Flavor Structure of Goldstone Bosons

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Contents

1. Introduction

The Goldstone Thereom: When a continuous globel symmetry is broken spontaneously, there are massless particles correspond to the broken generators. Examples: Axion, Majoron, Pion...

If the globel symmetry is gauged, the Goldstone boson become the corresponding gauge particle longitudinal component and the particle becomes massive. Examples: W and Z, Dark Photon...

If the Globel symmetry is explicitly broken, by tree level terms, or quantum loop anomalies, the particles corresponding broken generators will become massive. Examples: Axion, Pion...

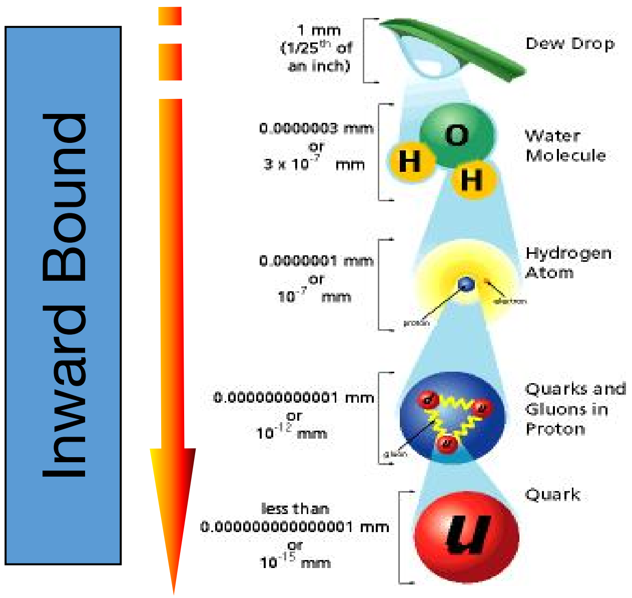
Gauge anomaly cancellation constrains mdoel building leading to charge quantization in the SM. Dirac monopole also provides a way to think about charge quantization.

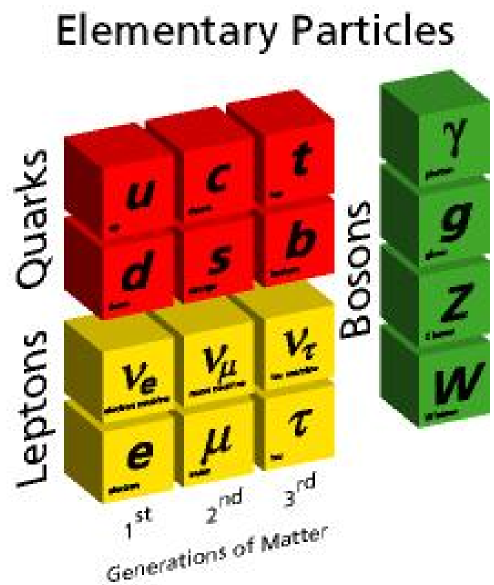
All the exotics, such as Axion, Majoron, Axion and Dark Photon particles are related and well motivated particles for experimental searches. They can also play different roles in particle and cosmology.

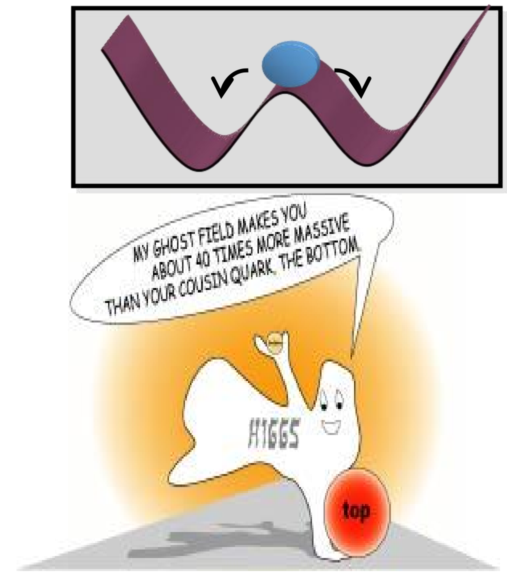
Experiments, go to find them or rule them out in all possible ways!

The standard model of strong and electroweak interactions

SU(3) x SU(2) x U(1) gauge theory for strong and electroweak interaction







Can one negeclects gravitation interaction when studying particle interactions?
The coulomb force between two protons: $F_c = e^2/r^2$,
And Gravitational force: $F_g = -Gm^2/r^2$ $\quad |F_g|/|F_c| = 7\times10^{-38}$

Gravitational force is much weaker than electromagnetism!

But when study cosmology，gravitational force always add up，but electromagnetism can cancel between positively and negatively charged particles!

2. Axion and Majoron Interactions

Axion

Some useful relations

The chiral anomaly relation, $\partial^\mu(\bar\psi\gamma_\mu\gamma_5\psi) = \frac{g_3^2}{16\pi^2}Tr(\tilde{G}G)$,

leads to a chiral rotation $\psi \to e^{i\alpha\gamma_5/2}\psi$ generates in the Lagrangian

$$\delta L_\alpha = -\alpha\frac{g_3^2}{16\pi^2}Tr(\tilde{G}G)\ .$$

An imaginary matter term $\delta L = -\bar\psi m(cos\alpha + isin\alpha\gamma_5)\psi$

can be transformed away by define $\psi' = e^{-\alpha\gamma_5/2}\psi$ and to

$$\delta L = -\bar\psi' m\psi' + \alpha\frac{g_3^2}{16\pi^2}Tr(\tilde{G}G)\ .$$

If one write with more than one ψ the mass matrices as $\psi_R M\psi_L$

In general M is complex. Then

$$\begin{aligned}\delta L &= -(\bar\psi_R M\psi_L + H.C.) - \theta\frac{g_3^2}{16\pi^2}Tr(\tilde{G}G)\\ &= -\bar\psi\hat{M}\psi - (\theta - Arg(Det(M))\frac{g_3^2}{16\pi^2}Tr(\tilde{G}G)\ .\end{aligned}$$

$\hat{M} = diag(m_1, m_2, ...)$, with $m_i > 0$

$$L_{\pi^i B_f B} = -\sqrt{2}\bar{N}_f\sigma^i(i\gamma_5 g_{\pi NN} + f_{\pi NN}|N>$$

$g_{\pi NN} \approx 14$ is CP conserving, and $f_{\pi NN}$ is CP violating coupling with

$$f_{\pi NN} = -2\frac{(m_\Xi - m_\Sigma)}{f_\pi}\frac{m_u m_d m_s\theta}{m_u m_d + m_u m_s + m_d m_s}\ ,$$

$D_n \sim -3.8\times 10^{-16}\theta$ ecm

Including all SU(3) octet contributions:

$2.5\times 10^{-16}\theta ecm < |D_n| < 4.6\times \theta ecm$

Using data $|D_n| < 3\times 10^{-27} ecm$, $|\theta| < 10^{-11}$!

Why θ is small is the strong CP problem.

1. One of the quark mass is zero, since D_n is proportional to $m_u m_d m_s$. But..?!
2. Making the theory left-right symmetric (parity conservation, θ is zero to start with).
3. Spontaneous CP violation, making θ equal to zero first. Need to check whether after symmetry breaking, θ is not generated.
4. Dynamic solution driving θ small by symmetry, the Peccei-Quinn symmetry. PQ symmetry is spontaneously broke by Higgs vev, results a Goldstone

This is the Axion which has not been discovered, but.....

Peccei-Qiunn symmetry and Axion

$U(1)_A$ chiral model of QP symmetry for strong CP problem

$L = L_{SM} + \delta L_\theta$, $\delta L_\theta = -\theta(g_3^2/16\pi^2)Tr(\tilde{G}G)$

$u_R \to e^{i\alpha}u_R$, $d_R \to e^{i\alpha}$, $Q_L \to Q_L$, $L_L \to L_l$ and $e_R \to e^{i\alpha}e_R$

$\bar{\theta} = \theta \to \theta - 2\alpha$,

If L_{SM} is symmetric under $U(1)_A$, $L \to L_{SM} + \delta L_{\bar{\theta}=\theta-2\alpha}$

For L_{SM}, α is arbitrary, choose one such that $\bar{\theta} = \theta - 2\alpha = 0$.

No strong CP term!

Extend the Higgs secotor to have two Higgs doublets H_1 and H_2

$H_1 \to e^{i\alpha}H_1$ and $H_2 \to e^{-i\alpha}H_2$,

Then $L_Y = -\bar{Q}_L Y_u \tilde{H}_1 u_R - \bar{Q}_L Y_d H_2 d_R$

Should make the potential $V(H_1, H_2)$ invariant.

Both H_1 and H_2 should have non-zero vev, v_1 and v_2

The PQ symmetry in $V(H_1, H_2)$ is spontaneously broken by v_i,

There is a massless GOLDSTONE boson, Axion.

