The open string pair production & its use

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The talk is based on the following papers:

- J. X. Lu, B. Ning, R. Wei and S. S. Xu, "Interaction between two non-threshold bound states, "Phys. Rev. D **79**, 126002 (2009)
- J. X. Lu and S. S. Xu, "The Open string pair-production rate enhancement by a magnetic flux," JHEP **0909**, 093 (2009)
- J. X. Lu and S. S. Xu, "Remarks on D(p) and D(p-2) with each carrying a flux," Phys. Lett. B **680**, 387 (2009)
- J. X. Lu,"Magnetically-enhanced open string pair production," JHEP **1712**, 076 (2017)
- J. X. Lu, "Some aspects of interaction amplitudes of D branes carrying worldvolume fluxes," Nucl. Phys. B **934**, 39 (2018)
- J. X. Lu, "A possible signature of extra-dimensions: The enhanced open string pair production," Phys. Lett. B **788**, 480 (2019)
- Q. Jia and J. X. Lu,"Remark on the open string pair production enhancement," Phys. Lett. B **789**, 568 (2019)
- J. X. Lu, "A note on the open string pair production of the D3/D1 system," JHEP **1910**, 238 (2019)
- Q. Jia, J. X. Lu, Z. Wu and X. Zhu, "On D-brane interaction & its related properties,"Nucl. Phys. B 953 (2020)114947
- 卢建新 & 张楠: 物理学报特邀综述文章: http://wulixb.iphy.ac.cn/article/doi/10.7498/aps.69.20200037
- J. X. Lu and Nan Zhang, "More on the open string pair production" , Nucl. Phys. B **977** (2022) 115721

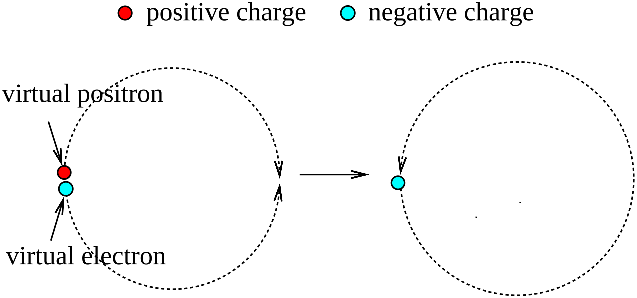
Outline

- Motivation/introduction
- The pair production, its enhancement and the physics behind
- The D3/(D3, (F, D1)) system, a potentially testable rate
- Summary

QED Vacuum Fluctuations

VACUUM FLUCTUATION!

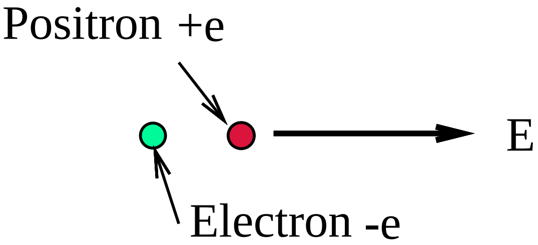
An anti-charge moving forward in time equivalent to a charge moving backward in time



$$(\delta t \sim 10^{-21} s)$$

QED Vacuum Fluctuations

Applying a constant E to QED vacuum, there is certain probability to create real electron and positron pairs from the vacuum fluctuations, called Schwinger pair production (1951).



$$2eE\frac{1}{m_e} \approx 2m_e \to \boxed{\mathrm{e\ E = m}_e^2} \to E = \frac{m_e^2}{e} \sim 10^{18}\ V/m \qquad (1.1)$$

The current lab E-field limit: $\sim 10^{10}\ V/m$

D-branes in Type II

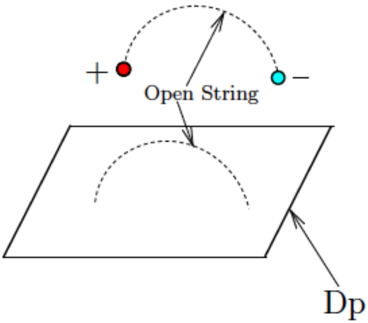
The question is then: Does there exist an analogous process in string theory?

The answer is simply yes!

