低能强子部分对g-2的贡献

戴凌云
Hunan University

Based on: PRD88 (2013) 056001; PLB736(2014)11;
PRD90 (2014) 036004; PRD94 (2016) 116061;
PRD95 (2017) 056007; PRD97 (2018) 036012;
JHEP03(2021)092, RPP84(2021)076201, *et.al.*

粒子与核理论冬季学校
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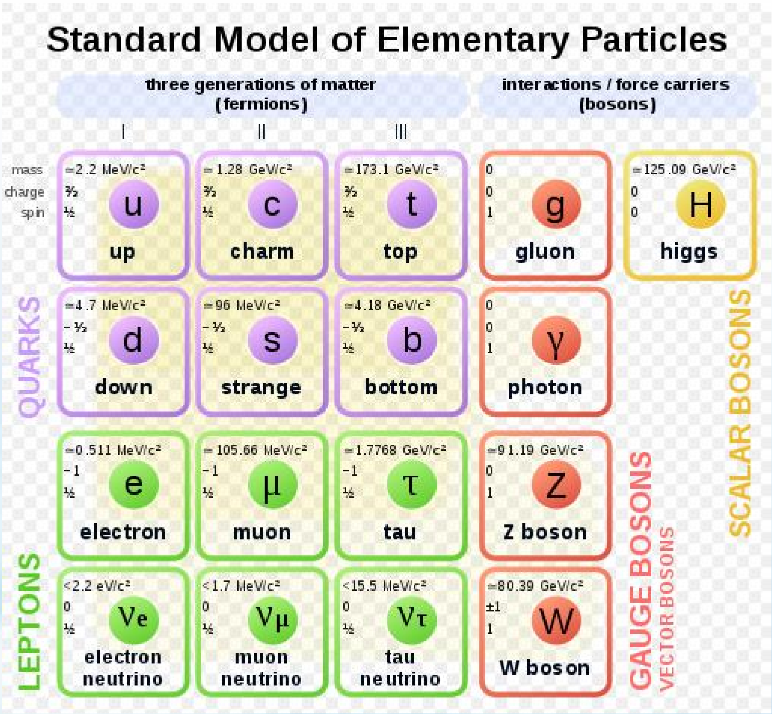
湖南大学
HUNAN UNIVERSITY

Outlines

1. Introduction
2. HVP
3. HLBL
4. Summary

Introduction: muon

- Muon: The heavier cousin of the electron. Supposed to be elementary.



$$\tau = 2.2 * 10^{-6};$$

$$v = \sqrt{1 - \left(\frac{m\mu}{3.097}\right)^2}$$

$$t\mu = 2.2 * 10^{-6} * \frac{3.097}{m\mu}$$

$$L = v * t\mu * c$$

$$L / (14 * \pi)$$

0.999417

0.0000644598

19313.3

439.115

- Why muon? life time is long: 2.2 μs, τ---$2.9 \times 10^{-7} \mu s$

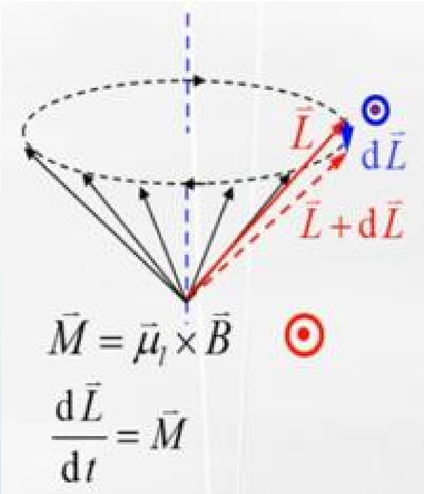
439 rounds in Fermi's ring !

Introduction: g factor

- Lande factor: relation between magnetic moment and angular momentum, Alfred Lande, anomalous Zeeman effect, 1921

$$g = 1 + \frac{J(J+1) - L(L+1) + S(S+1)}{2J(J+1)}$$

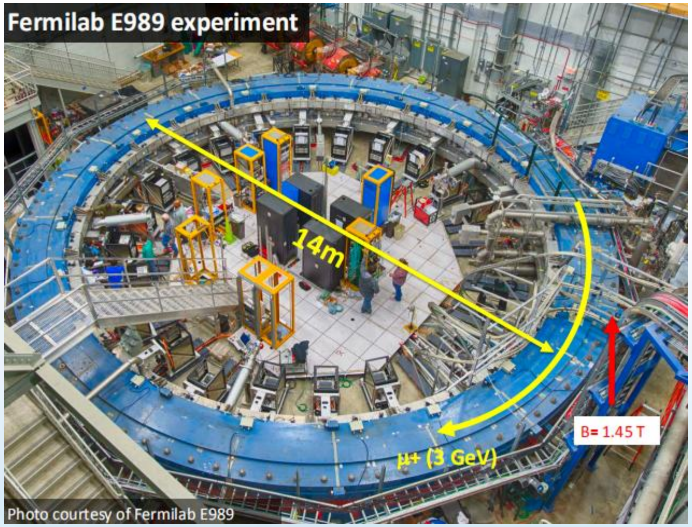
$$\mu_J = g\frac{e}{2m}\vec{J}$$

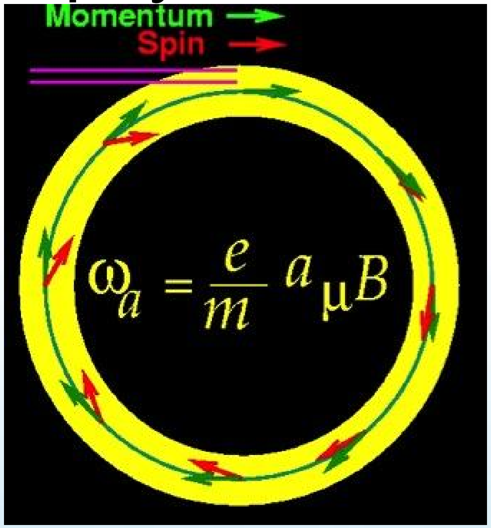


- Elementary particle: g is close to 2.
  - Electron: g=2.00231930436152(56) [PDG2022], it is close to $g = 2[1 + \frac{\alpha}{4\pi} + O(\alpha^2)]$
- Composite particle: g=5.6 for proton and g=-3.8 for neutron.

Introduction: muon g-2

- The most precise indicator of new physics





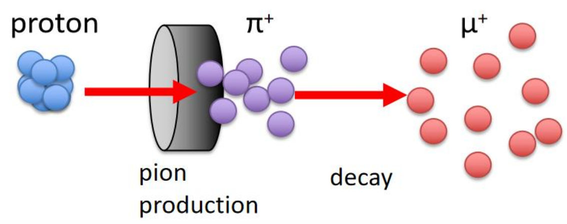
- muon spin precession $\omega_a = \frac{e}{m_\mu} a_\mu B$
- proton spin precession $\omega_p = \mu_p B$
- muon magnetic moment $\mu_\mu = g \frac{e}{2m_\mu} = (1 + a_\mu) \frac{e}{m_\mu}$

$$\vec{\mu}_S = g \frac{q}{2m} \vec{S} \quad a = \frac{g-2}{2}$$

$$a_\mu = \frac{\omega_a/\omega_p}{\omega_a/\omega_p - \mu_\mu/\mu_p}$$

Tsutomu Mibe, talk at g-2 Theory Initiative

Introduction: muon



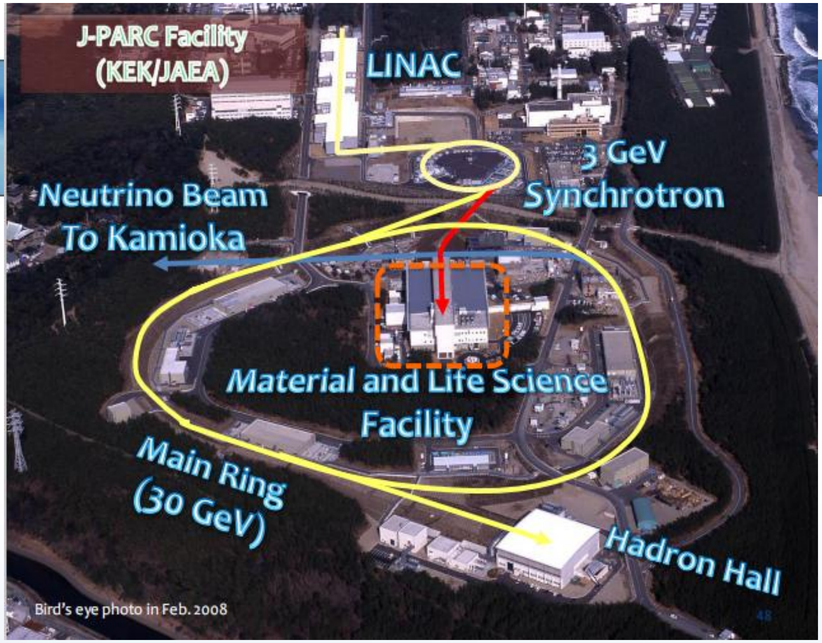
- Measurements:
- $\vec{\omega}_a = \vec{\omega}_\mu - \vec{\omega}_C$

E=0 at any γ, J-PARC approach

$$= -\frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c} + \frac{\eta}{2}\left(\vec{\beta}\times\vec{B} + \frac{\vec{E}}{c}\right)\right]$$

It is vanished at $\gamma = 30$, $p_\mu = 3.094$GeV, magic momentum

- $\mu \to e + \bar{\nu}_e + \nu_\mu$, the frequency of variation of the electron's energy corresponds to the a_μ



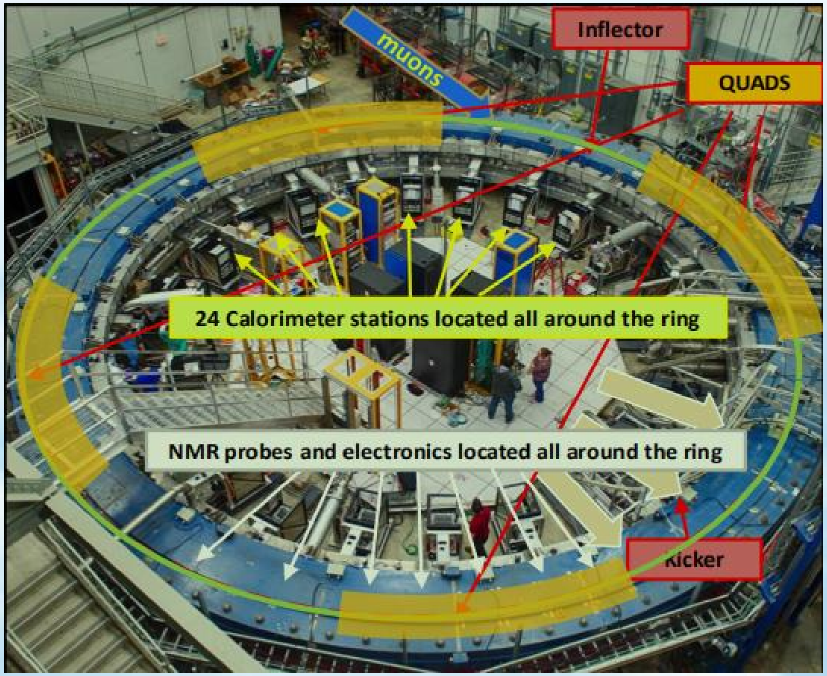
J-PARC

BNL E821 J-PARC E3

g-2: 0.46 ppm → 0.37 ppm (→0.1ppm)

50 times of number of events as large as BNL's to 0.46ppm

2001，2009，2025?



FNAL

Run1: only 6% of full statistics used now

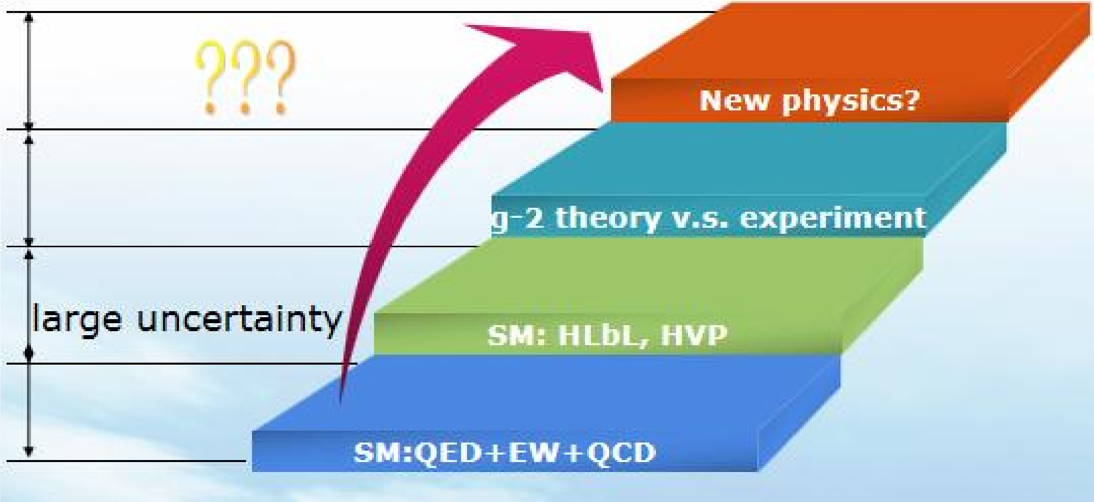
Run2-3: analyzing, factor 2 improvment

Run4: 13 times as large as BNL's

Run5: 20 times as large as BNL's

2017，2021，2025……

uncertainty from SM



$$a_\mu = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{EW}} + a_\mu^{\mathrm{QCD}}$$

- HVP, HLbL?

	values ($\times 10^{-11}$)
QED	116584718.931(104)
EW	153.6(1.0)
HVP	6845(40)
HLBL	92(18)
SM	116591810(43)
exp.(BNL)	116592089(63)
exp.(FNAL)	116592040(54)
exp.(avg.)	116592061(41)
a_μ^{SM}-a_μ^{exp}	251(59)

Phys.Rev.Lett.126, 141801 (2021)
Phys.Rev.D 73, 072003 (2006).

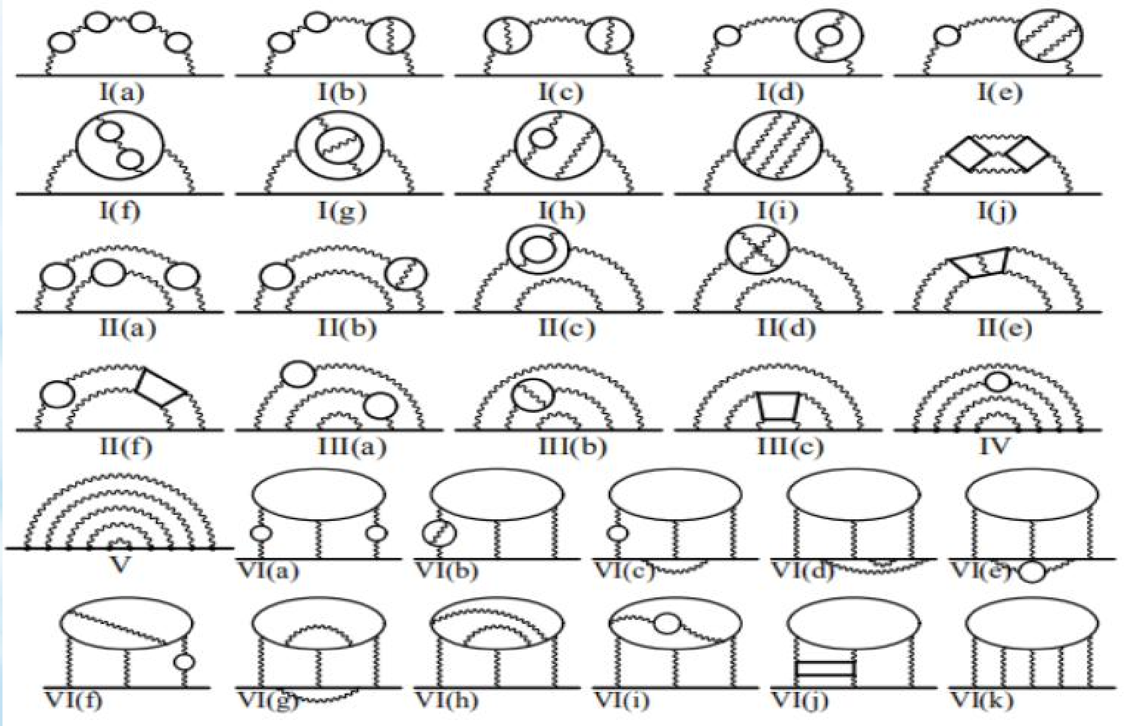
Phys.Rept.887(2020)1

Science and Technology Cooperation Program in High Energy Physics. This review benefited from discussions with O. Catà, N. Christ, L.Y. Dai, H. Davoudiasl, S. Fayer, S. Ganguly, A. Gasparian, S. Hashimoto, T. Iijima, K. Kampf, D. Kawall, I. Larin, Z. Pagel, M. Petschlies, A. Rebhan, K. Schilcher, K. Shimomura, E. Shintani, D. Steffen, S. Tracz, C. Tu, and T. Yamazaki.

