暗物质理论介绍

Jia Liu (刘佳)
北京大学物理学院理论物理所

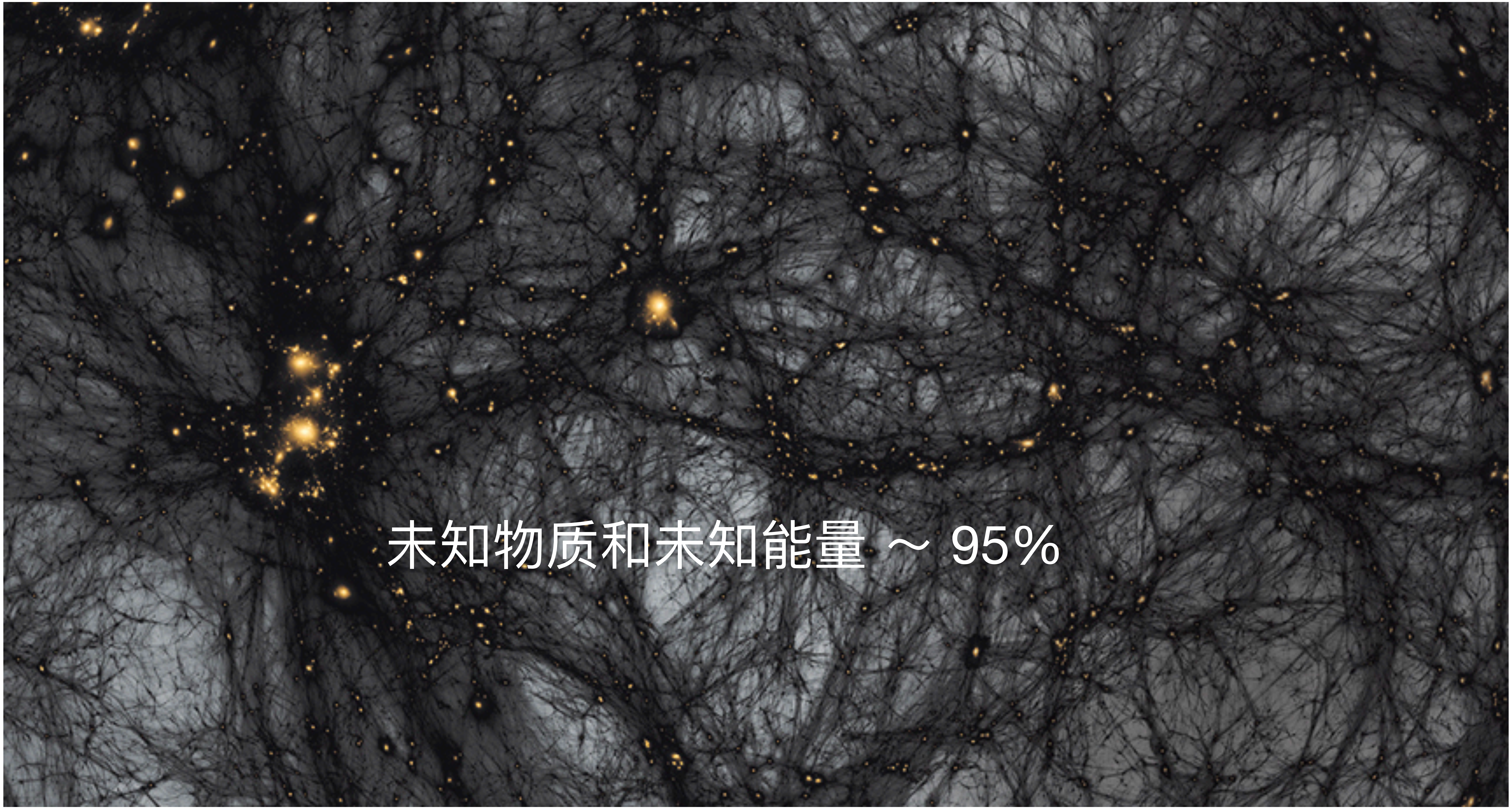
2022年粒子与核物理冬季研讨班
四川大学
【14:30-16:10】
2022-12-26

Outlines

- 暗物质的起源
    - 暗物质的天文观测证据
    - 宇宙学标准模型 ΛCDM
    - 暗物质分布
- 暗物质的物理模型
    - 可能的暗物质候选者
    - WIMP暗物质
- 热退耦暗物质残余丰度计算
- WIMP暗物质现状
    - WIMP暗物质的直接探测危机
        - 解决危机的多种办法
    - 暗物质对撞机探测的互补性
    - 暗物质的间接探测限制
        - 避开限制的办法
- 总结

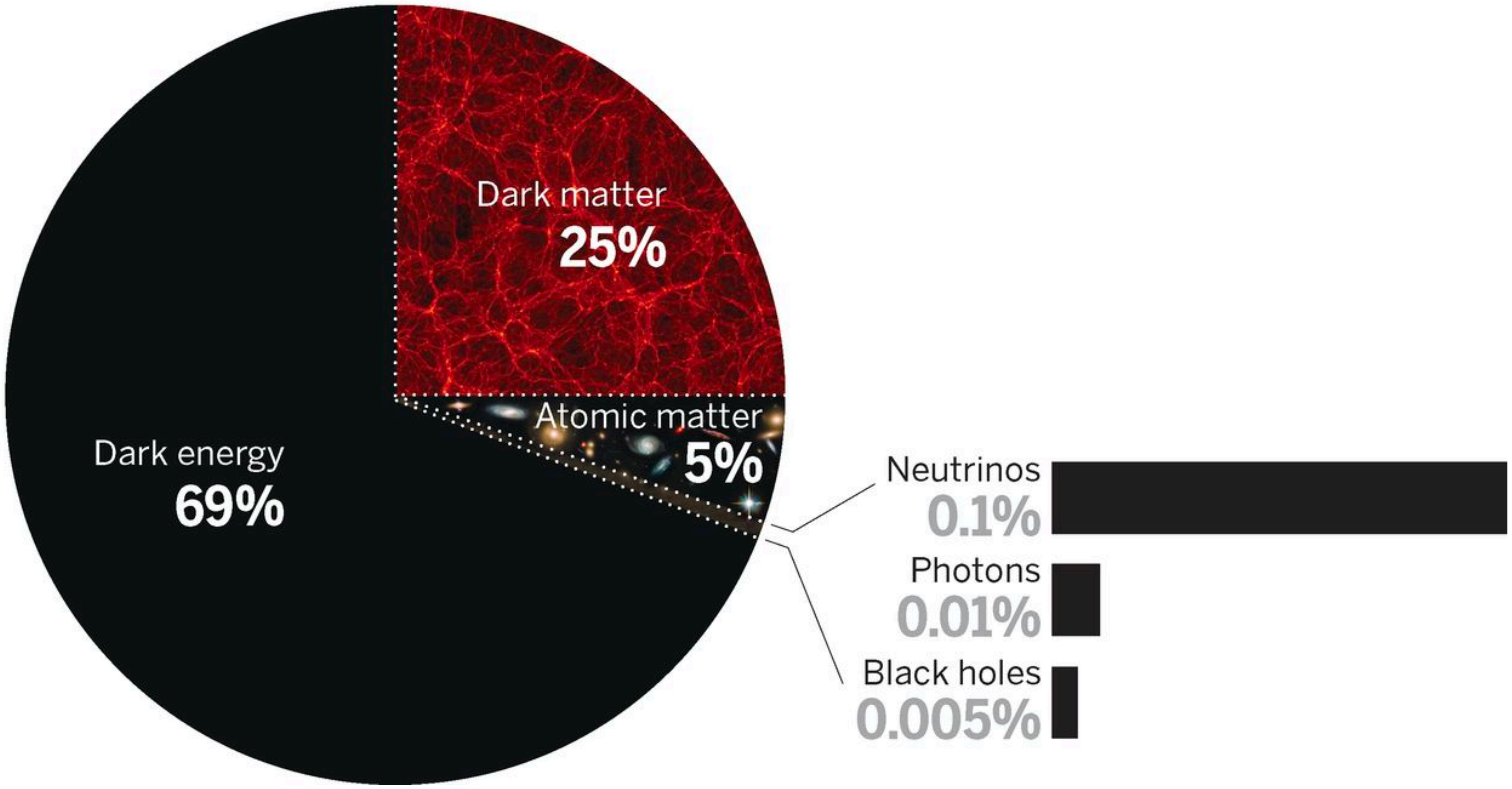
什么是暗物质?

什么是暗物质?



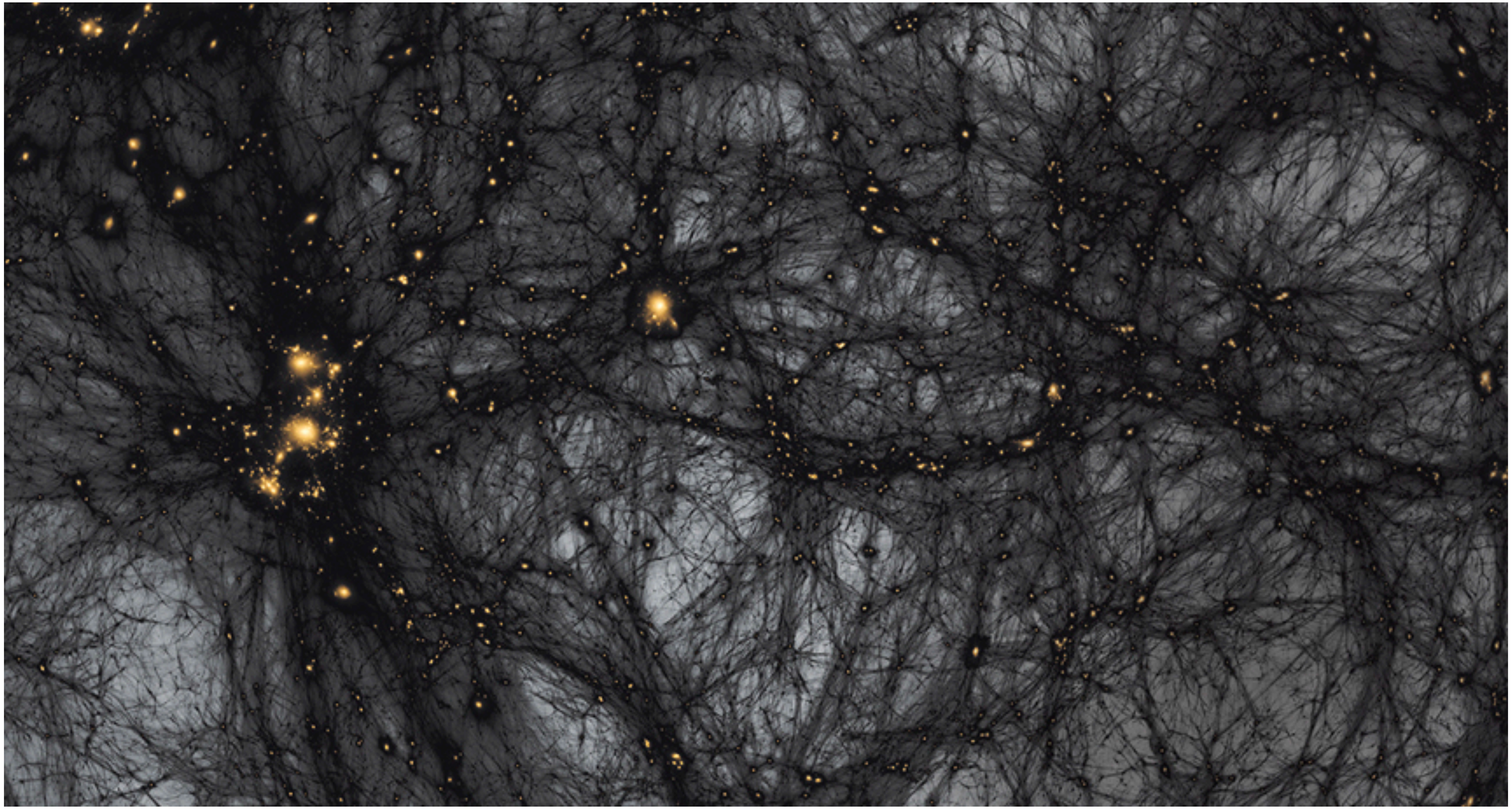
什么是暗物质?

- 已知的可见物质 ~ 5%
- 未知物质和未知能量 ~ 95%



什么是暗物质?

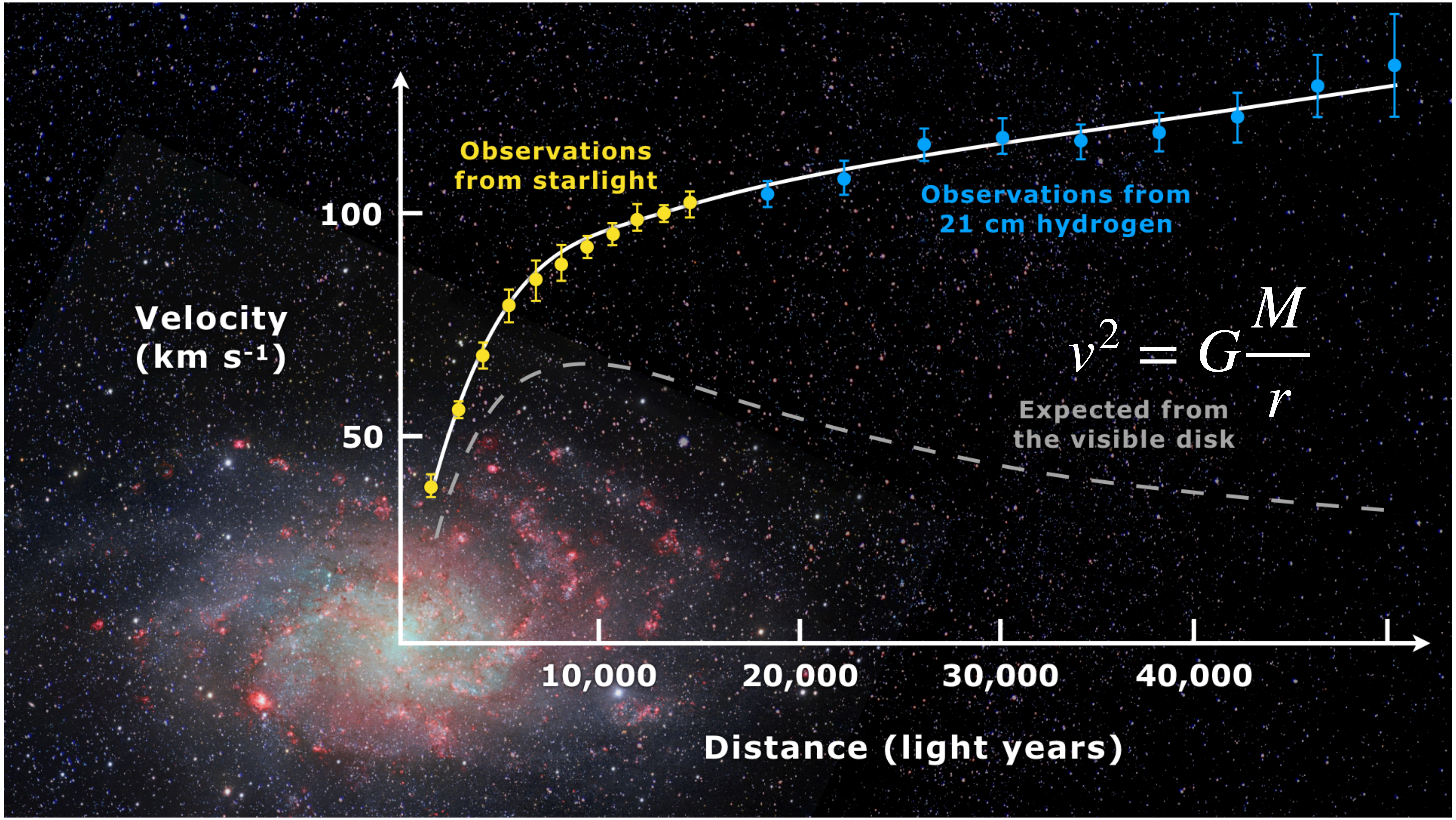
- 暗物质是理论上提出的可能存在于宇宙中的一种不可见的物质，它可能是宇宙物质的主要组成部分，但又不属于构成可见天体的任何一种已知的物质。



- 中性不带电
  - 和可见物质相互作用小的
- 稳定
- 有质量的
- 冷的

Observational evidence for DM

- Galaxy rotation curves
- Bullet cluster
- Gravitational Lensing
- Structure formation
- Cosmic Microwave Background

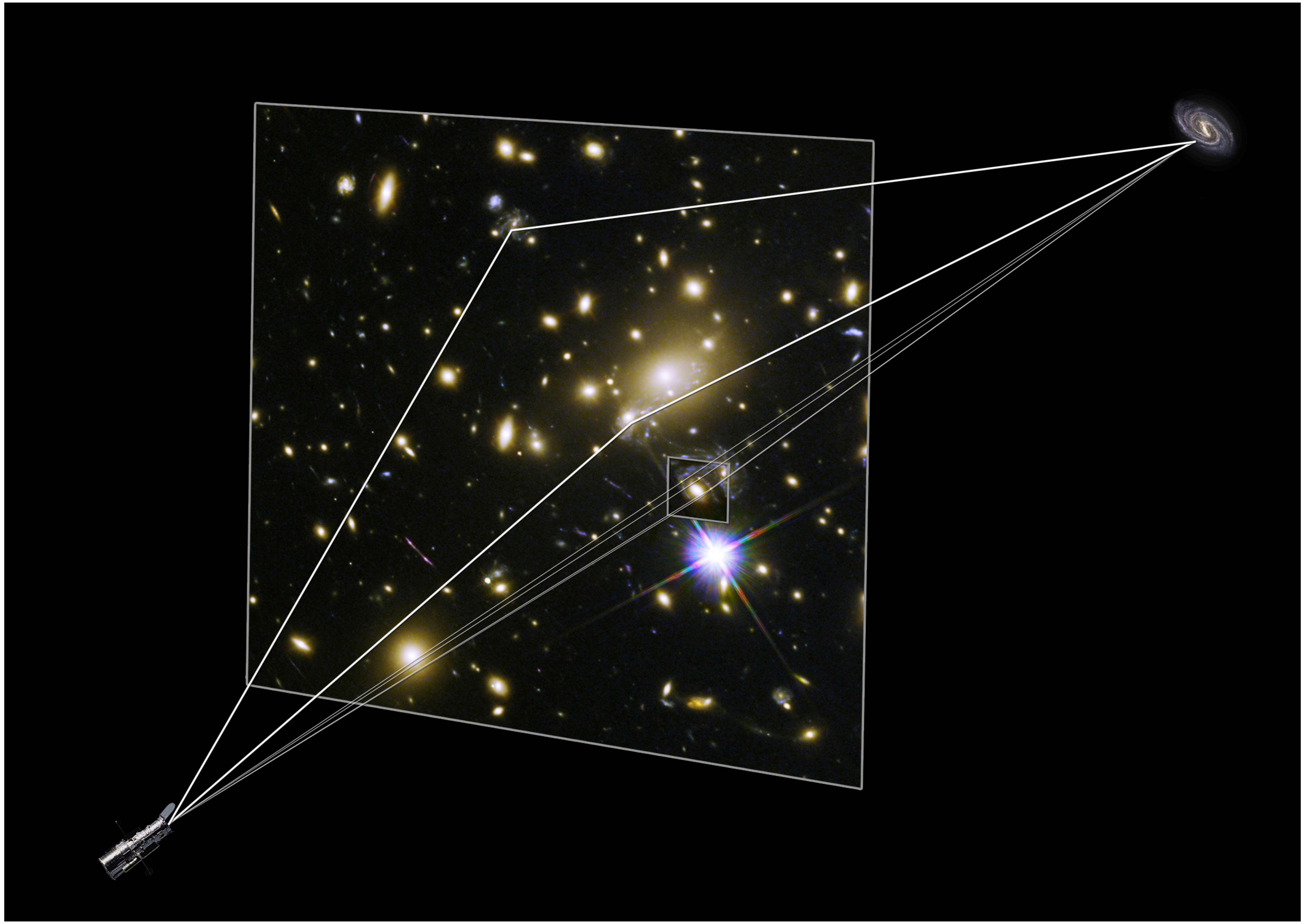


Observational evidence for DM

- Galaxy rotation curves
- Bullet cluster
- Gravitational Lensing
- Structure formation
- Cosmic Microwave Background

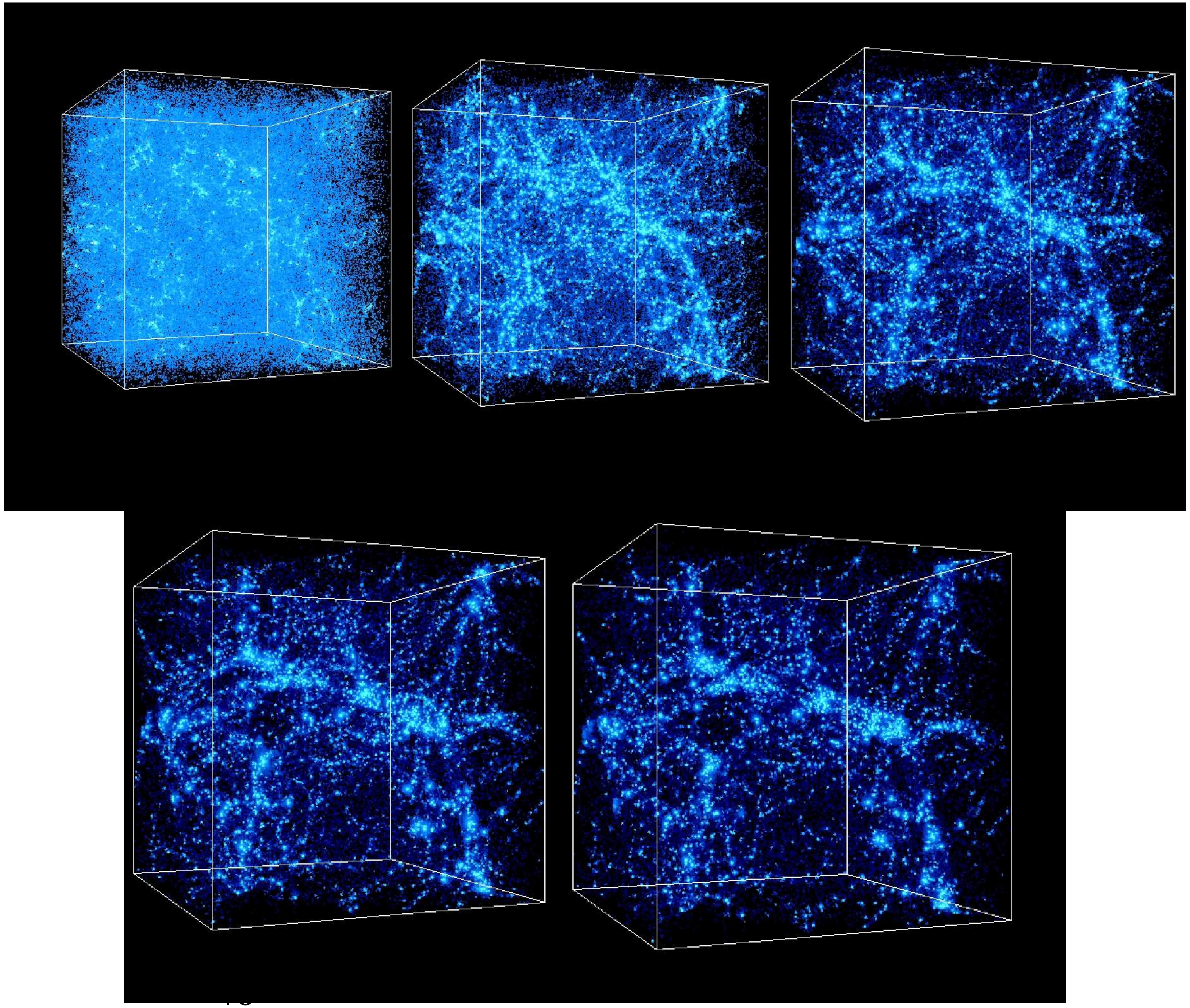
Observational evidence for DM

- Galaxy rotation curves
- Bullet cluster
- Gravitational Lensing
- Structure formation
- Cosmic Microwave Background



Observational evidence for DM

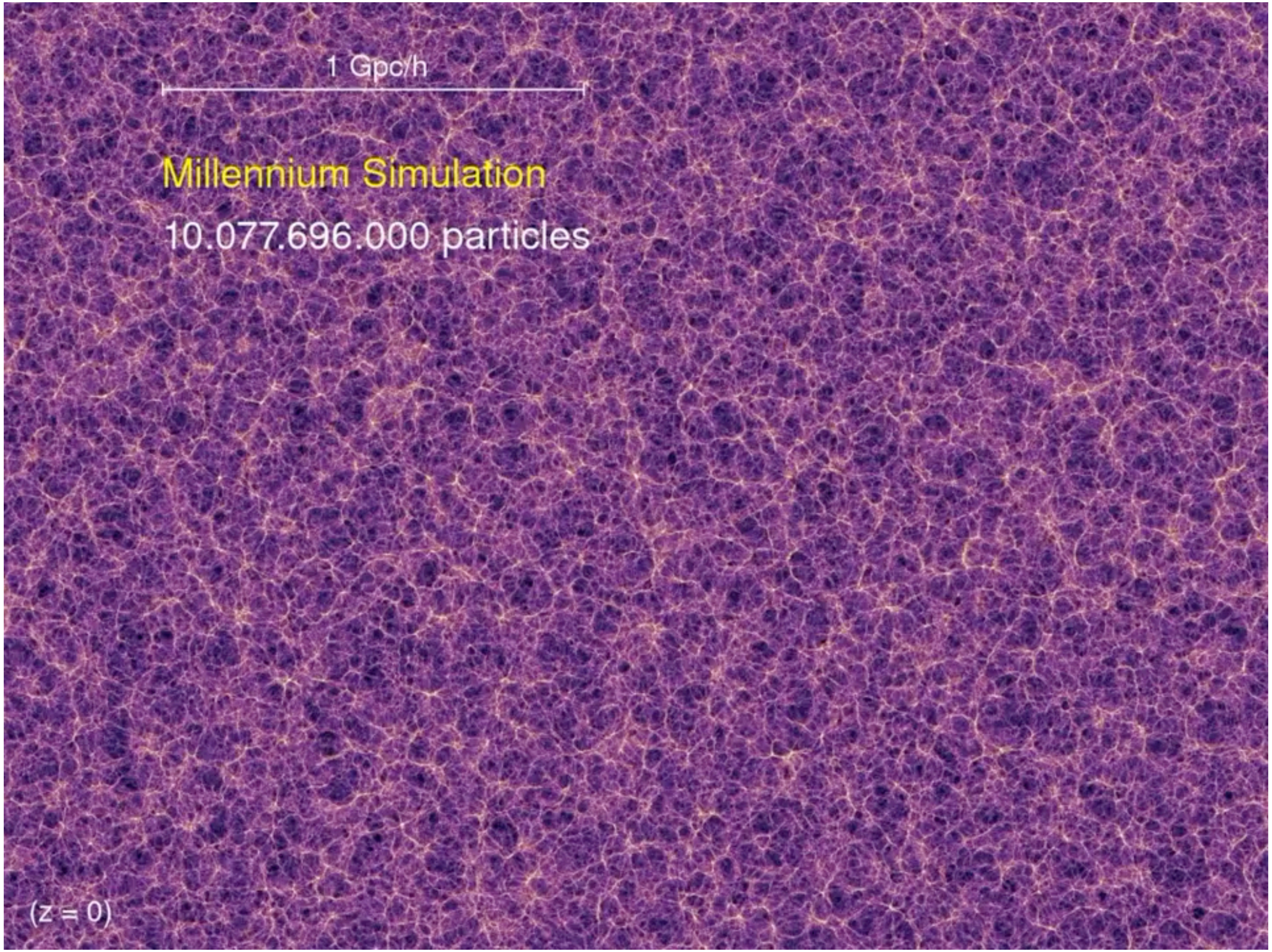
- Galaxy rotation curves
- Bullet cluster
- Gravitational Lensing
- Structure formation
- Cosmic Microwave Background
- …



Observational evidence for DM

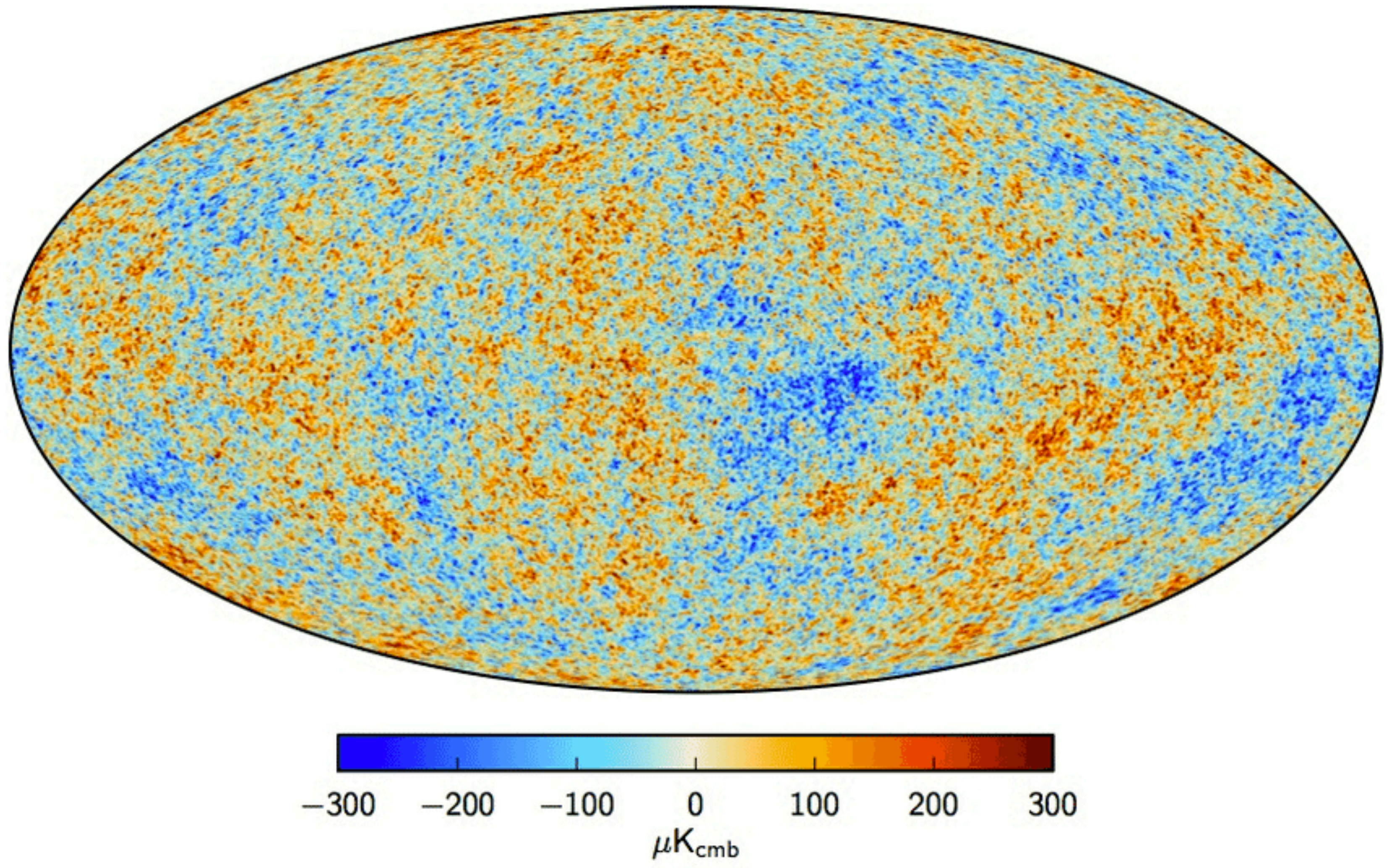
- Galaxy rotation curves
- Bullet cluster
- Gravitational Lensing
- Structure formation
- Cosmic Microwave Background
- …

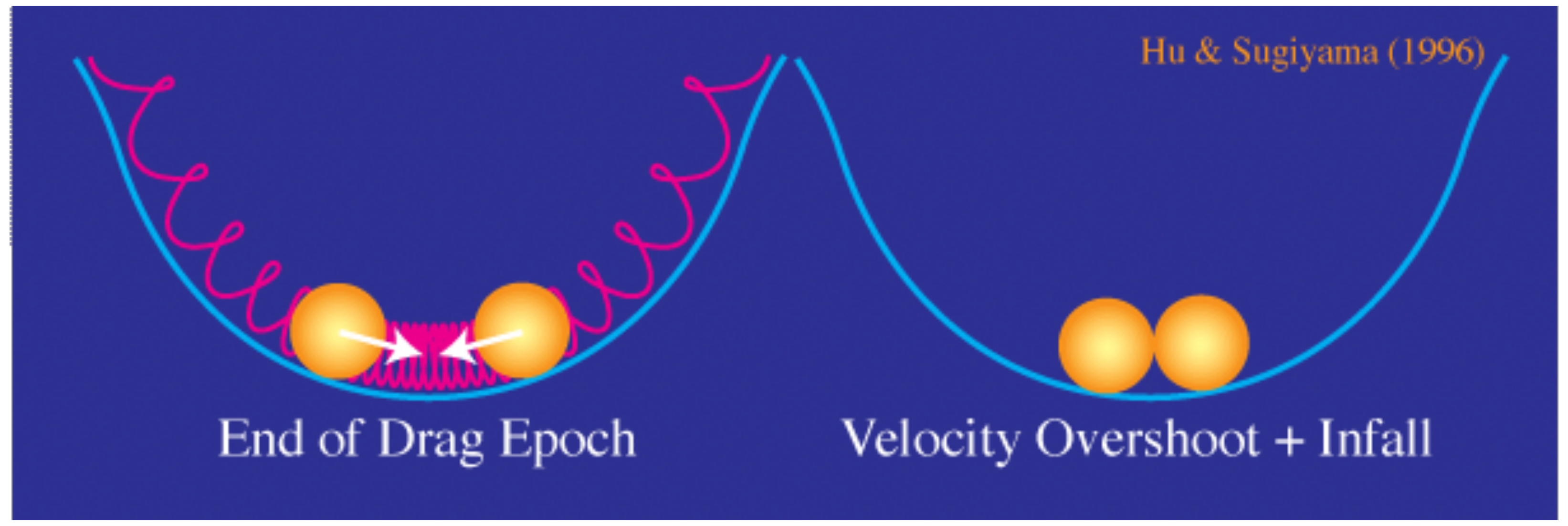
The *Millennium Run* used more than 10 billion particles to trace the evolution of the matter distribution in a cubic region of the Universe over 2 billion light-years on a side.