The invisible Axion Model: H_1 (1,2, 1/2)(X_1), H_2 (1,2,1/2)(X_2) S (1,1,0)($-X_1+X_2$) In order to have a non-zero α,

X1+X2 not equal to zero,

to solve strong CP problem. DFSZ

$$H_i = \begin{pmatrix} h_i^+ \\ \frac{1}{\sqrt{2}}(v_i + h_i + iI_i) \end{pmatrix}, \quad S = \frac{1}{\sqrt{2}}(v_s + h_s + iI_s).$$

$$Q_L : 0, \quad U_R : X_u = X_1, \quad D_R : X_d = -X_2, \quad L_L : 0, \quad E_R : X_e = -X_2$$

linear combination I_1, I_2, I_s provide the golstone modes, need to fined out which one is the Axion from broken genrators

$$z : (v_1, \ v_2, \ 0), \quad A : (X_1 v_1, \ X_2 v_2, \ X_s v_s).$$

$$\begin{pmatrix} z \\ a \\ p \end{pmatrix} = \begin{pmatrix} \frac{v_1}{v} & \frac{v_2}{v} & 0 \\ \frac{v_2^2 v_1}{v\sqrt{v_1^2v_2^2+v^2v_s^2}} & -\frac{v_1^2 v_2}{v\sqrt{v_1^2v_2^2+v^2v_s^2}} & -\frac{v^2 v_s}{v\sqrt{v_1^2v_2^2+v^2v_s^2}} \\ \frac{v_2 v_s}{\sqrt{v^2v_s^2+v_1^2v_2^2}} & -\frac{v_1 v_s}{\sqrt{v^2v_s^2+v_1^2v_2^2}} & \frac{v_1 v_2}{\sqrt{v^2v_s^2+v_1^2v_2^2}} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_s \end{pmatrix}$$

$$L_{Y-a} = i\frac{a}{v\sqrt{v_1^2v_2^2+v^2v_s^2}}(v_1^2\bar{U}M_u\gamma_5 U + v_2^2\bar{D}M_d\gamma_5 D + v_2^2\bar{E}M_e\gamma_5 E)$$

$$L_{agg} = N\frac{g_3^2}{16\pi^2}\frac{1}{v\sqrt{v_1^2v_2^2+v^2v_s^2}}(v_1^2+v_2^2)aT(q)G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a = \frac{\alpha_s}{8\pi}\frac{a}{f_a}G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a ,$$

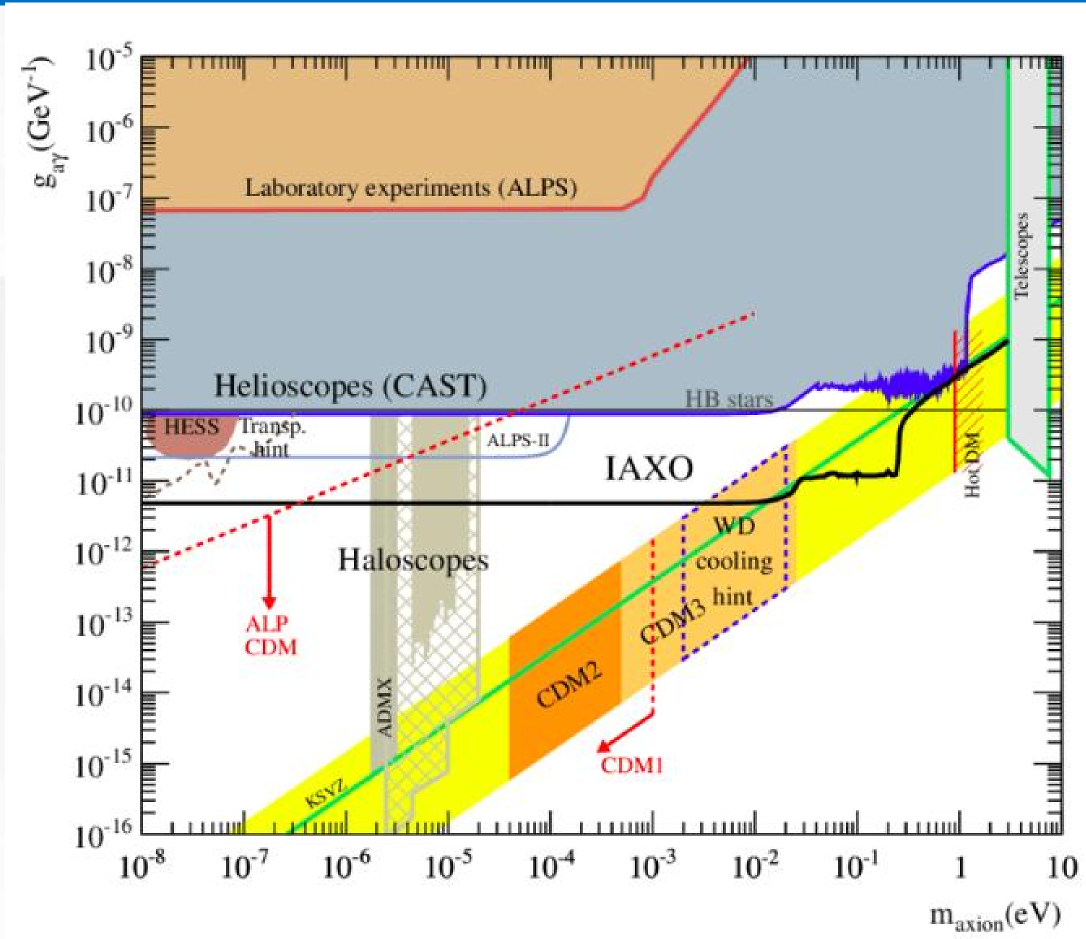
$$L_{a\gamma\gamma} = N\frac{e^2}{16\pi^2}\frac{1}{v\sqrt{v_1^2v_2^2+v^2v_s^2}}((v_1^2Q_u^2+v_2^2Q_d^2)N_c + v_2^2Q_e^2)aF_{\mu\nu}\tilde{F}^{\mu\nu} = \frac{1}{4}ag^0_{a\gamma}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

QCD anomaly from loop generated aGG coupliongs, breaks PQ symmetry

$$m_a^2 = \frac{f_\pi^2}{f_a^2}m_{\pi^0}^2\frac{m_um_dm_s}{(m_u+m_d)(m_um_d+m_um_s+m_dm_s)} \approx \frac{f_\pi^2}{f_a^2}m_{\pi^0}^2\frac{m_um_d}{(m_u+m_d)^2}$$

v_s much larger than v_i Axion becomes invisible, $v_s > 10^9$ GeV

A lot of interesting phenomenology, also can play the role of dark matter



No experimental evidences for Axion. All flavor conserving interactions.

How to obtain flavor changing interactions to test Axion models?

Jin Sun, Xiao-Gang He, PLB811(2020)135881), Jin Sun, Yu Cheng, Xiao-Gang He, JHEP04(2021)141

Majoron

Majoron: resulted from spontaneous symmetry breaking of global lepton for neutrion mass generation.

Experimentally observed neutrino oscillations show that neutrinos have masses. This is a firm evidence of new physics beyond SM.

A popular way of generating by the introcudtion of a Majorana mass for neutrinos. Seesaw models are most studied models which provide explanation by neutrinos have small masses compared with their charged leton partners..

Type I seesaw model: ν_R $(1,1)(0)$ neutrinos, $-\bar{L}_L Y_\nu \tilde{H} \nu_R - (1/2) m_R \bar{\nu}_R^c \nu_R$,

Type II seesaw model: $\chi(1,3)(-1)$ small vev v_χ, $-L_L Y_\nu \chi L_L^c \rightarrow -\nu_L (Y^\nu v_\chi/\sqrt{2}) \nu_L^c$

Type III seesaw model: N_R (1,3)(0),- $\bar{L}_L Y_\nu \tilde{H} N_R - (1/2) m_R \bar{N}_R^c N_R$,

The Majorana mass terms break lepton number by two units explicitely!

Type III seesaw. Yu, Cheng, Cheng-Wei Chiang, Xiao-Gang He, Jin Sun, PRD104(2021)013001.

Besdies SM leptons and Higgs L_{Li} (1,2, -1/2)(1), E_{Ri} (1,1,-1)(1), H (1,2,1/2)(0)

$$H = (h^+, (v + h + iI)/\sqrt{2})^T : (1, 2, 1/2, 0)$$

Introduce right handed leptron triplet: Σ_{iR} (1,3, 0)(1)

$$\Sigma_R = \begin{pmatrix} \frac{\Sigma_L^{0\ c}}{\sqrt{2}} & \Sigma_L^{-\ c} \\ \Sigma_L^{+\ c} & -\frac{\Sigma_L^{0\ c}}{\sqrt{2}} \end{pmatrix}, \quad \Sigma_L = \begin{pmatrix} \frac{\Sigma_L^0}{\sqrt{2}} & \Sigma_L^+ \\ \Sigma_L^- & -\frac{\Sigma_L^0}{\sqrt{2}} \end{pmatrix}$$

Also a singlet $S = (1/\sqrt{2})(v_s + h_s + iI_s) : (1, 1, 0, -2).$ $f_J = v_s$

Lepton number is broken by v_s, I_s corresponds the broken genrator,

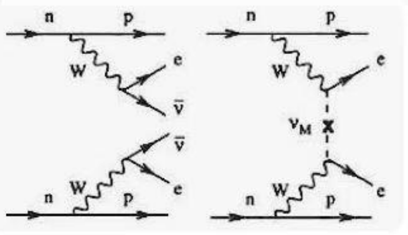
it is the Goldstone: Majoron

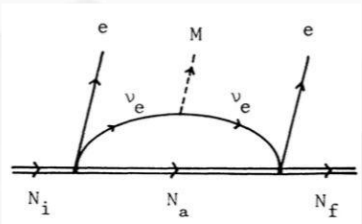
$$\mathcal{L}_Y = -\overline{\ell_\ell} Y_e H E_R - \overline{\ell_\ell}\sqrt{2} Y_\nu \Sigma_R \tilde{H} - \frac{1}{2}\mathrm{Tr}\,\overline{\Sigma_R^c} Y_s S \Sigma_R$$

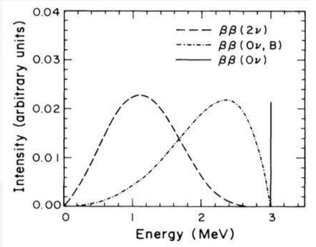
$$-\frac{1}{2}\left(\overline{\nu_L}, \overline{\nu_R^c}\right) M_\nu \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} - \left(\overline{E_L}, \overline{\psi_L}\right) M_c \begin{pmatrix} E_R \\ \psi_R \end{pmatrix}$$
$$-i\frac{J}{2f_J}\left[\overline{\nu_R^c} M_R \nu_R - 2\overline{\psi_L} M_R \psi_R\right] + \mathrm{H.c.}\,,$$

$$M_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}, \quad M_c = \begin{pmatrix} M_e & \sqrt{2} M_D \\ 0 & M_R \end{pmatrix}$$
$$M_e = \frac{Y_e v}{\sqrt{2}}, \quad M_D = \frac{Y_\nu v}{\sqrt{2}}, \quad M_R = \frac{Y_s v_s}{\sqrt{2}},$$

