

轻介子手征微扰论简介 2022.12.

5.1. 手征对称性及其自发破缺

轻味夸克: $q = \begin{pmatrix} u \\ d \end{pmatrix}$ or $q = \begin{pmatrix} u \\ s \end{pmatrix}$

QCD 拉氏量:

$$\mathcal{L}_{QCD} = i\bar{q}\not{\partial}q - \bar{q}M_q q + \dots \quad \text{其中 } M_q = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$$

引入 $q_R = \frac{1}{2}(1+\gamma_5)q$, $q_L = \frac{1}{2}(1-\gamma_5)q$

$$\mathcal{L}_{QCD} = i\bar{q}_L\not{\partial}q_L + i\bar{q}_R\not{\partial}q_R - \bar{q}_R M_q q_L - \bar{q}_L M_q q_R + \dots$$

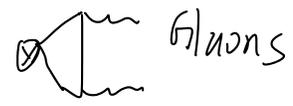
取手征极限 ($M_q \rightarrow 0$):

$$\mathcal{L}_{QCD}^\chi = i\bar{q}_L\not{\partial}q_L + i\bar{q}_R\not{\partial}q_R + \dots$$

当 $\left\{ \begin{array}{l} q_R \rightarrow \underline{V}_R q_R, \quad \underline{V}_R \in SU(3)_R \\ q_L \rightarrow \underline{V}_L q_L, \quad \underline{V}_L \in SU(3)_L \end{array} \right.$ 时, \mathcal{L}_{QCD}^χ 不变

另外: $\left\{ \begin{array}{l} ① q \rightarrow e^{i\alpha} q \quad [U(1)_V \text{ 变换}] \quad \mathcal{L}_{QCD}^\chi \text{ 不变} \\ ② q \rightarrow e^{i\beta\gamma_5} q \quad [U(1)_A \text{ 变换}]: \quad q_R \rightarrow e^{i\beta} q_R, \quad q_L \rightarrow e^{-i\beta} q_L \end{array} \right.$

经典水平上: \mathcal{L}_{QCD}^χ 不变, 但是其存在 量子反常, $\therefore U(1)_A$ 被破坏



即 \mathcal{L}_{QCD}^χ 对称群: $G^\chi = SU(3)_L \otimes SU(3)_R \times U(1)_V$

\Downarrow
手征对称群

\searrow
重子数守恒

味空间的整体对称性: Noether 定理, 写出守恒流, \rightarrow 关系: 流代数

Gasser, Leutwyler 1984: 外源场

含有外源场的 QCD: (external source)

$$\mathcal{L}_{\text{QCD}}^{\text{ext}} = \mathcal{L}_{\text{QCD}}^{\pi} + \bar{q} \gamma^{\mu} (V_{\mu} + a_{\mu} \gamma_5) q - \bar{q} \underbrace{(S - i \gamma_5)}_{\uparrow} q \quad (1)$$

其中: 外源场 V_{μ} , a_{μ} , S , P 均定义在味空间.

例: $V_{\mu} = \sum_{a=1}^{\infty} \lambda^a V_{a\mu}$, \dots $\text{Tr}(V_{\mu}) = 0$, \dots

特别地: 取 $V_{\mu} = 0$, $a_{\mu} = 0$, $P = 0$, $S = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$ 时, $-\bar{q} M_q q$,

$$\mathcal{L}_{\text{QCD}}^{\text{ext}} \rightarrow \mathcal{L}_{\text{QCD}}$$

(1) 改写为:

$$\mathcal{L}_{\text{QCD}}^{\text{ext}} = \mathcal{L}_{\text{QCD}}^{\pi} + \bar{q}_L \gamma^{\mu} l_{\mu} q_L + \bar{q}_R \gamma^{\mu} r_{\mu} q_R - \bar{q}_L \underbrace{(S + i P)}_{\text{---}} q_L - \bar{q}_R \underbrace{(S - i P)}_{\text{---}} q_R$$