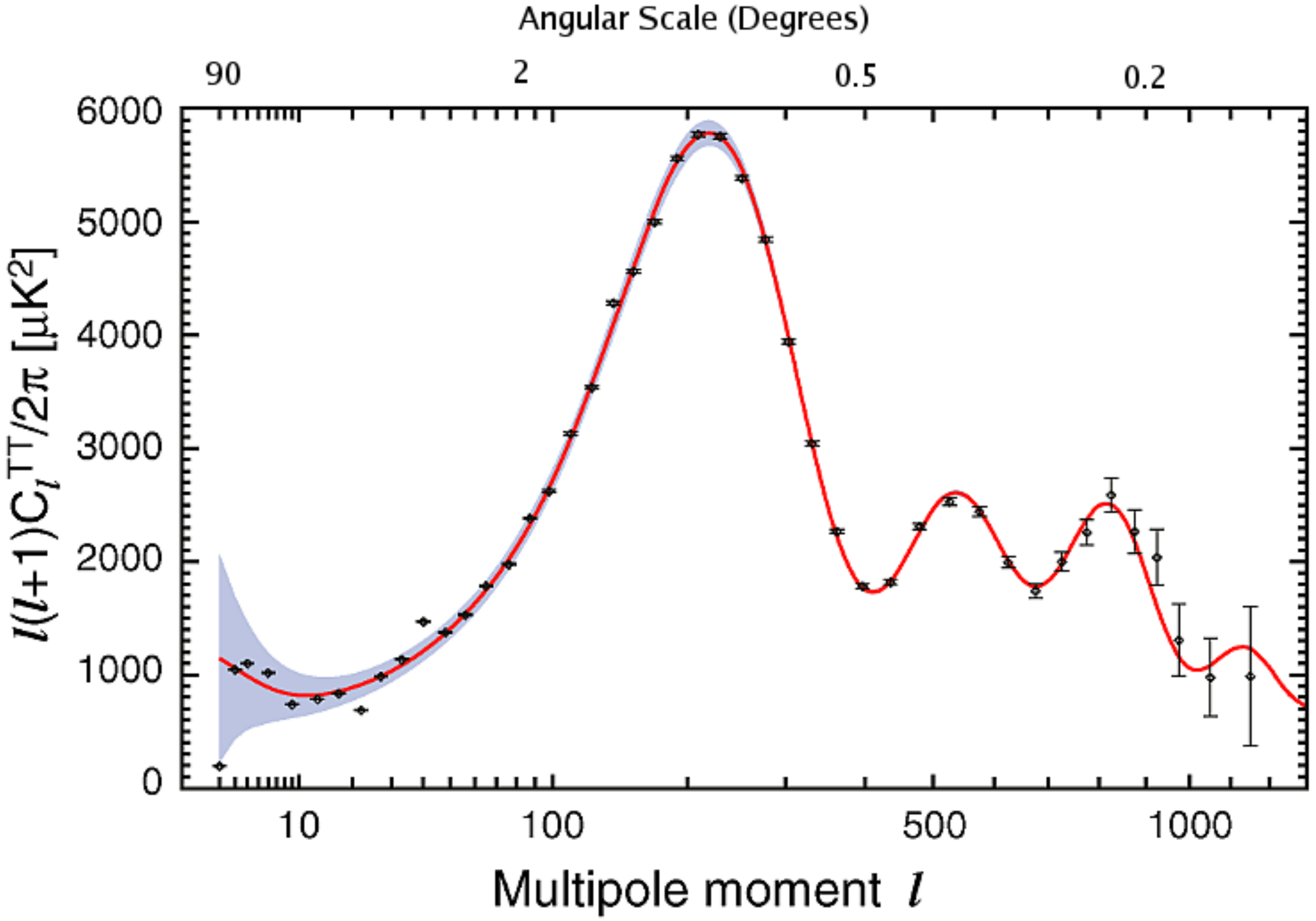


Observational evidence for DM

- Galaxy rotation curves
- Bullet cluster
- Gravitational Lensing
- Structure formation
- Cosmic Microwave Background
- …





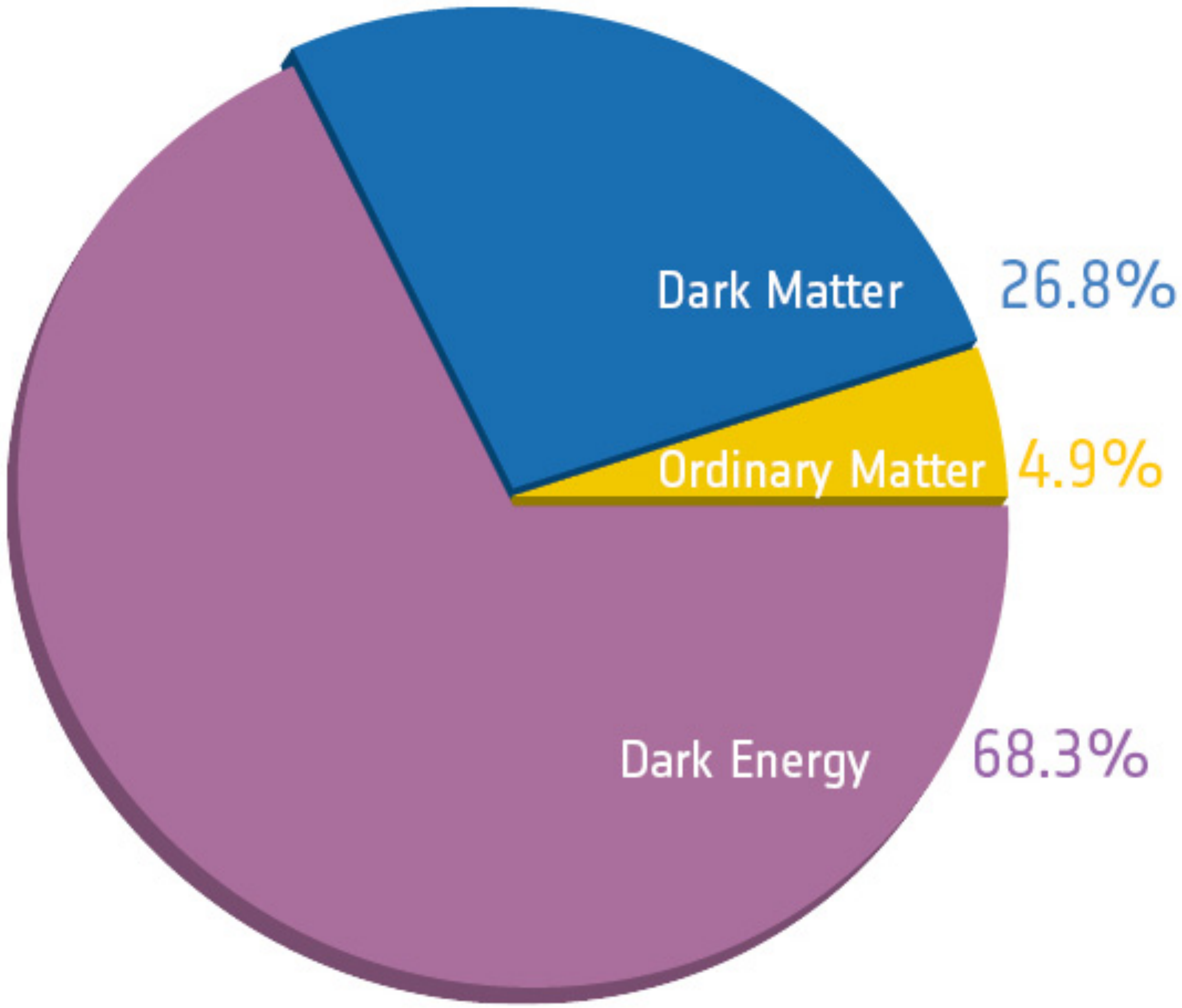


Outlines

- 暗物质的起源
  - 暗物质的天文观测证据
  - 宇宙学标准模型 ΛCDM
  - 暗物质分布
- 暗物质的物理模型
  - 可能的暗物质候选者
  - WIMP暗物质
- 热退耦暗物质残余丰度计算
- WIMP暗物质现状
  - WIMP暗物质的直接探测危机
    - 解决危机的多种办法
  - 暗物质对撞机探测的互补性
  - 暗物质的间接探测限制
    - 避开限制的办法
- 总结

The success of the Lambda cold dark matter Model

- The standard model of Big Bang cosmology
  - Λ, dark energy; CDM, cold dark matter; Matter, SM particles



HISTORY OF THE UNIVERSE
Dark energy accelerated expansion
Cosmic Microwave Background radiation is visible
Structure formation
RHIC & LHC heavy ions
Accelerators
LHC protons
High-energy cosmic rays
Size of visible universe
TODAY
Inflation
Big Bang
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
t = 10^-36 s
E = 10^16 GeV
t = 10^-10 s
E = 10^2 GeV
t = 10^-4 s
E = 10^-1 GeV
t = 10^2 s
E = 10^-4 GeV
t = 3x10^5 y
E = 3x10^-10 GeV
t = 10^9 y
E = 10^-12 GeV
t = 13.8x10^9 y
E = 2.3x10^-13 GeV
t = Time (seconds, years)
E = Energy (GeV)
Key
quark
gluon
electron
muon
tau
neutrino
bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
Particle Data Group, LBNL © 2014
Supported by DOE

STARGAZING LIVE

THE UNIVERSE THROUGH TIME

THE BEGINNING

The Universe begins 13.7 billion years ago with an event known as the Big Bang.

Both time and space are created in this event.

UNOBSERVABLE UNIVERSE (PAST)

FRACTION OF A SECOND

Rapid expansion occurs during a billionth of a billionth of a billionth of a billionth of a second – the visible Universe is the size of a grapefruit.

1 SECOND

The Large Hadron Collider at CERN is recreating the conditions that prevailed a fraction of a second after the Big Bang.

100 – 1000 SECONDS

Nuclei of hydrogen, helium, lithium and other light elements form.

300,000 YEARS

We can detect radiation from the early formation of the Universe back as far as this point. Before this, the Universe is opaque: it's as if a veil has been pulled over it.

POTENTIALLY OBSERVABLE UNIVERSE (PAST)

A FEW HUNDRED MILLION YEARS

Matter clumps together under its own gravity forming the first protogalaxies and within them, the first stars.

Stars are nuclear furnaces in which heavier elements such as carbon, oxygen, silicon and iron are formed. Massive stars exploding as supernovae create even heavier elements. Such explosions send material into space ready to be incorporated into future generations of stars and planets.

A FEW BILLION YEARS

Initially, the expansion of the Universe decelerated – but a few billion years after the Big Bang, the expansion began to accelerate. The acceleration is caused by a mysterious force known as 'dark energy', the nature of which is completely unknown.

9 BILLION YEARS

The Sun, along with its eight planets, and all the asteroids, comets and Kuiper Belt objects, such as Pluto, form from the debris left behind by earlier generations of stars.

10 BILLION YEARS

The first life appears on Earth in the form of simple cells. Impacting comets and asteroids might have contributed organic molecules to Earth. Life spreads across the globe.

TODAY

13.7 BILLION YEARS

This is where we are today. Using our own ingenuity, humanity is probing the depths of the Universe and trying to unravel its mysteries, from our tiny, home planet, Earth. The visible Universe contains billions of galaxies, each comprising billions of stars. Within our own Galaxy, hundreds of exoplanets have been discovered orbiting other stars.

FUTURE

20 BILLION YEARS

In a few billion years the Sun's outer layers will expand as it turns into a Red Giant star. Life on Earth will become impossible.

Expansion of the Universe will continue to accelerate.

10^{100} YEARS

Stars no longer form; matter is trapped in black holes or dead stars. Protons decay and black holes evaporate, leaving the Universe to its ultimate fate as cold, dead, empty space, containing only radiation, which itself too will eventually disperse.

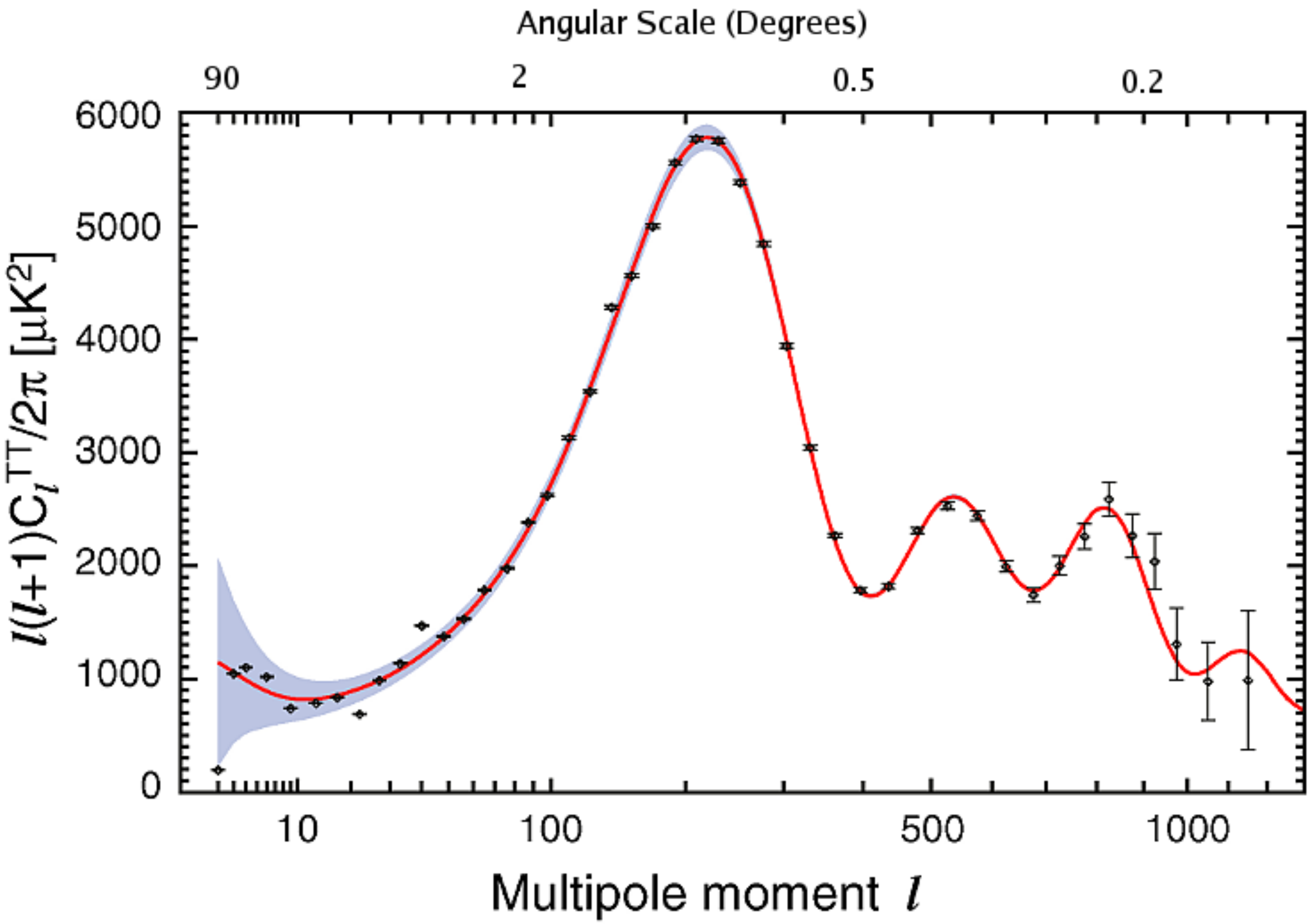
Stargazing LIVE is a BBC and Open University co-production. Credit: Photography sourced from NASA.

The success of the Lambda Cold Dark Matter Model

- The standard model of Big Bang cosmology
  - Λ, dark energy; CDM, cold dark matter; Matter, SM particles
- 6 parameter for the Universe: Baryon matter density, DM density, lifetime of the Universe …
  - Explain the structure of the CMB
  - Large-scale structure in the distribution of the galaxies
  - The observed abundance of H, D, He and Li
  - Accelerating expansion of the Universe

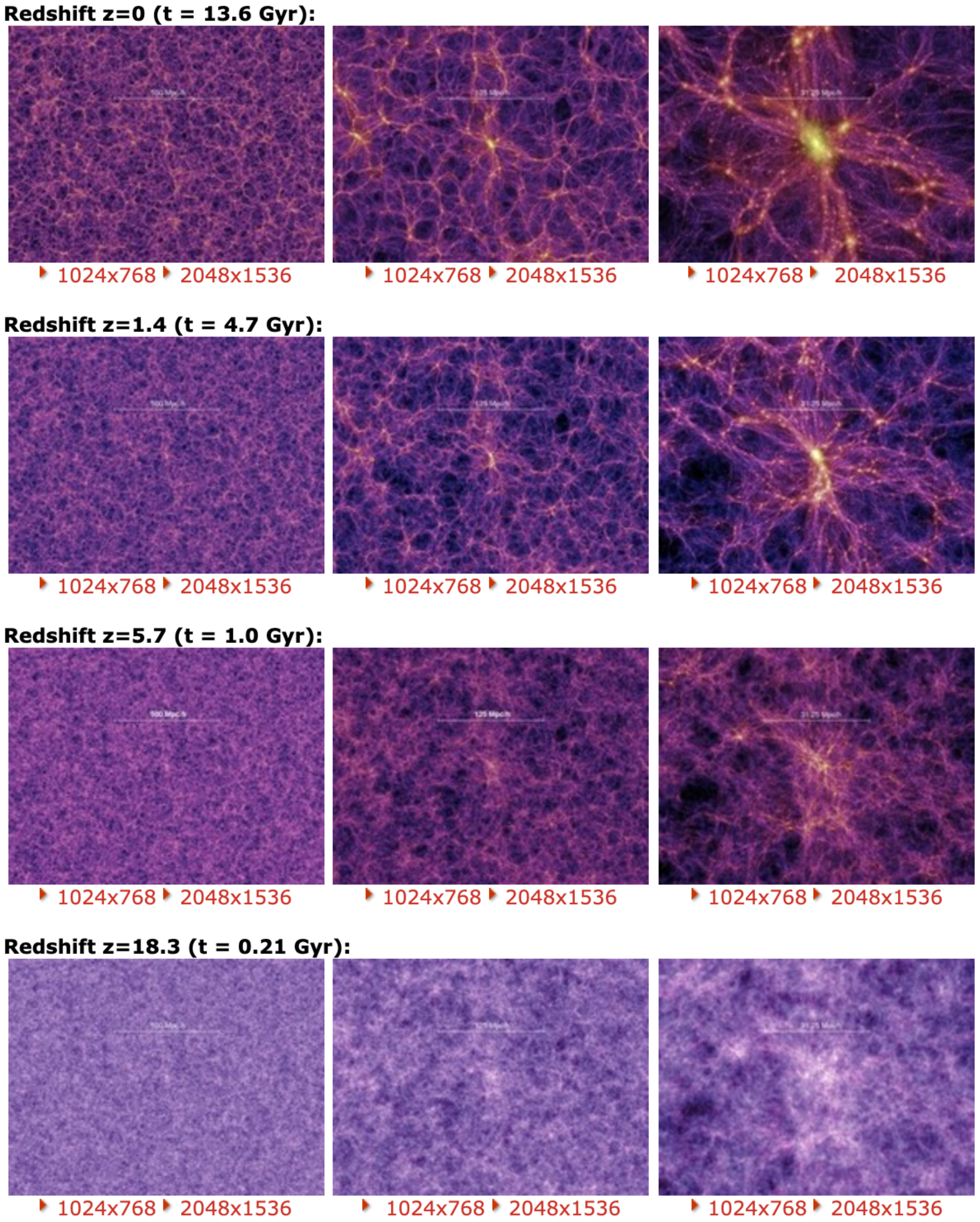
The success of the Lambda Cold Dark Matter Model

- The standard model of Big Bang cosmology
  - Λ, dark energy; CDM, cold dark matter; Matter, SM particles
- 6 parameter for the Universe: Baryon matter density, DM density, lifetime of the Universe …
  - Explain the structure of the CMB
  - Large-scale structure in the distribution of the galaxies
  - The observed abundance of H, D, He and Li
  - Accelerating expansion of the Universe



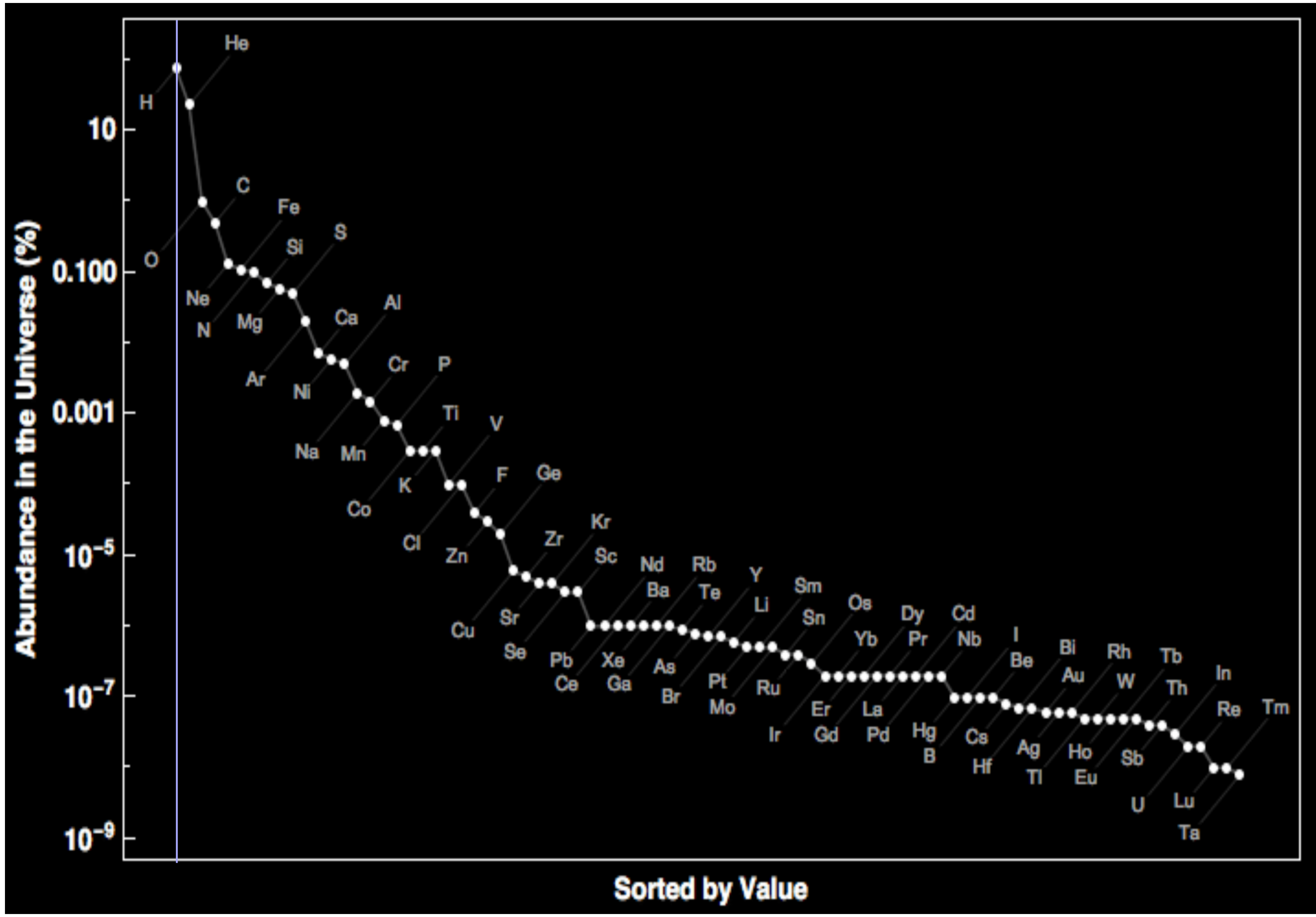
The success of the Lambda Cold Dark Matter Model

- The standard model of Big Bang cosmology
  - Λ, dark energy; CDM, cold dark matter; Matter, SM particles
- 6 parameter for the Universe: Baryon matter density, DM density, lifetime of the Universe …
  - Explain the structure of the CMB
  - Large-scale structure in the distribution of the galaxies
  - The observed abundance of H, D, He and Li
  - Accelerating expansion of the Universe



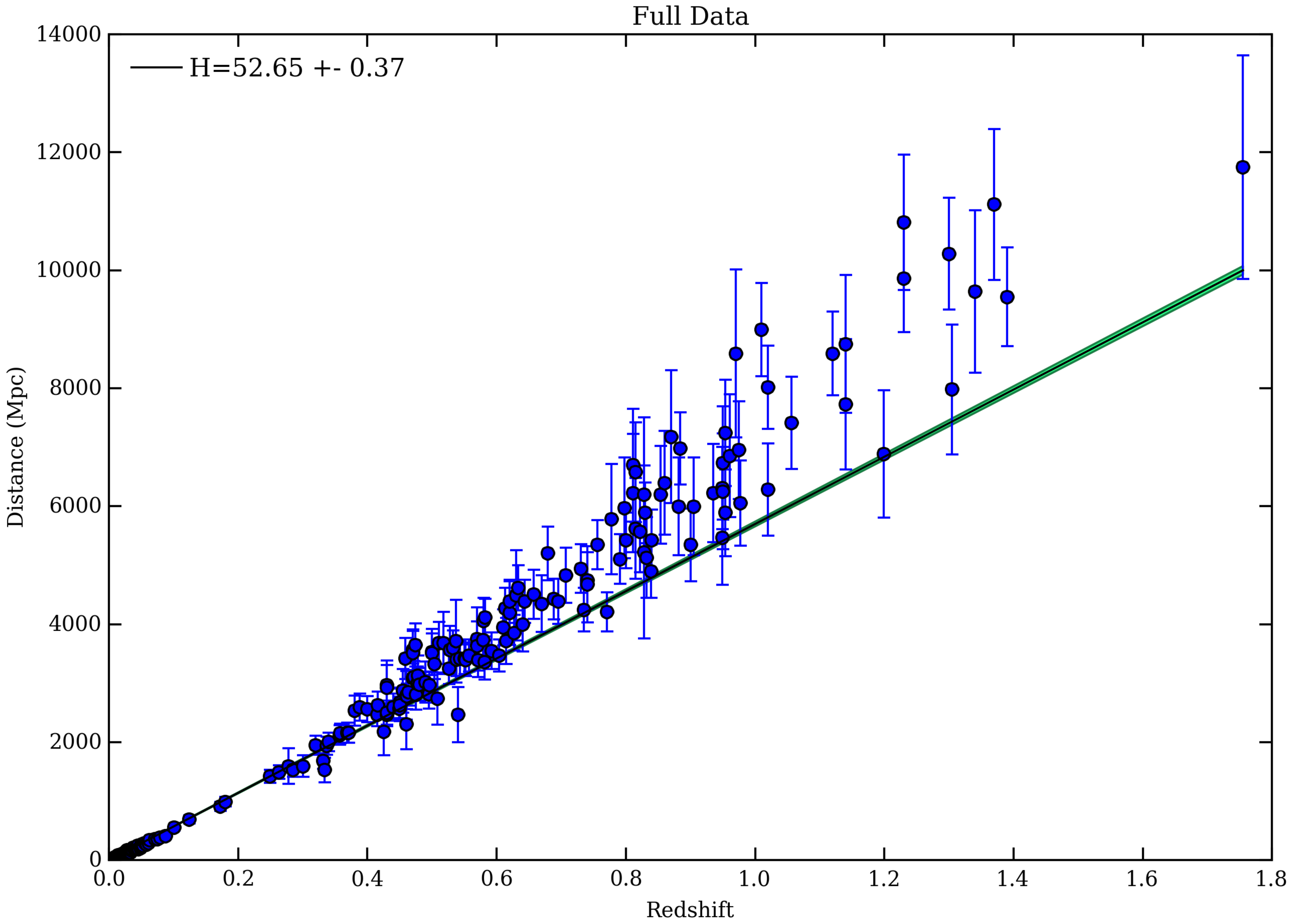
The success of the Lambda Cold Dark Matter Model

- The standard model of Big Bang cosmology
  - Λ, dark energy; CDM, cold dark matter; Matter, SM particles
- 6 parameter for the Universe: Baryon matter density, DM density, lifetime of the Universe …
  - Explain the structure of the CMB
  - Large-scale structure in the distribution of the galaxies
  - The observed abundance of H, D, He and Li
  - Accelerating expansion of the Universe



The success of the Lambda Cold Dark Matter Model

- The standard model of Big Bang cosmology
  - Λ, dark energy; CDM, cold dark matter; Matter, SM particles
- 6 parameter for the Universe: Baryon matter density, DM density, lifetime of the Universe …
  - Explain the structure of the CMB
  - Large-scale structure in the distribution of the galaxies
  - The observed abundance of H, D, He and Li
  - Accelerating expansion of the Universe

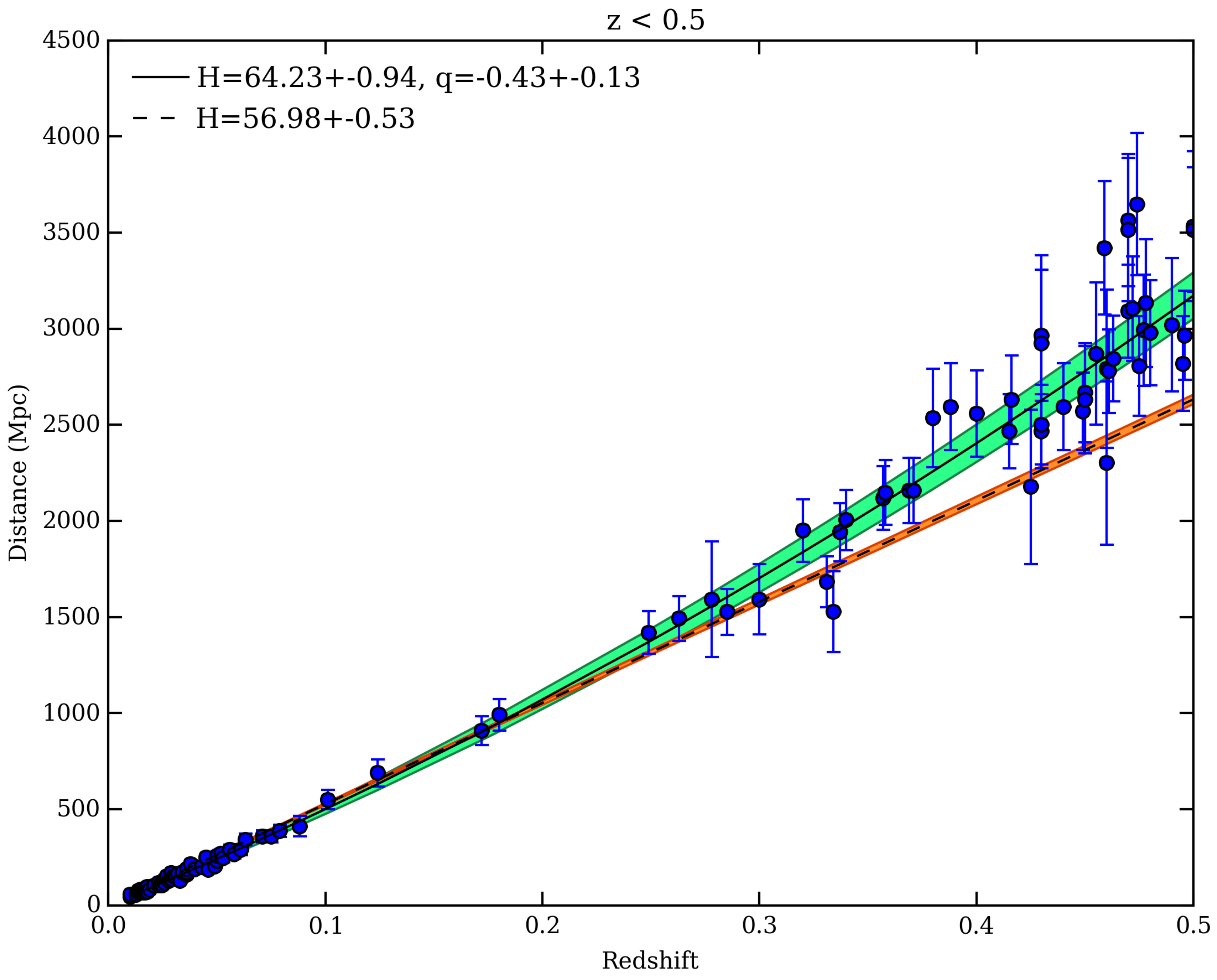


(Re)Discovering Dark Energy and the Expanding Universe

Credit: 2016, Adam Dempsey. CIERA, Northwestern University

The success of the Lambda Cold Dark Matter Model

- The standard model of Big Bang cosmology
  - Λ, dark energy; CDM, cold dark matter; Matter, SM particles
- 6 parameter for the Universe: Baryon matter density, DM density, lifetime of the Universe …
  - Explain the structure of the CMB
  - Large-scale structure in the distribution of the galaxies
  - The observed abundance of H, D, He and Li
  - Accelerating expansion of the Universe



(Re)Discovering Dark Energy and the Expanding Universe

Credit: 2016, Adam Dempsey. CIERA, Northwestern University

The dark matter in astrophysics/cosmology

- Energy density scales as $\rho \propto a^{-3}$, others $\rho_r \propto a^{-4}$, $\rho_{cc} \propto a^0$
- Massive, interacting gravitationally
- Neutral, not quite interacting with others, collision-less
- Stable
- Local DM energy density $\rho_{\rm DM} \sim 0.4\ {\rm GeV\ cm}^3$

Outlines

- 暗物质的起源
  - 暗物质的天文观测证据
  - 宇宙学标准模型 ΛCDM
  - 暗物质分布
- 暗物质的物理模型
  - 可能的暗物质候选者
  - WIMP暗物质
- 热退耦暗物质残余丰度计算
- WIMP暗物质现状
  - WIMP暗物质的直接探测危机
    - 解决危机的多种办法
  - 暗物质对撞机探测的互补性
  - 暗物质的间接探测限制
    - 避开限制的办法
- 总结

The dark matter distribution

- Astrophysicist knows the distribution of DM by simulation
- Navarro-Frenk-White profile: $$\rho(r) = \frac{\rho_0}{\frac{r}{R_s}\left(1 + \frac{r}{R_s}\right)^2}$$
- R_s is the "scale radius", $\{\rho_0, R_s\}$ varies from halo to halo
- Integrated mass: $$M = \int_0^{R_{\max}} 4\pi r^2 \rho(r)\, dr = 4\pi\rho_0 R_s^3 \left[\ln\left(\frac{R_s + R_{\max}}{R_s}\right) + \frac{R_s}{R_s + R_{\max}} - 1\right]$$
    - Virial radius R_{vir}: $R_{\mathrm{vir}} = cR_s$, with c called "concentration parameter"
    - Typical c: Milky Way 10~15, others 4~40 for various size of halos
    - Total mass within R_{vir}: $$M = \int_0^{R_{\mathrm{vir}}} 4\pi r^2 \rho(r)\, dr = 4\pi\rho_0 R_s^3 \left[\ln(1 + c) - \frac{c}{1 + c}\right]$$

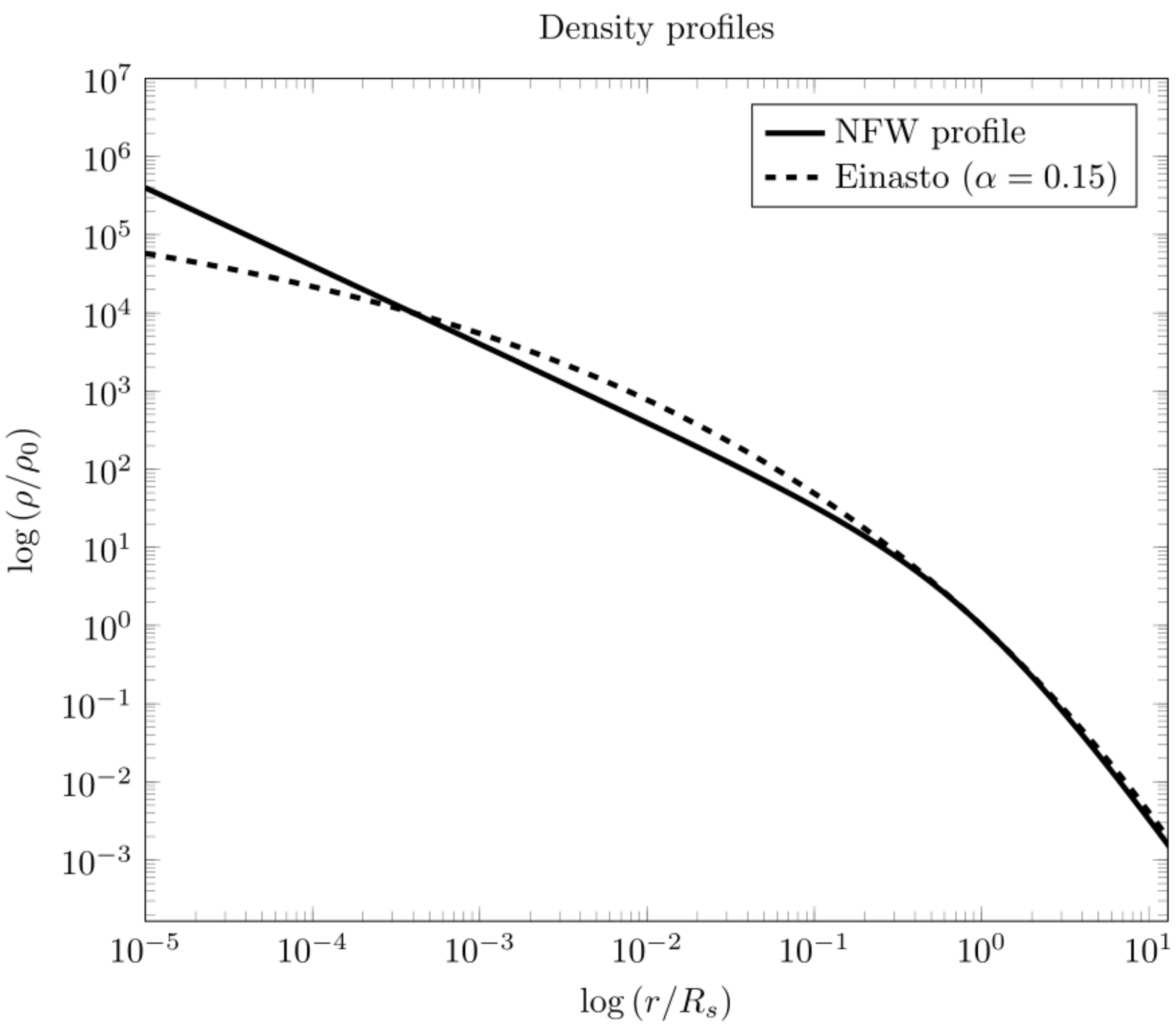
The dark matter distribution

- Astrophysicist knows the distribution of DM by N-body simulation

- Navarro-Frenk-White profile:

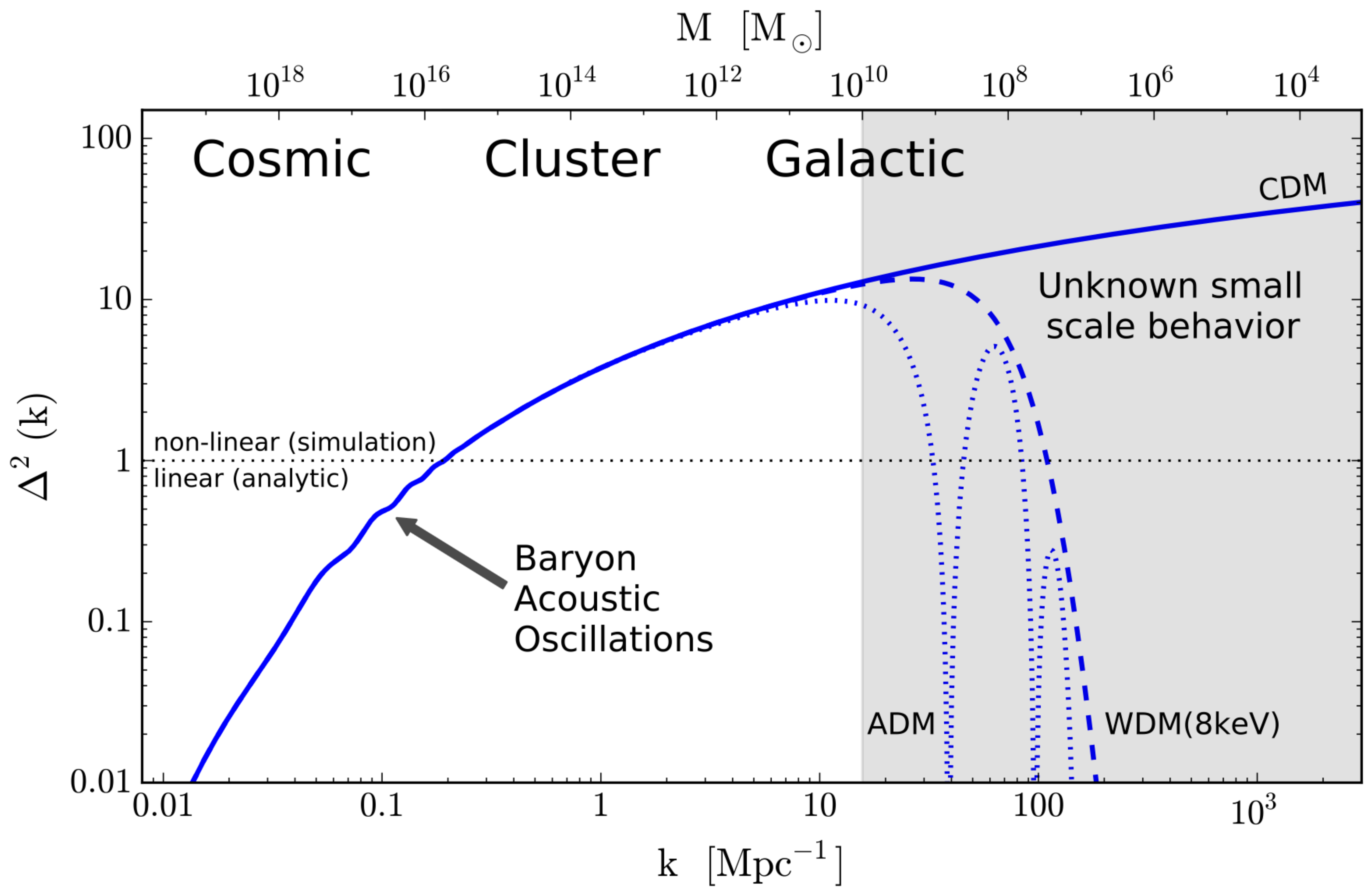
$$\rho(r) = \frac{\rho_0}{\frac{r}{R_s}\left(1 + \frac{r}{R_s}\right)^2}$$

- Other competing profile: Einasto



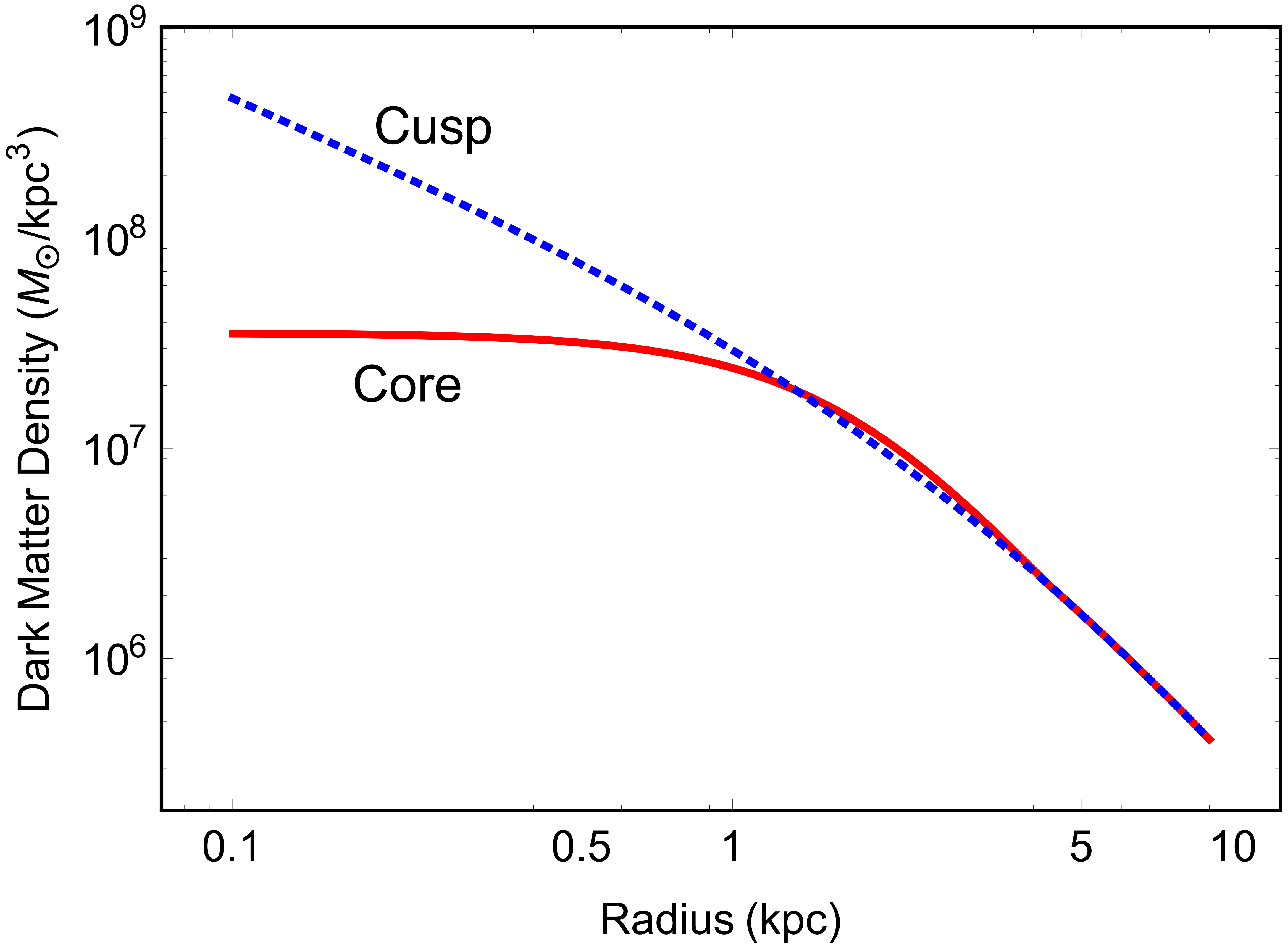
冷暗物质小尺度结构问题

- Astrophysicist knows the distribution of DM by N-body simulation
- Navarro-Frenk-White profile:
- $$\rho(r) = \frac{\rho_0}{\frac{r}{R_s}\left(1 + \frac{r}{R_s}\right)^2}$$
- CDM: very good for large scale, but problems at galactic scale



小尺度结构：尖峰-核分布问题

- Astrophysicist knows the distribution of DM by simulation
- Navarro-Frenk-White profile:
- $$\rho(r) = \frac{\rho_0}{\frac{r}{R_s}\left(1 + \frac{r}{R_s}\right)^2}$$
- CDM: very good for large scale, but problems at galactic scale
  - Core-Cusp problem of cold dark matter



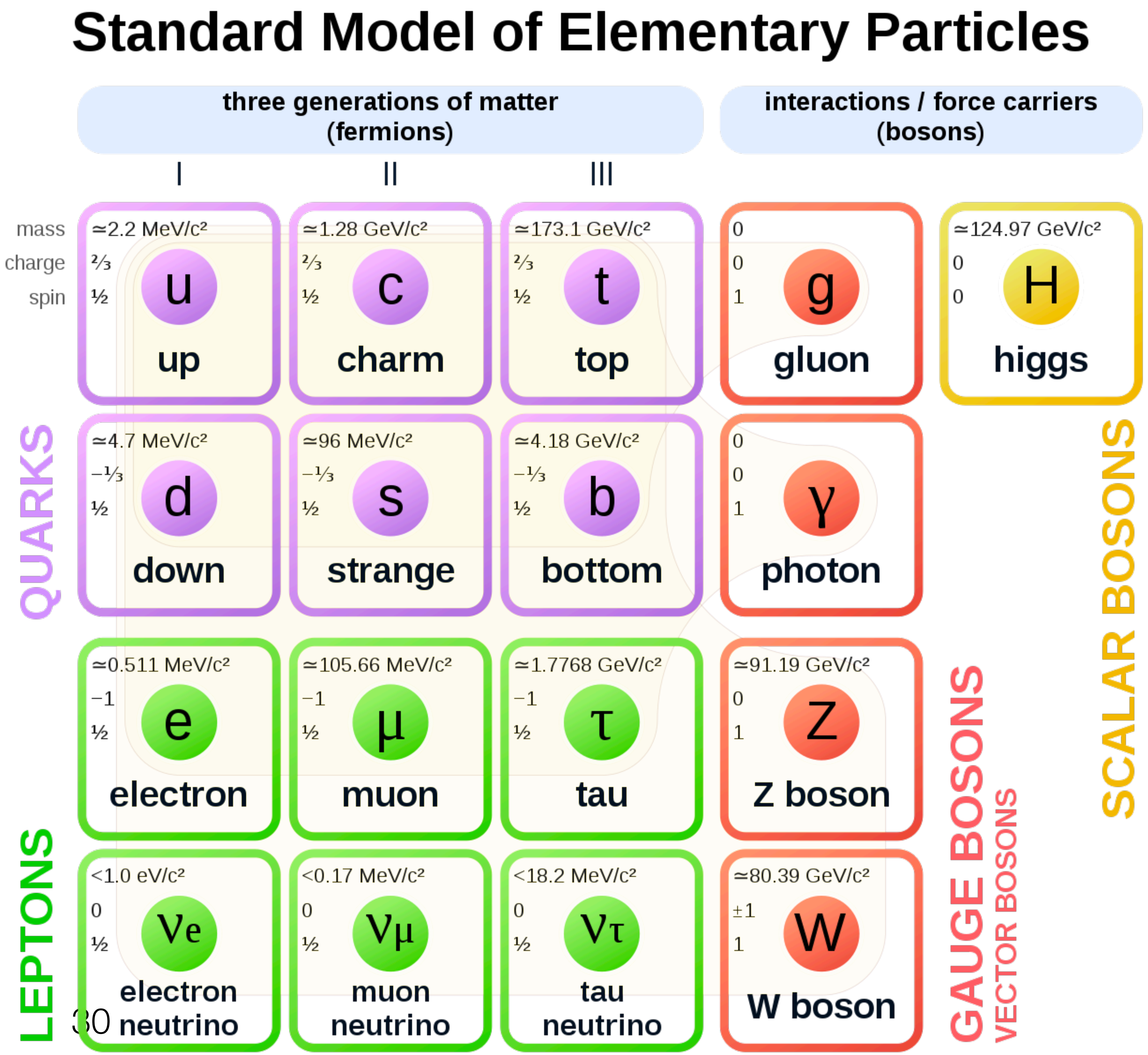
Outlines

- 暗物质的起源
  - 暗物质的天文观测证据
  - 宇宙学标准模型 ΛCDM
  - 暗物质分布
- 暗物质的物理模型
  - 可能的暗物质候选者
  - WIMP暗物质
- 热退耦暗物质残余丰度计算
- WIMP暗物质现状
  - WIMP暗物质的直接探测危机
    - 解决危机的多种办法
  - 暗物质对撞机探测的互补性
  - 暗物质的间接探测限制
    - 避开限制的办法
- 总结

粒子物理标准模型与暗物质

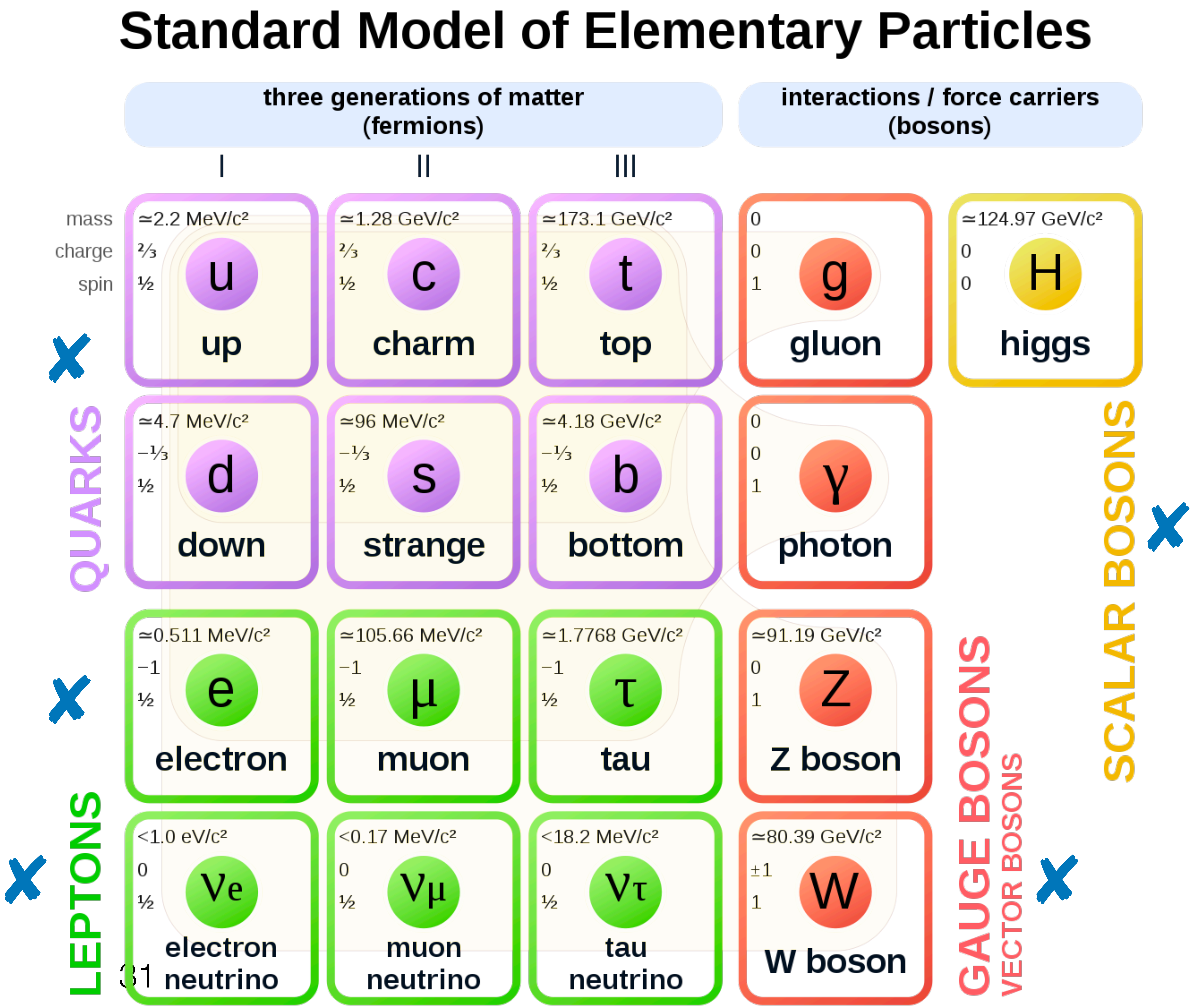
- No body knows what DM is
- Not in Standard Model
- There are good guesses

Not neutrinos ✗



粒子物理标准模型与暗物质

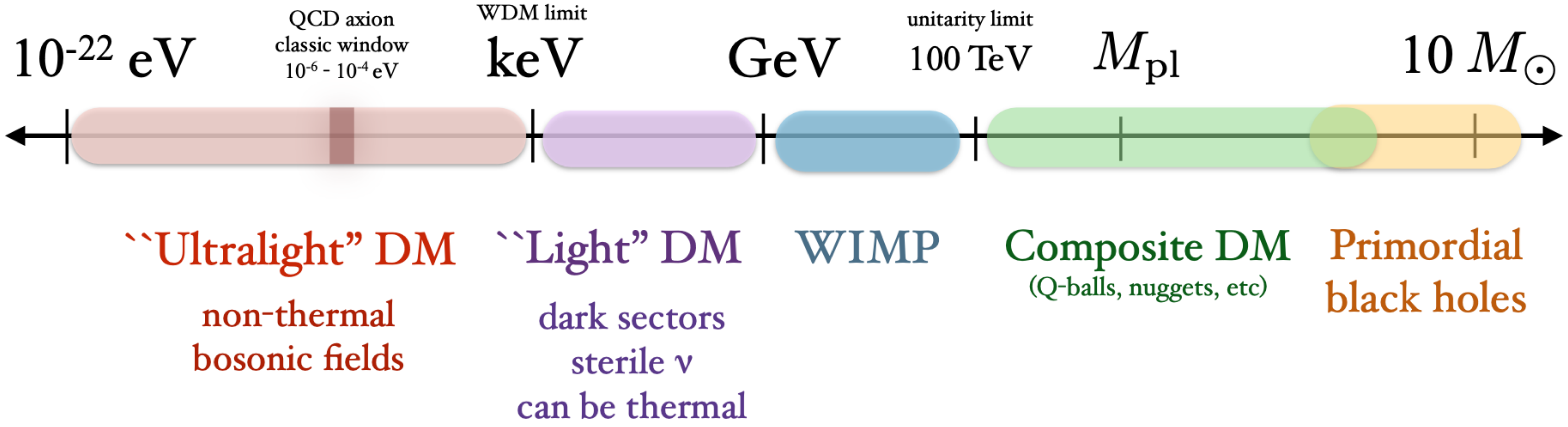
- No body knows what DM is
- Not in Standard Model
- There are good guesses



Not neutrinos ✘

暗物质的候选者模型

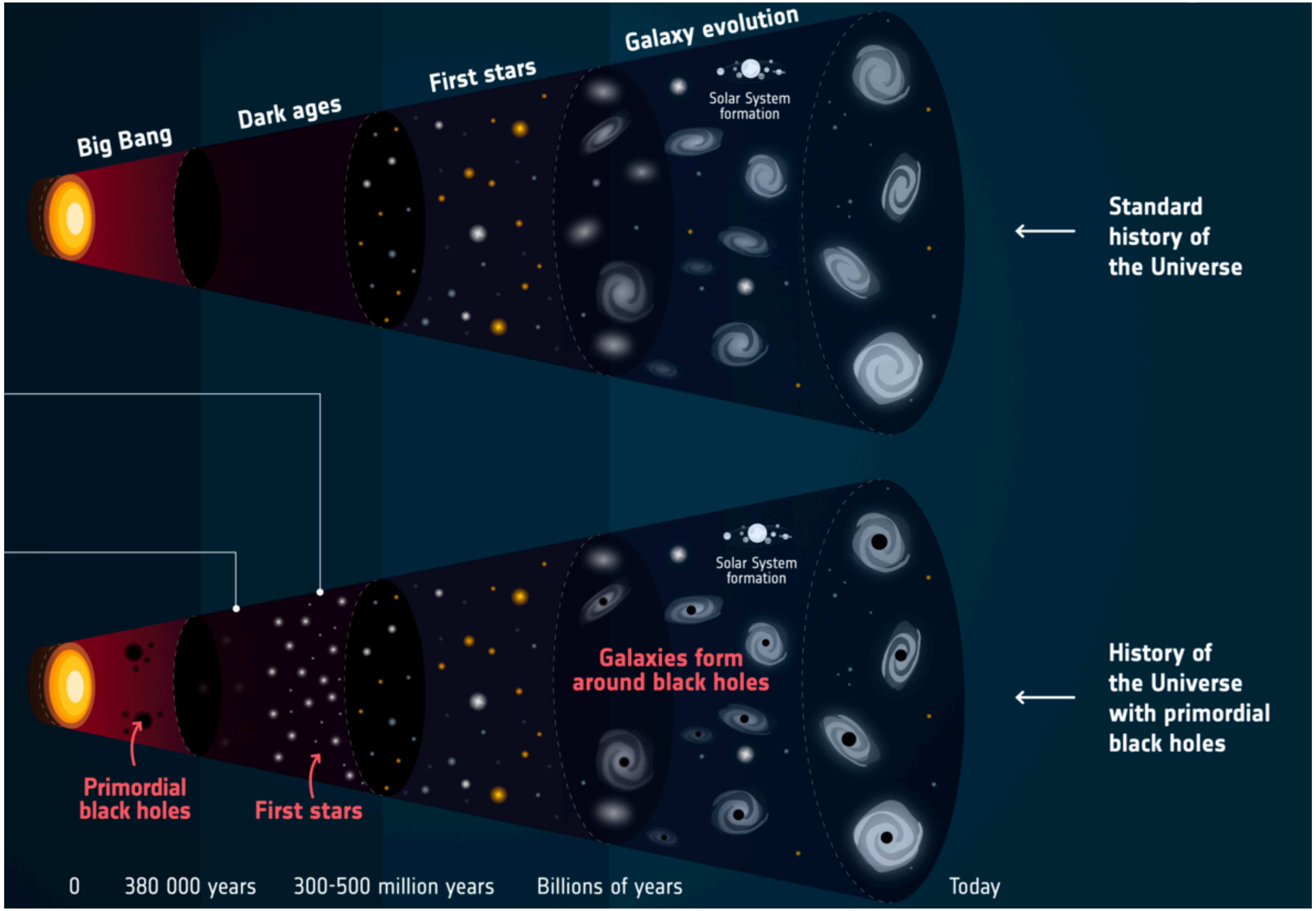
1904.07915，TASI lecture



- 原初黑洞 (Primordial Black Hole, PBH)
- 超轻波动型暗物质 (Ultralight Dark Matter)
- 具有弱相互作用的有质量粒子 (Weakly Interacting Massive Particle, WIMP)

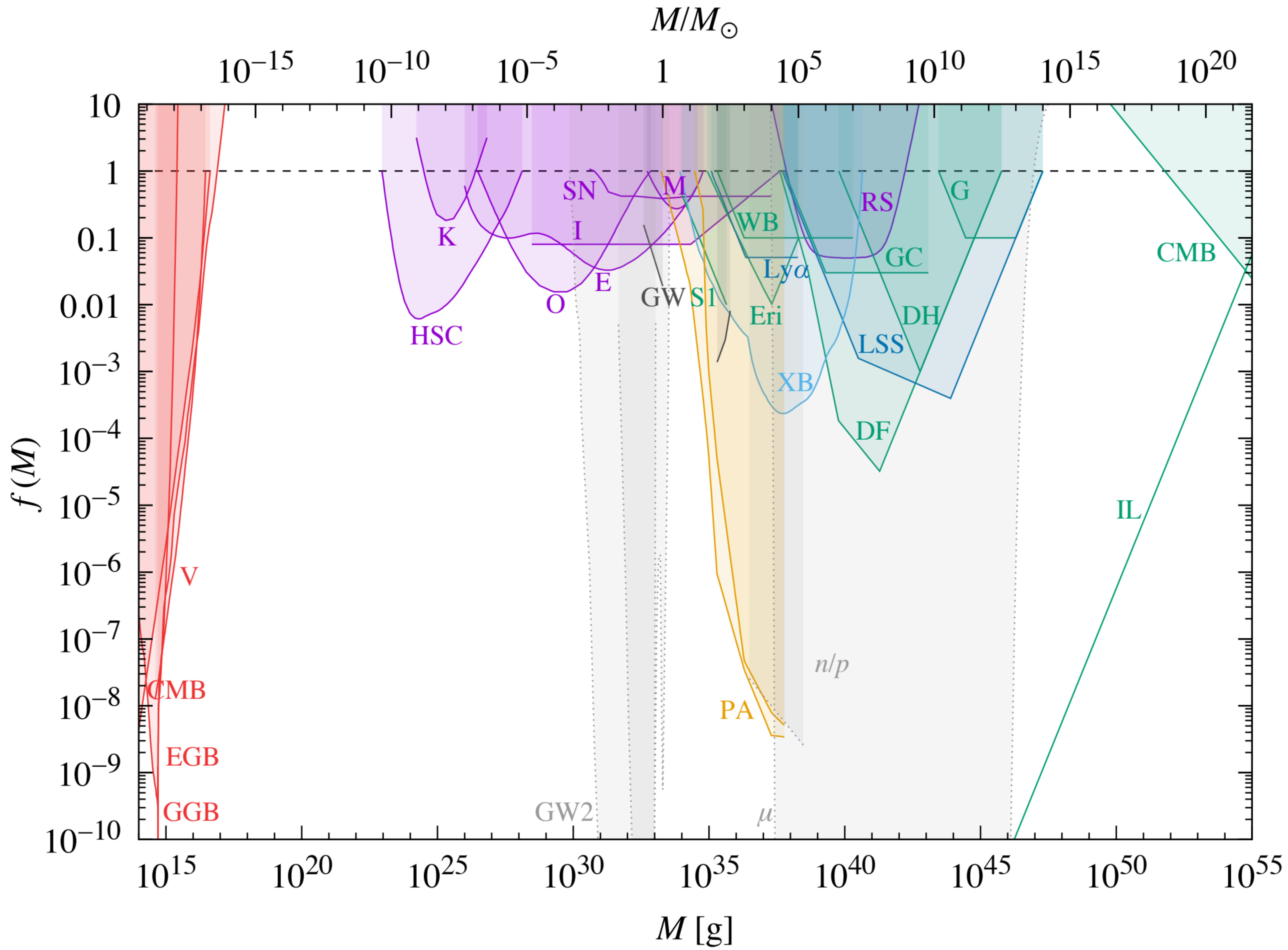
原初黑洞暗物质

- 宏观客体
- 先天黑洞
- 小行星质量大小的原初黑洞可以作为暗物质



原初黑洞暗物质

- 宏观客体
- 先天黑洞
- 小行星质量大小的原初黑洞可以作为暗物质
- 限制： evaporation (red), lensing (magenta), dynamical effects (green), gravitational waves (black), accretion (light blue), CMB distortions (orange), large-scale structure (dark blue) and background effects (grey).



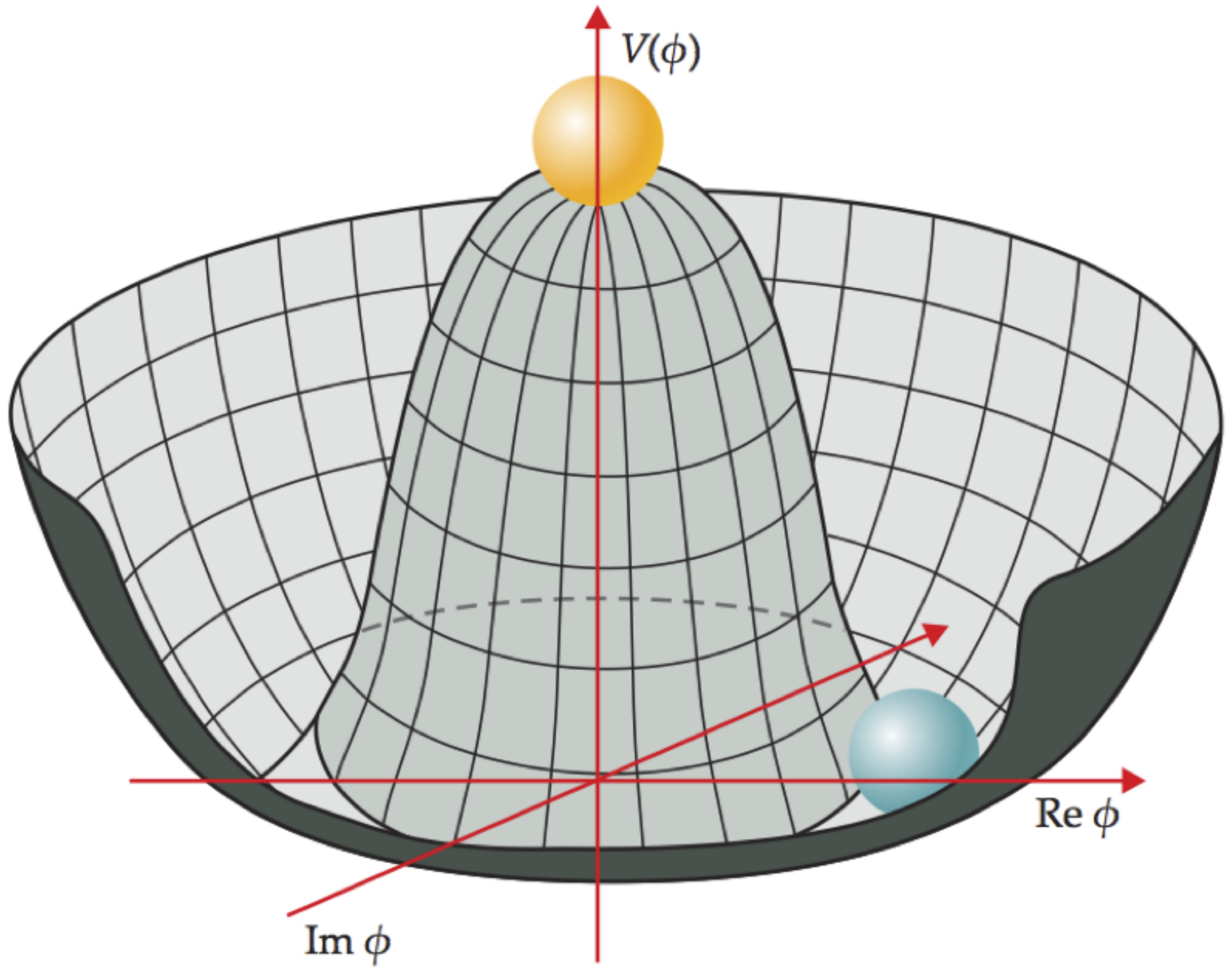
2002.12778 [Rept.Prog.Phys.]

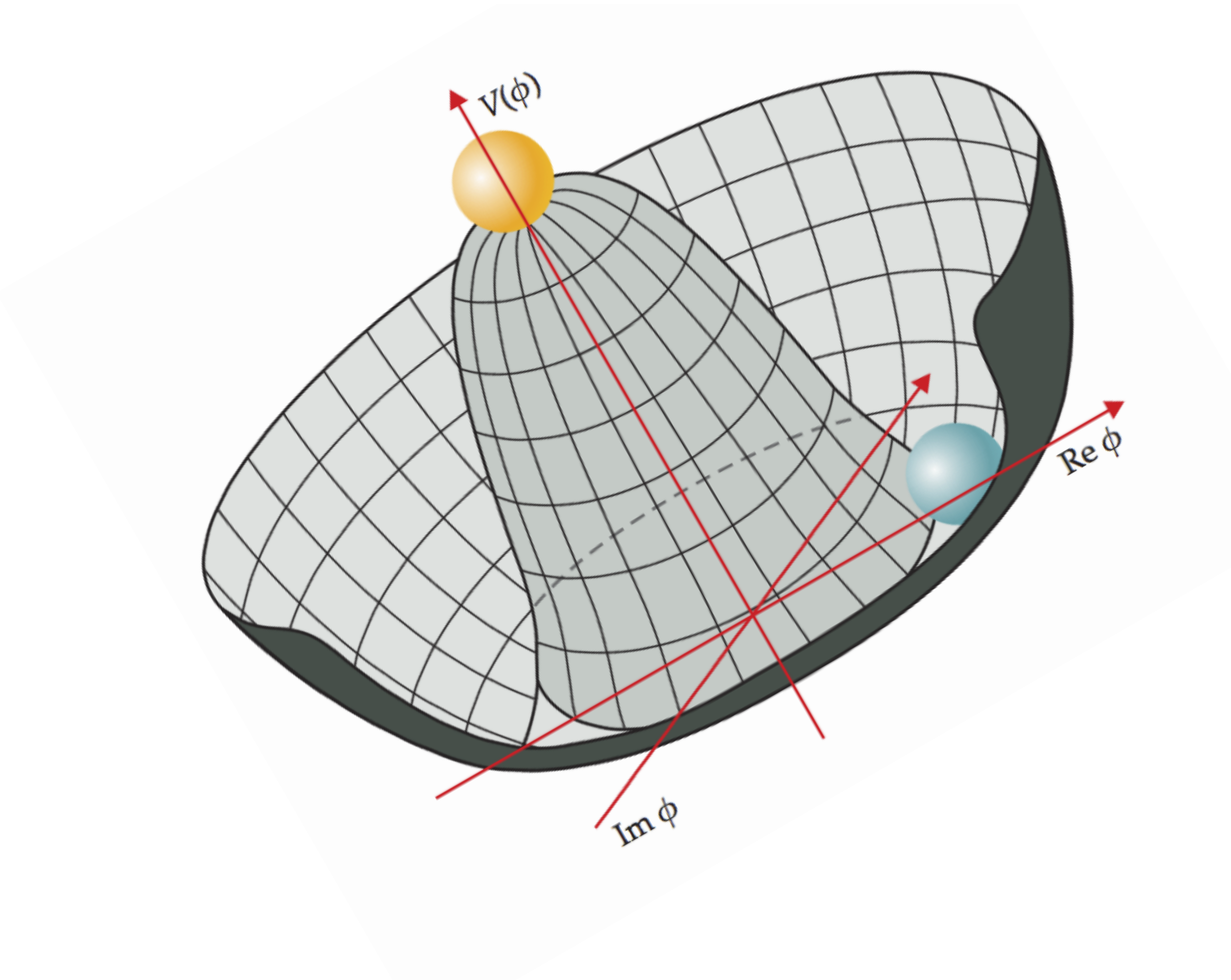
超轻波动型暗物质

- 超轻波动型暗物质 (Ultralight Dark Matter)
  - QCD轴子、类轴子、暗光子、暗希格斯粒子等等…
  - 产生机制：例如 Misalignment

超轻波动型暗物质

- 超轻波动型暗物质 (Ultralight Dark Matter)
  - QCD轴子，类轴子，暗光子等等…
  - 产生机制：例如 Misalignment
    - $\ddot{a} + 3H\dot{a} + m_a^2 a = 0$

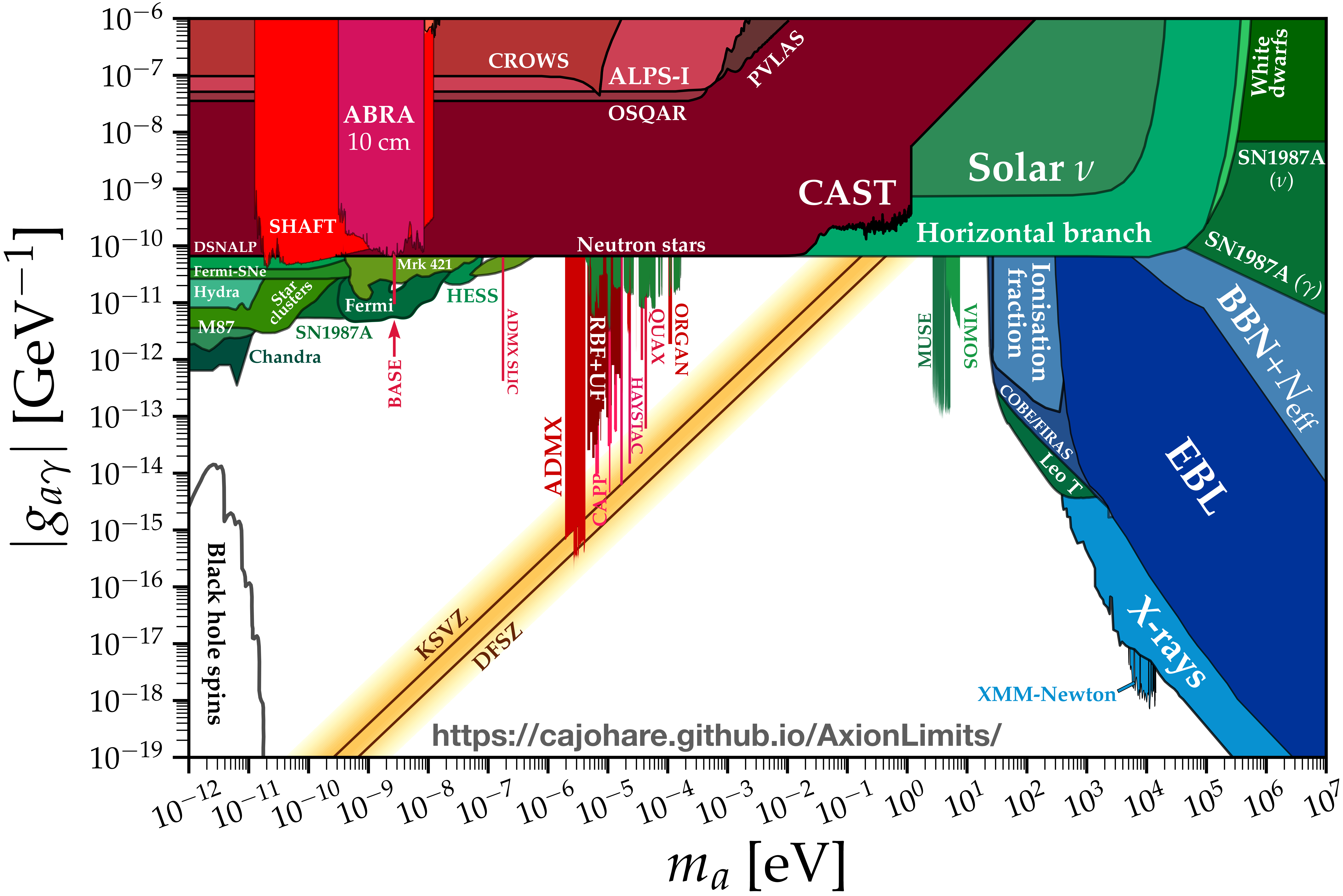




超轻波动型暗物质的探测

- 超轻波动型暗物质 (Ultralight Dark Matter)
  - QCD轴子，类轴子，暗光子等等…
  - 产生机制：例如 Misalignment
    - $\ddot{a} + 3H\dot{a} + m_a^2 a = 0$

