Distinguishing different axion models with low energy couplings

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KC, Im, Seong and Kim, JHEP 08 (2021) 058 (arXiv:2106.05816)

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Axions or axion-like particles (ALPs) are light pseudo-scalar bosons postulated in many well-motivated models for BSM physics:

Strong CP problem,
Dark matter,
Inflation,
Baryogenesis,
String theory, ...

Axions have a finite field range (periodic):

$$a(x) \cong a(x) + 2\pi f_a \quad (f_a = \text{axion decay constant})$$

Axions can be naturally light as their interactions are constrained by an approximate PQ-symmetry non-linearly realized in low energy effective theory:

$$U(1)_{\rm PQ}: \quad a(x) \;\to\; a(x) + \text{constant}$$

Classification of axions by the origin of the field variable

Field-theoretic axions from the phase of complex scalar fields:

PQWW, KSVZ, DFSZ, ...

$$\sigma(x) = \rho(x) e^{ia(x)/f_a} \quad \left(\frac{f_a}{\sqrt{2}} = \langle \rho \rangle\right)$$

String-theoretic axions from p-form gauge fields in string theory:

(or in any theory with extra-dim)

Witten '84, Svrcek & Witten '06, ...

$$a(x) = \int_{\Sigma_p} C^{(p)}_{[m_1,..,m_p]}(x,y)\, dy^{m_1}...dy^{m_p} \quad (\Sigma_p = p\text{-cycle in extra-dim})$$

or $\partial_\mu a = \epsilon_{\mu\nu\rho\sigma}(\partial^\nu B^{\rho\sigma} + ...)$ $\quad (B_{\mu\nu} = \text{2-form gauge field in 4D spacetime})$

Classification by low energy couplings

For model-independent discussion of axion couplings, it is convenient to use a field basis for which all fields other than the axion are PQ-invariant:

Georgi, Kaplan, Randall '85

$$U(1)_{\rm PQ}: \;\; a(x) \;\rightarrow\; a(x) + {\rm constant}, \quad \Phi = (\psi, H, ...) \;\rightarrow\; \Phi$$

Axion couplings to the SM at high scales around $\mathbf{f_a}$:

PQ-invariant derivative couplings to matter fields described by real-valued $\mathbf{c_\psi}$ & $\mathbf{c_H}$

$$\mathcal{L}_{\rm eff} = \frac{1}{2} f_a^2 (\partial_\mu \theta)^2 + \partial_\mu \theta \Big(\sum_\psi c_\psi \bar{\psi} \sigma^\mu \psi + i c_H (H^\dagger D^\mu H - D^\mu H^\dagger H) \Big) \quad \text{(PQ-invariant)}$$

$$+ \sum_{F^A = G,W,B} c_A \frac{g_A^2}{32\pi^2} \theta(x) F^{A\mu\nu} \tilde{F}^A_{\mu\nu} - V_0(\theta(x)) - m_H^2(\theta(x)) |H|^2 + \ldots \quad \text{(PQ-breaking)}$$

Bare axion potential

Couplings to the SM gauge fields normalized in such a way that $\mathbf{c_G}$, $\mathbf{c_W}$, $\mathbf{c_B}$ are rational numbers or integers.

Non-derivative coupling to $|H|^2$ (axion-dependent Higgs mass)

$$\theta(x) = \frac{a(x)}{f_a} \cong \theta(x) + 2\pi$$

➔ Axion couplings $\propto \dfrac{1}{f_a}$

QCD axion postulated to solve the strong CP problem

$U(1)_{\mathrm{PQ}}$ is broken dominantly by the coupling to the gluon anomaly:

Peccei & Quinn

$$\frac{\mathbf{c}_G}{32\pi^2}\frac{a(x)}{f_a}G^{\mu\nu}\tilde{G}_{\mu\nu}$$

PQ-breaking couplings other than the anomalous couplings to the SM gauge fields are highly suppressed as

$$\frac{\partial}{\partial\theta}\left(V_0(\theta),\frac{1}{16\pi^2}m_H^2(\theta)\Lambda^2,\ldots\right)\lesssim 10^{-10}f_\pi^2 m_\pi^2$$

➔ $$V_{\rm axion}(\theta)=-\frac{f_\pi^2 m_\pi^2}{m_u+m_d}\sqrt{m_u^2+m_d^2+2m_u m_d\cos(c_G\theta)}+\mathcal{O}(10^{-10}f_\pi^2 m_\pi^2)$$

P- and CP-invariant QCD: $|\theta_{\rm QCD}|\equiv|c_G\left\langle\frac{a}{f_a}\right\rangle|\lesssim 10^{-10}$

$$m_a\simeq 5.7\,\mathbf{c}_G\left(\frac{10^{12}\mathrm{GeV}}{f_a}\right)\mu\mathrm{eV}$$

Ultra-light ALP which may constitute (part of) dark matter

(e.g. fuzzy dark matter with $m_a \sim 10^{-20} - 10^{-21}\,\mathrm{eV}$)

No coupling to the gluon anomaly (**c_G** = 0) , but may have non-zero coupling to the EW anomalies

$$\frac{1}{32\pi^2}\frac{a(x)}{f_a}\left(c_B B^{\mu\nu}\tilde{B}_{\mu\nu} + c_W W^{\mu\nu}\tilde{W}_{\mu\nu}\right)$$

All other PQ-breakings are so tiny, yielding

$$m_a \ll m_{a_{\rm QCD}} \sim \frac{m_\pi f_\pi}{f_a}$$

Low energy axion couplings:

At $\mu = \mathcal{O}(1)$ GeV

photon　　gluons　　light quarks and electron

$$\frac{1}{32\pi^2}\frac{a(x)}{f_a}\left(c_\gamma F^{\mu\nu}\tilde{F}_{\mu\nu} + c_G G^{\alpha\mu\nu}G^\alpha_{\mu\nu}\right) + \sum_{\Psi=u,d,e}\frac{\partial_\mu a}{2f_a}C_\Psi\bar{\Psi}\gamma^\mu\gamma_5\Psi$$

$$c_\gamma = c_W + c_B, \quad C_\Psi = C^0_\Psi + \Delta C_\Psi$$

Rational numbers which are typically of O(1).

Tree level values

Radiative corrections

$$C^0_u = c_{q_1}(f_a) + c_{u^c_1}(f_a) + c_H(f_a),$$
$$C^0_d = c_{q_1}(f_a) + c_{d^c_1}(f_a) - c_H(f_a),$$
$$C^0_e = c_{\ell_1}(f_a) + c_{e^c_1}(f_a) - c_H(f_a).$$

Measurable axion couplings in low energy experiments:

$$\frac{1}{2}g_{a\gamma}a(x)\vec{E}\cdot\vec{B}+\partial_\mu a(x)\left(\frac{g_{ap}}{2m_p}\bar{p}\gamma^\mu\gamma_5 p+\frac{g_{an}}{2m_n}\bar{n}\gamma^\mu\gamma_5 n+\frac{g_{ae}}{2m_e}\bar{e}\gamma^\mu\gamma_5 e\right)$$

$$g_{a\gamma} \simeq \frac{\alpha_{\rm em}}{2\pi}\frac{1}{f_a}\left(c_\gamma-\frac{2}{3}\left(\frac{m_u+4m_d}{m_u+m_d}\right)c_G\right)$$

$$g_{ap} \simeq \frac{m_p}{f_a}\left(C_u\Delta u+C_d\Delta d-\left(\frac{m_d}{m_u+m_d}\Delta u+\frac{m_u}{m_u+m_d}\Delta d\right)c_G\right)$$

$$\simeq \frac{m_p}{f_a}\left(0.88C_u-0.39C_d-0.47c_G\right)$$

$$g_{an} \simeq \frac{m_n}{f_a}\left(C_d\Delta u+C_u\Delta d-\left(\frac{m_u}{m_u+m_d}\Delta u+\frac{m_d}{m_u+m_d}\Delta d\right)c_G\right)$$

$$\simeq \frac{m_n}{f_a}\left(-0.39C_u+0.88C_d-0.02c_G\right)$$

The parts depending on $\mathbf{c_G}$ are mostly from the axion-meson mixing.

