

The First International Conference on Axion Physics and Experiment

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Lepton-flavor-violating decays into axion-like particles

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mainly based on LC, D. Redigolo, R. Ziegler, J. Zupan,
[JHEP 09 \(2021\) 173](#)

Assume there is a *light, invisible*, new particle “ a ”
with *flavour-violating couplings* to leptons

Light:

$$m_a < m_\mu, m_\tau$$

Invisible:

- Neutral
- Feebly coupled (long-lived)

CLFV modes would then be $\mu \rightarrow e a$, $\tau \rightarrow \mu a$, $\mu \rightarrow e \gamma a$, etc.

Interesting interplay with cosmo/astro:

- DM candidate? (if long-lived enough)
- Bounds from star cooling/supernovae (if light and feeble enough)

Lepton-flavour-violating ALPs

Why should a be light and feebly-coupled?

That's natural, if it is the (pseudo) Nambu-Goldstone boson (PNGB) of a broken global U(1), *aka* an axion-like particle (ALP)

Examples:

Global symmetry:	PNGB:
• Lepton Number	Majoron
• Peccei-Quinn	Axion
• Flavour symmetry	Familon
...	

[Wilczek '82](#)
[Pilaftsis '93](#)
[Feng et al. '97](#)
[LC Goertz Redigolo](#)
[Ziegler Zupan '16](#)
[Di Luzio et al. '17, '19](#)

...

Equivalent possibility: light Z' of a local U(1), e.g. L_i-L_j (with $g \ll 1$)

[Heeck '16](#)

General couplings to leptons:

Shift symmetry (PNGB!) $\rightarrow m_a$ from (small) explicit U(1) breaking

$$\mathcal{L}_{all} = \frac{\partial^\mu a}{2f_a} \left(C_{ij}^V \bar{\ell}_i \gamma_\mu \ell_j + C_{ij}^A \bar{\ell}_i \gamma_\mu \gamma_5 \ell_j \right)$$

U(1)-breaking scale \rightarrow coupling suppression

Where does *lepton flavour violation* come from?

- If lepton U(1) charges are flavour non-universal
 \Rightarrow naturally flavour-violating couplings
- Alternatively, loop-induced flavour-violating couplings

Explicit examples at the end...

$$\mathcal{L}_{all} = \frac{\partial^\mu a}{2f_a} (C_{ij}^V \bar{l}_i \gamma_\mu l_j + C_{ij}^A \bar{l}_i \gamma_\mu \gamma_5 l_j)$$

This generic Lagrangian induces 2-body LFV decays such as:

$$\Gamma(l_i \rightarrow l_j a) = \frac{1}{16\pi} \frac{m_{l_i}^3}{F_{ij}^2} \left(1 - \frac{m_a^2}{m_{l_i}^2}\right)^2 \quad F_{ij} \equiv \frac{2f_a}{\sqrt{|C_{ij}^V|^2 + |C_{ij}^A|^2}}$$

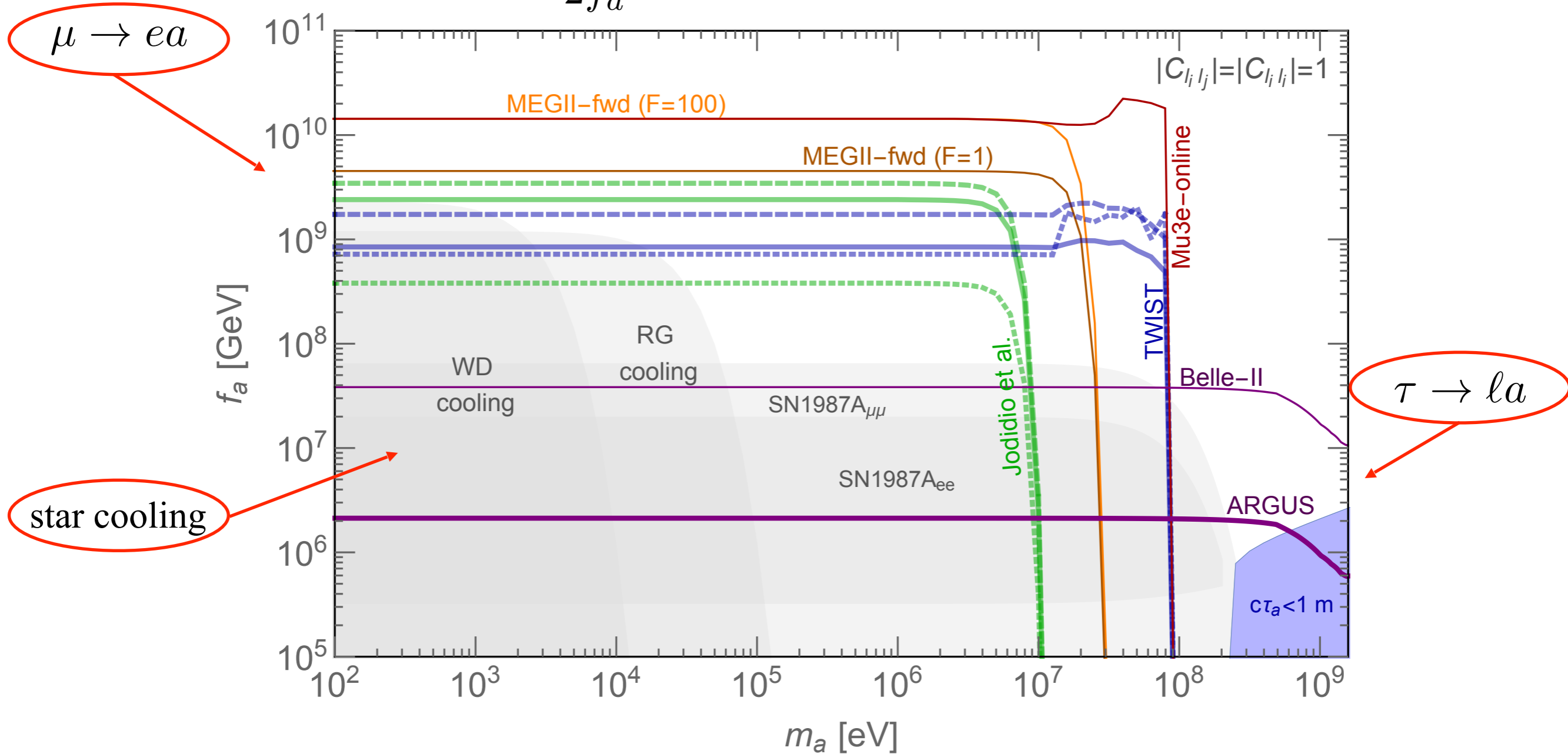
[Feng et al. '97](#)

Goal: constrain the effective LFV scales F_{ij} using experimental data

- Which experiments?
- What are the future prospects?

Summary plot

$$\mathcal{L}_{all} = \frac{\partial^\mu a}{2f_a} (C_{ij}^V \bar{l}_i \gamma_\mu l_j + C_{ij}^A \bar{l}_i \gamma_\mu \gamma_5 l_j)$$



Decays mediated by dim-5 operators: much larger NP scales can be reached than $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$, $\mu \rightarrow e$ conv. (from dim-6 ops, NP scale reach $\sim 10^7 - 10^8$ GeV)

LFV experiments

$\mu \rightarrow e a$: signal and background

Signal: monochromatic positron with $p_e = \sqrt{\left(\frac{m_\mu^2 - m_a^2 + m_e^2}{2m_\mu}\right)^2 - m_e^2}$

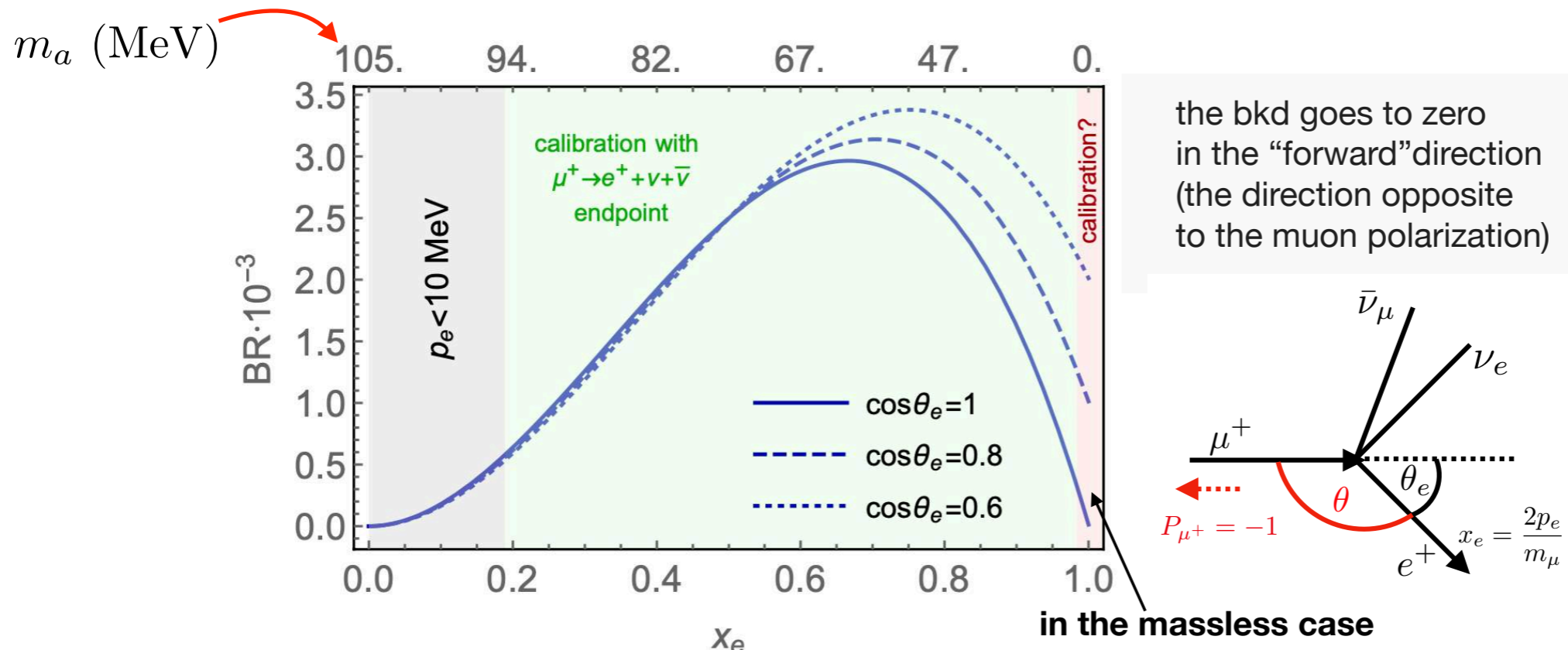
Differential decay rate: $\frac{d\Gamma(l_i \rightarrow l_j a)}{d\cos\theta} = \frac{m_{l_i}^3}{32\pi F_{l_i l_j}^2} \left(1 - \frac{m_a^2}{m_{l_i}^2}\right)^2 \left[1 + 2P_{l_i} \cos\theta \frac{C_{l_i l_j}^V C_{l_i l_j}^A}{(C_{l_i l_j}^V)^2 + (C_{l_i l_j}^A)^2}\right]$

signal depends on the chirality of the couplings

Michel spectrum: $\frac{d^2\Gamma(\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu)}{dx_e d\cos\theta} \simeq \Gamma_\mu ((3 - 2x_e) - P_\mu (2x_e - 1) \cos\theta) x_e^2$ $x_e = \frac{2p_e}{m_\mu}$

μ polarization

And “surface” muons are highly polarized (produced by pion decays at rest on the surface of the production target) \rightarrow the SM background can be suppressed



Past searches: $\mu \rightarrow e a$

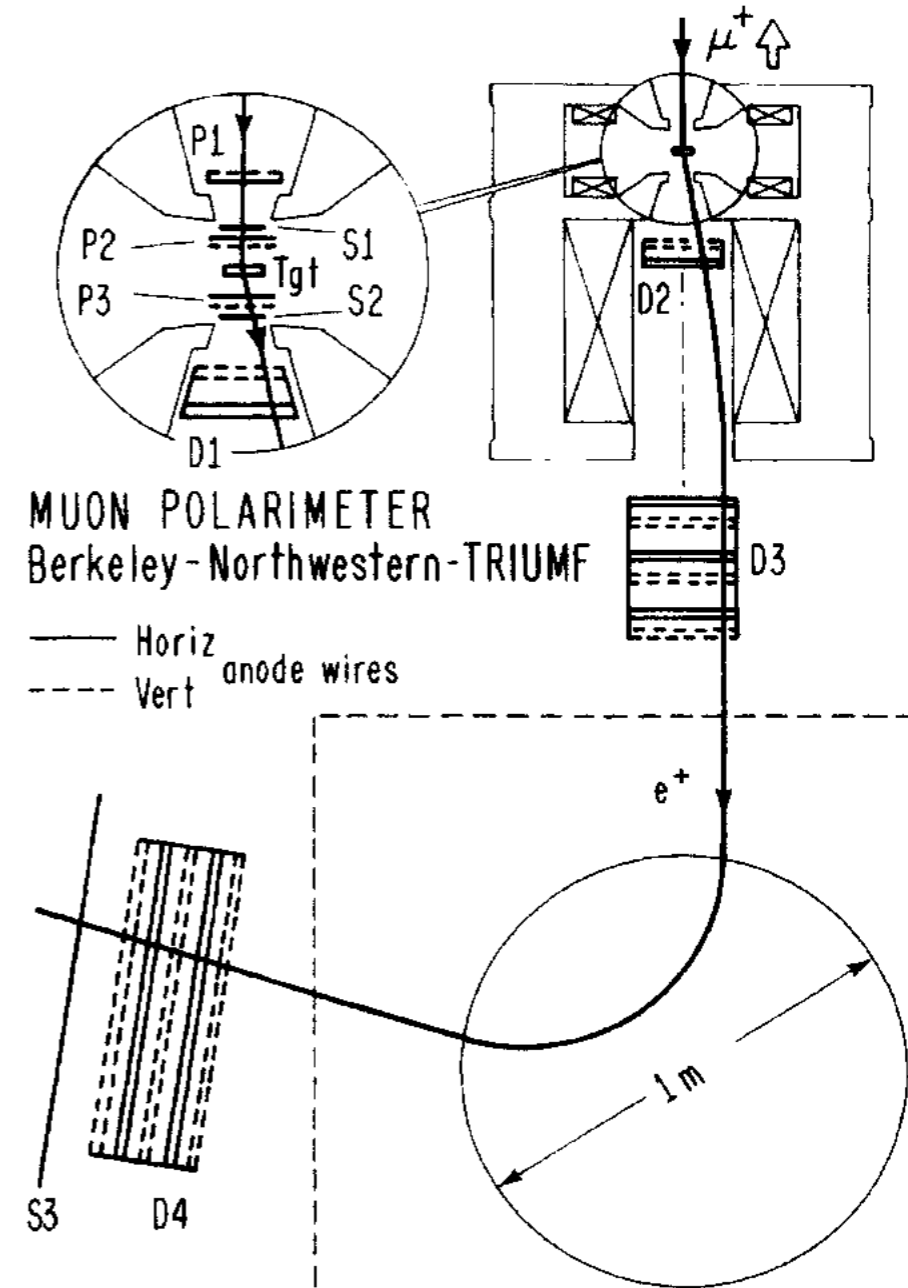
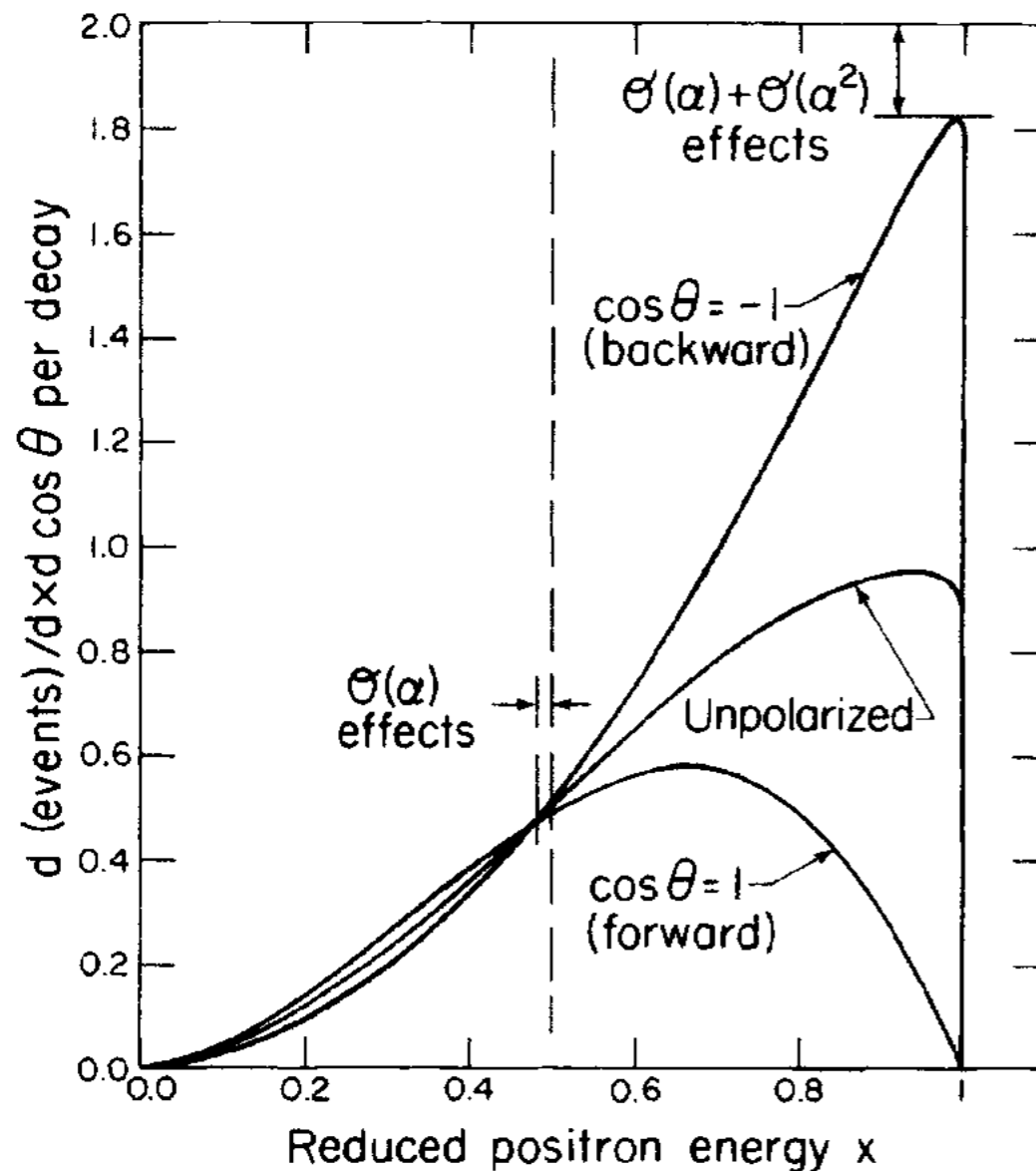
- [Jodidio et al. \(TRIUMF\) 1986](#)