轴子暗物质的探测

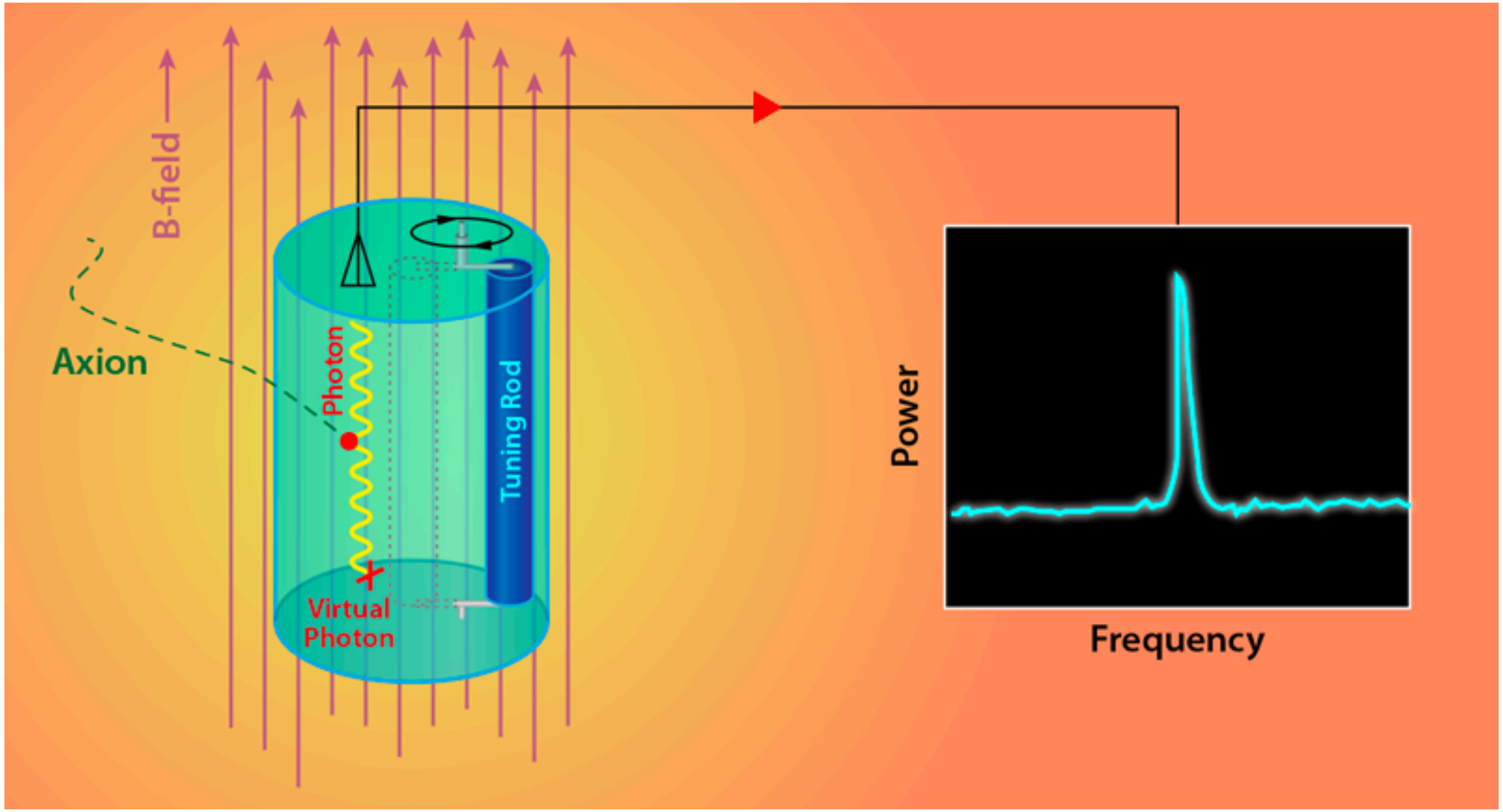


CROWS
ALPS-I
PVLAS
OSQAR
ABRA
10 cm
CAST
Solar ν
White dwarfs
SN1987A (ν)
Horizontal branch
SN1987A (γ)
SHAFT
DSNALP
Neutron stars
Fermi-SNe
Mrk 421
Hydra
Star clusters
Fermi
HESS
M87
SN1987A
Chandra
BASE
ADMX SLIC
RBF+UF
QUAX
ORGAN
HAYSTAC
ADMX
CAPP
MUSE
VIMOS
Ionisation fraction
BBN+N_eff
COBE/FIRAS
Leo T
EBL
X-rays
XMM-Newton
Black hole spins
KSVZ
DFSZ
https://cajohare.github.io/AxionLimits/
|g_aγ| [GeV^−1]
m_a [eV]

超轻波动型暗物质的实验室探测

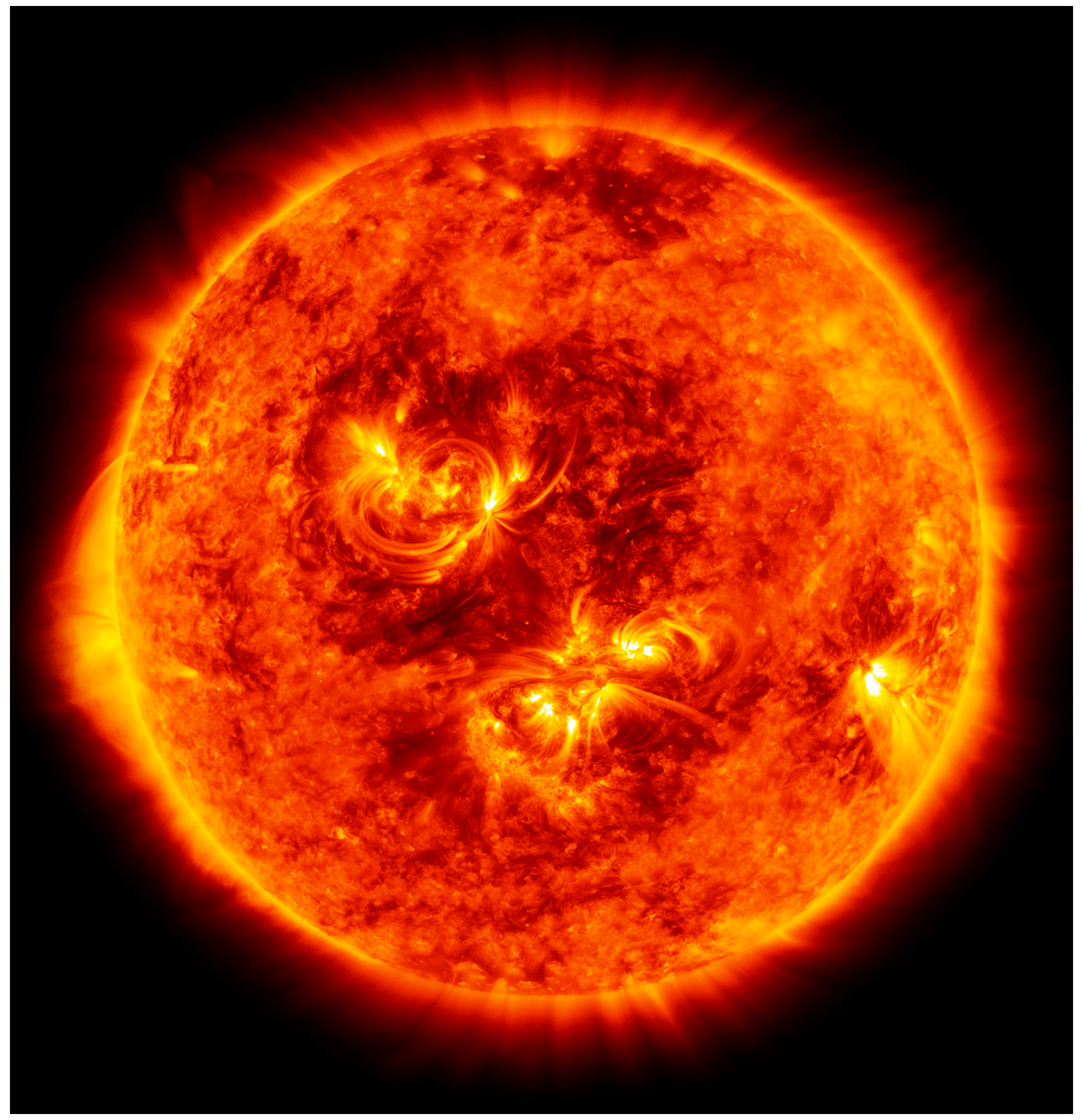
ADMX 实验

- 谐振腔技术



$$g_{a\gamma\gamma} a F_{\mu\nu} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \sim g_{a\gamma\gamma} a \vec{E} \cdot \vec{B}$$

超轻波动型暗物质的天文探测



- 等离子体中的暗光子共振转化
- 天文学与粒子物理的结合

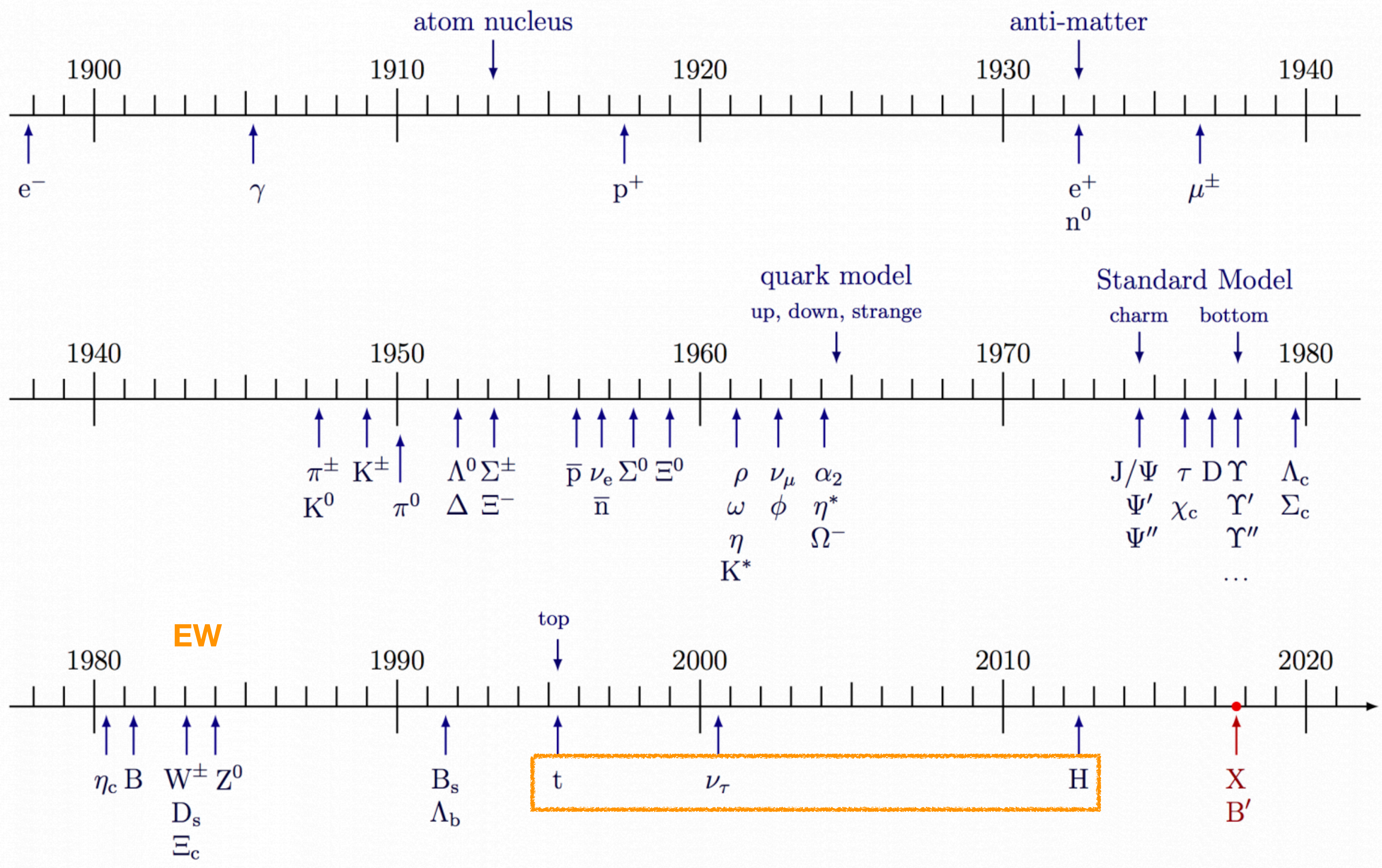
Outlines

- 暗物质的起源
  - 暗物质的天文观测证据
  - 宇宙学标准模型 ΛCDM
  - 暗物质分布
- 暗物质的物理模型
  - 可能的暗物质候选者
  - WIMP暗物质
- 热退耦暗物质残余丰度计算
- WIMP暗物质现状
  - WIMP暗物质的直接探测危机
    - 解决危机的多种办法
  - 暗物质对撞机探测的互补性
  - 暗物质的间接探测限制
    - 避开限制的办法
- 总结

今日主角：WIMP暗物质

- 为什么WIMP暗物质重要?
  - 理想的化身
  - 优秀的产生机制
  - 优秀的实验预期信号

粒子发现编年史



atom nucleus
anti-matter
1900
1910
1920
1930
1940
e^-
γ
p^+
e^+
n^0
$\mu^\pm$
quark model
up, down, strange
Standard Model
charm
bottom
1940
1950
1960
1970
1980
$\pi^\pm$ $K^\pm$
K^0 π^0
Λ^0 $\Sigma^\pm$
Δ Ξ^-
$\bar{p}$ ν_e Σ^0 Ξ^0
$\bar{n}$
ρ ω η K^*
ν_μ ϕ
α_2 η^* Ω^-
J/Ψ Ψ' Ψ''
τ χ_c
D
Υ Υ' Υ'' ...
Λ_c Σ_c
EW
top
1980
1990
2000
2010
2020
η_c B
$W^\pm$ Z^0 D_s Ξ_c
B_s Λ_b
t
ν_τ
H
X
B′

粒子物理标准模型的菜单

Top quark
The heaviest fermion
3rd gen up-type quark
1995

Tau neutrino
The last discovered neutrino
3rd gen neutrino
2000

Higgs
It gives masses to other particles
The heaviest scalar particle
2012

- We always hold a menu
- What about next?

理想的化身

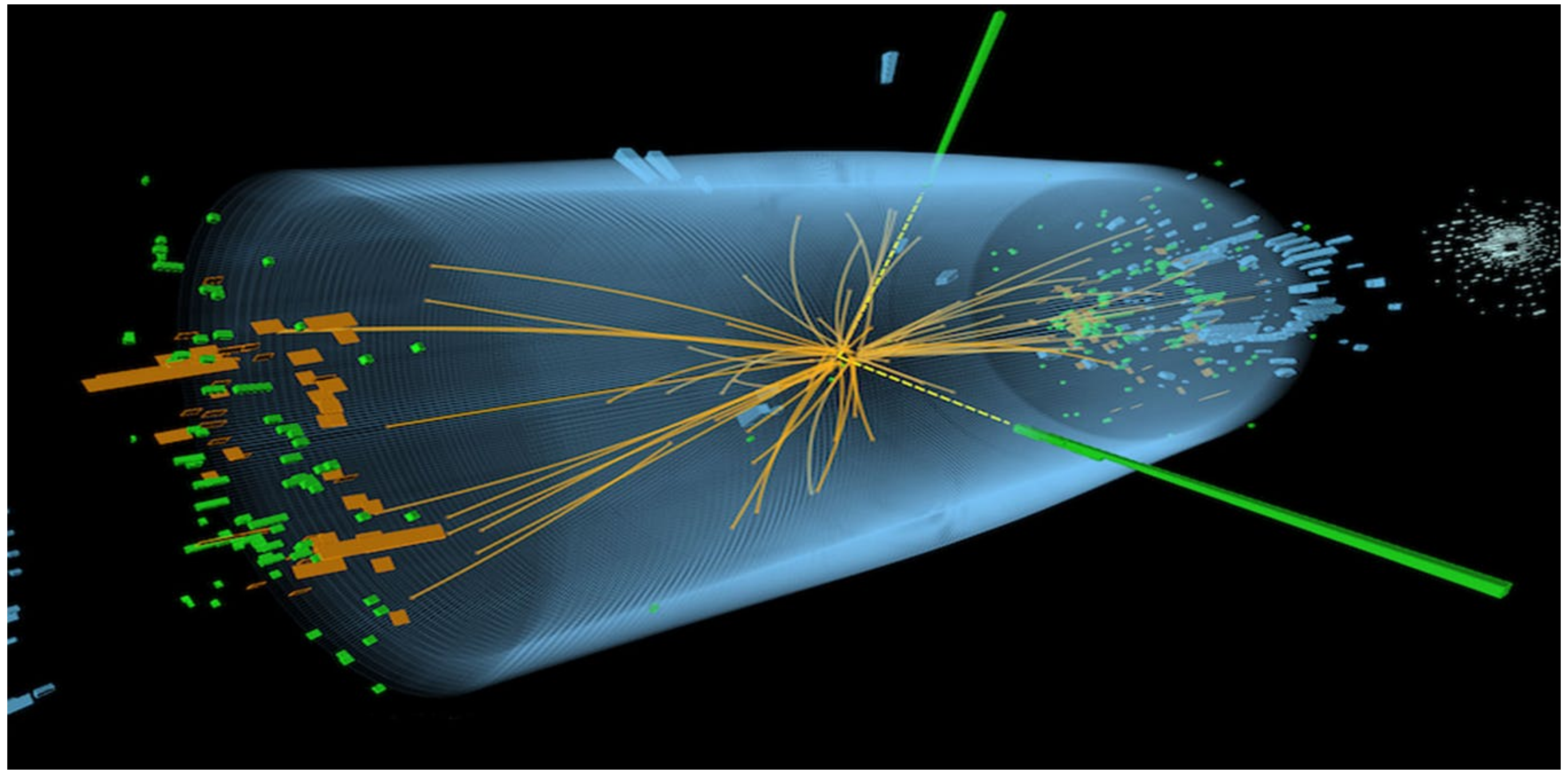
- 至尊私房菜：超对称

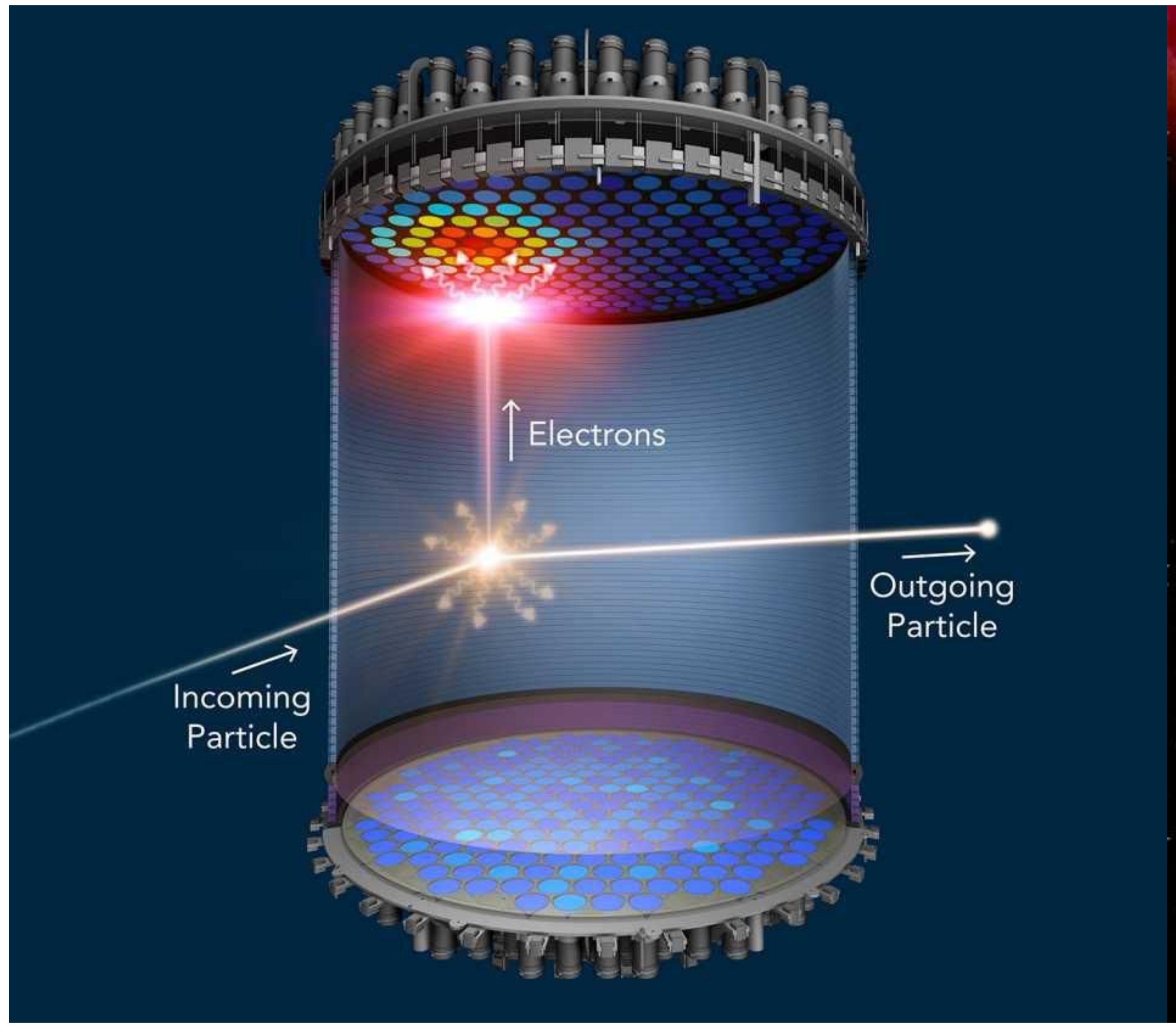
- 中性微子暗物质

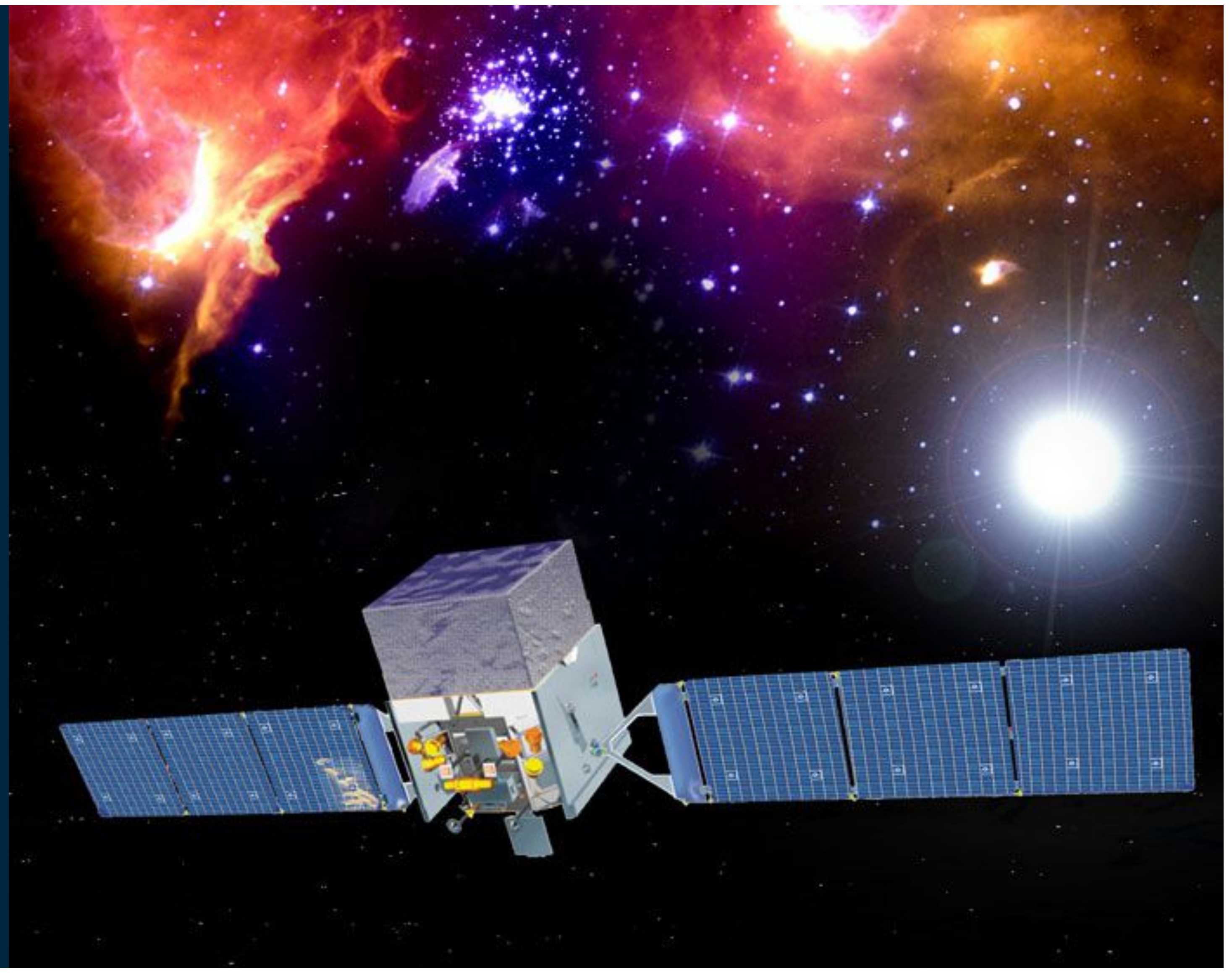
- 超对称暗物质的理想化身：WIMP暗物质

优秀的实验信号预期

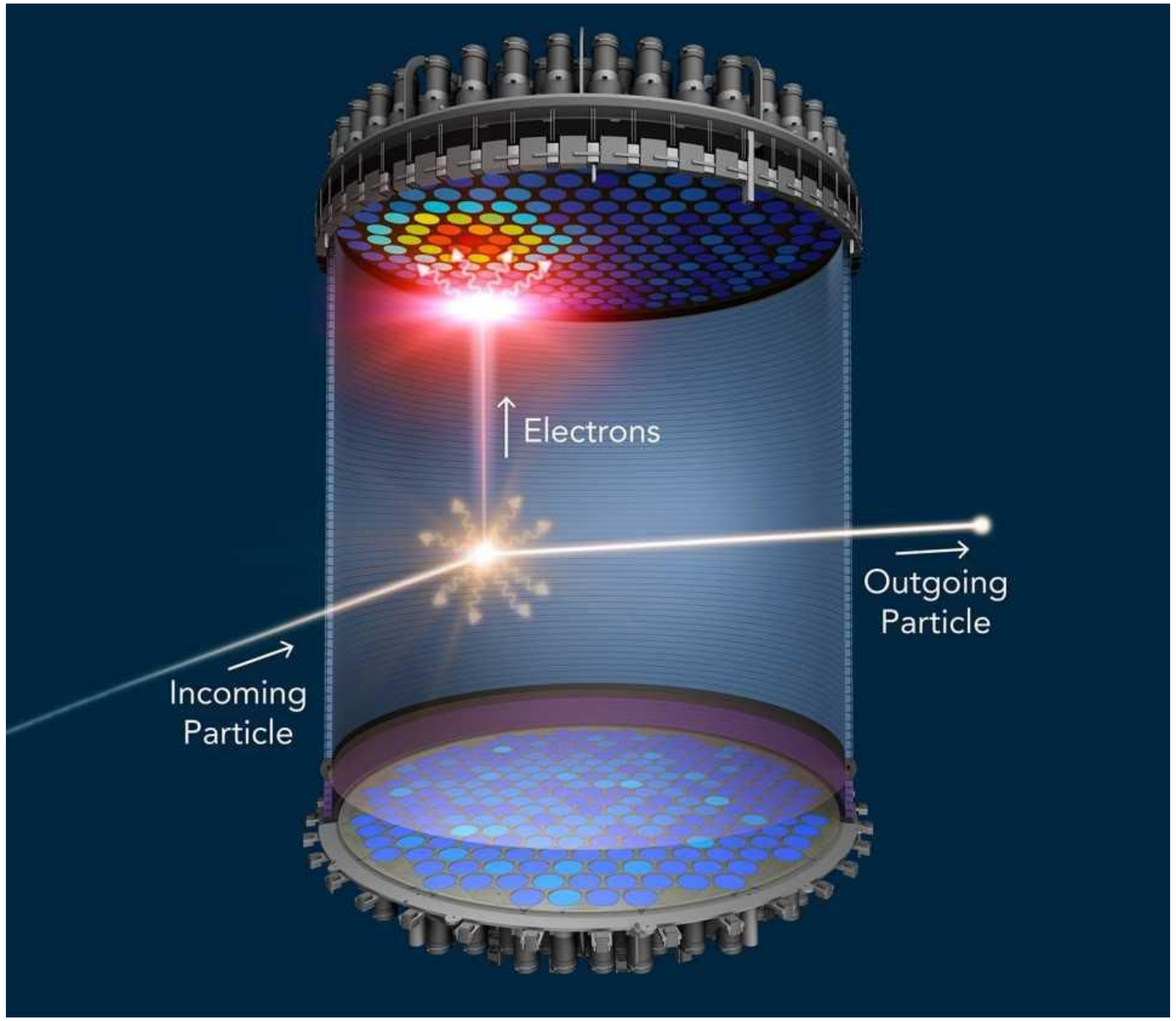
- WIMP暗物质和标准模型有较大的相互作用
  - 直接探测实验 SM + DM > SM + DM
  - 间接探测实验 DM + DM > SM + SM
  - 对撞机实验 SM + SM > DM + DM

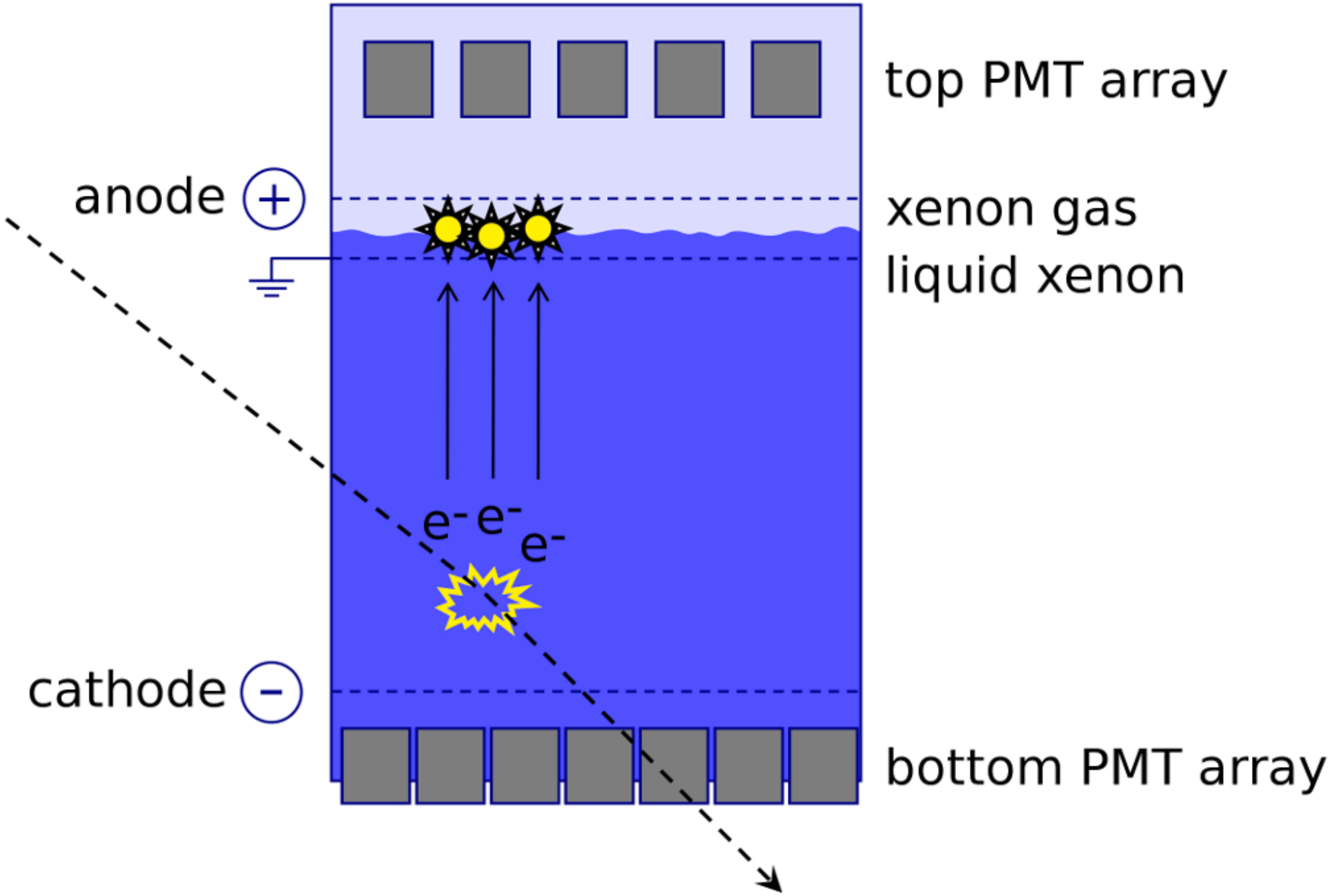






暗物质直接探测实验



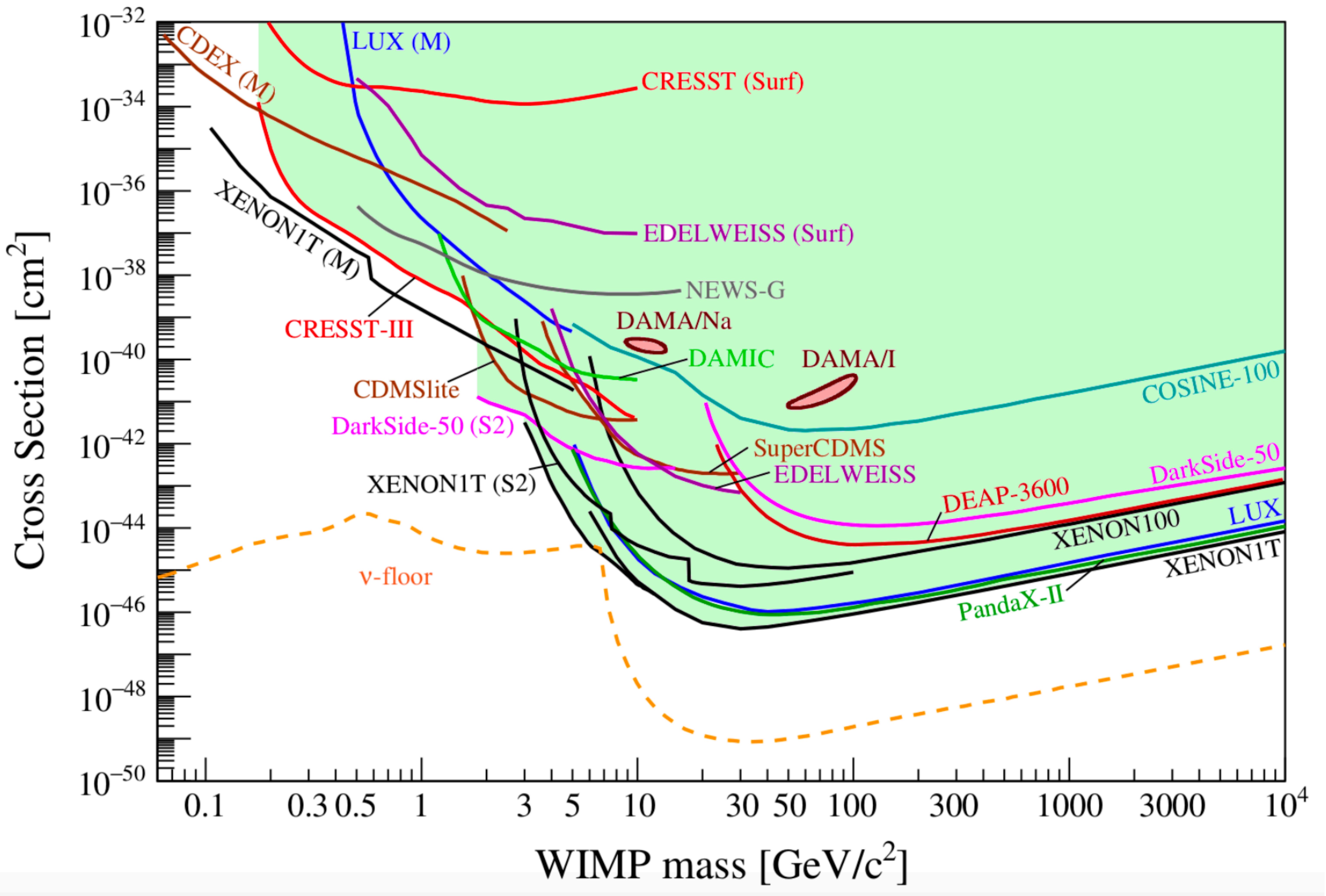


- DM + SM > DM + SM, 暗物质碰撞产生动能转移

暗物质直接探测实验

- 中国相关实验：CDEX, PANDA-X
- 实验探测到的事例数

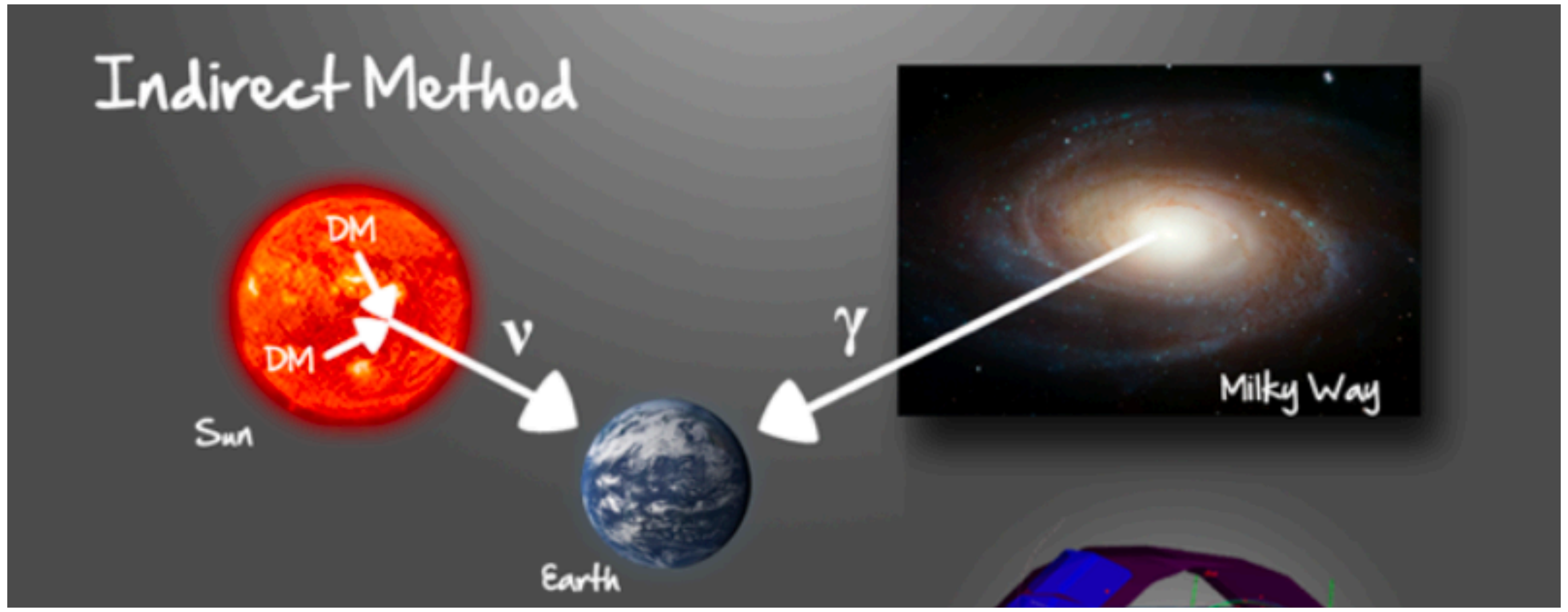
$$N = n_{\rm DM} N_{\rm target} \sigma v_{\rm DM} t_{\rm obs}$$

