



GW & WDM

Ultralight DM

Axion-like DM
BH Superradiance

PTA and DM

spin-0 DM
spin-2 DM

Summary

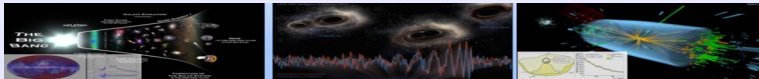
Pulsar Timing Residuals induced by Wideband Ultralight Dark Matter

Speaker: Yun-Long Zhang (NAOC)

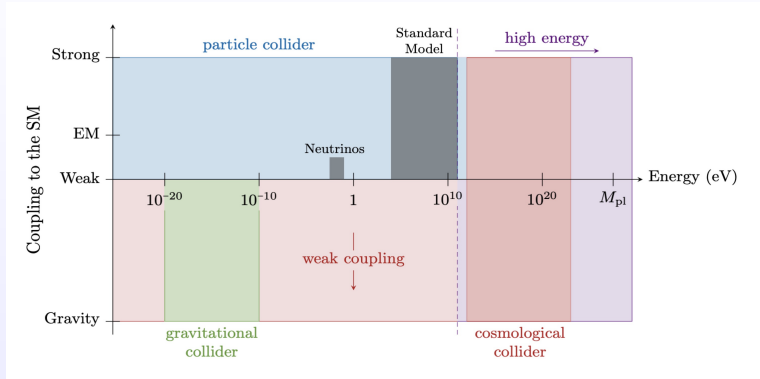
National Astronomical Observatories,
Chinese Academy of Sciences

Phys.Rev.D 106, 066006(2022) with S. Sun, Xing-Yu Yang(ITP-CAS)
Phys.Rev.D 104, 103009(2021) with Sichun Sun(Beijing Inst.Tech.)

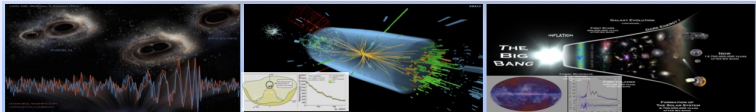
Nov.22@Axion2022 [Email: zhangyunlong@nao.cas.cn]



Motivation: new physics in ultra-low energy

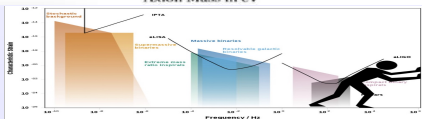
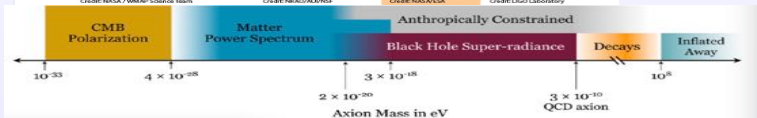
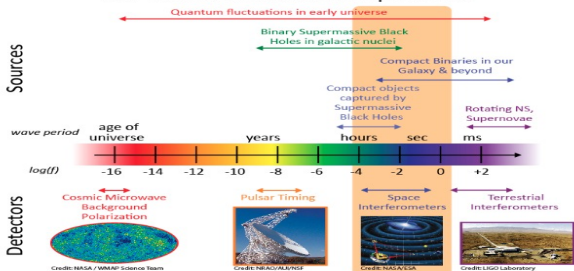


[cf. Baumann-Chia-Porto-Stout, Gravitational Collider Physics, 2019]



Spectrum of Gravitational Wave and Axion Mass

The Gravitational Wave Spectrum



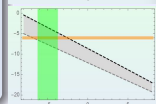
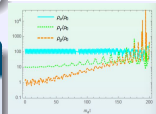
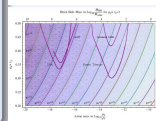
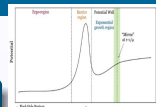
[cf. LISA/Ultra-High-Frequency Gravitational Waves Initiative]