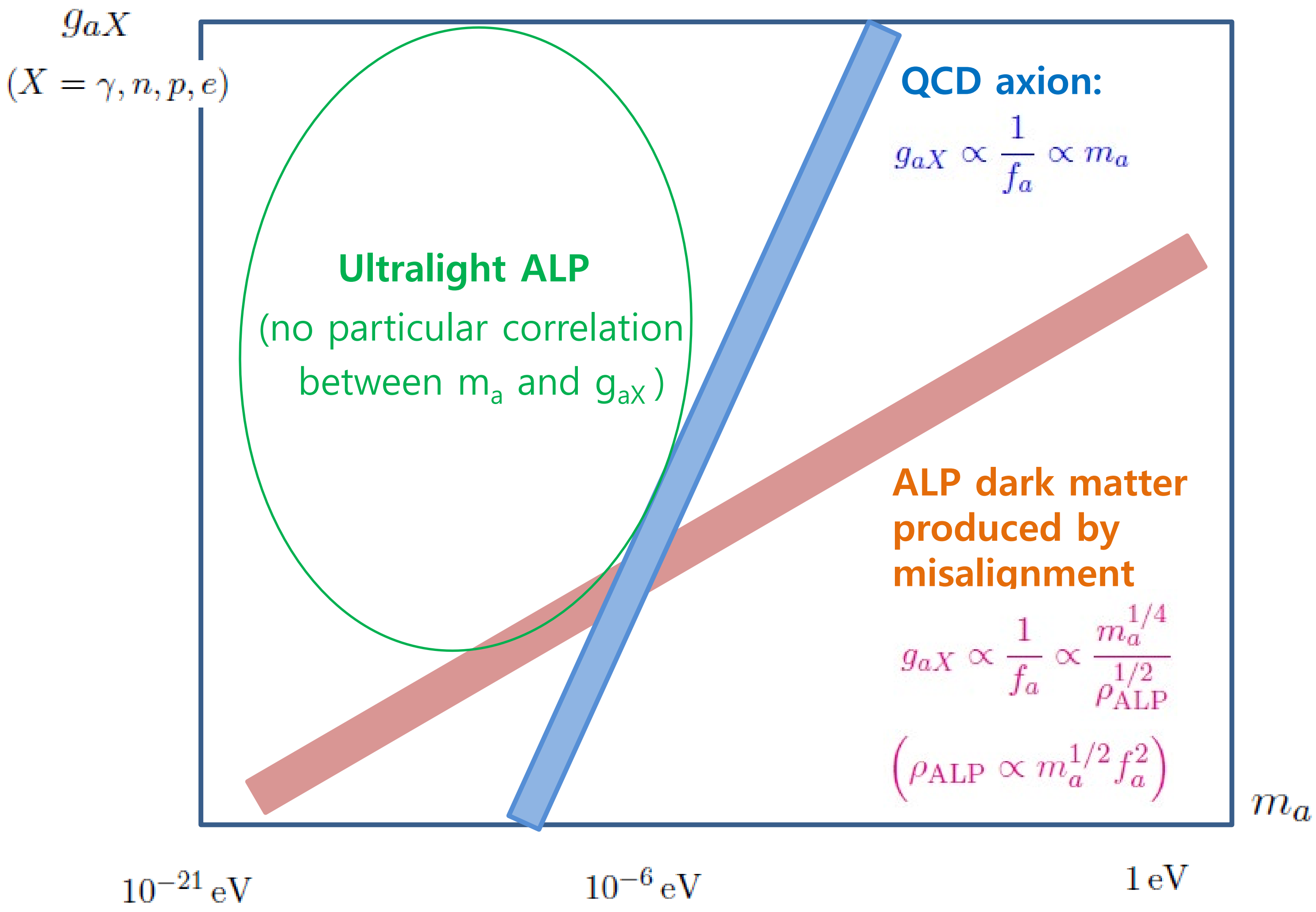
$$g_{ae} \simeq \frac{m_e}{f_a}C_e$$

$\Delta u = 0.897(27),\ \Delta d = -0.376(27)$ for $C_{u,d}$ at $\mu = 2$ GeV

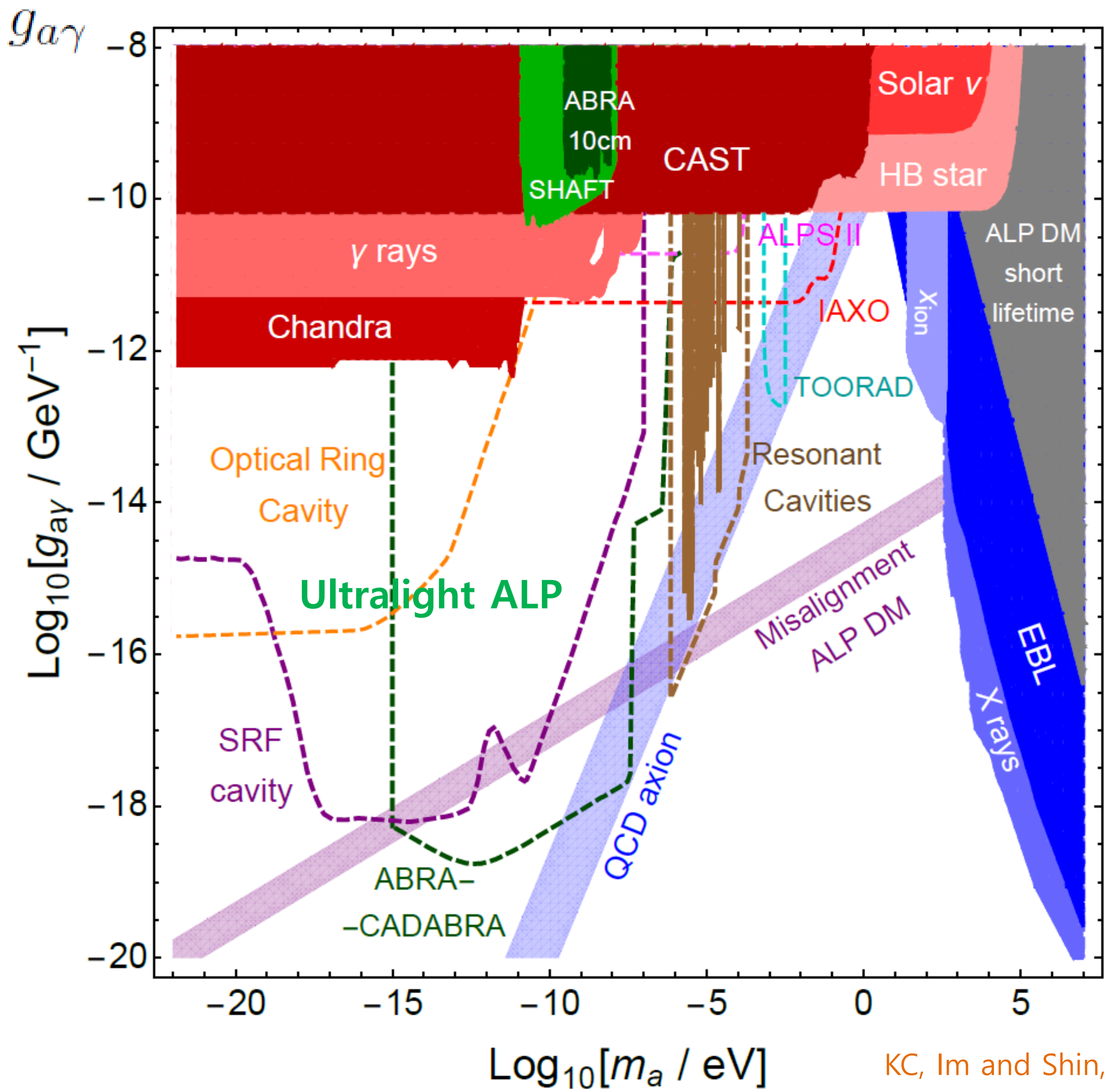
$\left(s^\mu\Delta q = \langle p|\bar{q}\gamma^\mu\gamma_5 q|p\rangle\ \ (q=u,d)\right)$

