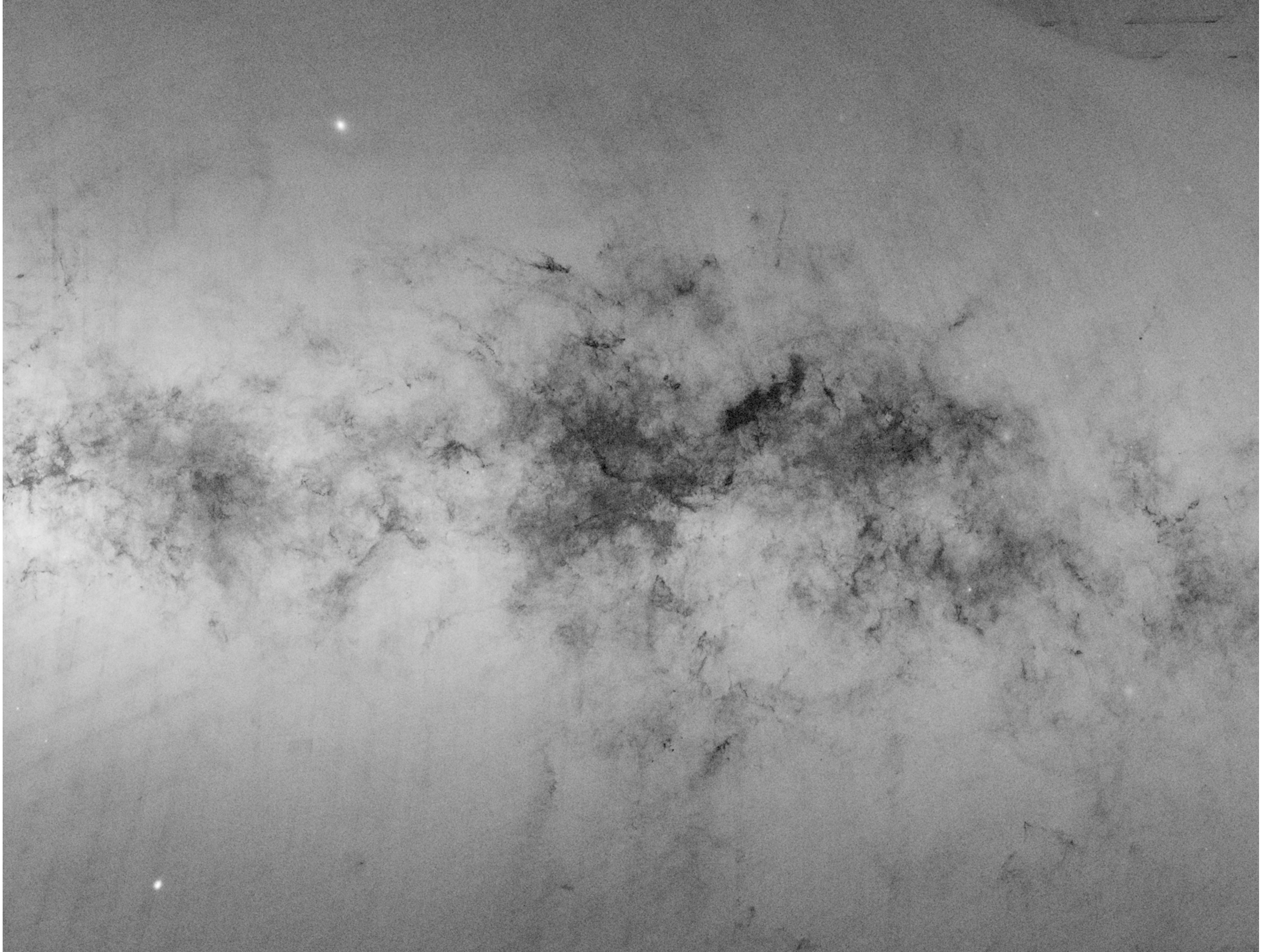


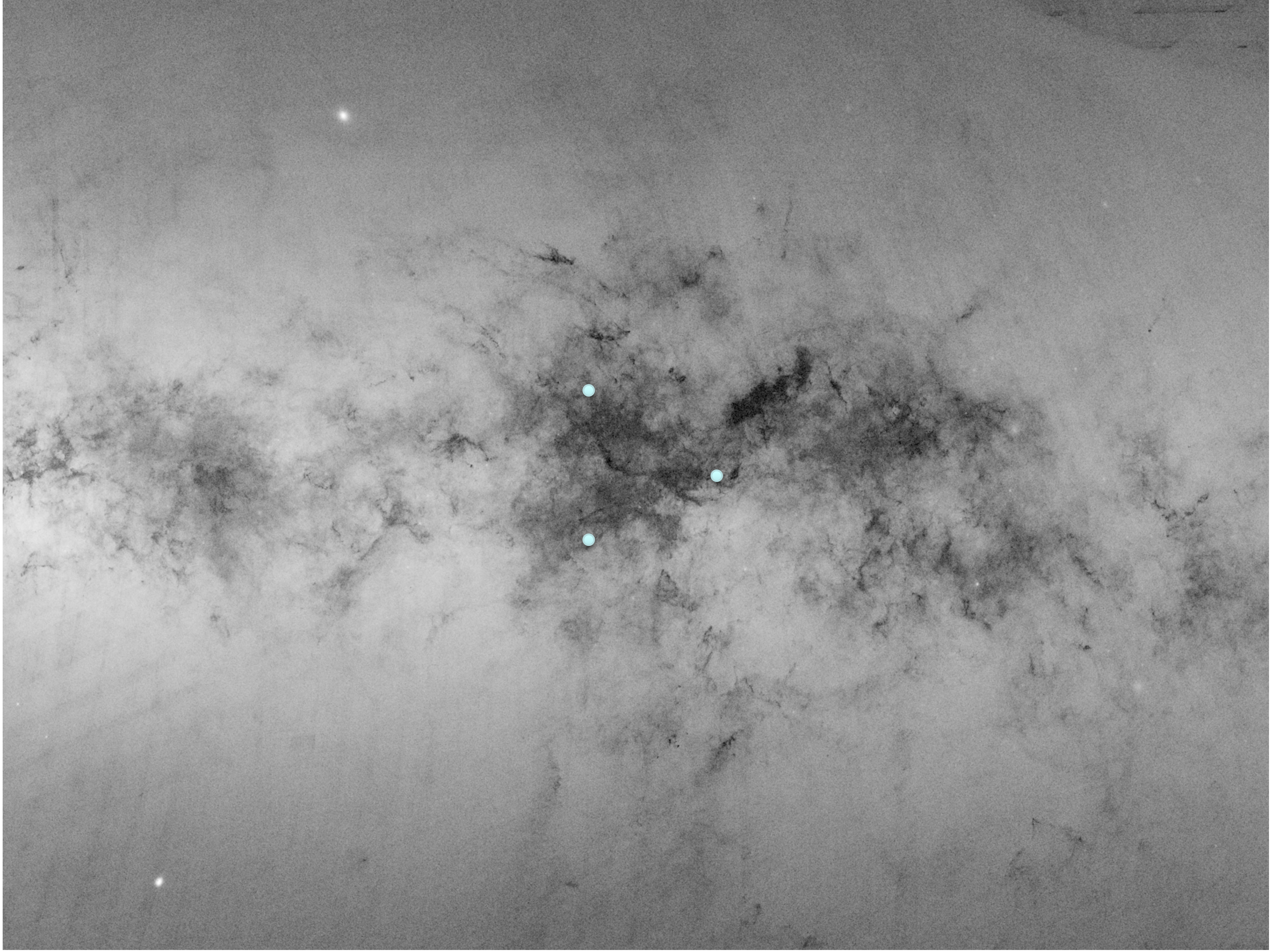


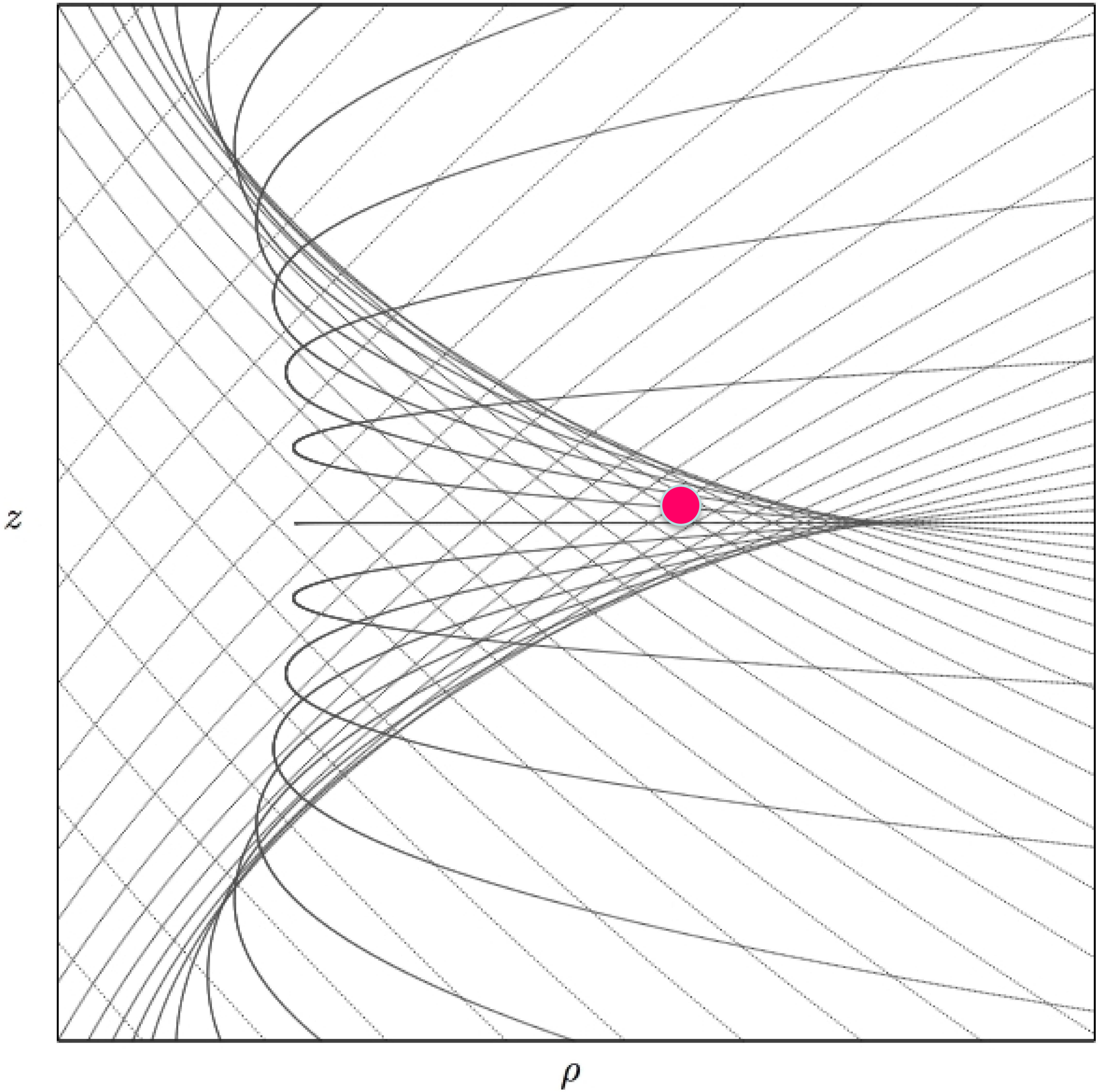