$(l_{\mu} = V_{\mu} - a_{\mu}, r_{\mu} = V_{\mu} + a_{\mu})$

下一步: 整体对称性作“定域化”, 即 $\mathcal{L}_{\text{QCD}}^{\text{ext}}$ 在定域变换下不变:

$$q_L \rightarrow V_L(x) q_L, \quad q_R \rightarrow V_R(x) q_R, \quad \partial q \rightarrow \underbrace{\partial V_L q + V_L \partial q}$$

$$l_{\mu} \rightarrow V_L l_{\mu} V_L^{\dagger} + i V_L \partial_{\mu} V_L^{\dagger}, \quad r_{\mu} \rightarrow V_R r_{\mu} V_R^{\dagger} + i V_R \partial_{\mu} V_R^{\dagger}$$

$$S + i P \rightarrow V_R (S + i P) V_L^{\dagger}$$

外源场势能:

$$\begin{aligned} e^{iZ(V_{\mu}, a_{\mu}, S, P)} &= \int D\bar{q} Dq D\sigma_{\mu} e^{i \int d^4x \mathcal{L}_{\text{QCD}} | \bar{q}, q, \sigma_{\mu}, V_{\mu}, a_{\mu}, S, P } \\ &= \int Dq e^{i \int d^4x \mathcal{L}_{\text{EFT}}(U, V_{\mu}, a_{\mu}, S, P)} \end{aligned}$$

手征对称性自发破缺

若 $U(1)$ 具有 $G^X = SU(3)_L \otimes SU(3)_R$ 对称性, 预言 parity doublets, 与实验强子谱不符.

于是, 猜测发生自发破缺:

$$G^X = SU(3)_L \otimes SU(3)_R \longrightarrow H = SU(3)_V$$

由此产生 8 个 Nambu-Goldstone boson (NGB) $\Leftrightarrow \pi^\pm, \pi^0, K^\pm, K^0, \bar{K}^0, \dots$

Goldstone 粒子空间具有与 H 相同的结构, 在 H 变换不变.

用 U 场表示 NGB 场, $U \in SU(3)$, 现在流行的方式:

$$U = e^{i \frac{\sqrt{2}}{F} \Phi}$$

式中: F 待定常数, $\Phi = \sum_{a=1}^8 \frac{\lambda_a}{\sqrt{2}} \phi_a = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta^8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta^8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}} \eta^8 \end{pmatrix}$

说明: (a) U 在 G^X 下的变换行为: $U \xrightarrow{G^X} V_L U V_R^+$

$$0 \neq \langle \bar{q} q \rangle : \quad \langle \bar{q}_L q_L \rangle + \langle \bar{q}_R q_R \rangle$$

(b) $U \in SU(3)$, $\left. \begin{array}{l} U^+ U = 1 \\ \det U = 1 \end{array} \right\} U = e^{i \frac{\sqrt{2}}{F} \Phi}, \quad \Phi^+ = \Phi$
 $\log \det X = \text{Tr} \log X$

$$0 = \ln \det U = \text{Tr}(\ln U) = i \frac{\sqrt{2}}{F} \text{Tr}(\Phi), \quad \text{即 } \Phi \text{ 为无迹的.}$$

§2. 手征拉氏量的构造

自由度: U, V_L, L_M, S, P .

当不考虑外源场时, 也不考虑微商时, 由 U 构造的在 G^x 下不变的项有:

$$U \rightarrow \underbrace{V_R}_\sim U \underbrace{V_L^+}_\sim$$

$$U^+ U \xrightarrow{G^x} V_L U^+ V_R^+ V_R U V_L^+ = V_L U^+ U V_L^+$$

求 trace($\langle \rangle$): $\langle U^+ U \rangle \rightarrow \langle U^+ U \rangle \quad \checkmark$

$$\langle U^+ U \rangle, \det U, \det U^+$$

$$\text{又: } \langle U^+ U \rangle = 3, \det U = 1, \det U^+ = 1.$$

表明在没有微商算符时, N 与 B 无相互作用

下一步: 引入微商及外源场

$$\partial_\mu U \xrightarrow{G^x} V_R \partial_\mu U V_L^+ + \partial_\mu V_R U V_L^+ + V_R U \partial_\mu V_L^+ \neq V_R \partial_\mu U V_L^+$$