In this talk, we will address three things:

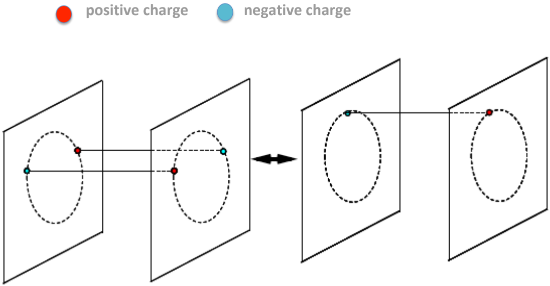
- the open string pair production in Type II superstring theories and its enhancement,
- the relation between the present rate and the relevant rates in QED,
- and the potentially-testable rate.

D-branes in Type II

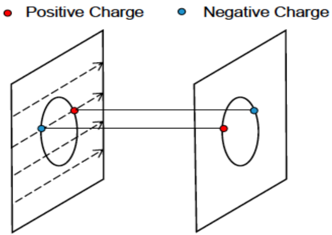


The open string pair production

A simple setup for this is to consider two Dp branes in Type II string theory, placed parallel at a separation.



The open string pair production



- Stringy computations show indeed a non-vanishing pair production rate for this setup. However, this rate is usually vanishing small for any realistic electric fields Lu'17.
- This rate can be greatly enhanced if we add in addition a magnetic flux in a particular manner on each Dp for $p \geq 3$ Lu'19, Jia & Lu' 19.
- The largest rate occurs for $p = 3$, a $(1 + 3)$-dimensional world like ours, if the same electric and magnetic fields are applied Lu'19.

So from now on, we just focus on the D3/D3-system

The D3/D3 rate

For this purpose, consider the electric/magnetic tensor $\hat{F}^1$ on one D3 brane and the $\hat{F}^2$ on the other D3 brane, respectively, as

$$\hat{F}^a_{\mu\nu} = \begin{pmatrix} 0 & -\hat{f}_a & 0 & 0 \\ \hat{f}_a & 0 & 0 & 0 \\ 0 & 0 & 0 & -\hat{g}_a \\ 0 & 0 & \hat{g}_a & 0 \end{pmatrix}, \tag{2.1}$$

where $\hat{f}_a$ denotes the dimensionless electric field ($|\hat{f}_a| < 1$) while g_a the dimensionless magnetic one ($|\hat{g}_a| < \infty$) with $a = 1, 2$. Note $\hat{F}_{\mu\nu} = 2\pi\alpha' F_{\mu\nu}$. Note $[\alpha'] = -2$, $[F] = 2 \to [\hat{F}] = 0$.

The D3/D3 rate

The pair production rate can be computed to be Lu'17 ,

$$\mathcal{W}^{(1)} = \frac{8\,|\hat f_1 - \hat f_2||\hat g_1 - \hat g_2|}{(8\pi^2\alpha')^2}\, e^{-\frac{y^2}{2\pi\nu_0\alpha'}}\, \frac{\left[\cosh\frac{\pi\nu_0'}{\nu_0} + 1\right]^2}{\sinh\frac{\pi\nu_0'}{\nu_0}} Z_1(\nu_0,\nu_0'), \qquad (2.2)$$

where

$$\begin{aligned} Z_1(\nu_0,\nu_0') &= \prod_{n=1}^{\infty} \frac{\left[1 + 2e^{-\frac{2n\pi}{\nu_0}}\cosh\frac{\pi\nu_0'}{\nu_0} + e^{-\frac{4n\pi}{\nu_0}}\right]^4}{\left[1 - e^{-\frac{2n\pi}{\nu_0}}\right]^6 \left[1 - e^{-\frac{2\pi}{\nu_0}(n-\nu_0')}\right]\left[1 - e^{-\frac{2\pi}{\nu_0}(n+\nu_0')}\right]}. \\ &= 1 + 4\left[1 + \cosh\frac{\pi\nu_0'}{\nu_0}\right]^2 e^{-\frac{2\pi}{\nu_0}} + \cdots \end{aligned} \qquad (2.3)$$

In the above, the parameters $\nu_0 \in [0,\infty)$ and $\nu_0' \in [0,1)$ are

$$\tanh\pi\nu_0 = \frac{|\hat f_1 - \hat f_2|}{1 - \hat f_1 \hat f_2}, \quad \tan\pi\nu_0' = \frac{|\hat g_1 - \hat g_2|}{1 + \hat g_1\hat g_2}. \qquad (2.4)$$