QED

- The most contribution
- Precise prediction
- At 10-th order, $O(\alpha^5)$

$$a_\mu = 116\,584\,718.951\ (0.080)\ \times\ 10^{-11}$$



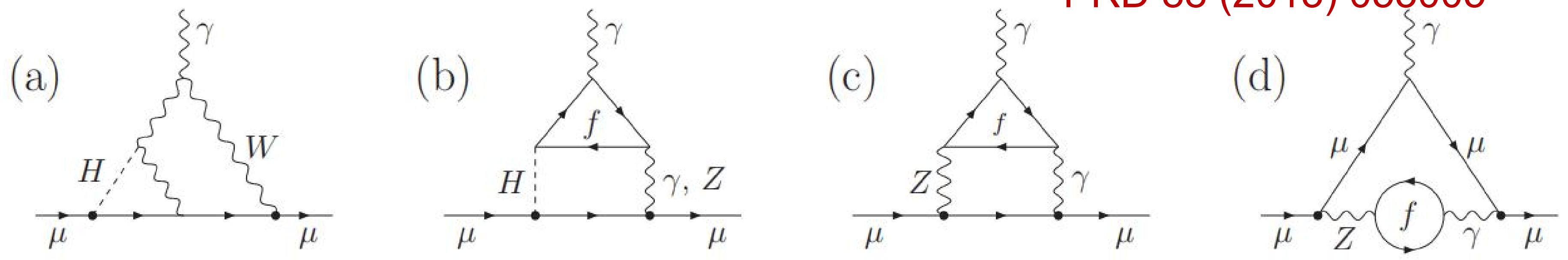
Aoyama *et.al.*,
PRL109 (2012) 111808

EW+Strong interactions

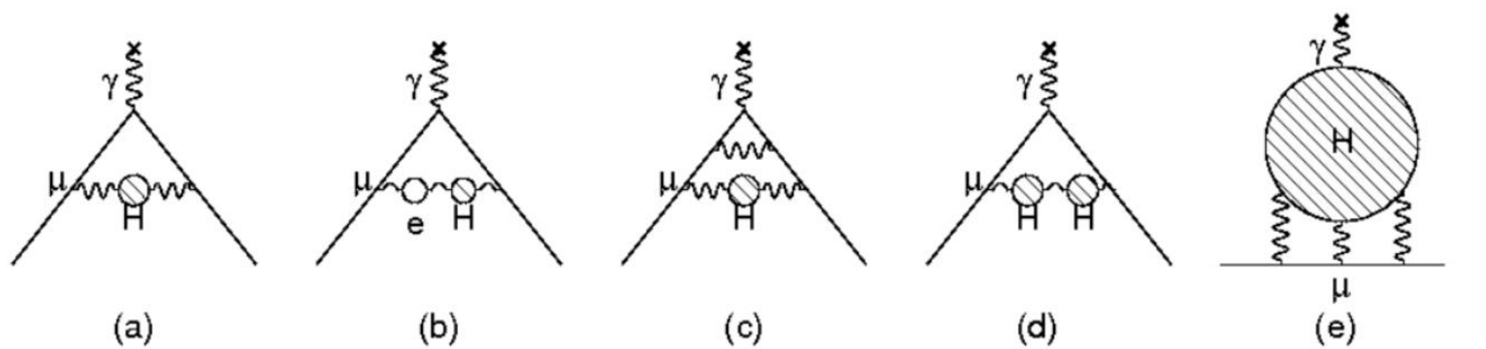
- Precise prediction
- At two-loop level

$$a_\mu = 153.6\,(1.0) \times 10^{-11}$$

Gnendiger *et.al.*,
PRD 88 (2013) 053005



- Strong interactions: pQCD---high energy region



Phys.Rept.887(2020)1

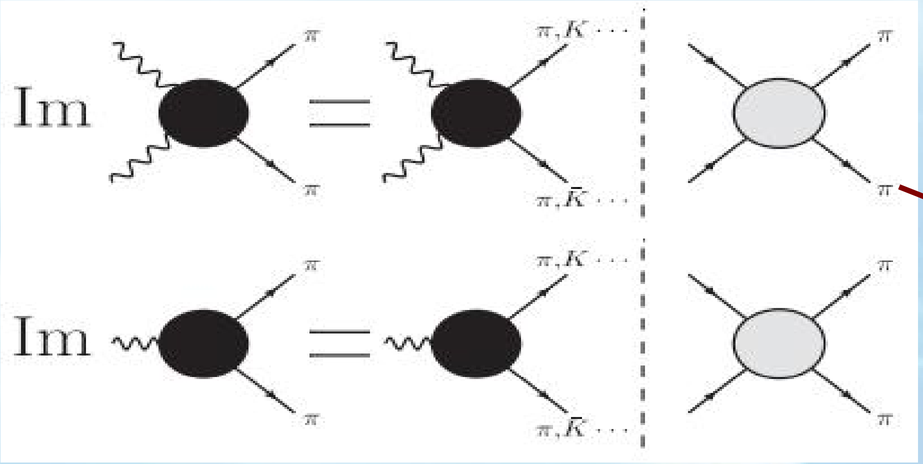
Hadronic Part: Methods from SM

- LQCD
- Data-driven solutions from experiment
- Amplitude analysis: model independent

  - Only one physical amplitude!
  - It should satisfy the fundamental QFT principles
  - It should be compatible with the exp results

Amplitude analysis: FSI

- Most resonances decays into light pseudoscalars
- FSI needs to be taken into account to perform an amplitude analysis
- Methods: KM, N/D, AMP, Roy equation, PKU, Pade, LSE, BSE, ChEFT, *et.al.*



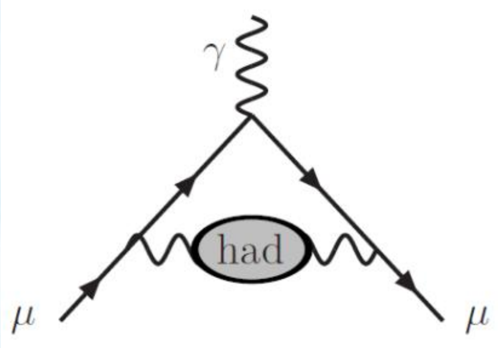
Yao, Dai#, Zheng, Zhou, RPP84(2021)076201

FSI: application

- Scattering, decaying amplitudes: extracting resonance information
- Check the working range of ChEFT
- Scalar? The same quantum number with QCD vacuum. Dynamics?
- HVP, HLBL

2、HVP

- QCD: high energy region
- Dispersive approach: Roy, KT, PKU, etc., difficult to deal with multi-body rescattering
- ChPT: works in the very low energy region
- RChT: extend to a bit higher energy region



$$a_\mu^{\mathrm{had}} = \left(\frac{\alpha_e(0) m_\mu}{3\pi}\right)^2 \int_{s_{\mathrm{thr}}}^{\infty} \mathrm{d}s \frac{\hat{K}(s)}{s^2} R_{\mathrm{h}}(s)$$

Low energy physics dominates

RChT: Constraints from QCD

- resonances included as new degrees of freedom

$$R \equiv \frac{1}{\sqrt{2}} \sum_{i=1}^{8} \lambda_i \phi_R^i$$

- Construct Lagrangians by discrete and chiral symmetries

$$\mathcal{L}_{\rm kin}^R = -\frac{1}{2}\langle \nabla^\lambda R_{\lambda\mu} \nabla_\nu R^{\nu\mu} \rangle + \frac{M_R^2}{4}\langle R_{\mu\nu} R^{\mu\nu} \rangle \,, \qquad R = V, A \,,$$

$$\mathcal{L}_{\rm kin}^R = \frac{1}{2}\langle \nabla^\mu R \nabla_\nu R - M_R^2 R^2 \rangle \,, \qquad R = S, P \,.$$

$$\mathcal{L}_{(4)}^R = \sum_{i=1}^{22} \lambda_i^V \mathcal{O}_i^V + \sum_{i=1}^{17} \lambda_i^A \mathcal{O}_i^A + \sum_{i=1}^{18} \lambda_i^S \mathcal{O}_i^S + \sum_{i=1}^{13} \lambda_i^P \mathcal{O}_i^P$$

$$\mathcal{L}_{(2)}^{RR} = \sum_{(ij)n} \lambda_n^{R_i R_j} \mathcal{O}_n^{R_i R_j} \,,$$

$$\mathcal{L}_{(0)}^{RRR} = \sum_{(ijk)} \lambda^{R_i R_j R_k} \mathcal{O}^{R_i R_j R_k} \,.$$

i	Operator $\mathcal{O}_i^{RR}$, $R = V, A$	Operator $\mathcal{O}_i^{SS}$	Operator $\mathcal{O}_i^{PP}$
1	$\langle R_{\mu\nu} R^{\mu\nu} u^\alpha u_\alpha \rangle$	$\langle S S u_\mu u^\mu \rangle$	$\langle P P u_\mu u^\mu \rangle$
2	$\langle R_{\mu\nu} u^\alpha R^{\mu\nu} u_\alpha \rangle$	$\langle S u_\mu S u^\mu \rangle$	$\langle P u_\mu P u^\mu \rangle$
3	$\langle R_{\mu\alpha} R^{\nu\alpha} u^\mu u_\nu \rangle$	$\langle S S \chi_+ \rangle$	$\langle P P \chi_+ \rangle$
4	$\langle R_{\mu\alpha} R^{\nu\alpha} u_\nu u^\mu \rangle$		
5	$\langle R_{\mu\alpha} (u^\alpha R^{\mu\beta} u_\beta + u_\beta R^{\mu\beta} u^\alpha) \rangle$		
6	$\langle R_{\mu\nu} R^{\mu\nu} \chi_+ \rangle$		
7	$i \langle R_{\mu\alpha} R^{\alpha\nu} f_{+\beta\nu} \rangle g^{\beta\mu}$		

Matching GF: SVV,SAA

- Matching GF between QCD and ChEFT in the high energy region, using large Nc and OPE.

$$\left(\Pi^{ijk}_{SAA}\right)_{\mu\nu} = i^2 \int d^4x\, d^4y\, e^{i(p_1\cdot x+p_2\cdot y)} \langle 0|T\left\{S^i(0)A^j_\mu(x)A^k_\nu(y)\right\}|0\rangle$$

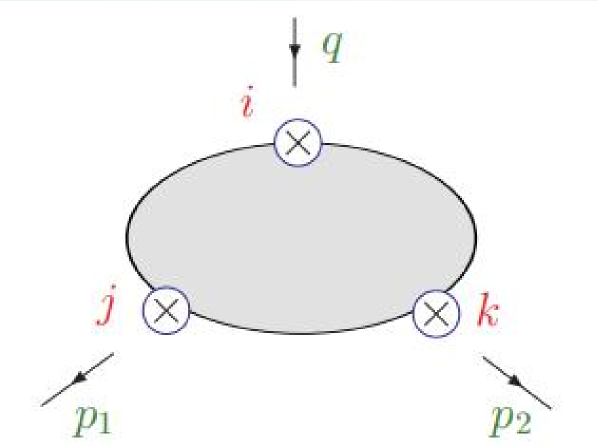
$$\left(\Pi^{ijk}_{SVV}\right)_{\mu\nu} = i^2 \int d^4x\, d^4y\, e^{i(p_1\cdot x+p_2\cdot y)} \langle 0|T\left\{S^i(0)V^j_\mu(x)V^k_\nu(y)\right\}|0\rangle$$

$$S^i(x) = \left(\bar{q}\lambda^i q\right)(x) \qquad V^i_\mu(x) = \left(\bar{q}\gamma_\mu\frac{\lambda^i}{2}q\right)(x) \qquad A^i_\mu(x) = \left(\bar{q}\gamma_\mu\gamma_5\frac{\lambda^i}{2}q\right)(x)$$

- Ward identity

$$p_1^\mu\left(\Pi^{ijk}_{SAA}\right)_{\mu\nu} = -2\,d^{ijk}B_0F^2\frac{(p_2)_\nu}{p_2^2} \qquad p_1^\mu\left(\Pi^{ijk}_{SVV}\right)_{\mu\nu} = 0$$

$$p_2^\nu\left(\Pi^{ijk}_{SAA}\right)_{\mu\nu} = -2\,d^{ijk}B_0F^2\frac{(p_1)_\mu}{p_1^2} \qquad p_2^\nu\left(\Pi^{ijk}_{SVV}\right)_{\mu\nu} = 0$$



Dai *et.al.*, PRD99 (2019) 114015

SAA

- P and Q are the Lorentz structure of momentum, they vanish by timing $p_{1\mu}$ and $p_{2\nu}$.

$$\left(\Pi^{ijk}_{SAA}\right)_{\mu\nu} = d^{ijk} B_0 \left[-2\, F^2 \frac{(p_1)_\mu (p_2)_\nu}{p_1^2 p_2^2} + \mathcal{F}_A\left(p_1^2, p_2^2, q^2\right) P_{\mu\nu} + \mathcal{G}_A\left(p_1^2, p_2^2, q^2\right) Q_{\mu\nu} \right]$$

$$P_{\mu\nu} = (p_2)_\mu (p_1)_\nu - p_1 \cdot p_2\, g_{\mu\nu}\,,$$

$$Q_{\mu\nu} = p_1^2 (p_2)_\mu (p_2)_\nu + p_2^2 (p_1)_\mu (p_1)_\nu - p_1 \cdot p_2 (p_1)_\mu (p_2)_\nu - p_1^2 p_2^2\, g_{\mu\nu}$$

$$\lim_{\lambda\to\infty} \left(\Pi^{ijk}_{SAA}\right)_{\mu\nu} (\lambda p_1, \lambda p_2) = -2\, d^{ijk} B_0 F^2 \frac{1}{\lambda^2} \frac{1}{p_1^2\, p_2^2\, q^2} \left[q^2 (p_1)_\mu (p_2)_\nu + Q_{\mu\nu} - p_1 \cdot p_2\, P_{\mu\nu} \right] + \mathcal{O}\left(\frac{1}{\lambda^3}\right)$$

$$\lim_{\lambda\to\infty} \left(\Pi^{ijk}_{SAA}\right)_{\mu\nu} (\lambda p_1, p_2) = -2\, d^{ijk} B_0 F^2 \frac{1}{\lambda} \frac{(p_1)_\mu (p_2)_\nu}{p_1^2 p_2^2} + \mathcal{O}\left(\frac{1}{\lambda^2}\right)$$

$$\lim_{\lambda\to\infty} \left(\Pi^{ijk}_{SAA}\right)_{\mu\nu} (p_1, \lambda p_2) = -2\, d^{ijk} B_0 F^2 \frac{1}{\lambda} \frac{(p_1)_\mu (p_2)_\nu}{p_1^2 p_2^2} + \mathcal{O}\left(\frac{1}{\lambda^2}\right)$$

$$\lim_{\lambda\to\infty} \left(\Pi^{ijk}_{SAA}\right)_{\mu\nu} (\lambda p_1, q - \lambda p_1) = \mathcal{O}\left(\frac{1}{\lambda^2}\right)$$

SAA matching

- constrains

$$\hat{L}_5 = \hat{C}_{12} = \hat{C}_{80} = \hat{C}_{85} = 0,$$
$$\lambda_6^A = \lambda_{16}^A = \lambda_{12}^S = \lambda_{16}^S = 0,$$
$$\lambda_6^{AA} = -\frac{F^2}{16 F_A^2},$$
$$\lambda_1^{SA} = \frac{1}{\sqrt{2} F_A}\left(c_d - \frac{F^2}{8 c_m}\right),$$
$$\lambda_2^{SA} = -\frac{c_d}{2\sqrt{2} F_A}.$$

- 15 couplings, 4 of them remain λ_{17}^A λ_{17}^S λ_{18}^S λ^{SAA}
- also from $\Pi_{SS-PP}^{ij}(t)$ $F_S^{ij}(t)$, one can knows three more couplings, only 1 remain

$$\lambda_{17}^S = \lambda_{18}^S = 0,$$
$$\lambda_{17}^A = 0,$$

V. Cirigliano, et.al., NPB753 (2006) 179
G. Ecker, PLB223 (1989) 425

RChT

- 1/Nc expansion,
  - loop diagrams are suppressed
  - uncertainty ~1/3

- 'chiral counting' by integrating out resonances
  - Those generating O(p^6) ChPT Lagrangians

$$\langle R_a \chi(p^4) \rangle, \langle R_a R_b \chi(p^2) \rangle \text{ and } \langle R_a R_b R_c \rangle.$$

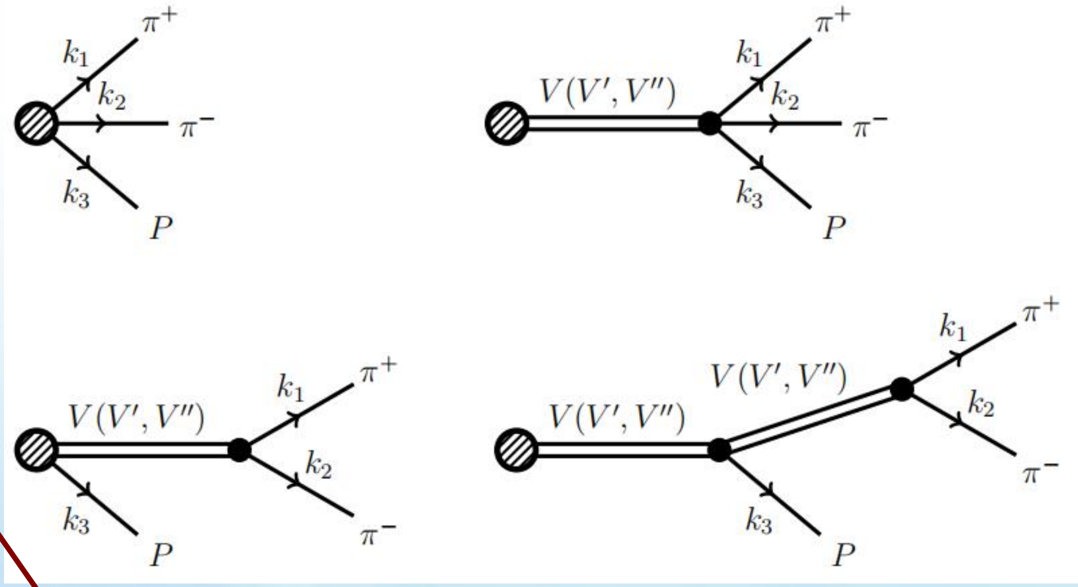
Dai *et.al.*, PRD99 (2019) 114015

Building amplitudes

- RChT in the resonance region, excited states?