定义协变微商: $D_\mu U = \partial_\mu U - i \chi_\mu U + i U L_\mu$

$$\text{自行证明: } D_\mu U \xrightarrow{G^x} V_R D_\mu U V_L^+$$

S, P 外源场: 按照 G_L 的约定, 引入: $\chi = 2B(S + iP) \quad [B \text{ 为待定常数}]$

$$\chi \xrightarrow{G^x} V_R \chi V_L^+$$

现在有: $U, D_n U, \chi \quad \left[\xrightarrow{G^X} V_R \oplus V_L^+ \right]$

在 G^X 下不变的厄米算符有:

$$\mathcal{L}_2 = a_1 \langle (D_n U)^+ D^n U \rangle + a_2 \langle \chi U^+ + U \chi^+ \rangle + i a_3 \langle \chi U^+ - U \chi^+ \rangle$$

说明: (a) $\langle \chi U^+ \rangle \rightarrow \langle V_R \chi U^+ V_R^+ \rangle = \langle \chi U^+ \rangle, \dots$

$$(b) \langle U^+ D_n U \rangle \langle U^+ D_n U \rangle = 0$$

$$\begin{aligned} \langle U^+ D_n U \rangle &= \langle U^+ \partial_n U - i k_n U U^+ + i l_n U U^+ \rangle \\ &= \langle U^+ \partial_n U \rangle - i \underbrace{\langle k_n \rangle}_{=0} + i \underbrace{\langle l_n \rangle}_{=0} \end{aligned}$$

$$\langle U^+ \partial_n U \rangle = \partial_n \langle \ln U \rangle \sim \partial_n \langle \Phi \rangle = 0$$

(c) 在 P 宇称下: $L \leftrightarrow R$, 意味 $U \rightarrow U^+, \chi \rightarrow \chi^+$

$$i a_3 \langle \chi U^+ - U \chi^+ \rangle \xrightarrow{P} -i a_3 \langle \chi U^+ - U \chi^+ \rangle, \text{ 不再考虑}$$

(d) a_1, a_2 取值: $\partial_n \phi_n \partial^n \phi_n$ 的系数 $\frac{1}{2}$, 以及构造的质量项

$$a_1 = a_2 = \frac{F^2}{4}$$

$$\text{于是 } \mathcal{L}_2 = \frac{F^2}{4} \langle (D_n U)^+ D^n U \rangle + \frac{F^2}{4} \langle \chi U^+ + U \chi^+ \rangle$$

§2.1 最低阶手征拉氏量

$$\mathcal{L}_2 = \frac{F^2}{4} \langle (D_\mu U)^\dagger D^\mu U \rangle + \frac{F^2}{4} \langle \chi U^\dagger + U \chi^\dagger \rangle \quad \leftarrow$$

式中: $U = e^{i \frac{\sqrt{2}}{F} \Phi} = 1 + i \frac{\sqrt{2}}{F} \Phi - \frac{1}{F^2} \Phi^2 + \dots$

$$D_\mu U = i \frac{\sqrt{2}}{F} \partial_\mu \Phi - \frac{1}{F^2} (\partial_\mu \Phi \Phi + \Phi \partial_\mu \Phi) - 2i a_\mu + V_\mu (\partial^\nu \Phi \Phi - \Phi \partial^\nu \Phi) + \dots$$

$$\chi = 2B(\text{Stip}) \stackrel{\text{UCCD}}{=} 2B M_q \stackrel{\text{isospinV}}{=} 2B \begin{pmatrix} m_q & & \\ & m_q & \\ & & m_s \end{pmatrix}$$

于是 $\mathcal{L}_2 = \frac{1}{2} \partial_\mu \pi^0 \partial^\mu \pi^0 + \partial_\mu \pi^+ \partial^\mu \pi^- + \dots$

$$- B m_q \pi^0 \pi^0 - 2B m_q \pi^+ \pi^- - B (m_q + m_s) (\bar{k}^0 k^0 + k^+ k^-) - B \frac{m_q + 2m_s}{3} \eta \eta + \dots$$

$$\Rightarrow m_\pi^2 = 2B m_q, \quad m_K^2 = B(m_q + m_s),$$

$$m_\eta^2 = 2B \frac{m_q + 2m_s}{3} = \frac{4m_K^2 - m_\pi^2}{3} \quad [\text{Gell-Mann-Okubo 关系}]$$

