Quantum interferometry and axion haloscope

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The argument of existing QCD axion is very strong.

- A classical field configuration of QCD vacuum: $A_\mu = i/gU\partial_\mu U^\dagger$

have a winding number n

The vacuum cannot be smoothly deformed into others with a different winding number without passing energy barriers

$$n = -\frac{1}{24\pi^2}\int d^3x \epsilon^{ijk}\, Tr[(U\partial_i U^\dagger)(U\partial_j U^\dagger)(U\partial_k U^\dagger)].$$

- However these field configurations with different winding numbers can tunnel to each other due to instantons.

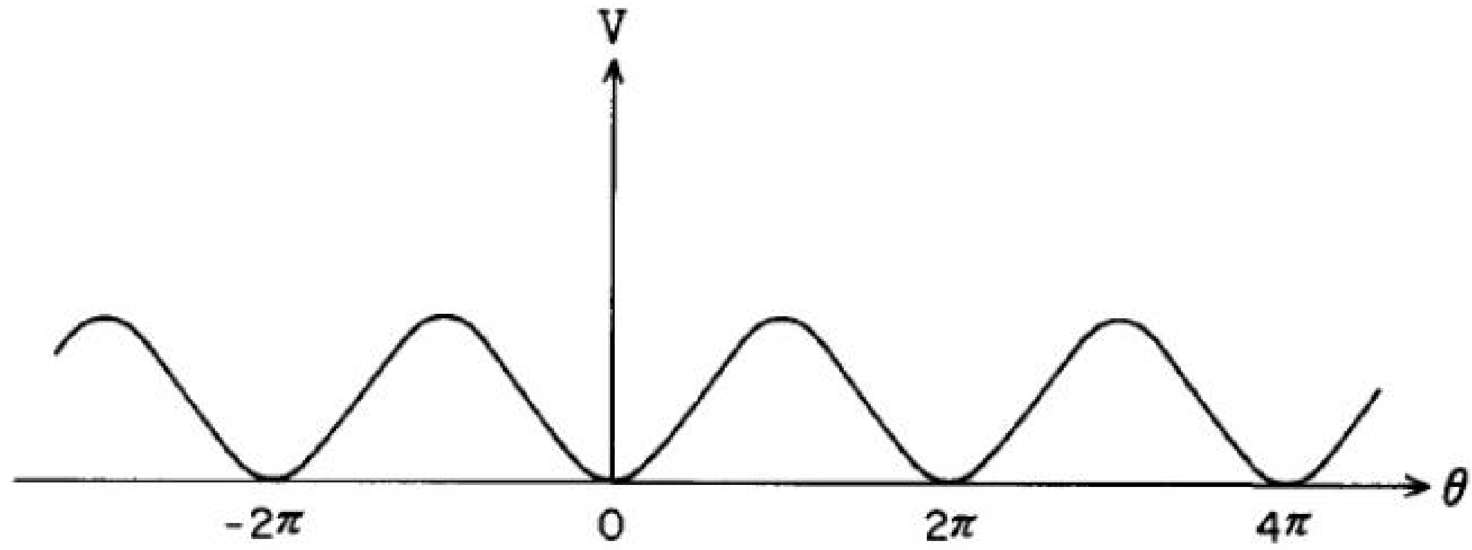
- So the physical vacuum has to include field configuration with all possible winding numbers thus has the form:

$$|\omega >= \sum_n e^{in\theta}|n >.$$

- This term violates CP invariance if $\theta \neq 0$
- The measurement of electric dipole moment of neutron gives a upper limit:

$$|\theta| \leq 10^{-9}$$

- This fine turning problem is harder to explain because the anthropic theory cannot solve it.



The energy due to the vacuum angle

To solve the strong CP problem, one introduces the $U(1)_{PQ}$ symmetry which is spontaneously broken

$$L = -1/4g^2 Tr(G_{\mu\nu} G_{\mu\nu}) + \sum \bar{q}_(D_\mu \gamma_\mu + m_i) q_i$$
$$+ \theta/32\pi^2 Tr G_{\mu\nu} \tilde{G}_{\mu\nu} + 1/2 \partial_\mu a \partial^\mu a + a/(f_a 32\pi^2) Tr G_{\mu\nu} \tilde{G}_{\mu\nu},$$

$\theta + a/f_a \rightarrow 0$ relaxes to zero during QCD phase transition.