Cortona et al, arXiv:1511.02867

Typical parameter space for theoretically well-motivated light axions which may have good potential to be experimentally detected:



Observational bounds and the sensitivities of planned experiments



Resonant cavity:
Sikivie '83,
Semertzidis et al '09

ABRACADABRA:
Kahn et al '16

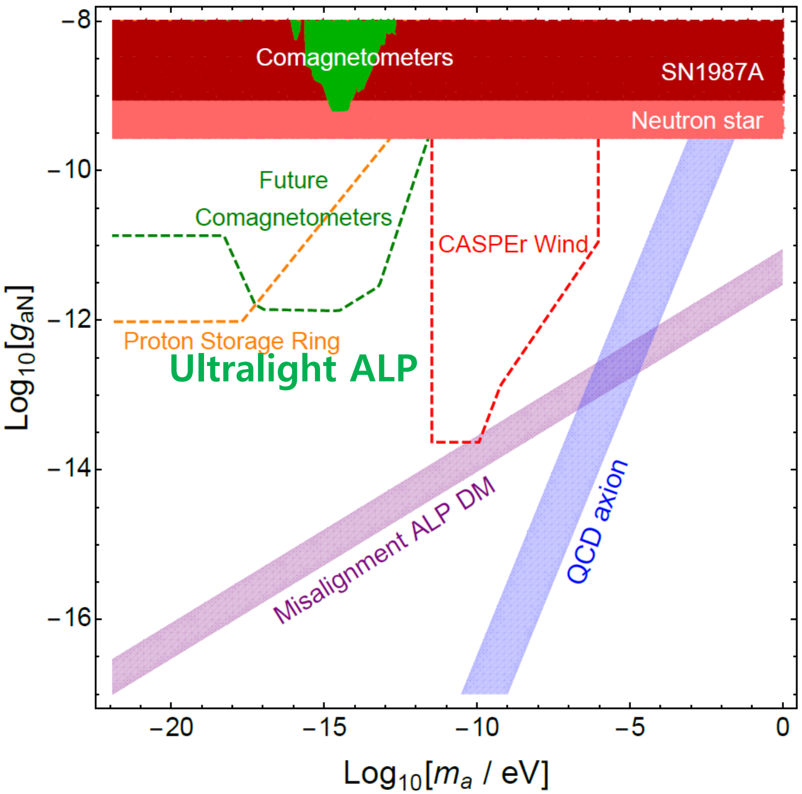
Optical ring cavity:
Obata et al '18

TOORAD:
Marsh et al '19

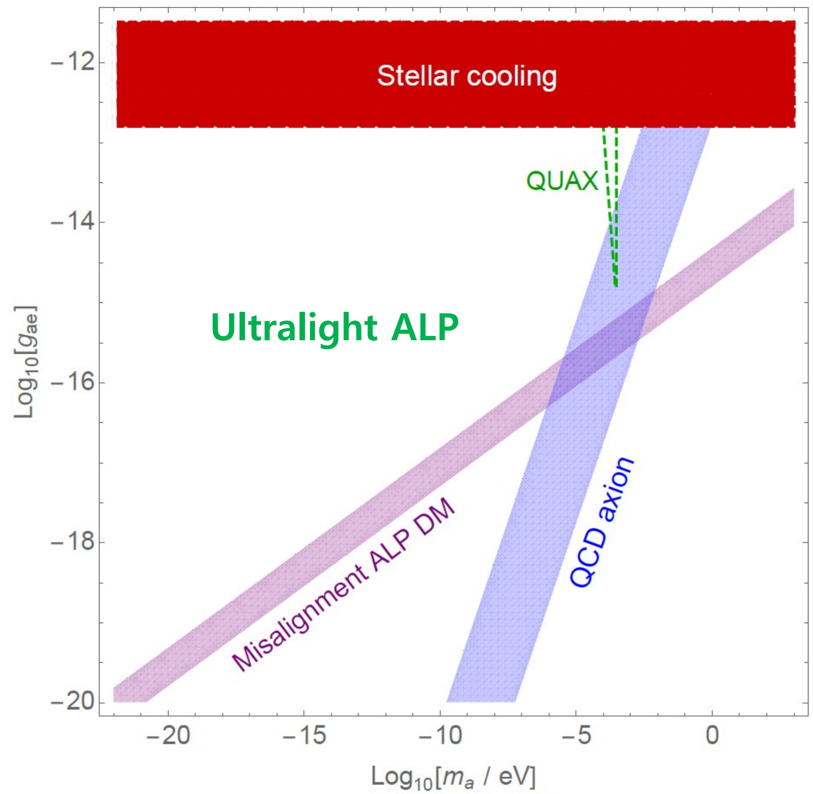
SRF cavity:
Berlin et al '20

KC, Im and Shin, arXiv:2012.02029

g_{aN}



g_{ae}



Once an axion is discovered by any means, so its mass is known, we might be able to experimentally determine all of g_{aX} $(X = \gamma, n, p, e)$ in some future.