• V', V'' has the same topologies as the ground states

• ππ-KK FSI part by matching with Omens functions



Guerrero, et.al., PLB 412 (1997) 382

$$\frac{1}{M_V^2 - x} \rightarrow \frac{1}{M_V^2 - x} + \frac{\beta'_\pi}{M_{V'}^2 - x} + \frac{\beta''_\pi}{M_{V''}^2 - x}$$

Dai, et.al., PRD88 (2013) 056001

Building amplitudes

- We give a combined analysis on four channels:

$$\pi^+\pi^-, K^+K^-, \pi^+\pi^-\pi^0, \pi^+\pi^-\eta$$

- ππ-KK FSI part by matching with Omnes function
- ρ-ω mixing, origined from Gasser&Leutwyler's

$$F_V^\pi = \Big(1 + \frac{F_V G_V}{F^2} Q^2 \,(BW(M_\rho, \Gamma_{\rho,}, Q^2) + \beta'_{\pi\pi} BW(M_{\rho'}, \Gamma_{\rho',}, Q^2) + \beta''_{\pi\pi} BW(M_{\rho''}, \Gamma_{\rho'',}, Q^2)\Big) (\frac{1}{\sqrt{3}} \sin\theta_V \sin\delta^\rho + \cos\delta) \cos\delta - \frac{F_V G_V}{F^2} Q^2 \Big(BW(M_\omega, \Gamma_{\omega,}, Q^2) + \beta'_{\pi\pi} BW(M_{\omega'}, \Gamma_{\omega',}, Q^2) + \beta'''_{\pi\pi} BW(M_{\omega''}, \Gamma_{\omega'',}, Q^2)\Big) (\frac{1}{\sqrt{3}} \sin\theta_V \cos\delta - \sin\delta^\omega) \sin\delta^\omega\Big) \exp\left[\frac{-s}{96\pi^2 F^2} \Big(\mathrm{Re}\left[A[m_\pi, M_\rho, Q^2] + \frac{1}{2} A[m_K, M_\rho, Q^2]\right]\Big)\right]$$

- Not much freedom for Fit

=1, from QCD as well as disersion relation constraints

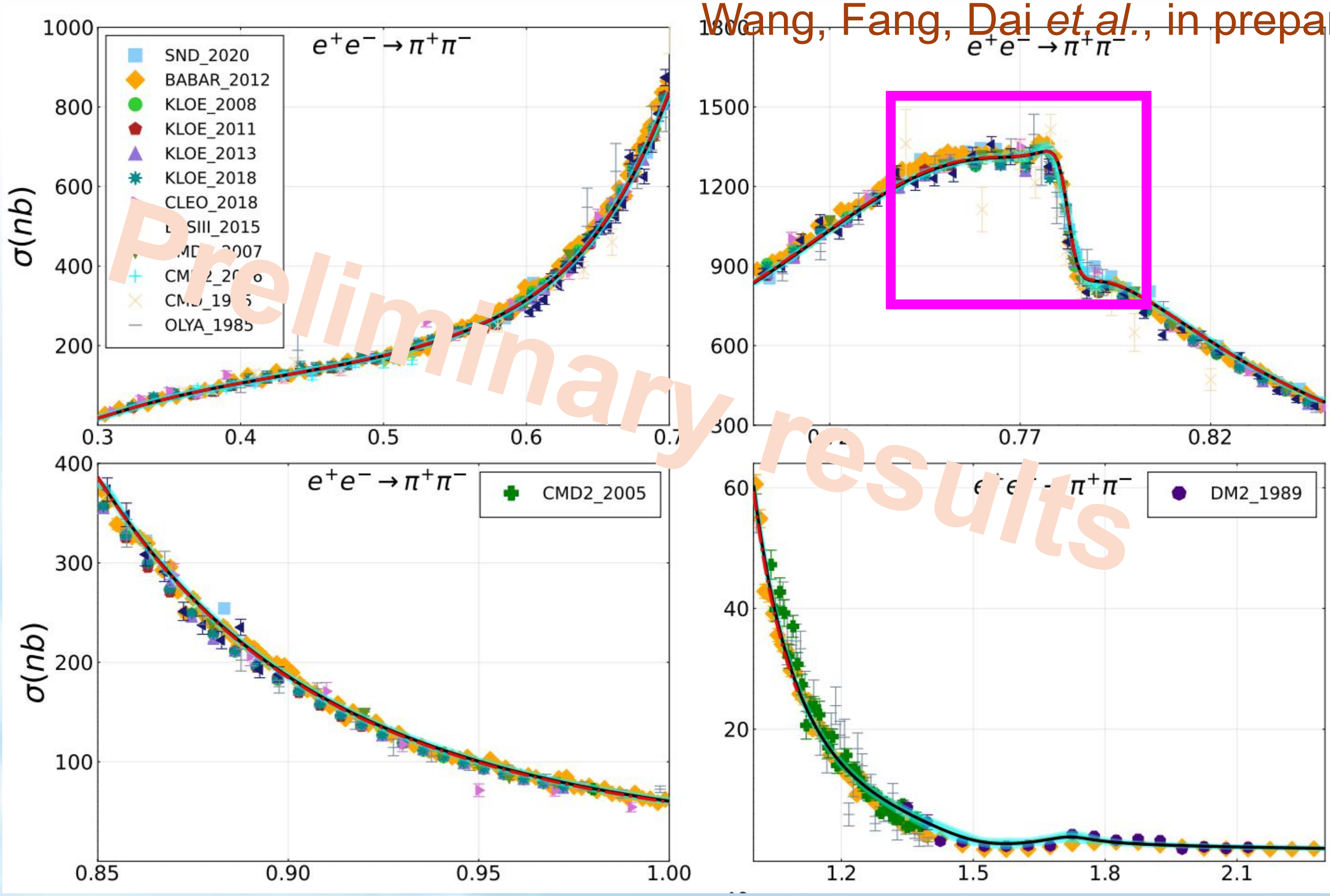
Gasser&Leutwyler, Phys.Rept.87 (1982) 77

Guerrero&Pich, PLB 412 (1997) 382

$\pi\pi$

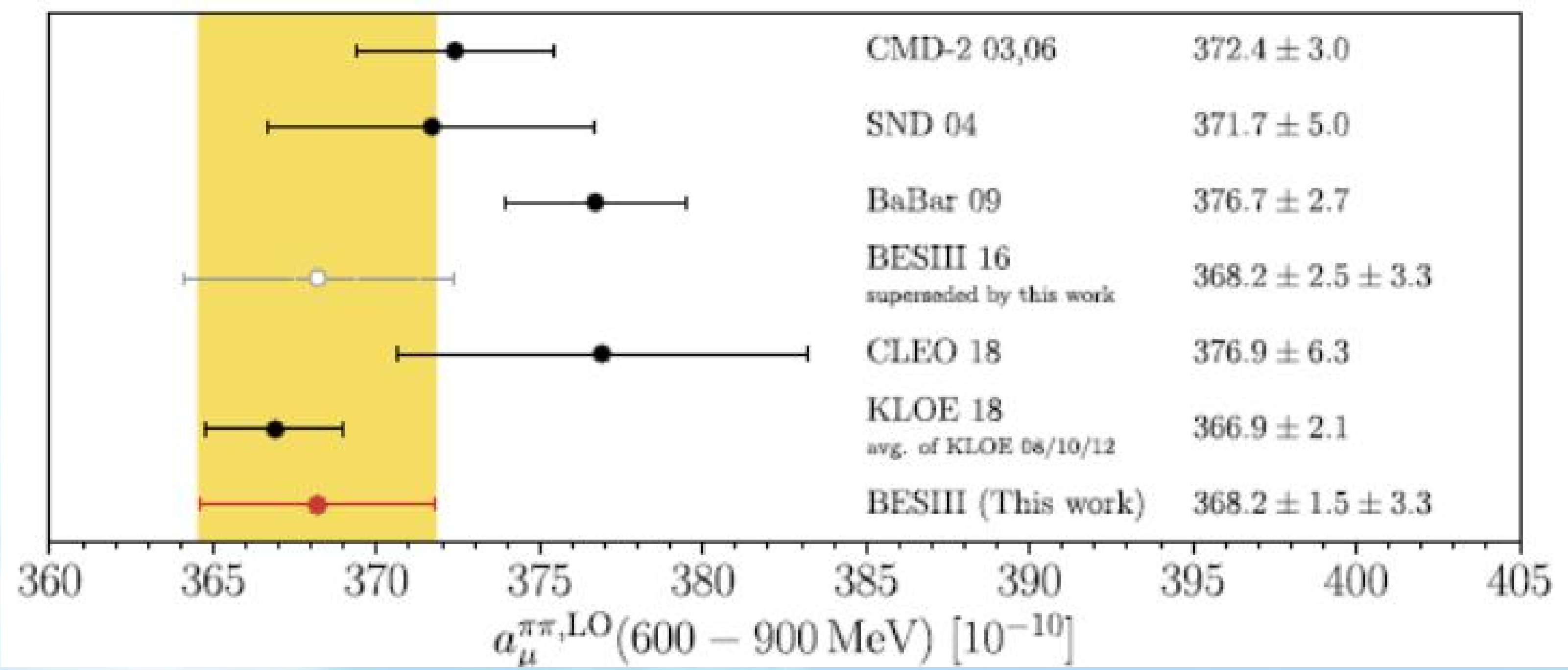
- $\pi\pi$: Now closer to KLOE and BESIII's

Wang, Fang, Dai *et al.*, in preparation



Experiment

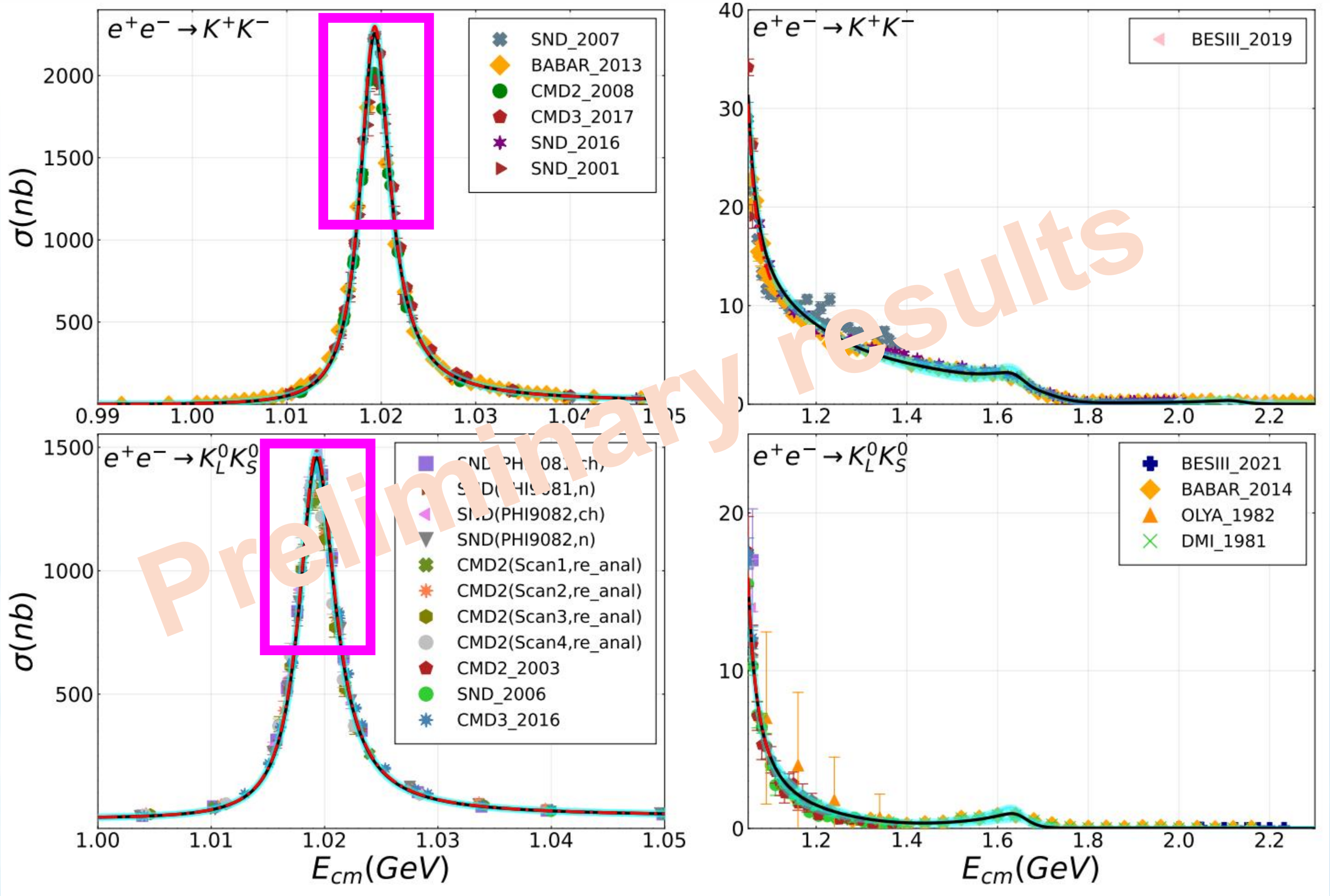
- Future experiments?



Guangshun Huang, talk at HNU

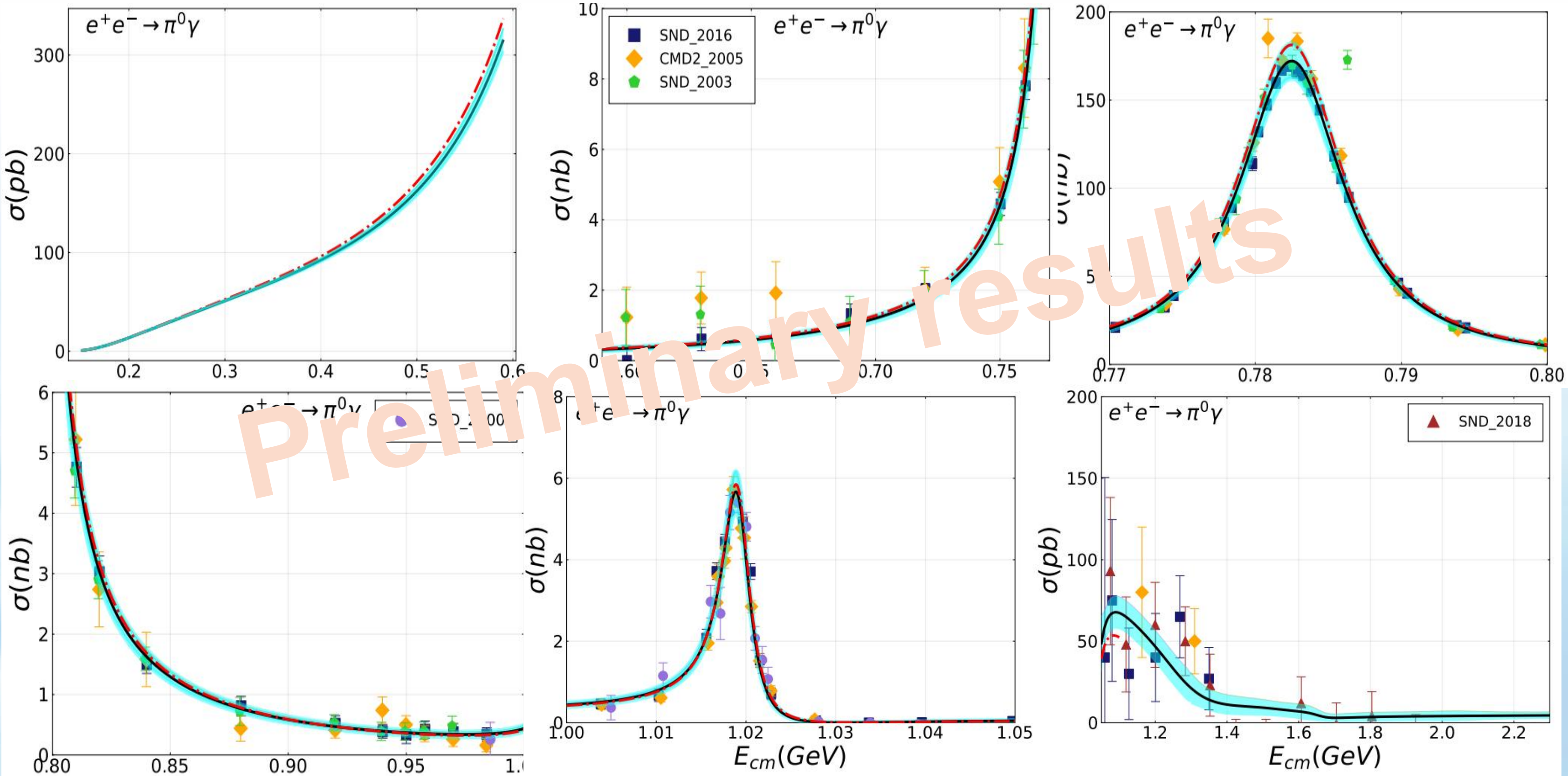
KK

- KK: data in the ϕ 'peak' have large discrepancy
- $K_L K_S$: further direct constraints on $\pi\pi$, KK channels