An example: the KSVZ axion

- One introduces an new complex scalar and a new heavy quark Q.

$$L_{Yu} = -fQ_L^\dagger \sigma Q_R - f^* Q_R^\dagger \sigma^* Q_R$$

$$V = -\mu_\sigma^2 \sigma^* \sigma + \lambda_\sigma (\sigma^* \sigma)^2$$

=> $\sigma = (v + \rho) exp(i\frac{a}{v}).$

$$\begin{aligned} U(1)_{PQ}: \quad a &\to a + f_a \alpha \\ \sigma &\to exp(iq\alpha)\sigma \\ Q_L &\to exp(iQ\alpha/2) Q_L \\ Q_R &\to exp(-iQ\alpha/2) Q_R \end{aligned}$$

Axion like particles

- alps arises due to compactification of the antisymmetric tensor fields

$$B = \frac{1}{2\pi} \sum b^i(x)\omega_i(y) + \dots ,$$

- the x are non-compact coordinate, y are compact coordinates.

Axion like particles

- the zero mode acquires a potential due to non-perturbative effects on the compactification cycle.
- The effective Lagrangian in four dimension:

$$\mathcal{L} = \frac{f_{ALPs}^2}{2}(\partial a)^2 - \Lambda_{ALPs}^4 U(a)$$

So the logic could be:

Compactified extra dimensions -> many U(1) symmetries -> one of them happened couples with QCD sector -> strong CP problem solved

Strong CP even is a consequence of extra dimensions

How to find the QCD axion?

We need to “know” the QCD axion mass

Assume the axion is one of the majority components of DM

Axions produced during the QCD phase transition needs to be abundant

Axion fluctuations are correlated to the CMB fluctuation.

$$\langle \delta S_a^2 \rangle = \frac{2\sigma_\theta^2(2\theta_0^2 + \sigma_\theta^2)}{(\theta_0^2 + \sigma_\theta^2)^2}$$

- The observed CMB isocurvature fluctuation is small

$$\left\langle \left(\frac{\delta T}{T}\right)_{\rm iso}^2 \right\rangle \sim \langle \delta S_a^2 \rangle \lesssim \mathcal{O}(10^{-11}).$$

$$m_0 \approx 6 \times 10^{-5}\mathrm{eV}\left(\frac{10^{11}\mathrm{GeV}}{f_a}\right)$$