练习: m_s 为物理值时, 估计当 $m_\pi^{\text{lat}} = 300 \text{ MeV}$ 时, $m_K^{\text{lat}} = ?$

参数 B 的定义:

在 χ PT 下 UCCD, 求基态的能量密度

$$\chi\text{PT: } \phi_a = 0 \Rightarrow U = U^\dagger = 1, \quad \langle H_2 \rangle_{\text{VZU}} = -BF^2 \langle M_q \rangle = -BF^2 (m_u + m_d + m_s)$$

$$\text{UCCD: } \langle H_{\text{UCCD}} \rangle = \langle \bar{q} M_q q \rangle = \langle \bar{q} q \rangle (m_u + m_d + m_s) \quad [\langle \bar{u} u \rangle = \langle \bar{d} d \rangle = \langle \bar{s} s \rangle]$$

$$\Rightarrow 0 \neq \langle \bar{q} q \rangle = -BF^2$$

↑

参数 F 的定义:

$$\text{在 } L_2 \text{ 中抽取轴矢流: } \mathcal{L}_2 = \frac{F^2}{4} \langle (D_\mu U)^\dagger (D^\mu U) \rangle = -\sqrt{2} F \langle a_\mu \partial^\mu \phi \rangle + \dots$$

$$\langle 0 | \bar{u} \gamma^\mu \gamma_5 u | \pi^+ \rangle = - \langle 0 | -\sqrt{2} F a_\mu \pi^+ | \pi^+ \rangle = i\sqrt{2} P_0 F$$

$\therefore F$ 对应 π 的 weak decay constant,

F_π 的物理值 92 MeV, [有的文献 $\bar{F}_\pi = \sqrt{2} F_\pi = 130 \text{ MeV}$.]

PNBB 相互作用:

$$\mathcal{L}_2 = \frac{1}{6F^2} \langle \partial_\mu \phi \phi \partial^\mu \phi \phi - \partial_\mu \phi \partial^\mu \phi \phi \phi \rangle + \frac{B}{6F^2} \langle M_q \phi^4 \rangle$$

例子: $\pi^+ \pi^- \pi^0 \pi^0$ 顶点:

$$\frac{1}{3F^2} (\partial_\mu \pi^+ \partial^\mu \pi^0 \pi^- \pi^0 + \partial_\mu \pi^- \partial^\mu \pi^0 \pi^+ \pi^0 - \partial_\mu \pi^+ \partial^\mu \pi^- \pi^0 \pi^0 - \partial_\mu \pi^0 \partial^\mu \pi^0 \pi^+ \pi^-)$$

$$+ \frac{m_\pi^2}{6F^2} \pi^+ \pi^- \pi^0 \pi^0$$

$$T_{\pi^+ \pi^- \rightarrow \pi^0 \pi^0} = \frac{S - m_\pi^2}{F^2} \quad [S = (p_+ + p_-)^2] \quad [\text{自验证}]$$

说明: π^+ 相互作用量纲为 6, 不重整, 无穷多抵消项。 Power counting 救场!

§ 2.2 手征数幂规则 (Chiral power counting)

power counting: 1) 理论的有效性: 即无穷多参数的重要性

2) 逐阶重整化

手征展开: 1) 小动量 P 展开

2) m_π , $m_\pi \sim P$, $m_\pi \sim O(P^2)$

原因: $M_\pi^2 = 2B M_\pi \sim P^2$ [B 常数, $\langle \bar{q}q \rangle \sim BF^2$]

3) 外源场幂次: $D_\mu U = \partial_\mu U - i\gamma_\mu U + i k_\mu U$

$\partial_\mu \sim P_\mu$, $\partial_\mu \sim \gamma_\mu \sim k_\mu$ 地位等价, $\therefore \gamma_\mu \sim k_\mu \sim O(P)$