Note $\nu_0 \ll 1, \quad Z_1(\nu_0,\nu_0') \approx 1$

The D3/D3 rate

For any realistic applied electric and magnetic fields, $|\hat{f}_a| \sim |\hat{g}_a| \sim 10^{-21} \ll 1$ with $a = 1, 2$, giving $\nu_0 \ll 1$ & $\nu_0' \ll 1$ $(Z_1(\nu_0, \nu_0') \approx 1)$.

We first consider the case $\hat{f}_2 = \hat{g}_2 = 0$ on the hidden D3 while on our own D3, in terms of the lab. field E and B via

$$\hat{f}_1 = 2\pi\alpha' eE \ll 1, \qquad \hat{g}_1 = 2\pi\alpha' eB \ll 1, \tag{2.5}$$

the pair production rate(2.2) for D3 brane is now

$$\mathcal{W}^{(1)} = \frac{2(eE)(eB)}{(2\pi)^2} \frac{\left[\cosh\frac{\pi B}{E} + 1\right]^2}{\sinh\frac{\pi B}{E}} e^{-\frac{\pi m^2(y)}{eE}}, \tag{2.6}$$

where we have introduced a mass scale

$$m(y) = T_f y = \frac{y}{2\pi\alpha'}. \tag{2.7}$$

Keep in mind, we need to have a nearby hidden D3 brane for this rate!

The D3/D3 rate

The rate enhancement:

$$\frac{\mathcal{W}^{(1)}(B \neq 0)}{\mathcal{W}^{(1)}(B = 0)} = \frac{1}{4}\frac{\pi B}{E}\frac{\left[\cosh\frac{\pi B}{E} + 1\right]^2}{\sinh\frac{\pi B}{E}} \tag{2.8}$$

which is larger than unity when $B/E \sim \mathcal{O}(1)$ and becomes

$$\frac{1}{8}\frac{\pi B}{E}\, e^{\frac{\pi B}{E}} \gg 1, \tag{2.9}$$

when $B/E \gg 1$.

The D3/D3 rate

Let us try to understand (2.6) a bit more.

In the absence of both E and B, the mass spectrum for the open string connecting the two D3 is

$$\alpha' M^2 = -\alpha' p^2 = \begin{cases} \frac{y^2}{4\pi^2\alpha'} + N_{\rm R} & ({\rm R-sector}), \\ \frac{y^2}{4\pi^2\alpha'} + N_{\rm NS} - \frac{1}{2} & ({\rm NS-sector}), \end{cases} \quad (2.10)$$

where $p = (k, 0)$ with k the momentum along the brane worldvolume directions, $N_{\rm R}$ and $N_{\rm NS}$ are the standard number operators in the R-sector and NS-sector, respectively, as

$$\begin{aligned} N_{\rm R} &= \sum_{n=1}^{\infty} (\alpha_{-n}\cdot\alpha_n + n d_{-n}\cdot d_n), \\ N_{\rm NS} &= \sum_{n=1}^{\infty} \alpha_{-n}\cdot\alpha_n + \sum_{r=1/2}^{\infty} r d_{-r}\cdot d_r. \end{aligned} \quad (2.11)$$

The D3/D3 rate

The R-sector gives fermions with $N_{\rm R} \geq 0$ while the NS-sector gives bosons with $N_{\rm NS} \geq 1/2$. The $N_{\rm R} = 0$, $N_{\rm NS} = 1/2$ give the usual massless $4(8_{\rm F} + 8_{\rm B})$ degrees of freedom (The 4D N = 4 U(2) SYM) when $y = 0$.

Among these, $2(8_{\rm F} + 8_{\rm B})$ become massive ones, all with mass $T_f y = y/(2\pi\alpha')$ due to unbroken SUSY, when $y \neq 0$. This just reflects $U(2) \to U(1) \times U(1)$ when $y = 0 \to y \neq 0$. The two broken generators give 16 pairs of charged/anti-charged massive DOF with respect to the brane observer (5 scalar pairs, 4 spinor pairs and one vector pair).