Couplings of field-theoretic axions

UV completion within 4D EFT with linearly realized PQWW, KSVZ, DFSZ, ..

$$U(1)_{\rm PQ}: \ \ \Phi \ \to \ e^{iX_\Phi \alpha}\Phi \quad (X_\Phi = \text{quantized PQ-charges})$$

Couplings of field-theoretic axions (in the unit of $1/f_a$) are determined mostly by the quantized PQ-charges:

* Couplings to the photon and gluons:

$$c_\gamma = \sum_\psi X_\psi Q^2_{\rm em}(\psi), \quad c_G = \sum_\psi X_\psi Q^2_{\rm color}(\psi) \quad (\psi = \text{chiral fermions})$$

* Tree-level couplings to the light quarks and electron:

$$C^0_u = X_Q + X_{u^c} + ...$$
$$C^0_d = X_Q + X_{d^c} + ...,$$
$$C^0_e = X_L + X_{e^c} + ...$$

Minimal **DFSZ:** SM fields have flavor-universal non-zero $U(1)_{\rm PQ}$ charges as

Dine-Fischler-Srednicki-Zhitnitsky

$$c_{H_u} = c_{H_d} = -\frac{1}{2}, \quad c_\psi = \frac{1}{4}\ (\psi \in {\rm SM}), \quad c_G = c_W = \frac{3}{5} c_B = 3$$

$$\rightarrow \quad C_u^0 = \cos^2\beta, \quad C_d^0 = C_e^0 = \sin^2\beta \quad \left(\tan\beta = H_u/H_d\right)$$

Minimal **KSVZ:** All SM fields are neutral under $U(1)_{\rm PQ}$, but there exist exotic PQ-charged quark Q & Q^c

Kim-Shifman-Vainshtein-Zakharov

$$c_Q = c_{Q^c} = -\frac{1}{2}, \quad c_\psi = 0\ (\psi \in {\rm SM}), \quad c_G = 1, \quad c_W = 0, \quad c_B = 6Y_Q^2$$

$$\rightarrow \quad C_\Psi^0 = 0 \quad (\Psi = u, d, e)$$

Generic field-theoretic axions may be categorized into two classes:

DFSZ-type : SM fields are PQ-charged

➔ $$c_G \ \text{and/or} \ c_{W,B} = \mathcal{O}(1)$$

$$C^0_\Psi = \mathcal{O}(1) \quad (\Psi = u, d, e)$$

KSVZ-type: SM fields are all PQ-neutral, but there exist some exotic PQ-charged fermions charged under the SM gauge group

➔ $$c_G \ \text{and/or} \ c_{W,B} = \mathcal{O}(1)$$

$$C^0_\Psi = 0 \quad (\Psi = u, d, e)$$

Couplings of string-theoretic axions

$$a(x) = \int_{\Sigma_p} C^{(p)}_{[m_1,..,m_p]}(x,y)\, dy^{m_1}...dy^{m_p}$$

or $$\partial_\mu a = \epsilon_{\mu\nu\rho\sigma}(\partial^\nu B^{\rho\sigma} + ...)$$

For each string-theoretic axion, there exists a modulus partner which forms an N=1 chiral superfield together with the axion:

$$T = \tau(x) + i\frac{a(x)}{f_a}$$

τ = Modulus partner of an angular axion field $\frac{a(x)}{f_a} \cong \frac{a(x)}{f_a} + 2\pi$

= <u>Euclidean action (**S**$_{\text{ins}}$) of the brane instanton</u> which is a low energy consequence of the brane which couples to the underlying p-form gauge field

$$\propto \text{Vol}(\Sigma_p) \;\; \text{and/or} \;\; \frac{1}{g_{\text{st}}^n} \quad (g_{\text{st}} = \text{string coupling})$$

4D effective SUGRA

Kahler potential: $K = K_0(T+T^*) + Z_I(T+T^*)\Phi_I^*\Phi_I + ...,$

Gauge kinetic function: $\mathcal{F}_A = \frac{c_A}{8\pi^2}T + ...$ $\left(c_A = \text{rational numbers of order unity}\right)$

$$\left(-\frac{1}{4}\mathrm{Re}(\mathcal{F}_A)F^{A\mu\nu}F^A_{\mu\nu} + \frac{1}{4}\mathrm{Im}(\mathcal{F}_A)F^{A\mu\nu}\tilde{F}^A_{\mu\nu} \right)$$

Couplings of string-theoretic axions (in the unit of $1/f_a$) at $\mu \sim f_a \sim M_{\rm st}$ are determined by the modulus-dependence of the 4D gauge couplings and the matter wavefunction coefficients:

$$\frac{1}{2}\partial_\mu a \partial^\mu a + \frac{\partial_\mu a}{f_a}\left[ic_\phi(\phi^* D^\mu \phi - \phi D^\mu \phi^*) + c_\psi \bar{\psi}\bar{\sigma}^\mu \psi\right] + \frac{c_A}{32\pi^2}\frac{a(x)}{f_a}F^{A\mu\nu}\tilde{F}^A_{\mu\nu}$$

$$\frac{1}{2}f_a^2 = M_P^2 \frac{\partial^2 K_0}{\partial T \partial T^*}$$

$$c_{\phi_I} = \frac{\partial \ln Z_I}{\partial T}, \quad c_{\psi_I} = \frac{\partial \ln(e^{-K_0/2}Z_I)}{\partial T}, \quad c_A = 8\pi^2 \frac{\partial}{\partial T}\mathcal{F}_A,$$

Moduli vacuum value:

* Axion weak gravity conjecture: $\tau = S_{\rm ins} \lesssim \mathcal{O}\left(\frac{M_P}{f_a}\right)$ Arkani-Hamed et al '07

* Axion potential induced by the brane-instanton: Dine et al '85; Blumenhagen et al '09; …

$$\delta V = e^{-\tau}\Lambda^2 M_P^2 \cos\left(\frac{a}{f_a}\right) \quad \text{with} \quad \Lambda^2 \sim m_{3/2}M_P \;\text{ or }\; m_{3/2}^2$$

For QCD axion, $\delta V \lesssim 10^{-10} m_\pi^2 f_\pi^2$ ➔ $\tau \gtrsim \ln\left(10^{10}\Lambda^2/f_\pi m_\pi\right)$

For ultra-light ALP, $\delta V \lesssim m_a^2 f_a^2$ ➔ $\tau \gtrsim 2\ln\left(\Lambda/m_a\right)$

(Note that this lower bound is meaningful only for $m_a \ll m_{3/2}$.)

* To avoid a too large $\frac{1}{g_{\rm GUT}^2} = {\rm Re}(\mathcal{F}_A) = c_A\frac{\tau}{8\pi^2} + \ldots$ ➔ $\tau \lesssim \mathcal{O}\left(\frac{8\pi^2}{g_{\rm GUT}^2}\right)$

➔ $\tau = \mathcal{O}\left(\frac{8\pi^2}{g_{\rm GUT}^2}\right)$

For $\tau = S_{\rm ins} = \mathcal{O}\left(\frac{8\pi^2}{g_{\rm GUT}^2}\right) \gg 1$,

$$\frac{\partial K_0}{\partial \tau} \sim \tau \frac{\partial^2 K_0}{\partial \tau^2} \sim \tau \frac{f_a^2}{M_P^2} \lesssim \mathcal{O}\Big(\frac{g_{\rm GUT}^2}{8\pi^2}\Big),$$

$$\frac{\partial \ln Z_I}{\partial \tau} \sim \frac{1}{\tau} = \mathcal{O}\Big(\frac{g_{\rm GUT}^2}{8\pi^2}\Big),$$

➔ $$c_\psi \sim c_\phi = \mathcal{O}\left(\frac{1}{\tau}\right) = \mathcal{O}\left(\frac{g_{\rm GUT}^2}{8\pi^2}\right)$$

➔ $$C_\Psi^0 = \mathcal{O}\left(\frac{1}{\tau}\right) = \mathcal{O}\left(\frac{g_{\rm GUT}^2}{8\pi^2}\right) \quad (\Psi = u, d, e)$$

for string-theoretic axions

Overall strength of the axion couplings is determined by f_a .

Is there any range of f_a favored by string-theoretic axions?