Search for RH currents with 1.8×10^7 polarized μ^+

Ordinary $\mu \rightarrow e \bar{\nu} \nu$

$$\frac{d^2\Gamma}{dx d\cos\theta} = \Gamma_\mu ((3 - 2x) - P(2x - 1) \cos\theta) x^2$$

$$x = 2E_e/m_\mu$$



Very good e^+ momentum resolution
(~ 70 KeV at the e.p.)

Past searches: $\mu \rightarrow e a$

- [Jodidio et al. \(TRIUMF\) 1986](#)

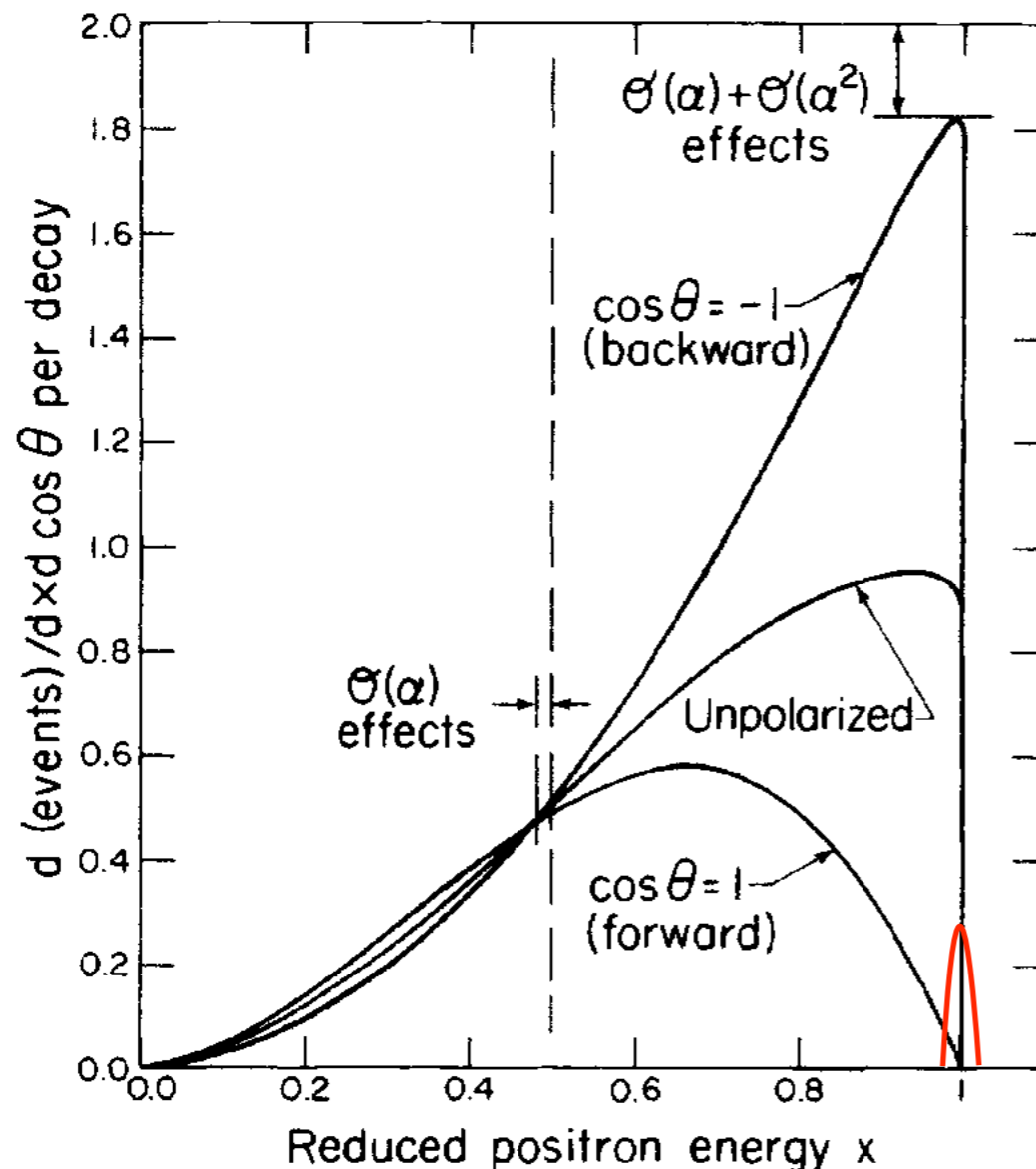
Search for RH currents with 1.8×10^7 polarized μ^+ interpreted in terms of $\mu \rightarrow e a$ too

Ordinary $\mu \rightarrow e \bar{\nu} \nu$

$$\frac{d^2\Gamma}{dx d\cos\theta} = \Gamma_\mu \left((3 - 2x) - P(2x - 1) \cos\theta \right) x^2$$

$$x = 2E_e/m_\mu$$

$\mu \rightarrow e a$ signal for $m_a \approx 0$:
monochromatic e^+ at $m_\mu/2$



Unless it couples (V-A) like in the SM:

$$\frac{d\Gamma(\mu^+ \rightarrow e^+ a)}{d\cos\theta} = \frac{\Gamma_{\mu \rightarrow e a}}{2} \left[1 + 2P \cos\theta \frac{C_{e\mu}^V C_{e\mu}^A}{(C_{e\mu}^V)^2 + (C_{e\mu}^A)^2} \right]$$

for the *isotropic* case, they set the limit

$$\Rightarrow \text{BR}(\mu^+ \rightarrow e^+ a) < 2.6 \times 10^{-6}$$

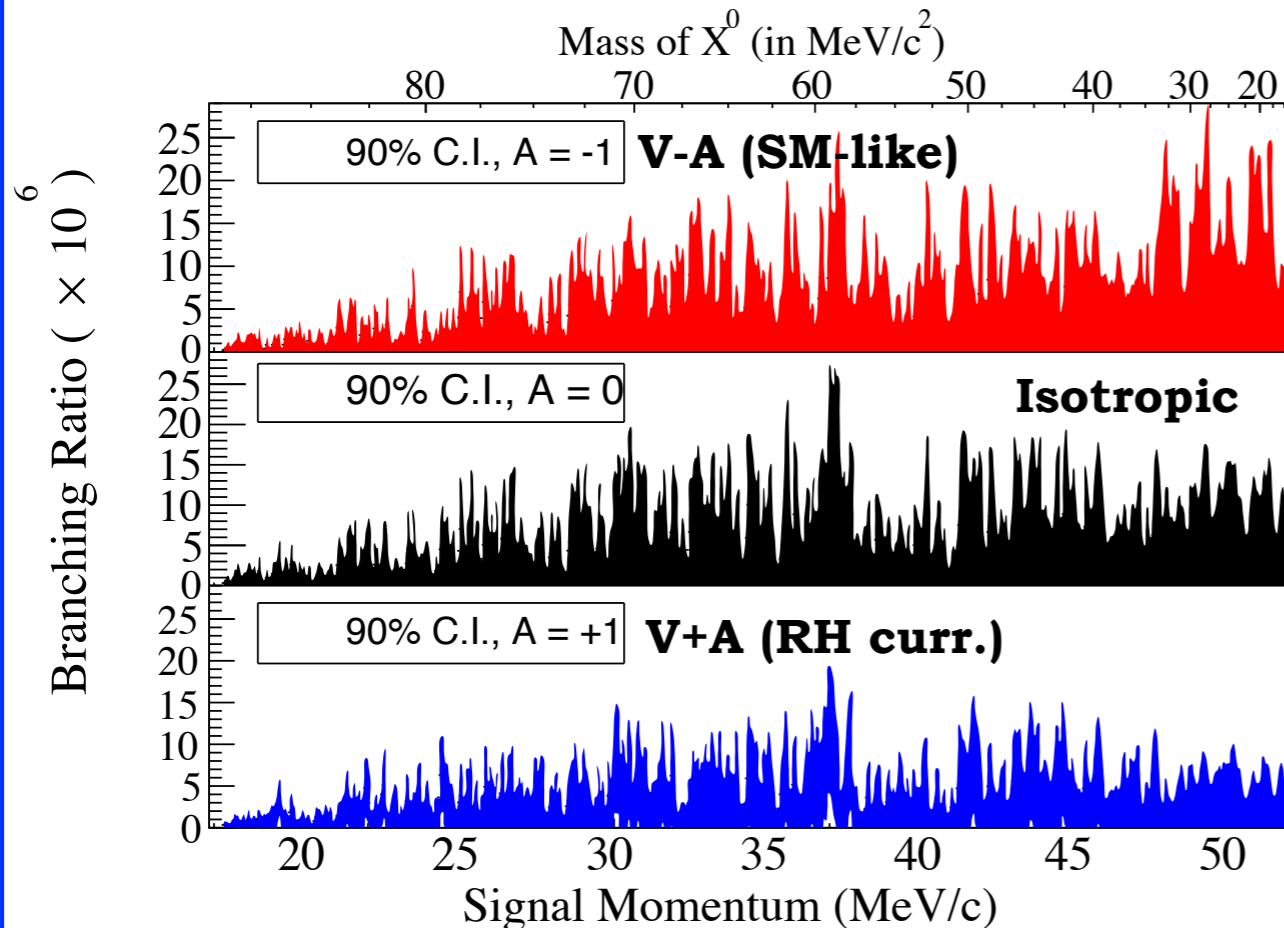
thus one gets

$$\Rightarrow F_{\mu e} > 4.8 \times 10^9 \text{ GeV}$$

Past searches: $\mu \rightarrow e a$

- **TWIST 2014** Precise measurement of Michel parameters plus dedicated search for $\mu \rightarrow e a$ in the whole m_a range considering anisotropy of the signal

Limits (with $5.8 \times 10^8 \mu^+$):



Decay Signal		90% C.L. (in ppm)	p-value
$A = 0$	Average	9	
	$p = 37.03 \text{ MeV}/c$ Endpoint	26	0.66
$A = -1$ SM-like	Average	10	
	$p = 37.28 \text{ MeV}/c$ Endpoint	26	0.60
$A = +1$	Average	6	
	$p = 19.13 \text{ MeV}/c$ Endpoint	6	0.59
		10	0.90

For V-A coupl. and $m_a \approx 0$: $\text{BR}(\mu \rightarrow e a) < 5.8 \times 10^{-5}$

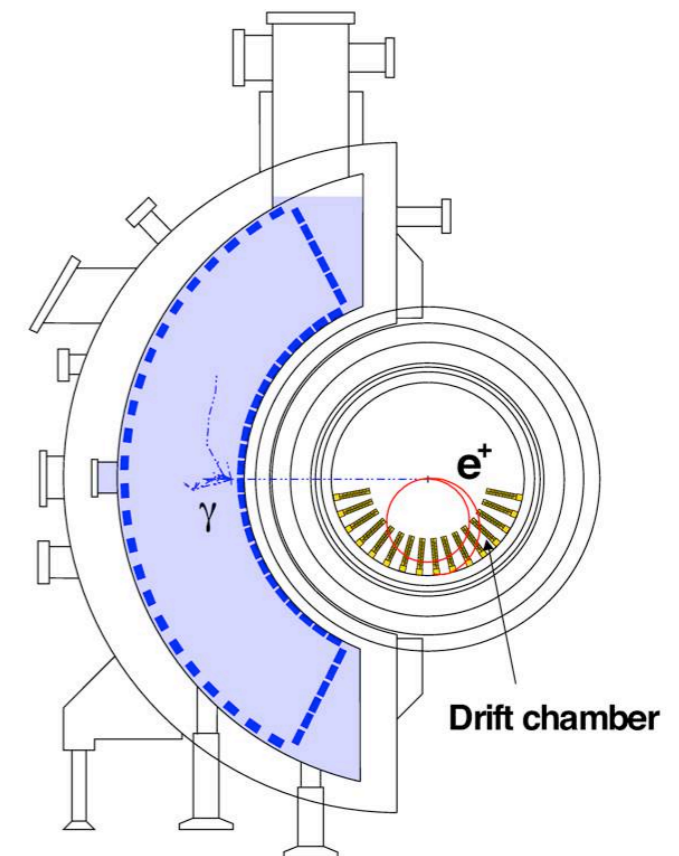
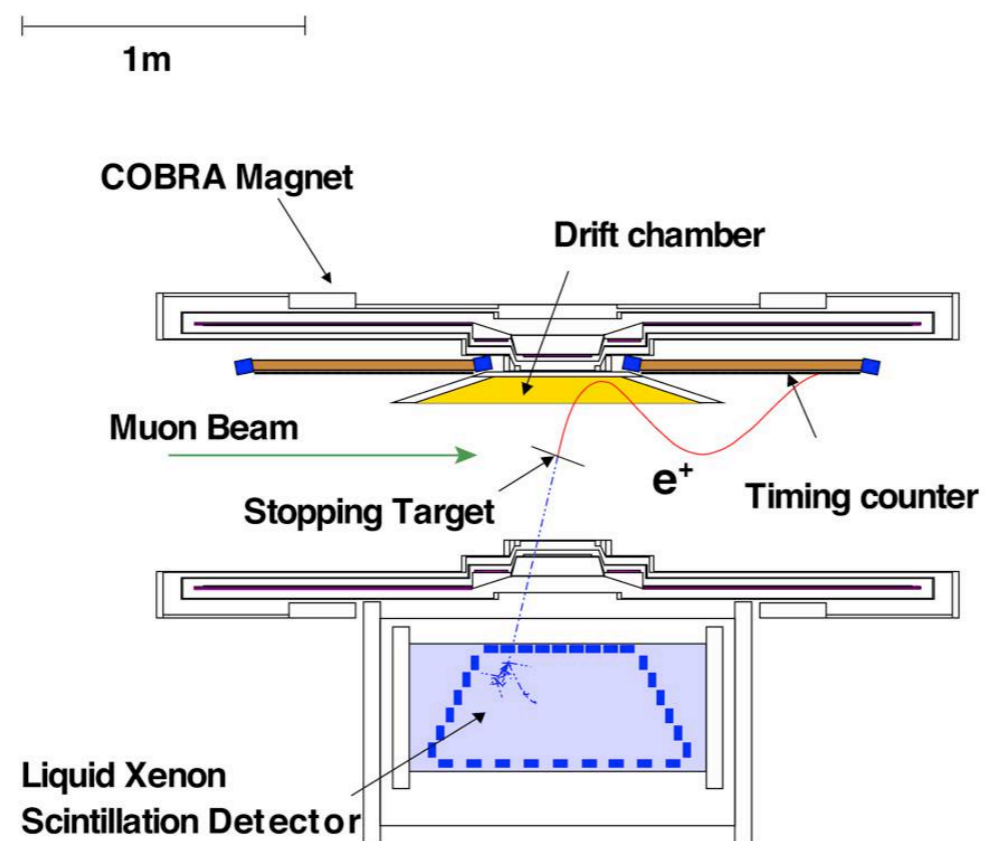
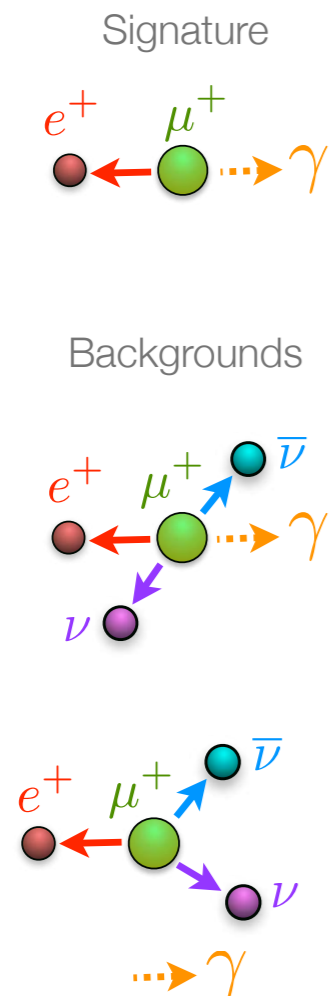
$$\Rightarrow F_{\mu e} > 1.0 \times 10^9 \text{ GeV}$$

Present bounds based on old experiments and/or not-so-high luminosities ($<10^9$ total muon decays)

$\pi E5$ beamline at PSI (where MEGII and Mu3e are located)
can deliver $>10^8$ muons *per second*:
next generation experiment must do better!