The pair production rate (2.6) is obtained in the weak field limit and all massive other than the lowest 16 charged/anti-charged pairs of dof are dropped since $Z_1 \approx 1$.

In other words, only these 16 pairs of dof actually contribute to this rate or the one for the N = 4 massive SYM.

The D3/D3 rate

We now compare the open string pair production rate (2.6) with QED charged scalar, spinor and W-boson pair production rate with the same E and B. The present rate is

$$\mathcal{W}^{(1)} = \frac{2(eE)(eB)}{(2\pi)^2} \frac{\left[\cosh \frac{\pi B}{E} + 1\right]^2}{\sinh \frac{\pi B}{E}} \, e^{-\frac{\pi m^2}{eE}}, \tag{2.12}$$

while for the QED massive charged scalar pair Nikishov'70

$$\mathcal{W}_{\mathrm{scalar}} = \frac{(eE)(eB)}{2(2\pi)^2} \operatorname{csch}\left(\frac{\pi B}{E}\right) \, e^{-\frac{\pi m_0^2}{eE}}, \tag{2.13}$$

for massive charged spinor pair

$$\mathcal{W}_{\mathrm{spinor}} = \frac{(eE)(eB)}{(2\pi)^2} \coth\left(\frac{\pi B}{E}\right) \, e^{-\frac{\pi m_{1/2}^2}{eE}}, \tag{2.14}$$

and for massive charged vector pair Kruglov'01,

$$\mathcal{W}_{\mathrm{vector}} = \frac{(eE)(eB)}{2(2\pi)^2} \frac{2\cosh \frac{2\pi B}{E} + 1}{\sinh \frac{\pi B}{E}} \, e^{-\frac{\pi m_1^2}{eE}}. \tag{2.15}$$

The D3/D3 rate

Observations:

- Identifying $m_0 = m_{1/2} = m_1 = m$ and when $B = 0$, we have

$$\begin{aligned} \mathcal{W}_{\rm vector} &= 3\mathcal{W}_{\rm scalar}, \quad \mathcal{W}_{\rm spinor} = 2\mathcal{W}_{\rm scalar}, \\ \mathcal{W}^{(1)} &= 16\,\mathcal{W}_{\rm scalar} = 8\,\mathcal{W}_{\rm spinor} \\ &= \frac{16}{3}\,\mathcal{W}_{\rm vector} = \frac{8(eE)^2}{(2\pi)^2}\,e^{-\frac{\pi m^2}{eE}}. \end{aligned} \tag{2.16}$$

- While for large B/E (or $B \neq 0, E \sim 0$),

$$\begin{aligned} &\mathcal{W}^{(1)} \approx \frac{(eE)(eB)}{(2\pi)^2}\,e^{-\frac{\pi(m^2 - eB)}{eE}}, \quad \mathcal{W}_{\rm vector} \approx \frac{(eE)(eB)}{(2\pi)^2}\,e^{-\frac{\pi(m_1^2 - eB)}{eE}}, \\ &\mathcal{W}_{\rm scalar} \approx \frac{(eE)(eB)}{(2\pi)^2}\,e^{-\frac{\pi(m_0^2 + eB)}{eE}}, \quad \mathcal{W}_{\rm spinor} \approx \frac{(eE)(eB)}{(2\pi)^2}\,e^{-\frac{\pi m_{1/2}^2}{eE}}. \end{aligned} \tag{2.17}$$

The D3/D3 rate

The pre-factor for vector, spinor and the present rate is the same as that for the scalar but the exponential suppressing factor is different for different case. How to understand this?