Black Hole Superradiance & GW signal

- Axion annihilation $\vartheta + \vartheta \rightarrow h$ (Stochastic GW)
Energy transition $\vartheta^+ \rightarrow \vartheta^- + h$ (Monochromatic)
- Superradiance $\alpha \equiv \frac{R_{BH}}{\lambda_{\vartheta}} \simeq \left(\frac{M_{BH}}{M_{\odot}} \right) \left(\frac{m_{\vartheta}}{10^{-10} \text{eV}} \right)$
- Fast Radio Burst from Axion $\sim \vartheta F \tilde{F}$ ($\vartheta \rightarrow \gamma\gamma$)
- GW burst from Axion $\sim \vartheta R \tilde{R}$ ($\vartheta \rightarrow hh$)

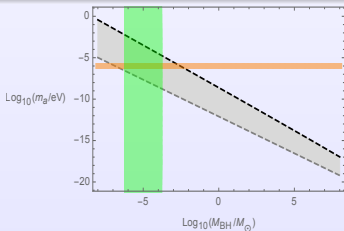
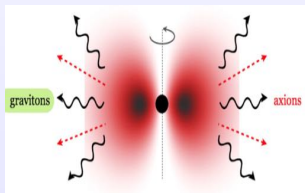
Ultra-light DM and multi band GW detection

- Tabletop exp: Axion star & GW burst (\sim GHz)
- LISA & LVK: Superradiance (\sim mHz - kHz)
- FAST & SKA : Ultra-light dark matter (\sim nHz)



Schwarzschild $R_{BH} = G_N M_{BH} / c^2$, de Broglie wave length $\lambda_{\vartheta} = \hbar / (m_{\vartheta} v_{\vartheta})$

- Characteristic $\alpha \equiv \frac{R_{BH}}{\lambda_{\vartheta}} \simeq \left(\frac{M_{BH}}{M_{\odot}} \right) \left(\frac{m_{\vartheta}}{10^{-10} \text{eV}} \right) \left(\frac{v_{\vartheta}}{c} \right)$.
- Formation time $\tau_{\vartheta} < \text{Universe's age } \tau_U \simeq 10^{23} \left(\frac{M_{\odot}}{M_{BH}} \right) R_{BH}$
- $\tau_{\vartheta\uparrow} \simeq 10^7 e^{1.84\alpha} R_{BH}$, $\alpha \gg 1$, $\tau_{\vartheta\downarrow} \simeq 24 \alpha^{-9} R_{BH}$, $\alpha \ll 1$.



GW & WDM

Ultralight DM

Axion-like DM

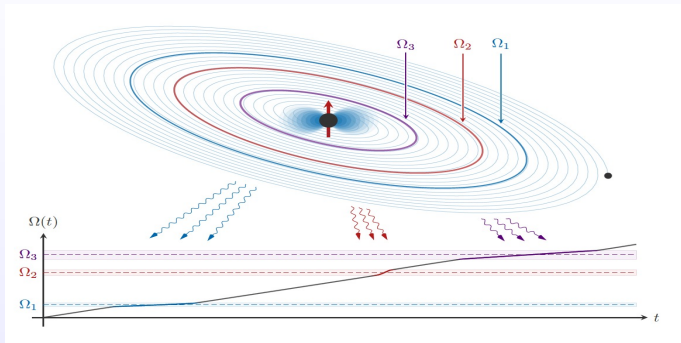
BH Superradiance

PTA and DM

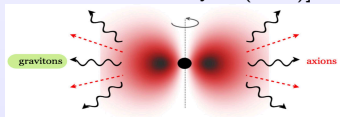
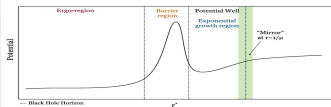
spin-0 DM

spin-2 DM

Summary



[cf. Baumann-Chia-Porto-Stout, Gravitational Collider Physics(2019)]



[cf. Arvanitaki-Dubovsky, String Axiverse -2011]