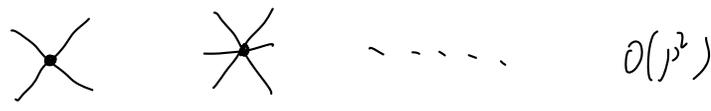
Stip: $\bar{q}(Stip)q \xrightarrow{S \rightarrow M_\pi} + \bar{q} M_\pi q$, $S \sim B M_\pi \sim O(P^2)$, 于是 $Stip \sim O(P^2)$

4) U 场: $O(P^0)$, $F \sim O(P^0)$, $D_\mu U \sim O(P)$, $\chi = 2B(Stip) \sim O(P^2)$

审核 $I_2 = \frac{F^2}{4} \langle (D_\mu U)^\dagger D^\mu U \rangle + \frac{F^2}{4} \langle \chi U^\dagger + U \chi^\dagger \rangle \sim O(P^2)$

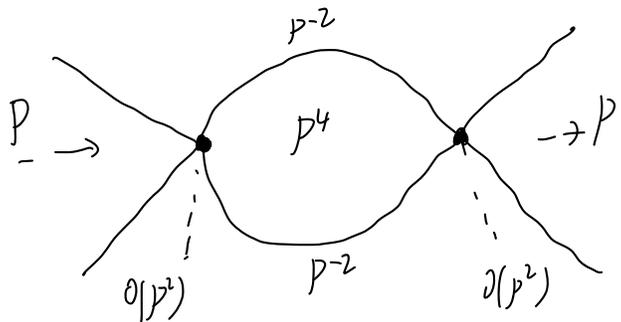
\uparrow \downarrow \downarrow \uparrow
 $O(P^2)$ $O(P)$ $O(P)$ $O(P^2)$

I_2 : 给出所有的作用顶点均为 $O(P^2)$, [与中场的幂次无关]

例如:  $O(P^2)$

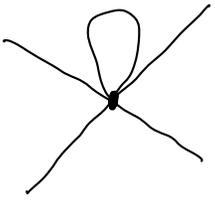
传播: $\frac{i}{P^2 - m_\pi^2} \sim O(P^{-2})$

$\pi\pi \rightarrow \pi\pi$ 单圈的手征幂次



$P^2 \cdot P^2 \cdot (P^{-2})(P^{-2}) \cdot P^4 \sim P^4$

$\int \frac{d^4k}{(k^2 - m_\pi^2) [(k+P)^2 - m_\pi^2]} \sim f(P^2, m_\pi^2)$



$$p^2 \cdot \frac{1}{p^2} \cdot p^4 \sim p^4$$

一般性: N_p 个传播子, N_L 个圈, N_n 个第 n 阶 p^n 的顶点, 费曼图的幂次:

$$N_{Tot} = -2N_p + 4N_L + \sum_{n \geq 2} n N_n$$

利用拓扑学知识: $N_p = N_L + N_V - 1$ $[N_V = \sum_n N_n]$
(内线) (圈) (顶点)

$$\Rightarrow N_{Tot} = 2 + 2N_L + \sum_{n \geq 2} (n-2) N_n$$

分析: (1) $N_{Tot} = 2$ 时, $N_L = 0, N = 2$, 即 \mathcal{L}_2 贡献的树图

(2) $N_{Tot} = 4$ 时, $\left. \begin{array}{l} N_L = 1, N = 2 : \mathcal{L}_2 \text{ 顶点贡献的一圈图} \\ N_L = 0, N = 4, N_n = 1, \text{ 由 } \mathcal{L}_4 \text{ 贡献的树图} \end{array} \right\}$

一般性结论: 理论中的发散可以逐渐抵消.

§2.3 高斯拉氏量的构造

步骤: (1) 写出所有对称性允许的算符 (2) 利用条件限制找出独立的算符.