In string compactifications without a big hierarchy between the string scale and the 4D Planck scale, KC & Kim, '85 Svrcek & Witten '06

$$f_a = \mathcal{O}\left(\frac{g_{\rm GUT}^2}{8\pi^2} M_P\right) \sim 10^{16} - 10^{17}\,{\rm GeV}$$

However, in string compactifications with a large compactification volume and/or a large warp factor which would generate a big scale hierarchy, f_a of string-theoretic axions can in principle be anywhere in the range

$$10^7 - 10^8\ {\rm GeV} \lesssim f_a \lesssim \mathcal{O}\left(\frac{g_{\rm GUT}^2}{8\pi^2} M_P\right)$$

Burgess, Ibanez, Quevedo '99,
Conclon '06,
Cicoli et al '12;
…

Astrophysics — Weak Gravity Conjecture

The same is also true for the f_a of generic field-theoretic axions.

Three types of axions with different pattern of couplings:

$$\frac{C_\Psi^0}{c_A} = \frac{\text{tree-level axion-matter}}{\text{quantized axion-gauge}} \qquad (\Psi = u, d, e; \;\; A = \gamma, G)$$

$$\frac{1}{32\pi^2}\frac{a(x)}{f_a}\left(c_\gamma F^{\mu\nu}\tilde{F}_{\mu\nu} + c_G G^{\alpha\mu\nu}G^\alpha_{\mu\nu}\right) + \sum_{\Psi=u,d,e}\frac{\partial_\mu a}{2f_a}C_\Psi\bar{\Psi}\gamma^\mu\gamma_5\Psi$$

$$c_\gamma = c_W + c_B, \quad C_\Psi = C_\Psi^0 + \Delta C_\Psi$$

c_A =rational numbers (A=γ, G)

Tree level values

Radiative corrections

DFSZ-type: $\frac{C_\Psi^0}{c_A} = \mathcal{O}(1)$ **KSVZ-type:** $\frac{C_\Psi^0}{c_A} = 0$

String-theoretic: $\frac{C_\Psi^0}{c_A} \sim \frac{1}{\tau} = \frac{1}{S_{\rm ins}} = \mathcal{O}\left(\frac{g_{\rm GUT}^2}{8\pi^2}\right)$

All three types of axions can have f_a anywhere in the range

$$10^7 - 10^8 \text{ GeV} \lesssim f_a \lesssim \mathcal{O}\left(\frac{g_{\rm GUT}^2}{8\pi^2}M_P\right)$$

Astrophysics Weak Gravity Conjecture

Would it be possible to distinguish these three types of axions by experimentally measurable $\mathrm{g_{aX}}$ $(\mathbf{X} = \gamma, \mathbf{p}, \mathbf{n}, \mathbf{e})$?

$$\frac{1}{2} g_{a\gamma} a(x) \vec{E} \cdot \vec{B} + \partial_\mu a(x) \left(\frac{g_{ap}}{2m_p} \bar{p} \gamma^\mu \gamma_5 p + \frac{g_{an}}{2m_n} \bar{n} \gamma^\mu \gamma_5 n + \frac{g_{ae}}{2m_e} \bar{e} \gamma^\mu \gamma_5 e \right)$$

Both KSVZ-type and string-theoretic axions have $C_\Psi^0 / c_A \ll 1$, therefore one needs to study radiative corrections ΔC_Ψ $(\Psi = u, d, e)$ to see if they have a distinguishable pattern of $\mathrm{g_{aX}}$ $(\mathbf{X} = \gamma, \mathbf{p}, \mathbf{n}, \mathbf{e})$.

RG evolution of axion couplings

High scale axion couplings at $\mu \sim f_a$:

$$\frac{\partial_\mu a}{f_a}\left[ic_\phi(\phi^* D^\mu \phi - \phi D^\mu \phi^*) + c_\psi \bar{\psi}\bar{\sigma}^\mu \psi\right] + c_A \frac{g_A^2}{32\pi^2}\frac{a(x)}{f_a} F^{A\mu\nu}\tilde{F}^A_{\mu\nu}$$

$$\phi = \{H_u, H_d, ...\} \quad \psi = \{Q_i, u_i^c, d_i^c, L_i, e_i^c\} \quad F^A_{\mu\nu} = (G^\alpha_{\mu\nu}, W^i_{\mu\nu}, B_{\mu\nu})$$

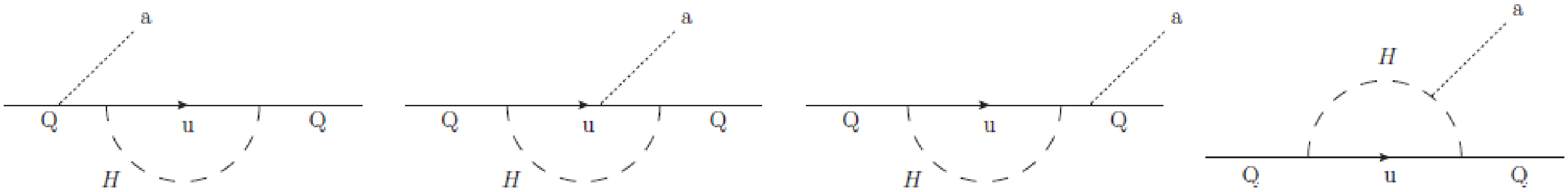
$$c_A = \{c_G, c_W, c_B\}: \text{ rational numbers}$$

$$c_\phi(f_a) = c_\phi^0, \quad c_\psi(f_a) = c_\psi^0$$

RG evolution of axion couplings

Yukawa-induced 1-loop running:

KC, Im, Park, Yun, 1708.00021



$$\left.\frac{d\mathbf{c}_F}{d\ln\mu}\right|_{1-\text{loop}} = \frac{\xi_y}{16\pi^2}\sum_{f,\alpha}\left(\frac{1}{2}\{\mathbf{c}_F, \mathbf{y}^\dagger_{fF\alpha}\mathbf{y}_{fF\alpha}\} + \mathbf{y}^\dagger_{fF\alpha}\mathbf{c}^T_f\mathbf{y}_{fF\alpha} + c_{H_\alpha}\mathbf{y}^\dagger_{fF\alpha}\mathbf{y}_{fF\alpha}\right),$$

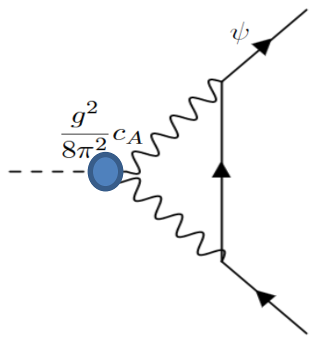
$$\left.\frac{d\mathbf{c}^T_f}{d\ln\mu}\right|_{1-\text{loop}} = \frac{\xi_y}{16\pi^2}\sum_{F,\alpha}\left(\frac{1}{2}\{\mathbf{c}^T_f, \mathbf{y}_{fF\alpha}\mathbf{y}^\dagger_{fF\alpha}\} + \mathbf{y}_{fF\alpha}\mathbf{c}_F\mathbf{y}^\dagger_{fF\alpha} + c_{H_\alpha}\mathbf{y}_{fF\alpha}\mathbf{y}^\dagger_{fF\alpha}\right),$$