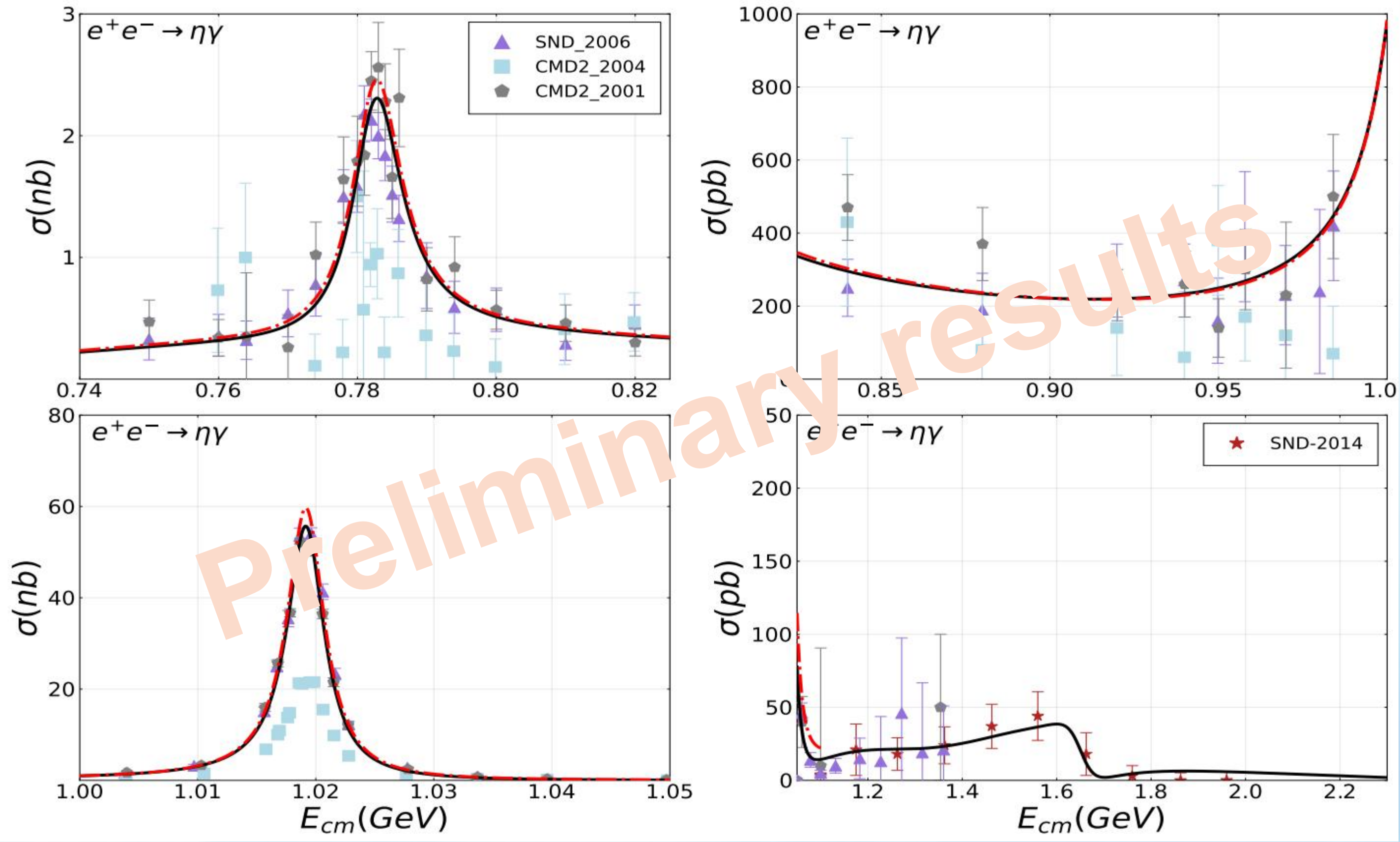
πγ

- πγ: helps to constrain ππ, KK channels, masses of ρ, ω, ϕ



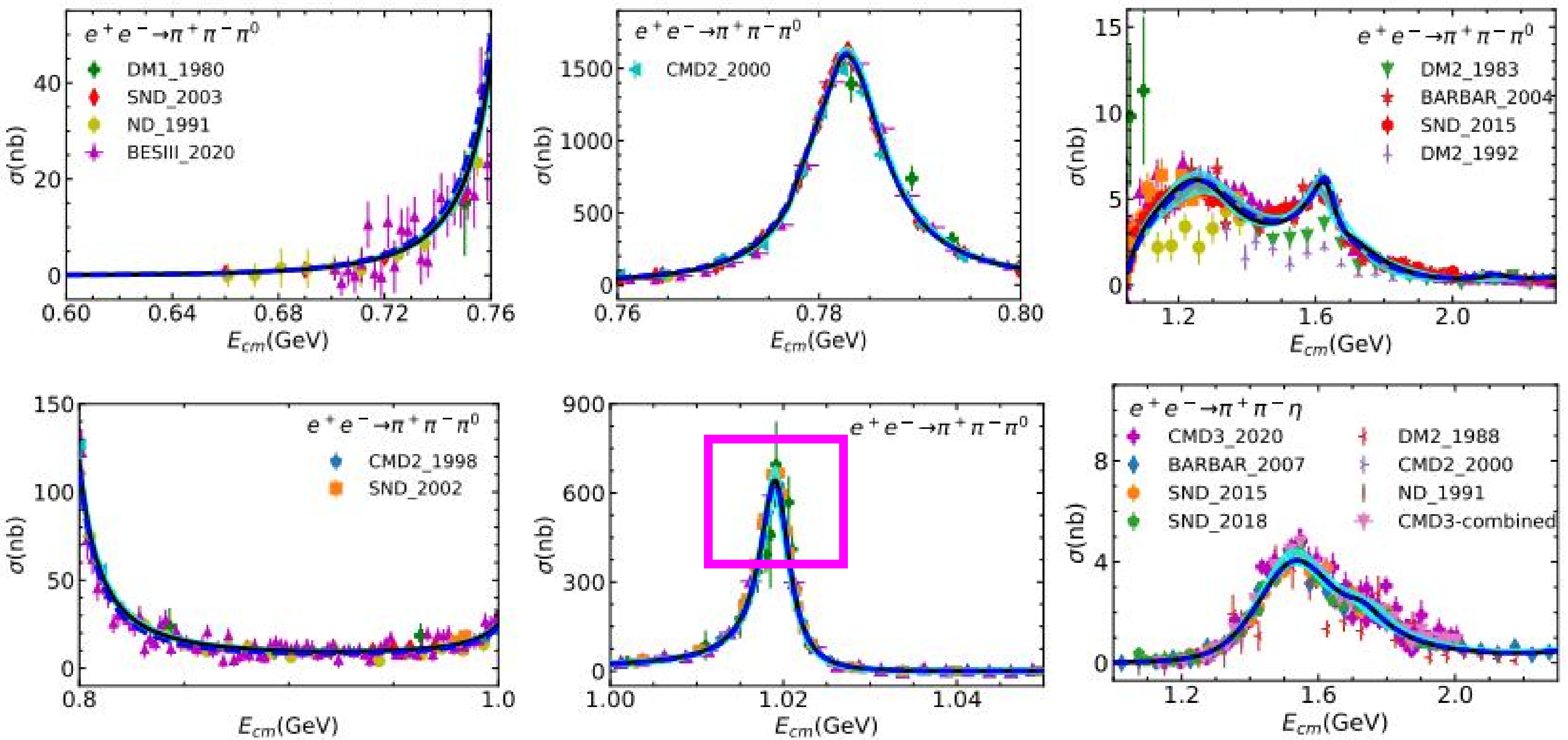
ηγ

- ηγ: helps to constrain KK, and masses of ρ, ω, ϕ



ππππ, ππη

- πππ: needs more precise data in the ω ϕ region
- ππη: check our model

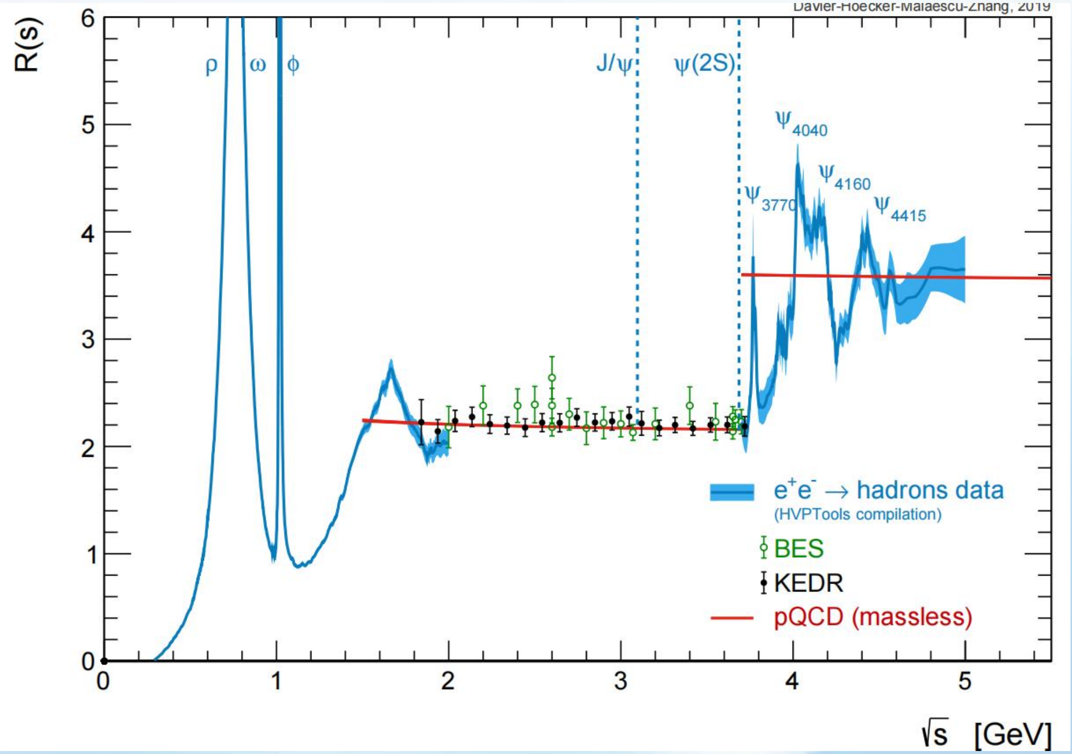


R value

- Cross sections needs to be corrected

$$R_{\mathrm{h}}(s) = \frac{3s}{4\pi\alpha_e^2(s)}\sigma\left(e^+e^- \to \text{ hadrons }\right) \qquad \mathrm{Re}\Pi_{\mathrm{had}}(s) = -\frac{\alpha_e(0)s}{3\pi}\mathrm{P}\int_{s_{\mathrm{th}}}^{\infty}\frac{R(s')}{s'(s'-s)}ds'$$

- R values are input from PDG



Davier *et.al.*,
EPJC 80 (2020) 3, 241

g-2: HVP-LO

- Other channels are taken from data-driven or QCD

J/ψ (BW integral)	6.28 ± 0.07
$\psi(2S)$ (BW integral)	1.57 ± 0.03
R data [3.7 – 5.0] GeV	$7.29 \pm 0.05 \pm 0.30 \pm 0.00$
$R_{\rm QCD}$ [1.8 – 3.7 GeV]$_{uds}$	$33.45 \pm 0.28 \pm 0.65_{\rm dual}$
$R_{\rm QCD}$ [5.0 – 9.3 GeV]$_{udsc}$	6.86 ± 0.04
$R_{\rm QCD}$ [9.3 – 12.0 GeV]$_{udscb}$	1.21 ± 0.01
$R_{\rm QCD}$ [12.0 – 40.0 GeV]$_{udscb}$	1.64 ± 0.00
$R_{\rm QCD}$ [> 40.0 GeV]$_{udscb}$	0.16 ± 0.00
$R_{\rm QCD}$ [> 40.0 GeV]$_{t}$	0.00 ± 0.00

- HVP-LO: $693.85 \pm 3.38 \times 10^{-10}$

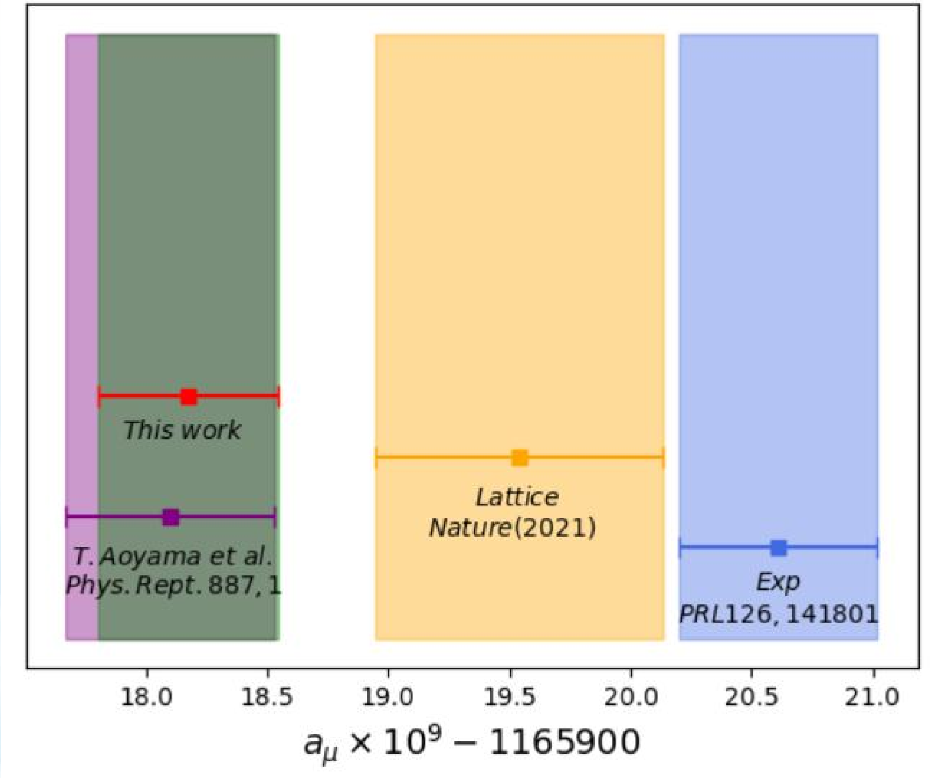
$708.7(5.3) \times 10^{-10}$

Nature 593 (2021) 7857, 51-55

- Ours: $a_\mu = 11659181.7 \pm 3.7 \times 10^{-11}$

HVP

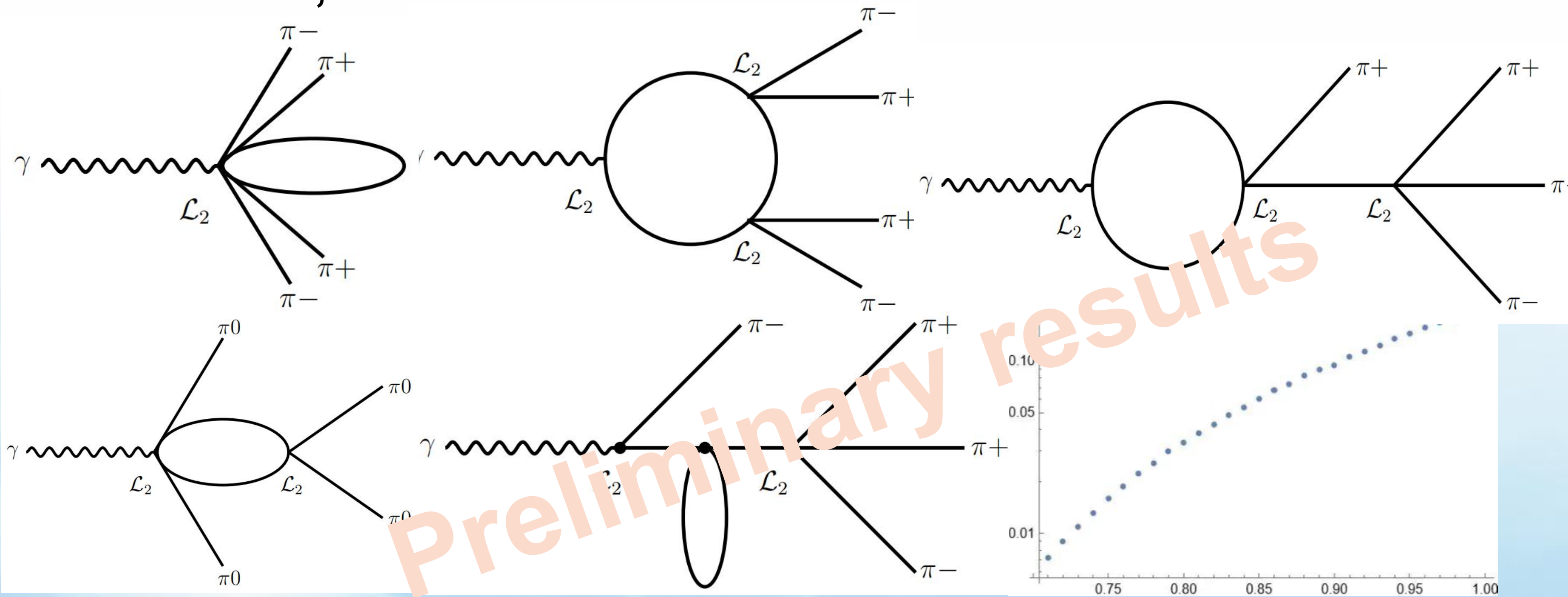
- Ours: a_μ=11659181.7 ±3.7× 10^{-11}
- It differs 4.4σ from latest experiment's



Wang, Fang, Dai, in preparation

Four body final states?

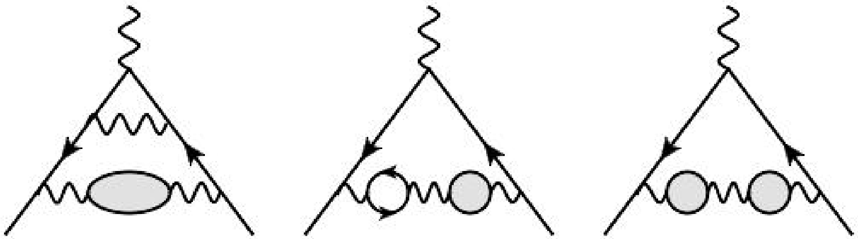
Four body final states are important: ππππ, ππKK channels,etc.

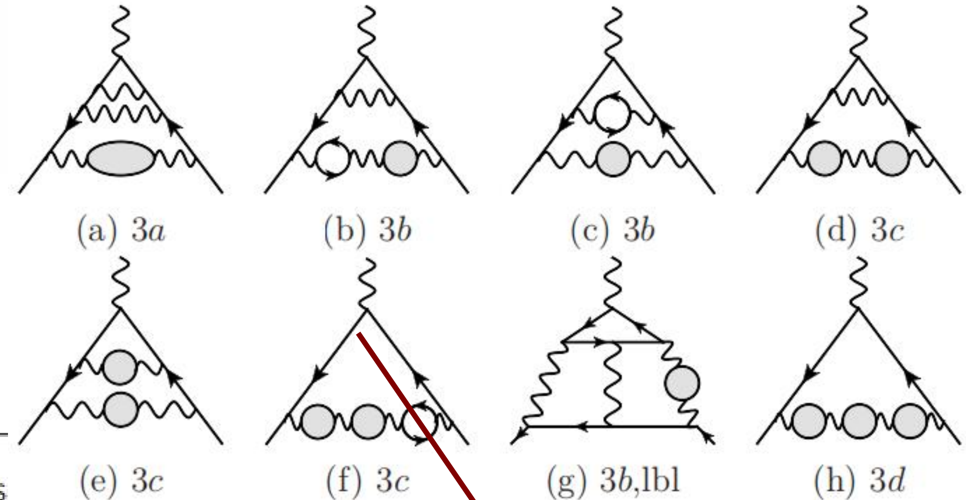


- ChPT's << data, in resonance energy region
- FSI?
- Resonances?