GW & WDM

Ultralight DM

Axion-like DM

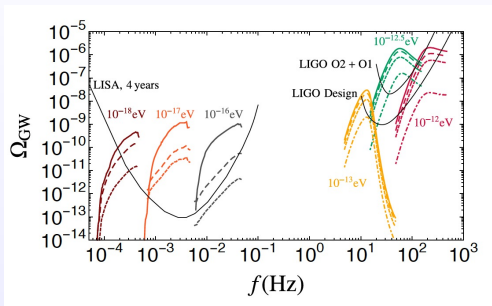
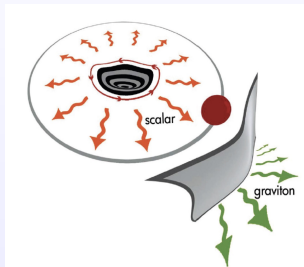
BH Superradiance

PTA and DM

spin-0 DM

spin-2 DM

Summary



- Axion annihilation $\vartheta + \vartheta \rightarrow h$, Strain $h \sim 10^{-21} - 10^{-32}$.
- Stochastic GW [cf. Brito-Cardoso-Pani, Superradiance 2020]

GW & WDM

Ultralight DM

Axion-like DM

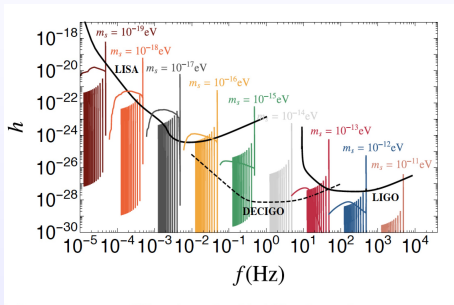
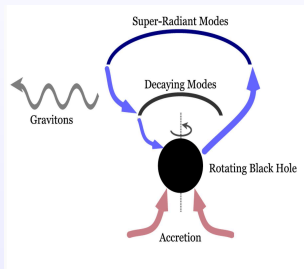
BH Superradiance

PTA and DM

spin-0 DM

spin-2 DM

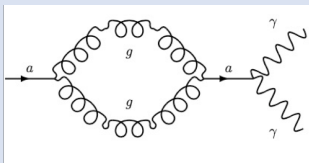
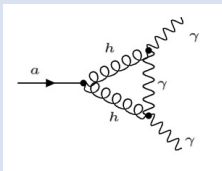
Summary



- Energy transition $\vartheta^+ \rightarrow \vartheta^- + h$, Strain $h \sim 10^{-19} - 10^{-27}$
- Monochromatic GW [cf. Brito-Cardoso-Pani, Superradiance 2020]

- The triangle Feynman diagram: where the axion-photon coupling is generated from Chern-Simon gravity coupling.

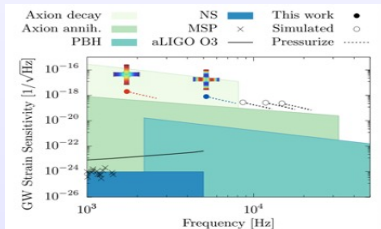
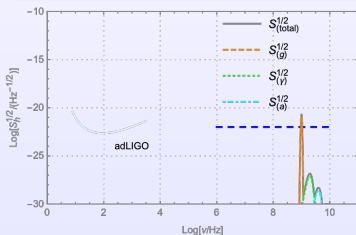
$$\mathcal{L}_{\partial F \tilde{F}} = -\frac{\alpha_\gamma}{4} \partial F_{\mu\nu} \tilde{F}^{\mu\nu}, \quad \mathcal{L}_{\partial R \tilde{R}} = \frac{\alpha_g}{4} \partial R^\beta_{\alpha\gamma\delta} \tilde{R}^\alpha{}_{\beta\gamma\delta}.$$



- The triangle diagram is divergent as $\alpha_\gamma \sim \alpha_g (\Lambda_{CS}/M_{pl})^4$, where Λ_{CS} is the cut-off for Chern-Simons theory.
- Two powers of M_{pl} from $h_{\mu\nu} T^{\mu\nu}$ coupling.