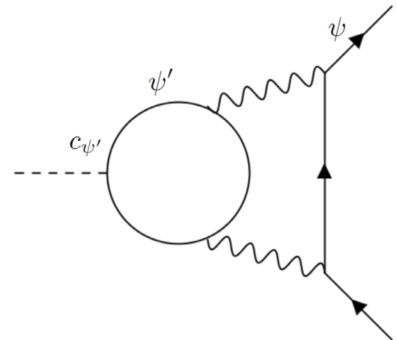
$$\left.\frac{dc_{H_\alpha}}{d\ln\mu}\right|_{1-\text{loop}} = \frac{1}{8\pi^2}\sum_{f,F}\left(c_{H_\alpha}\text{tr}(\mathbf{y}^\dagger_{fF\alpha}\mathbf{y}_{fF\alpha}) + \text{tr}(\mathbf{y}_{fF\alpha}\mathbf{c}_F\mathbf{y}^\dagger_{fF\alpha}) + \text{tr}(\mathbf{y}^\dagger_{fF\alpha}\mathbf{c}^T_f\mathbf{y}_{fF\alpha})\right),$$

$$F_i = \{q_i, \ell_i\} \qquad f_i = \{u^c_i, d^c_i, e^c_i\}$$

$$\text{non-SUSY}: \ \xi_y = 1 \qquad \text{SUSY}: \ \xi_y = 2$$

Gauge-induced 2-loop (computationally 1-loop) running:





KC, Im, Shin, 2012.05029
Baur et al, 2012.12272
KC, Im, Seong, Kim, arXiv:2106.05816

$$\left.\frac{d\mathbf{c}_\psi}{d\ln\mu}\right|_{2-\mathrm{loop}} = -\xi_g \sum_A \frac{3}{2}\left(\frac{g_A^2}{8\pi^2}\right)^2 \mathbb{C}_A(\psi)\Big(c_A - 2\sum_{\psi'}\mathrm{tr}(\mathbf{c}_{\psi'})\mathbb{T}_A(\psi')\Big)\mathbb{1},$$

$$\left.\frac{dc_{H_\alpha}}{d\ln\mu}\right|_{2-\mathrm{loop}} = -\xi_H \sum_A \frac{3}{2}\left(\frac{g_A^2}{8\pi^2}\right)^2 \mathbb{C}_A(H_\alpha)\Big(c_A - 2\sum_{\psi'}\mathrm{tr}(\mathbf{c}_{\psi'})\mathbb{T}_A(\psi')\Big),$$

$\mathbb{C}_A(\Phi)$ = quadratic Casimir $\mathbb{T}_A(\Phi)$ = Dynkin index

$$\text{non-SUSY}: \ \xi_g = 1, \ \xi_H = 0 \qquad \text{SUSY}: \ \xi_g = \xi_H = \frac{2}{3}$$

MSSM:
$$\frac{d\mathbf{c}_Q}{d\ln\mu} = \frac{\xi_y}{16\pi^2}\left(\frac{1}{2}\{\mathbf{c}_Q, \tilde{\mathbf{y}}_u^\dagger\tilde{\mathbf{y}}_u + \tilde{\mathbf{y}}_d^\dagger\tilde{\mathbf{y}}_d\} + \tilde{\mathbf{y}}_u^\dagger\mathbf{c}_{u^c}^T\tilde{\mathbf{y}}_u + \tilde{\mathbf{y}}_d^\dagger\mathbf{c}_{d^c}^T\tilde{\mathbf{y}}_d + c_{H_u}\tilde{\mathbf{y}}_u^\dagger\tilde{\mathbf{y}}_u + c_{H_d}\tilde{\mathbf{y}}_d^\dagger\tilde{\mathbf{y}}_d\right)$$
$$-\xi_g\left(\frac{\alpha_s^2}{2\pi^2}\tilde{c}_G + \frac{9\alpha_2^2}{32\pi^2}\tilde{c}_W + \frac{\alpha_1^2}{96\pi^2}\tilde{c}_B\right)\mathbb{1},$$
$$\frac{d\mathbf{c}_{u^c}^T}{d\ln\mu} = \frac{\xi_y}{8\pi^2}\left(\frac{1}{2}\{\mathbf{c}_{u^c}^T, \tilde{\mathbf{y}}_u\tilde{\mathbf{y}}_u^\dagger\} + \tilde{\mathbf{y}}_u\mathbf{c}_Q\tilde{\mathbf{y}}_u^\dagger + c_{H_u}\tilde{\mathbf{y}}_u\tilde{\mathbf{y}}_u^\dagger\right) - \xi_g\left(\frac{\alpha_s^2}{2\pi^2}\tilde{c}_G + \frac{\alpha_1^2}{6\pi^2}\tilde{c}_B\right)\mathbb{1},$$
$$\frac{d\mathbf{c}_{d^c}^T}{d\ln\mu} = \frac{\xi_y}{8\pi^2}\left(\frac{1}{2}\{\mathbf{c}_{d^c}^T, \tilde{\mathbf{y}}_d\tilde{\mathbf{y}}_d^\dagger\} + \tilde{\mathbf{y}}_d\mathbf{c}_Q\tilde{\mathbf{y}}_d^\dagger + c_{H_d}\tilde{\mathbf{y}}_d\tilde{\mathbf{y}}_d^\dagger\right) - \xi_g\left(\frac{\alpha_s^2}{2\pi^2}\tilde{c}_G + \frac{\alpha_1^2}{24\pi^2}\tilde{c}_B\right)\mathbb{1},$$
$$\frac{d\mathbf{c}_L}{d\ln\mu} = \frac{\xi_y}{16\pi^2}\left(\frac{1}{2}\{\mathbf{c}_L, \tilde{\mathbf{y}}_e^\dagger\tilde{\mathbf{y}}_e\} + \tilde{\mathbf{y}}_e^\dagger\mathbf{c}_{e^c}^T\tilde{\mathbf{y}}_e + c_{H_e}\tilde{\mathbf{y}}_e^\dagger\tilde{\mathbf{y}}_e\right) - \xi_g\left(\frac{9\alpha_2^2}{32\pi^2}\tilde{c}_W + \frac{3\alpha_1^2}{32\pi^2}\tilde{c}_B\right)\mathbb{1},$$
$$\frac{d\mathbf{c}_{e^c}^T}{d\ln\mu} = \frac{\xi_y}{8\pi^2}\left(\frac{1}{2}\{\mathbf{c}_{e^c}^T, \tilde{\mathbf{y}}_e\tilde{\mathbf{y}}_e^\dagger\} + \tilde{\mathbf{y}}_e\mathbf{c}_L\tilde{\mathbf{y}}_e^\dagger + c_{H_e}\tilde{\mathbf{y}}_e\tilde{\mathbf{y}}_e^\dagger\right) - \xi_g\frac{3\alpha_1^2}{8\pi^2}\tilde{c}_B\mathbb{1},$$
$$\frac{dc_{H_u}}{d\ln\mu} = \frac{3}{8\pi^2}\left(c_{H_u}\mathrm{tr}(\tilde{\mathbf{y}}_u^\dagger\tilde{\mathbf{y}}_u) + \mathrm{tr}(\tilde{\mathbf{y}}_u\mathbf{c}_Q\tilde{\mathbf{y}}_u^\dagger) + \mathrm{tr}(\tilde{\mathbf{y}}_u^\dagger\mathbf{c}_{u^c}^T\tilde{\mathbf{y}}_u)\right) - \xi_H\left(\frac{9\alpha_2^2}{32\pi^2}\tilde{c}_W + \frac{3\alpha_1^2}{32\pi^2}\tilde{c}_B\right),$$
$$\frac{dc_{H_d}}{d\ln\mu} = \frac{3}{8\pi^2}\left(c_{H_d}\mathrm{tr}(\tilde{\mathbf{y}}_d^\dagger\tilde{\mathbf{y}}_d) + \mathrm{tr}(\tilde{\mathbf{y}}_d\mathbf{c}_Q\tilde{\mathbf{y}}_d^\dagger) + \mathrm{tr}(\tilde{\mathbf{y}}_d^\dagger\mathbf{c}_{d^c}^T\tilde{\mathbf{y}}_d)\right)$$
$$+\frac{1}{8\pi^2}\left(c_{H_d}\mathrm{tr}(\tilde{\mathbf{y}}_e^\dagger\tilde{\mathbf{y}}_e) + \mathrm{tr}(\tilde{\mathbf{y}}_e\mathbf{c}_L\tilde{\mathbf{y}}_e^\dagger) + \mathrm{tr}(\tilde{\mathbf{y}}_e^\dagger\mathbf{c}_{e^c}^T\tilde{\mathbf{y}}_e)\right) - \xi_H\left(\frac{9\alpha_2^2}{32\pi^2}\tilde{c}_W + \frac{3\alpha_1^2}{32\pi^2}\tilde{c}_B\right),$$