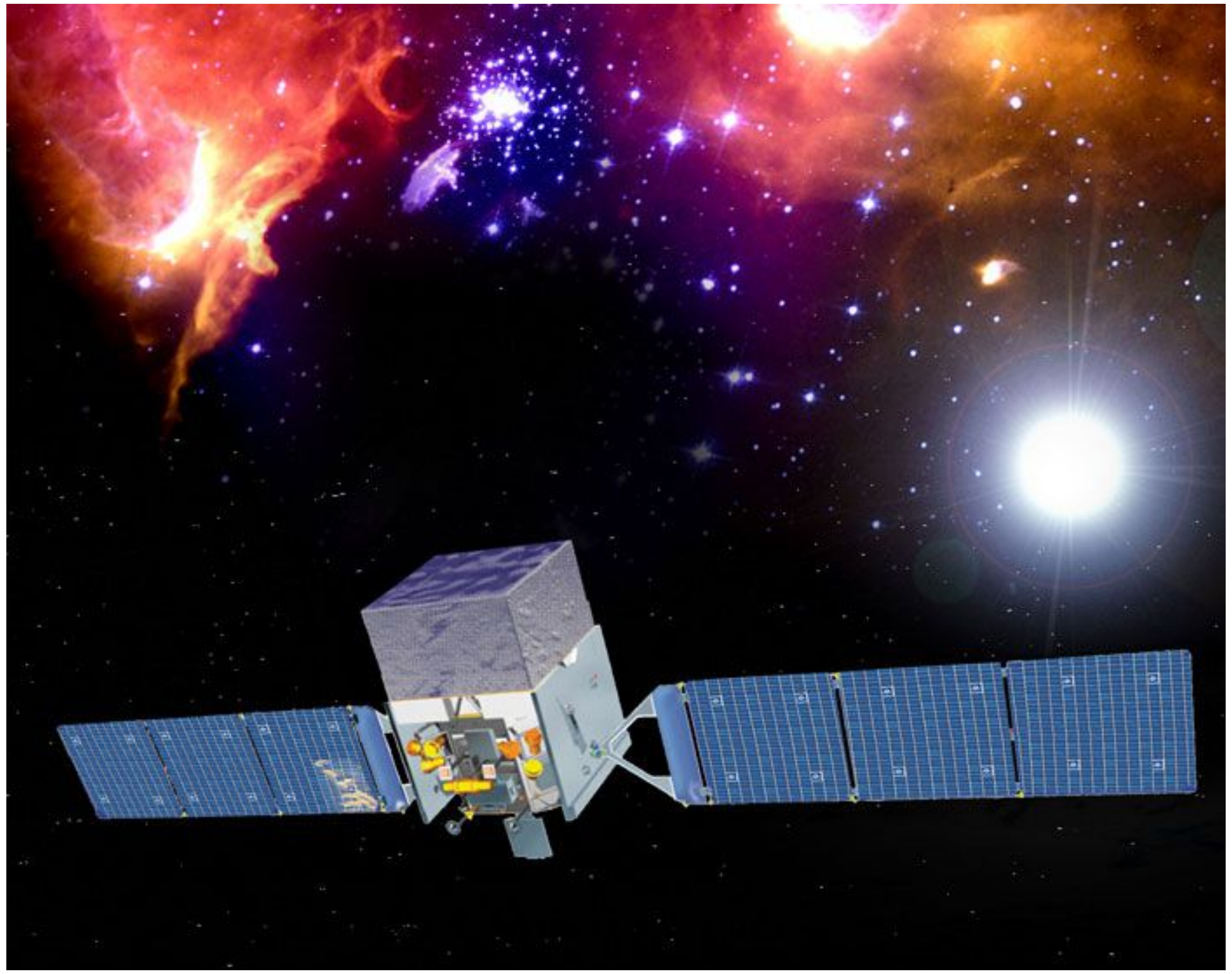


APPEC Committee Report: 2104.07634

暗物质间接探测实验

Credit: HAP/ A. Chantelauze





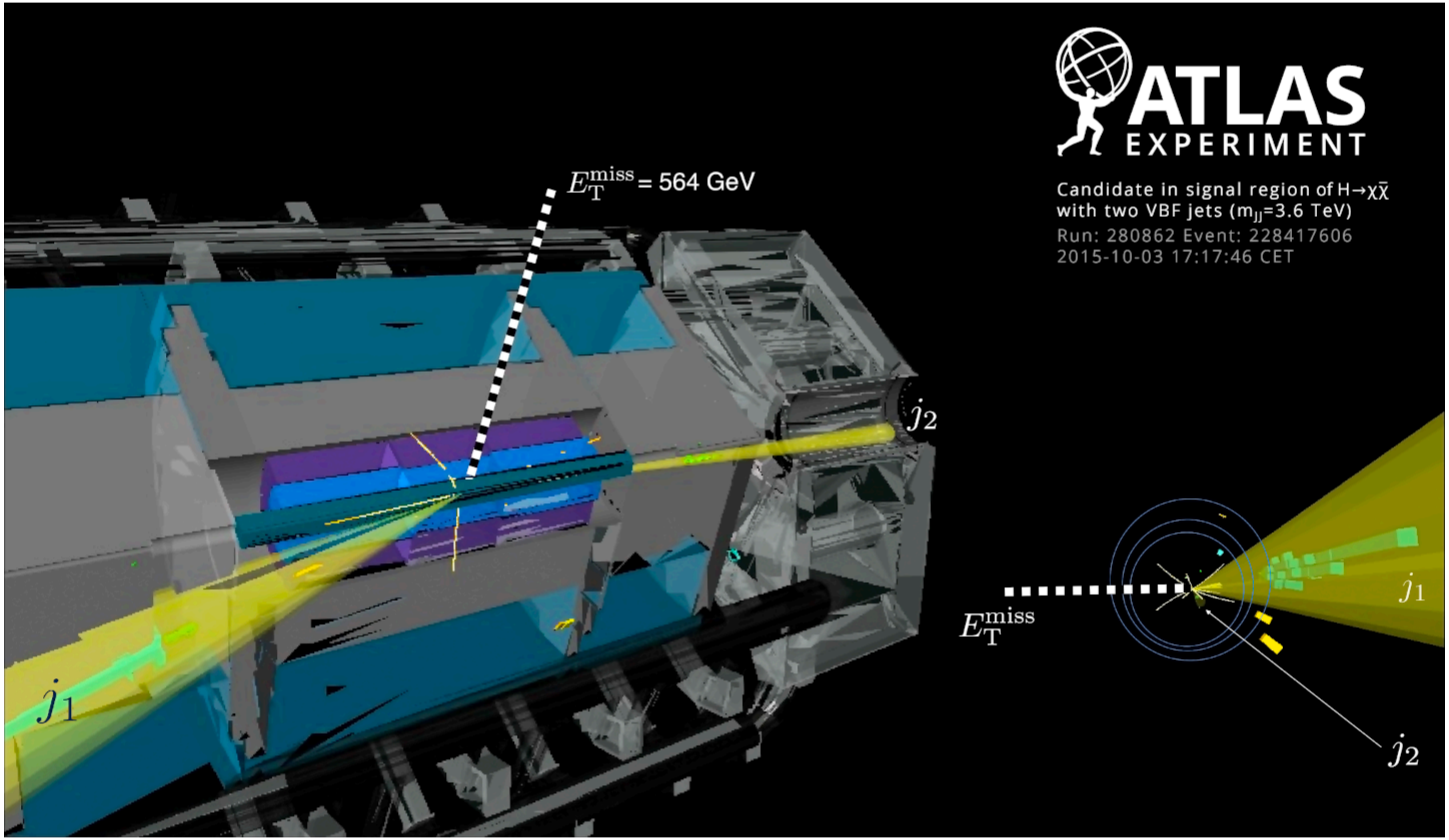
- 天体或者星系中心暗物质湮灭产生的次级粒子
  - DM + DM > SM + SM
- 中国相关实验：DAMPE, LHAASO
- 实验探测到的粒子流强

$$F = \frac{1}{4\pi}\frac{\langle \sigma v\rangle}{2m_{\mathrm{dm}}^2}\int_{\mathrm{l.o.s}} \rho_{\mathrm{dm}}^2 dl$$

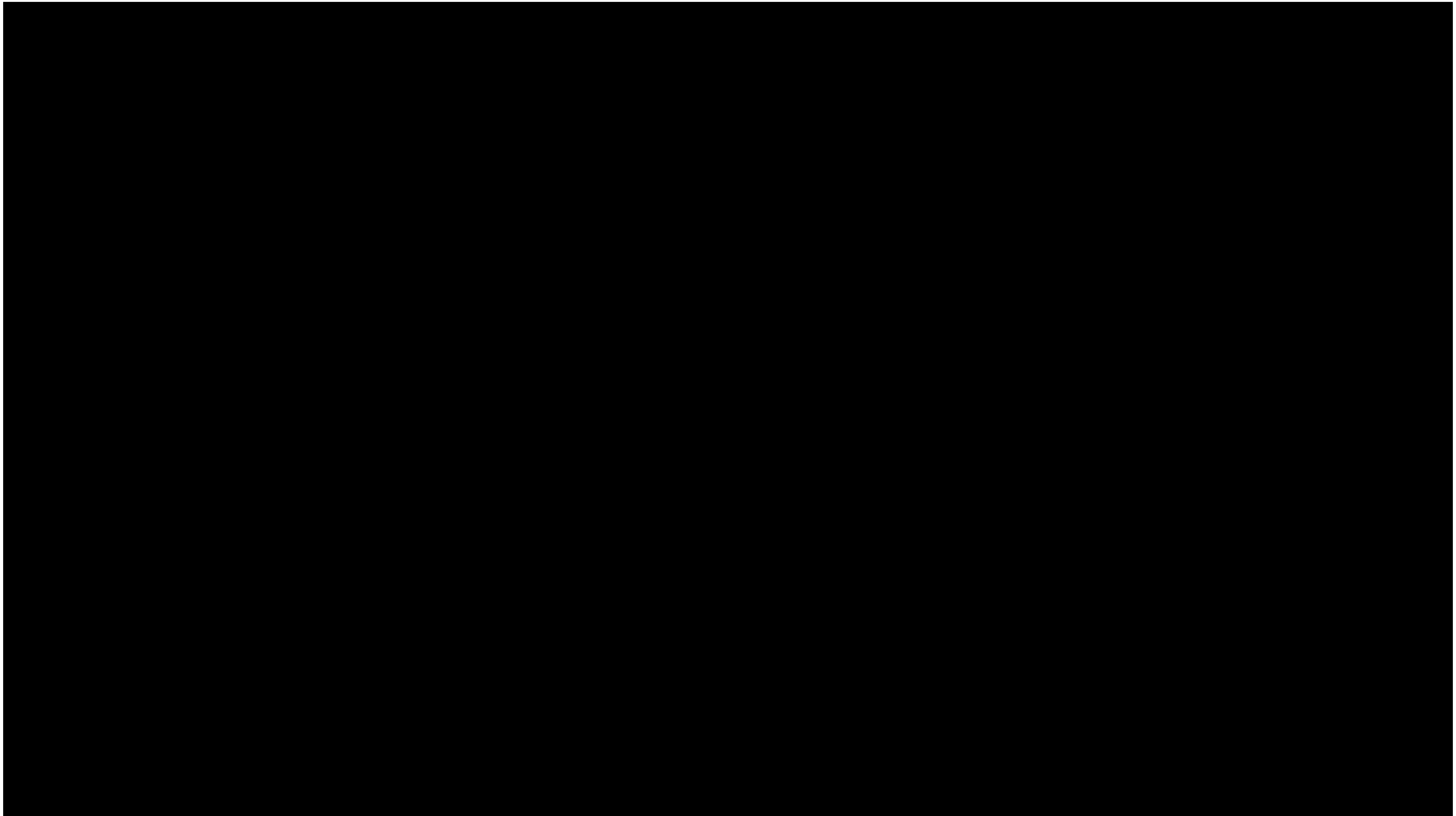
$$\left[\mathrm{cm}^2 \times \frac{\mathrm{cm}}{\mathrm{sec}} \times \mathrm{cm}^{-6} \times \mathrm{cm}\right] = \left[\mathrm{cm}^{-2}\mathrm{sec}^{-1}\right]$$

暗物质对撞机探测实验

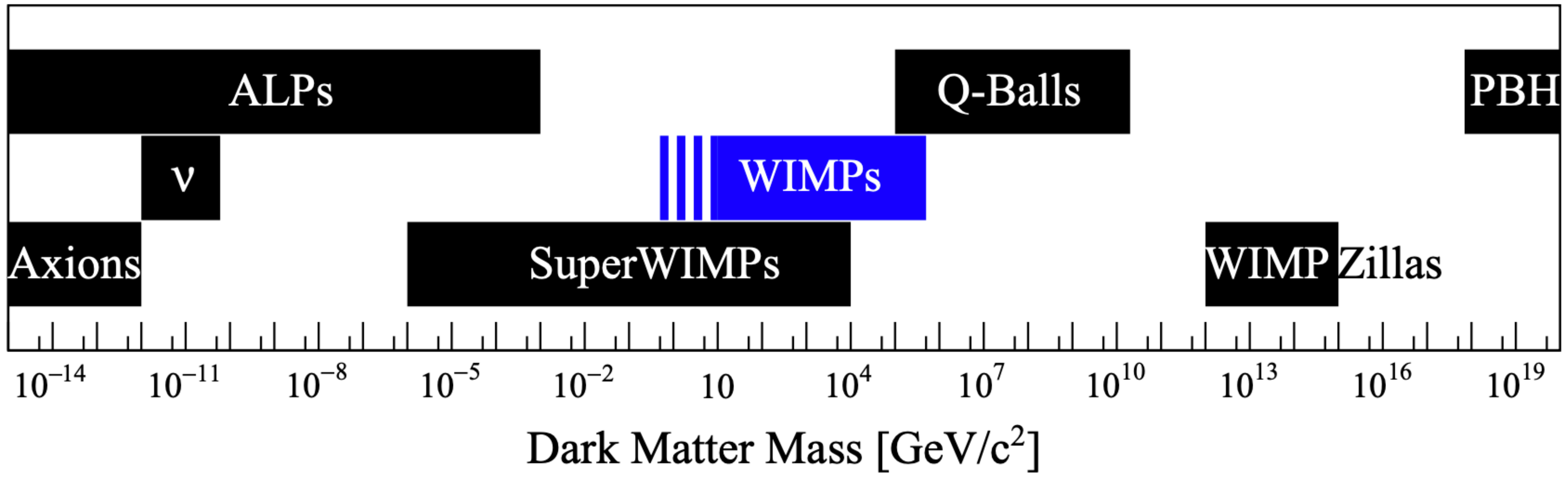
- SM + SM > DM pair + SM
- 暗物质在对撞机无法探测
- 根据能动量守恒反推其存在，类似中微子之于beta衰变



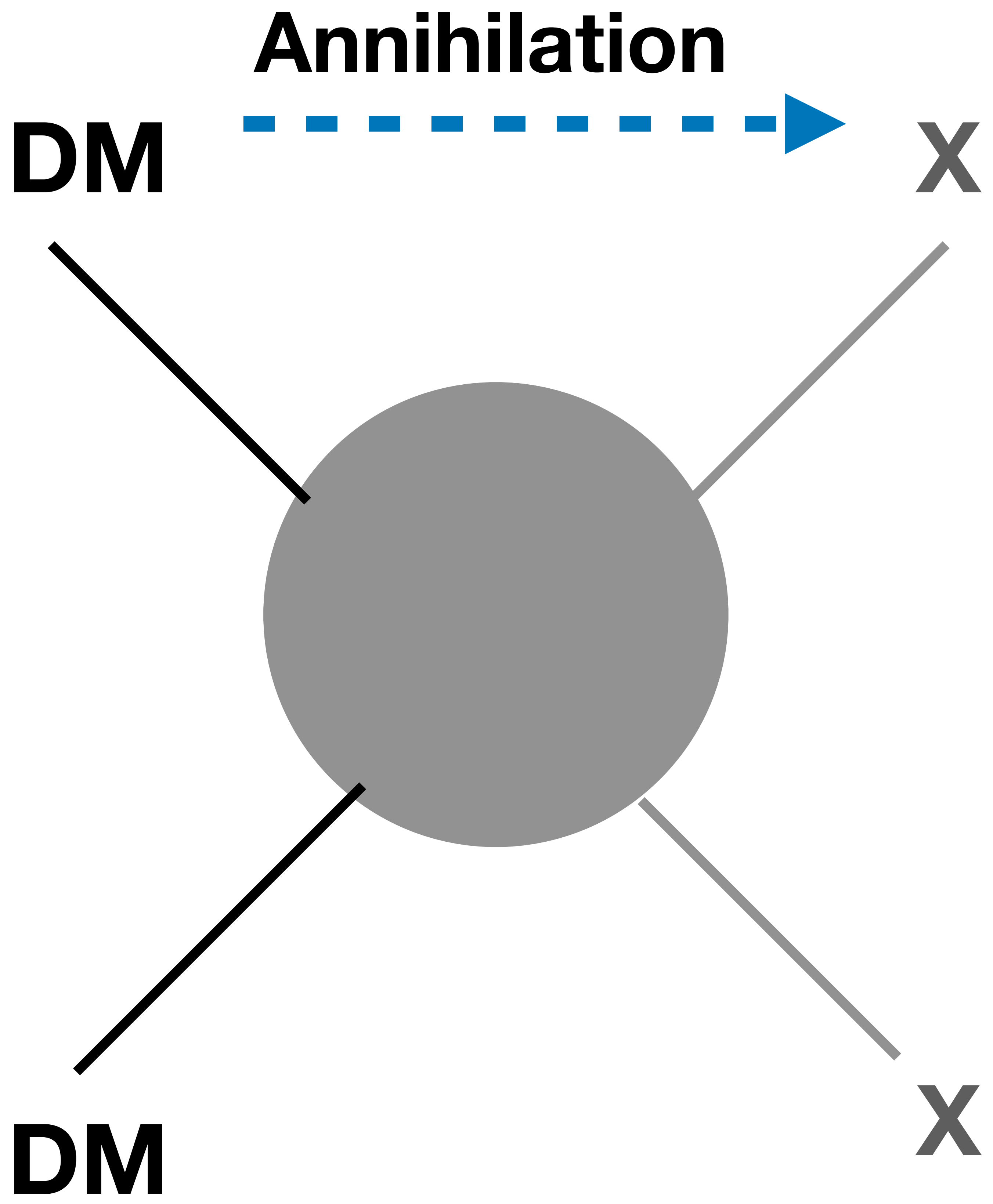
粒子物理的宇宙学前沿



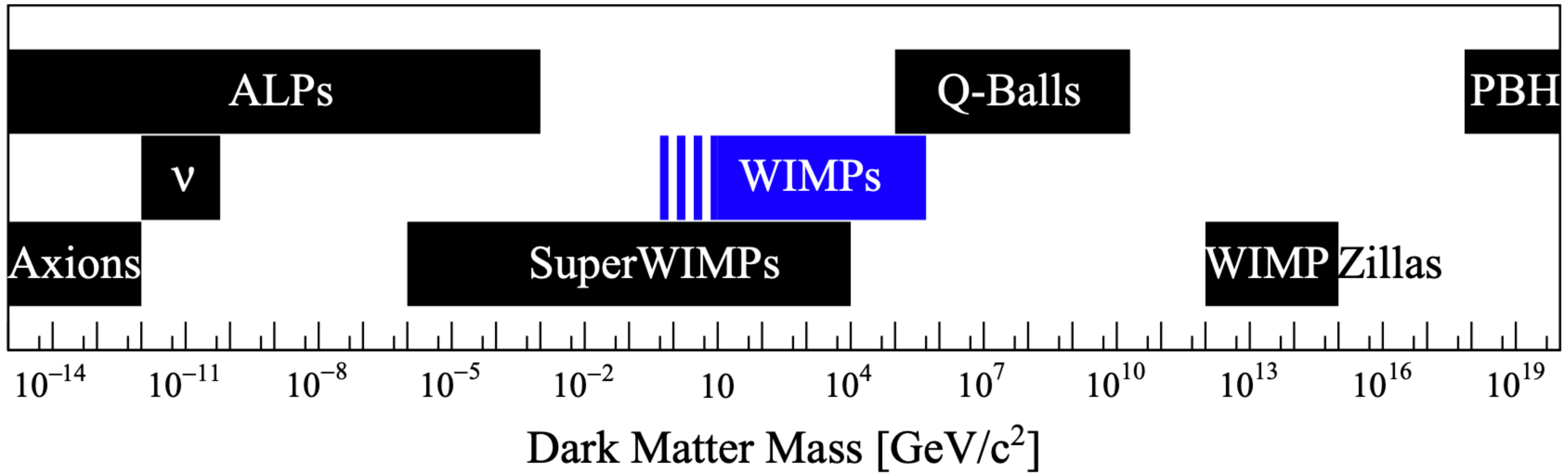
优秀的产生机制：热退耦合



- DM is a massive elementary particle
- DM has an electroweak-scale coupling
  - DM starts with thermal distribution
  - Relic abundance is determined by freeze-out mechanism
  - DM Annihilation into
    - X = Standard Model particles (direct coupling)



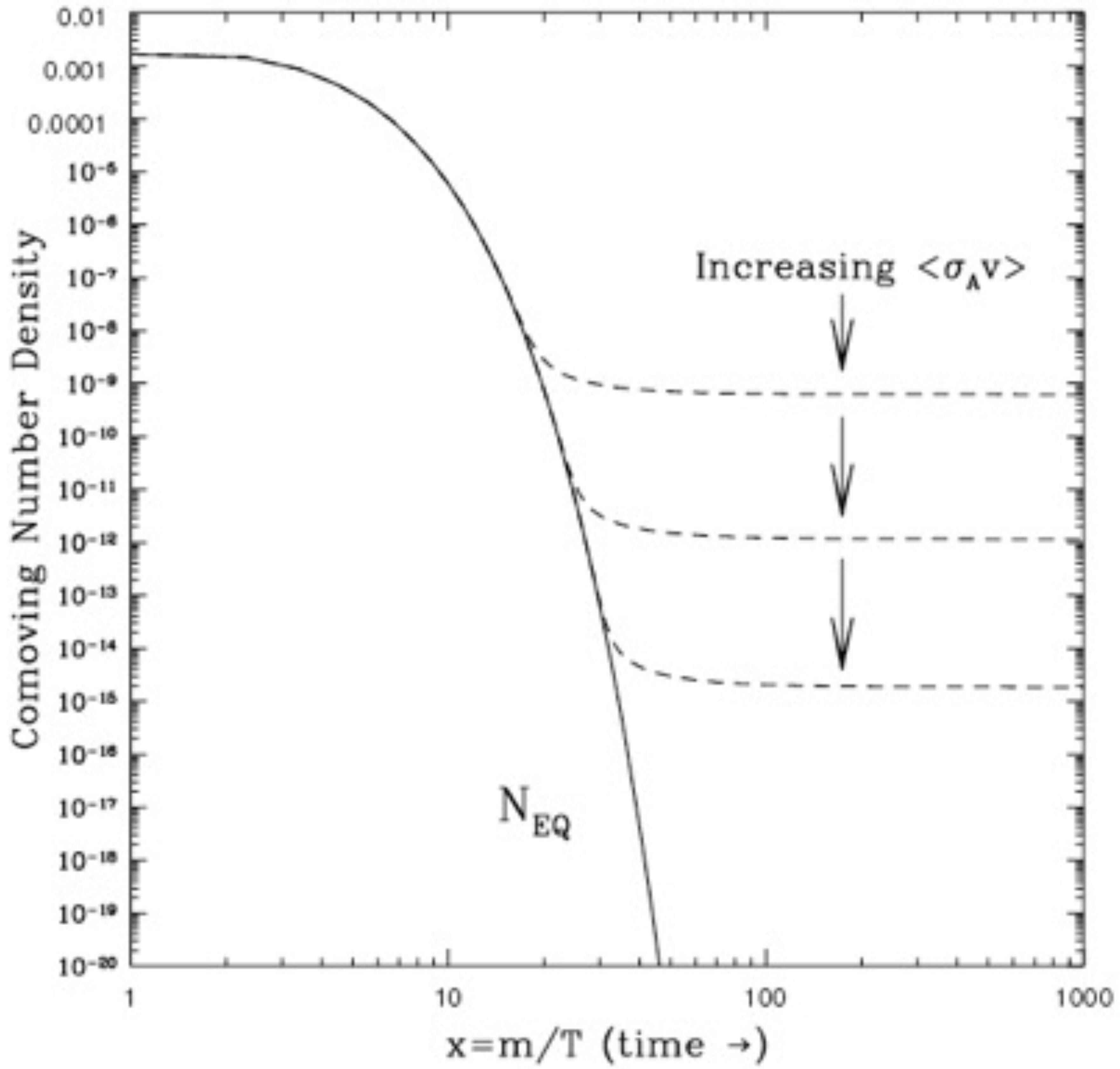
WIMP暗物质的热退耦合湮灭截面



- 热退耦合湮灭截面与电弱相互作用强度和能标吻合

$$\langle \sigma v \rangle \sim \frac{\alpha^2}{m_W^2} \sim 3 \times 10^{-26} \mathrm{cm}^3 \mathrm{s}^{-1}$$

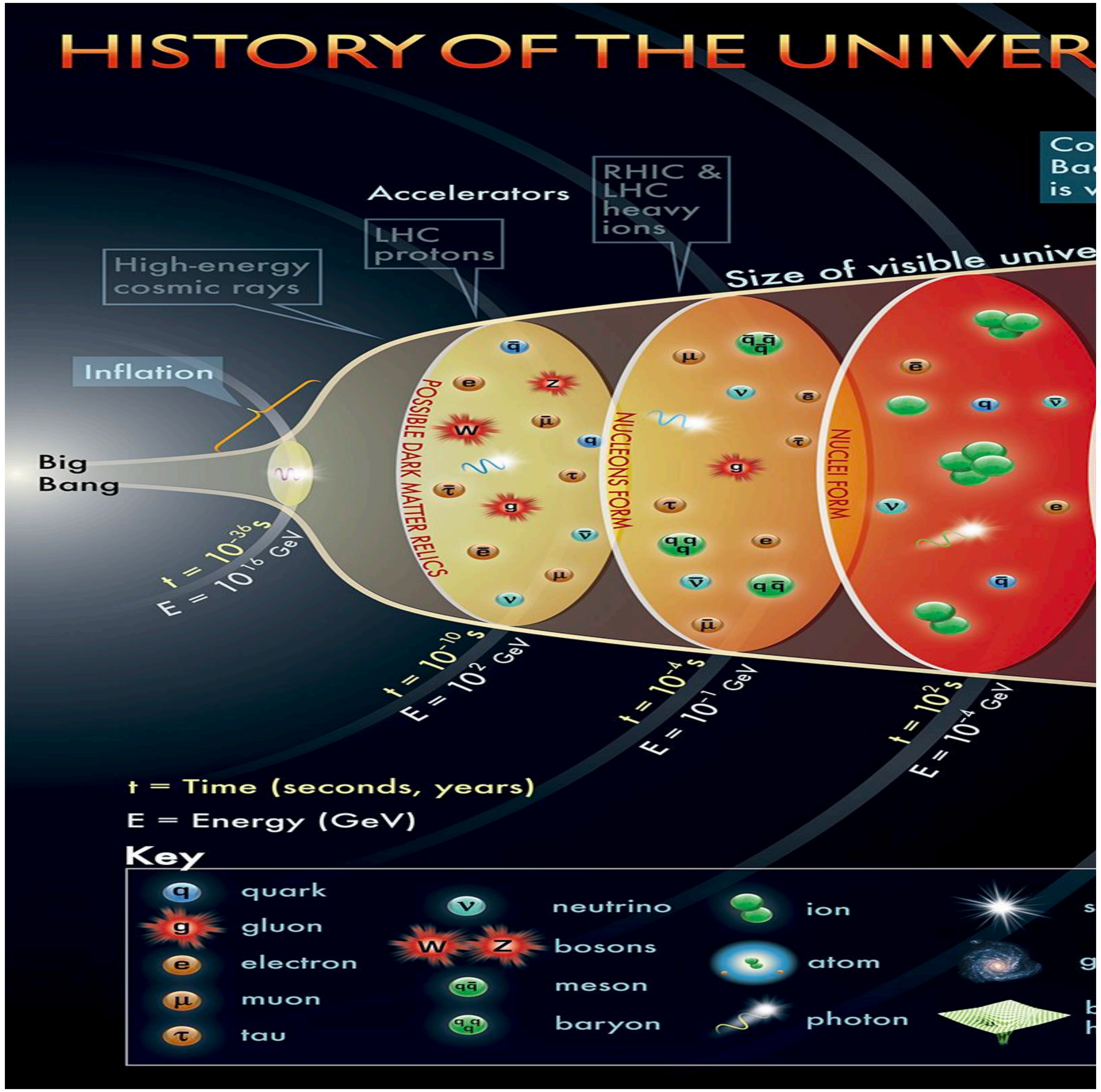
人们称该吻合为 WIMP miracle



Jungman *et al* hep-ph/9506380

热退耦合：优秀！

- 自然的得到暗物质残余丰度
  - 不需要UV信息 (以热平衡分布开局)
  - 电弱能标的湮灭截面
  - 与标准模型其他粒子相似的故事
    - (ν decoupling, n_p/n_n ratio, nuclear elements)
  - 预言了直接/间接/对撞机的实验信号



Outlines

- 暗物质的起源
  - 暗物质的天文观测证据
  - 宇宙学标准模型 ΛCDM
  - 暗物质分布
- 暗物质的物理模型
  - 可能的暗物质候选者
  - WIMP暗物质
- 热退耦暗物质残余丰度计算
- WIMP暗物质现状
  - WIMP暗物质的直接探测危机
    - 解决危机的多种办法
  - 暗物质对撞机探测的互补性
  - 暗物质的间接探测限制
    - 避开限制的办法
- 总结

背景知识1：狭义相对论与粒子

- 协变动量和逆变动量：

$$p^\mu = \{E, \vec{p}\} = \{E, p_x, p_y, p_z\}$$

$$p_\mu = g_{\mu\nu} p^\mu = \{E, -\vec{p}\} = \{E, -p_x, -p_y, -p_z\}$$

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- 质能关系和相位:

$$p^\mu p_\mu = E^2 - p^2 = m^2$$ Invariant mass

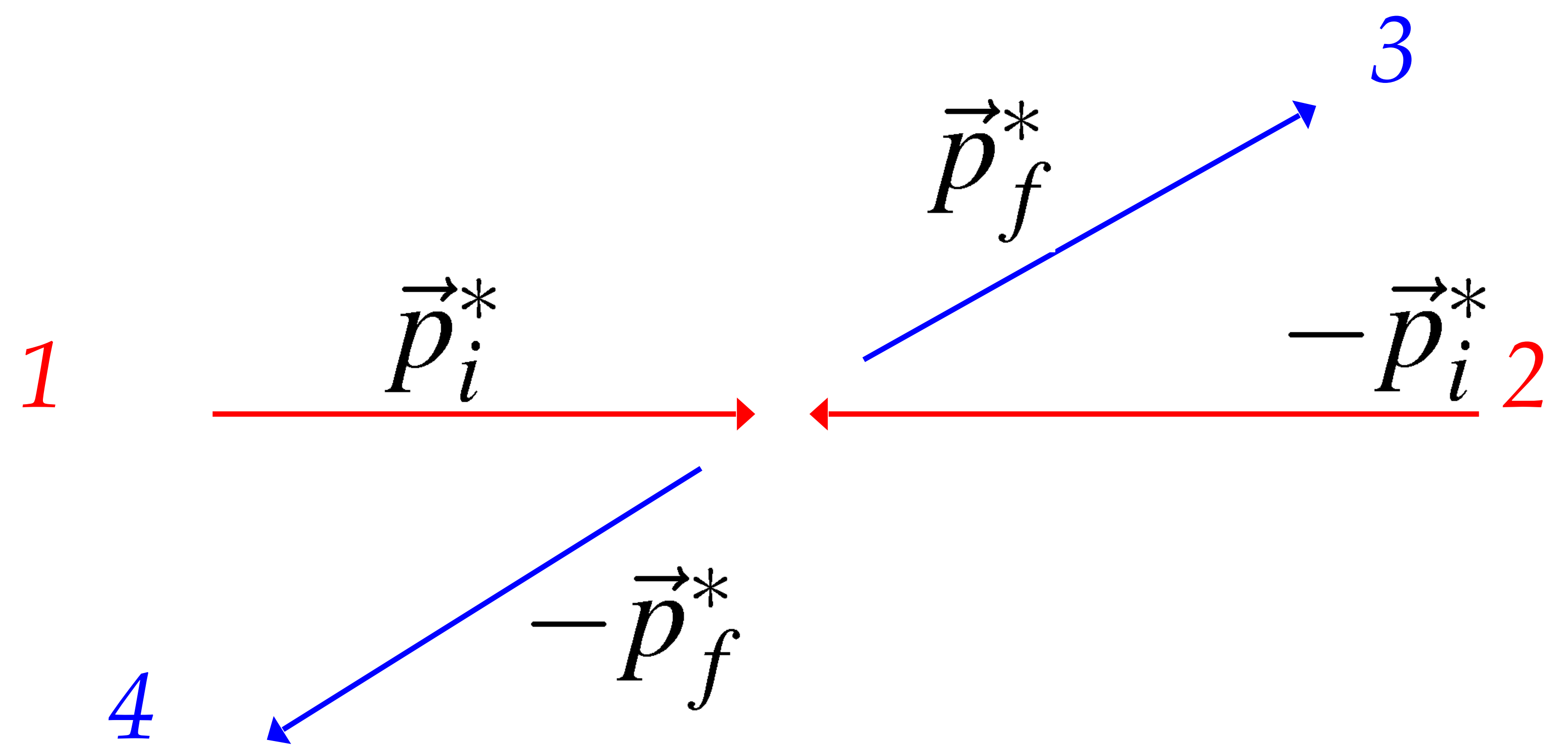
$$x^\mu p_\mu = Et - \vec{p}.\vec{r}$$ Phase