New search signal to Neutrinoless doublet beta decays







	Experimental group	Half-life (yr)
^{76}Ge	Osaka[a]	$>2\times10^{20}$
	Moscow (ITEP)[b]	$>2\times10^{20}$
	Batelle–South Carolina[c]	$(6\pm1)\times10^{20}$
	Caltech-SIN-Neuchatel[d]	$>12\times10^{20}$
	Santa Barbara–LBL[e]	$>14\times10^{20}$
^{82}Se	Irvine[f]	$>4.4\times10^{20}$
^{100}Mo	Osaka[g]	$>6\times10^{18}$
	Irvine[h]	$>7.5\times10^{18}$
^{128}Te/^{130}Te	Missouri[i]	$R_T=2\times10^3$
	Heidelberg[j]	$R_T^{-1}=(1.03\pm1.13)\times10^{-4}$
^{136}Xe	Milano[k]	$>1.6\times10^{19}$
	Moscow (INR)[l]	$>1.0\times10^{20}$
^{150}Nd	Moscow (INR)[s]	$>7.0\times10^{19}$

Early experimental searches: Elliott etal, PRL 59(1987)1649 Doi etal, PRD37(1988)2575

Flavor changing Majoron interaction

$$V^\nu = \begin{pmatrix} V^\nu_{LL} & V^\nu_{LR} \\ V^\nu_{RL} & V^\nu_{RR} \end{pmatrix}, \quad V^{e\,L(R)} = \begin{pmatrix} V^{e\,L(R)}_{LL} & V^{e\,L(R)}_{LR} \\ V^{e\,L(R)}_{RL} & V^{e\,L(R)}_{RR} \end{pmatrix}$$

$$\frac{\partial_\mu J}{2 f_J} \left[\overline{\ell_j} \gamma^\mu (c_V^{e\,ji} + c_A^{e\,ji} \gamma_5) \ell_i + \overline{\nu_{Lj}} \gamma^\mu c_L^{\nu\,ji} \nu_{Li} \right]$$

$$c^e_V = -c^e_A = -V^{e\,L}_{LR} V^{e\,L\dagger}_{LR} \text{ and } c^\nu_L = V^\nu_{LL} V^{\nu\,\dagger}_{LL} - V^\nu_{LR} V^{\nu\,\dagger}_{LR}$$

$$A \equiv \frac{\Gamma_{LL} + \Gamma_{RL} - \Gamma_{LR} - \Gamma_{RR}}{\Gamma_{LL} + \Gamma_{RL} + \Gamma_{LR} + \Gamma_{RR}} \approx -2 \frac{\mathrm{Re}(c_V^{\mu\tau} c_A^{\mu\tau *})}{|c_V^{\mu\tau}|^2 + |c_A^{\mu\tau}|^2}$$

$$V(H,S) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2 - \mu_s^2 S^\dagger S + \lambda_s (S^\dagger S)^2 + \lambda_{hs} (H^\dagger H)(S^\dagger S)$$

$$\Gamma(h \to JJ) = \lambda_{hs}^2 v^2 \cos^2\theta / 32\pi m_h \qquad \lambda_{hs} < 0.014.$$

TABLE I. Bounds on flavor-changing Majoron couplings with charged leptons. Each bound is obtained by keeping only one type of interaction at a time.

	Process	Experimental input	Bound (in units of f_J/TeV)
I	$M \to \overline{M}$	$P < 8.3 \times 10^{-11}/S_B(B_0)$ [15]	
		$S_B(B_0)_{SS} = 0.50$	$\lvert c_V^{\mu e}\rvert < 0.407$
		$S_B(B_0)_{PP} = 0.9$	$\lvert c_A^{\mu e}\rvert < 0.351$
		$S_B(B_0)_{(S\pm P)(S\pm P)} = 0.35$	$\lvert c_{V/A}^{\mu e}\rvert < 0.444$
II	$\mu \to eJ$	Br $< 2.6 \times 10^{-6}$ (90% CL) [16]	$\lvert c_{V/A}^{e\mu}\rvert < 3.64 \times 10^{-7}$
	$\tau \to \mu J$	Br $< 5.7 \times 10^{-3}$ (95% CL) [17]	$\lvert c_{V/A}^{\mu\tau}\rvert < 6.87 \times 10^{-4}$
	$\tau \to eJ$	Br $< 3.2 \times 10^{-3}$ (95% CL) [17]	$\lvert c_{V/A}^{e\tau}\rvert < 5.11 \times 10^{-4}$
III	$\tau \to \mu e \bar{\mu}$	Br $< 2.7 \times 10^{-8}$ (90% CL) [18]	$\sqrt{\lvert c_{V/A}^{\mu\tau}\rvert \lvert c_{V/A}^{e\mu}\rvert} < 0.379 - 0.405$
	$\tau \to \mu e \bar{e}$	Br $< 1.8 \times 10^{-8}$ (90% CL) [18]	$\sqrt{\lvert c_{V/A}^{e\tau}\rvert \lvert c_{V/A}^{\mu e}\rvert} < 0.353 - 0.355$
	$\tau \to \mu \mu \bar{e}$	Br $< 1.7 \times 10^{-8}$ (90% CL) [18]	$\sqrt{\lvert c_{V/A}^{\mu\tau}\rvert \lvert c_{V/A}^{\mu e}\rvert} < 0.346 - 0.349$
	$\tau \to e e \bar{\mu}$	Br $< 1.5 \times 10^{-8}$ (90% CL) [18]	$\sqrt{\lvert c_{V/A}^{e\tau}\rvert \lvert c_{V/A}^{e\mu}\rvert} < 0.346 - 0.347$
IV	$(g-2)_e$	$-(0.88 \pm 0.36) \times 10^{-12}$ [19]	$\lvert C_A^{e\mu}\rvert < 3.21$, $\lvert C_A^{e\tau}\rvert < 0.782$
	$(g-2)_\mu$	$(28.02 \pm 7.37) \times 10^{-10}$ [19]	$\lvert C_V^{\mu\tau}\rvert < 3.07$
V	$\mu \to e\gamma$	Br $< 4.2 \times 10^{-13}$ (90% CL) [20]	$\sqrt{\lvert C_{V/A}^{e\tau}\rvert \lvert C_{V/A}^{\tau\mu}\rvert} < 0.011$
	$\tau \to \mu\gamma$	Br $< 4.4 \times 10^{-8}$ (90% CL) [21]	$\sqrt{\lvert C_{V/A}^{\mu e}\rvert \lvert C_{V/A}^{e\tau}\rvert} < 5.14$
	$\tau \to e\gamma$	Br $< 3.3 \times 10^{-8}$ (90% CL) [21]	$\sqrt{\lvert C_{V/A}^{e\mu}\rvert \lvert C_{V/A}^{\mu\tau}\rvert} < 4.78$

Neutrinos also decay

New data to give more information

3. Flavor Changing Goldstone Boson Interactions

How to obtain flavor changing interactions?

Jin Sun, Xiao-Gang He, PLB811(2020)135881); Jin Sun, Yu Cheng, Xiao-Gang He, JHEP04(2021)141

1. Making the Majoron also play the role of Axion. Just make the lepton number part ofthe PQ charge.

Change the Lepton number in seesaw model to be PQ charge and keepting the quarks and charged lepton as in the previously discussed TypeIII DFSZ model. The Axion current is given by

$$\frac{\partial_\mu J}{2f_J}\left[\overline{\ell_j}\gamma^\mu(c_V^{e\,ji}+c_A^{e\,ji}\gamma_5)\ell_i+\overline{\nu_{Lj}}\gamma^\mu c_L^{\nu\,ji}\nu_{Li}\right]$$

$$c_V^e = -c_A^e = -V^{e\,L}_{LR}V^{e\,L\dagger}_{LR} \text{ and } c_L^\nu = V^\nu_{LL}V^{\nu\,\dagger}_{LL} - V^\nu_{LR}V^{\nu\,\dagger}_{LR}$$

$$\begin{aligned} j^\mu_{a\nu} =& \frac{\bar{v}^2}{N_\alpha}(\bar{\nu}_L V^\nu_{LL}+\bar{\nu}^c_R V^\nu_{RL})\gamma^\mu(V^{\nu\dagger}_{LL}\nu_L+V^{\nu\dagger}_{RL}\nu^c_R) \\ &+\frac{v^2}{N_\alpha}\Big((\bar{\nu}_L V^\nu_{LL}+\bar{\nu}^c_R V^\nu_{RL})X^\nu_L\gamma^\mu(V^{\nu\dagger}_{LL}\nu_L+V^{\nu\dagger}_{RL}\nu^c_R)+(\bar{\nu}^c_L V^{\nu *}_{LR}+\bar{\nu}_R V^{\nu *}_{RR})X^\nu_R\gamma^\mu(V^{\nu T}_{LR}\nu^c_L+V^{\nu T}_{RR}\nu_R)\Big) \end{aligned}$$

This has plavor changing interactions for Axion, but only in lepton sector.

Seesaw scale is the same as PQ symmetry breaking scale, 10^9 GeV.

2. Also make different genrations of quarks, lpetons having different PQ charges.

How does one achieve this?