MEG: Signature and experimental setup

- The MEG experiment aims to search for $\mu^+ \rightarrow e^+ \gamma$ with a sensitivity of $\sim 10^{-13}$ (previous upper limit $BR(\mu^+ \rightarrow e^+ \gamma) \leq 1.2 \times 10^{-11}$ @90 C.L. by MEGA experiment)
- Five observables (E_γ , E_e , t_{eg} , ϑ_{eg} , ϕ_{eg}) to characterize $\mu \rightarrow e\gamma$ events



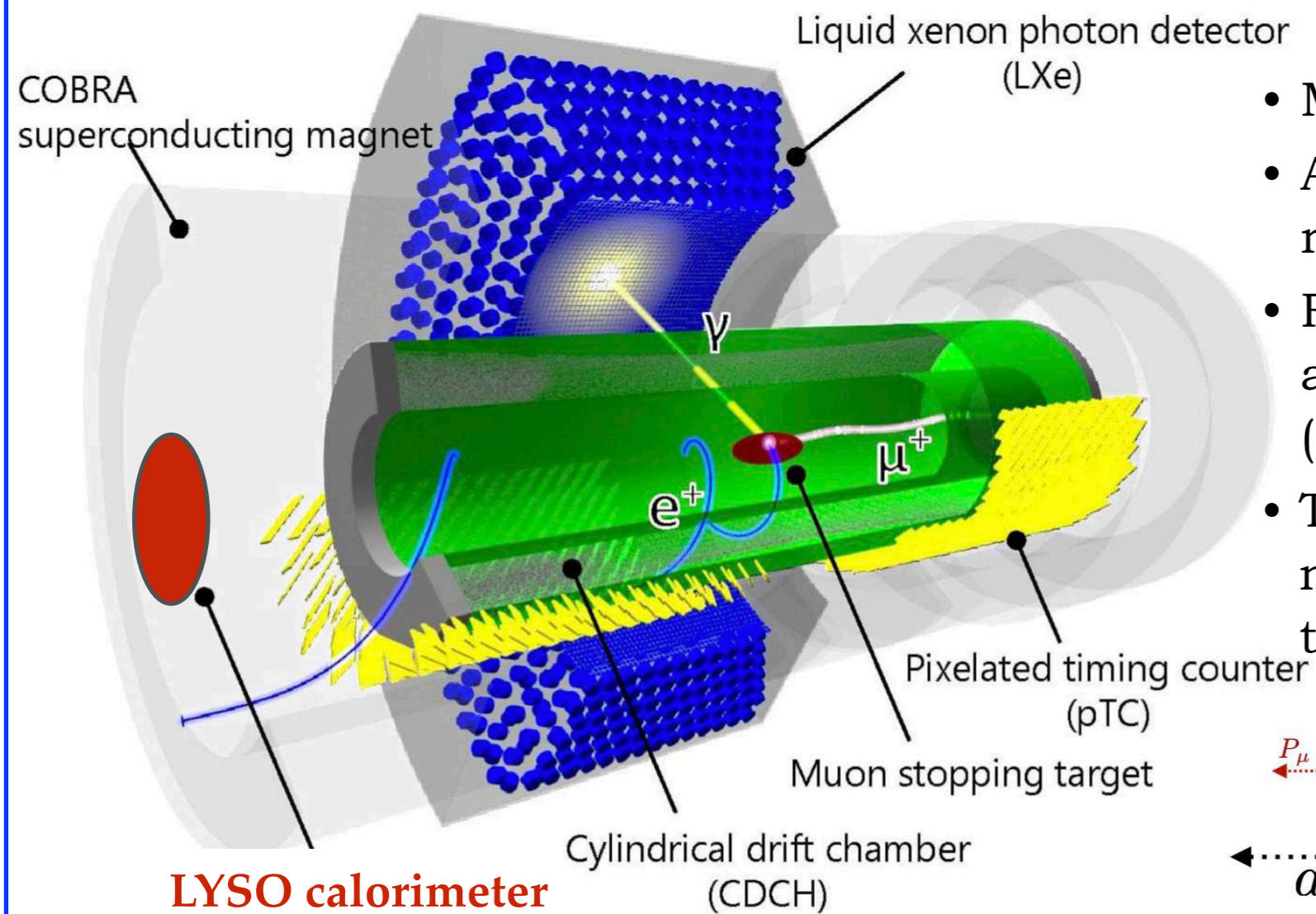
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Final result (with 7.5×10^{14} μ^+ on target): $BR(\mu \rightarrow e\gamma) < 4.2 \times 10^{-13}$ (90% CL)

Future prospects: MEG II

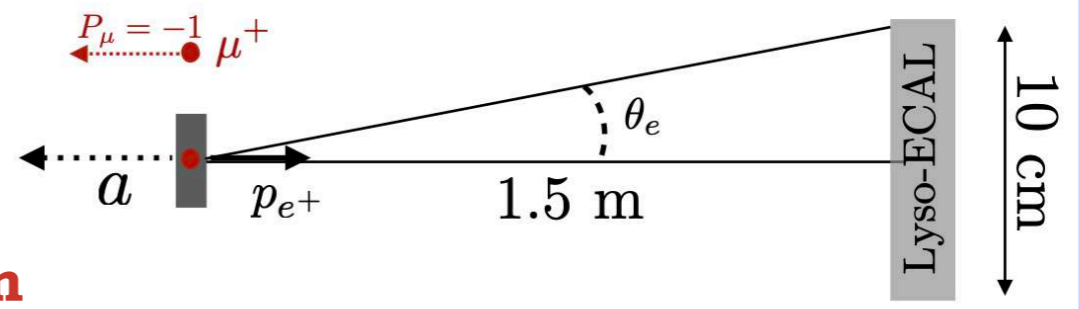
- Prospect at MEG II for $\mu \rightarrow e a$

What about a Jodidio-like search at MEG II for $m_a \approx 0$ with a *forward calorimeter*? We propose a modified setup of MEG II (“MEGII-fwd”) and ~2 weeks dedicated run *idea from discussions with A. Papa and G. Signorelli, thanks!*



LYSO calorimeter
~1.5 m from the target, diameter ~ 10 cm

- Muon beam already polarized
- A suitable magnetic field can reduce depolarization effects
- Reconfiguring the field we can also increase the acceptance (“magnetic focusing” up to $F \sim 100$)
- Two weeks of run after MEG II main run are enough to improve the bound (even with $F=0$)



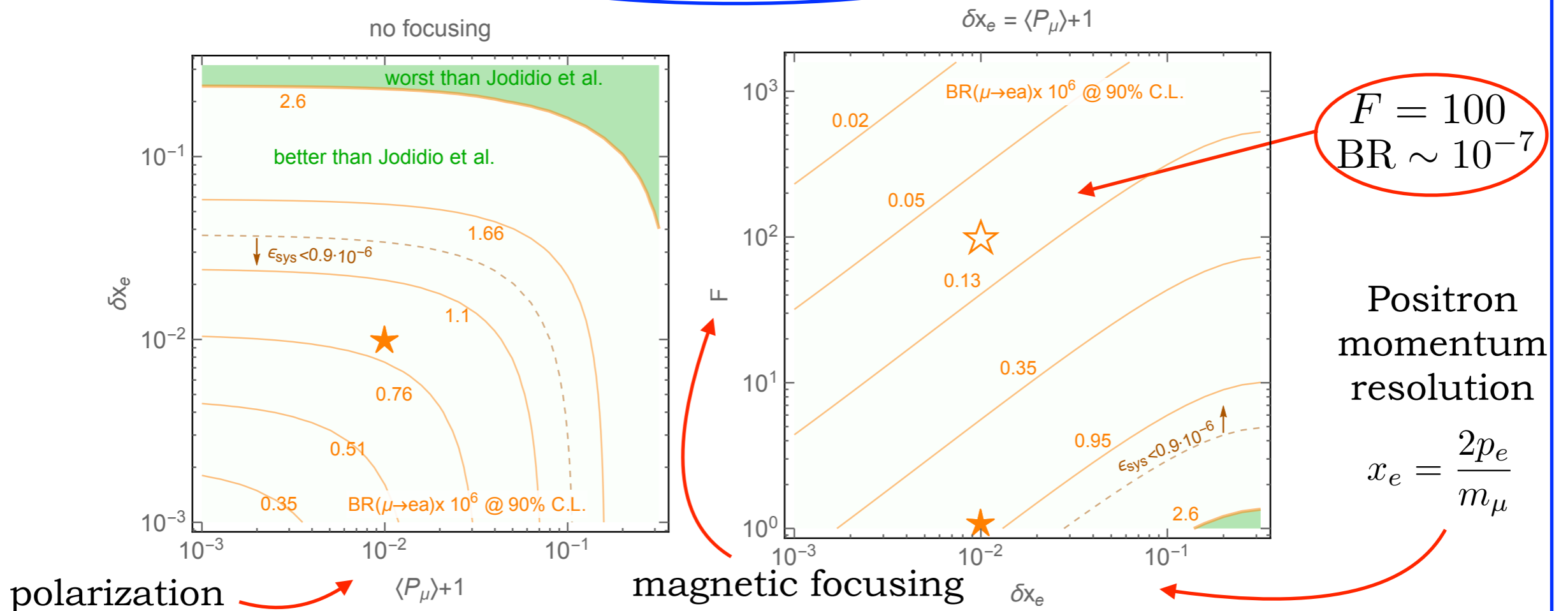
Future prospects: MEG II

- Prospect at MEG II for $\mu \rightarrow e a$

What about a Jodidio-like search at MEG II for $m_a \approx 0$ with a *forward calorimeter*? We propose a modified setup of MEG II (“MEGII-fwd”) and ~ 2 weeks dedicated run *idea from discussions with A. Papa and G. Signorelli, thanks!*

Our estimate of the sensitivity of a dedicate run (2 weeks with $10^8 \mu^+ / s$):

$F = 0$ MEGII-fwd : $10^{14} \mu^+$

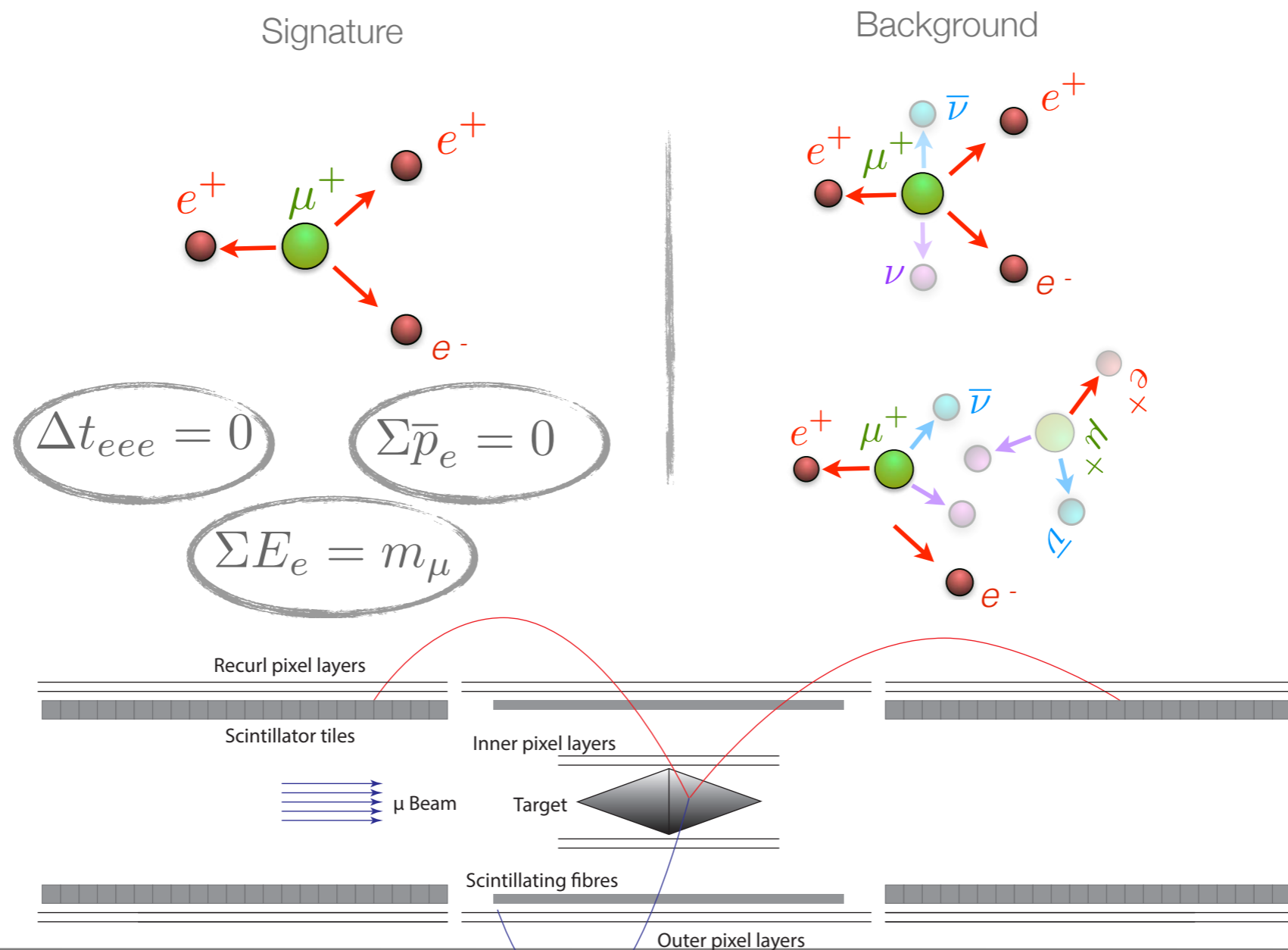


Future prospects: Mu3e

Mu3e: The $\mu^+ \rightarrow e^+ e^+ e^-$ search

slide borrowed from A. Papa

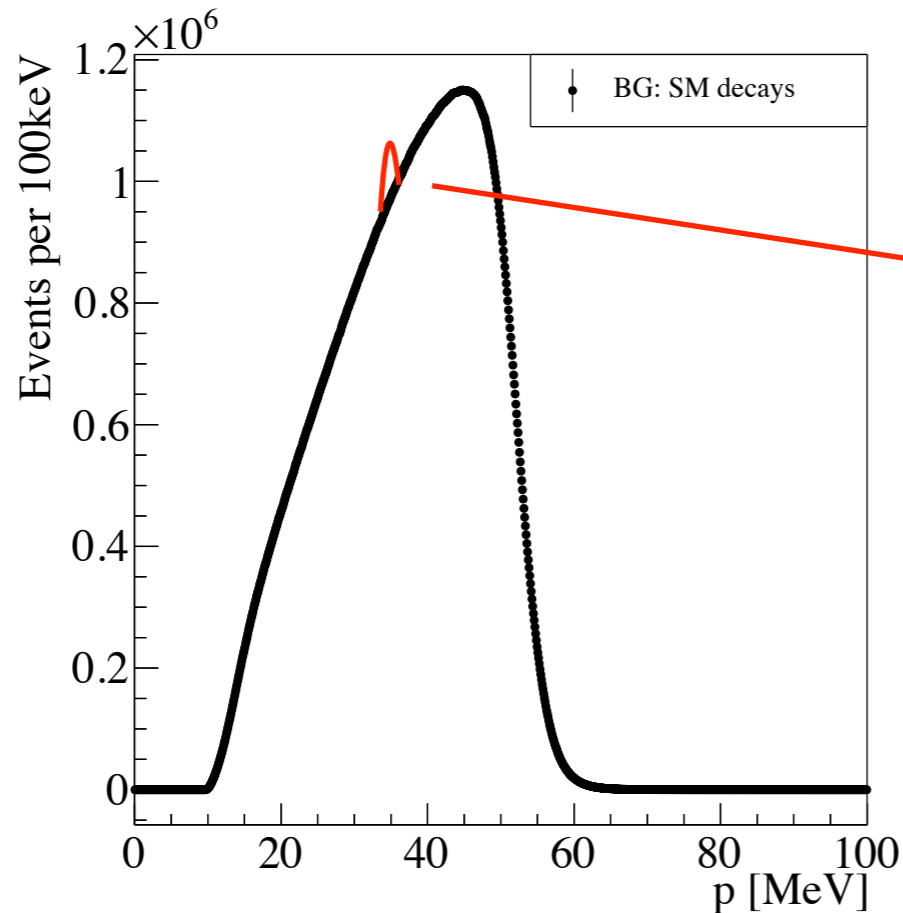
- The Mu3e experiment aims to search for $\mu^+ \rightarrow e^+ e^+ e^-$ with a sensitivity of $\sim 10^{-15}$ (Phase I) up to down $\sim 10^{-16}$ (Phase II). Previous upper limit $BR(\mu^+ \rightarrow e^+ e^+ e^-) \leq 1 \times 10^{-12}$ @90 C.L. by SINDRUM experiment)
- Observables (E_e , t_e , vertex) to characterize $\mu \rightarrow eee$ events