Branch Ratio and GWs

- $\frac{\text{Br}(\vartheta \rightarrow \text{gg})}{\text{Br}(\vartheta \rightarrow \gamma\gamma)} \simeq \frac{\alpha_{\text{pl}}^2}{\alpha_\gamma^2} \simeq \left(\frac{M_{\text{pl}}}{\Lambda_{\text{CS}}}\right)^8$, (Power of FRB $P_{(\gamma)} \sim 10^{42}$ ergs/s).
- High frequency $h_{(\text{g})} \sim 10^{-26} \left(\frac{1\text{GHz}}{\nu}\right) \left(\frac{P_{(\text{g})}}{P_{(\gamma)}}\right)^{1/2} \left(\frac{1\text{kpc}}{L}\right)$
- Low freq. $h_{(\text{g})} \sim 10^{-21} \left(\frac{10^{-2}\text{Hz}}{\nu}\right)^{1/2} \left(\frac{M_{\text{BH}}}{10^7 M_\odot}\right)^{1/2} \left(\frac{1\text{kpc}}{L}\right)$

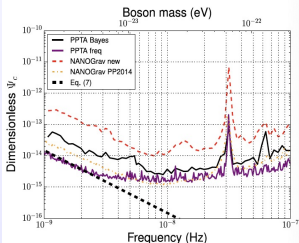
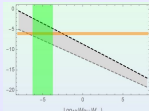
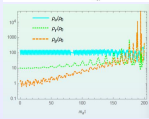
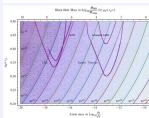
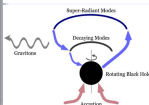
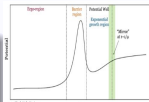


cf. PRD'21, S. Sun, Y. L. Zhang, Gravitational Wave Burst from Axion Clumps.

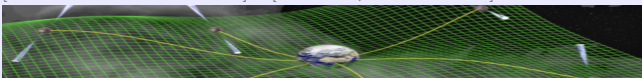
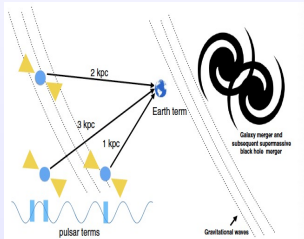
PRD'21, V. Vadakkumbatt et al, Prototype superfluid gravitational wave detector.

Ultra light dark matter

- PTA & SKA: ultra light DM (\sim nHz)
- $\lambda_{dB} = \frac{2\pi\hbar}{mv} \simeq 4\text{kpc} \left(\frac{10^{-23}\text{eV}}{m} \right) \left(\frac{10^{-3}}{v} \right)$
- $f_c = \frac{m}{\pi} \simeq 4.8\text{ nHz} \left(\frac{m}{10^{-23}\text{eV}} \right)$



[cf. X. Xue, X. J. Zhu et al. 2018] & [cf. Burke-Spolaor, et al. 2019]



The astrophysics of nanohertz gravitational waves

GW & WDM

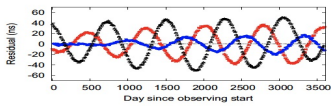
Ultralight DM

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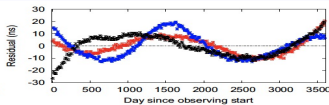
PTA and DM

spin-0 DM
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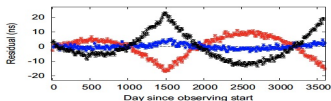
Summary