Model construction: Assuming SM particles have PQ changes

$Q_L^j \;:\; (3,2,1/6)(X_L^{qj})$ $X_L^{uj} = X_L^{dj} = X_L^{qj}$ $U_R^j \;:\; (3,1,2/3)(X_R^{uj}),$ or $D_R^j \;:\; (3,1,-1/3)(X_R^{dj})$

$L_L^j \;:\; (1,2,-1/2)(X_L^{lj}),$ $X_L^{\nu j} = X_L^{ej} = X_L^{lj}$ $E_R^j : (1,1,-1)(X_R^{ej}).$

$X_{L,R}^f$ PQ charge matrix $\mathrm{diag}(X^{f1}_{L,R}, X^{f2}_{L,R}, X^{f3}_{L,R})$

pluse multi Higgs doublets $H_{jk}^{u,d,e,\nu}$ transforming as $(1,2,1/2)(X_L^{q,l\,j} - X_R^{u,d,e,\nu\,k})$

singlets S_{jk} are introduced with $U(1)_G$ charge $-(X_R^{\nu j} + X_R^{\nu k})$

$$L_Y = -\bar{Q}_L^j Y_u^{jk} \tilde{H}_{jk}^u U_R^k - \bar{Q}_L^j Y_d^{jk} H_{jk}^d D_R^k - \bar{L}_L^j Y_e^{jk} H_{jk}^e E_R^k - \bar{L}_L^j Y_\nu^{jk} \tilde{H}_{jk}^\nu \nu_R^k - (1/2)\bar{\nu}_R^{cj} Y_s^{jk} S_{jk} \nu_R^k.$$

$$H_{jk}^a = \begin{pmatrix} h_{jk}^{a+} \\ \frac{1}{\sqrt{2}}(v_{jk}^a + h_{jk}^a + iI_{jk}^a) \end{pmatrix} \qquad S_{jk} = (1/\sqrt{2})(v_{jk}^s + R_{jk}^s + iI_{jk}^s).$$

To solve strong CP problem, one must have $Tr(X_R^u - X_L^u) + Tr(X_R^d - X_L^d) \neq 0$

vevs of H_i break both SM and PQ symmetries, and S_i break PQ symmetry

How to identify Axion (or more general the Goldstone boson)?

The vevs of H_i breaks SM

The vector z "eaten" by Z boson, in the basis $\vec{I} = (I^u_{jk}, I^d_{jk}, I^e_{jk}, I^\nu_{jk}, I^s_{jk})$, is given by

$$\vec{z} = (v^u_{jk},\ v^d_{jk},\ v^e_{jk},\ v^\nu_{jk},\ 0)\ ,$$

$U(1)_G$ broken generator vector A is given by

$$\vec{A} = \left(-(X^{uj}_L - X^{uk}_R)v^u_{jk},\ (X^{dj}_L - X^{dk}_R)v^d_{jk},\ (X^{ej}_L - X^{ek}_R)v^e_{jk},\ -(X^{\nu j}_L - X^{\nu k}_R)v^\nu_{jk},\ -(X^{\nu j}_R + X^{\nu k}_R)v^s_{jk}\right)$$

But it is not yet the Axion, one needs to find a linear combination of z and A

which is orthorgonal to z

$$\vec{a} = \frac{1}{N_\alpha}(\bar{v}^2\vec{z} - v^2\vec{A})\ ,$$

N_α is a normalization constant to ensure $\vec{a}\cdot\vec{a}^T = 1$, and

$$v^2 = \vec{z}\cdot\vec{z}^T = (v^u_{jk})^2 + (v^d_{jk})^2 + (v^e_{jk})^2 + (v^\nu_{jk})^2\ ,$$
$$\bar{v}^2 = \vec{A}\cdot\vec{z}^T = -(X^{uj}_L - X^{uk}_R)(v^u_{jk})^2 + (X^{dj}_L - X^{dk}_R)(v^d_{jk})^2 + (X^{ej}_L - X^{ek}_R)(v^e_{jk})^2 - (X^{\nu j}_L - X^{\nu k}_R)(v^\nu_{jk})^2\ .$$

the physical GB, $a = \vec{a}\cdot\vec{I}^T$, in terms of I^a_{jk} , we have

$$a = \frac{1}{N_\alpha}\left[\left((X^{pl}_L - X^{pm}_R) - (X^{qj}_L - X^{qk}_R)\right)(v^p_{lm})^2 v^q_{jk}\, sign(q) I^q_{jk} + (X^{\nu j}_R + X^{\nu k}_R)(v^p_{lm})^2 v^s_{jk} I^s_{jk}\right]\ .$$

sign(q) takes "−" for $q = u,\ \nu$ and "+" for $q = d, e$.

$$L_Y = -\bar{U}^j_L M^{jk}_u \left[1 + ia\frac{v^2}{N_\alpha}\left(-\frac{\bar{v}^2}{v^2} - (X^{uj}_L - X^{uk}_R)\right)\right] U^k_R - \bar{D}^j_L M^{jk}_d \left[1 + ia\frac{v^2}{N_\alpha}\left(\frac{\bar{v}^2}{v^2} - (X^{dj}_L - X^{dk}_R)\right)\right] D^k_R$$
$$-\bar{E}^j_L M^{jk}_e \left[1 + ia\frac{v^2}{N_\alpha}\left(\frac{\bar{v}^2}{v^2} - (X^{ej}_L - X^{ek}_R)\right)\right] E^k_R - \bar{\nu}^j_L M^{jk}_D \left[1 + ia\frac{v^2}{N_\alpha}\left(-\frac{\bar{v}^2}{v^2} - (X^{\nu j}_L - X^{\nu k}_R)\right)\right] \nu^k_R$$
$$-\frac{1}{2}\bar{\nu}^{cj}_R M^{jk}_R \left(1 + ia\frac{v^2}{N_\alpha}(X^{\nu j}_R + X^{\nu k}_R)\right) \nu^k_R + \text{H.c.} ,$$

$$M^{jk}_u = \frac{Y^{jk}_u v^u_{jk}}{\sqrt{2}} , \qquad M^{jk}_d = \frac{Y^{jk}_d v^d_{jk}}{\sqrt{2}} , \qquad M^{jk}_e = \frac{Y^{jk}_e v^e_{jk}}{\sqrt{2}} ,$$
$$M^{jk}_\nu = \begin{pmatrix} 0 & M_D \\ M^T_D & M_R \end{pmatrix}^{jk} , \text{ with } M^{jk}_D = \frac{Y^{jk}_\nu v^\nu_{jk}}{\sqrt{2}} , \quad M^{jk}_R = \frac{Y^{jk}_s v^s_{jk}}{\sqrt{2}} .$$

$$j^\mu_{ac} = -\frac{\bar{v}^2}{2N_\alpha}(\bar{U}^m\gamma^\mu\gamma_5 U^m - \bar{D}^m\gamma^\mu\gamma_5 D^m - \bar{E}^m\gamma^\mu\gamma_5 E^m) + \frac{v^2}{N_\alpha}(\bar{U}^m_L V^u_L X^u_L V^{u\dagger}_L \gamma^\mu U^m_L + \bar{U}^m_R V^u_R X^u_R V^{u\dagger}_R \gamma^\mu U^m_R)$$
$$+\frac{v^2}{N_\alpha}(\bar{D}^m_L V^d_L X^d_L V^{d\dagger}_L \gamma^\mu D^m_L + \bar{D}^m_R V^d_R X^d_R V^{d\dagger}_R \gamma^\mu D^m_R) + \frac{v^2}{N_\alpha}(\bar{E}^m_L V^e_L X^e_L V^{e\dagger}_L \gamma^\mu E^m_L + \bar{E}^m_R V^e_R X^e_R V^{e\dagger}_R \gamma^\mu E^m_R) .$$

Since X are not unit matrix, in general flavor changing interactions are generated for Axion (Goldstone bosons)

$$L_{ag} = a\frac{g_3^2}{16\pi^2}N(X)T(q)\tilde{G}^{a\mu\nu}G^a_{\mu\nu} = \frac{\alpha_s}{8\pi}\frac{a}{f_a}\tilde{G}^{a\mu\nu}G^a_{\mu\nu}\,,$$

$$L_{a\gamma} = a\frac{e^2}{16\pi^2}\tilde{E}(X)\tilde{F}^{\mu\nu}F_{\mu\nu}$$

$$Tr(T^aT^b) = T(q)\delta^{ab} = \delta^{ab}/2. \quad N(X) = N^u(X) + N^d(X)$$

$$N(X) = (v^2/N_\alpha)Tr(X^u_R - X^u_L + X^d_R - X^d_L)$$

$$\tilde{E}(X) = E^u(X) + E^d(X) + E^e(X)$$

$$N^q_c = 3 \text{ and } N^e_c = 1$$

$$E^u(X) = Q_u^2 N_c^q N^u(X)\,, \quad E^d(X) = Q_d^2 N_c^q N^d(X)\,,$$
$$E^e(X) = Q_e^2 N_c^e\left(-N_G\frac{\bar{v}^2}{N_\alpha} + \frac{v^2}{N_\alpha}Tr(X^e_R - X^e_L)\right)\,.$$

Can look for flavor violating axion interactions

Axion with spontaneous CP violtion, no new CP violating interaction for Axion

$$\vec{z} = (v^u_{jk}e^{i\theta^u_{jk}},\ v^d_{jk}e^{i\theta^d_{jk}},\ v^e_{jk}e^{i\theta^e_{jk}},\ v^\nu_{jk}e^{i\theta^\nu_{jk}},\ 0)\,,$$
$$\vec{A} = (-(X^{uj}_L - X^{uk}_R)v^u_{jk}e^{i\theta^u_{jk}},\ (X^{dj}_L - X^{dk}_R)v^d_{jk}e^{i\theta^d_{jk}},\ (X^{ej}_L - X^{ek}_R)v^e_{jk}e^{i\theta^e_{jk}},$$
$$-(X^{\nu j}_L - X^{\nu k}_R)v^\nu_{jk}e^{i\theta^\nu_{jk}},\ -(X^{\nu j}_R + X^{\nu k}_R)v^s_{jk}e^{i\theta^s_{jk}})\,.$$

$$a = \frac{1}{N_\alpha}Im\left[\left((X^{pl}_L - X^{pm}_R) - (X^{qj}_L - X^{qk}_R)\right)(v^p_{lm})^2 v^q_{jk}e^{i\theta^q_{jk}}sign(q)(h^q_{ij} + iI^q_{ij}) + (v^p_{lm})^2(X^{\nu j}_R + X^{\nu k}_R)v^s_{jk}e^{\theta^s_{jk}}(h^s_{jk} + iI^s_{jk})\right]$$

j_a is the same as before without spontaneous CP violation

Type II Seasaw: One Triplet scalr (1,3, 1)(-2) having a small vev to give neutrino mass and break lepton number by two units.