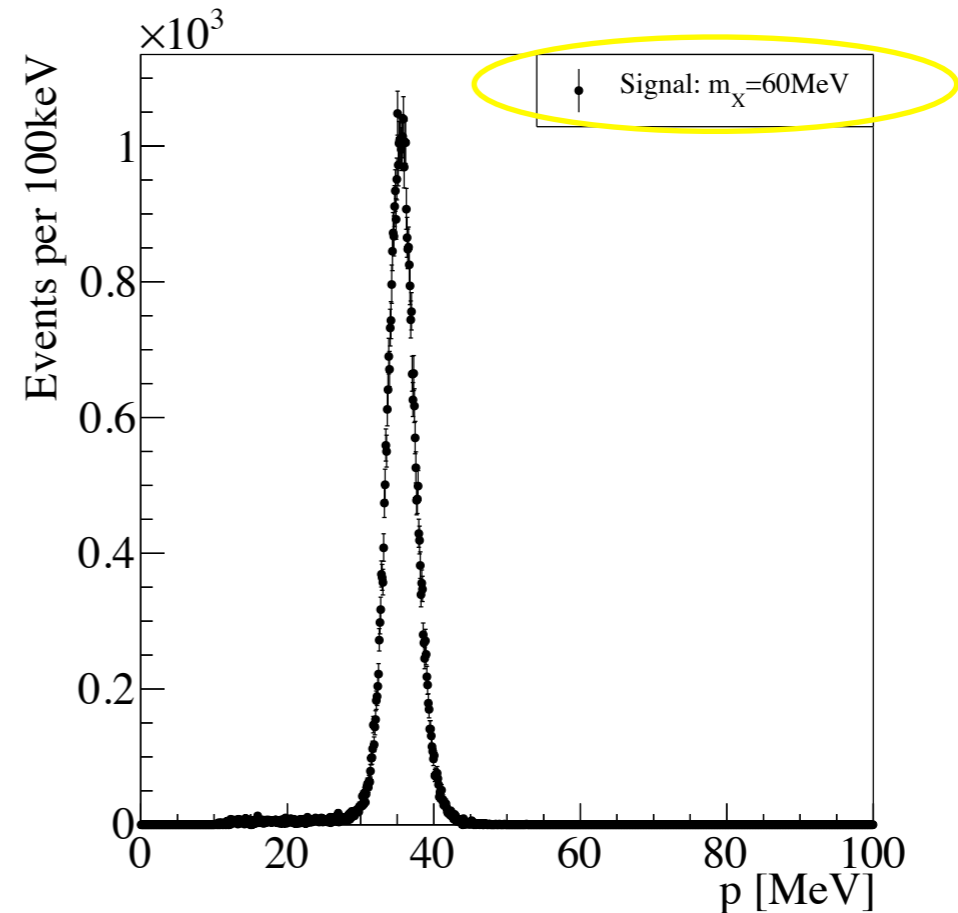
Future prospects: Mu3e

- Mu3e prospect for $\mu \rightarrow e a$ (Perrevoort '18)

Potential search for performed on positron momentum histograms filled with *online* reconstructed short tracks



(a) Simulated background events.



(b) Simulated $\mu \rightarrow eX$ signal events.

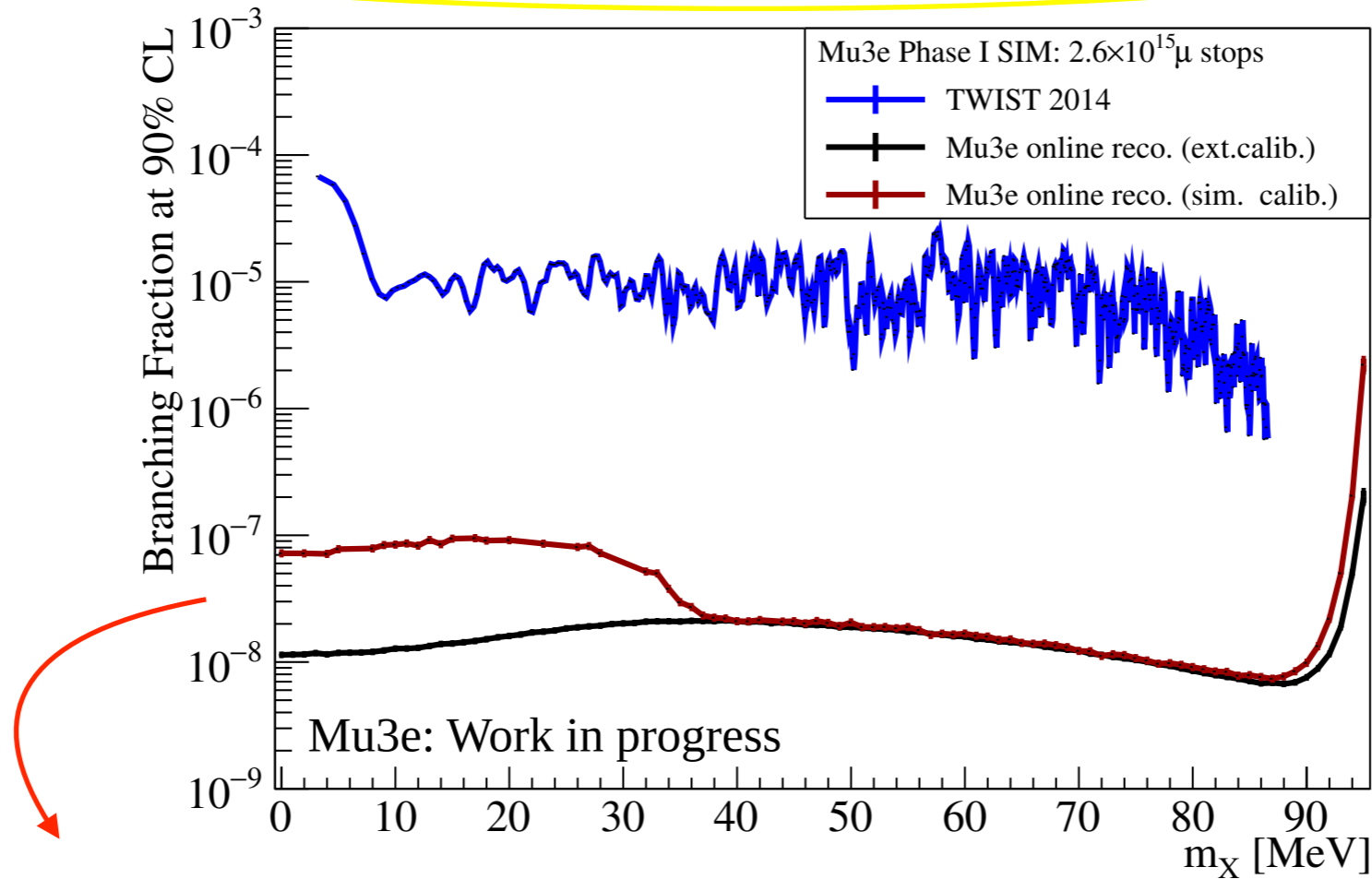
Perrevoort (Mu3e) '18

Future prospects: Mu3e

- Mu3e prospect for $\mu \rightarrow e a$ ([Perrevoort '18](#))

Potential search for performed on positron momentum histograms filled with *online* reconstructed short tracks

Expected limit for phase I ($2.6 \times 10^{15} \mu^+$):



[Perrevoort \(Mu3e\) '18](#)

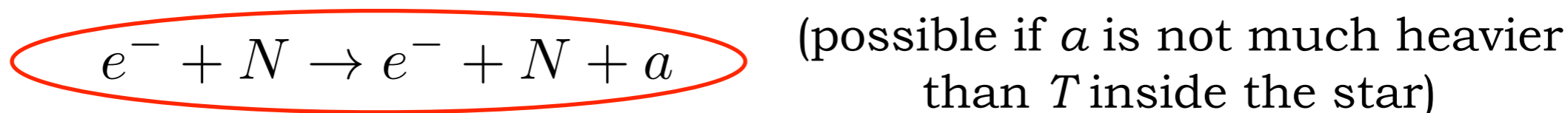
$$m_a \approx 0 : \quad \text{BR}(\mu \rightarrow e a) < 7 \times 10^{-8} \quad \Rightarrow \quad F_{\mu e} \gtrsim 3 \times 10^{10} \text{ GeV}$$

Astrophysics and cosmology

Astrophysical bounds

Well-known bounds on ALP-electron couplings from energy loss in star systems [red giants (RG), white dwarfs (WD)] due to processes like:

Raffelt Weiss '94



$$\Rightarrow F_{ee}^A \gtrsim 4.6 \times 10^9 \text{ GeV (WD)} \quad F_{ee}^A \gtrsim 2.4 \times 10^9 \text{ GeV (RG)} \quad (m_a \lesssim 1 - 10 \text{ keV})$$

Bertolami et al '14

Viaux et al '13

Hints ($\sim 3\sigma$) for non-standard WD cooling require: $F_{ee}^A \approx 6 \times 10^9 \text{ GeV}$

Giannotti et al '17

We extend the bounds to the case of massive ALP: Boltzmann suppression we need to rescale the energy loss rate by the ratio

$$R(m_a, T) \equiv \mathcal{E}_a(m_a, T) / \mathcal{E}_a(0, T)$$

Raffelt Phys. Rept. '90

$$\text{energy density: } \mathcal{E}_a(m_a, T) = \frac{1}{2\pi^2} \int_{m_a}^{\infty} \frac{E^2 \sqrt{E^2 - m_a^2}}{e^{E/T} - 1} dE = \begin{cases} \frac{\pi^2}{30} T^4 & m_a \ll T \\ \frac{1}{(2\pi)^{3/2}} T^4 \left(\frac{m_a}{T}\right)^{5/2} e^{-m_a/T} & m_a \gg T \end{cases}$$

$$T_{\text{RG}} \approx 10^8 K \approx 8.6 \text{ keV} \quad T_{\text{WD}} \approx 10^7 K \approx 0.8 \text{ keV}$$

Astrophysical bounds

Above ~ 0.1 MeV, supernova bounds become important

We get a bound from the cooling of the proto-neutron star in the SN1978A explosion to the ALP coupling to electrons (new!)

energy loss in highly-degenerate conditions:

$$\epsilon = \frac{\pi}{15} \alpha_{\text{em}}^2 \frac{T^4}{m_n (F_{ee}^A)^2} Y_p F = 1.2 \times 10^{20} \frac{\text{erg}}{\text{g s}} \left(\frac{10^7 \text{ GeV}}{F_{ee}^A} \right)^2$$

angular integral (including plasma screening)

$m_a \lesssim T_{SN} \approx 30$ MeV : $F_{ee}^A \gtrsim 3.4 \times 10^7$ GeV (SN1987A)

Similarly, bounds can be obtained for the $\mu\mu$ coupling and the μe coupling

Summary of astro-bounds ($m_a \approx 0$)

Process	Decay constant	Bound (GeV)	Experiment
Star cooling	F_{ee}^A	4.6×10^9	WD
	F_{ee}^A	2.4×10^9	RG
	F_{ee}^A	3.4×10^7	SN1987A _{ee}
	$F_{\mu\mu}^A$	1.3×10^8	SN1987A _{$\mu\mu$}
	$F_{\mu e}$	1.4×10^8	SN1987A _{μe}

[Bollig et al '20](#)

equivalent to $\text{BR}(\mu \rightarrow e a) \lesssim 4 \times 10^{-3}$

- Obvious requirement, cosmological lifetime

$$\frac{H_0}{\Gamma_{\text{tot}}} = H_0 \tau_a > 1$$

- More stringent bound: extragalactic background light (from $a \rightarrow \gamma\gamma$)

Coupling to photons ($m_a \ll m_{\ell_i}$): $\mathcal{L}_{\text{eff}} = E_{\text{UV}} \frac{\alpha_{\text{em}}}{4\pi} \frac{a}{f_a} F \tilde{F}$

depends on UV completion,
e.g. anomaly coefficient (QCD axion: $E_{\text{UV}} = E/2N$)

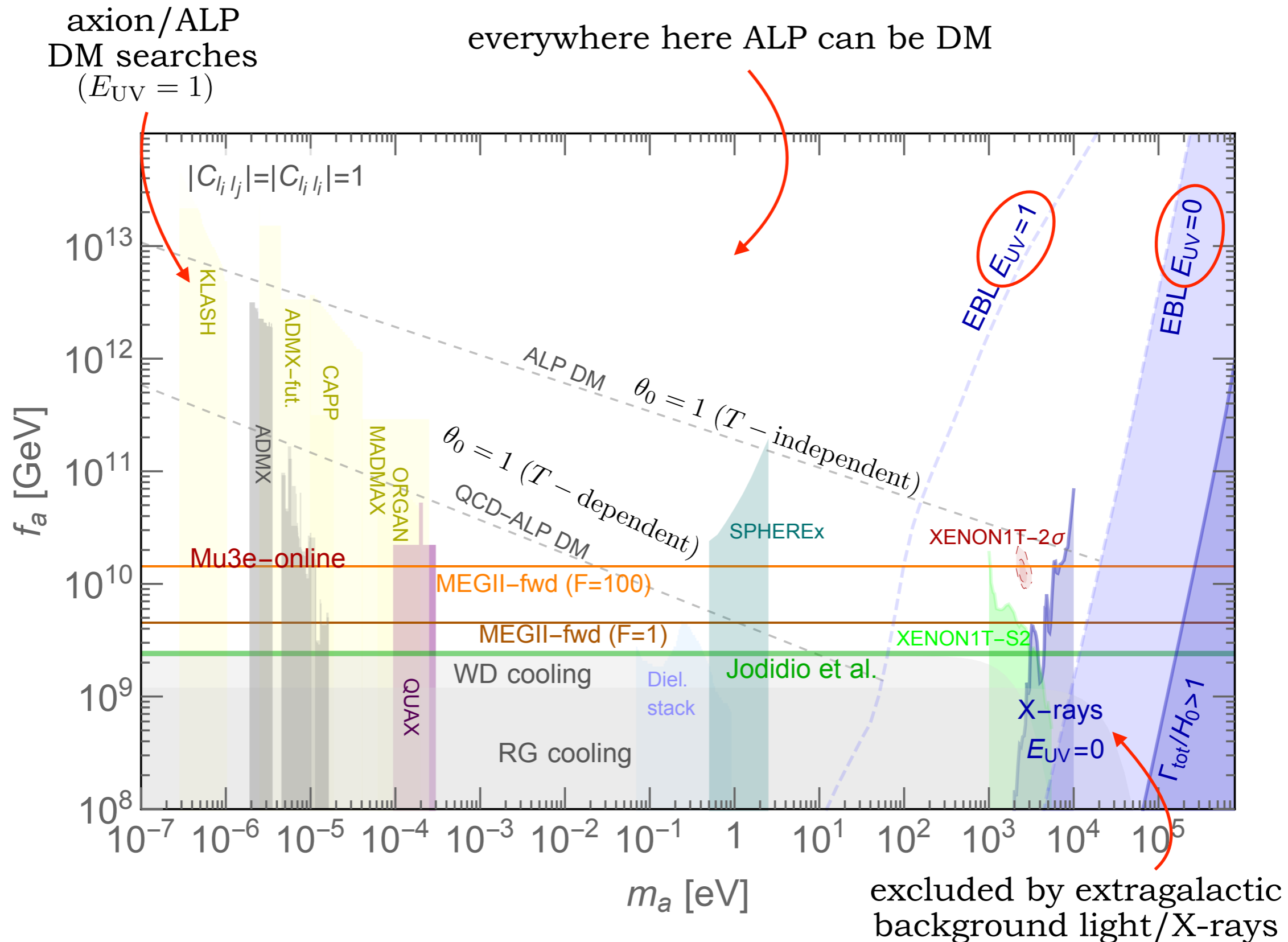
- Production: misalignment mechanism

$$\Omega_a^{T\text{-indep.}} h^2 = 0.12 \times 10^{-2} \sqrt{\frac{m_a}{\text{eV}}} \left(\frac{f_a}{10^{10} \text{GeV}} \right)^2 \left(\frac{\theta_0}{\pi} \right)^2 \left(\frac{90}{g_*(T_{\text{osc}})} \right)^{1/4}$$

misalign. angle

it can be enhanced if ALP mass suppressed at
finite temperature (e.g. QCD axion)