背景知识2：相空间与截面

- 粒子的相空间
  - 4D 洛伦兹不变的角度：单个on-shell 粒子的相空间

$$dPS = \Theta(E)\delta(p\cdot p - m^2)d^4p = \frac{d^3\vec{p}}{(2\pi)^3 2E}$$

  - 为什么老是有$(2E)^{-1}$的因子?
  - Normalize to 2E particle in the volume



- 相互作用截面： $1+2\to 3+4$ (DM + DM > SM SM)

$$\sigma = \frac{1}{2E_1 2E_2\left|v_1 - v_2\right|}\int\left(\prod_f \frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}\right)\times\left|\mathcal{M}\left(p_1,p_2\to\left\{p_f\right\}\right)\right|^2(2\pi)^4\delta^{(4)}\left(p_1+p_2-\sum p_f\right)$$

背景知识3：粒子的态密度

- 粒子态相空间函数 $f(\vec{x}, \vec{p}, t) d\vec{x} d\vec{p}$
- 热平衡分布: $f_{\rm eq} = \dfrac{1}{e^{E/T} \pm 1} \approx e^{-E/T}$
- 粒子数密度 $n_{\rm eq} = \displaystyle\int d\vec{p} f_{\rm eq} = \int \frac{d\vec{p}}{(2\pi)^3} e^{-\frac{E}{T}}$
  - 高温$T \gg m$（相对论）极限 $n_{\rm eq} = T^3$
  - 低温$T \ll m$（非相对论）极限 $n_{\rm eq} = \left(\dfrac{mT}{2\pi}\right)^{3/2} e^{-\frac{m}{T}}$

背景知识4：宇宙度规与辐射为主的宇宙

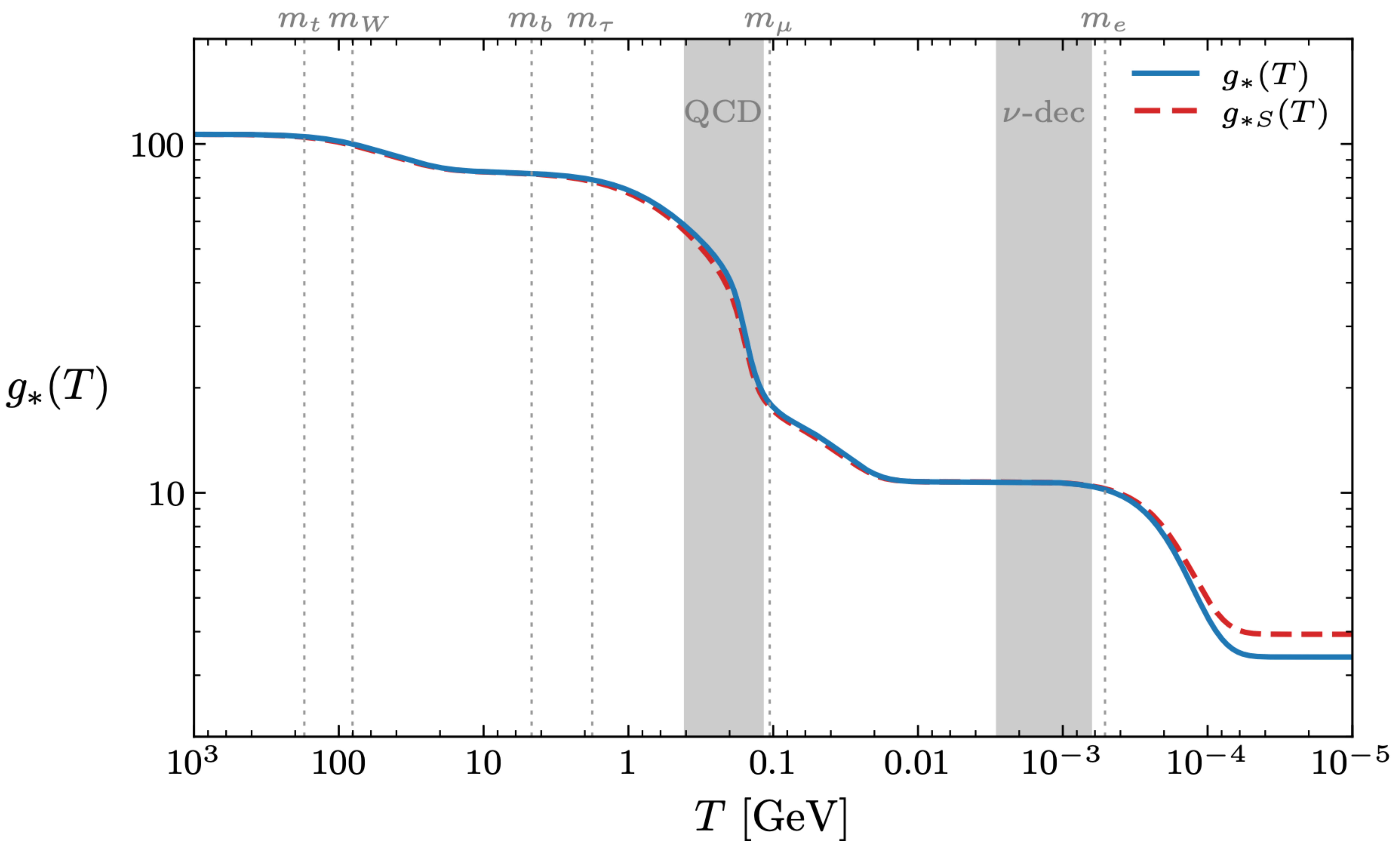
- 辐射为主的宇宙的膨胀速率

$$H_{\rm rad}^2 = \frac{8\pi^3}{90}\frac{g_* T^4}{m_{\rm PL}^2}$$

- 辐射为主的宇宙的温度红移

$$T \propto a(t)^{-1}$$

$$\rho_{\rm rad} \propto a(t)^{-4}$$



暗物质热退耦Boltzmann方程

- 直接从数密度出发 $\mathrm{DM}_1 + \mathrm{DM}_2 \to \mathrm{SM}_3 + \mathrm{SM}_4$

$$a^{-3}\frac{d(n_1 a^3)}{dt} = \int \Pi_{i=1}^{4} d\mathrm{PS}_i \times (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) |\mathscr{M}|^2$$

$$\times \left[f_3 f_4 (1 \pm f_1)(1 \pm f_2) - f_1 f_2 (1 \pm f_3)(1 \pm f_4)\right]$$

- 化简

暗物质热退耦Boltzmann方程

- 最终的暗物质演化Boltzmann方程

$$a^{-3}\frac{d(na^3)}{dt} = n_1^{\rm eq} n_2^{\rm eq}\langle\sigma v\rangle\left(\frac{n_3 n_4}{n_3^{\rm eq} n_4^{\rm eq}} - \frac{n_1 n_2}{n_1^{\rm eq} n_2^{\rm eq}}\right)$$

$$\dot{n} + 3Hn = \langle\sigma v\rangle(n_{\rm eq}^2 - n^2)$$

- 暗物质湮灭截面的热平均公式

$$\langle\sigma v\rangle \equiv \frac{1}{n_1^{\rm eq} n_2^{\rm eq}}\int \Pi_{i=1}^4 d{\rm PS}_i \times (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4)|\mathscr{M}|^2 \times e^{-\frac{E_1+E_2}{T}}$$

求解—暗物质热退耦Boltzmann方程

- 粒子数密度的行为

$$n_{\rm eq}^{\rm rad} \sim T^3 \sim a^{-3}, \quad n_{\rm eq}^{\rm mat} \sim (mT)^{3/2} e^{-m/T}$$

$$n_{\rm freeze-out} \sim a^{-3}$$

- 一个实用的变量 DM Yield和采用温度变量x

$$Y_{\rm dm} \equiv n_{\rm dm}/s, \quad x \equiv m_{\rm dm}/T$$

- 暗物质演化Boltzmann方程 $dx/dt = (8\pi^3 g_\star/(90 m_{\rm Pl}^2))^{1/2} m^2/x$

$$\frac{dY}{dx} = \frac{\langle \sigma v \rangle x s}{\sqrt{\frac{8\pi^3 g_\star}{90 m_{\rm Pl}^2}}\, m^2} \left(Y_{\rm eq}^2 - Y^2\right)$$

求解—暗物质热退耦Boltzmann方程

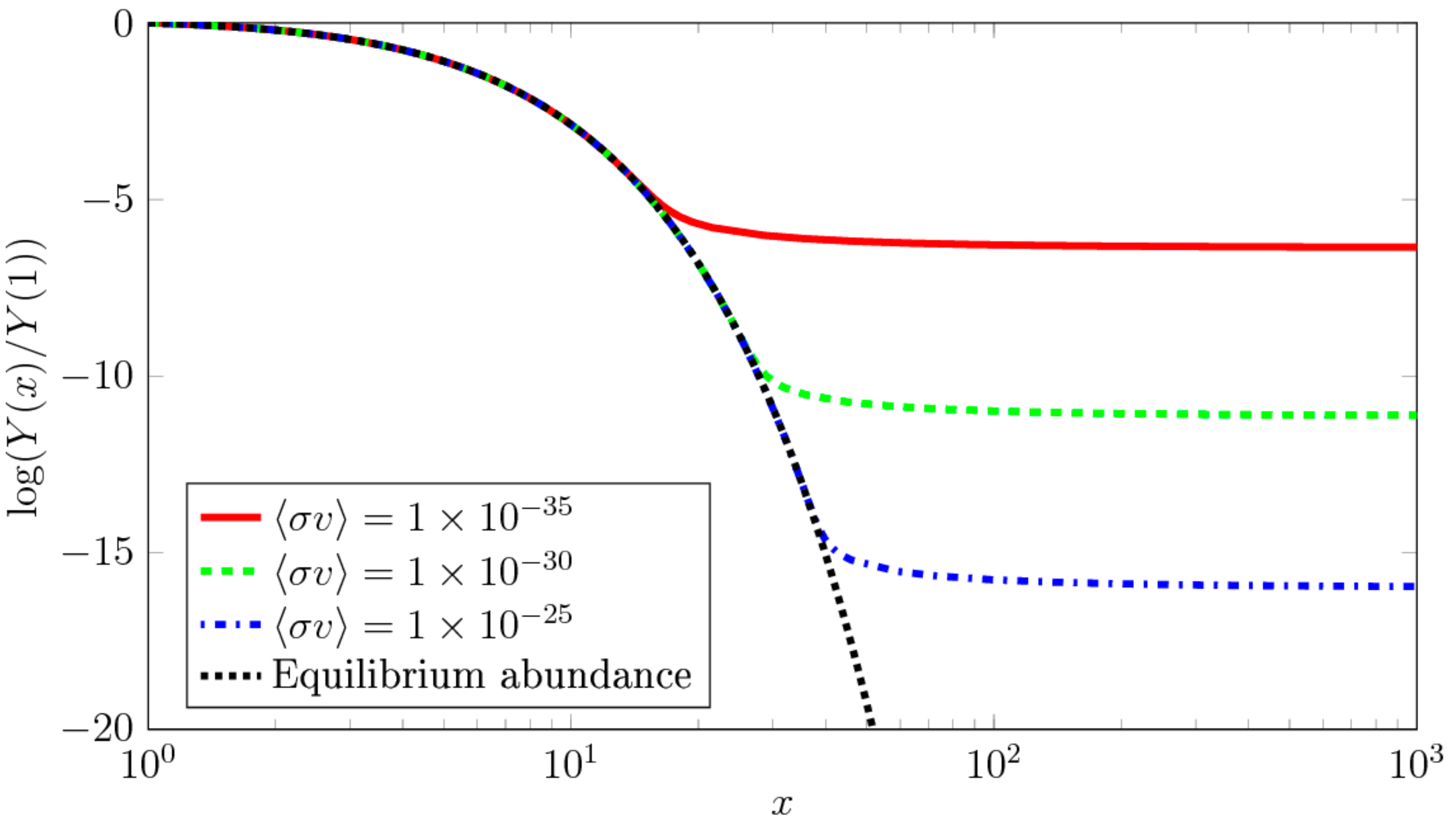
- 暗物质演化Boltzmann方程

$$\frac{dY}{dx} = \frac{\langle \sigma v \rangle x s}{\sqrt{\frac{8\pi^3 g_\star}{90 m_{\rm Pl}^2}}\, m^2} \left(Y_{\rm eq}^2 - Y^2\right)$$

- 暗物质热退耦温度

$$n_{\rm fo} \langle \sigma v \rangle \approx H_{\rm fo}$$

$$x_{\rm fo} \sim 25$$



近似求解—暗物质热退耦Boltzmann方程

- 近似求解Boltzmann方程

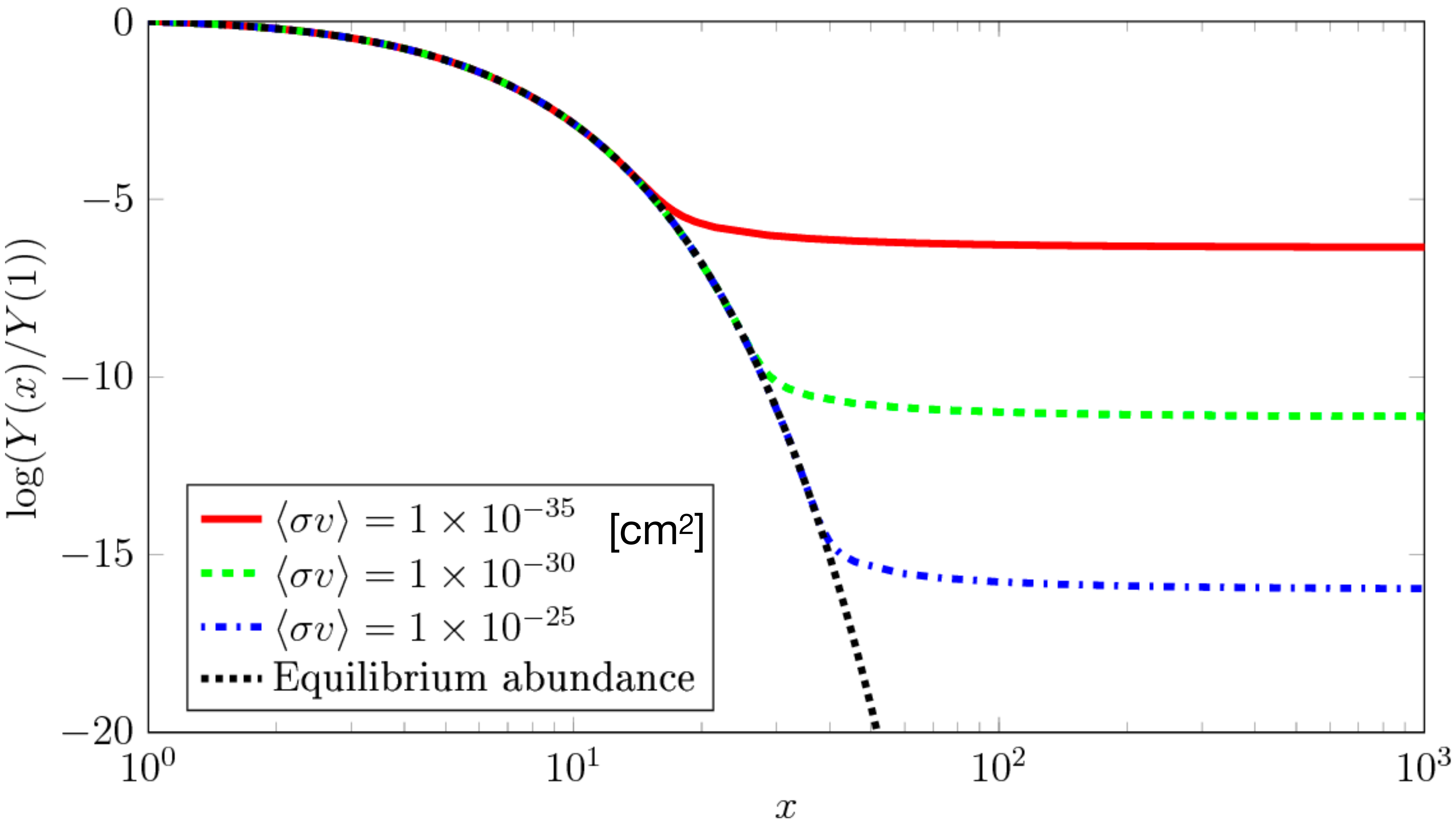
$$\frac{dY}{dx} = -\frac{\lambda}{x^2}(Y^2 - Y_{\mathrm{eq}}^2)$$

$$\frac{\lambda}{x^2} \equiv \langle \sigma v \rangle \frac{xs}{H_{\mathrm{rad}}(T = m_{\mathrm{DM}})}$$

- 对于 $x \gg 1$

$$\frac{dY}{dx} \approx -\frac{\lambda}{x^2} Y^2$$

$$\Longrightarrow Y_\infty^{-1} - Y_{\mathrm{fo}}^{-1} = \frac{\lambda}{x_{\mathrm{fo}}} \Longrightarrow Y_\infty^{-1} = \frac{\lambda}{x_{\mathrm{fo}}}$$



近似求解—暗物质热退耦Boltzmann方程

- 近似求解Boltzmann方程 $Y_\infty^{-1} = \frac{\lambda}{x_{\rm fo}}$

- 今天的暗物质能量密度占比 $\Omega_{\rm dm} = 26.8\,\%$

$$\Omega_{\rm dm} h^2 = \frac{Y_0 s_0 m_{\rm dm}}{\rho_{\rm cr}} h^2 \approx \frac{Y_\infty s_0 m_{\rm dm}}{\rho_{\rm cr}} h^2 \approx 0.3 \left(\frac{m_{\rm dm}}{\rm eV} \right) Y_\infty$$

$$\rho_{\rm cr} = 3H_0^2 m_{\rm Pl}^2/8\pi \approx 8 \times 10^{-47} h^2\,{\rm GeV}^4 \text{ and } s_0 \approx 2970\,{\rm cm}^{-3}$$

- 暗物质热退耦湮灭截面大小

$$\Omega h^2 \approx 0.1 \left(\frac{x_f}{25}\right) \left(\frac{g_\star}{80}\right)^{-1} \left(\frac{3\times 10^{-26}{\rm cm}^3{\rm s}^{-1}}{\langle\sigma v\rangle}\right)$$

WIMP 暗物质奇迹

- 暗物质热退耦湮灭截面大小

$$\Omega h^2 \approx 0.1 \left(\frac{x_f}{25}\right)\left(\frac{g_\star}{80}\right)^{-1}\left(\frac{3\times 10^{-26}\mathrm{cm}^3\mathrm{s}^{-1}}{\langle\sigma v\rangle}\right)$$

$$\langle \sigma v\rangle \sim 3\times 10^{-26} cm^3/s$$

$$\sim 10^{-8}\ \mathrm{GeV}^{-2} \sim \frac{\alpha^2}{m_W^2}$$

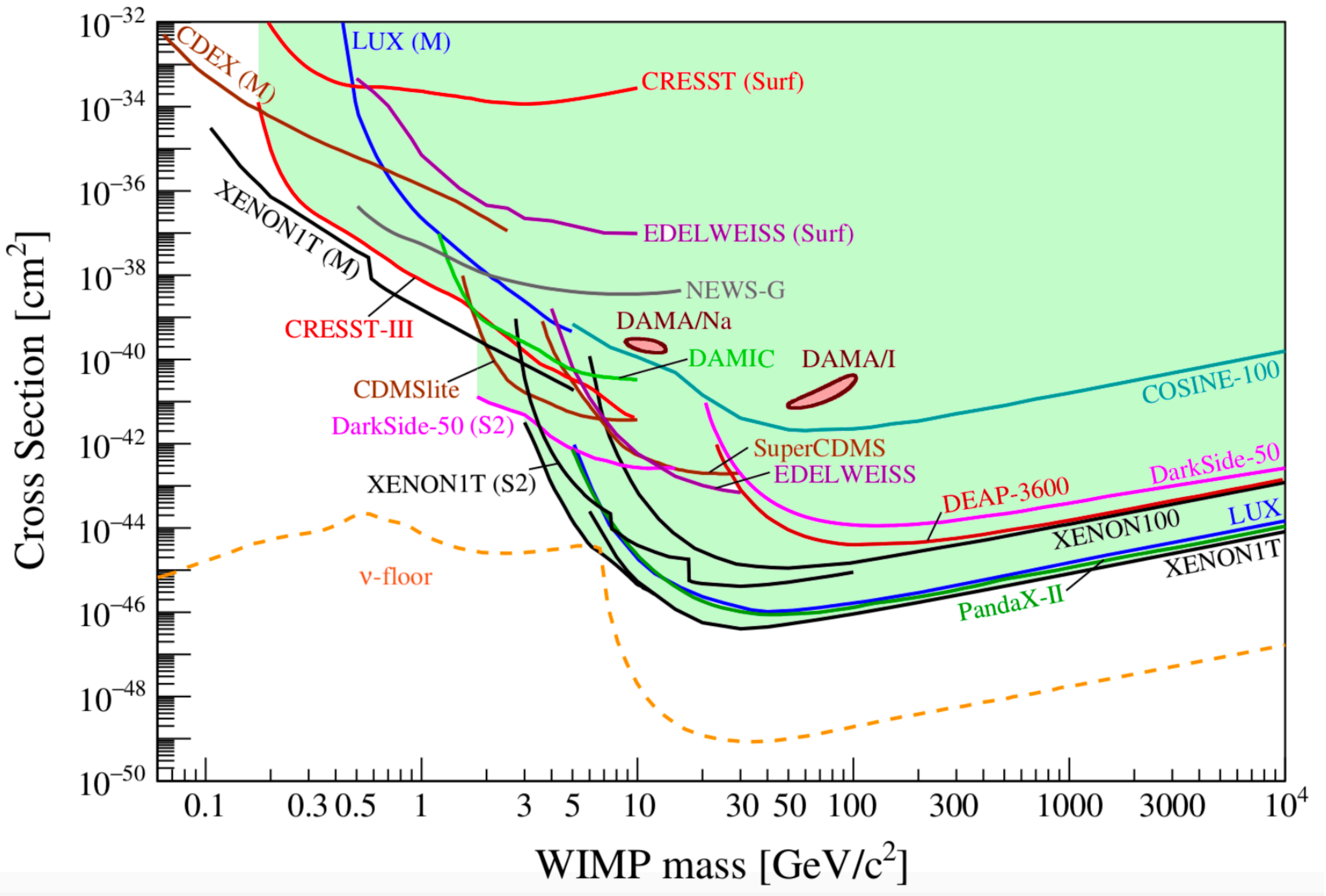
- 暗物质可能和弱相互作用能标相关联

Outlines

- 暗物质的起源
  - 暗物质的天文观测证据
  - 宇宙学标准模型 ΛCDM
  - 暗物质分布
- 暗物质的物理模型
  - 可能的暗物质候选者
  - WIMP暗物质
- 热退耦暗物质残余丰度计算
- WIMP暗物质现状
  - WIMP暗物质的直接探测危机
    - 解决危机的多种办法
  - 暗物质对撞机探测的互补性
  - 暗物质的间接探测限制
    - 避开限制的办法
- 总结

WIMP暗物质危机

- Null result from direct detection
  - Maybe discovery in the corner?
  - Neutrino floor and beyond: directional ..
  - The rise of light dark matter ($\lesssim$ 10 GeV)

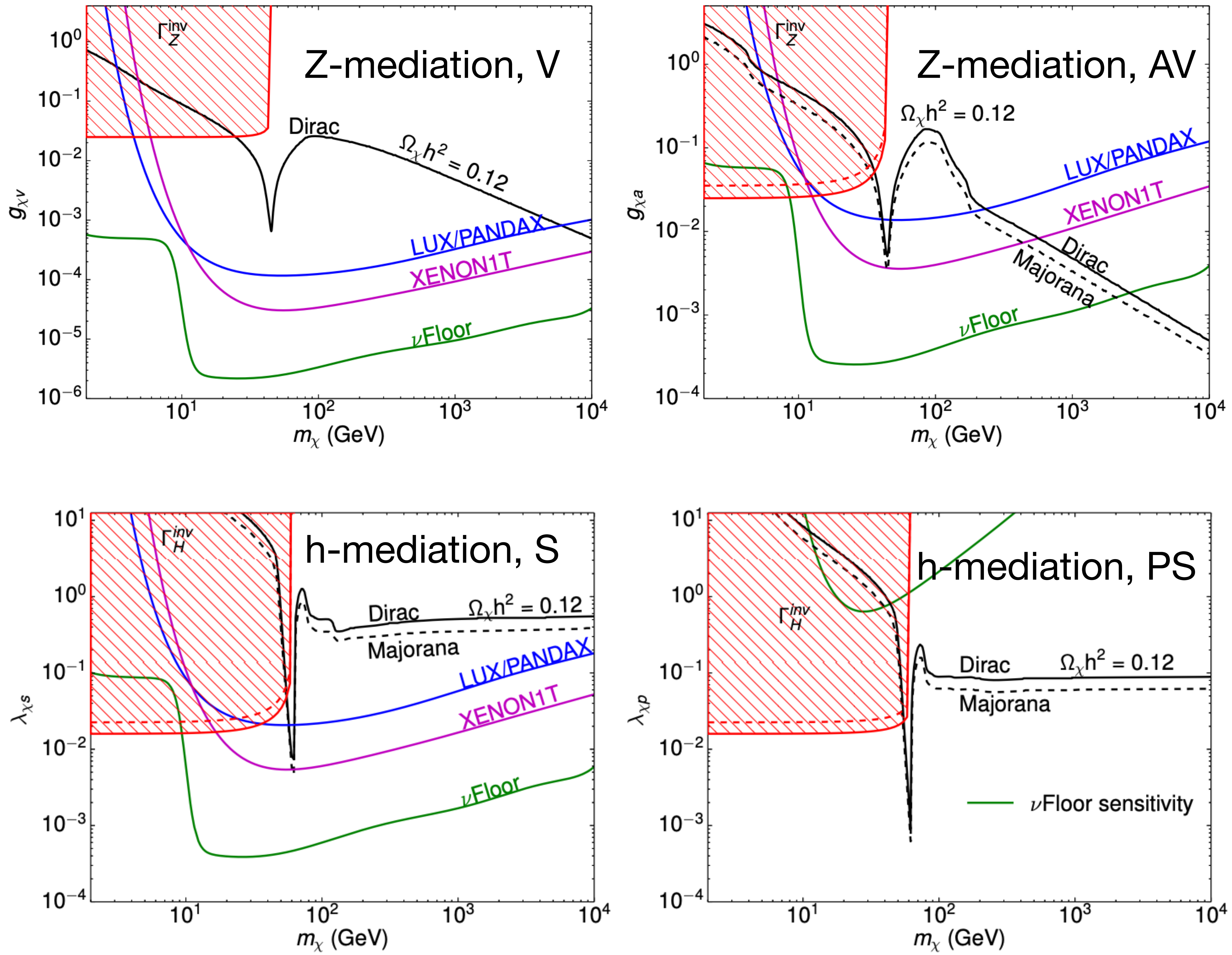


APPEC Committee Report: 2104.07634

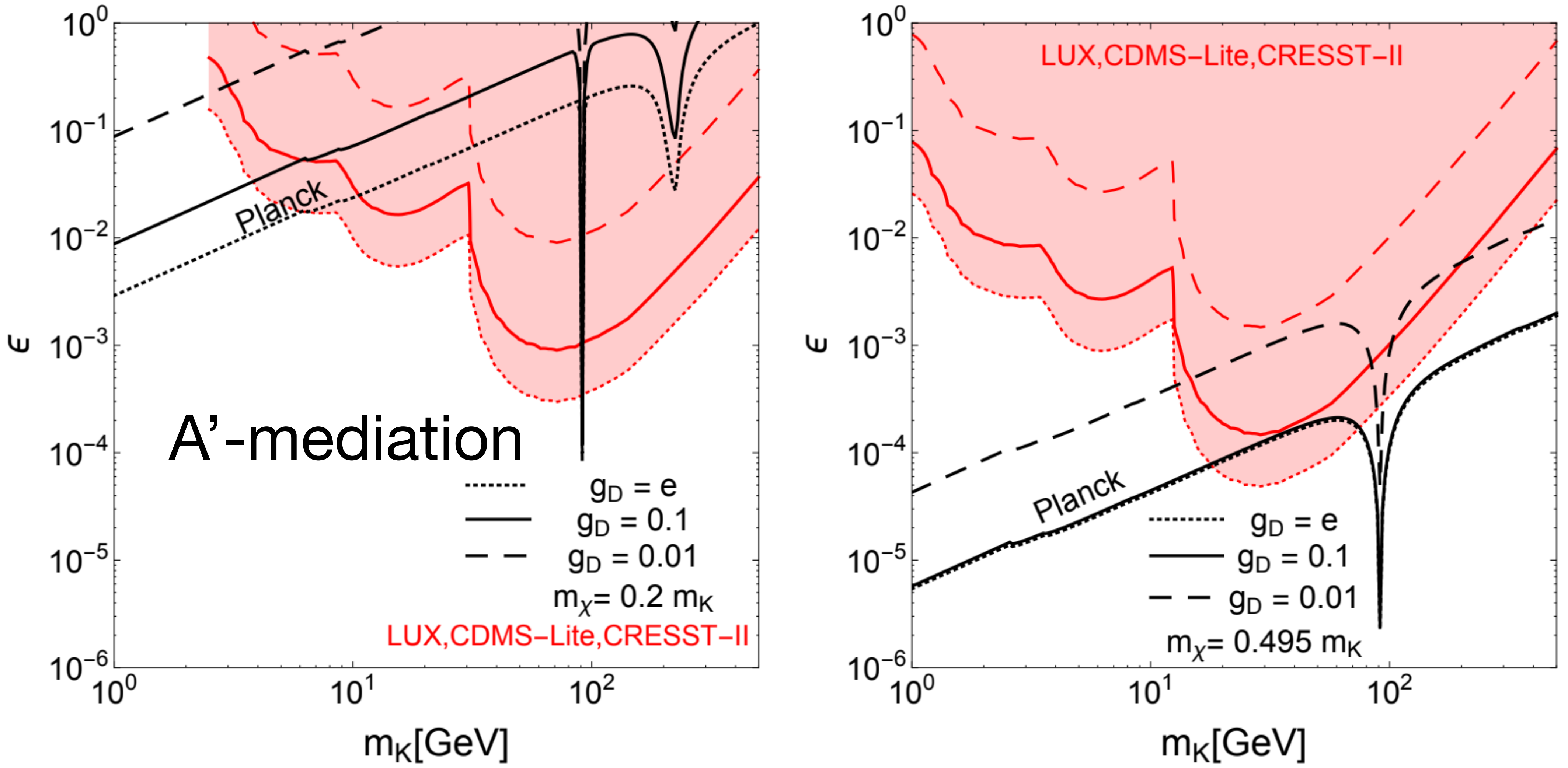
The WIMP crisis from direct detection

- SM Higgs and Z mediated scenario are highly constrained
- Other mediators without DD suppression is also highly constrained, e.g. A’
    - Unless in the resonant region

Toward (Finally!) Ruling Out Z and Higgs Mediated Dark Matter Models
Hooper et al, ArXiv: 1609.09079, JCAP



JL, X.P. Wang, F. Yu, 1704.00730, JHEP



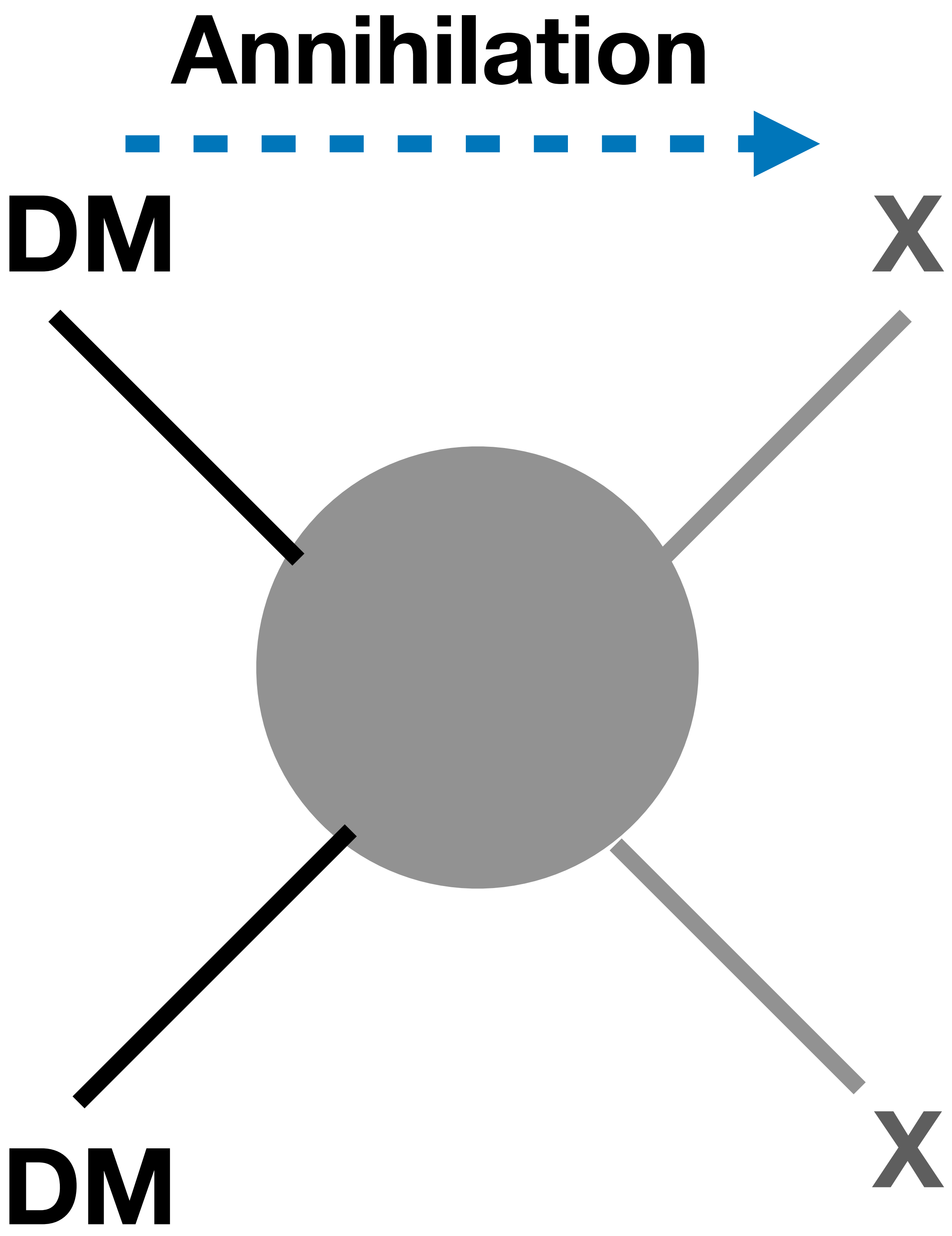
Outlines

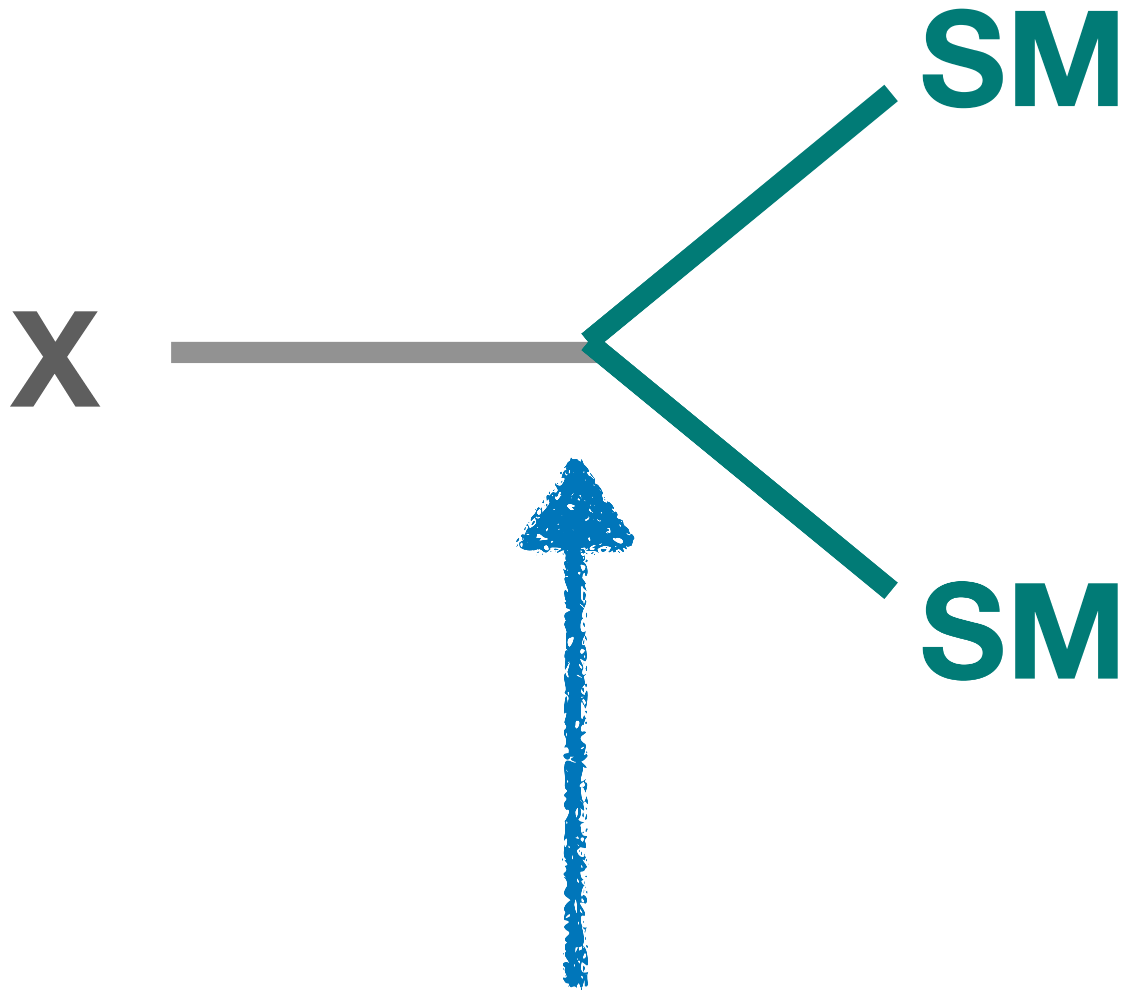
- 暗物质的起源
  - 暗物质的天文观测证据
  - 宇宙学标准模型 ΛCDM
  - 暗物质分布
- 暗物质的物理模型
  - 可能的暗物质候选者
  - WIMP暗物质
- 热退耦暗物质残余丰度计算
- WIMP暗物质现状
  - WIMP暗物质的直接探测危机
    - 解决危机的多种办法
  - 暗物质对撞机探测的互补性
  - 暗物质的间接探测限制
    - 避开限制的办法
- 总结