$$\tilde{c}_G = c_G - \mathrm{tr}(2\mathbf{c}_{\mathrm{Q}} + \mathbf{c}_{\mathrm{u^c}} + \mathbf{c}_{\mathrm{d^c}}),$$
$$\tilde{c}_W = c_W - \mathrm{tr}(3\mathbf{c}_{\mathrm{Q}} + \mathbf{c}_{\mathrm{L}}) - \frac{3}{2}\xi_{\mathrm{H}}(\mathrm{c}_{\mathrm{H_u}} + \mathrm{c}_{\mathrm{H_d}}),$$
$$\tilde{c}_B = c_B - \mathrm{tr}\left(\frac{1}{3}(\mathbf{c}_{\mathrm{Q}} + 8\,\mathbf{c}_{\mathrm{u^c}} + 2\,\mathbf{c}_{\mathrm{d^c}}) + \mathbf{c}_{\mathrm{L}} + 2\mathbf{c}_{\mathrm{e^c}}\right) - \frac{3}{2}\xi_{\mathrm{H}}(\mathrm{c}_{\mathrm{H_u}} + \mathrm{c}_{\mathrm{H_d}})$$

One can systematically integrate the RG evolution, while including the relevant threshold corrections to compute $\Delta C_\Psi \; (\Psi = u, d, e)$ at $\mu = 2 \text{ GeV}$, and then use

$$\frac{1}{2} g_{a\gamma} a(x) \vec{E} \cdot \vec{B} + \partial_\mu a(x) \left(\frac{g_{ap}}{2m_p} \bar{p}\gamma^\mu \gamma_5 p + \frac{g_{an}}{2m_n} \bar{n}\gamma^\mu \gamma_5 n + \frac{g_{ae}}{2m_e} \bar{e}\gamma^\mu \gamma_5 e \right)$$

$$g_{a\gamma} \simeq \frac{\alpha_{\rm em}}{2\pi} \frac{1}{f_a} \left(c_\gamma - \frac{2}{3} \left(\frac{m_u + 4m_d}{m_u + m_d} \right) c_G \right)$$

$$g_{ap} \simeq \frac{m_p}{f_a} \left(C_u \Delta u + C_d \Delta d - \left(\frac{m_d}{m_u + m_d} \Delta u + \frac{m_u}{m_u + m_d} \Delta d \right) c_G \right)$$

$$\simeq \frac{m_p}{f_a} \left(0.88 C_u - 0.39 C_d - 0.47 c_G \right)$$

$$g_{an} \simeq \frac{m_n}{f_a} \left(C_d \Delta u + C_u \Delta d - \left(\frac{m_u}{m_u + m_d} \Delta u + \frac{m_d}{m_u + m_d} \Delta d \right) c_G \right)$$

$$\simeq \frac{m_n}{f_a} \left(-0.39 C_u + 0.88 C_d - 0.02 c_G \right)$$

$$g_{ae} \simeq \frac{m_e}{f_a} C_e$$

accidental cancellation for the contribution from $\mathbf{c_G}$

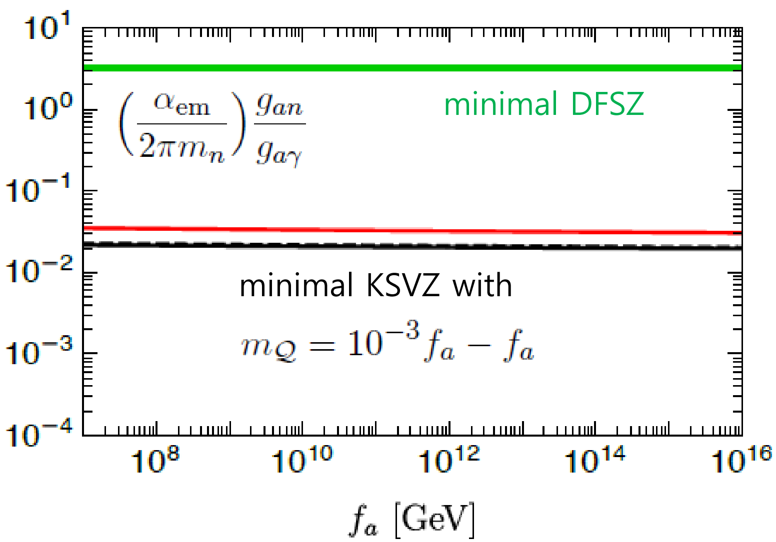
$g_{aX}/g_{a\gamma}$ $(X = p, n, e)$ including radiative corrections

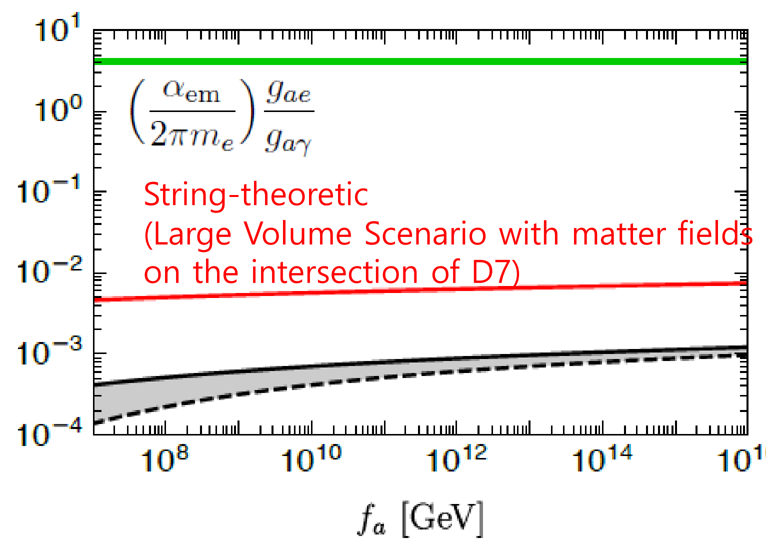
KC, Im, Seong, Kim, arXiv:2106.05816

QCD axion $(c_G \neq 0)$

* All three-types of QCD axions have a similar value of $g_{ap}/g_{a\gamma}$ because of the contribution from the axion-meson mixing induced by $\mathbf{c_G}$.

* As the axion-meson mixing contribution to g_{an} & g_{ae} are small, these three-type of QCD axions have distinguishable pattern of $g_{an}/g_{a\gamma}$ and $g_{ae}/g_{a\gamma}$.