$O(p^4)$: Next-to-leading order (NLO)

可以用 U, SU, X 来讨论

另一种: $u, U = u^2, [U \rightarrow V_R U V_L^+]$

$$u \xrightarrow{G^X} V_R u H^+ = H u V_L^+ \quad [H \text{ 为矩阵}, H \in SU(3)_V, H = H(V_L, V_R, u)]$$

由此定义: $u_m = i [u^+ (\partial_m - i \chi_m) u - u (\partial_m - i b_m) u^+]$

$$\Rightarrow u_m \xrightarrow{G^X} \underline{H u_m H^+} \quad [H \in SU(3)_V]$$

类似地: $\chi_{\pm} = u^+ \chi u^+ \pm u \chi^+ u$

$$f_{\pm, \mu\nu} = u F_{\pm}^{\mu\nu} u^+ \pm u^+ F_{\pm}^{\mu\nu} u \quad [F_{\pm}^{\mu\nu}, F_{\pm}^{\mu\nu}, b_{\mu\nu}, \chi_{\pm} \text{ 场强分量}]$$

$$\underline{u_m, \chi_{\pm}, f_{\pm, \mu\nu}} \longrightarrow H \square H^+$$

↓
X

$$\nabla_m X = \partial_m X + [\Gamma_m, X]$$

$$\Gamma_m = \frac{i}{2} \{ u^+ (\partial_m - i \chi_m) u + u (\partial_m - i b_m) u^+ \}$$

$$\nabla_m X \longrightarrow H (\nabla_m X) H^+$$

重写 \mathcal{L}_2 : $\mathcal{L}_2 = \frac{F^2}{4} \langle u_m u^m \rangle + \frac{F^2}{4} \langle \chi_+ \rangle$

OP⁴): $\langle u_m u^m u^\nu u_\nu \rangle, \langle u_m u^m \rangle \langle u_\nu u^\nu \rangle, \langle u_m u^m \rangle \langle \chi_+ \rangle, \dots$

$$\begin{aligned} \mathcal{L}_4 = & \mathcal{L}_1 \langle u^m u_m \rangle \langle u^\nu u_\nu \rangle + \mathcal{L}_2 \langle u^m u^\nu \rangle \langle u_m u_\nu \rangle + \mathcal{L}_3 \langle (u^m u_m) (u_\nu u^\nu) \rangle \\ & + \mathcal{L}_4 \langle u_m u^m \rangle \langle \chi_+ \rangle + \mathcal{L}_5 \langle u_m u^m \chi_+ \rangle + \mathcal{L}_6 \langle \chi_+ \rangle \langle \chi_+ \rangle + \mathcal{L}_7 \langle \chi_- \rangle \langle \chi_- \rangle \\ & + \frac{\mathcal{L}_8}{2} \langle \chi_+ \chi_+ + \chi_- \chi_- \rangle - i \mathcal{L}_9 \langle f_+^{\mu\nu} u_m u_\nu \rangle + \frac{\mathcal{L}_{10}}{4} \langle f_+^{\mu\nu} f_{+\mu\nu} - f_-^{\mu\nu} f_{-\mu\nu} \rangle + \dots \end{aligned}$$

说明: ^(a) SU(3) 规范.

$$\langle \underline{u^m u^\nu u_m u_\nu} \rangle = -2 \langle u_m u^m u_\nu u^\nu \rangle + \frac{1}{2} \langle u_m u^m \rangle \langle u^\nu u_\nu \rangle + \langle u^m u_\nu \rangle \langle u_m u_\nu \rangle$$

$\underbrace{\hspace{10em}}_{\mathcal{L}_3} \qquad \qquad \mathcal{L}_1 \qquad \qquad \qquad \mathcal{L}_2$

(b): L_i 的 N_c 阶数

- 一般规则: 1个夸克圈 \Leftrightarrow 夸克圈的 1个 trace [XPT]

每多一个 trace 带来 N_c^{-1} 压低

$$L_2: O(N_c); \quad F \sim O(\sqrt{N_c})$$

$$L_3, L_5, L_8, L_9, L_{10} \sim O(N_c)$$

$$L_4, L_6, L_7 \sim O(N_c^0)$$

$$L_0 \langle U^{\mu\nu} U^{\nu\mu} U_{\mu\nu} U_{\nu\mu} \rangle: \quad L_1 \sim \frac{L_0}{2}, \quad L_2 \sim L_0, \quad L_3 \sim -2L_0$$

$$L_1, L_2 \sim O(N_c), \quad 2L_1 - L_2 \sim O(N_c^0)$$

$$(c) \quad L_i = L_i^r + L_i^\infty$$

L_i 有限部分: 唯象学、共振态估计 (resonance chiral theory),

Gr. Ecker, et al, 1988 NPB

§2.4 重整化

总体原则: 具体到某一阶, 圈图发散可以用有限项的拉氏量抵消.