HVP: NLO, NNLO?

- More channels (also high energy ones) to give a complete estimation?





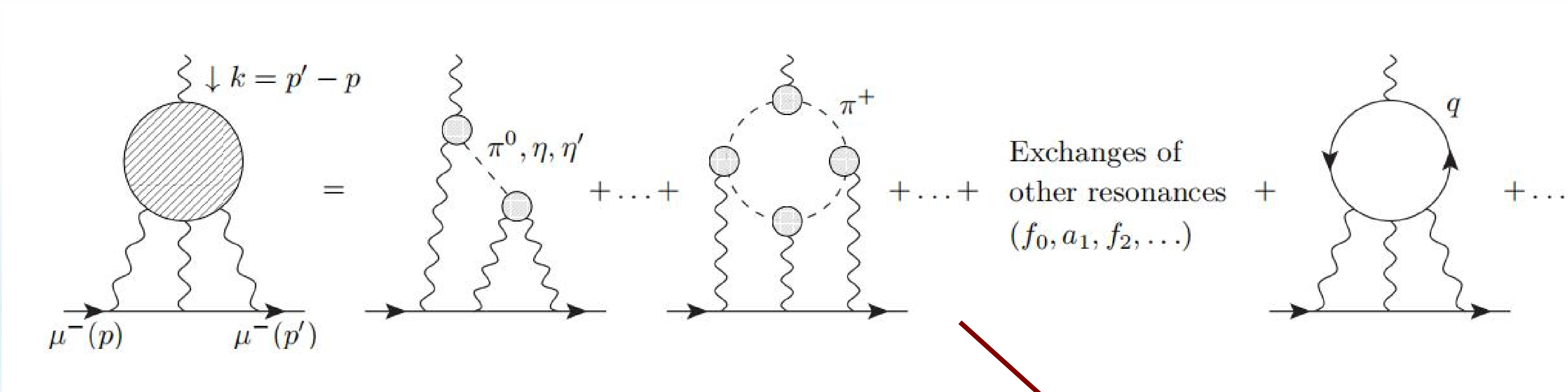
$\times 10^{-12}$	$\pi\pi$	KK	$\pi\pi\pi$	$\pi\pi\eta$	S	(PLB 734,144)
2a	-1369±8	-79.8±2.8	-145±3	-5.93±0.46	-1600±9	-2090
2b	776±5	37.6±1.3	74.7±1.8	2.37±0.18	891±5	1068
2c		22.4±0.2			22.4±0.2	35
a_μ^{NLO}					-687±10	-987±9
3a	45.4±0.3	3.11±0.11	5.20±0.12	0.267±0.021	54.0±0.3	80
3b	-24.8±0.2	-1.62±0.06	-2.78±0.06	-0.131±0.010	-29.3±0.2	-41
3bLBL	58.0±0.3	3.47±0.12	6.19±0.14	0.268±0.021	67.9±0.4	91
3c		-2.34±0.02			-2.34±0.02	-6
3d		0.0249±0.0004			0.0249±0.0004	0.05
a_μ^{NNLO}					90.3±0.5	124±1

Kurz, et.al.
PLB 734 (2014) 144

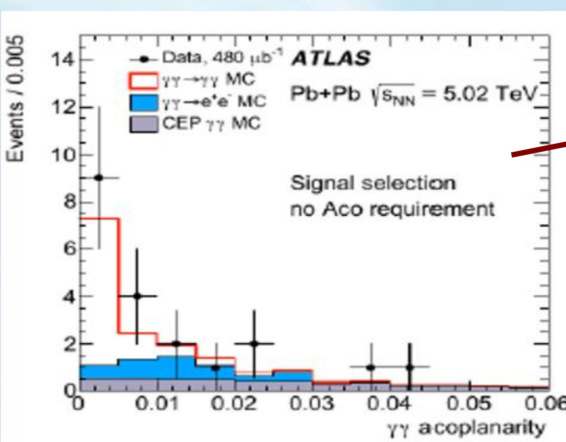
Refine our results by considering other channels of three, four body final states .

3、HLBL

- $\gamma\gamma^* \rightarrow \gamma^*\gamma^*$ has the clean background, a typical example for amplitude analysis



Phys.Rept.887(2020)1



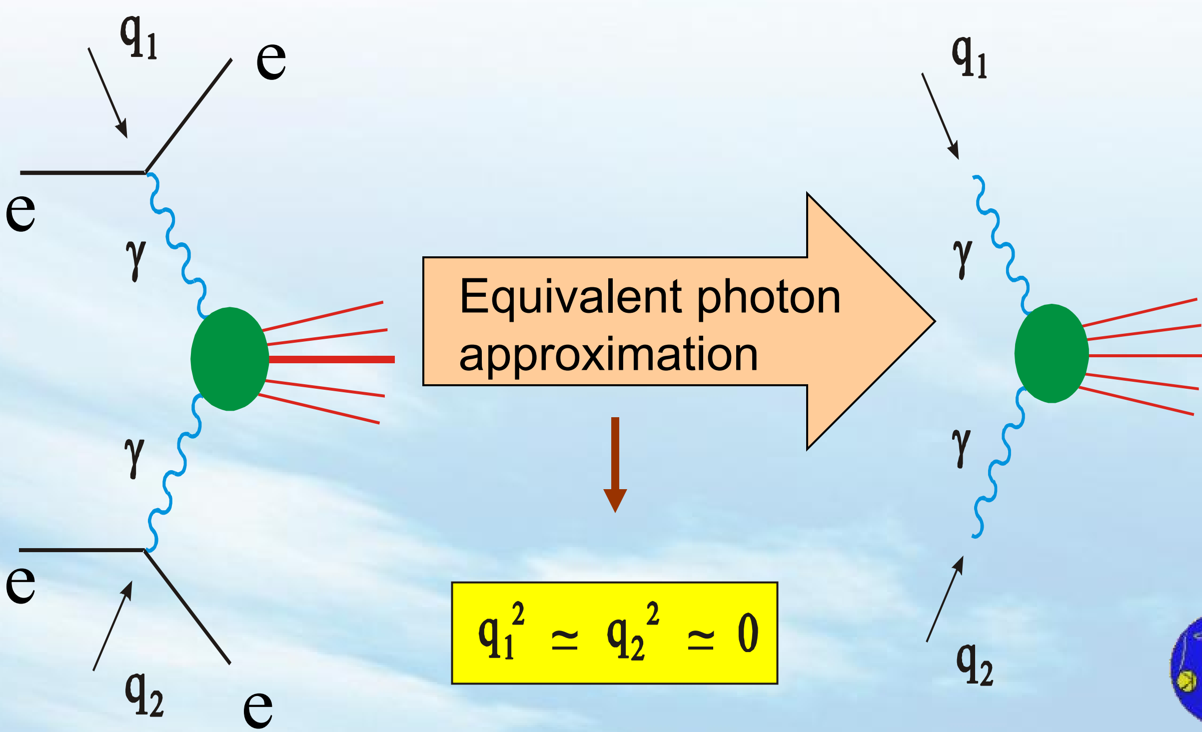
Natrure Phys. 13 (2017) 852,
$\gamma\gamma \rightarrow \gamma\gamma$, only 13 events

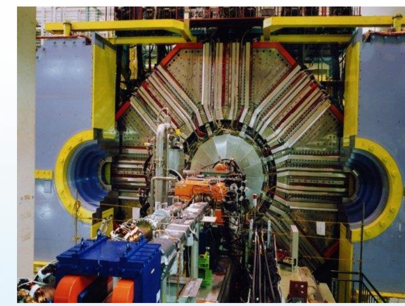
Small yeild, but the result has already been used to set new limits on the Born infeld extension of the SM

Phys. Rev. Lett. 118 (2017) 261802

3、HLBL

- $\gamma\gamma \rightarrow$MM has the clean background, a typical example for amplitude analysis
- $\gamma\gamma \rightarrow$MM contributes significantly to LbL sumrule





π^+ π^-	π^0 π^0	K^+ K^-	$\bar{K}^0$ K^0

Strategy

Hadron → Photon → constraint

Hadron

Fit to get $\pi\pi$ - KK coupled channel scattering amplitudes

Photon

With Final State Interaction Theorem we construct di-photon amplitudes and fit to the data

constraint

Constraints to LbL sumrule, polarizabilities. ..

Hadronic amplitudes

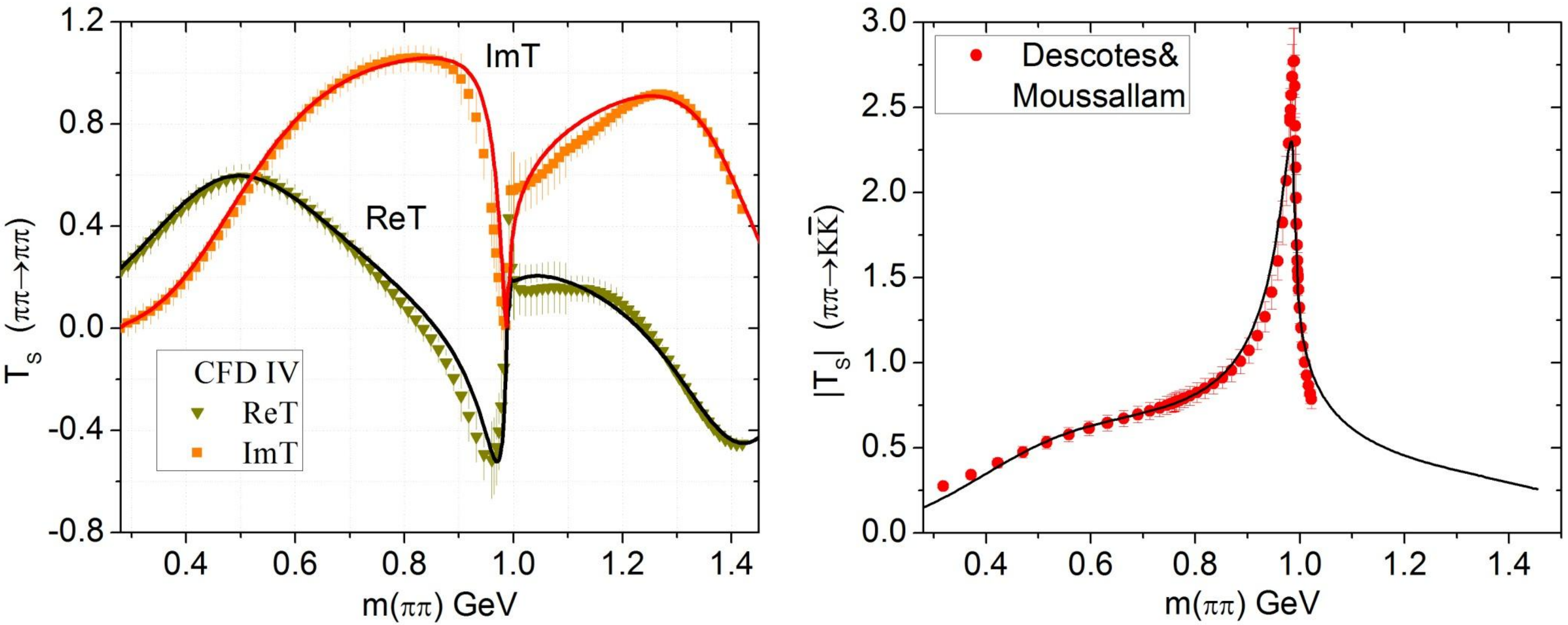
- **$\pi\pi$ - KK scattering inputs**
  - K-matrix to represent S and D partial waves
  - Data on Phase shifts and inelasticities of $\pi\pi$ - KK coupled channel scattering.
  - BABAR's Dalitz plot analysis of $D_s^+ \to (\pi^+\pi^-)\pi^+$ and $D_s^+ \to (K^+K^-)\pi^+$ process. BES's analysis on $J/\psi \to \pi^+\pi^-\phi$ and $J/\psi \to K^+K^-\phi$.
  - Dispersion analysis:
    - T-matrix of $\pi\pi$ scattering by CFDIV .
    - $\pi\pi \to KK$ amplitudes given by Roy-Steiner Equation.

Descotes *et.al*
EPJC33 (2004) 409

Pelaez *et al.*
PRD83 (2011) 074004

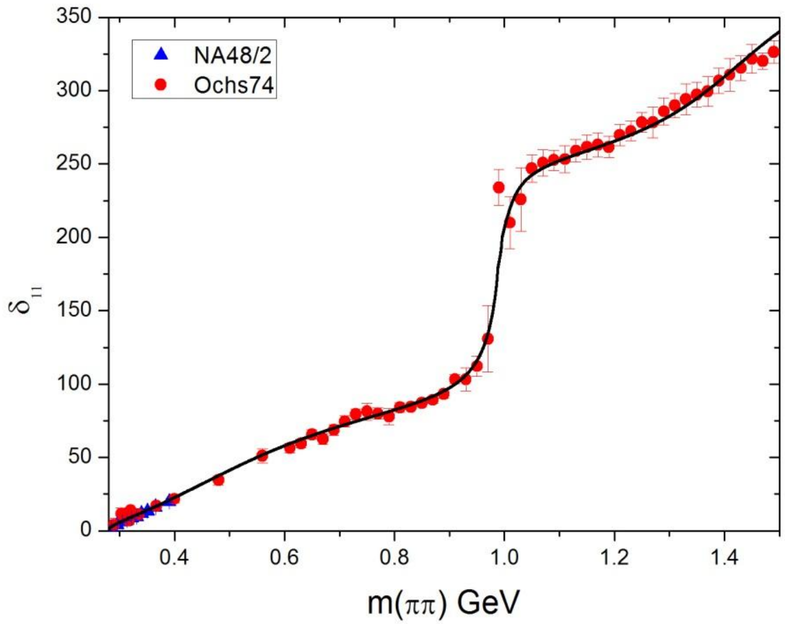
Dispersion analysis constraints

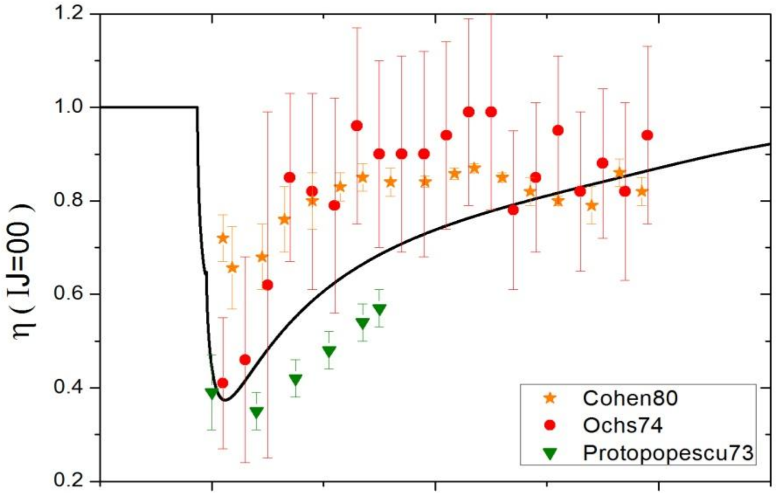
- They use Roy like equation and take crossing symmetry, unitarity into account.