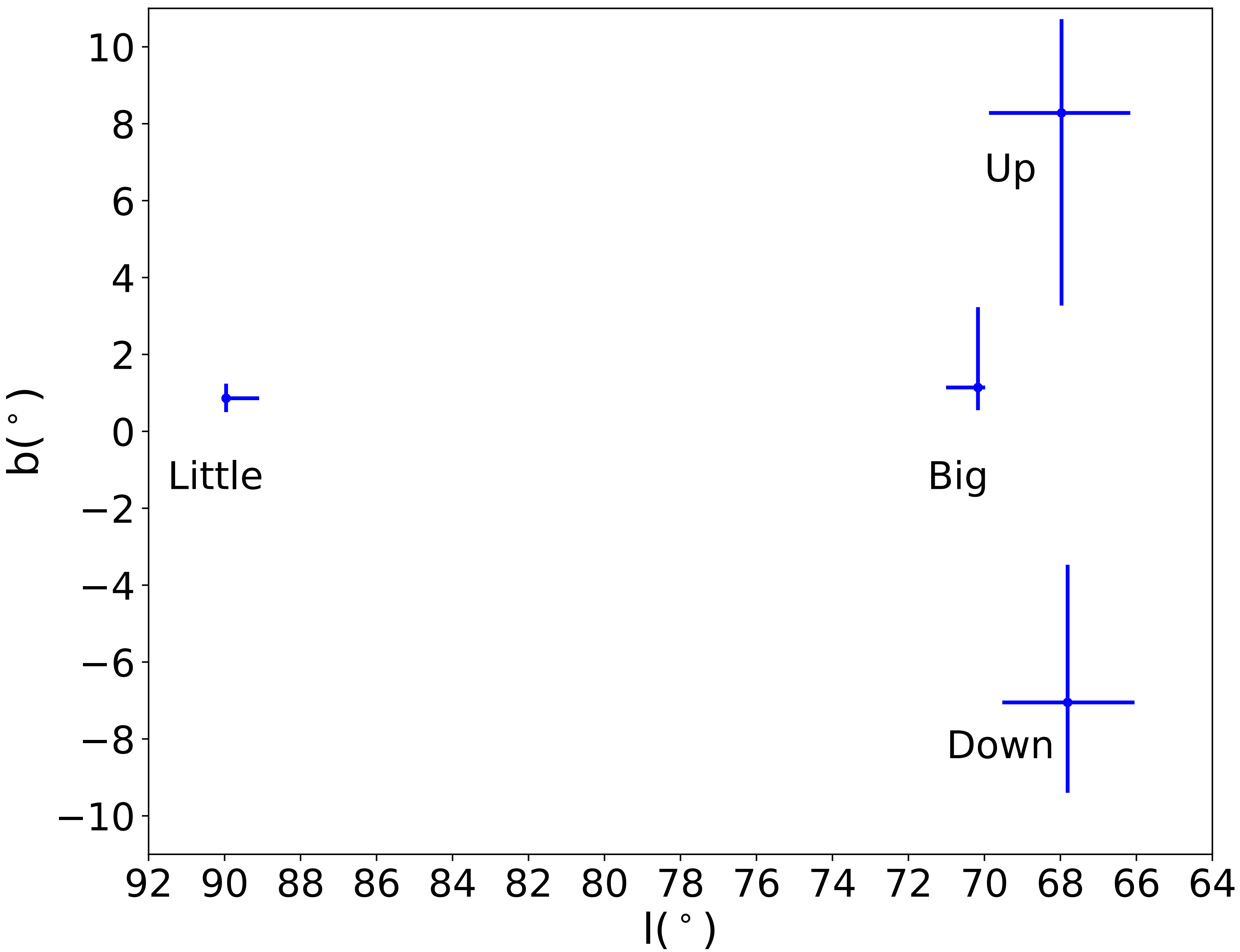
GAIA sky map








z
ρ



Up
Big
Little
Down
b(°)
l(°)
10
8
6
4
2
0
−2
−4
−6
−8
−10
92
90
88
86
84
82
80
78
76
74
72
70
68
66
64

Conclusions

- Axions differ from the other cold dark matter candidates because they thermalize by gravitational self-interactions and form a Bose-Einstein condensate

- Axion BEC explains the evidence for caustic rings of dark matter