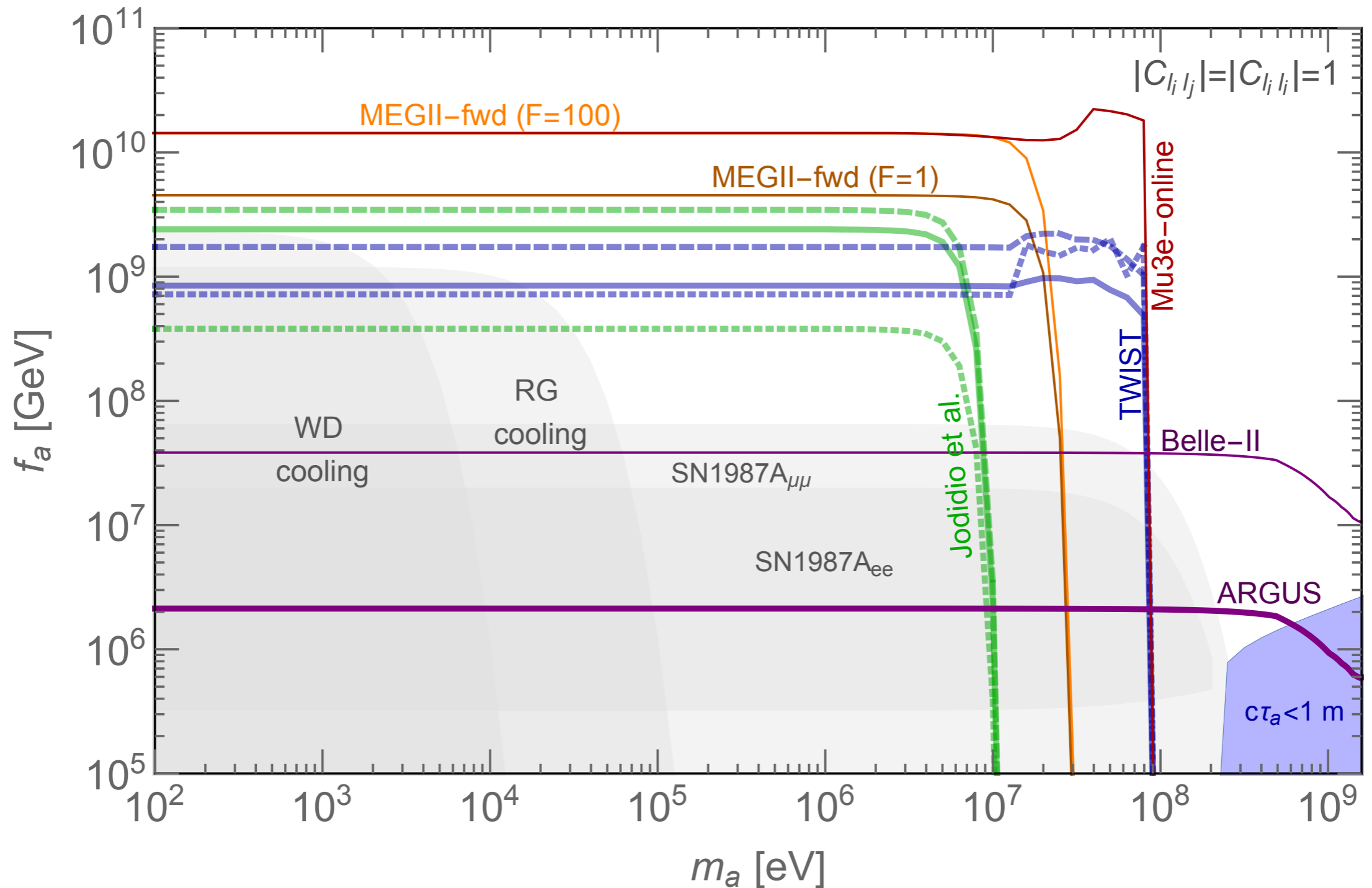
ALP dark matter



Putting everything together...

Summary of model-independent bounds

$$\mathcal{L}_{all} = \frac{\partial^\mu a}{2f_a} (C_{ij}^V \bar{l}_i \gamma_\mu l_j + C_{ij}^A \bar{l}_i \gamma_\mu \gamma_5 l_j)$$



Models

- How generic is a PNCB with flavour-violating couplings to leptons?
- Can we test ALPs with LFV *beyond stars*?
- That is, how are FC and FV couplings related (F_{ee} , $F_{\mu e}$, etc.) ?

To answer these questions, we need to consider specific models

- LFV QCD axion:

QCD axion (DSFZ type) with leptons carrying non-universal PQ

- LFV axiflavor:

QCD axion obtained by identifying PQ = Froggatt-Nielsen U(1)

(FV axion-quark couplings suppressed by an additional flavour SU(2))

- Leptonic familon

PNCB from spontaneously broken Froggatt-Nielsen U(1) (acting on leptons only)

- Majoron

spontaneously broken lepton number (in the context of low-energy seesaw)

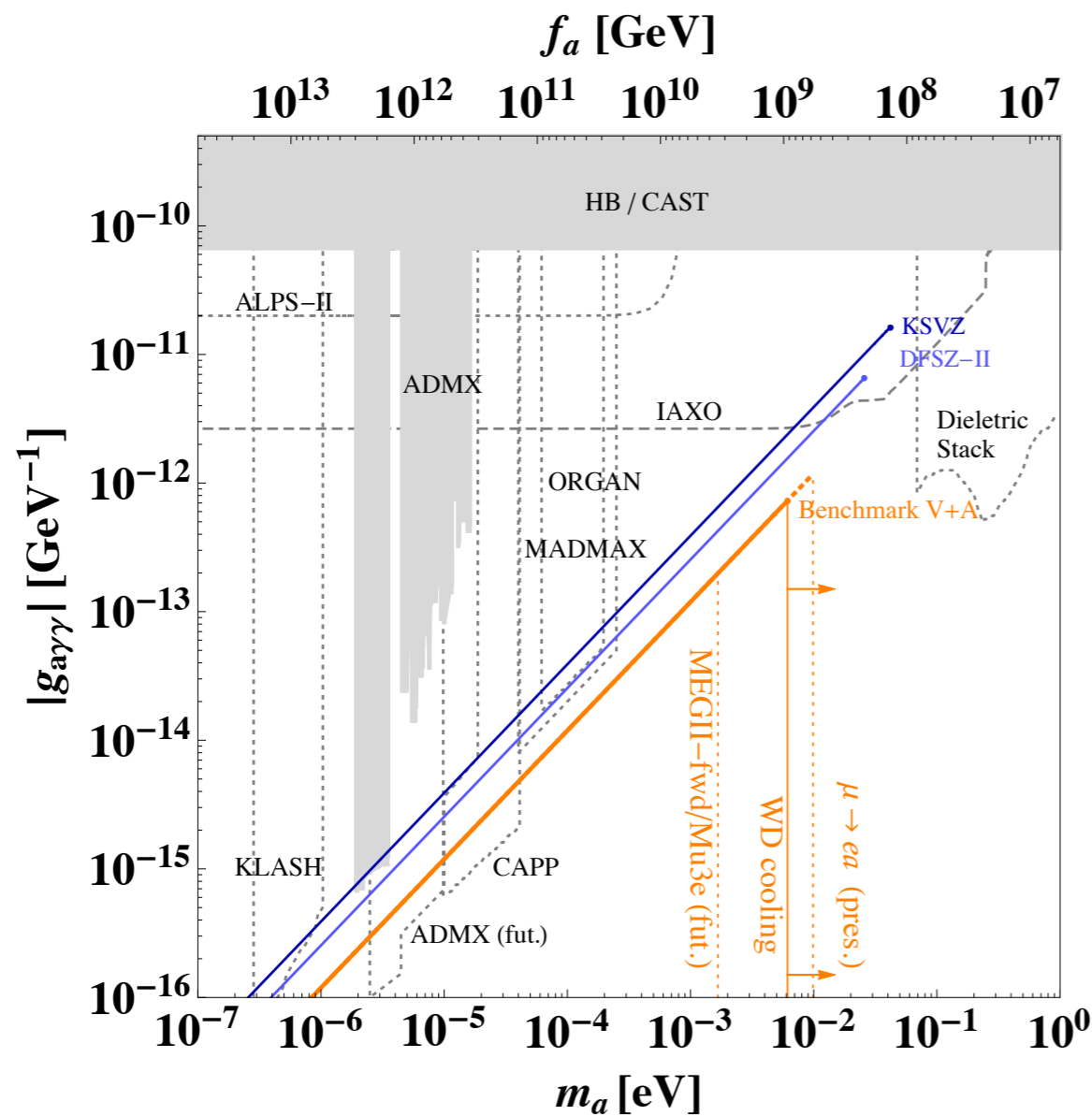
LFV QCD axion

flavor non-universal charges
 → flavor-violating couplings

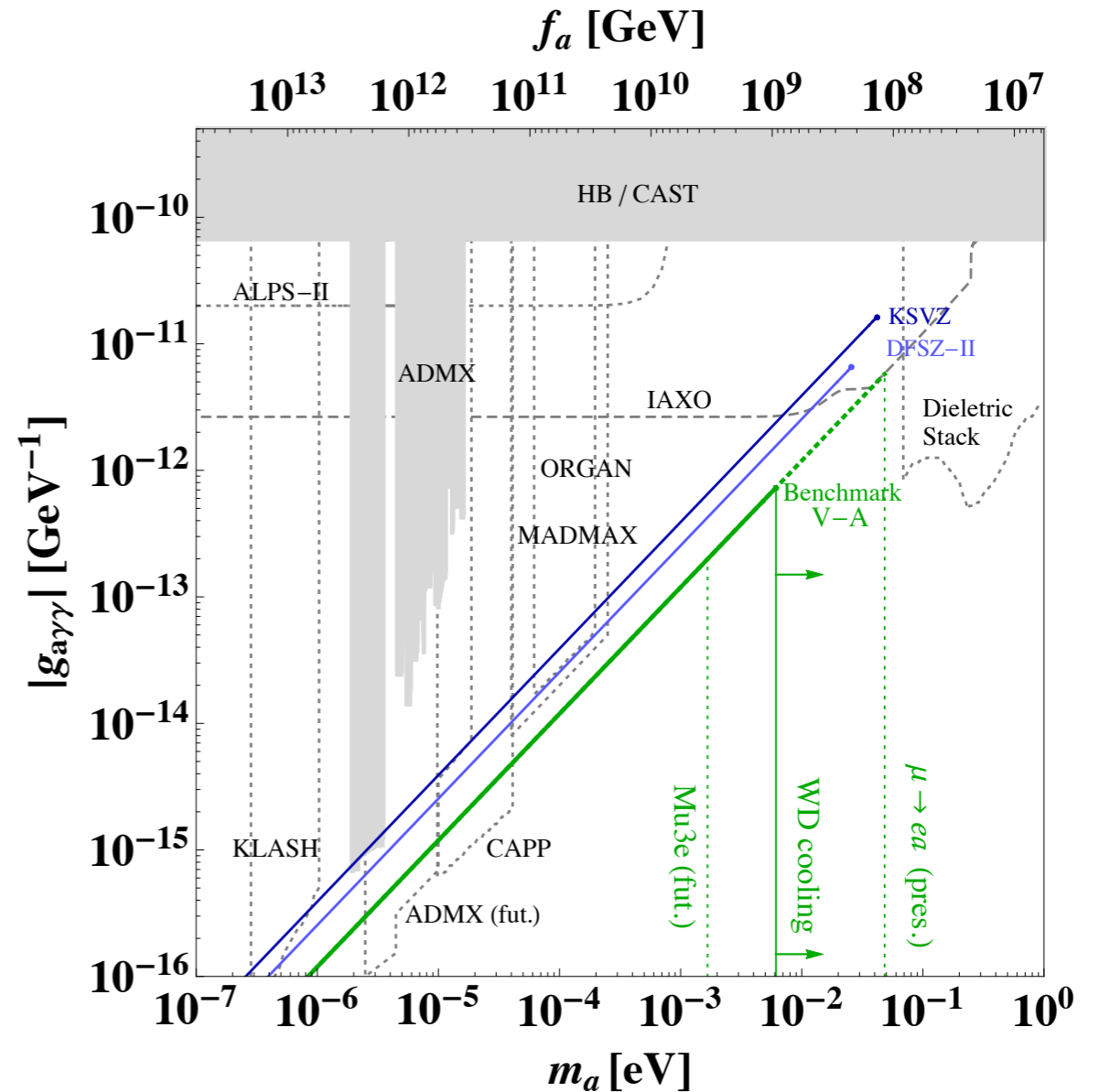
$$C_{fi f_j}^{V,A} = \frac{1}{2N} \left(V_R^{f\dagger} X_{f_R} V_R^f \pm V_L^{f\dagger} X_{f_L} V_L^f \right)_{ij}$$

$V_L^\dagger Y^e V_R = Y_{diag}^e$ L and R unitary rotations to the lepton mass basis

matrices of PQ charges



V+A axion (large R rotations)



V-A axion (large L rotations)