The way-out from direct detection limits

- 1. Secluded dark matter (dark sector)

  - Very small coupling to SM sector





Dark mediator
with very small coupling to SM
$\epsilon \gtrsim 10^{-7}$ **to thermalize dark sector**

Pospelov, Ritz, Voloshin, 0711.4866 [PLB]
Arkani-Hamed, Finkbeiner, Slatyer, Weiner, 0810.0713 [PRD]

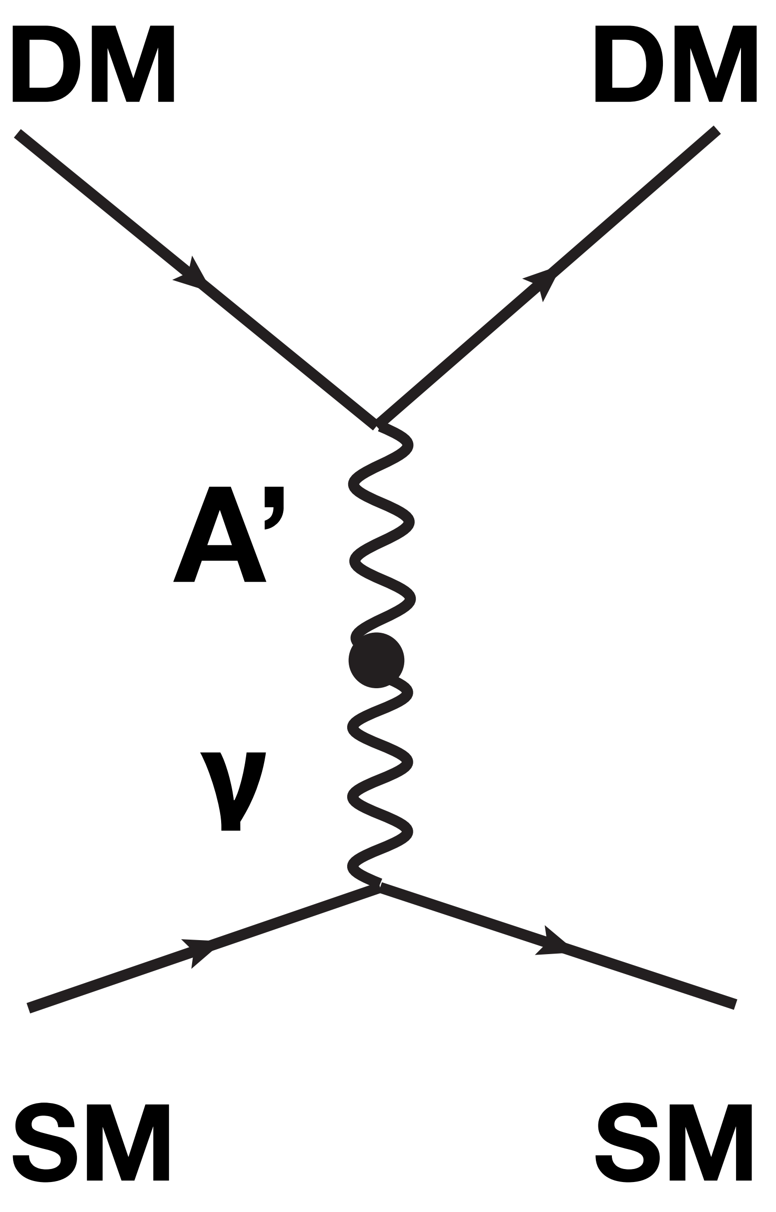
The way-out from direct detection limits

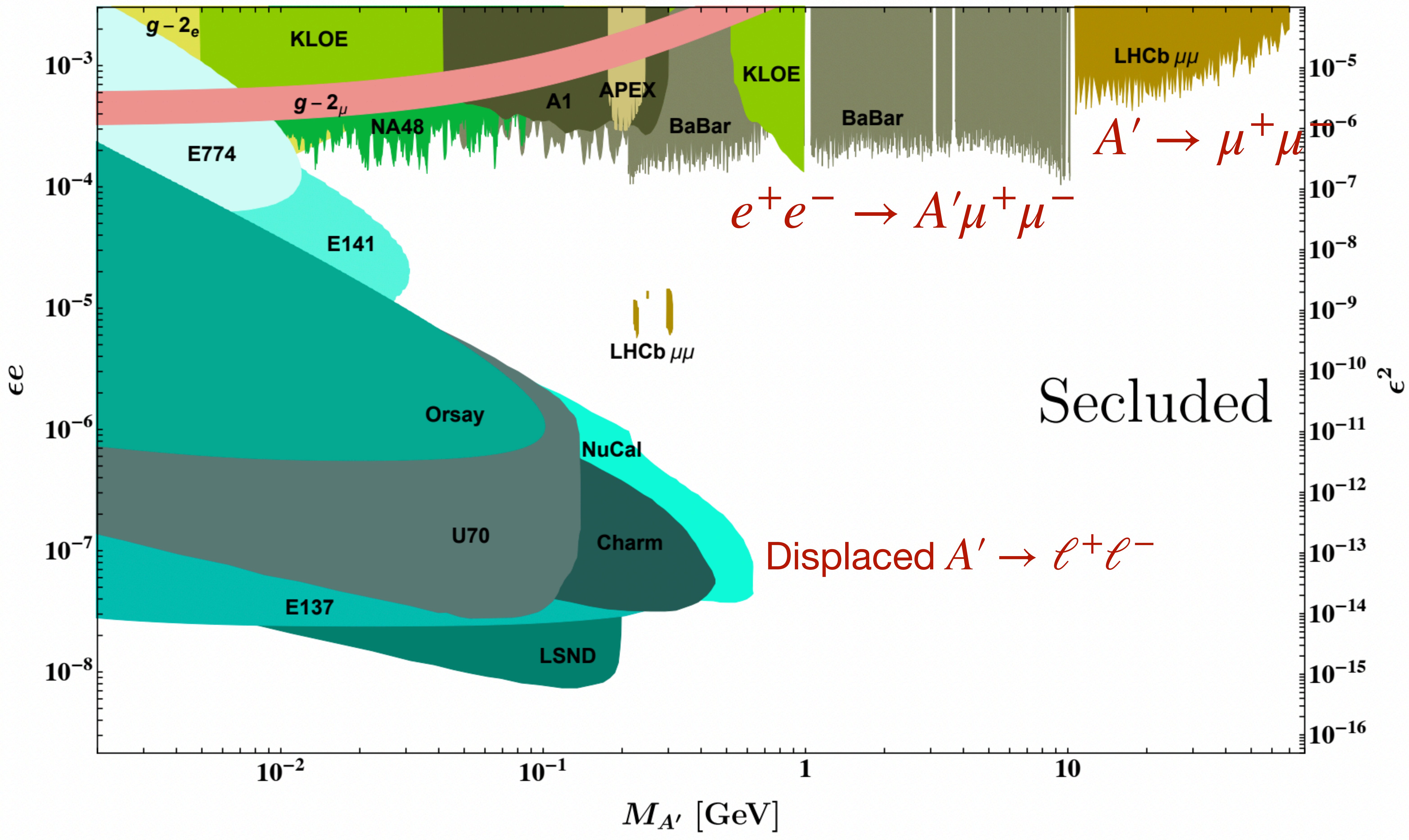
- 1. Secluded dark matter (dark sector)
  - Looking for mediator X is easier than DM

Bauer et al: 1803.05466 (JHEP)

Dark photon A' example: visible

$$\epsilon F'_{\mu\nu}B^{\mu\nu}: \ A' \to \ell^+\ell^-$$





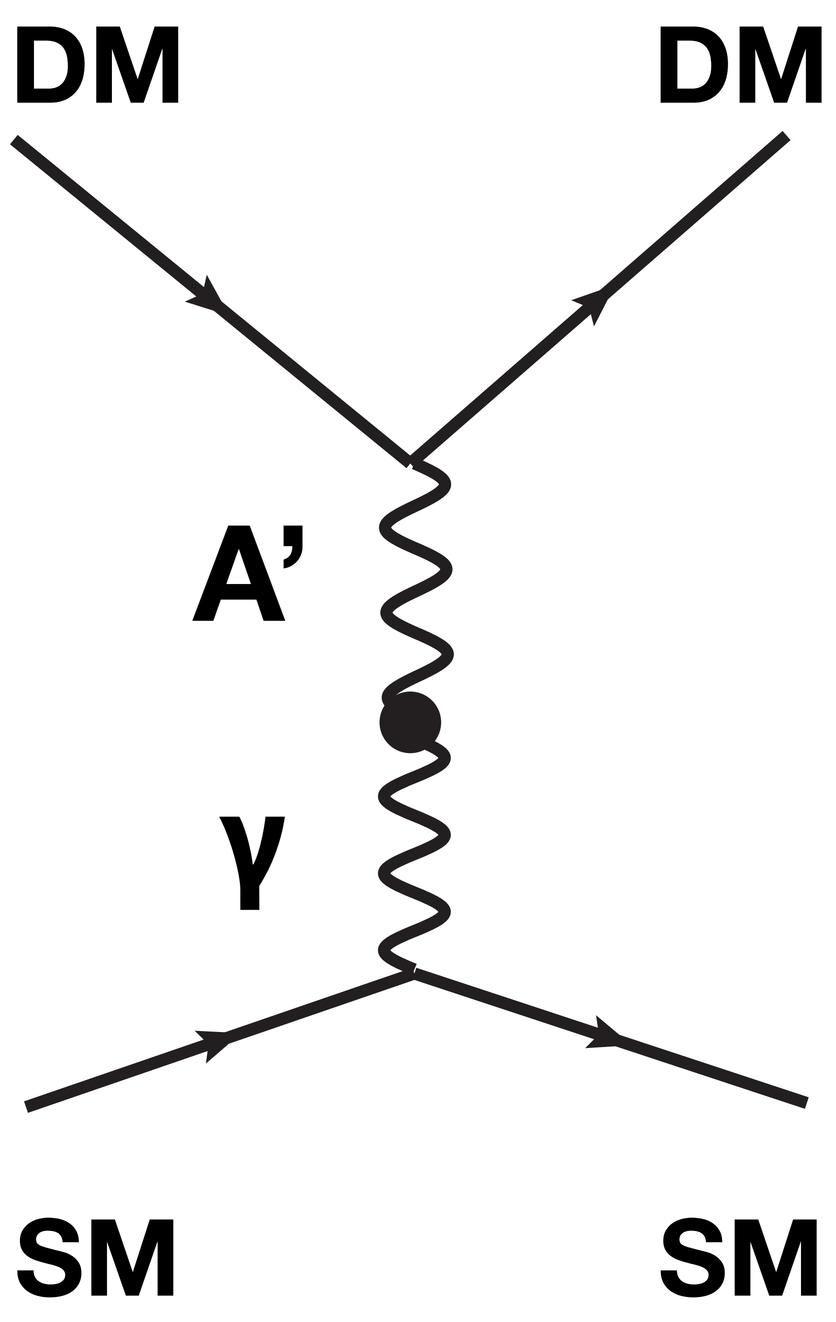
刘佳

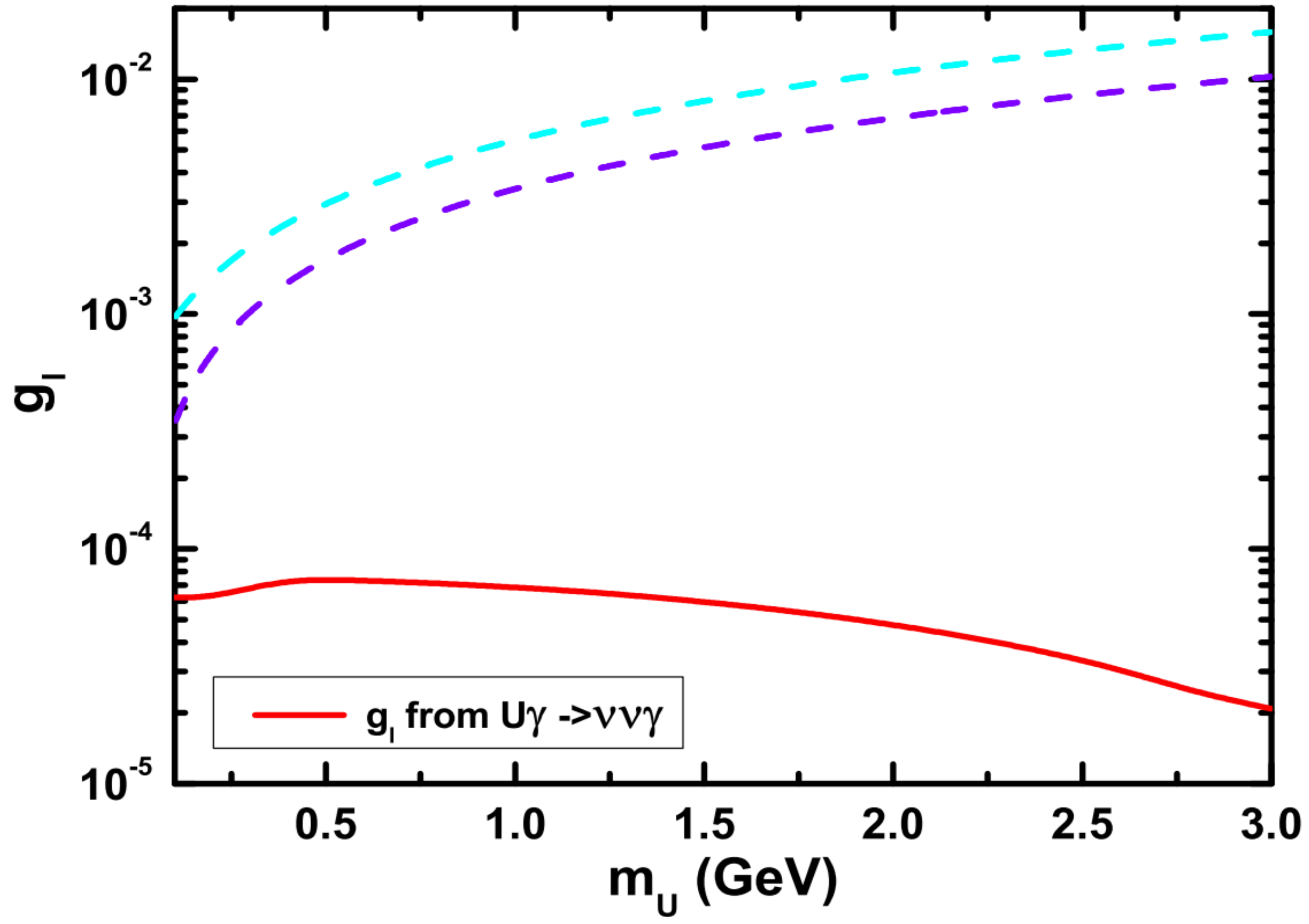
The way-out from direct detection limits

- 1. Secluded dark matter (dark sector)

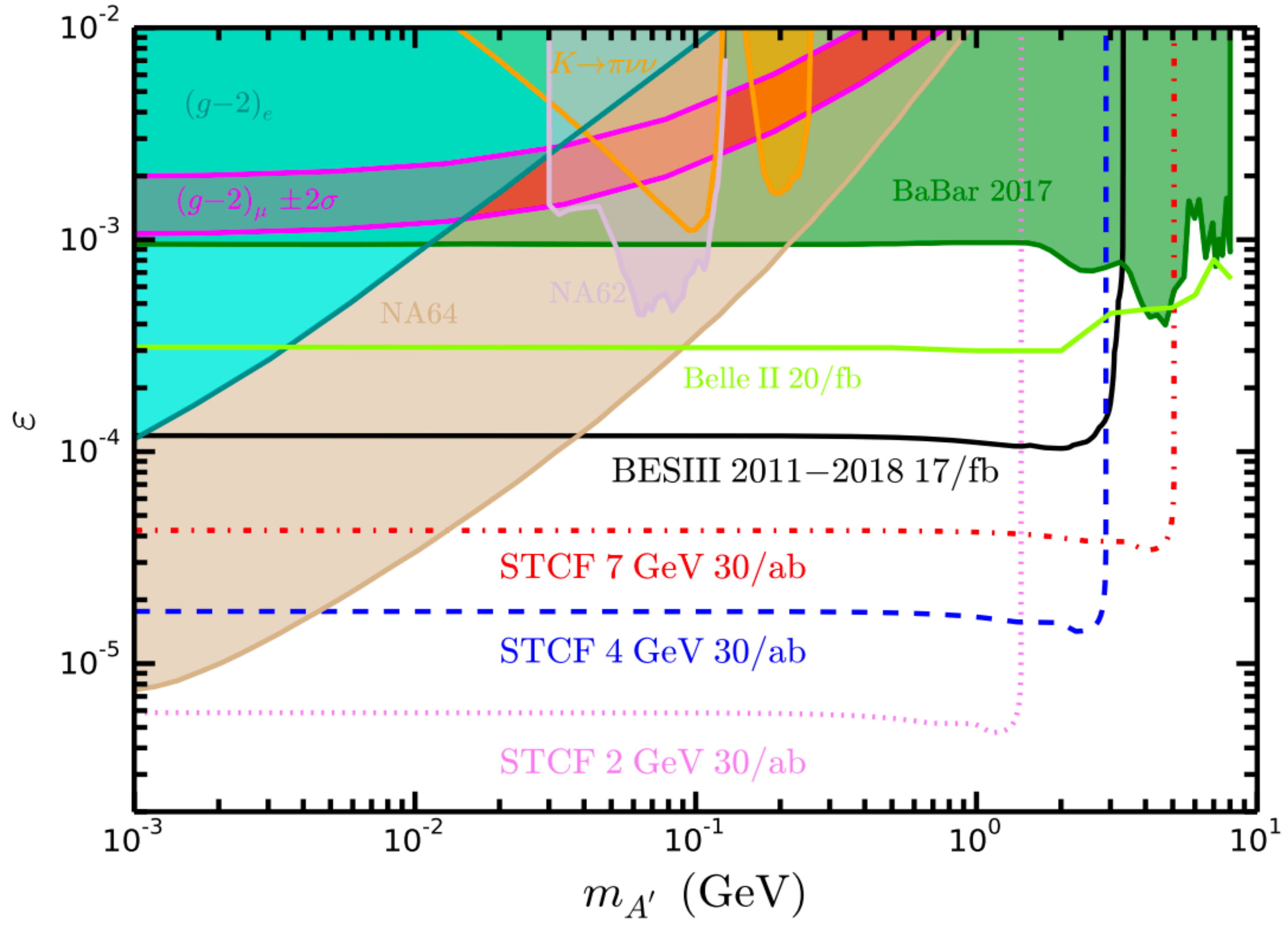
  - Looking for mediator X is easier than DM

Dark photon A' example: invisible $A' \to \mathrm{DM} + \mathrm{DM},\ \bar{\nu}\nu$





PF Yin, JL, SH Zhu: 0904.4644 (PRD)



BESIII: 1907.07046 (PRD)

The way-out from direct detection limits

- 2. Suppressed scattering cross-section:
  - By velocity or momentum transfer

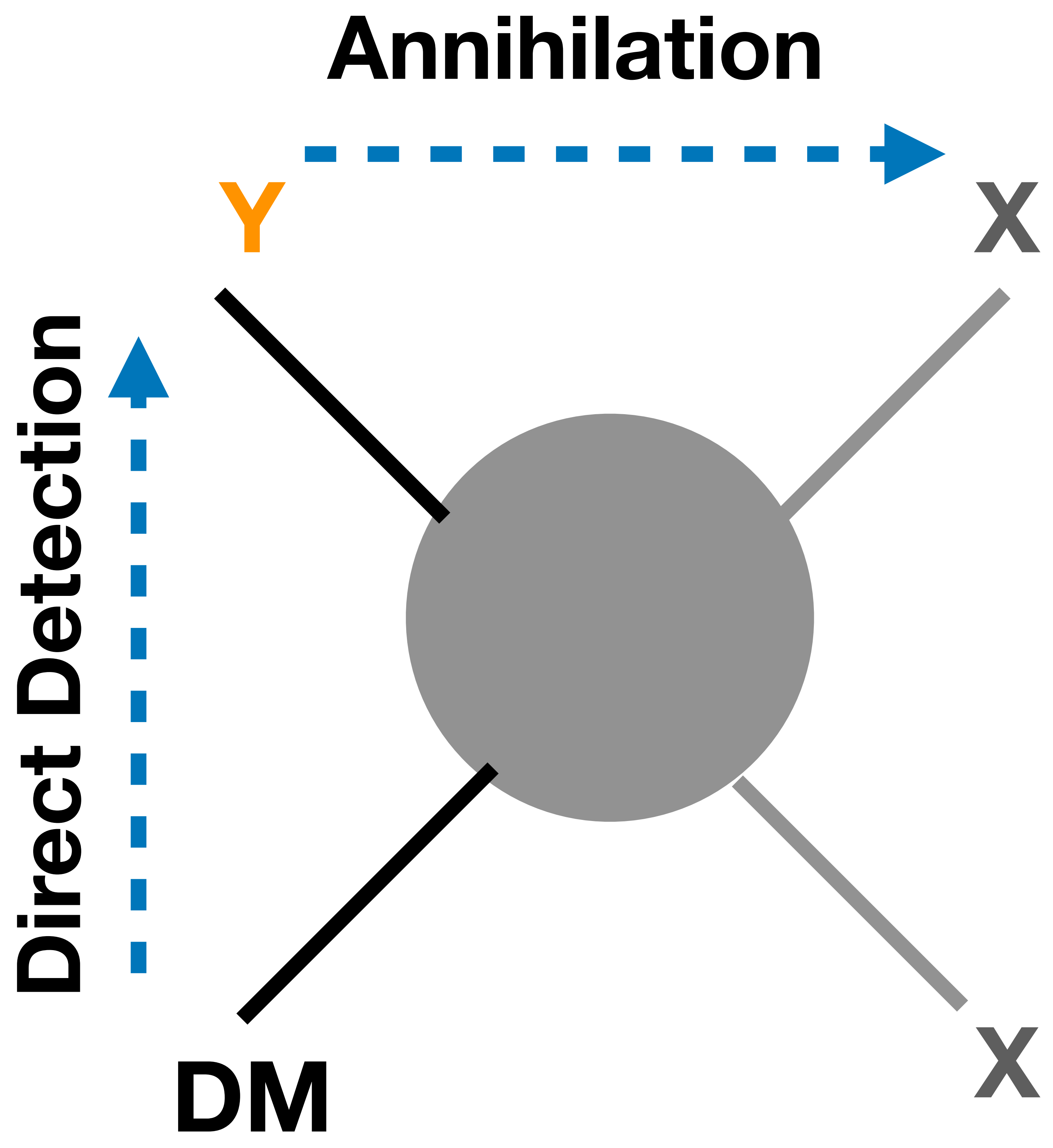
Case for Fermionic DM

Kumar & Marfatia:1305.1611 (PRD)

	Name	Interaction Structure	$\sigma_{\rm SI}$ suppression	$\sigma_{\rm SD}$ suppression	s-wave?
Scalar	F1	$\bar{X}X\bar{q}q$	1	$q^2v^{\perp 2}$ (SM)	No
	F2	$\bar{X}\gamma^5X\bar{q}q$	q^2 (DM)	$q^2v^{\perp 2}$ (SM); q^2 (DM)	Yes
	F3	$\bar{X}X\bar{q}\gamma^5q$	0	q^2 (SM)	No
Pseudoscalar	F4	$\bar{X}\gamma^5X\bar{q}\gamma^5q$	0	q^2 (SM); q^2 (DM)	Yes
Vector	F5	$\bar{X}\gamma^\mu X\bar{q}\gamma_\mu q$ (vanishes for Majorana X)	1	$q^2v^{\perp 2}$ (SM) q^2 (SM); q^2 or $v^{\perp 2}$ (DM)	Yes
Anapole	F6	$\bar{X}\gamma^\mu\gamma^5X\bar{q}\gamma_\mu q$	$v^{\perp 2}$ (SM or DM)	q^2 (SM)	No
	F7	$\bar{X}\gamma^\mu X\bar{q}\gamma_\mu\gamma^5q$ (vanishes for Majorana X)	$q^2v^{\perp 2}$ (SM); q^2 (DM)	$v^{\perp 2}$ (SM) $v^{\perp 2}$ or q^2 (DM)	Yes
	F8	$\bar{X}\gamma^\mu\gamma^5X\bar{q}\gamma_\mu\gamma^5q$	$q^2v^{\perp 2}$ (SM)	1	$\propto m_f^2/m_X^2$
	F9	$\bar{X}\sigma^{\mu\nu}X\bar{q}\sigma_{\mu\nu}q$ (vanishes for Majorana X)	q^2 (SM); q^2 or $v^{\perp 2}$ (DM) $q^2v^{\perp 2}$ (SM)	1	Yes
	F10	$\bar{X}\sigma^{\mu\nu}\gamma^5X\bar{q}\sigma_{\mu\nu}q$ (vanishes for Majorana X)	q^2 (SM)	$v^{\perp 2}$ (SM) q^2 or $v^{\perp 2}$ (DM)	Yes

The way-out from direct detection limits

- 3. Coannihilation mechanism



$m_{\mathrm{DM}} \lesssim m_Y$

- Y has a close mass with DM
  - Y is not populated today due to decay
  - Charged Y: near degenerate spectrum of SUSY, AMSB
  - Neutral Y: Inelastic Dark Matter

Fermionic DM with kinetic mixing A' mediator

$$\mathscr{L} = \bar{\psi} i \gamma_\mu D^\mu \psi + m \bar{\psi}\psi + \delta \overline{\psi^c} \psi / 2$$

$$\overline{\psi}\gamma_\mu\psi \simeq i(\overline{\chi}_1\overline{\sigma}_\mu\chi_2 - \overline{\chi}_2\overline{\sigma}_\mu\chi_1) + \frac{\delta}{2m}(\overline{\chi}_2\overline{\sigma}_\mu\chi_2 - \overline{\chi}_1\overline{\sigma}_\mu\chi_1).$$

$$m_{\chi_1} = m - \delta;\ \ m_{\chi_2} = m + \delta$$

Smith, Weiner: hep-ph/0101138 (PRD)

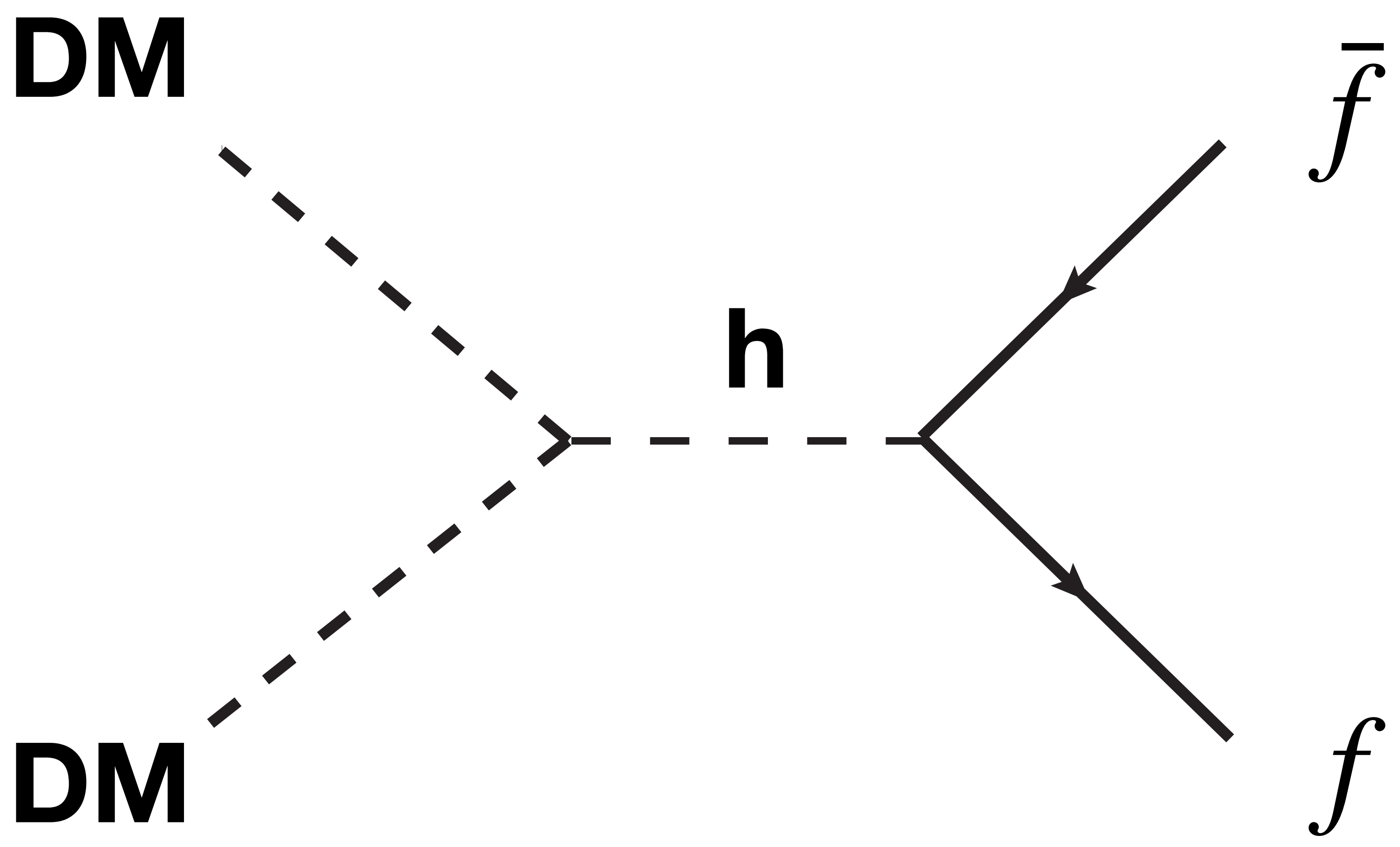
The way-out from direct detection limits

- 4. Resonant annihilation

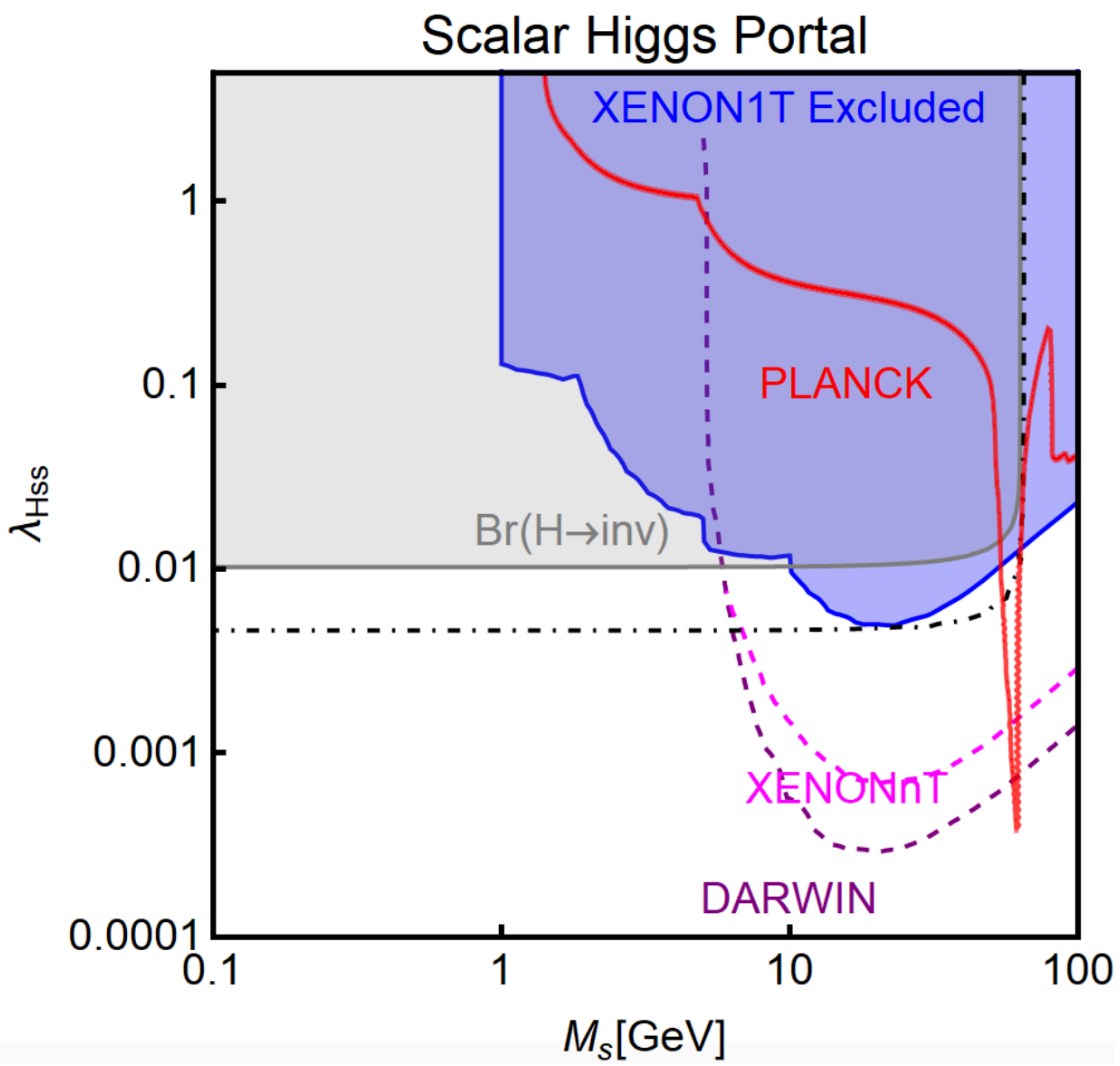
  - $2m_{\rm DM} \approx m_X$

Scalar DM (s) with a Higgs portal coupling

$$\Delta\mathcal{L}_s = -\frac{1}{2}m_s^2 s^2 - \frac{1}{4}\lambda_s s^4 - \frac{1}{4}\lambda_{Hss}\phi^\dagger\phi s^2$$