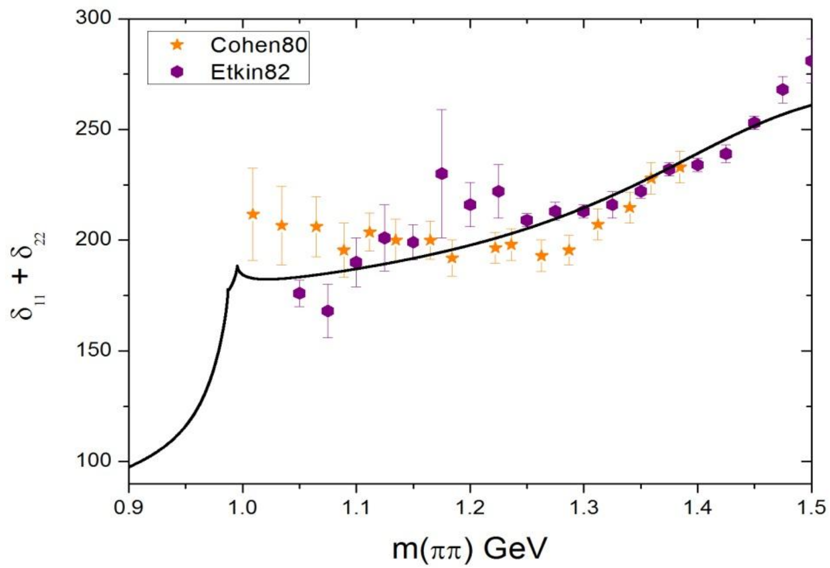


Data: phase shift and inelasticity

- ππ→ππ,KK phase shift and inelasticity

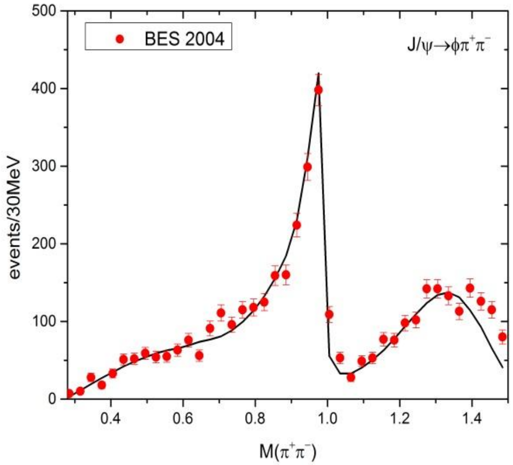


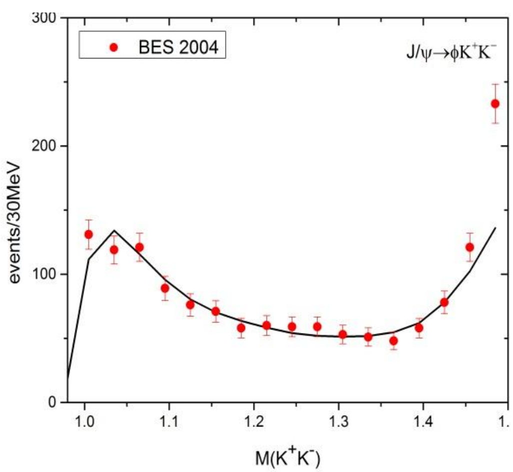


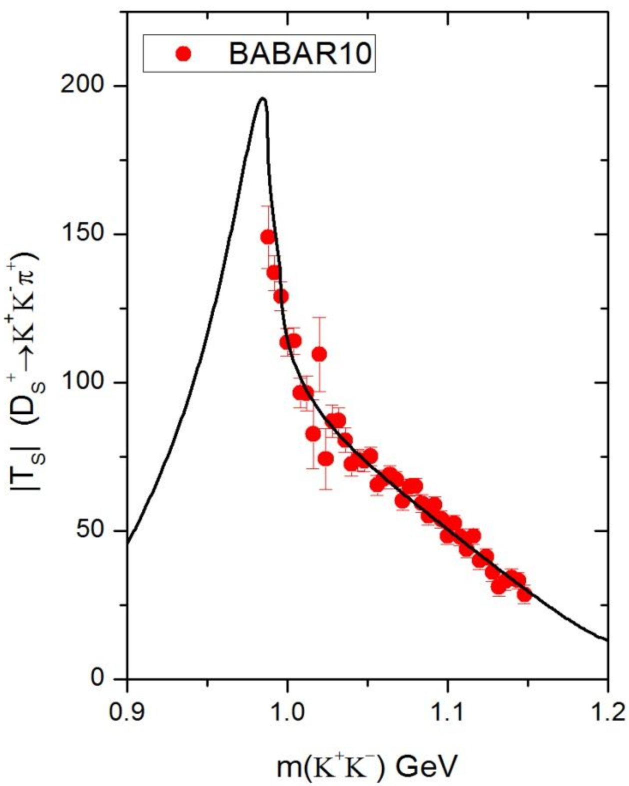


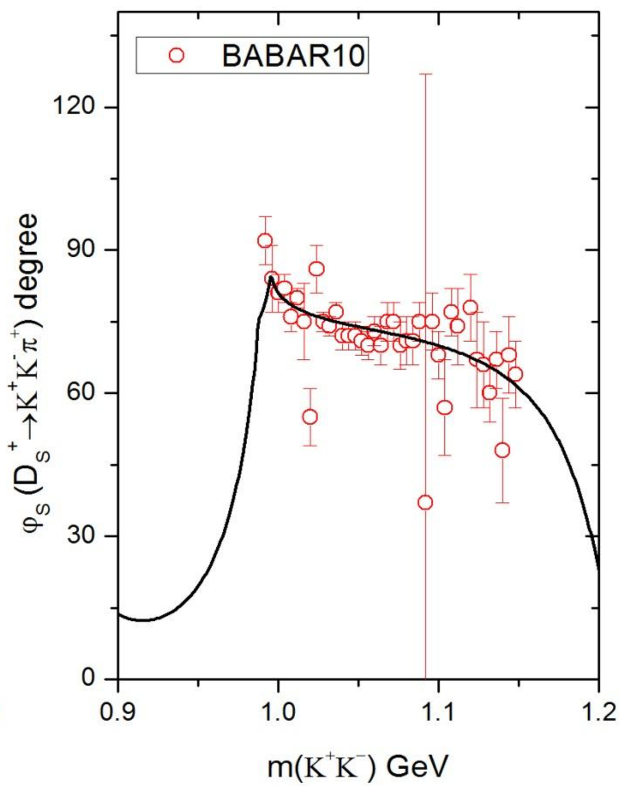
BABAR && BES

- $\pi\pi$ - KK **scattering inputs**
  - KK threshold region is important as it is around $f_0(980)$.



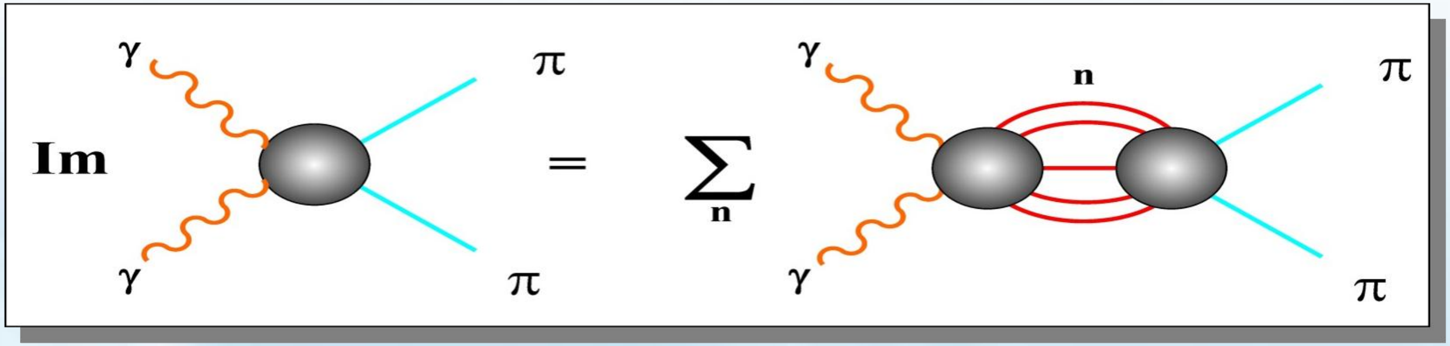






building amplitudes

- Final State Interaction Theorem
- Dispersion relations
- ChPT constraints

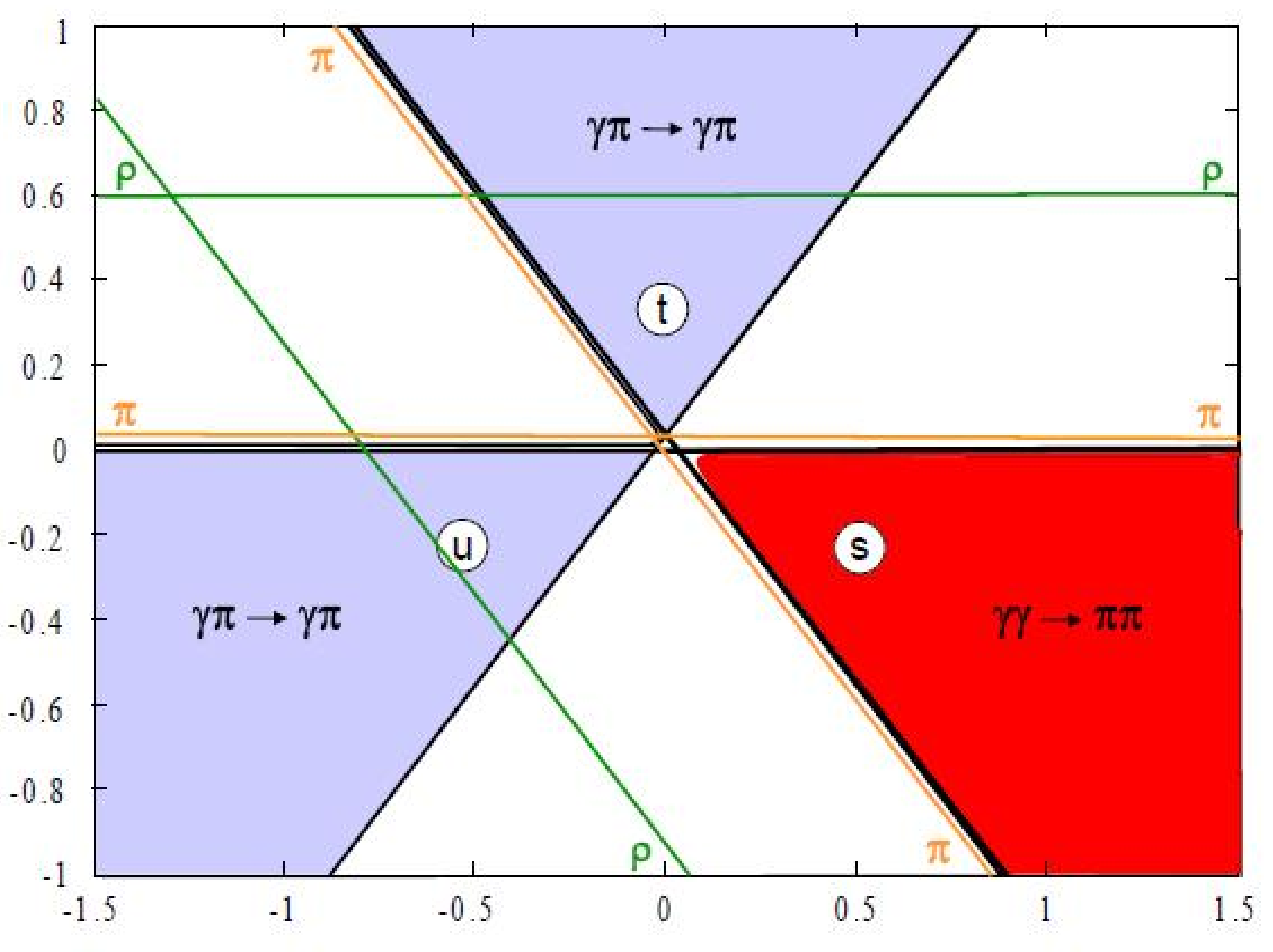


$$\mathcal{F}^I_{00}(s) = \mathcal{B}^I_{00}(s) + b^I s\, \Omega^I_{00}(s) + \frac{s^2\, \Omega^I_{00}(s)}{\pi} \int_L ds' \frac{\mathrm{Im}\left[\mathcal{L}^I_{00}(s')\right] \Omega^I_{00}(s')^{-1}}{s'^2(s'-s)} - \frac{s^2\, \Omega^I_{00}(s)}{\pi} \int_R ds' \frac{\mathcal{B}^I_{00}(s')\, \mathrm{Im}\left[\Omega^I_{00}(s')^{-1}\right]}{s'^2(s'-s)}$$

Solved by ChPT

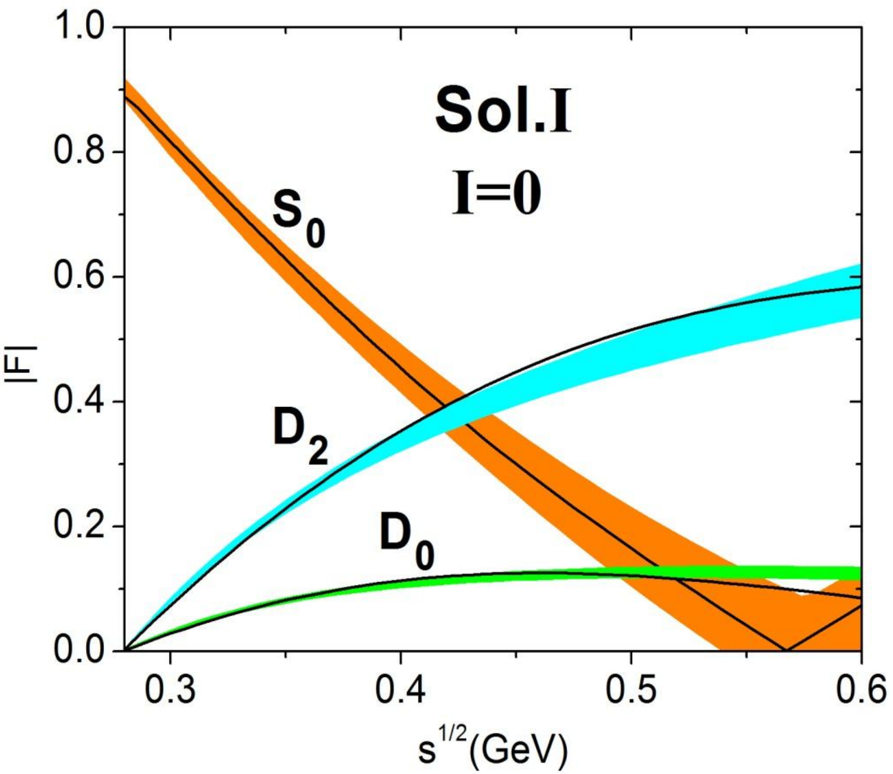
Vector, Axial-Vector, Tensor contributions

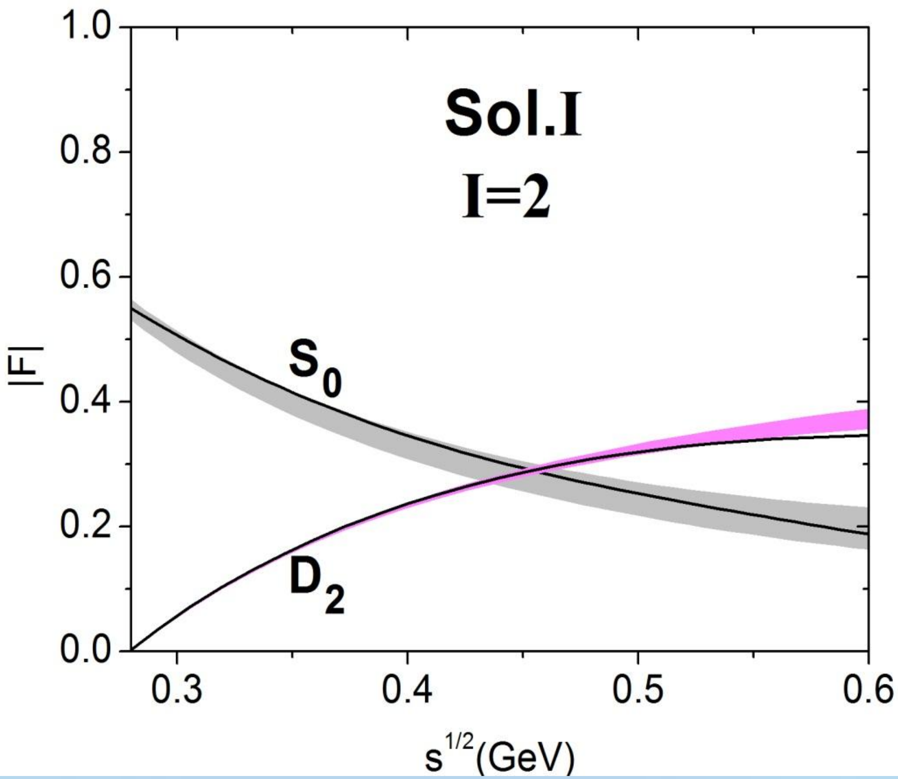
- LHCs of ρ, ω, a_1, b_1, h_1 give an error band of low energy amplitudes,
- Remain parts are parametrized as an effective pole 'T'.



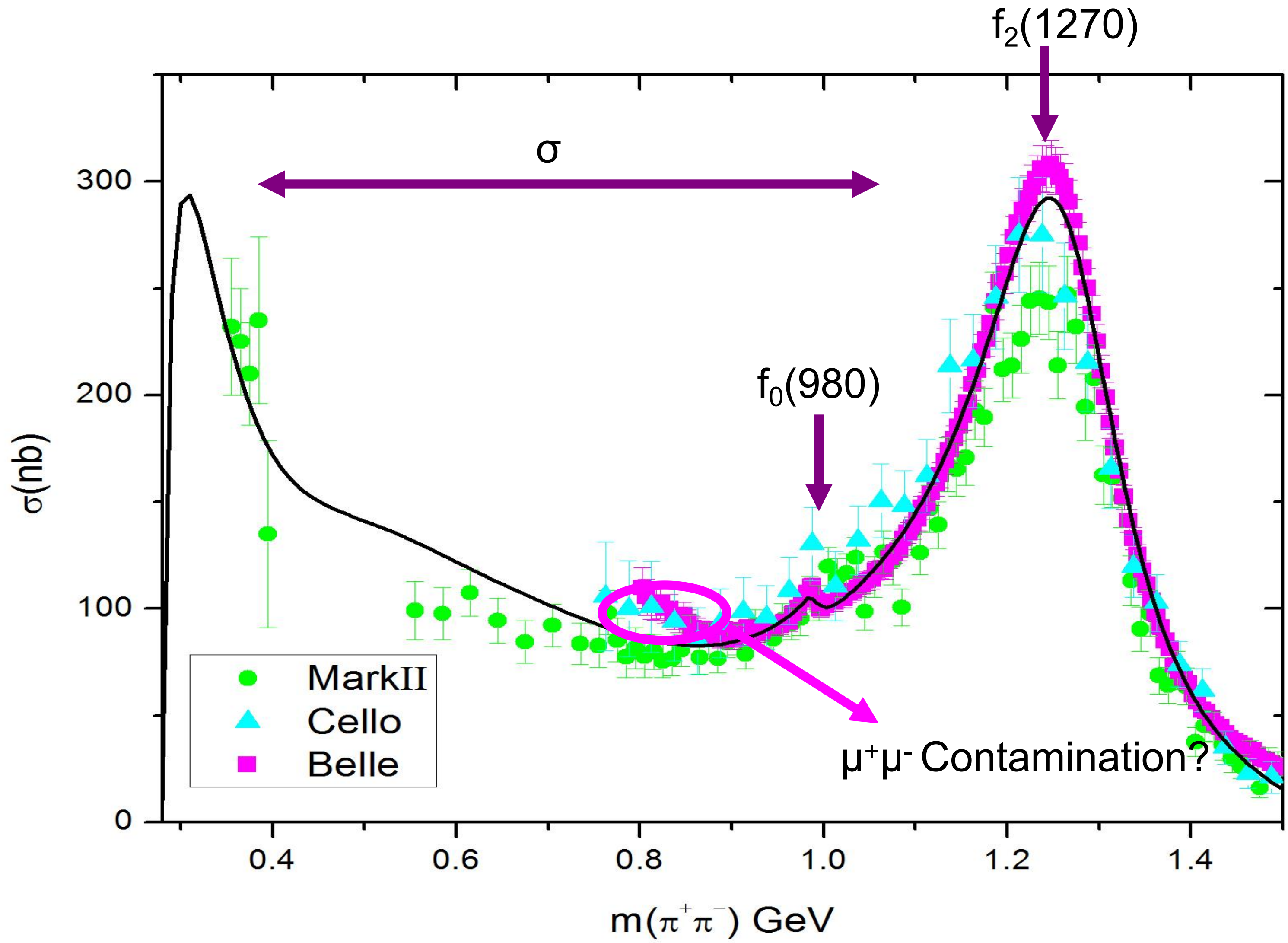
Constraints on low energy amplitudes

- Finally we have the bands given by dispersion relations:

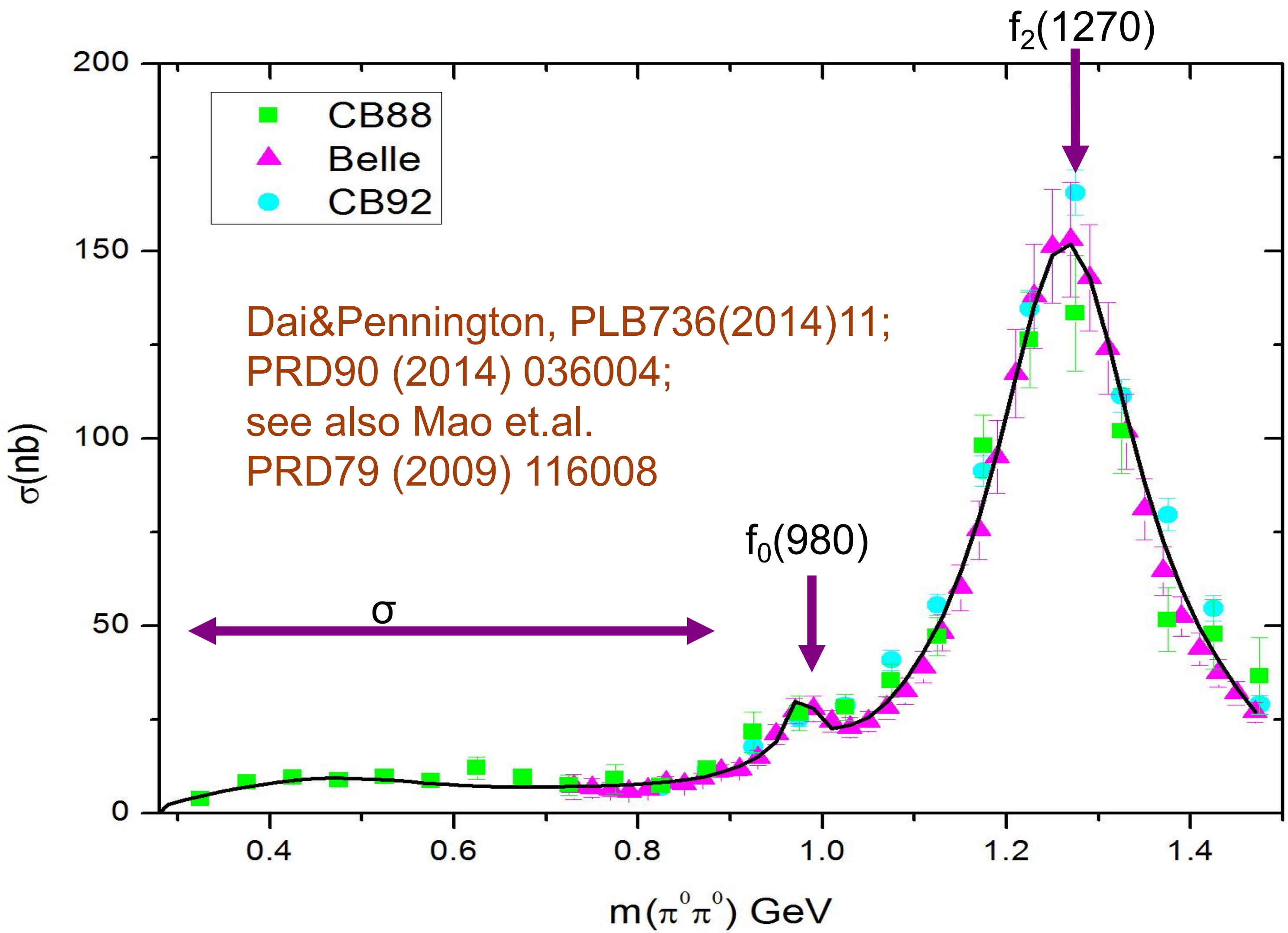




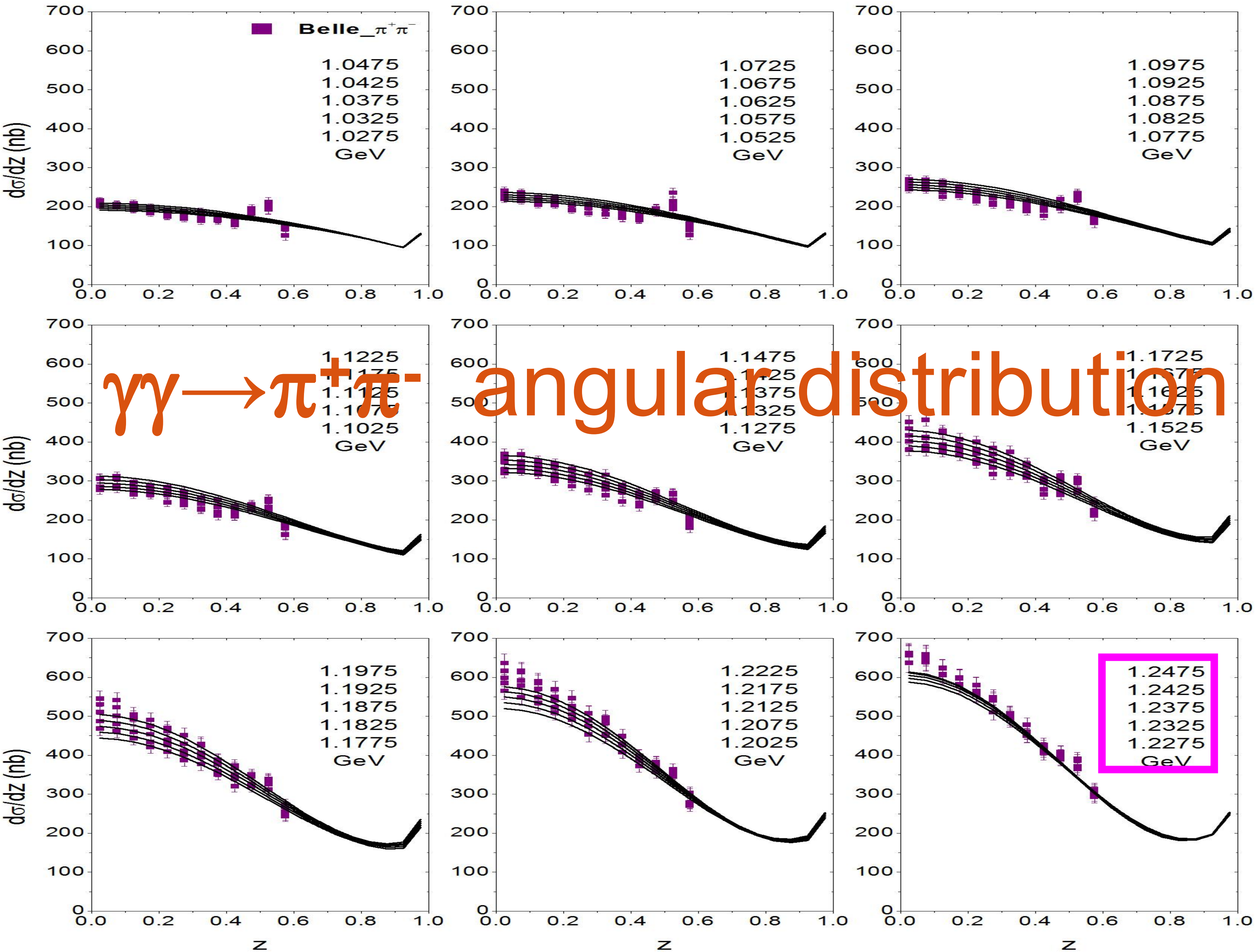
$\gamma\gamma \to \pi^+\pi^-$ integrated cross section



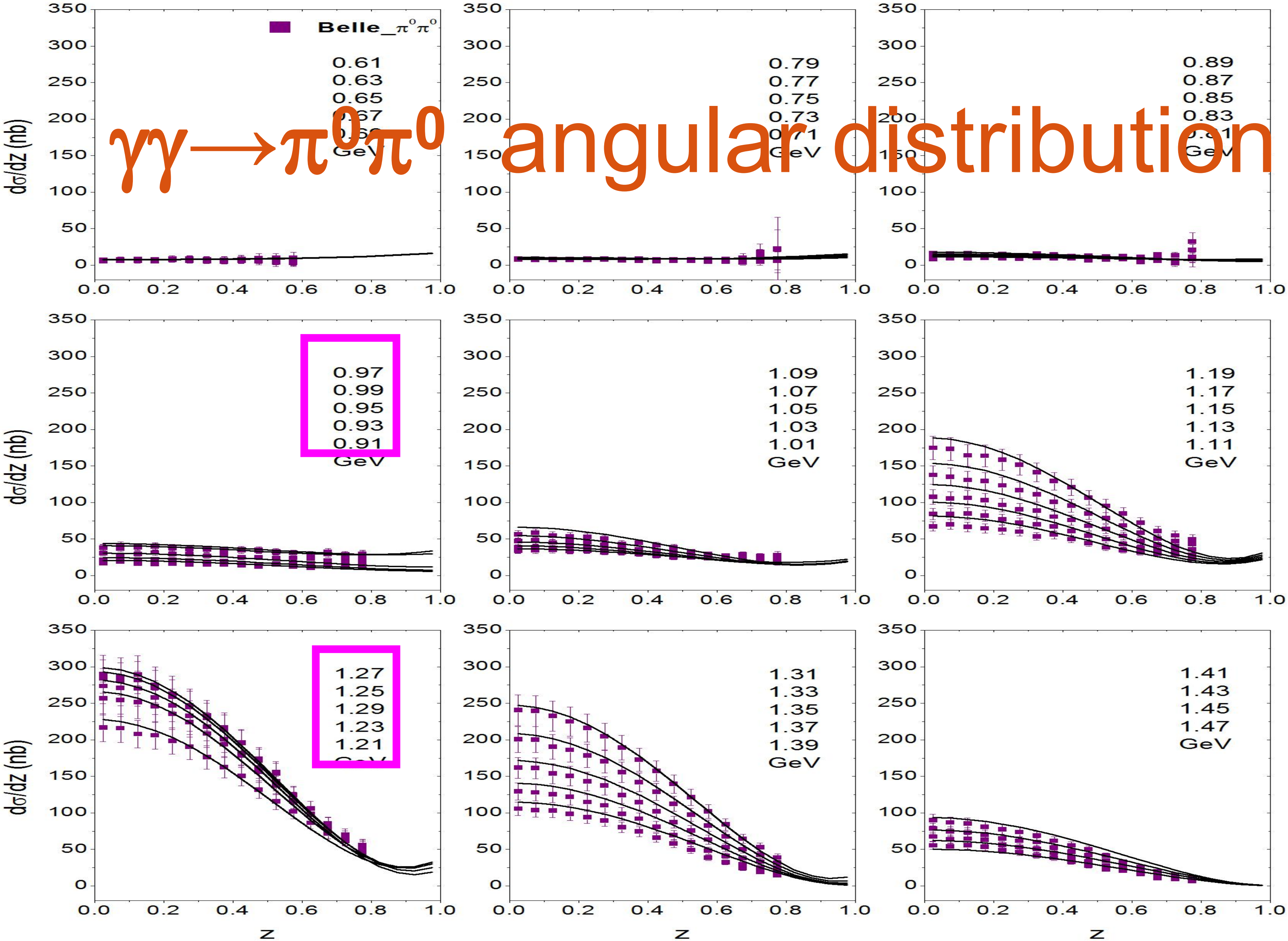
$\gamma\gamma \to \pi^0\pi^0$ integrated cross section



$\gamma\gamma \to \pi^+\pi^-$ angular distribution

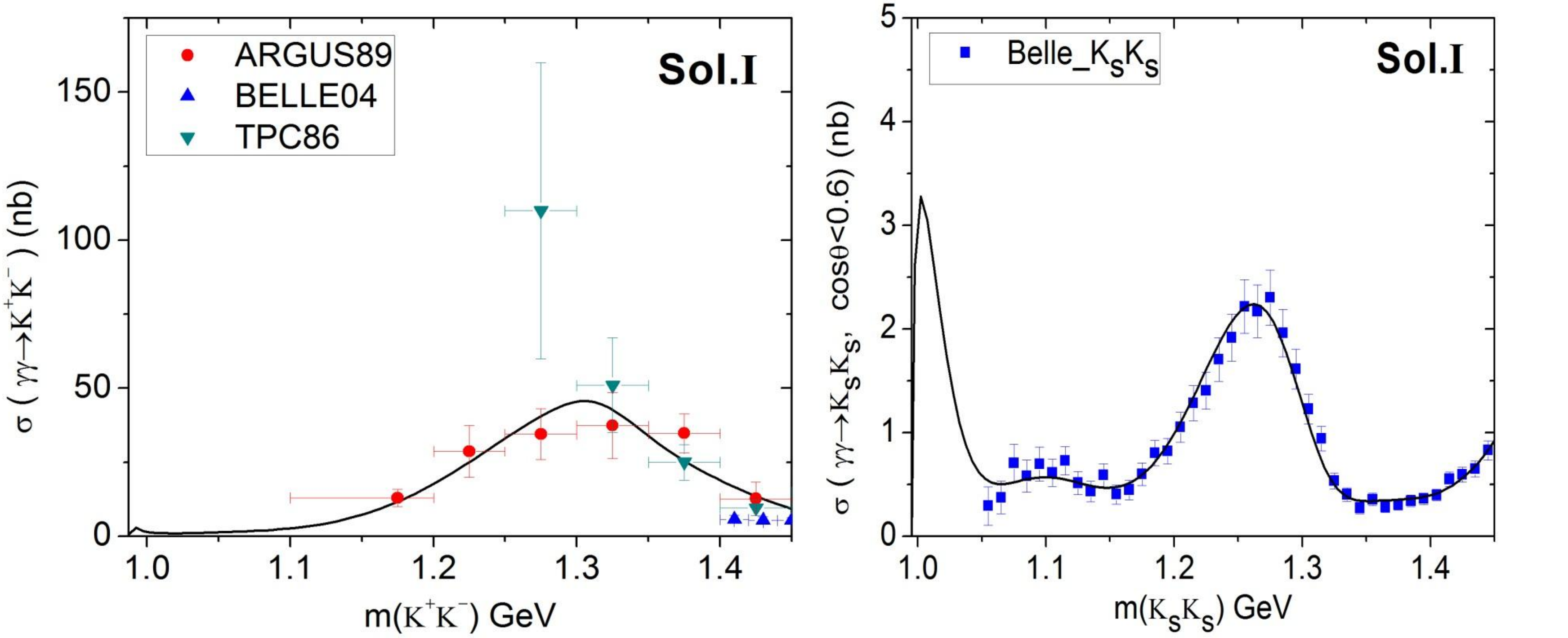


The angular distribution is helpful to seperate each partial wave.