Further if set $m_0 = m_{1/2} = m_1 = m$, we have

$$\frac{\mathcal{W}_{\rm scalar}}{\mathcal{W}_{\rm vector}} = e^{-\frac{2\pi B}{E}} \to 0, \quad \frac{\mathcal{W}_{\rm spinor}}{\mathcal{W}_{\rm vector}} = e^{-\frac{\pi B}{E}} \to 0,$$

$$\mathcal{W}^{(1)} = \mathcal{W}_{\rm vector} = \frac{(eE)(eB)}{(2\pi)^2}\, e^{-\frac{\pi(m^2 - eB)}{eE}} \tag{2.18}$$

The D3/D3 rate

It is well-known that an electrically charged particle with mass m_S and spin S in a weak magnetic field B background has energy

$$E^2_{(S,S_z)} = (2N+1)eB - g_S eB \cdot S + m_S^2, \tag{2.19}$$

with g_S the gyromagnetic ratio ($g_S = 2$) and N the Landau level. So for the lowest Landau level ($N = 0$), we have the following mass splittings

S	0	1/2	1
$E^2_{(S,S_z)}$	$E^2_{(0,0)} = m_0^2 + eB$	$E^2_{(\frac{1}{2},-\frac{1}{2})} = m^2_{\frac{1}{2}} + 2eB$ $E^2_{(\frac{1}{2},\frac{1}{2})} = m^2_{\frac{1}{2}}$	$E^2_{(1,-1)} = m_1^2 + 3eB$ $E^2_{(1,0)} = m_1^2 + eB$ $E^2_{(1,1)} = m_1^2 - eB$

So for large B/E and from the scalar rate in (2.17), we have for each spin polarization

$$\mathcal{W}_{(S,S_z)} \approx \frac{(eE)(eB)}{(2\pi)^2} e^{-\frac{\pi E^2_{(S,S_z)}}{eE}}. \tag{2.20}$$

The D3/D3 rate

The above explains why only the lowest energy polarization survives when B/E is large. For example,

$$\frac{\mathcal{W}_{(1,0)}}{\mathcal{W}_{(1,1)}} = e^{-\frac{2\pi B}{E}} \to 0. \tag{2.21}$$

For general B/E, we also expect to have,

$$\mathcal{W}^{(1)} = 5\, W_{\text{scalar}} + 4\, W_{\text{spinor}} + W_{\text{vector}}, \tag{2.22}$$

when all the modes with the same mass.

One can check this holds indeed true and it explains the previous results for $B = 0$ and large B/E, respectively.

It is also very satisfied to have this since they are computed completely differently, one in string theory and the other in QFT.

The D3/D3 rate

Can the rate (2.12), rewritten below, be useful for actual detection in practice?

$$\mathcal{W}^{(1)} = \frac{2(eE)(eB)}{(2\pi)^2} \frac{\left[\cosh \frac{\pi B}{E} + 1\right]^2}{\sinh \frac{\pi B}{E}} e^{-\frac{\pi m^2}{eE}}. \tag{2.23}$$

The answer is simply no since we expect $m > \mathrm{TeV}$, due to the unbroken SUSY, for all the modes contributing to the above while $eE \sim eB \sim 10^{-8} m_e^2 \sim 10^{-21} \mathrm{TeV}^2$.

$$eE \ll m^2. \tag{2.24}$$

The D3/(D3, (F, D1)) rate

Now the previous hidden D3 is replaced by the so-called 1/2 BPS (D3, (F, D1)) non-threshold bound state of D3 with a delocalized (F, D1) non-threshold bound state (Lu & Shibaji' 00) in the following sense

	0	1	2	3
(F, D1)	$\times$	$\times$		
D3	$\times$	$\times$	$\times$	$\times$

$\equiv$

	0	1	2	3
$\hat{F}^2_{\alpha\beta}$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
D3	$\times$	$\times$	$\times$	$\times$

In other words, the delocalized (F, D1) along 23-directions can be represented by the electric and magnetic fluxes as given earlier with now the quantized fluxes

$$\hat{f}_2 = \frac{p}{\sqrt{p^2 + \frac{q^2+n^2}{g_s^2}}}, \quad \hat{g}_2 = \frac{q}{n}, \tag{3.1}$$

where the three integers n, p, q stand for the number of D3 branes, the quantized electric flux and the quantized magnetic flux, respectively, without a common divisor.