+ 2 diagrams to hh



Arcadi et al: 2101.02507

See also WL Guo, LY Wu et al 2010; B Li, YF Zhou 2015

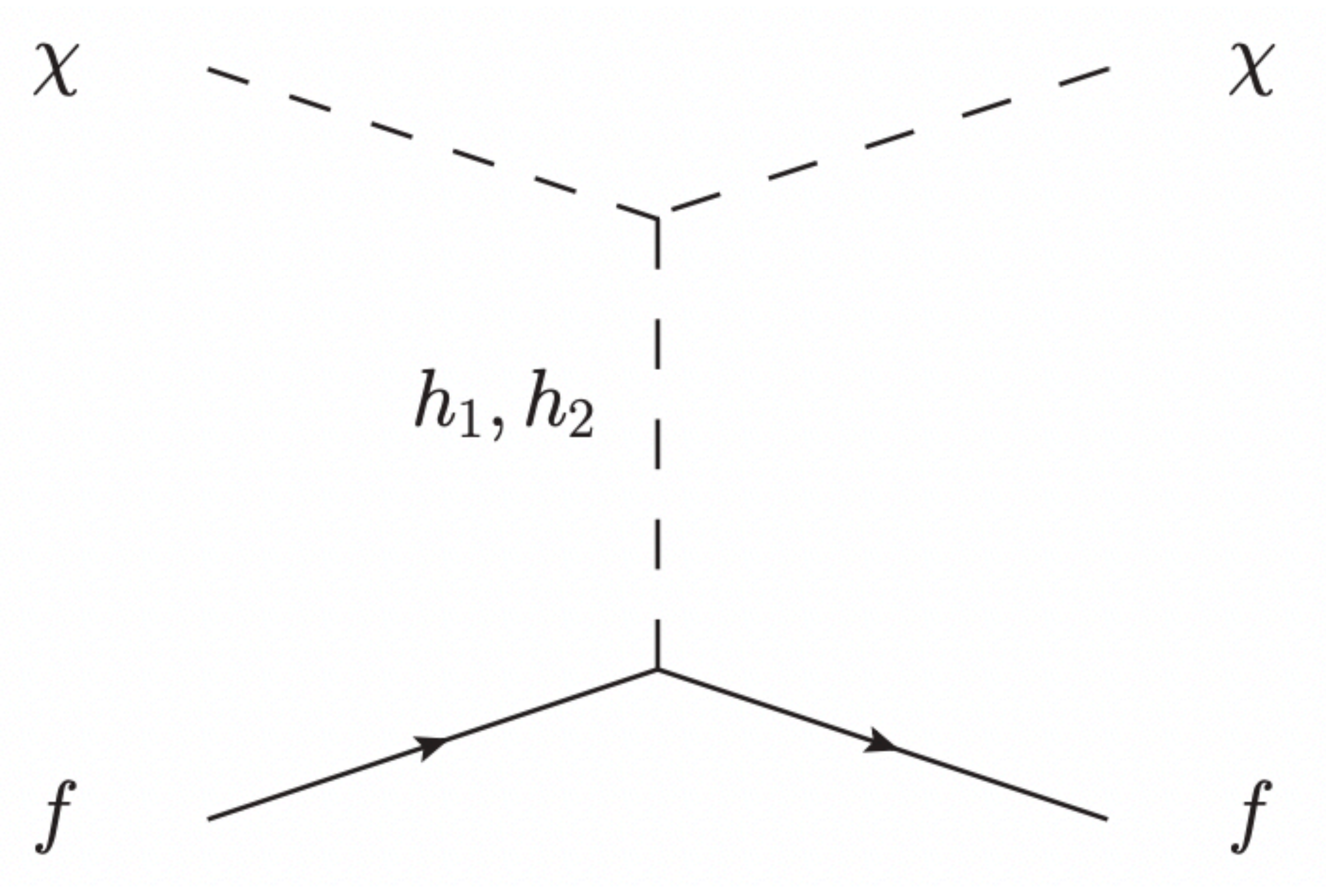
刘佳

The way-out from direct detection limits

- 5. Cancellation effect in scattering cross-section

  - SM Higgs - Dark scalar mediator cancellation

Gross, Lebedev, Toma: 1708.02253 (PRL)



$$V_0 = -\frac{\mu_H^2}{2}|H|^2 - \frac{\mu_S^2}{2}|S|^2 + \frac{\lambda_H}{2}|H|^4 + \lambda_{HS}|H|^2|S|^2 + \frac{\lambda_S}{2}|S|^4$$

$$V_{\rm soft} = -\frac{\mu_S'^2}{4} S^2 + \text{h.c.} \qquad \text{symmetry} : S \leftrightarrow S^*$$

$$S = (v_s + s + i\chi)/\sqrt{2}$$ Pseudoscalar DM

CP-even scalar mixing (s, h) $\rightarrow (h_1,\ h_2)$

$$\mathcal{L} \supset -(h_1 \cos\theta + h_2 \sin\theta) \sum_f \frac{m_f}{v} \bar{f} f \qquad \mathscr{L} \supset \frac{\chi^2}{2v_s}\left(m_{h_1}^2 \sin\theta h_1 - m_{h_2}^2 \cos\theta h_2\right)$$

$$\mathcal{A}_{dd}(t) \propto \sin\theta\cos\theta\left(\frac{m_{h_2}^2}{t - m_{h_2}^2} - \frac{m_{h_1}^2}{t - m_{h_1}^2}\right) \simeq \sin\theta\cos\theta\,\frac{t\,(m_{h_2}^2 - m_{h_1}^2)}{m_{h_1}^2\, m_{h_2}^2} \simeq 0$$

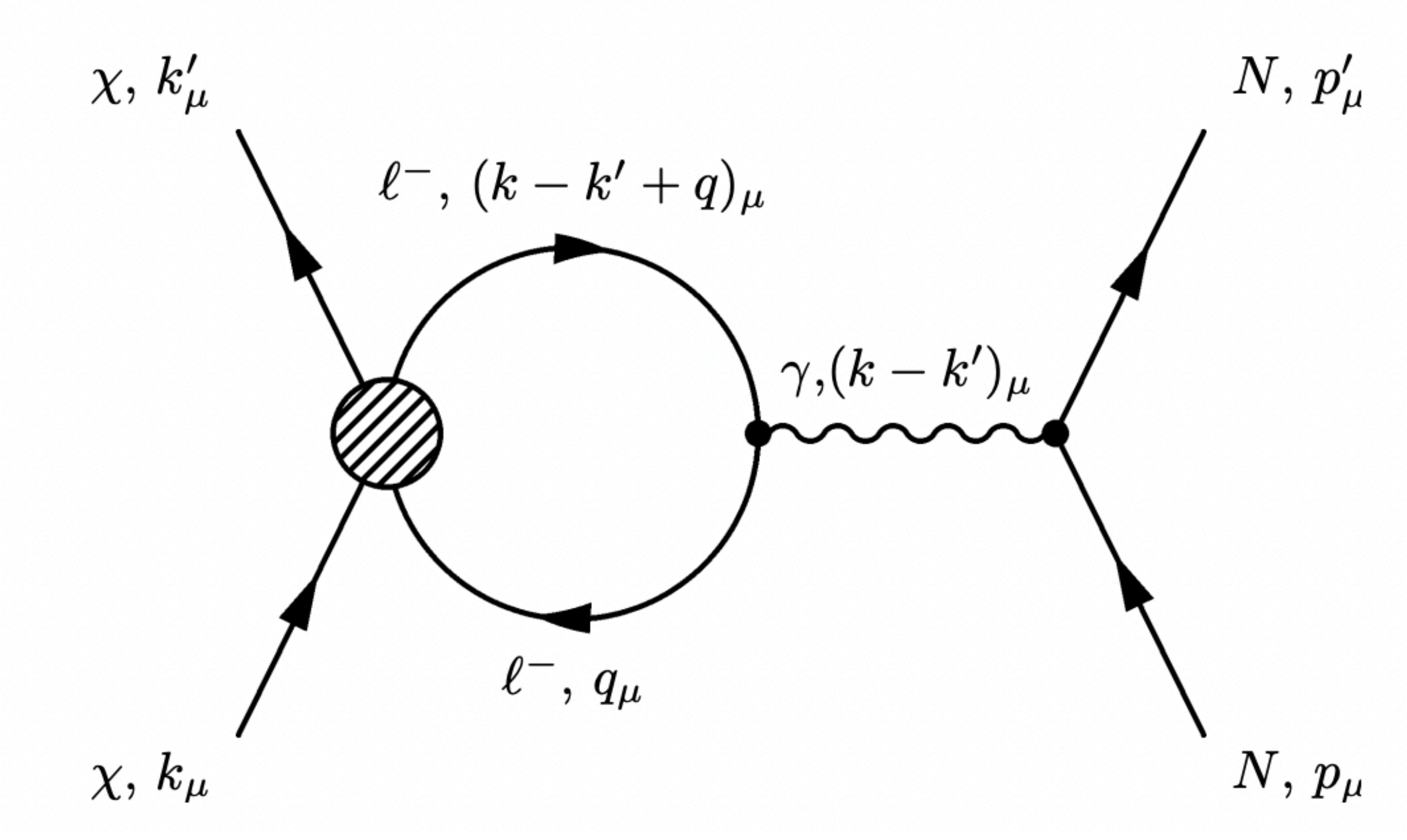
See JL, XP Wang and F Yu 1704.00730 (JHEP), for cancellation between A' - Z boson in kinetic mixing dark photon model

The amplitude is suppressed by q^2 from pseudo-goldstone nature

See an extension from Honghao Zhang et al, 2109.11499

The way-out from direct detection limits

- 6. Leptophilic models
  - Only couples to electrons, couples to nucleons at 1-loop
    - For light DM, e-DM recoils can have stringent limits (e.g. XENON1T, PANDAX, CDEX, LZ)
    - For heavy DM, neucleus-DM recoils wins over e-DM recoil



$$R^{\rm WAS} : R^{\rm WES} : R^{\rm WNS} \sim \epsilon_{\rm WAS} : \epsilon_{\rm WES}\,\frac{m_e}{m_N} : \left(\frac{\alpha_{\rm em} Z}{\pi}\right)^2 \sim 10^{-17} : 10^{-10} : 1$$

WAS = e kicked out

WES = e to higher energy level

WNS = nucleus recoil

The probability to find a high p electron in the wave function is highly suppressed!

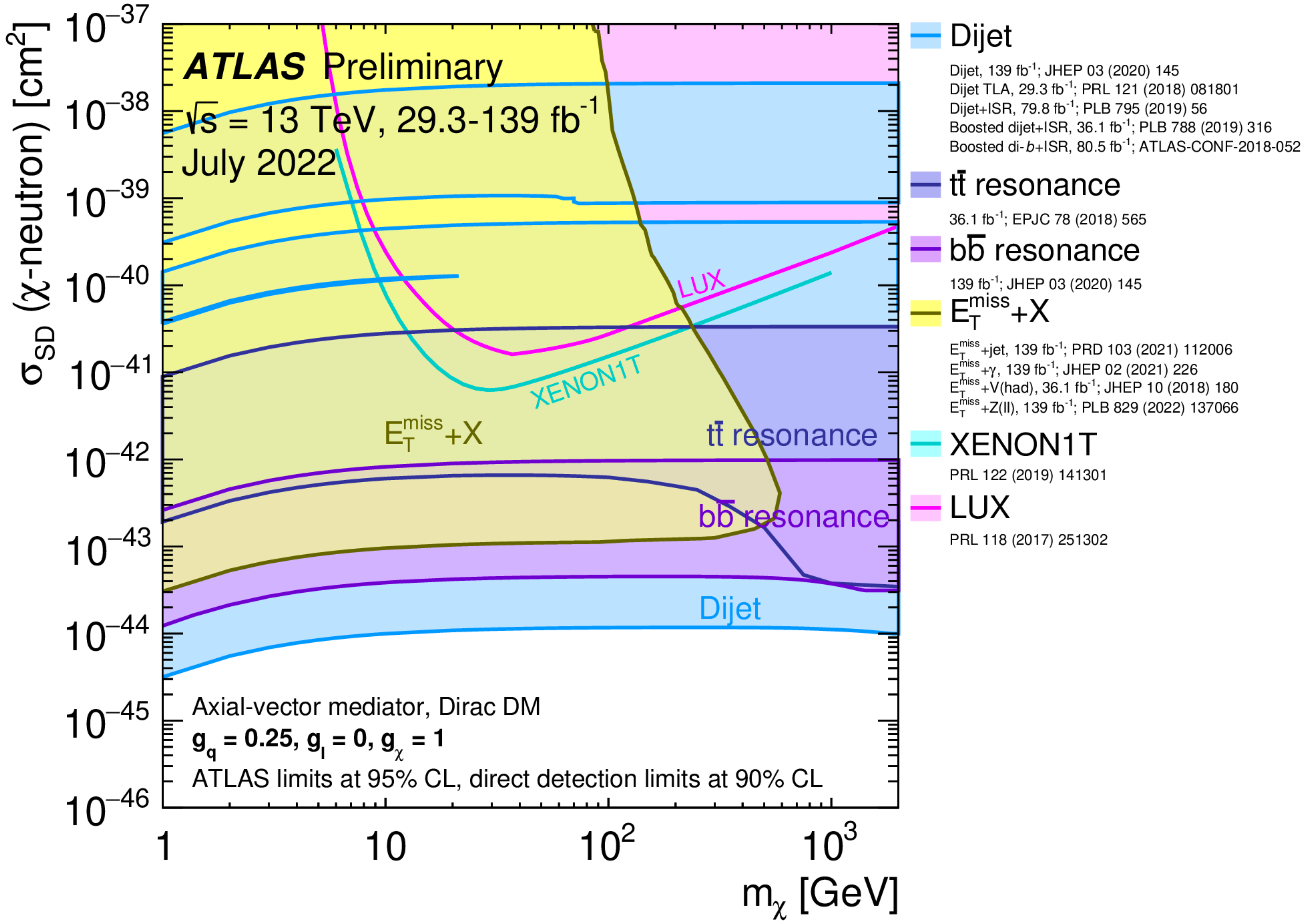
Kopp et al: 0907.3159 (PRD)

Outlines

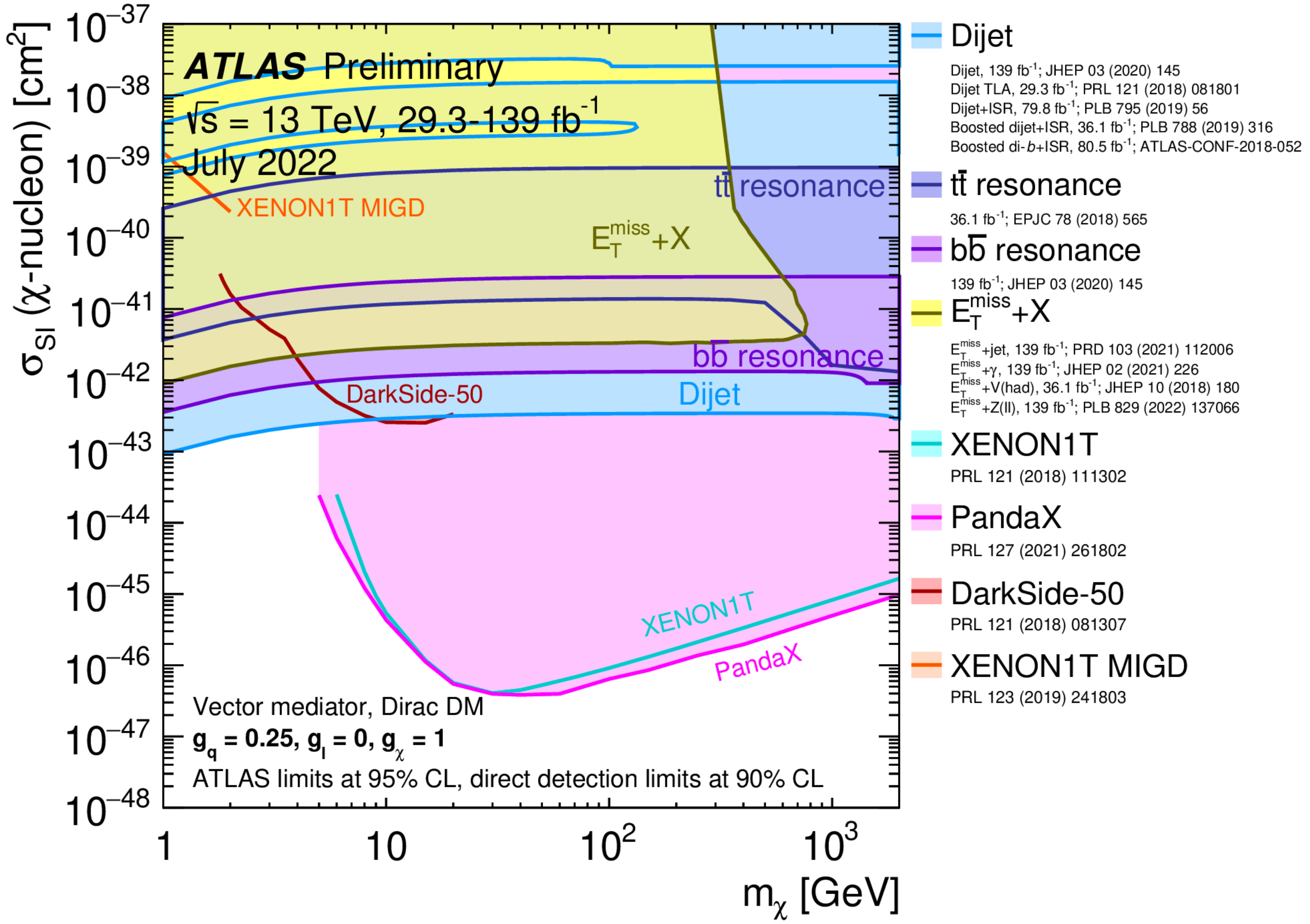
- 暗物质的起源
  - 暗物质的天文观测证据
  - 宇宙学标准模型 ΛCDM
  - 暗物质分布
- 暗物质的物理模型
  - 可能的暗物质候选者
  - WIMP暗物质
- 热退耦暗物质残余丰度计算
- WIMP暗物质现状
  - WIMP暗物质的直接探测危机
    - 解决危机的多种办法
  - 暗物质对撞机探测的互补性
  - 暗物质的间接探测限制
    - 避开限制的办法
- 总结

The complementarity between direct detection and collider searches

- Collider searches
  - Not suppressed by small velocity or small momentum transfer
  - Not suppressed by small dark matter mass



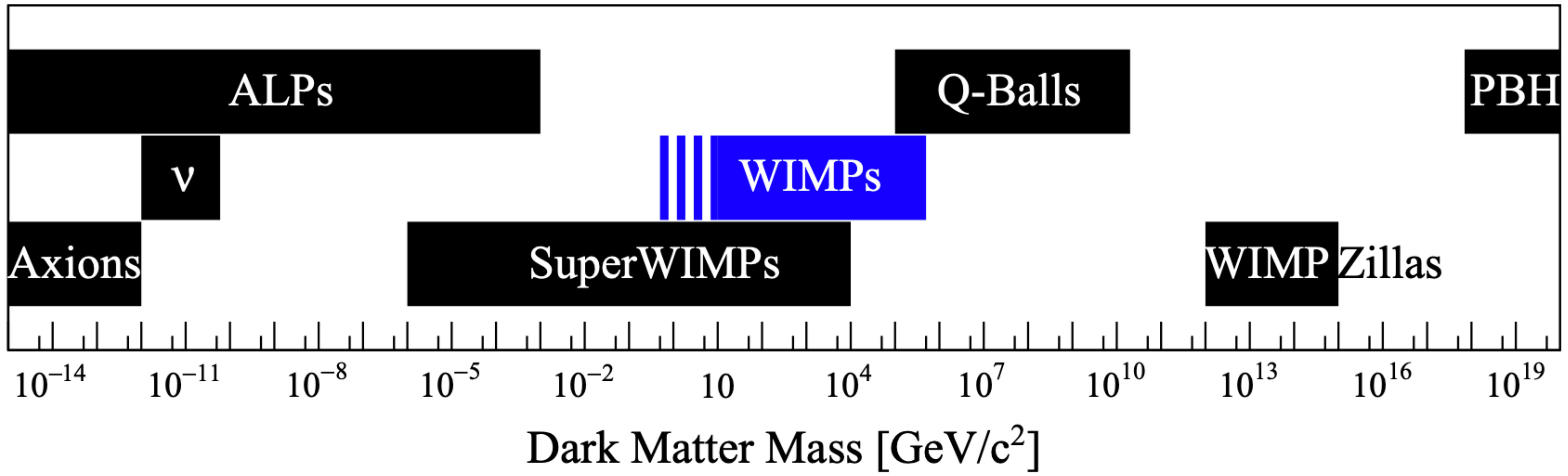
- Future: Collider + Direct detection searches
  - 15 years data from LHC
  - All the way down to neutrino floor



Outlines

- 暗物质的起源
  - 暗物质的天文观测证据
  - 宇宙学标准模型 ΛCDM
  - 暗物质分布
- 暗物质的物理模型
  - 可能的暗物质候选者
  - WIMP暗物质
- 热退耦暗物质残余丰度计算
- WIMP暗物质现状
  - WIMP暗物质的直接探测危机
    - 解决危机的多种办法
  - 暗物质对撞机探测的互补性
  - 暗物质的间接探测限制
    - 避开限制的办法
- 总结

The indirect detection limits from DM annihilation



- DM starts with thermal distribution
- DM has electroweak-scale coupling
- Relic abundance is determined by freeze-out mechanism
- DM Annihilation into
  - X = Standard Model particles (direct coupling)
  - X = Dark Sector particles (secluded DM models)

} The entropy of DM goes into SM sector most of the time!

(Secluded X → SM + SM)

Lower mass bound for thermal DM

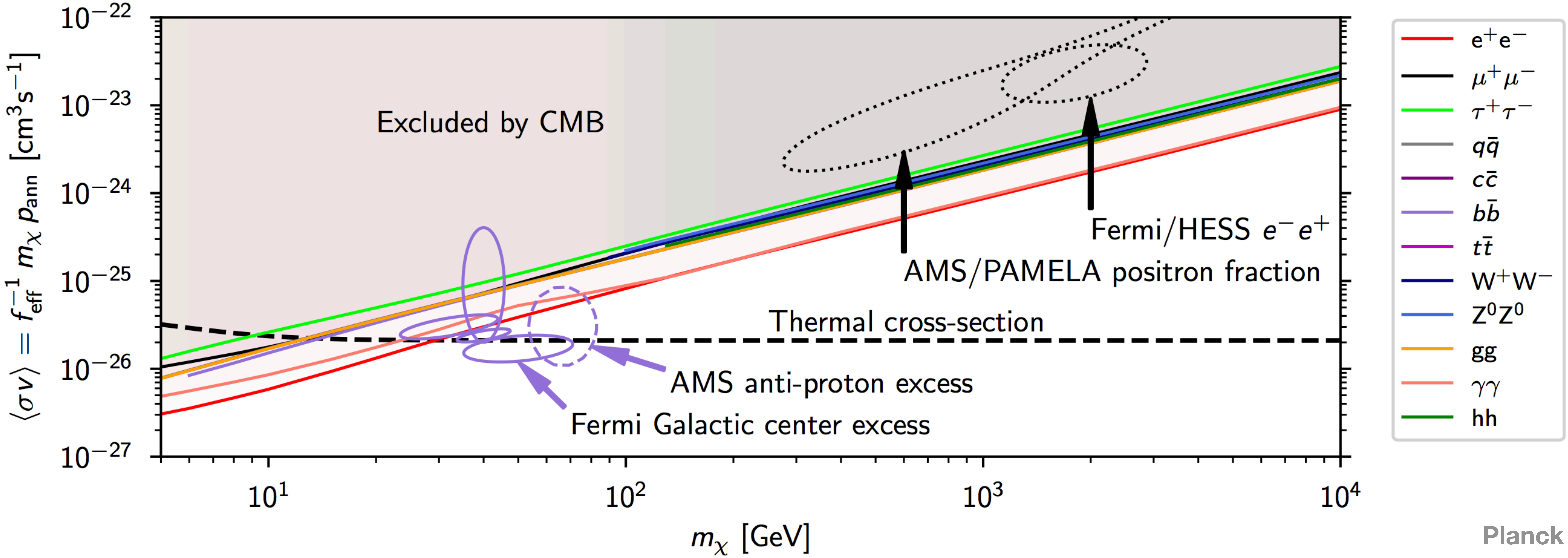
- Lower bound from N_{eff} at CMB
  - Light DM freeze-out after neutrino decoupling at $T_D \approx 2.3$ MeV
  - Normally $T_{fo} \sim m_{\rm DM}/20$
  - DM entropy goes into neutrinos or e/γ, will modify T_ν/T_γ
  - DM mass $\gtrsim 5$ MeV, depending on d.o.f.

Annihilation constraints from CMB

- The annihilation: $\mathrm{DM} + \mathrm{DM} \rightarrow \mathrm{SM} + \mathrm{SM}$
- The rate DM energy density converted into EM energy

$$\frac{d\rho_{\mathrm{DM}}}{dt} = m_{\mathrm{DM}} n_{\mathrm{DM}}^2 \langle \sigma v \rangle \times f_{\mathrm{eff}}$$

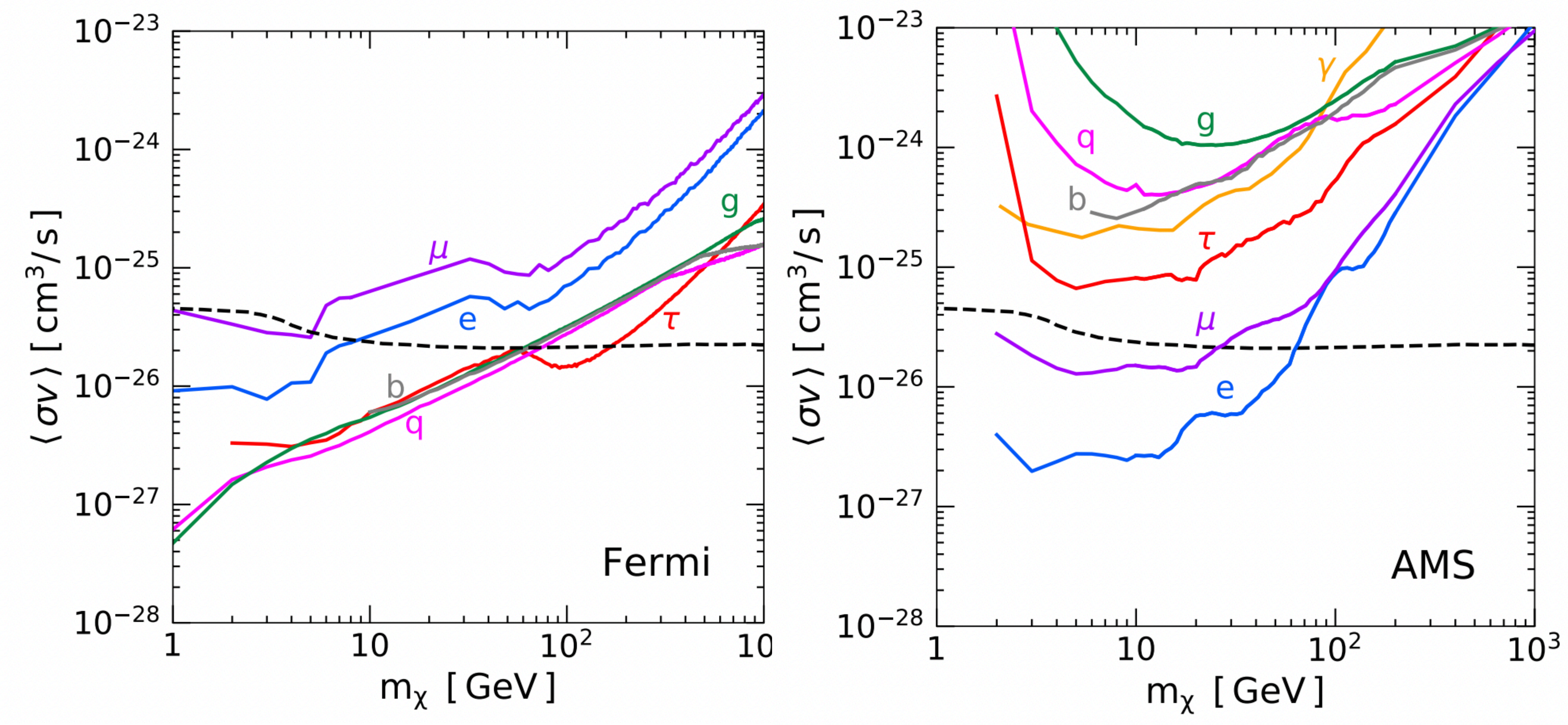
- f_{eff} : the efficiency with which the energy released in DM annihilation is absorbed by the primordial plasma



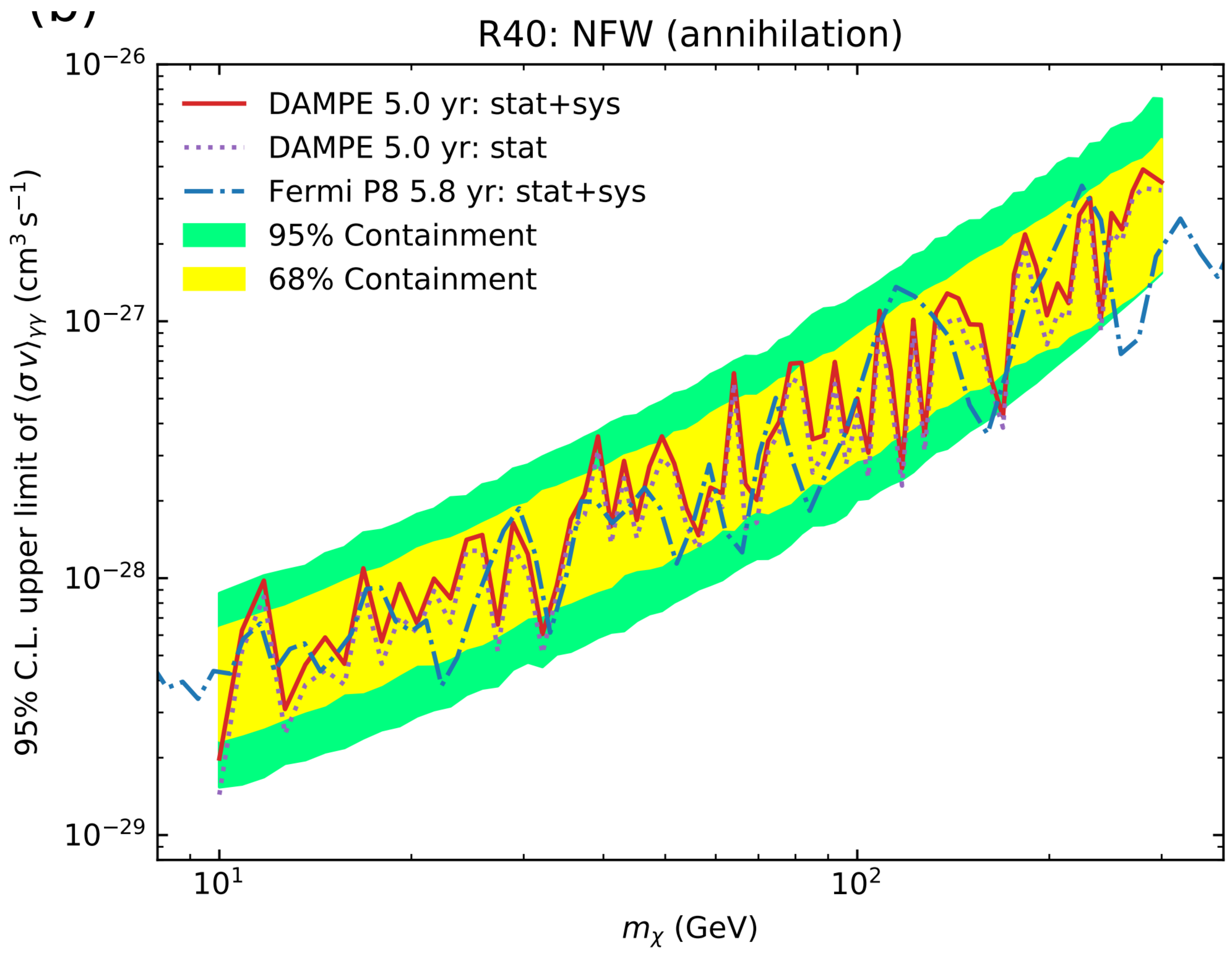
Planck 2018

卫星间接探测实验限制

- CMB limits only works for DM mass $\lesssim 10$ GeV
- Indirect limits from AMS-02, DAMPE(悟空卫星), Fermi-LAT



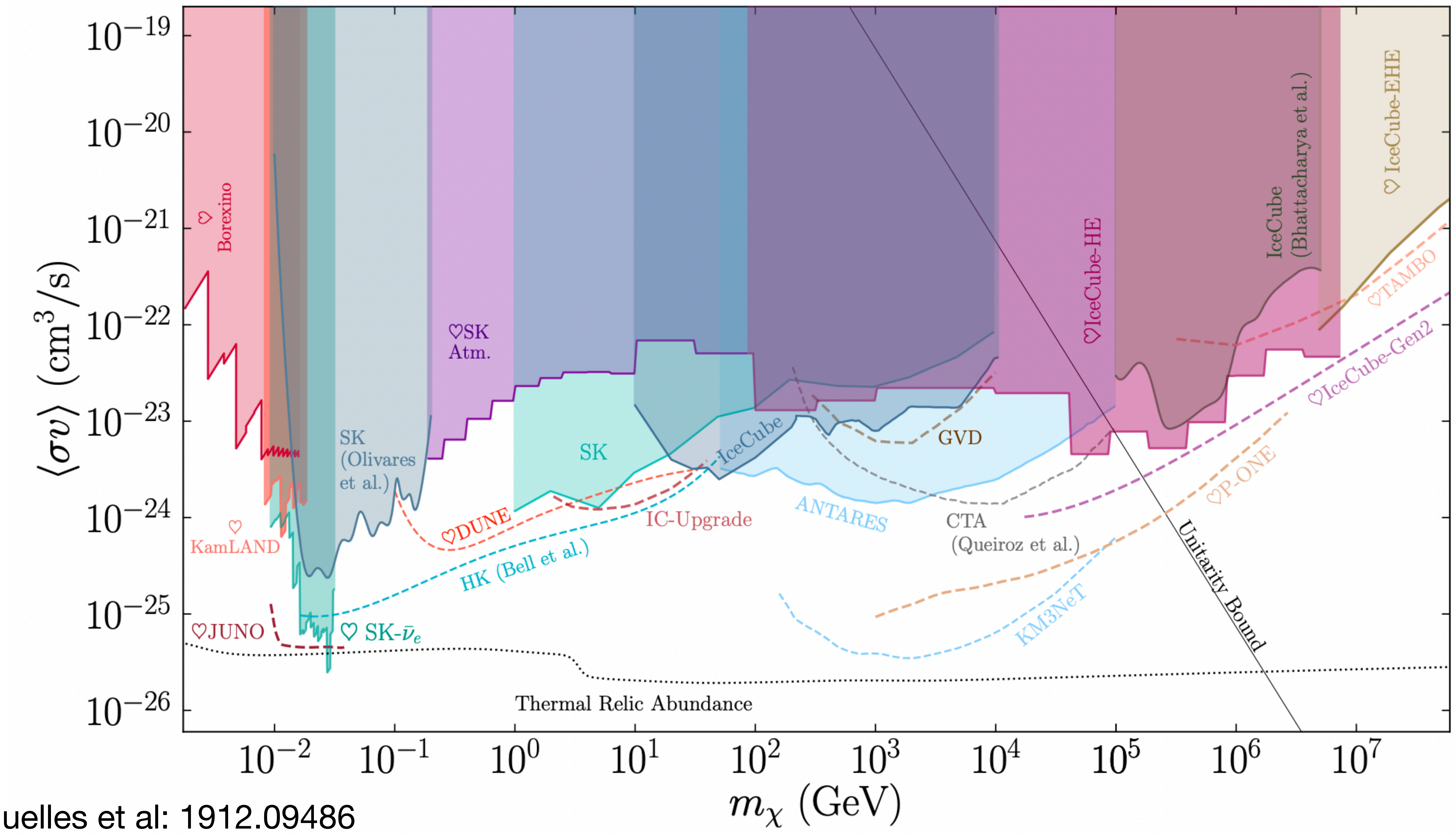
Leane et al: 1805.10305 (PRD)



悟空卫星: 2112.08860 (Science Bulletin)

How to escape CMB constraints?

- Annihilation to neutrinos ($2\mathrm{DM} \to \bar{\nu}\nu$): $f_{\mathrm{eff}} = 0$



Arguelles et al: 1912.09486

How to escape CMB constraints?

- P-wave annihilation or no annihilation (asymmetric DM) but no indirect detection signal

- Expansion over velocity
  - S-wave
  - P-wave (L=1)
  - D-wave (L=2), due to extra chiral suppression
  - Linear v dependence?
    - Final state phase space suppression ($m_{\rm DM} \approx m_X$) from symmetry reason

$$\sigma v \sim \sigma_s + \sigma_p v^2 + \sigma_d v^4 + \ldots$$

- The value of velocities at different time
  - Freeze-out: $v^2 \sim 0.25$
  - CMB: $v^2 \sim {\rm eV}/m_{\rm DM} \sim 10^{-5}$
  - Today: $v \sim 10^{-3} c$

J Kopp, JL, T Slatyer, XP Wang, W Xue: 1609.02147 (JHEP)

暗物质总结

- Darkness in the Universe (Credit DMUK)
  - https://youtu.be/xcZjpAaZREo

总结

- 暗物质是一种不与可见物质相互作用的有质量的未知物质
  - 大量的天文证据以及ΛCDM的重要拼图
  - 暗物质存在各种候选者
    - 超轻波动型暗物质是以经典场形式存在的低质量玻色子暗物质
      - 粒子类型、物理动机、探测方式、多学科交叉
    - WIMP暗物质是电弱能标的具有极强竞争力的模型
      - 暗物质的三种探测方法
    - 热退耦暗物质的残余丰度计算
- WIMP暗物质现状
  - 直接探测限制使得人们非常焦虑，开始思考其它的暗物质模型
    - （模型依赖）多种方式避免直接探测限制
  - 对撞机探测+直接探测互补性强：LHC未来15年数据+搜寻至中微子地板
  - 间接探测限制，需要暗物质质量 $\gtrsim \mathcal{O}(10)$ GeV
    - 低质量暗物质避免限制：p波湮灭截面、湮灭产物不参与电磁相互作用
  - WIMP暗物质依然存活
- 暗物质问题是当前天文学和粒子物理学科的最重要问题之一

谢谢！

Backup slides

热暗物质的问题

- 热暗物质在极端相对论的情况下热退耦

$$n_\chi(T=\mathrm{MeV}) = \frac{135\,\zeta(3)g}{8\pi^4 g_{*,S}(T_{\mathrm{fo}})} s(T=\mathrm{MeV}^+)$$

$$s(T=\mathrm{MeV}^+) = (2+4\times 7/8+6\times 7/8)\frac{2\pi^2}{45}T^3 = \frac{43}{4}\frac{2\pi^2}{45}T^3$$

- 热暗物质的Yield

$$Y_\chi = \frac{n_\chi(T=\mathrm{MeV})}{s_\gamma(T=\mathrm{MeV}^-)} = \frac{135\,\zeta(3)g}{8\pi^4 g_{*,S}(T_{\mathrm{fo}})}\frac{43}{22}$$

$$\Omega_\chi h^2 \simeq 0.12\times\frac{g}{g_{*,S}(T_{\mathrm{fo}})}\left(\frac{m_\chi}{2\,\mathrm{eV}}\right)$$