If impose lepton number symmetry to be respectied, there is a Majoron.

To make it invisible, needs a singlet having a large vev so that the other light degrees of freedom will couple weakly with SM particle.

To have flavor changing interactions, need at least two triplets.

Left-Right symmetric model: $SU(3)_C x SU(2)_L x SU(2)_R x U(1)_{B-L}$. Again to have flavor changing Majoron, two copies of triplets are needed. Also two bi-doublets are need to have quark and charged letpon masses to be able to fit data.

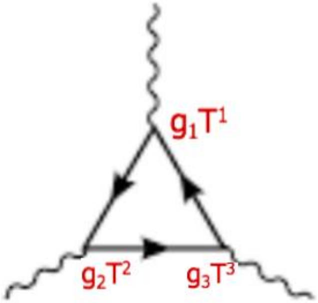
4. Would-be Goldstone bosons and Dark Photons

What if a global symmetry is gauged?

Axion models have problem to be gauged because it rquires to be anomalous of QCD.

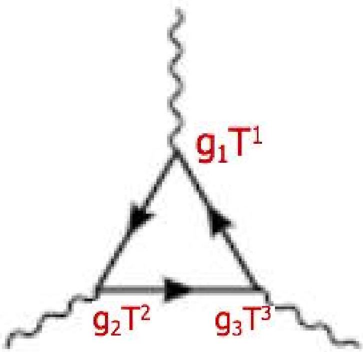
Gauging the a particular symmetry, one should have gauge anomaly to be zero when sum over all contributions.

Normalizing the contributions by right handed $1+\gamma_5$ chiral fermion in the loop to be positive proportional to the couplings, then left handed $1-\gamma_5$ chiral fermion in the loop will be negative. The total sign also depends on the couplings $g_1T^1\, g_2T^2\, g_3T^3$.



The cancellation can happen by summing up left and right handed Fermion with appropriate couplings contributions. If vector fermion, no anomalies generated.

Gauge anomaly cancellation in the SM



The standard model of strong and electroweak interaction has gauge group

$SU(3)_C \times SU(2)_L \times U(1)_Y$ with gauge bosons

8 $SU(3)_C$ Gluons : $G^\mu = \frac{\lambda^a}{2} G^\mu_a, \;\; Tr(\frac{\lambda^a}{2}\frac{\lambda^b}{2}) = \frac{\delta^{ab}}{2}$.

3 $SU(2)_L$ W-bosons : $W^\mu = \frac{\sigma^i}{2} W^\mu_i, \;\; Tr(\frac{\sigma^i}{2}\frac{\sigma^j}{2}) = \frac{\delta^{ij}}{2}$.

1 $U(1)_Y$ B boson : B^μ

The building blocks of fermions are chiral fields $f_{L,R} = \frac{1 \mp \gamma_5}{2} f$

The SM fermions are leptons L_L, E_R and quarks Q_L, U_R and D_R

$L_L = (\nu_L, e_L : (1,2)(-1/2)^T \;, \;\; e_R : (1,1)(-1) \;,$

$Q_L = (u_L, d_L)^T : (3,2)(1/6) \;, \;\; u_R : (3,1)(2/3) \;, \;\; d_R : (3,1)(-1/3) \;.$

Type of anomalies:

GGG (3 grkuons): automatically zero, because under $SU(3)_C$ all fermions are vector like.

WWW: also automatically zero, because $T_i = \sigma_i$ $Tr((\sigma_i\sigma_j+\sigma_j\sigma_i)\sigma_k)=0$

GGW, WWG, BBG, BBW, GWB all are zero due to trace of one single T_i is zero.

Nonzero ones: GGB, WWB, BBB, and ggB for individual fermion in the loop

Two gravitation gg and a B

One generation of SM fermion contributions to gauge anomalies

	u_R	d_R	u_L	d_L	e_R	ν_L	e_L	sum
GGB	2/3	-1/3	-(1/6)	-(1/6)	0	0	0	0
WWB	0	0	-3(1/6)	-3(1/6)	0	-(-1/2)	-(-1/2).	0
BBB	$3(2/3)^3$.	$3(-1/3)^3$.	$-3(1/6)^3$.	$-3(1/6)^3$.	$(-1)^3$	$-(-1/2)^3$	$-(-1/2)^3$	0
ggB	2/3	-1/3	-(1/6)	-(1/6)	-1	-(-1/2)	-(-1/2)	0

All anomalies are automatically cancelled!

One of the reasons for having two Higgs doublets H_1 (1,2)(-1/2) and H_2(1,2)(1/2)
Because Higgsino is a chiral fermion, it produce gauge anomalies
WWB -1/2 + 1/2= 0; BBB $(-1/2)^3 + (1/2)^3 = 0$; ggB (-1/2) + (1/2) = 0!

Another example: $SU(3)_C x SU(2)_L x U(1)_Y x U(1)_{\mu-\tau}$

(PRD 43 (1991) R22; PRD 44(1991) 2118)

$U(1)_{\mu-\tau}$ charges: 0 for u_R, d_R, u_L d_L and e_R e_L,
+1 for μ_R, $\nu_{\mu L}$, μ_L, -1 for τ_R $\nu_{\tau L}$, τ_L

New anomalies (indicate $U(1)_{\mu-\tau}$ gauge boson as Z')

	μ_R	$\nu_{\mu L}$	μ_L	τ_R	$\nu_{\tau L}$,	τ_L	sum
WWZ'	1	-1	-1	-1	-(-1)	-(-1)	0
BBZ'	$(-1)^2$x1	$-(-1/2)^2$x(1)	$-(-1/2)^2$x(1).	$(-1)^2$x(-1)	$-(-1/2)^2$x(-1)	$-(-1/2)^2$x(-1)	0
Z'Z'B	$(1)^2$x(-1)	$-1^2(-1/2)$	$-1^2(-1/2)$	$(-1)^2$x(-1)	$-(-1)^2(-1/2)$	$-(-1)^2(-1/2)$	0
Z'Z'Z'	1^3	-1^3	-1^3	$(-1)^3$.	$-(-1)^3$	$-(-1)^3$.	0
ggZ'	1	-1	-1	-1	-(-1)	-(-1)	0

Gauge anomaly free. The simplest model with a new Z' model!

New $U(1)_X$ gauge gropu can mixing with $U(1)_Y$
New consequences: Kinetic mixing of U(1) gauge gropus. Dark Photon...

Gauge anomaly cancellation and charge quNtization

$Q_L(u_L,d_L)^T$: Y_Q, u_R: y_u, d_R: y_d, $L_L(\nu_L, e_L)^T$: Y_L, e_R: y_e.

	u_R	d_R	u_L	d_L	e_R	ν_L	e_L	sum
GGB	y_u	y_d	Y_Q	Y_Q	0	0	0	0
WWB	0	0	$-3Y_Q$	$-3Y_Q$	0	$-Y_L$	$-Y_L$	0
BBB	$3(y_u)^3$.	$3(y_d)^3$.	$-3(Y_Q)^3$.	$-3(Y_Q)^3$.	$(y_e)^3$	$-(Y_L)^3$	$-(Y_L)^3$	0
ggB	y_u	y_d	$-Y_Q$	$-Y_Q$	Y_e	$-Y_L$	$-Y_L$	0

H (h+, (v + R + i I)/qart(2)): (1/2)

L = \bar u_L d_R H -> $y_d - y_u + 1/2 = 0$ conserve electric charge

$Y_Q=1/6$, $y_u=2/3$, $y_d=-1/3$, $Y_L = -1/2$, $y_e = -1$ quantized the way needed for SM!

C-Q. Geng, R. Marshak, PRD39(1989) 693; R. Foot, G. Joshi, H. Lew, R. Volkas, MPLA 5(1990)2721).

Enlighten Dark Photon by Kinetic Mixing

A photon and a pure dark photon

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$$

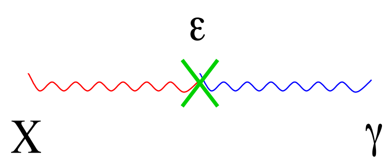
A theory of $U(1)_{em} x U(1)_X$ gauge group

$L = - ¼ F_{\mu\nu}F^{\mu\nu} – ¼ X_{\mu\nu}X^{\mu\nu} + A_\mu j^\mu_{em} + X_\mu j^\mu_X$

A is the usual photon field and X is a new gauge field X and $j^\mu_{em,X}$ currents
X may have or not have a finite mass $m^2_A X^\mu X_\mu/2$

If j^μ_X does not involve with SM particle, X is a photon like particle which cannot be probed using laboratory probes – a **Dark Photon**

$$\epsilon X_{\mu\nu} F^{\mu\nu}$$



Possible to add the following renormalizable and gauge invariant term.

This kinetic mixing term mixes photon and Dark Photon making dark photon to interact with SM particle, Dark Photon enlightened!

Holdom 1986, Foot and He 1991,....

Work with $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$

Kinetic mixing can happen between $U(1)_Y$ and $U(1)_X$

$$\mathcal{L} = -\frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{\sigma}{2}X_{\mu\nu}Y^{\mu\nu} - \frac{1}{4}Y_{\mu\nu}Y^{\mu\nu} + j^{\mu}_{Y}Y_{\mu} + j^{\mu}_{X}X_{\mu}$$

Need to re-write in the canonical form to identify physics gauge bosons. (mixing term removed!)

This may generate dark photon to interact with SM J^{μ}_{Y}

How to remove the mixing term?