The D3/(D3, (F, D1)) rate

As discussed in Lu & Zhang' 22, a typical generic case is to take $g_s = 0.01, n = q = 1, p = 10$ which gives

$$\hat{f}_2 = \frac{g_s p}{\sqrt{2}} = \frac{1}{10\sqrt{2}}, \quad \hat{g}_2 = 1. \tag{3.2}$$

With these and from (2.4), we have

$$\begin{aligned}
&\tanh \pi\nu_0 \approx \hat{f}_2 \rightarrow \nu_0 = \frac{\hat{f}_2}{\pi} = \frac{1}{10\pi\sqrt{2}} \ll 1 \rightarrow Z_1(\nu_0, \nu_0') \approx 1, \\
&\tan \pi\nu_0' \approx \hat{g}_2 = 1 \rightarrow \nu_0' = \frac{1}{4}, \\
&\frac{\pi\nu_0'}{\nu_0} = \frac{5\pi^2}{\sqrt{2}} \approx 35 \gg 1.
\end{aligned} \tag{3.3}$$

where we have set $\hat{f}_1 = \hat{g}_1 = 0$ for simplicity.

The D3/(D3, (F, D1)) rate

The corresponding pair production rate can now read from (2.2) as

$$\begin{aligned}\mathcal{W}^{(1)} &= \frac{\hat{f}_2}{4\pi^4\alpha'^2}\, e^{-\frac{y^2}{2\pi\nu_0\alpha'}+\frac{\pi}{4\nu_0}} \\ &= \frac{eE'}{2\pi^3\alpha'}\, e^{-\frac{\pi\left[m^2(y)-\frac{1}{8\alpha'}\right]}{eE'}} \\ &= \frac{eE'}{2\pi^3\alpha'}\, e^{-\frac{\pi m^2_{\rm eff}}{eE'}},\end{aligned} \tag{3.4}$$

where in the first equality we have $Z_1(\nu_0, \nu_0') \approx 1$, again due to $e^{-2\pi/\nu_0} = e^{-20\pi^2\sqrt{2}} \ll 1$, and replace the $\cosh(\pi\nu_0'/\nu_0)$ and $\sinh(\pi\nu_0'/\nu_0)$ factors each by $\frac{1}{2}\, e^{\pi\nu_0'/\nu_0}$, due to $\pi\nu_0'/\nu_0$ $= 5\pi^2/\sqrt{2} \approx 35 \gg 1$. In the second and third equalities, we set $\hat{f}_2 = 2\pi\alpha' eE'$ and define

$$m^2_{\rm eff}(y) = m^2(y) - \frac{1}{8\alpha'}, \tag{3.5}$$

with $m(y) = y/(2\pi\alpha')$. Note that in the above $eE' = M_s^2/(20\pi\sqrt{2})$.

The D3/(D3, (F, D1)) rate

Given our understanding of charged particle or charged open string moving in a magnetic field, it is clear, unlike the previous case, that only the pair of charged/anti-charged vector polarizations contributes to the rate (3.4).

The reason is simple that unlike the previous case, the underlying system D3/(D3, (F, D1)) is intrinsically non-SUSY and the above pair of vector polarizations is the lightest one among the lowest stringy modes.

The D3/(D3, (F, D1)) rate

In order to produce the pair detectable (similar to the discussion in Schwinger case), the applied electric field E' needs to satisfy

$$eE' = m_{\rm eff}^2. \tag{3.6}$$

In comparison with the previous case, this is much easier to hold since $eE' = M_s^2/(20\pi\sqrt{2})$, just about one order smaller than stringy one, giving $m_{\rm eff} \sim 0.1 M_s$. Concretely, we have (note $M_s = 1/\sqrt{\alpha'}$)

$$\frac{M_s^2}{20\pi\sqrt{2}} = m^2(y) - \frac{1}{8\alpha'}, \tag{3.7}$$

giving

$$m(y) \approx \frac{1}{2\sqrt{2}}\left(1 + \frac{1}{5\pi\sqrt{2}}\right) M_s \approx 0.37 M_s, \tag{3.8}$$

which can be satisfied without much difficulty so long we tune the brane separation to the right amount.

In other words, a sizable pair production rate can be reached in principle.

Summary

- We have discussed the open string pair production enhancement in the presence of a properly applied magnetic field and it is due to the lowering of energy of one particular pair of charged/anti-charged massive vector polarizations in such applied field,
- The stringy computed pair production rate for the lowest modes is found to agree completely with the relevant ones computed in QED if the same physics is considered, lending support to the consistency of string theory to the quantum field theory in low energy limit.
- A potentially testable open string pair production rate is considered .

THANK YOU!