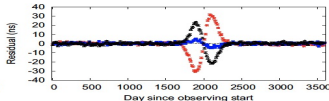
(a) Continuous wave



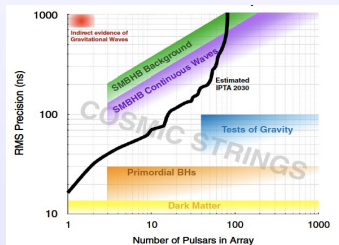
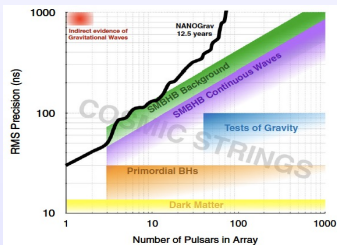
(b) Background



(c) Burst with Memory



(d) Burst



[cf. Burke-Spolaor, *et al.*, "The astrophysics of nanohertz gravitational waves"]



Pulsar timing residual and fuzzy dark matter

GW & WDM

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Summary

DM oscillation induced time residual

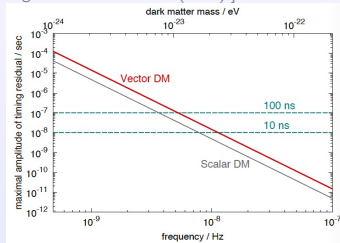
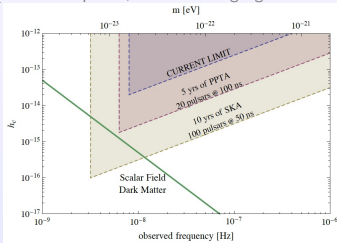
- Metric: $ds^2 = -(1 + 2\Phi) dt^2 + [(1 - 2\Psi) \delta_{ij} + h_{ij}] dx^i dx^j$.
- e.g. the scalar field $\phi(x, t) = \phi(x) \cos [mt + \theta_0(x)]$,
- Oscillating potential $\Psi \simeq \bar{\Psi}(x) + \Psi_\phi \cos [2(mt + \theta_0(x))]$
- Doppler effect: $z_\phi(t) \equiv \frac{\omega_0 - \omega_\phi(t)}{\omega_0} \simeq \Psi(x_\phi, t_\phi) - \Psi(x_0, t_0)$.
- Timing residual in the pulse $R_\phi(t) = \int_0^t z_\phi(t') dt'$
- Strain $h_\phi = 2\sqrt{3} \Psi_\phi = \frac{\sqrt{3}}{4M_{pl}^2} \frac{\rho_\phi}{m^2} \simeq 5.2 \times 10^{-17} \alpha_0 \left(\frac{f_{yr}}{f}\right)^2$,
- GW Timing residual: $R_c(f) \equiv \sqrt{\frac{S_c(f)}{T_s}} = \frac{1}{\sqrt{3}} \frac{h_c(f)}{2\pi f} \left(\frac{f_s}{f}\right)^{1/2}$

[cf. Burke-Spolaor, "Pulsar timing signal from ultralight scalar DM" JCAP(2014)]

DM oscillation induced time residual

- Spin-0: massive scalar field $\mathcal{L}_{(0)} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2$
- Spin-1: massive vector field $\mathcal{L}_{(1)} = -\frac{1}{4}F^2 - \frac{1}{4}m^2A^2$

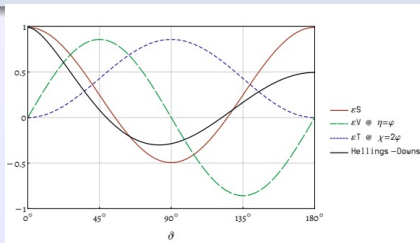
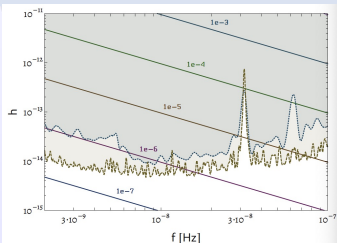
[cf. Burke-Spolaor, "Pulsar timing signal from ultralight scalar DM" JCAP(2014)]