M He, X-G He, G. Li, arXiv: 1807.00921

Not unique! Examples

$$Case\ \ a): \quad \mathcal{L}_a = -\frac{1}{4}\hat{X}_{\mu\nu}\hat{X}^{\mu\nu} - \frac{1}{4}\hat{Y}_{\mu\nu}\hat{Y}^{\mu\nu} + j^\mu_Y \frac{1}{\sqrt{1-\sigma^2}}\hat{Y}_\mu + j^\mu_X(\hat{X}_\mu - \frac{\sigma}{\sqrt{1-\sigma^2}}\hat{Y}_\mu)\ ,$$

$$\hat{Y}_\mu = \sqrt{1-\sigma^2}Y_\mu\ , \quad \hat{X}_\mu = \sigma Y_\mu + X_\mu\ ,$$

$$Case\ \ b): \quad \mathcal{L}_b = -\frac{1}{4}\hat{X}'_{\mu\nu}\hat{X}'^{\mu\nu} - \frac{1}{4}\hat{Y}'_{\mu\nu}\hat{Y}'^{\mu\nu} + j^\mu_Y(\hat{Y}'_\mu - \frac{\sigma}{\sqrt{1-\sigma^2}}\hat{X}'_\mu) + j^\mu_X \frac{1}{\sqrt{1-\sigma^2}}\hat{X}'_\mu\ ,$$

$$\hat{Y}'_\mu = Y_\mu + \sigma X_\mu\ , \quad \hat{X}'_\mu = \sqrt{1-\sigma^2}X_\mu\ .$$

Case a), Redefined X does not couple to $j^\mu{}_Y$ is still “dark”

Case b), Redefined X does not couple to $j^\mu{}_Y$ is not dark any more, but Y does not couple to $j^\mu{}_X$.

Which one is the correct one to choose?

Work with SM photon and dark photon

$$Y_\mu = c_W A_\mu - s_W Z_\mu \ , \ \ W^3_\mu = s_W A_\mu + c_W Z_\mu \ ,$$

$$\mathcal{L}_0 = -\frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{1}{4}A_{\mu\nu}A^{\mu\nu} - \frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} - \frac{1}{2}\sigma c_W X_{\mu\nu}A^{\mu\nu} + \frac{1}{2}\sigma s_W X_{\mu\nu}Z^{\mu\nu}$$
$$+ j^\mu_{em}A_\mu + j^\mu_Z Z_\mu + j^\mu_X X_\mu + \frac{1}{2}m_Z^2 Z_\mu Z^\mu \ ,$$

Write the above into canonical form requires

$$Case \ \ a): \quad \begin{pmatrix} A \\ Z \\ X \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{1-\sigma^2 c_W^2}} & \frac{-\sigma^2 s_W c_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2 c_W^2}} & 0 \\ 0 & \frac{\sqrt{1-\sigma^2 c_W^2}}{\sqrt{1-\sigma^2}} & 0 \\ \frac{-\sigma c_W}{\sqrt{1-\sigma^2 c_W^2}} & \frac{\sigma s_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2 c_W^2}} & 1 \end{pmatrix} \begin{pmatrix} \tilde{A} \\ \tilde{Z} \\ \tilde{X} \end{pmatrix} ,$$

$$Case \ \ b): \quad \begin{pmatrix} A \\ Z \\ X \end{pmatrix} = \begin{pmatrix} 1 & \frac{-\sigma^2 s_W c_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2 c_W^2}} & \frac{-\sigma c_W}{\sqrt{1-\sigma^2 c_W^2}} \\ 0 & \frac{\sqrt{1-\sigma^2 c_W^2}}{\sqrt{1-\sigma^2}} & 0 \\ 0 & \frac{\sigma s_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2 c_W^2}} & \frac{1}{\sqrt{1-\sigma^2 c_W^2}} \end{pmatrix} \begin{pmatrix} \tilde{A}' \\ \tilde{Z}' \\ \tilde{X}' \end{pmatrix} ,$$

$$\mathcal{L}_a = -\frac{1}{4}\tilde{X}_{\mu\nu}\tilde{X}^{\mu\nu} - \frac{1}{4}\tilde{A}_{\mu\nu}\tilde{A}^{\mu\nu} - \frac{1}{4}\tilde{Z}_{\mu\nu}\tilde{Z}^{\mu\nu} + \frac{1}{2}m_Z^2\frac{1-\sigma^2 c_W^2}{1-\sigma^2}\tilde{Z}_\mu\tilde{Z}^\mu$$
$$+j^\mu_{em}\left(\frac{1}{\sqrt{1-\sigma^2 c_W^2}}\tilde{A}_\mu - \frac{\sigma^2 s_W c_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2 c_W^2}}\tilde{Z}_\mu\right) + j^\mu_Z\left(\frac{\sqrt{1-\sigma^2 c_W^2}}{\sqrt{1-\sigma^2}}\tilde{Z}_\mu\right)$$
$$+j^\mu_X\left(\frac{-\sigma c_W}{\sqrt{1-\sigma^2 c_W^2}}\tilde{A}_\mu + \frac{\sigma s_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2 c_W^2}}\tilde{Z}_\mu + \tilde{X}_\mu\right),$$
$$\mathcal{L}_b = -\frac{1}{4}\tilde{X}'_{\mu\nu}\tilde{X}'^{\mu\nu} - \frac{1}{4}\tilde{A}'_{\mu\nu}\tilde{A}'^{\mu\nu} - \frac{1}{4}\tilde{Z}'_{\mu\nu}\tilde{Z}'^{\mu\nu} + \frac{1}{2}m_Z^2\frac{1-\sigma^2 c_W^2}{1-\sigma^2}\tilde{Z}'_\mu\tilde{Z}'^\mu$$
$$+j^\mu_{em}\left(\tilde{A}'_\mu - \frac{\sigma^2 s_W c_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2 c_W^2}}\tilde{Z}'_\mu - \frac{\sigma c_W}{\sqrt{1-\sigma^2 c_W^2}}\tilde{X}'_\mu\right)$$
$$+j^\mu_Z\left(\frac{\sqrt{1-\sigma^2 c_W^2}}{\sqrt{1-\sigma^2}}\tilde{Z}'_\mu\right) + j^\mu_X\left(\frac{\sigma s_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2 c_W^2}}\tilde{Z}'_\mu + \frac{1}{\sqrt{1-\sigma^2 c_W^2}}\tilde{X}'_\mu\right).$$

Which one to choose?

If dark photon is massive, easy to identify

X has a mass to start with: $(1/2)m_X^2 X^\mu X_\mu$

Example: get a mass from the vev of a scalar S with $U(1)_X$ charge but no SM charges.

$$Case\ \ a): \quad \frac{1}{2}m_X^2\left(\frac{-\sigma c_W}{\sqrt{1-\sigma^2 c_W^2}}\tilde{A}_\mu + \frac{\sigma s_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2 c_W^2}}\tilde{Z}_\mu + \tilde{X}_\mu\right)^2 ,$$

$$Case\ \ b): \quad \frac{1}{2}m_X^2\left(\frac{\sigma s_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2 c_W^2}}\tilde{Z}'_\mu + \frac{1}{\sqrt{1-\sigma^2 c_W^2}}\tilde{X}'_\mu\right)^2 .$$

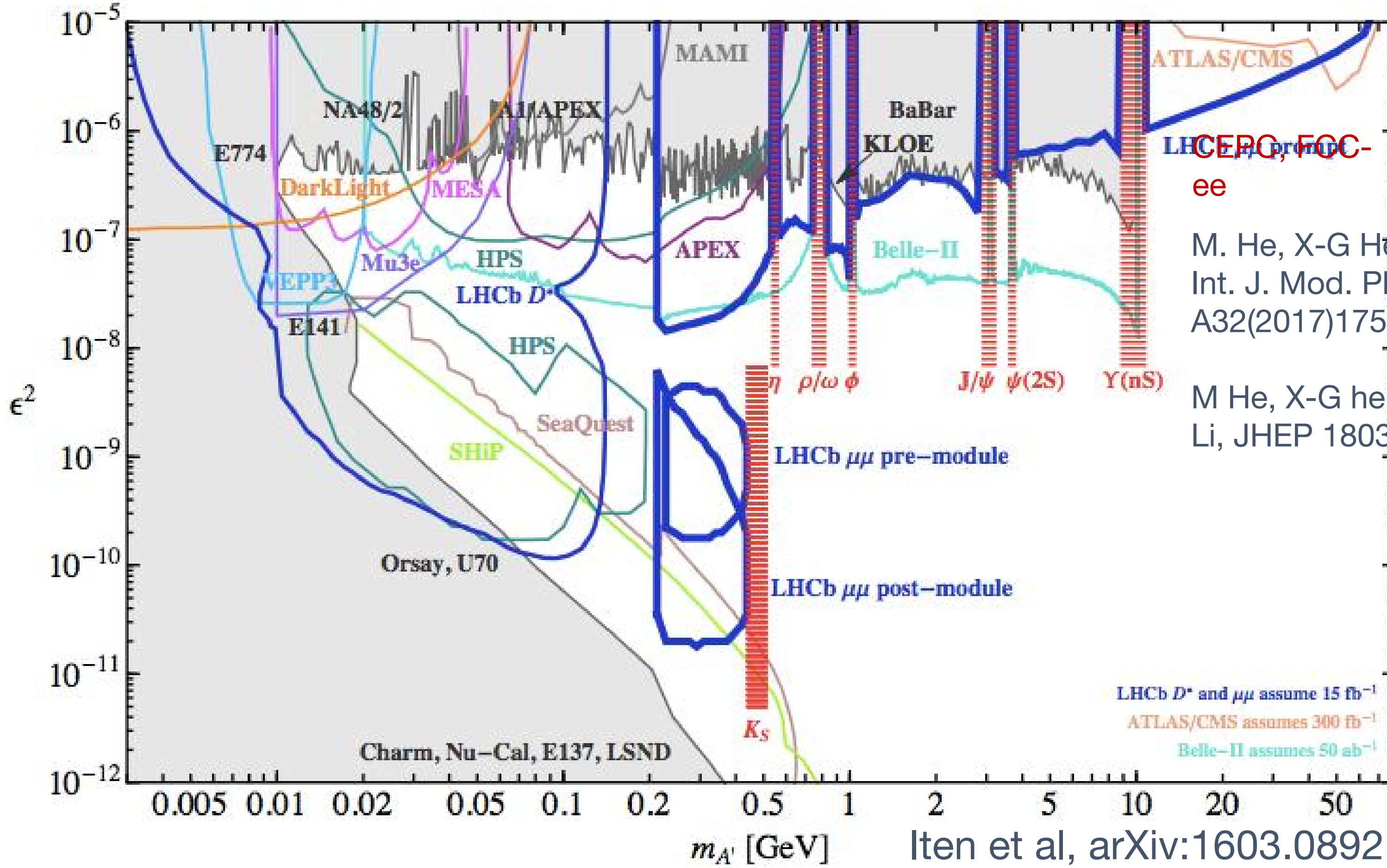
Case b) is more convenient to use, because tilde-A' already the physical massless photon, tilde-Z' and tilde X' mixing with each other