LFV QCD axion

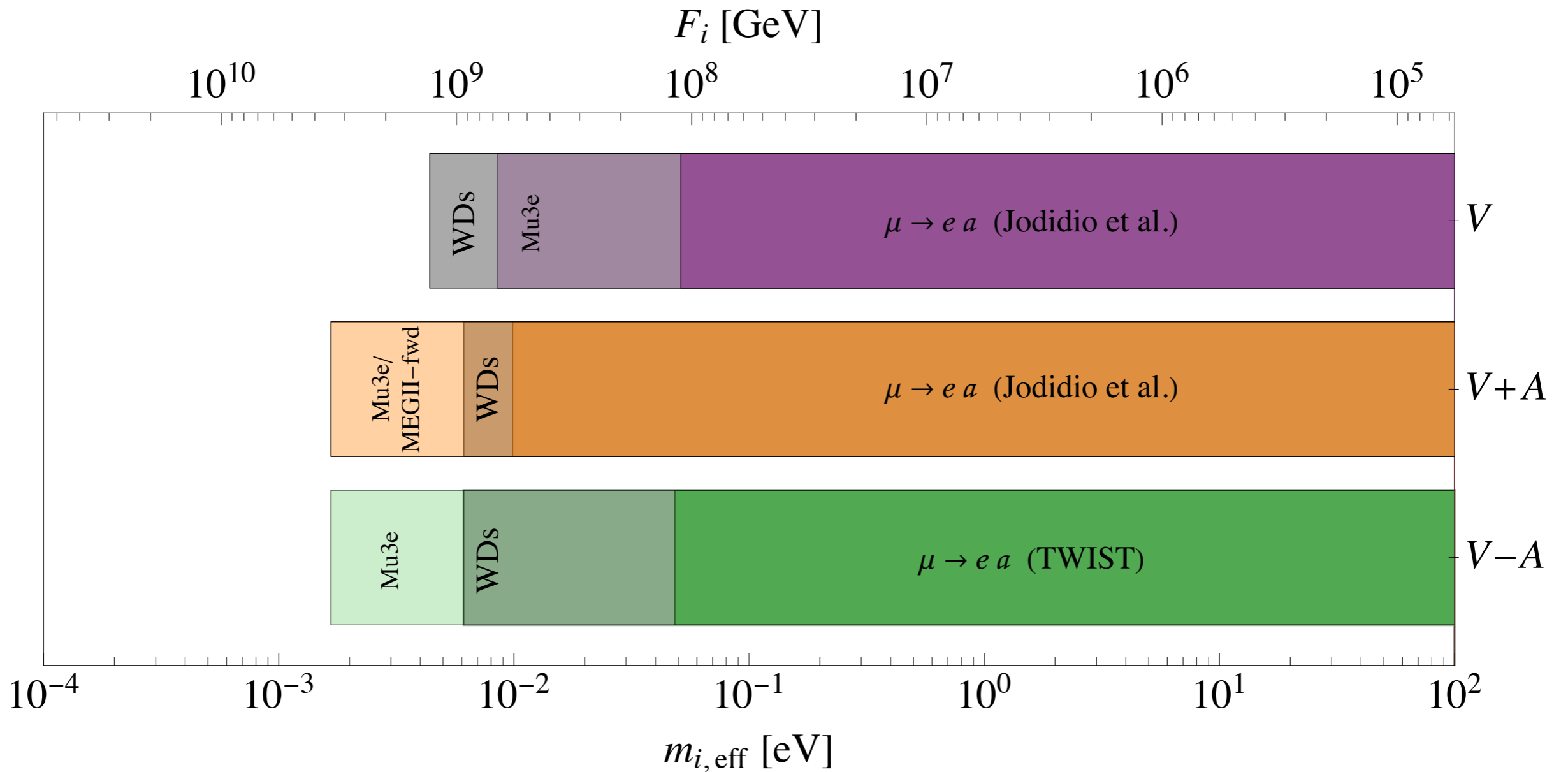
flavor non-universal charges
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$$V_L^\dagger Y^e V_R = Y_{diag}^e$$

L and R unitary rotations
to the lepton mass basis

matrices of
PQ charges



Majoron

Type I seesaw: $\mathcal{L} = \mathcal{L}_{\text{SM}} + i\bar{N}\not{\partial}N - \left(Y_N \bar{N} \tilde{\Phi}^\dagger L + \frac{1}{2} M_N \bar{N} N^c + \text{h.c.} \right)$

$\left(\frac{1}{2} M_N \bar{N} N^c + \text{h.c.} \right)$ ← L-breaking term

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & Y_N^T v / \sqrt{2} \\ Y_N v / \sqrt{2} & M_N \end{pmatrix} \xrightarrow{M_N \gg Y_N v} m_\nu = -\frac{v^2}{2} Y_N^T M_N^{-1} Y_N$$

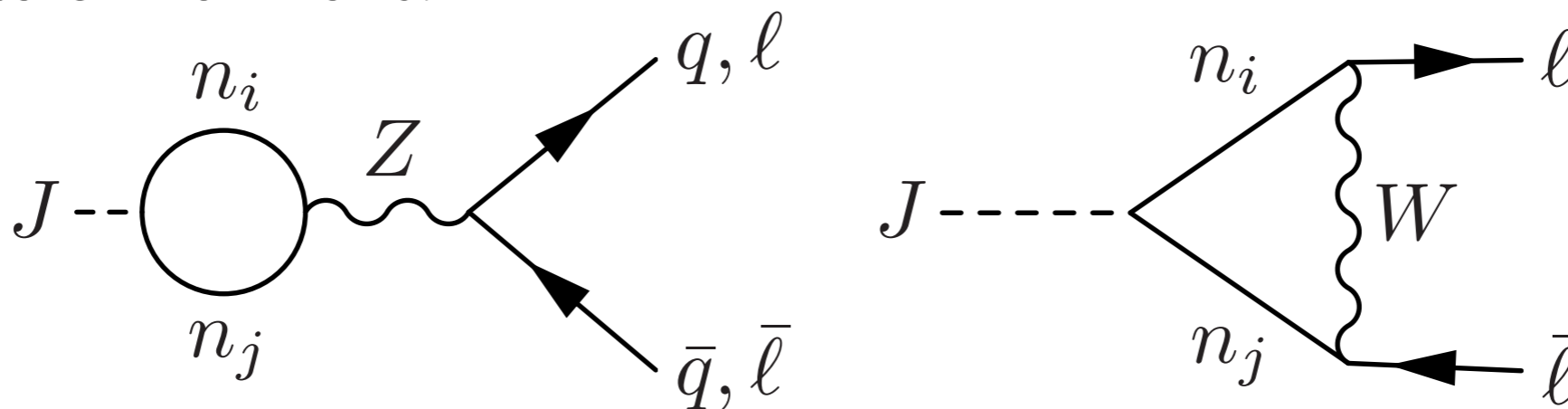
Spontaneous breaking of the lepton number:

$$\frac{1}{2} \lambda_N \sigma \bar{N}^c N, \quad \sigma = \frac{f_N + \hat{\sigma}}{\sqrt{2}} e^{iJ/f_N} \xrightarrow{\quad} M_N = \lambda_N f_N / \sqrt{2}$$

PNGB: Majoron!

Chikashige Mohapatra Peccei '80

Couplings to SM fermions:



Majoron

Type I seesaw: $\mathcal{L} = \mathcal{L}_{\text{SM}} + i\bar{N}\not{\partial}N - \left(Y_N \bar{N} \tilde{\Phi}^\dagger L + \frac{1}{2} M_N \bar{N} N^c + \text{h.c.} \right)$

\swarrow L-breaking term

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & Y_N^T v / \sqrt{2} \\ Y_N v / \sqrt{2} & M_N \end{pmatrix} \xrightarrow{M_N \gg Y_N v} m_\nu = -\frac{v^2}{2} Y_N^T M_N^{-1} Y_N$$

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PNGB: Majoron!

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Couplings to SM fermions:

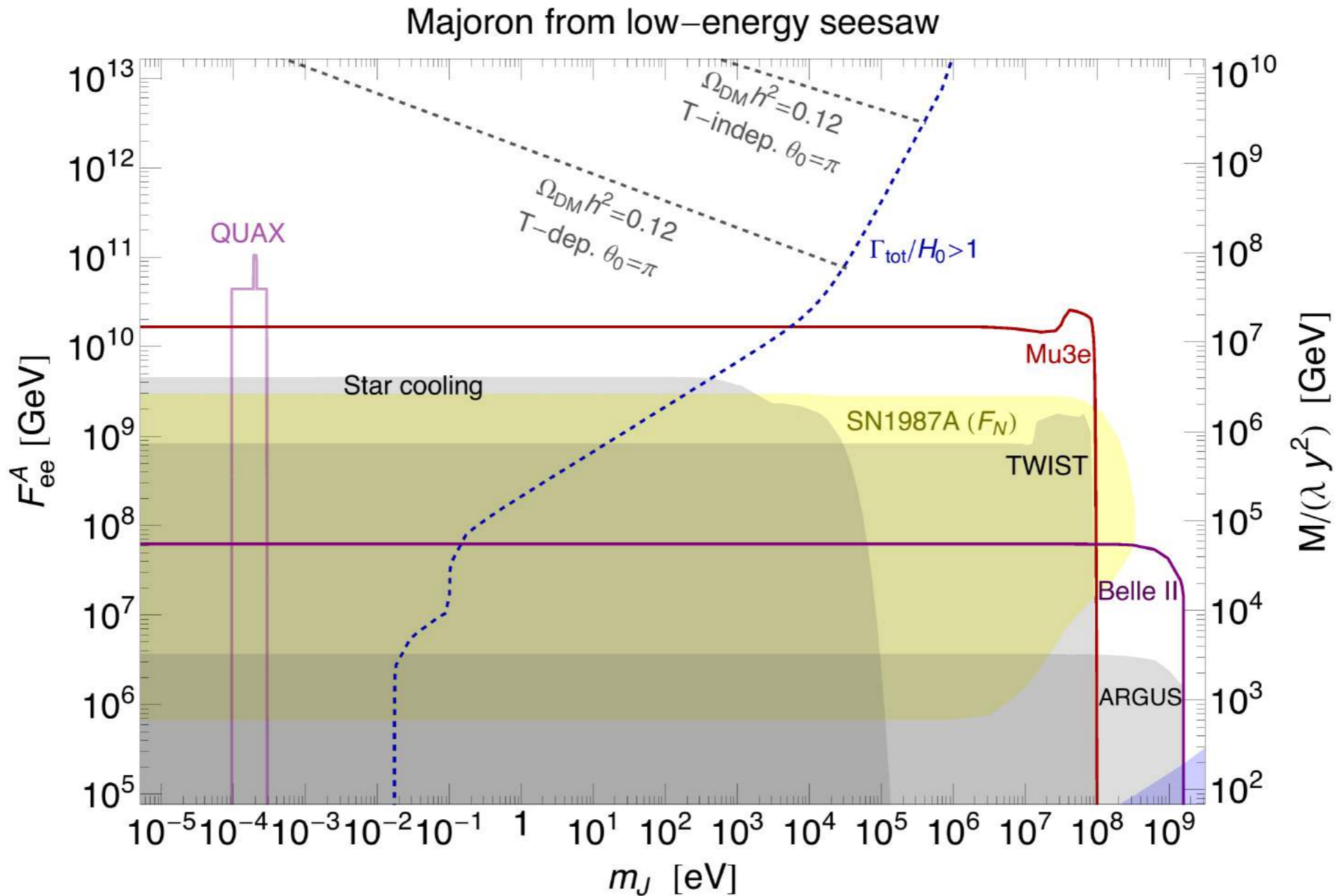
$$C_{q_i q_j}^V = 0, \quad C_{q_i q_j}^A = -\frac{T_3^q}{16\pi^2} \delta_{ij} \text{Tr} \left(Y_N Y_N^\dagger \right),$$

$$C_{l_i l_j}^V = \frac{1}{16\pi^2} \left(Y_N Y_N^\dagger \right)_{ij}, \quad C_{l_i l_j}^A = \frac{1}{16\pi^2} \left[\frac{\delta_{ij}}{2} \text{Tr} \left(Y_N Y_N^\dagger \right) - (Y_N Y_N^\dagger)_{ij} \right]$$

Generically flavour-violating, (V-A)

Pilaftsis '94
Garcia-Cely Heeck '17

Majoron



Lepton number anomaly free: suppressed coupling to photons ($E_{UV}=0$)

$$\Gamma(a \rightarrow \gamma\gamma) = \frac{\alpha_{em}^2 E_{eff}^2 m_a^3}{64\pi^3 f_a^2}, \quad m_a \ll m_{\ell_i} : E_{eff} \simeq E_{UV} \quad \mathcal{L}_{eff} = E_{UV} \frac{\alpha_{em}}{4\pi} \frac{a}{f_a} F \tilde{F}$$

Summary

PNGBs from non-universal global U(1)s (or due to loop effects) give rise to lepton-flavour-violating decays

We have huge room for improvement over the old limits
We propose to start with a MEGII-fwd phase of MEG II

Essential interplay among μ , τ , and astrophysical bounds

Very large symmetry-breaking scales can be probed

Future CLFV limits can supersede stellar bounds even for small ALP masses and start testing the ALP DM region

Thank you!

谢谢

Additional slides

Summary of the model-independent bounds

Comparison in the case $m_a \approx 0$

$$\mathcal{L}_{all} = \frac{\partial^\mu a}{2f_a} (C_{ij}^V \bar{l}_i \gamma_\mu l_j + C_{ij}^A \bar{l}_i \gamma_\mu \gamma_5 l_j) \quad F_{ij}^{V,A} \equiv \frac{2f_a}{C_{ij}^{V,A}} \quad F_{ij} \equiv \frac{2f_a}{\sqrt{|C_{ij}^V|^2 + |C_{ij}^A|^2}}$$

Present best limits				
Process	BR Limit	Decay constant	Bound (GeV)	Experiment
Star cooling	–	F_{ee}^A	4.6×10^9	WDs [44]
	–	$F_{\mu\mu}^A$	1.6×10^6	SN1987A $_{\mu\mu}$ [45]
	4×10^{-3}	$F_{\mu e}$	1.4×10^8	SN1987A $_{\mu e}$ (Sec. 6.1)
$\mu \rightarrow e a$	$2.6 \times 10^{-6*}$	$F_{\mu e}$ (V or A)	4.8×10^9	Jodidio et al. [9]
$\mu \rightarrow e a$	$2.5 \times 10^{-6*}$	$F_{\mu e}$ ($V + A$)	4.9×10^9	Jodidio et al. [9]
$\mu \rightarrow e a$	$5.8 \times 10^{-5*}$	$F_{\mu e}$ ($V - A$)	1.0×10^9	TWIST [10]
$\mu \rightarrow e a \gamma$	$1.1 \times 10^{-9*}$	$F_{\mu e}$	$5.1 \times 10^{8\#}$	Crystal Box [46]
$\tau \rightarrow e a$	$2.7 \times 10^{-3**}$	$F_{\tau e}$	4.3×10^6	ARGUS [43]
$\tau \rightarrow \mu a$	$4.5 \times 10^{-3**}$	$F_{\tau\mu}$	3.3×10^6	ARGUS [43]

Past searches: $\mu \rightarrow e \gamma a$

- Crystal Box 1988

PHYSICAL REVIEW D

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Search for rare muon decays with the Crystal Box detector

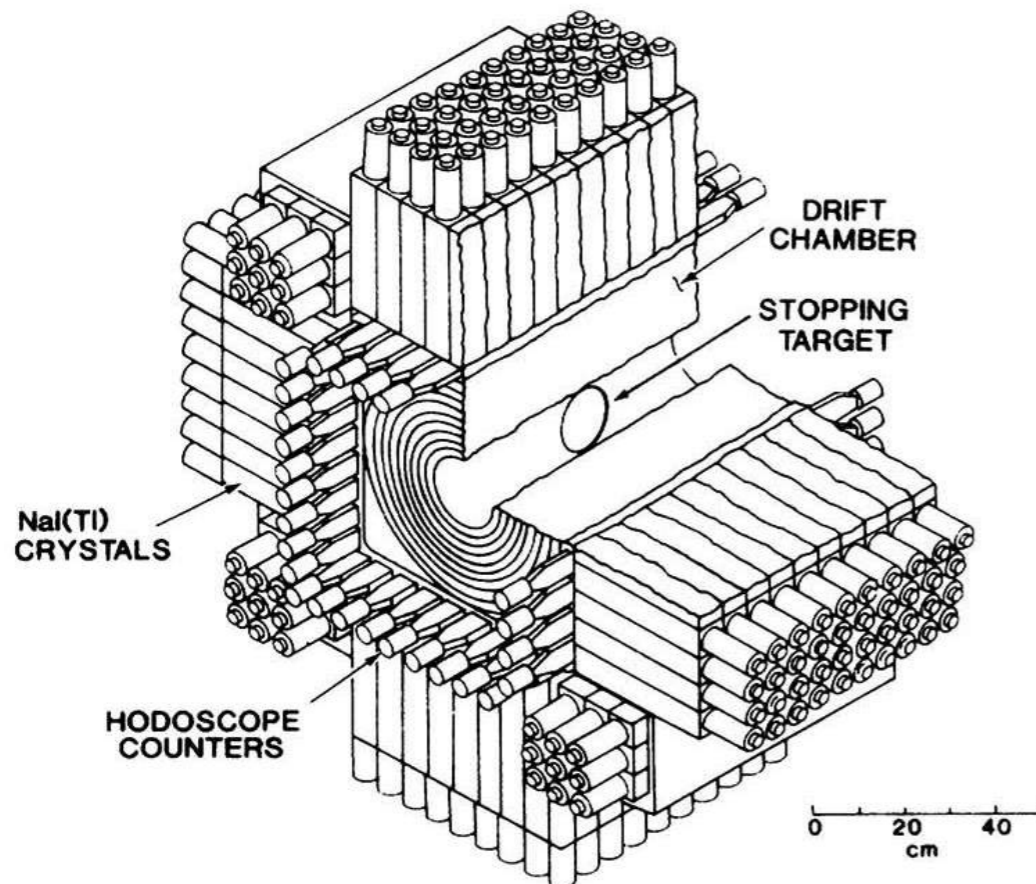


TABLE I. Types of events generated with the Monte Carlo program.