[cf. Nomura-Itoy-Soda, "Pulsar timing residual induced by ultralight vector DM" PRD(2020)]

spin-2 ultralight fields

- Spin-2: massive tensor field (Fierz-Pauli): Bi-metric gravity,
$$\mathcal{L}_{(2)} = \frac{1}{2} M_{\mu\nu} \mathcal{E}^{\mu\nu\rho\sigma} M_{\rho\sigma} - \frac{1}{4} m^2 (M_{\mu\nu} M^{\mu\nu} - M^2)$$
- The oscillating solution $M_{ij} = \mathcal{M} \cos [mt + \theta_2(x)] \varepsilon_{ij}$
- Effective metric perturbations: $\tilde{g}_{ij} = \delta_{ij} + \frac{\alpha_2}{M_{pl}} M_{ij}$
- The redshift $z(t) = \frac{\omega(t) - \omega_0}{\omega_0} = \frac{\alpha_2}{2M_{pl}} \int dt \omega_0 \partial_t M_{ij} n^i n^j$

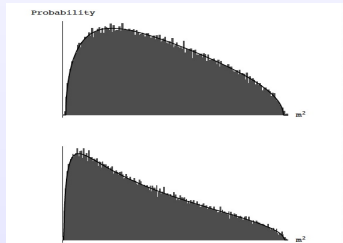
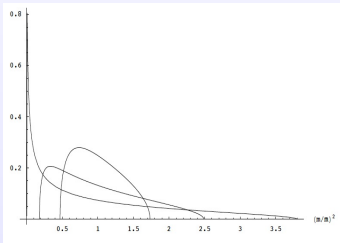


[cf. Armaleo-Nacir-Urbanb, "Pulsar timing array constraints on spin-2 ULDM" JCAP(2020)]

Mass spectrum and ultralight fields

- Marcenko-Pastur: $P_M(m^2) = \frac{\sqrt{(m^2 - m_-^2)(m_+^2 - m^2)}}{2\pi\beta m_0^2 m^2}$,
- Energy density: $\rho_\phi \equiv \int dm \tilde{\rho}(m) = \int dm \frac{1}{2} m^2 \tilde{\phi}(m)^2 P(m)$.
- Convenient choice: $\tilde{\rho}(m) \simeq \rho_\phi P(m)$, $\int dm P(m) = 1$.

[cf. Marcenko-Pastur, "Distributions of Eigenvalues for Some Sets of Random Matrices," (1967)]



[cf. Easthera-McAllister, "Random Matrices and the Spectrum of N-flation" JCAP(2006)
Cai-Hu-Piao, "Entropy Perturbations in N-flation" PRD(2009)]

Our phenomenological fitting results

GW & WDM

Ultralight DM

Axion-like DM
BH Superradiance

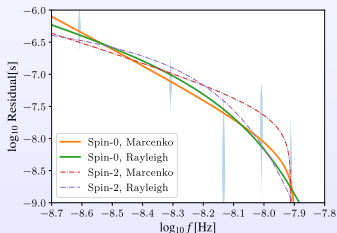
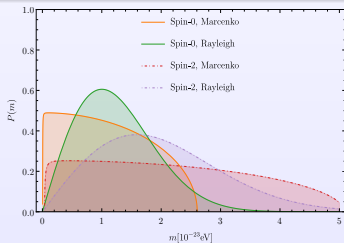
PTA and DM

spin-0 DM
spin-2 DM

Summary

Mass spectrum and ultralight fields

- Marcenko-Pastur: $P_M(m^2) = \frac{\sqrt{(m^2 - m_-^2)(m_+^2 - m^2)}}{2\pi\beta m_0^2 m^2}$,
- Rayleigh distribution: $P_\sigma(m) = \frac{m}{\sigma^2} e^{-\frac{m^2}{2\sigma^2}}$.