质量及波函数:

$$\text{---} \text{---} \text{---} \text{---} \text{---} = \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \text{---} + \dots$$

\uparrow
 $(-i\Sigma)$

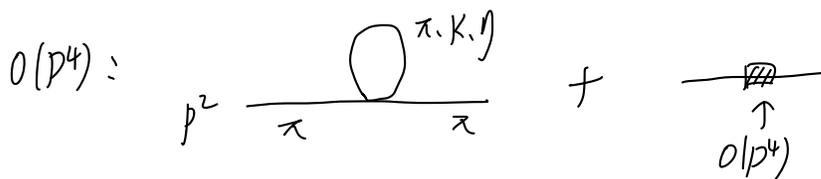
$$i\Delta_\phi^r = \frac{i}{p^2 - m_0^2 - \Sigma(p^2)}$$

$$= \frac{i z_\phi}{p^2 - m_{\text{phy}}^2} \leftarrow$$

$$\left. \begin{aligned} m^2 &= m_0^2 + \Sigma(m^2) \\ z_\phi &= 1 + \Sigma'(m^2) \end{aligned} \right\}$$

π 介子:

领头阶: $M_0^2 = \bar{M}_\pi^2 = 2B M_0$



$$-\Sigma_{loop}(s) = \frac{s-M_\pi^2}{3F^2} A_0(M_\pi^2) + \frac{2s+M_\pi^2}{6F^2} A_0(M_\pi^2) + \frac{s-M_\pi^2}{6F^2} A_0(M_K^2) + \frac{s-M_\pi^2}{6F^2} A_0(M_K^2) - \frac{M_\pi^2}{6F^2} A_0(M_\eta^2)$$

[π^\pm π^0 K^\pm K^0/\bar{K}^0 η]

$$A_0(M^2) = -i \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - M^2} = -\frac{M^2}{16\pi^2} [R + \ln \frac{M^2}{\mu^2}]$$

$$\left\{ R = 2 \left[\frac{1}{\epsilon} - \frac{1}{2} (\ln 4\pi + \Gamma'(1) + 1) \right] \right\}$$

$O(p^4)$ 树图

$$-\Sigma_{tree}(s) = \frac{8L_4}{F^2} (2M_K^2 + M_\pi^2) s + \frac{8L_5 M_\pi^2}{F^2} s - \frac{16L_6}{F^2} M_\pi^2 (2M_K^2 + M_\pi^2) - \frac{16L_8}{F^2} M_\pi^4$$

$$\Rightarrow M_\pi^2 = \bar{M}_\pi^2 + \Sigma_{loop}(s=M_\pi^2) + \Sigma_{tree}(s=M_\pi^2)$$

(物理)

[\bar{M}_π^2, M_π^2 差别) 只会引起 $O(p^6)$ 效应,]

波函数: $\Sigma'(M_\pi^2) = \frac{d\Sigma(s)}{ds} \Big|_{s=M_\pi^2} = -\frac{2}{3} A_0(M_\pi^2) - \frac{A_0}{3F^2} A_0(M_K^2) - \frac{8L_5 M_\pi^2}{F^2} - \frac{8L_4}{F^2} (2M_K^2 + M_\pi^2)$

讨论: [$Z_\pi = 1 + \Sigma'(M_\pi^2)$]

(1) M_π^2 物理量, 应有限:

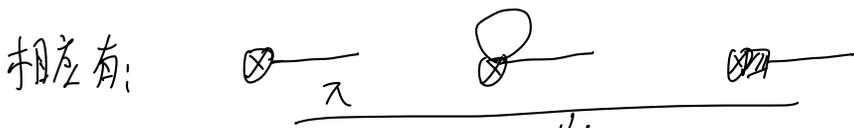
比较: [M_π^2, M_K^2, M_π^4] $\frac{1}{\epsilon}$ 系数, 得到:

$$L_4^\infty - 2L_6^\infty = -\frac{F}{16\pi^2} \frac{1}{F^2}, \quad -L_5^\infty + 2L_8^\infty = -\frac{F}{16\pi^2} \frac{1}{F^2}$$

注意: $\Sigma'(M_\pi^2)$ 非物理量, 可以发散!

π介子的衰变常数:

反定义: $\langle 0 | A_\mu | \pi \rangle = i\sqrt{2} F_\pi P_\mu$



$$F_\pi^r = \sqrt{2} \left(F + F_{loop}^{1PI} + F_{LEGS}^{1PI} \right)$$

$$= F + \frac{1}{F^2} A_0(m_\pi^2) + \frac{1}{2F^2} A_0(m_K^2) + \frac{4L_5}{F^2} m_\pi^2 + \frac{4L_4}{F^2} (2m_K^2 + m_\pi^2)$$

F_π^r 有限: 比较 m_π^2 , m_K^2 的^{1/2}系数

$$L_4^\omega = \frac{F}{16\pi^2} \frac{1}{16}, \quad L_5^\omega = \frac{F}{16\pi^2} \frac{3}{16}$$

于是有: $L_6^\omega = \frac{F}{16\pi^2} \frac{11}{208}, \quad L_8^\omega = \frac{F}{16\pi^2} \frac{5}{96}$, 与 G. L. SU(3) XPT一致!

重整化以后:

$$\bar{m}_\pi^2 = 2Bm_\pi^2$$

$$m_\pi^2 = \bar{m}_\pi^2 - \frac{m_\pi^2}{2F^2} A_0^r(m_\pi^2) + \dots + \frac{16L_8^r}{F^2} m_\pi^4 + \dots$$

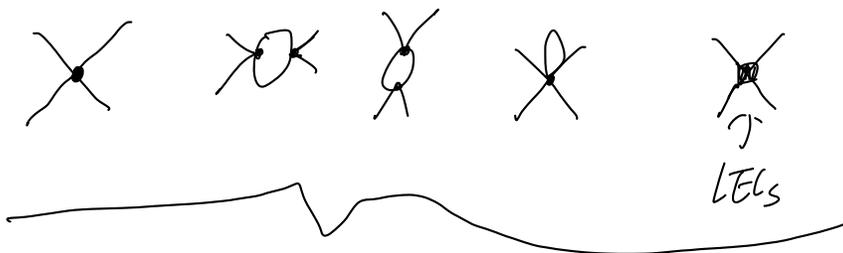
(物理)

\uparrow
 $-\frac{m_\pi^2}{16\pi^2} \ln \frac{m_\pi^2}{m^2}$

$$F_\pi = F + \frac{1}{F^2} A_0^r(m_\pi^2) + \dots$$

\uparrow

ππ散射:



$$T_{\pi\pi \rightarrow \pi\pi}^r = Z_\pi^2 T^{1PI}$$

$$\bar{m}_\pi^2, F \rightarrow m_\pi^2, F_\pi^r,$$

其他相关过程:

$\pi\pi$ 的矢量/标量形状因子: 

$$K_{13}: K \rightarrow \pi \ell \nu$$

$$K_{14}: K \rightarrow \pi \pi \ell \nu$$

$$\gamma\gamma \rightarrow \pi\pi: O(D^4) \text{ 有限}$$

$$\eta': \text{QCD } U_A(1) \text{ 反常, } SU(3) \rightarrow U(3), \quad \tilde{\rho} \sim m_\rho \sim \frac{1}{\Lambda_c}$$

Odd-intrinsic-parity 衰变: $\pi^0 \rightarrow \gamma\gamma$, $\phi, \omega \rightarrow \phi_3 \phi_4 \phi_5$

参考: Wess-Zumino-Witten 拉氏量.