$$\begin{pmatrix} \frac{m_Z^2(1-\sigma^2 c_W^2)^2+m_X^2\sigma^2 s_W^2}{(1-\sigma^2)(1-\sigma^2 c_W^2)} & \frac{m_X^2 \sigma s_W}{\sqrt{1-\sigma^2}(1-\sigma^2 c_W^2)} \\ \frac{m_X^2 \sigma s_W}{\sqrt{1-\sigma^2}(1-\sigma^2 c_W^2)} & \frac{m_X^2}{1-\sigma^2 c_W^2} \end{pmatrix} , \qquad \begin{pmatrix} Z^m \\ X^m \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \tilde{Z}' \\ \tilde{X}' \end{pmatrix}$$

$$\tan(2\theta) = \frac{2m_X^2 \sigma s_W \sqrt{1-\sigma^2}}{m_Z^2(1-\sigma^2 c_W^2)^2 - m_X^2[1-\sigma^2(1+s_W^2)]} .$$

Inconvenient to work with a) although finally one will reach the same interactions

Summary of constraints on the dark photon mass and coupling

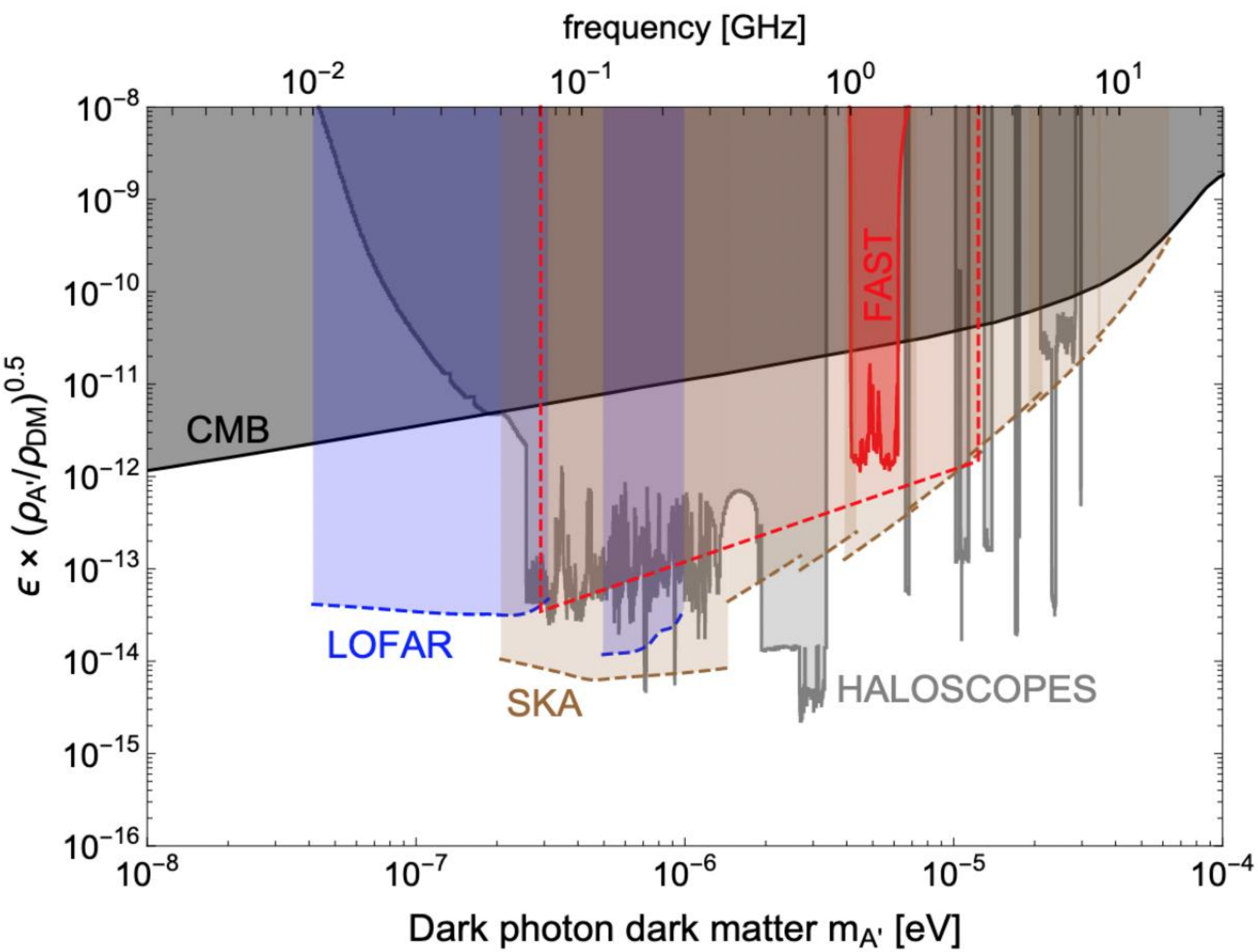


CEPC, FCC-ee

M. He, X-G He, C-K Huang, Int. J. Mod. Phys. A32(2017)1750138

M He, X-G he, C-K Huang, G Li, JHEP 1803(2018)139

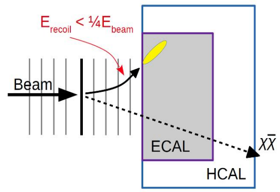
Iten et al, arXiv:1603.0892



frequency [GHz]
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
10^{-16}
$\epsilon \times (\rho_{A'}/\rho_{DM})^{0.5}$
FAST
CMB
LOFAR
SKA
HALOSCOPES
10^{-8}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
Dark photon dark matter $m_{A'}$ [eV]

“暗光计划”(Dark SHINE) — New Initiative for Dark Photon search at SHINE facility

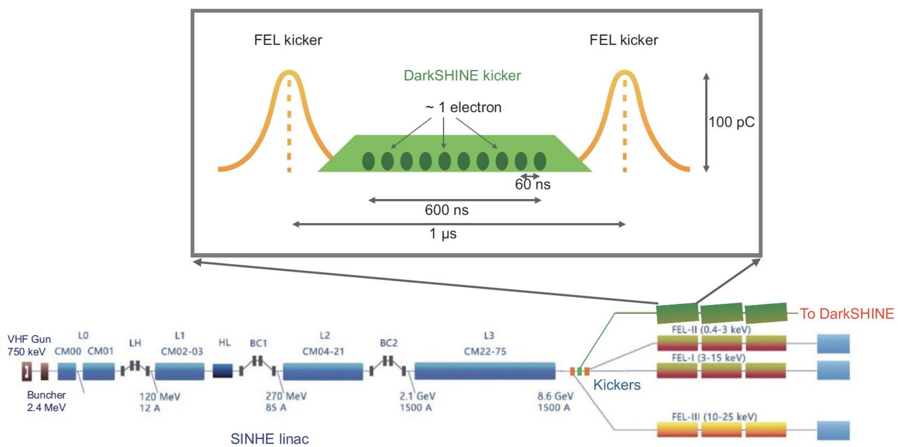
Innovation of Dark SHINE



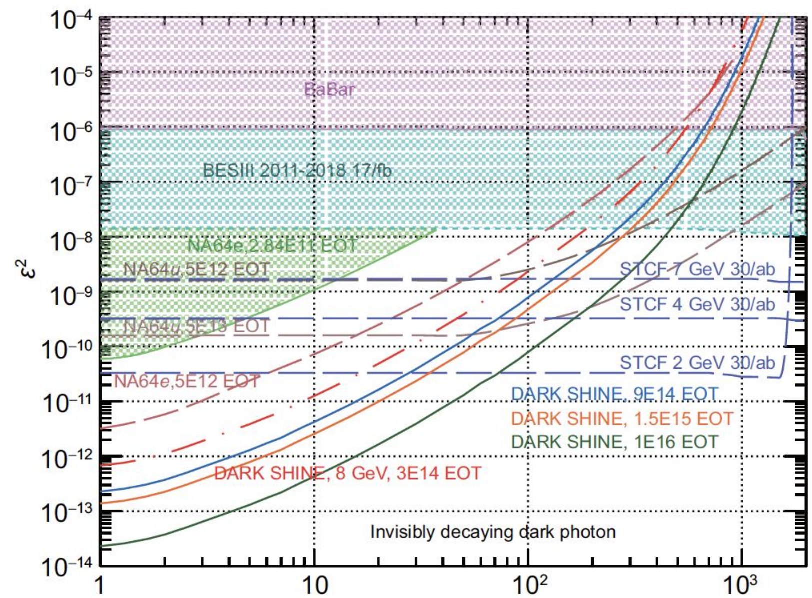
- High rate single electron beam
- “Momentum loss” method to use also angular info to enhance search sensitivity
- Being the first experiment in China of this type
- Explore the fundamental science potential of SHINE facility

- SHINE Under Construction：2018 – 2026
- High rate single electron Beam：8 GeV，1 MHz，~ 3x10^{14} EOT/yr
- SHINE Facility：Provide the desired beam line

SHINE LINAC and Dark SHINE Kicker illustration



Dark SHINE Expected 90% C.L. exclusion limits on ε^2 as a function of the dark photon mass



Sci. China-Phys. Mech. Astron., 66(1): 211062 (2023)

Non-abellian and Abellian gauge kinetic mixing

CP violating kinetic mixing allowed?