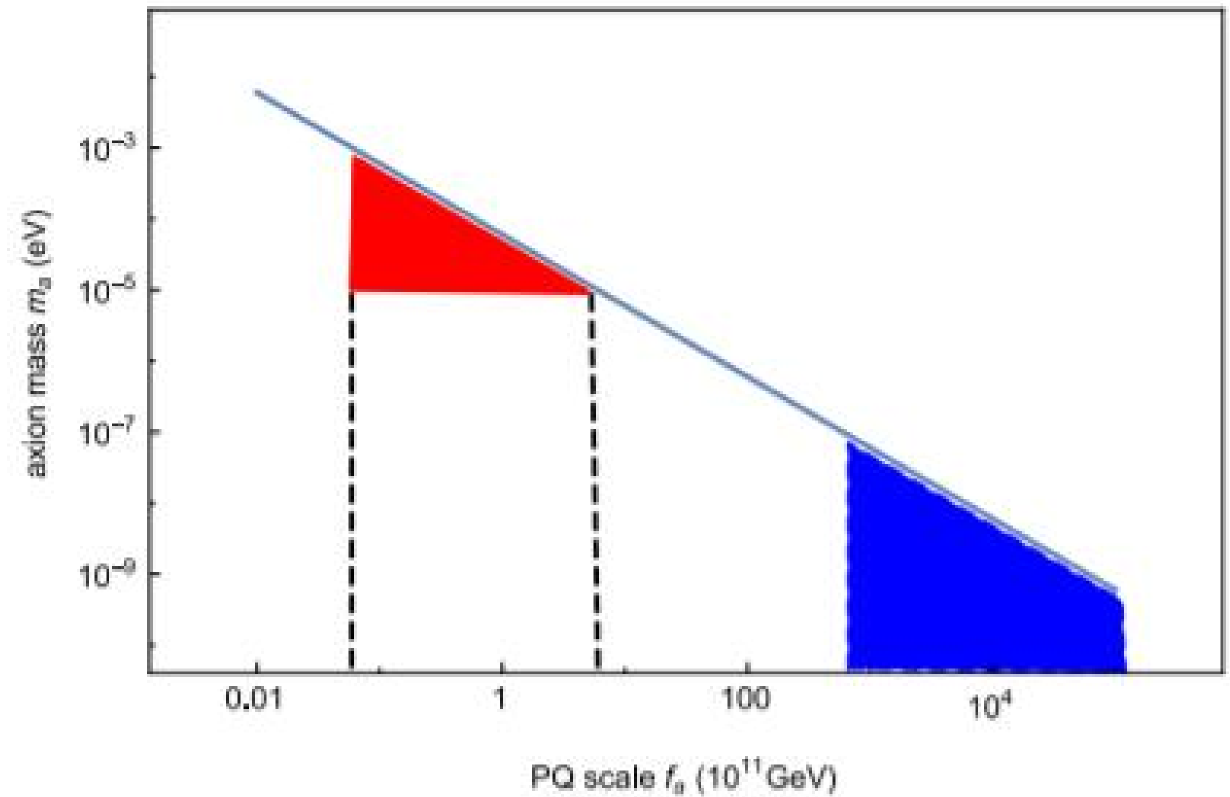


FIG. 1: The two possible windows of the dark matter axions. The upper-left one is often called the classical window and the lower-right one is the anthropic window assuming that $H_I < 10^{10}$GeV and the PQ symmetry was not restored after inflation.

The quantum properties of the Cavity

Modern cryogenic technology can sustain ~20mK or lower temperature

$$n(\omega_a, T) = \frac{1}{e^{\omega_a / k_B T} - 1}$$

The thermal photon has a very low occupation number n<<1 .

Thus it is useful to consider the quantum picture.

The Quantum picture of the Cavity (Feynman diagram method cannot be used here)

The axions has a long de-Broglie Wavelength and long coherent time, thus

$$a \approx a_0 \cos(\omega_a t) = \frac{\sqrt{2\rho_a}}{m_a} \cos(\omega_a t)$$

The Quantum picture of the Cavity

The axion photon coupling is

$$\mathcal{L}_{a\gamma\gamma} = -g_{a\gamma\gamma} a \vec{E} \cdot \vec{B}$$

The Quantum picture of the Cavity

The interaction Hamiltonian is

$$
\begin{aligned}
H_I &= -\int d^3x \mathcal{L}_{a\gamma\gamma} \\
&= \left(g_{a\gamma\gamma} \frac{\sqrt{2\rho_a}}{m_a} B_0 \int d^3x \hat{z} \cdot \vec{E} \right) \cos(\omega_a t)
\end{aligned}
$$

The Quantum picture of the Cavity

The electric field operator in the cavity can be expanded

$$\vec{E} = i\sum\sqrt{\frac{\omega_k}{2}}[a_k\vec{U}_k(\vec{r})e^{-i\omega_k t} - a_k^\dagger\vec{U}_k^*(\vec{r})e^{i\omega_k t}]$$

where $\vec{U}_k(\vec{r})$ is the cavity modes

The Quantum picture of the Cavity

The transition probability is

$$
\begin{aligned}
P &\approx \left| \langle 1 | \int_0^t dt H_I \, |0\rangle \right|^2 \\
&\approx g_{a\gamma\gamma}^2 \frac{\rho_a}{m_a^2} B_0^2 \sum_k \omega_k | \int d^3x \hat{z} \cdot \vec{U}_k^* |^2 \\
&\times \frac{\sin^2[(\omega_k - \omega_a)t/2]}{4[(\omega_k - \omega_a)/2]^2}
\end{aligned}
$$