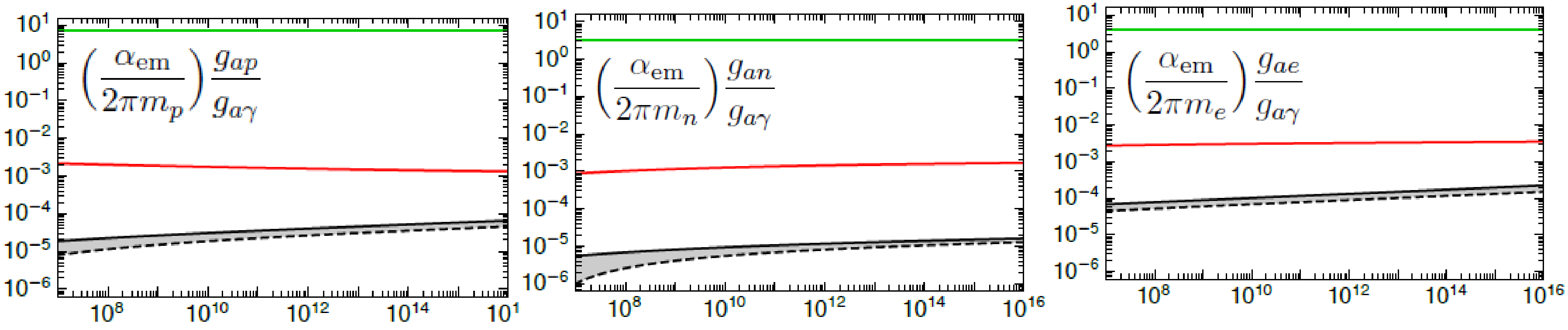
$$\left(\frac{g_{an}}{g_{a\gamma}}\right)_{\rm DFSZ} \gg \left(\frac{g_{an}}{g_{a\gamma}}\right)_{\rm string} \simeq \left(\frac{g_{an}}{g_{a\gamma}}\right)_{\rm KSVZ}$$

$$\left(\frac{g_{ae}}{g_{a\gamma}}\right)_{\rm DFSZ} \gg \left(\frac{g_{ae}}{g_{a\gamma}}\right)_{\rm string} \gg \left(\frac{g_{ae}}{g_{a\gamma}}\right)_{\rm KSVZ}$$

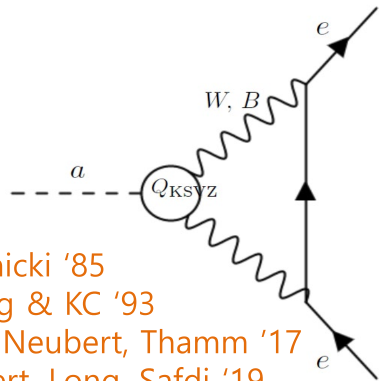
Ultra-light ALP $(c_G = 0,\ \ c_B \text{ or } c_W = 1)$

Different type of ALPs have clearly distinguishable pattern for all three coupling ratios:

$$\left(\frac{g_{aX}}{g_{a\gamma}}\right)_{\rm DFSZ} \gg \left(\frac{g_{aX}}{g_{a\gamma}}\right)_{\rm string} \gg \left(\frac{g_{aX}}{g_{a\gamma}}\right)_{\rm KSVZ} \qquad \left(X = p, n, e\right)$$

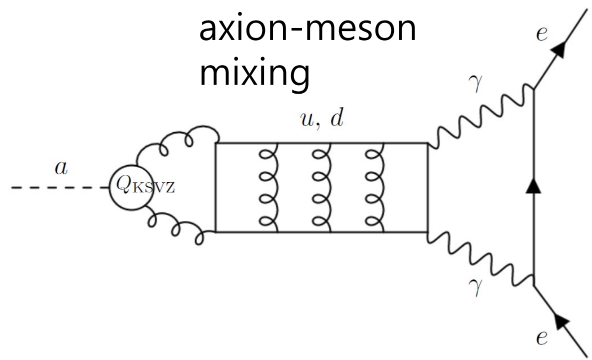


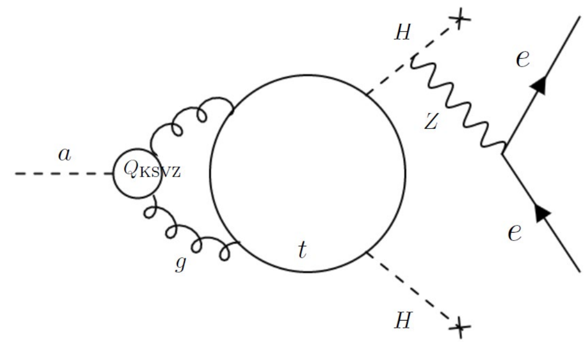
Coupling of KSVZ-type QCD axion to the electron



Srednicki '85
Chang & KC '93
Baur, Neubert, Thamm '17
Dessert, Long, Safdi '19

axion-meson mixing





Two-loop RG-running from the exotic heavy quark mass to m_e

$$\left(g_{ae}\right)_{\rm KSVZ} = \frac{m_e}{f_a}\Big[\frac{3\alpha_{\rm em}^2}{4\pi^2}\left(\frac{3}{8}\frac{c_W}{s_W^4}+\frac{5}{8}\frac{c_B}{c_W^4}\right)\ln\frac{M_{Q_{\rm KSVZ}}}{m_e} - \frac{\alpha_{\rm em}^2}{2\pi^2}\left(\frac{4m_d+m_u}{m_u+m_d}\right)c_G\ln\frac{4\pi f_\pi}{m_e}$$

$$+\,\mathcal{O}\left(\frac{y_t^2\alpha_s^2}{8\pi^4}\right)c_G\ln\frac{M_{Q_{\rm KSVZ}}}{m_t}\Big]$$

axion-meson-mixing

higher-loop

$$= 10^{-4}\frac{m_e}{f_a}\left[1.3\,c_B+5.4\,c_W-0.3\,c_G+8.3\,c_G\right] \quad \text{for} \quad f_a \gtrsim m_{Q_{\rm KSVZ}} = 10^{10}\,\text{GeV}$$

The higher loop contribution which was not considered before gives the dominant contribution.

Conclusion

- Considering the low energy couplings and the origin of axion field variable, axions can be categorized into three types:

  DFSZ-type, KSVZ-type, String-theoretic,

  having parametrically different ratios between the couplings to gauge fields and the couplings to matter fields at the UV scale.

- If an axion is discovered, so its mass is identified, we might be able to measure multiple number of low energy axion couplings and determine the coupling ratios $\mathbf{g_{aX} / g_{a\gamma}}$ **(X=p,n,e)** which would give us information on the underlying axion model.

  Compared to the other two types, DFSZ-type has a clearly distinct pattern of low energy couplings.

  However, to discriminate string-theoretic from KSVZ-type, we need an analysis taking into account the radiative corrections to axion couplings.

- For **QCD axions,** $g_{ae}/g_{a\gamma}$ of KSVZ-type and string-theoretic axions differ by about an order of magnitude, while $g_{aN}/g_{a\gamma}$ are similar to each other.

  For **ultra-light ALPs** without the coupling to gluons, different types of ALPs have clearly distinguishable pattern for all three coupling ratios **$g_{aX}/g_{a\gamma}$ (X=p,n,e)**.

- The coupling of KSVZ-type QCD axion to the electron is dominated by the higher-loop contribution involving the top quark and Higgs boson, which was ignored in the previous studies.

Thank you for your attention.