[Sun-Yang-Zhang, PRD(2022) "Pulsar Timing Residual induced by Wideband Ultralight Dark Matter"]

Corner Figures of Bayesian Fitting

GW & WDM

Ultralight DM

Axion-like DM

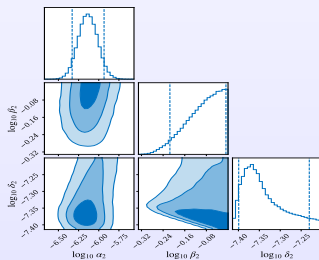
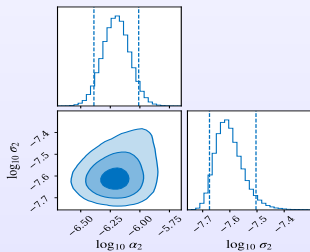
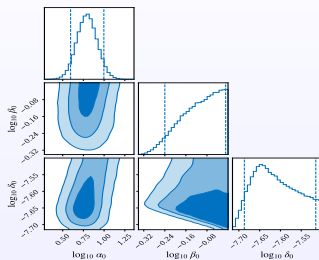
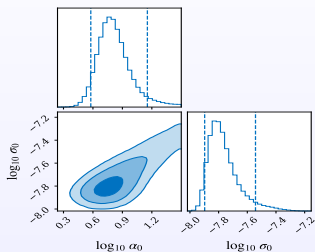
BH Superradiance

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Summary



The effective strain

- $$h_c^\phi(f) = \frac{\alpha_0}{M_{pl}^2} \frac{\sqrt{3}\rho_{DM}}{4\pi f} P(\pi f)$$
- $$h_c^M(f) = \frac{\alpha_2}{M_{pl}} \frac{mMP(m)}{\sqrt{5}} = \frac{\alpha_2}{M_{pl}} \frac{2\sqrt{\rho_M}}{\sqrt{5}} P(2\pi f).$$

	Parameters	spin-0	spin-1	spin-2
Marcenko	α_i	$5.9^{+1.9}_{-1.3}$	$\sim 3\alpha_0$	$7.6^{+2.2}_{-1.7} \times 10^{-7}$
	$m_-^i / (10^{-23} \text{eV})$	$2.9^{+3.6}_{-0.3} \times 10^{-3}$	$\sim \delta_0(1 - \sqrt{\beta_0})$	$6.3^{+6.0}_{-1.7} \times 10^{-3}$
	$m_+^i / (10^{-23} \text{eV})$	$2.61^{+0.21}_{-0.01}$	$\sim \delta_0(1 + \sqrt{\beta_0})$	$5.08^{+0.02}_{-0.01}$
Rayleigh	α_i	$5.6^{+3.8}_{-1.0}$	$\sim 3\alpha_0$	$6.1^{+2.1}_{-1.3} \times 10^{-7}$
	$\sigma_i / (10^{-23} \text{eV})$	$1.0^{+0.4}_{-0.1}$	$\sim \sigma_0$	$1.6^{+0.3}_{-0.1}$

[Sun-Yang-Zhang, PRD(2022), "Pulsar Timing Residual induced by Wideband Ultralight Dark Matter"]

Summary and Outlook

Black Hole Superradiance & Gravitational waves

- Superradiance $\alpha \equiv \frac{R_{BH}}{\lambda \vartheta} \simeq \left(\frac{M_{BH}}{M_{\odot}} \right) \left(\frac{m_{\vartheta}}{10^{-10} \text{eV}} \right)$
- Axion annihilation $\vartheta + \vartheta \rightarrow h$ (stochastic GW)
Energy transition $\vartheta^+ \rightarrow \vartheta^- + h$ (monochromatic)
- Fast Radio Burst from Axion $\sim \vartheta F \tilde{F}$ ($\vartheta \rightarrow \gamma\gamma$)
- Fast GW burst from Axion $\sim \vartheta R \tilde{R}$ ($\vartheta \rightarrow hh$)

Ultra-light DM and multi band GW detection

- Tabletop exp: GW burst & Axion clump (\sim GHz)
- LVK & LISA-Taiji: Superradiance (\sim mHz - kHz)
- PTA & SKA : Ultra-light dark matter (\sim nHz)

Thanks a lot for your attention!