The Quantum picture of the Cavity

The transition rate is

$$R \approx g_{a\gamma\gamma}^2 \frac{\rho_a}{m_a^2} B_0^2 V \sum_k C_k \omega_k \delta(\omega_k - \omega_a) \approx g_{a\gamma\gamma}^2 \frac{\rho_a}{m_a^2} B_0^2 C_{\omega_a} V Q$$

where

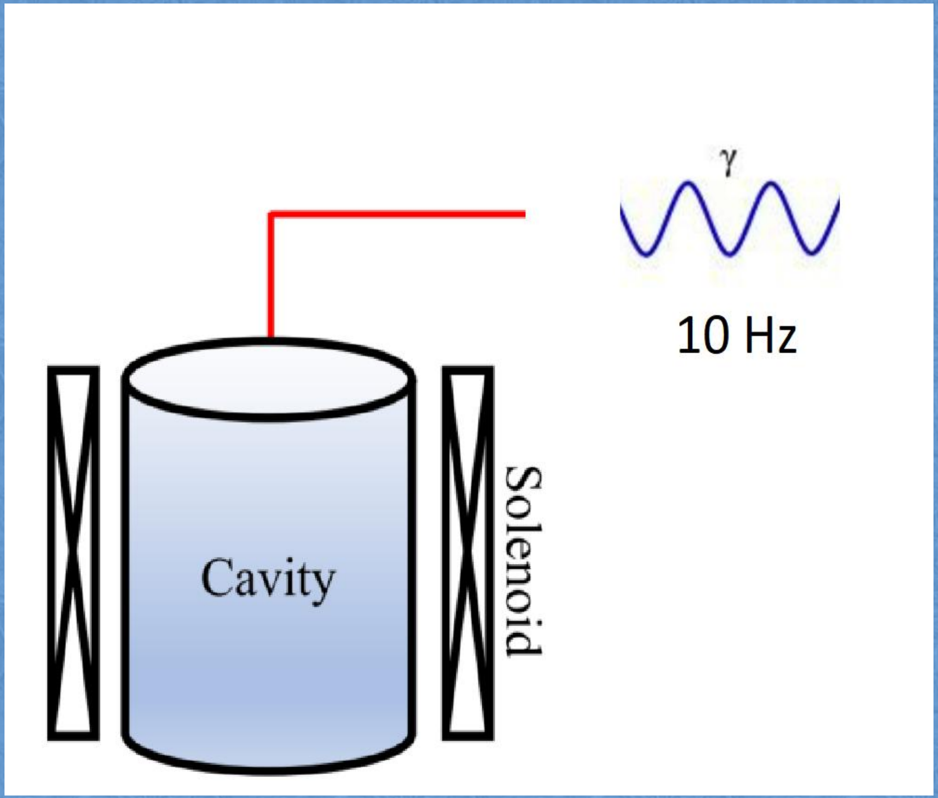
$$C_k = \frac{\left| \int d^3x \, \hat{z} \cdot \vec{U}_k \right|^2}{V \int d^3x |\vec{U}_k|^2}$$

The Quantum picture of the Cavity

The transition rate is enhanced by the cavity quality factor Q even for a single transition.

Typical photon emitting rate of the cavity is order of 10Hz.

The cavity at quantum level can be regarded as a single photon emitter with a slow rate~ 10Hz.