J. Cline and A. Frey, arXiv: 1408.0233; G. Barello abd s. Chang, PRD94(2016)055018C. Arguelles, X-G He, G. Ovanesyan, T Peng, M Ramsey-Musolf, PLB770(2017)101; K Fuyuto, X-G He, G. Li, M Ramsey-Musolf Phys. Rev.D101 (2020) 075016

For Abelian kinetic mixing

$Y^{\mu\nu} X_{\mu\nu},$ CP conserving

$Y^{\mu\nu} \tilde{X}_{\mu\nu}$, with $\tilde{X}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta} X^{\alpha\beta}$, CP violating

But

$$Y^{\mu\nu} \tilde{X}_{\mu\nu} = -\epsilon_{\mu\nu\alpha\beta}\partial^\alpha (Y^{\mu\nu} X^\beta)$$

It is a total derivative, can be dropped off. No physical effects.

For Non-Abelian kinetic mixing

$$\mathrm{Tr}(W_{\mu\nu}\Sigma)\tilde{X}^{\mu\nu}/\Lambda$$

$$\Sigma = \frac{1}{2}\begin{pmatrix} \Sigma^0 & \sqrt{2}\Sigma^+ \\ \sqrt{2}\Sigma^- & -\Sigma^0 \end{pmatrix}$$

$$(x_0 + \Sigma^0)\tilde{X}^{\mu\nu} W^+_\mu W^-_\nu$$

x_0 vev of Σ_0

Allowed! There are physical effects.

A renormalizable model

Yu Cheng, X-G He, M. Ramsey-Musolf, Jin Sun, PRD 105(2022)095010, arXiv:2104.11563

Generate

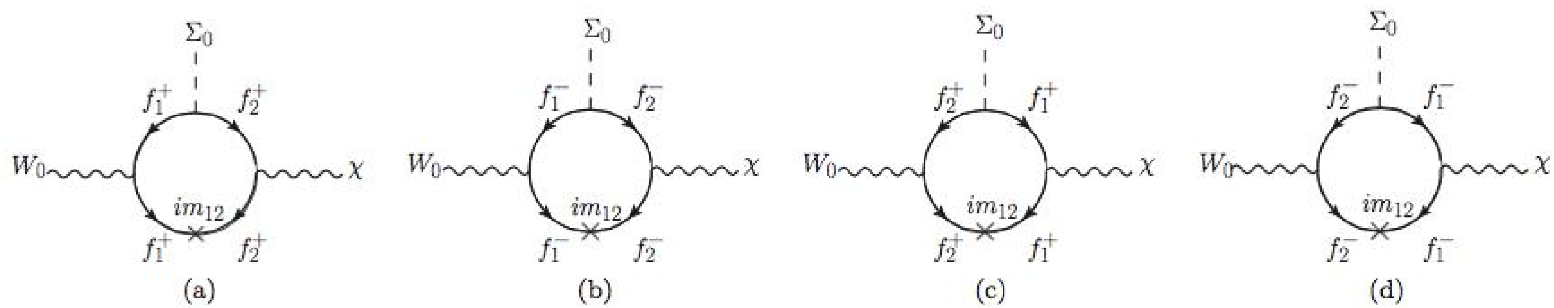
$$\mathcal{L}^{(d=5)} = -\frac{\beta}{\Lambda}\mathrm{Tr}\left[W_{\mu\nu}\Sigma\right] X^{\mu\nu} - \frac{\tilde{\beta}}{\Lambda}\mathrm{Tr}\left[W_{\mu\nu}\Sigma\right] \tilde{X}^{\mu\nu}$$

by loop to make the model renormalizable!

Type-III Seesaw come to be a natural extension

Right-handed neutrino f in the loop, non-trivial SU(2) representation needed, type-III seesaw,

f = (3,0) (x) -- new need x non-zero to connect X!



New particles: Σ:(3,0)(0)

$$\Sigma = \frac{\tau^a}{2}\Sigma^a = \frac{1}{2}\begin{pmatrix} \Sigma^3 & \sqrt{2}\Sigma^+ \\ \sqrt{2}\Sigma^- & -\Sigma^3 \end{pmatrix}$$

f: (3,0)(x)

$$f_R = \frac{1}{2}\sigma^a f_R^a = \frac{1}{2}\begin{pmatrix} f_R^0 & \sqrt{2}f_R^+ \\ \sqrt{2}f_R^- & -f_R^0 \end{pmatrix} \quad , \quad f_L = f_R^c = \frac{1}{2}\sigma^a f_L^a = \frac{1}{2}\begin{pmatrix} f_L^0 = (f_R^0)^c & \sqrt{2}f_L^+ = \sqrt{2}(f_R^-)^c \\ \sqrt{2}f_L^- = \sqrt{2}(f_R^+)^c & -f_L^0 = -(f_R^0)^c \end{pmatrix}$$

Anomaly cancellation: f_1 (3,0)(x_f), f_2: (3,0)($-x_f$), f3: (3,0)(0)

Also for generate a non-zero kinetic mixing.

S_X: (1,0) ($-2x_f$)

Seesaw neutrino mass $< S_X >= v_s/\sqrt{2}$ generate a dark photon mass $m_X^2 = x_f^2 g_X^2 v_s^2$.

Need two new Higgs doublet

$$L_m = -\frac{1}{2}(\bar{\nu}_L, \bar{\nu}_R^c)\begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}(\nu_L^c, \nu_R) - (\bar{E}_L, \bar{f}_L)\begin{pmatrix} m_e & \sqrt{2}M_D \\ 0 & M_R \end{pmatrix}(E_L, f_R),$$

H_1' : $(1,2)(-1/2, -x_f)$, H_2' : $(1,2)(-1/2, x_f)$

$-\bar{L}_L Y_{fL1} H_1' f_{R\,1} - \bar{L}_L Y_{fL2} H_2' f_{R\,2}$

$$M_D = \begin{pmatrix} \frac{Y_{fL11}v_1'}{\sqrt{2}} & \frac{Y_{fL12}v_2'}{\sqrt{2}} & \frac{Y_{fL13}v}{\sqrt{2}} \\ \frac{Y_{fL21}v_1'}{\sqrt{2}} & \frac{Y_{fL22}v_2'}{\sqrt{2}} & \frac{Y_{fL23}v}{\sqrt{2}} \\ \frac{Y_{fL31}v_1'}{\sqrt{2}} & \frac{Y_{fL32}v_2'}{\sqrt{2}} & \frac{Y_{fL33}v}{\sqrt{2}} \end{pmatrix}, \quad M_R = \begin{pmatrix} \frac{Y_{fs1}v_s}{\sqrt{2}} & m_{12} & 0 \\ m_{12} & \frac{Y_{fs2}v_s}{\sqrt{2}} & 0 \\ 0 & 0 & m_{33} \end{pmatrix},$$

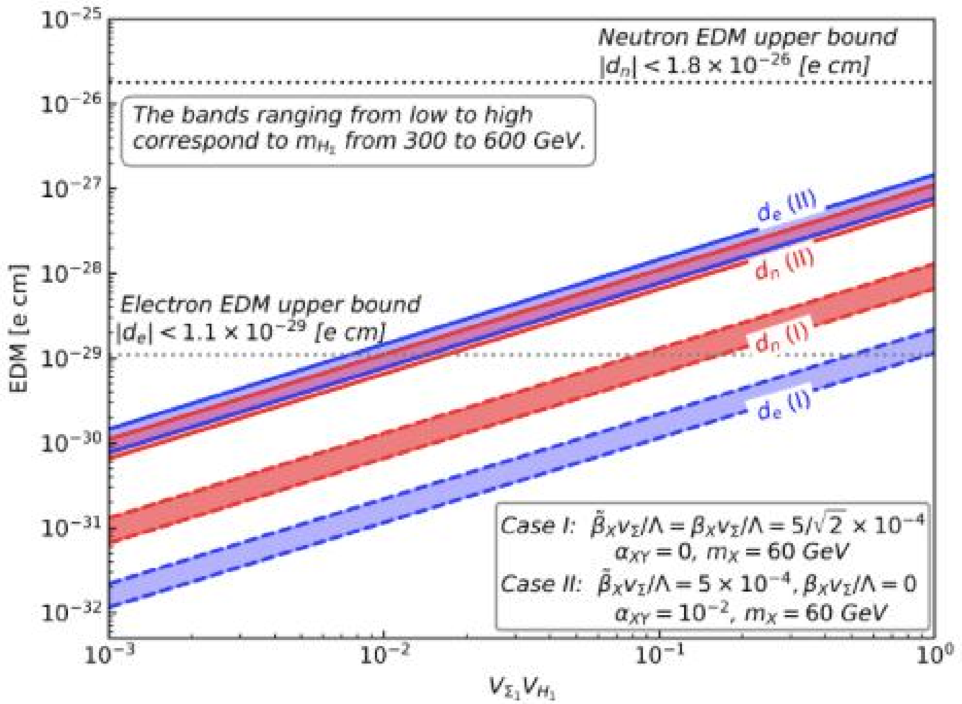


FIG. 3: The ranges for d_n and d_e for *Cases I* and *II*. The 90% C.L. upper limits for electron and neutron EDMs are from Ref. [17] and Ref. [18], respectively.

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Thanks

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