Process	Trigger
$\mu^+ \rightarrow e^+ \gamma$	$e-\gamma$
$\mu^+ \rightarrow e^+ \gamma \nu \bar{\nu}$	$e-\gamma, 1-\gamma$
$\mu^+ \rightarrow e^+ \gamma \gamma$	$e-\gamma-\gamma, e-\gamma$
$\mu^+ \rightarrow e^+ e^+ e^-$	$e-e-e$
$\mu^+ \rightarrow e^+ e^+ e^- \nu \bar{\nu}$	$e-e-e$
$\mu^+ \rightarrow e^+ \nu \bar{\nu}$	$1-e$
$\mu^+ \rightarrow e^+ \gamma f$ ($f = \text{familon}$)	$e-\gamma$
$\pi^0 \rightarrow \gamma \gamma$	$\gamma-\gamma, 1-\gamma$
$\pi^- p \rightarrow n \gamma$	$1-\gamma$

Past searches: $\mu \rightarrow e \gamma a$

- Crystal Box 1988

Analysis for massless familon $m_a \approx 0$
(with 1.4×10^{12} stopped μ^+) yields:

$$\text{BR}(\mu \rightarrow e a \gamma) < 1.1 \times 10^{-9} \quad (90\% \text{ CL})$$

$$\text{BR}(\mu \rightarrow e a \gamma) \approx \frac{\alpha_{\text{em}}}{2\pi} \mathcal{I}(x_{\text{min}}, y_{\text{min}}) \text{BR}(\mu \rightarrow e a)$$

[Hirsch et al. '09](#)

$$\mathcal{I}(x_{\text{min}}, y_{\text{min}}) = \int_{x_{\text{min}}, y_{\text{min}}}^1 dx dy \frac{(x-1)(2-xy-y)}{y^2(1-x-y)}$$

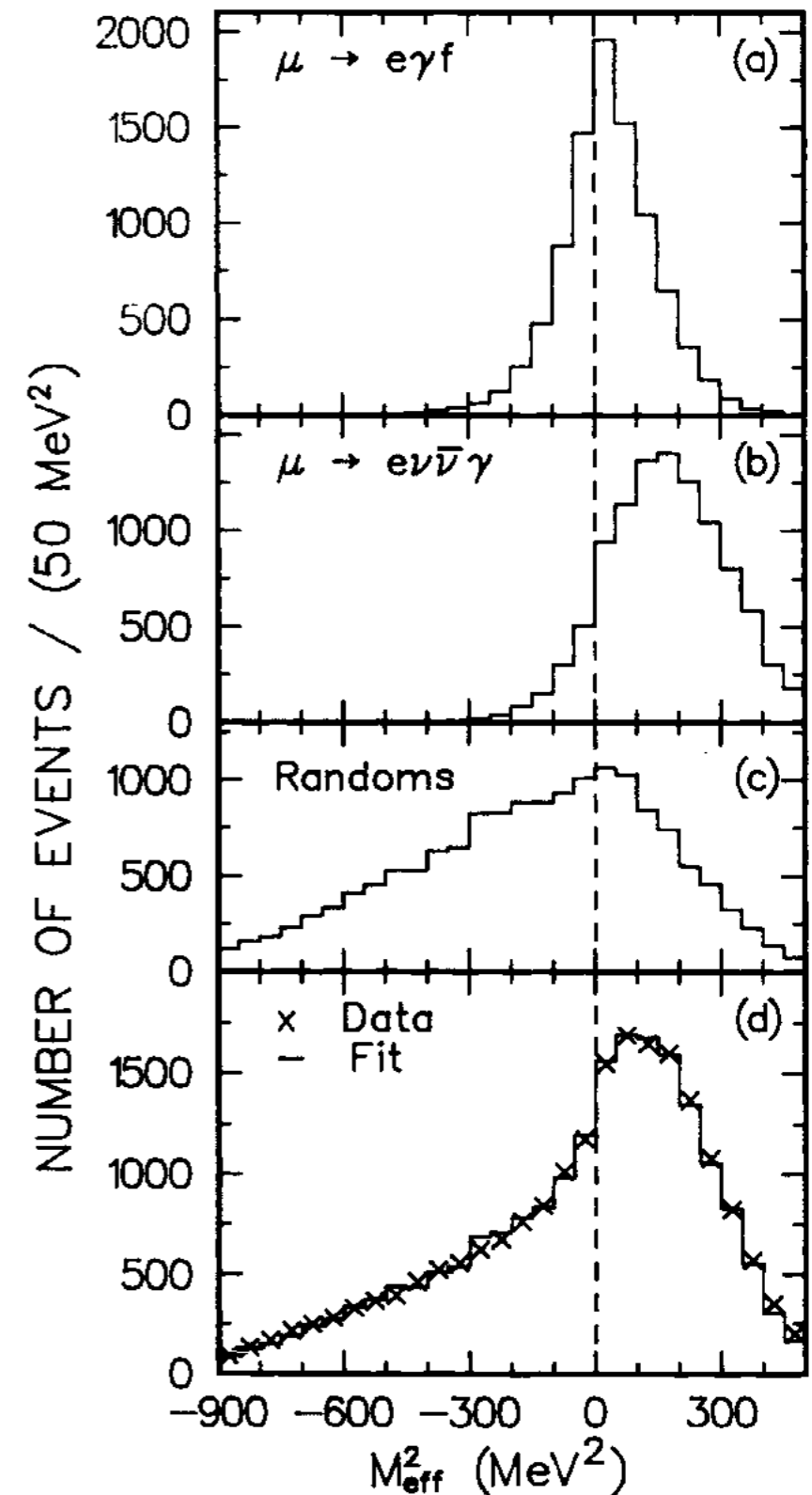
$$x = 2E_e/m_\mu \quad y = 2E_\gamma/m_\mu$$

Crystal Box energy thresholds:

$$E_e > 38 - 43 \text{ MeV}, \quad E_\gamma > 38 \text{ MeV} \quad \Rightarrow \quad x_{\text{min}} = 0.72 - 0.81, \quad y_{\text{min}} = 0.72$$

$$\Rightarrow F_{e\mu} > (5.1 - 8.3) \times 10^8 \text{ GeV}$$

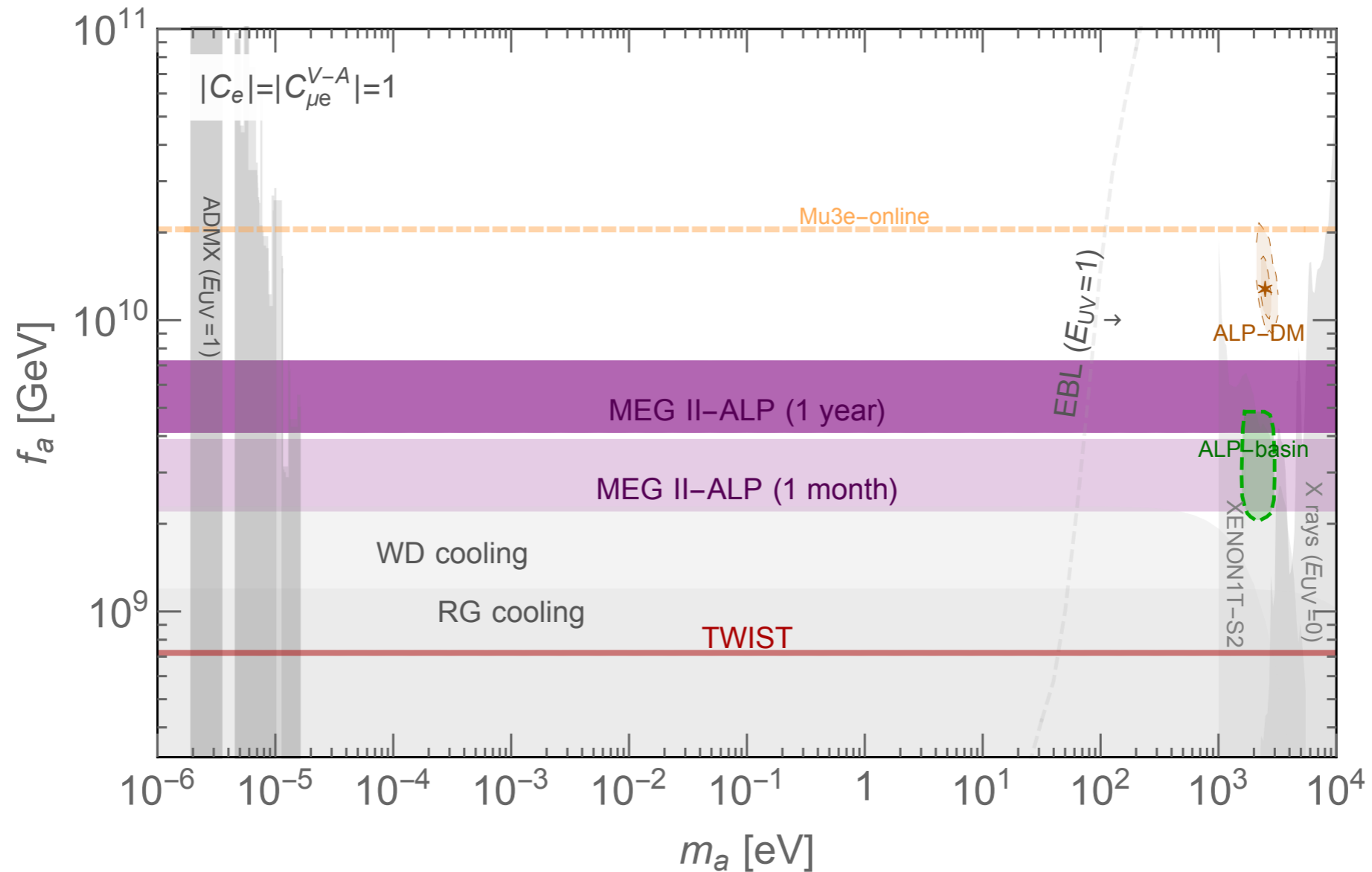
weaker but independent
of V/A nature of the couplings



Future prospects: MEG II

- Prospect at MEG II for $\mu \rightarrow e a \gamma$

Search sensitive to V-A couplings too, requires a dedicated trigger:



Jho Knapen Redigolo '22

Past searches: $\tau \rightarrow e a$, $\tau \rightarrow \mu a$

- ARGUS 1995**

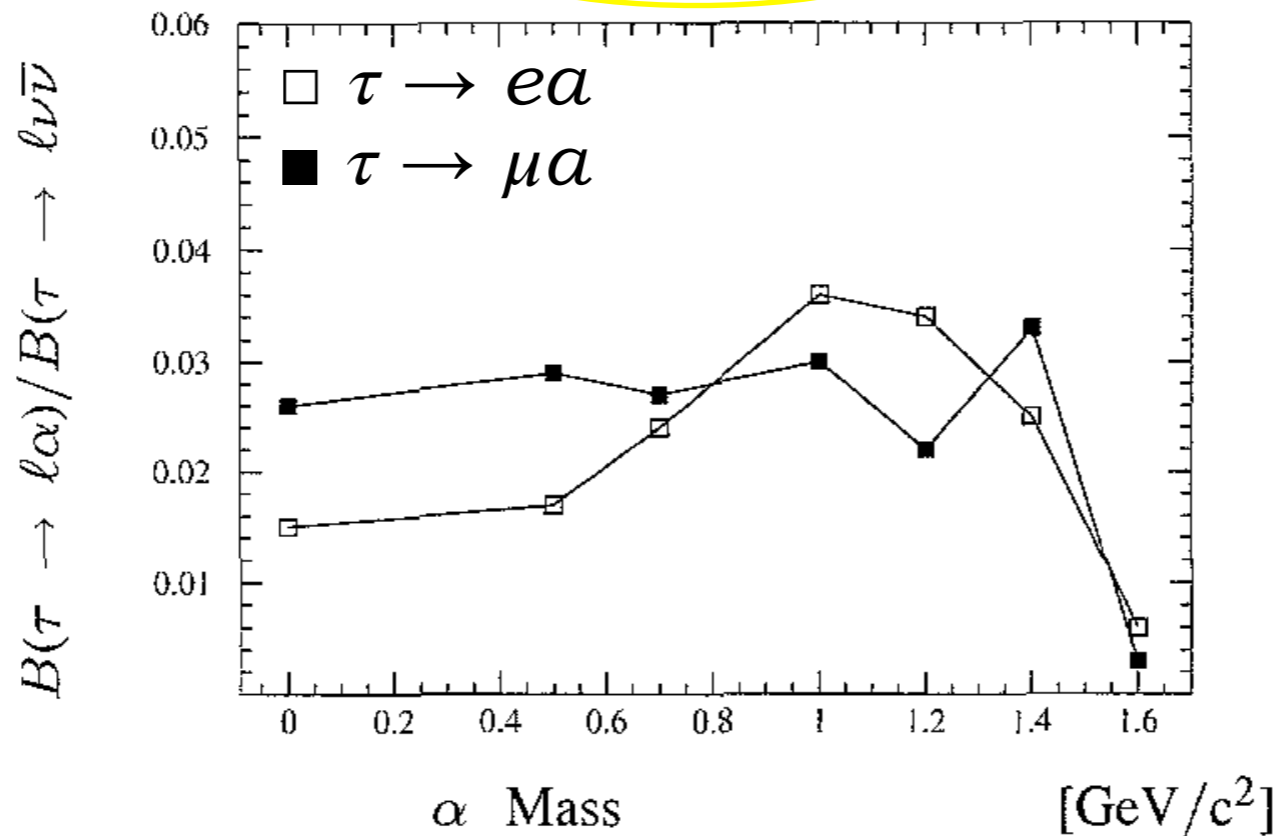
A search for the lepton-flavour violating decays

Z. Phys. C 68, 25–28 (1995)

$\tau \rightarrow e a$, $\tau \rightarrow \mu a$

ARGUS Collaboration

With 472 pb⁻¹:



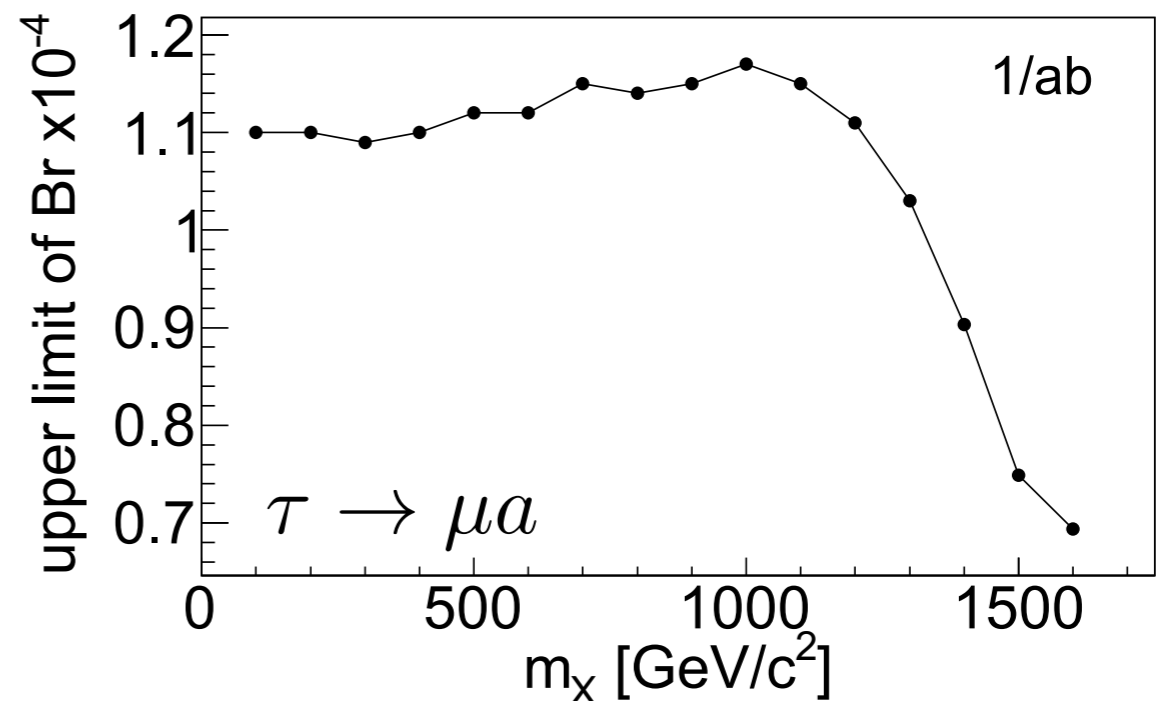
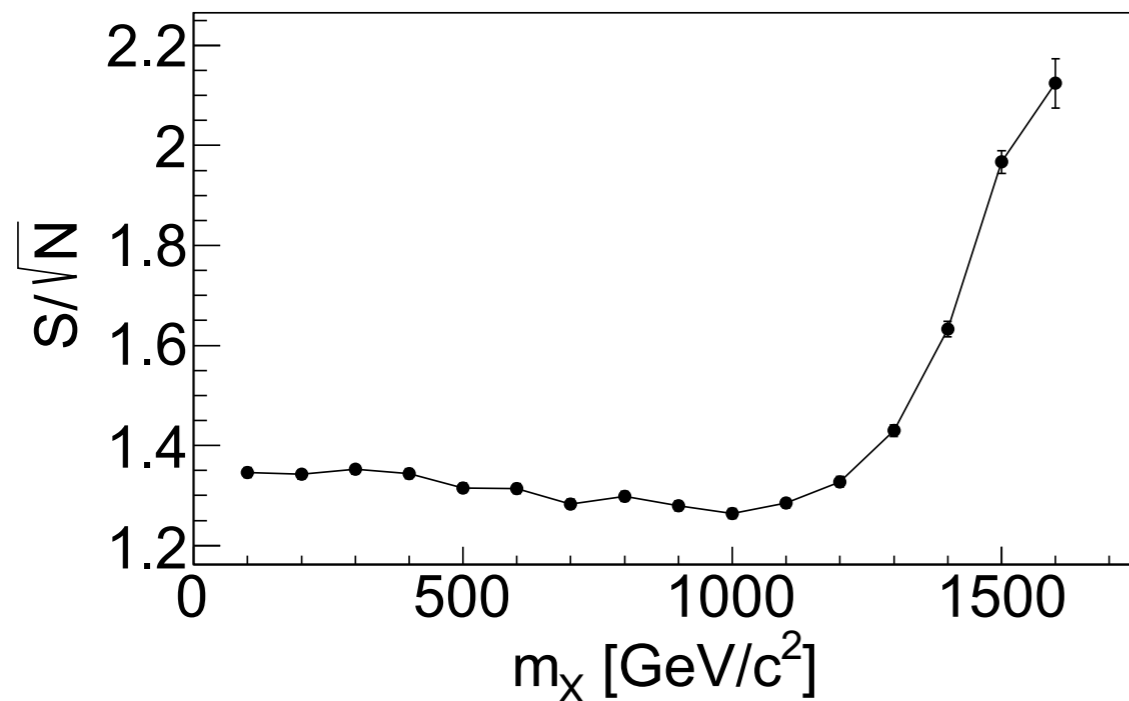
$m_a \approx 0$:

$$\begin{aligned} \text{BR}(\tau \rightarrow e a) < 2.7 \times 10^{-3} \quad (95\% \text{ CL}) &\Rightarrow F_{\tau e} \gtrsim 4.3 \times 10^6 \text{ GeV}, \\ \text{BR}(\tau \rightarrow \mu a) < 4.5 \times 10^{-3} \quad (95\% \text{ CL}) &\Rightarrow F_{\tau \mu} \gtrsim 3.3 \times 10^6 \text{ GeV}. \end{aligned}$$

Future prospects: B-factories/Belle-II

- Belle prospect for $\tau \rightarrow \mu a$ ([Yoshinobu Hayasaka '17](#))

Simulation of S and B and limit that can be set using the Belle data set (1/ab):



[Yoshinobu Hayasaka \(Belle\) '17](#)

$m_a \approx 0$:

Belle (1/ab) prospect: $\text{BR}(\tau \rightarrow \mu a) < 1.1 \times 10^{-4} \Rightarrow F_{\tau\mu} \gtrsim 2.1 \times 10^7 \text{ GeV}$

Belle-II (50/ab) prospect: $\text{BR}(\tau \rightarrow \mu a) < 2.0 \times 10^{-5} \Rightarrow F_{\tau\mu} \gtrsim 4.9 \times 10^7 \text{ GeV}$

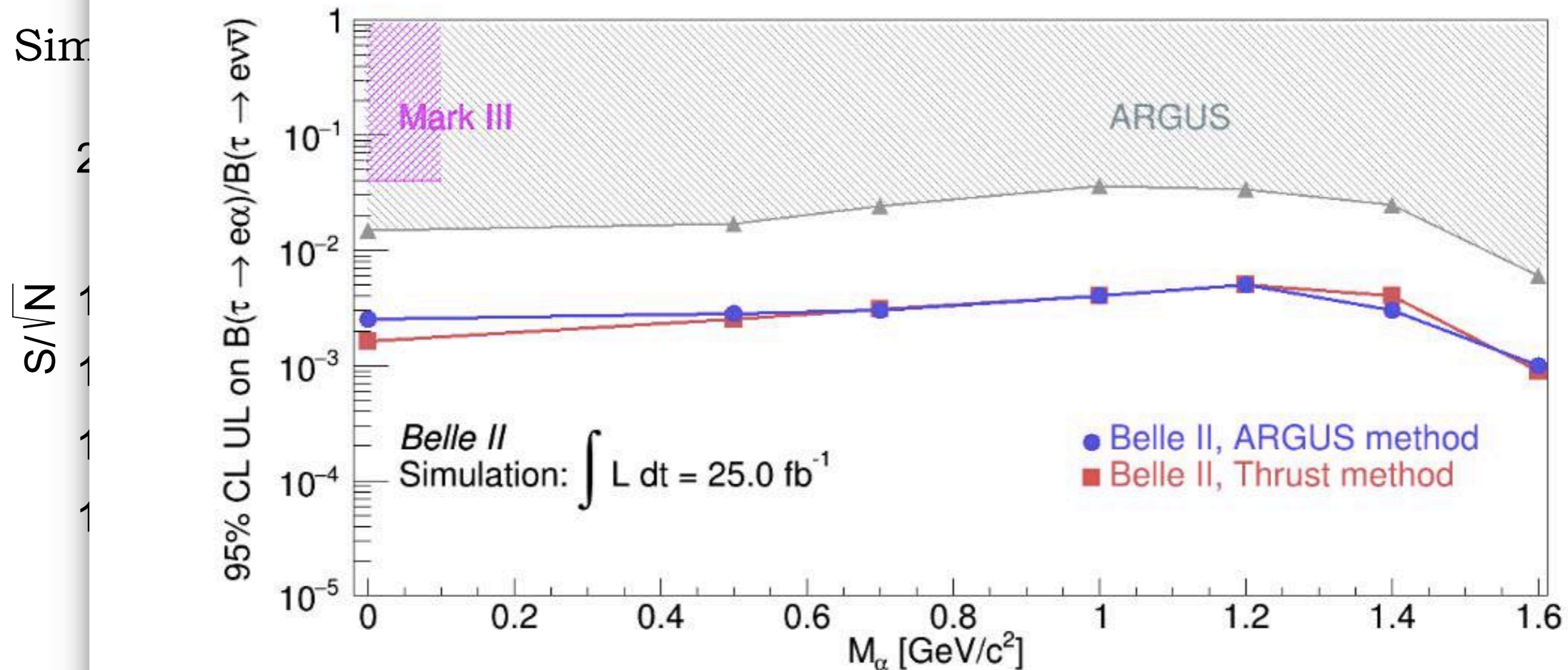
← Estimated by rescaling as $\sqrt{\mathcal{L}}$

Future prospects: B-factories/Belle-II

• Be

Recent Belle-II simulation

[Belle II note '20](#)



$m_a \approx 0$:

Belle (1/ab) prospect: $\text{BR}(\tau \rightarrow \mu a) < 1.1 \times 10^{-4} \Rightarrow F_{\tau\mu} \gtrsim 2.1 \times 10^7 \text{ GeV}$

Belle-II (50/ab) prospect: $\text{BR}(\tau \rightarrow \mu a) < 2.0 \times 10^{-5} \Rightarrow F_{\tau\mu} \gtrsim 4.9 \times 10^7 \text{ GeV}$

← Estimated by rescaling as $\sqrt{\mathcal{L}}$

“Low-energy seesaw” Majoron

Low-energy seesaw: pseudo-Dirac neutrinos \rightarrow approximately conserved (generalised) lepton number


Ibarra Molinaro Petcov '11

$$M_N = \begin{pmatrix} 0 & M \\ M & 0 \end{pmatrix} = \frac{\lambda}{\sqrt{2}} \begin{pmatrix} 0 & f_N \\ f_N & 0 \end{pmatrix} \quad y_N = \begin{pmatrix} y_{e1} & y_{e2} \\ y_{\mu 1} & y_{\mu 2} \\ y_{\tau 1} & y_{\tau 2} \end{pmatrix}$$

Global U(1) symmetry in the limit $y_{\ell 1} \rightarrow 0$

After imposing fit to neutrino obs., two free parameters: M , $y = \max [\text{eig}(y_N y_N^\dagger)]$

$$y_N y_N^\dagger \approx y^2 \frac{m_3}{m_2 + m_3} A_i^* A_j, \quad \text{where } A_i = U_{i3} + iU_{i2} \sqrt{m_2/m_3}$$

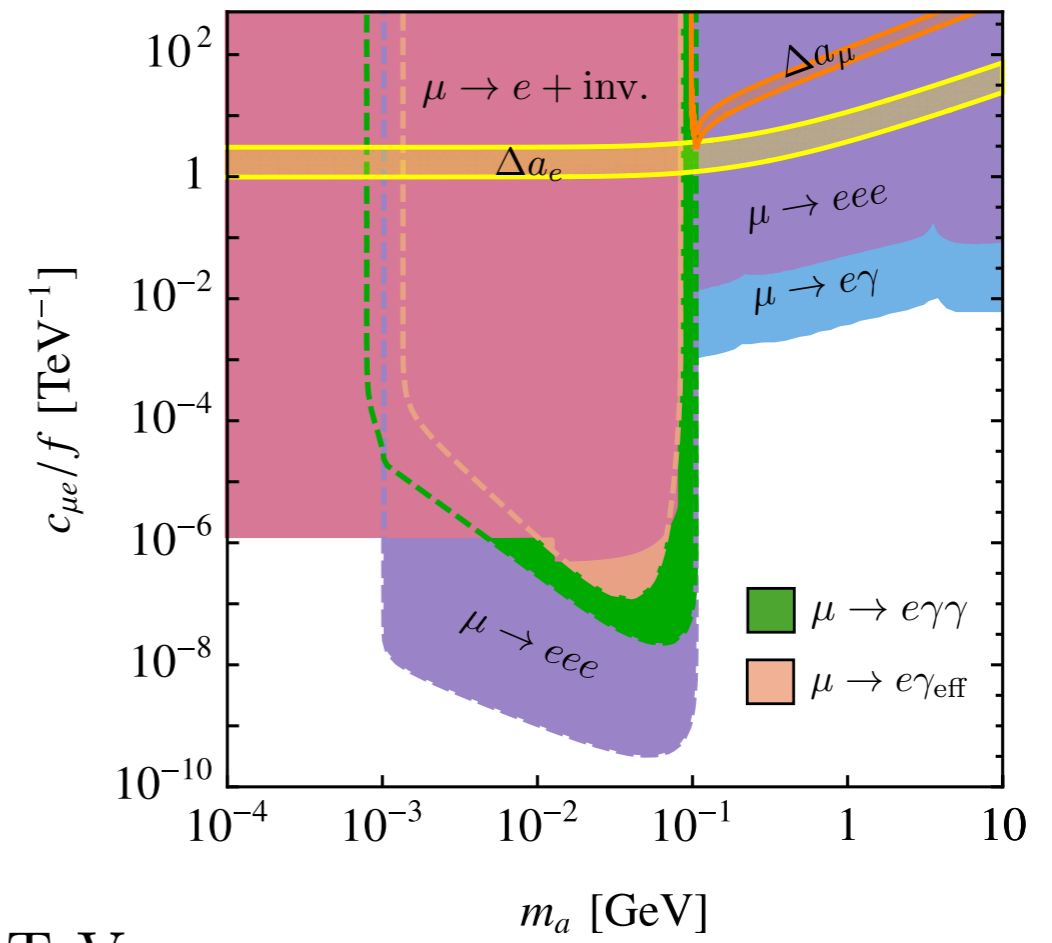
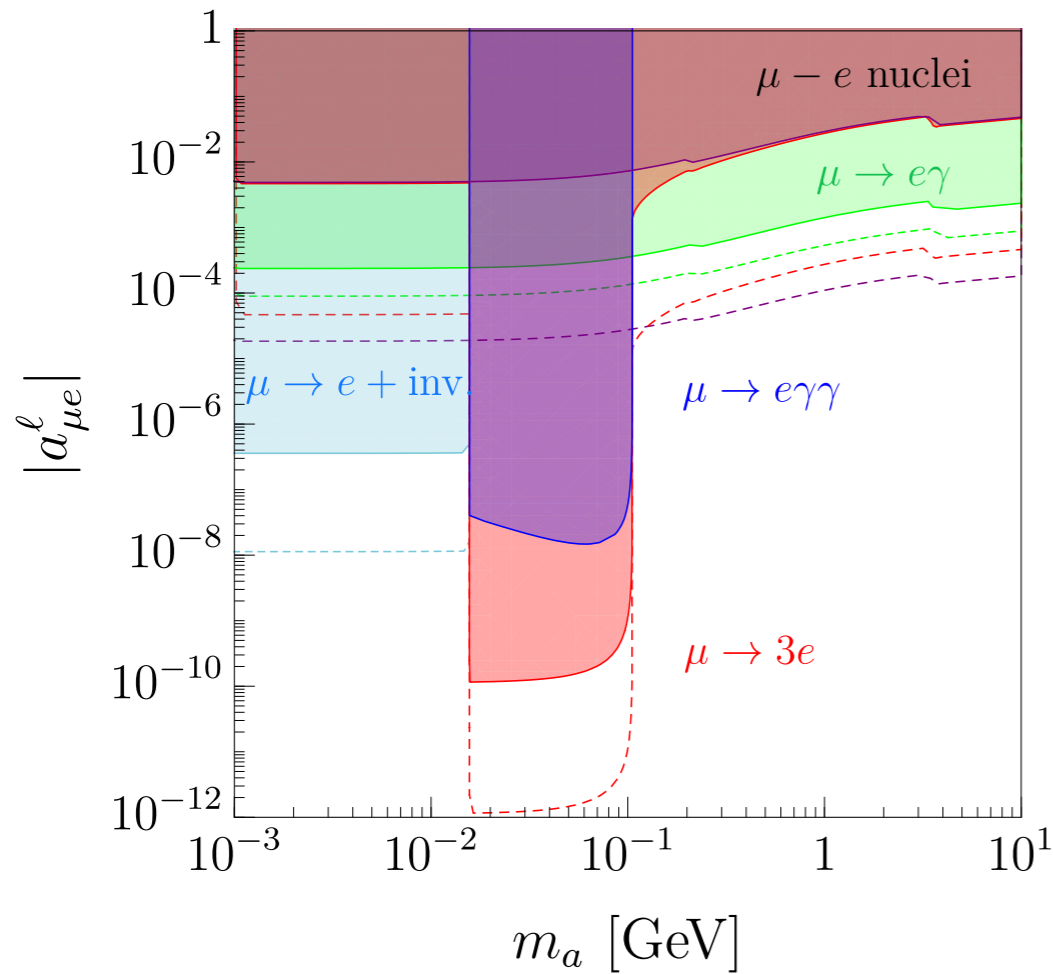


$$F_{ee}^A = \frac{1.1 \times 10^{10} \text{ GeV}}{\lambda y^2} \left(\frac{M}{10^7 \text{ GeV}} \right), \quad F_{\mu e} = \frac{1.4 \times 10^{10} \text{ GeV}}{\lambda y^2} \left(\frac{M}{10^7 \text{ GeV}} \right),$$

$$F_{\tau e} = \frac{1.6 \times 10^{10} \text{ GeV}}{\lambda y^2} \left(\frac{M}{10^7 \text{ GeV}} \right), \quad F_{\tau \mu} = \frac{0.71 \times 10^{10} \text{ GeV}}{\lambda y^2} \left(\frac{M}{10^7 \text{ GeV}} \right)$$

CLFV from short-lived ALPs

Bauer et al. '19
Cornella et al. '19



$c_{ee}/f = 1 \text{ TeV}$

$$\Gamma(a \rightarrow l_i l_j) = \frac{m_a}{2\pi} \left[\left(\frac{m_{l_i} - m_{l_j}}{F_{ij}^V} \right)^2 + \left(\frac{m_{l_i} + m_{l_j}}{F_{ij}^A} \right)^2 \right] \sqrt{1 - \frac{2(m_{l_i}^2 + m_{l_j}^2)}{m_a^2}}$$

$$\Gamma(a \rightarrow \gamma\gamma) = \frac{\alpha_{\text{em}}^2 E_{\text{eff}}^2 m_a^3}{64\pi^3 f_a^2}$$

$$E_{\text{eff}} = E_{\text{UV}} + \sum_f C_f^A B(\tau_f),$$

$$B(\tau) = \tau \arctan^2 \frac{1}{\sqrt{\tau - 1}} - 1$$

$$\tau_f = 4m_f^2/m_a^2 - i\epsilon$$