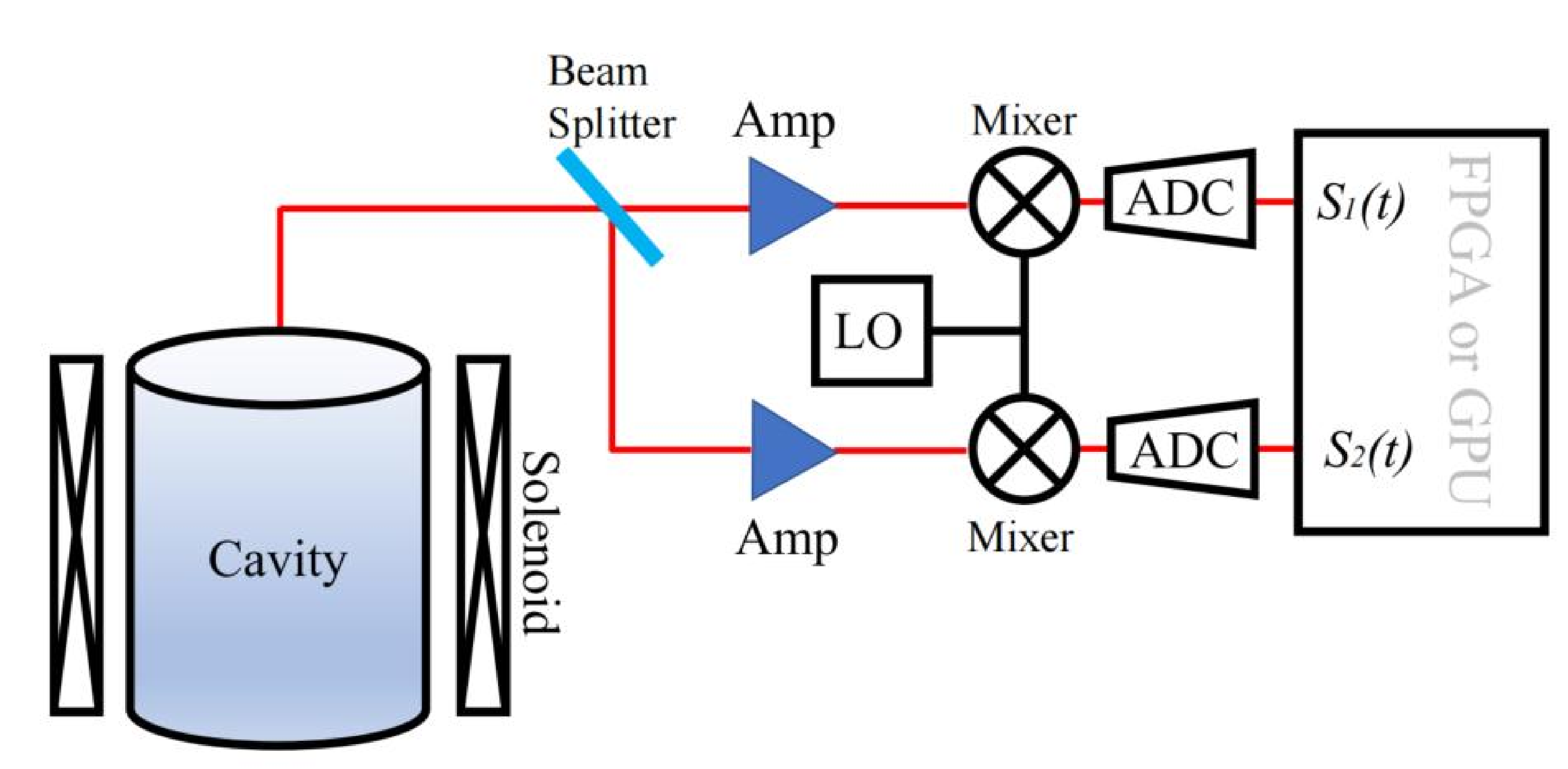


The bottleneck of the haloscope

The linear amplifier with a moderate bandwidth adds noise temperature order of T_eff=10K

$$\mathrm{SNR} = \frac{P_{sig}}{k_B T_{eff}} \sqrt{\frac{t}{b}}$$

The quantum interferometry



The quantum interferometry (HBT type)

The beam splitter gives two output fields in channel 1 and 2

$$\hat{r}_1 = (\hat{r} + \hat{\nu}_m)/\sqrt{2} \quad \text{and} \quad \hat{r}_2 = (\hat{r} - \hat{\nu}_m)/\sqrt{2}$$

$\hat{r}$ denotes the signal and $\hat{\nu}_m$ denotes the noises.

The quantum interferometry (HBT type)

Then measuring

$$\hat{I}_1 = (\hat{r}_1 + \hat{r}_1^+)/2$$

and

$$\hat{Q}_2 = -i(\hat{r}_2 - \hat{r}_2^+)/2$$

as the real and imaginary part of the field envelopes.

The quantum interferometry (HBT type)

After amplification and mixing, the two path read out is

$$S_i(t) = G_i \hat{r}(t) + \sqrt{G_i^2 - 1} h_i^{+}(t) + \nu_{m,i}^{+}(t)$$

The quantum interferometry (HBT type)

The two path instantaneous power function is

$$\langle S_1^*(t)S_2(t)\rangle = G_1G_2\left(\langle\hat{r}^+(t)\hat{r}(t)\rangle + N_{12}\right)$$

where N_{12} is the power of correlated noise in channel 1 and 2

The quantum interferometry (HBT type)

The two path instantaneous power function is

$$\langle S_1^*(t) S_2(t) \rangle = G_1 G_2 \left(\langle \hat{r}^+(t) \hat{r}(t) \rangle + N_{12} \right)$$

the effective temperature of N_{12} is typically 80 mK.

simulated signal