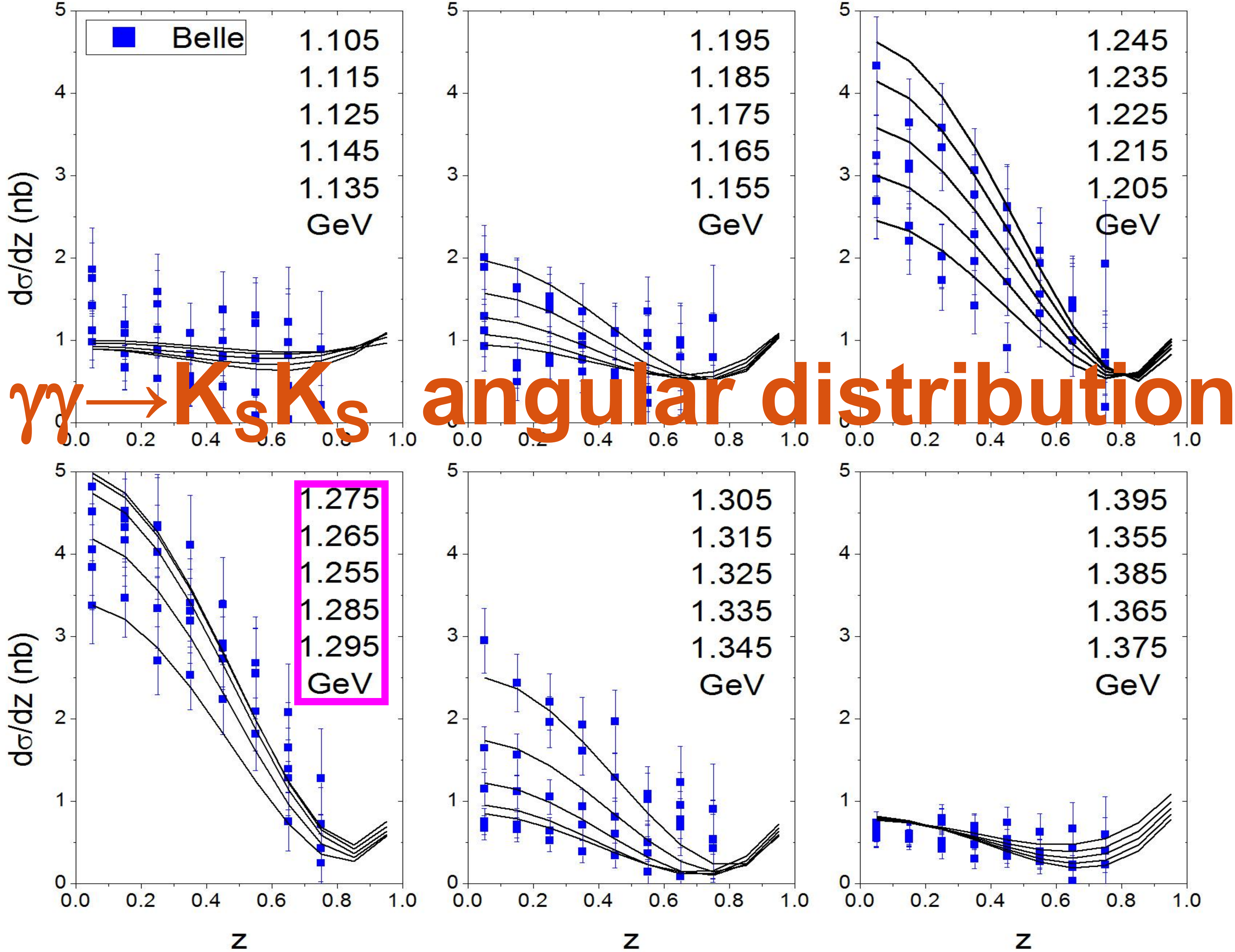


$\gamma\gamma \to KK$ integrated cross section

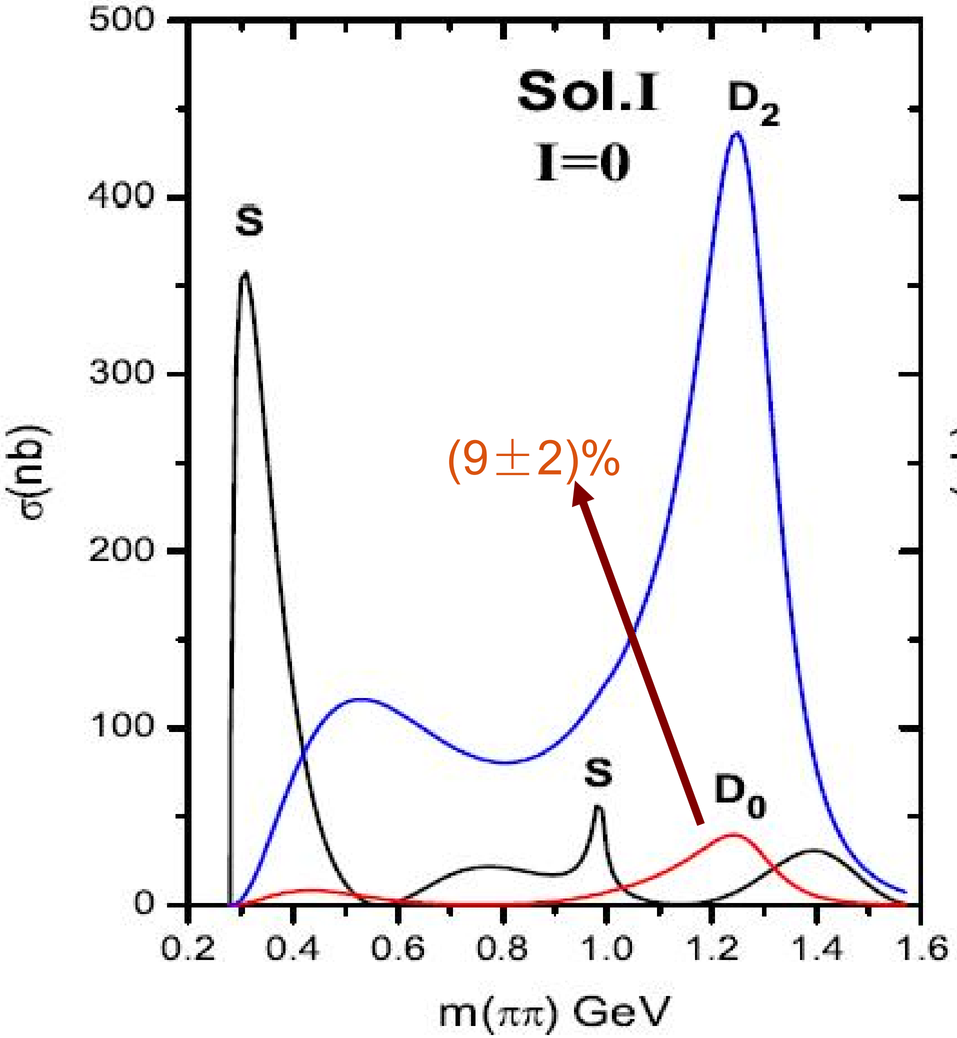
- If only fit to $\gamma\gamma \to \pi\pi$, we will get a region of solutions. $\gamma\gamma \to K\bar{K}$ data is helpful to select solutions.
- The latest $K_S K_S$ data of Belle make the accurate coupled channel analysis possible. Especially the angular distribution.

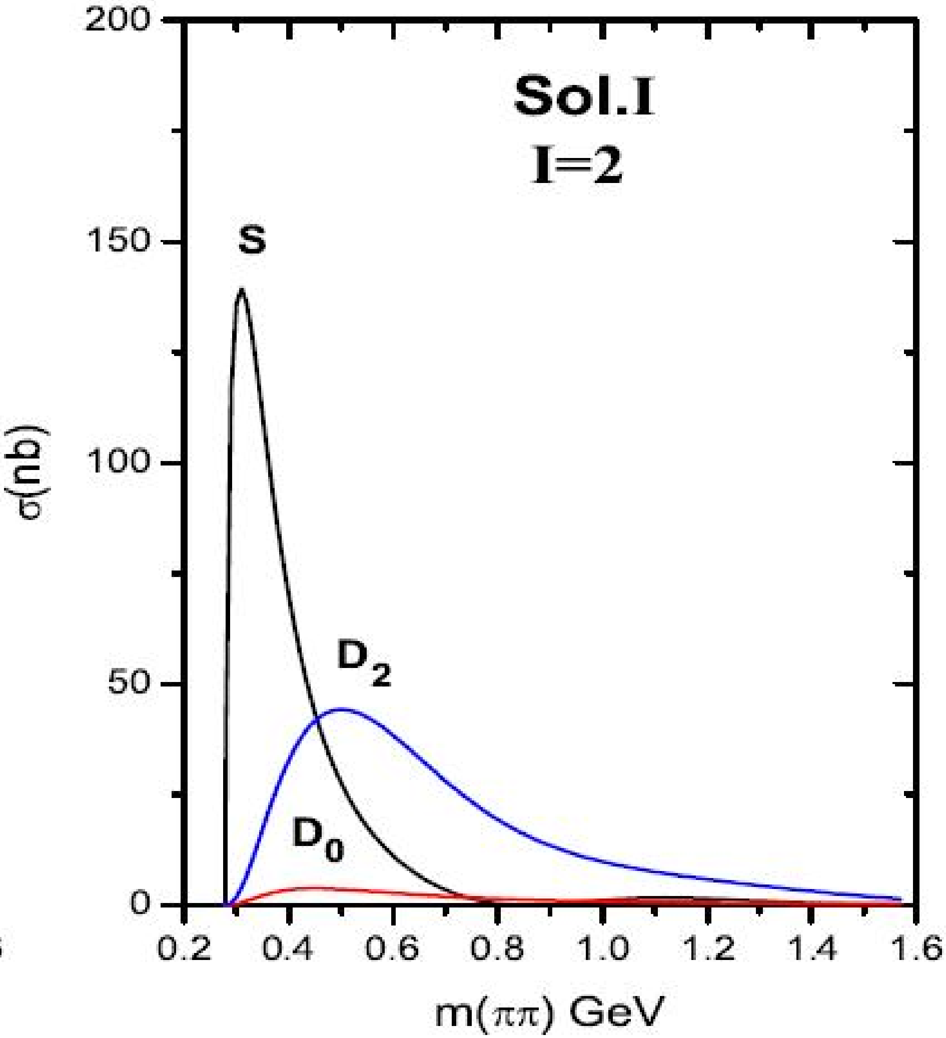


$\gamma\gamma \rightarrow K_S K_S$ angular distribution



γγ→ππ individual partial waves





$f_0(980) \to \gamma\gamma$

INSPIRE search

$\Gamma(f_0(980) \to \gamma\gamma)$ Γ_3

VALUE (keV)		DOCUMENT ID		TECN	COMMENT
$\mathbf{0.31^{+0.05}_{-0.04}}$	**OUR AVERAGE**				
0.32 ± 0.05	1	DAI	2014A	RVUE	Compilation
$0.286 \pm 0.017 {}^{+0.211}_{-0.070}$	2	UEHARA	2008A	BELL	10.6 $e^+ e^- \to e^+ e^- \pi^0 \pi^0$
$0.205 {}^{+0.095}_{-0.083} {}^{+0.147}_{-0.117}$	3	MORI	2007	BELL	10.6 $e^+ e^- \to e^+ e^- \pi^+ \pi^-$
$0.42 \pm 0.06 \pm 0.18$	4	OEST	1990	JADE	$e^+ e^- \to e^+ e^- \pi^0 \pi^0$
• • • We do not use the following data for averages, fits, limits, etc. • • •					
0.16 ± 0.01	5	MENNESSIER	2011	RVUE	
$0.29 \pm 0.21 {}^{+0.02}_{-0.07}$	6	MOUSSALLAM	2011	RVUE	Compilation
0.42	7, 8	PENNINGTON	2008	RVUE	Compilation
0.10	9, 8	PENNINGTON	2008	RVUE	Compilation
$0.28 {}^{+0.09}_{-0.13}$	10	BOGLIONE	1999	RVUE	$\gamma \gamma \to \pi^+ \pi^-, \pi^0 \pi^0$
$0.29 \pm 0.07 \pm 0.12$	11, 12	BOYER	1990	MRK2	$e^+ e^- \to e^+ e^- \pi^+ \pi^-$

$\Gamma(f_2(1270) \to \gamma\gamma)/\Gamma_{total}$ Γ_7/Γ

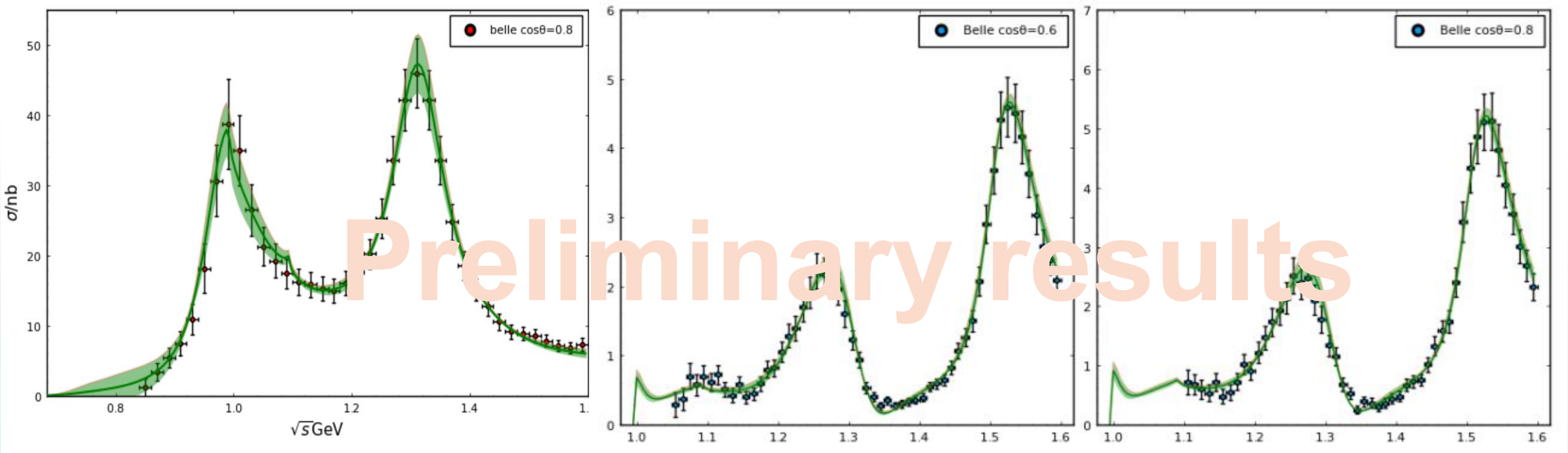
$\Gamma(f_2(1270) \to \gamma\gamma)$ Γ_7

The value of this width depends on the theoretical model used. Unitary approaches with scalars typically (with exception of PENNINGTON 2008) give values clustering around 2.6 keV; without an $S-wave$ contribution, values are systematically higher (typically around 3 keV).

VALUE (keV)	EVTS		DOCUMENT ID		TECN	COMMENT
2.6 ± 0.5	**OUR FIT** Error includes scale factor of 1.4.					
2.93 ±0.40		1	DAI	2014A	RVUE	Compilation
• • • We do not use the following data for averages, fits, limits, etc. • • •						
3.14 ±0.20		2, 3	PENNINGTON	2008	RVUE	Compilation
3.82 ±0.30		4, 3	PENNINGTON	2008	RVUE	Compilation
2.55 ±0.15	870	5	SCHEGELSKY	2006A	RVUE	$\gamma\gamma \to K^0_S K^0_S$
2.84 ±0.35			BOGLIONE	1999	RVUE	$\gamma\gamma \to \pi^+\pi^-$, $\pi^0\pi^0$
2.93 ±0.23 ±0.32		6	YABUKI	1995	VNS	
2.58 ±0.13 $^{+0.36}_{-0.27}$		7	BEHREND	1992	CELL	$e^+e^- \to e^+e^-\pi^+\pi^-$
3.10 ±0.35 ±0.35		8	BLINOV	1992	MD1	$e^+e^- \to e^+e^-\pi^+\pi^-$
2.27 ±0.47 ±0.11			ADACHI	1990D	TOPZ	$e^+e^- \to e^+e^-\pi^+\pi^-$

Other γγ collisions

- πη-KK-πη' coupled channel scatterings



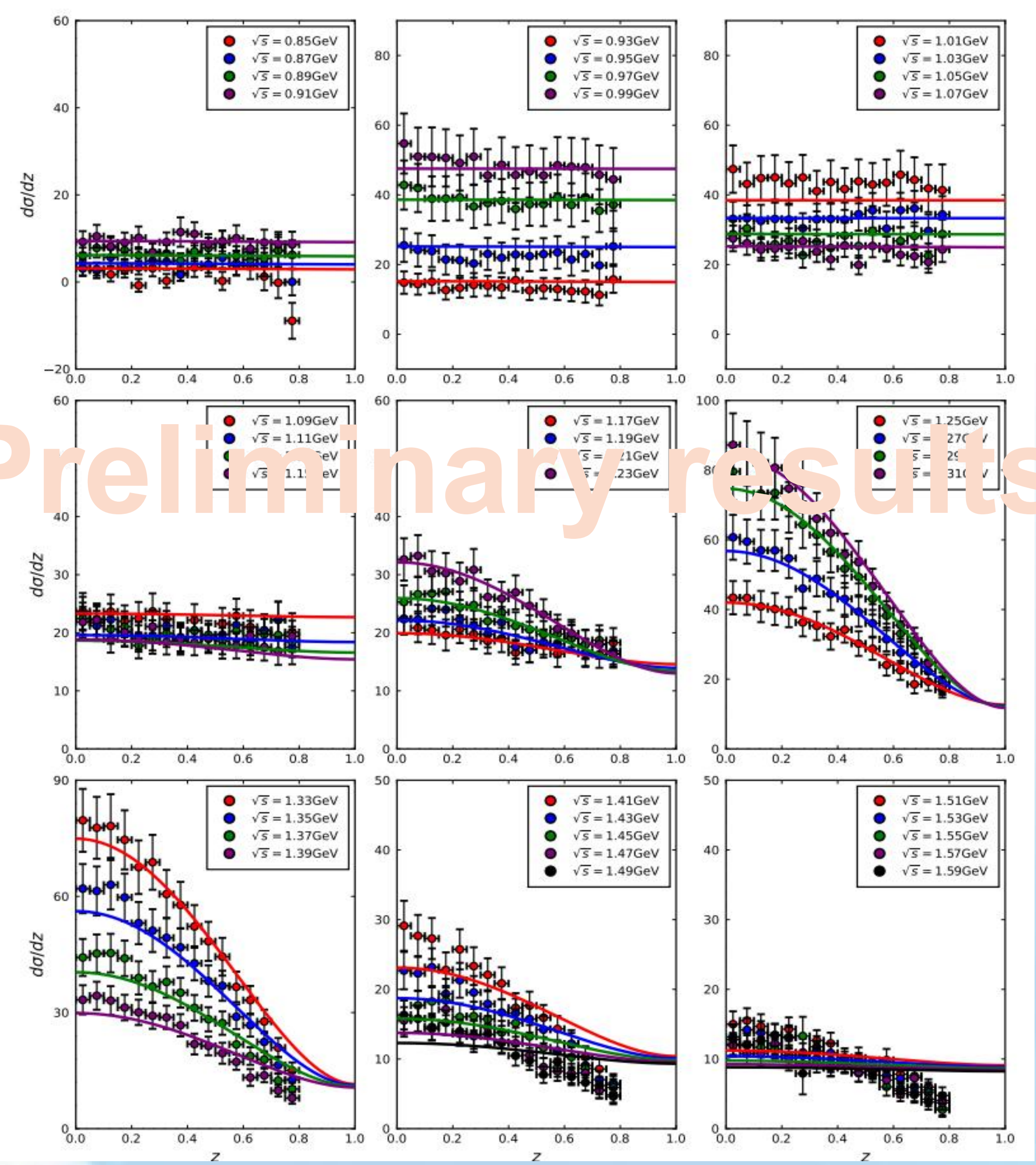
Kuang, Dai *et.al.*, in preparation

- DR+ChEFT constraints
- AMP: FSI

Experiment	Process	Data-points	χ^2_{average}
Belle/Crystal ball	$\gamma\gamma \to \pi^0\eta$	680	
CB(AGS)/A2 MAMI-B	$\eta \to \pi^0\gamma\gamma$	21	
TPC/Argus/Belle	$\gamma\gamma \to K^+K^-$	18	
TASSO/CELLO	$\gamma\gamma \to \bar{K}^0K^0$	5	
Belle	$\gamma\gamma \to \bar{K}^0_SK^0_S$	315	
BESIII	$\eta' \to \pi^0\gamma\gamma$	13	

angular distribution

- $a_0(980)$?
- HLBL constraints for I=1



Constraints to light-by-light sumrule

- For LbL one needs photons with virtualities. Our massless photon amplitudes are boundary values when $Q^2 = 0$.
- Narrow resonance estimates from the tensor mesons are not a good approximation.
- Test the Pascalutsa-Vanderhaeghen sumrule.：

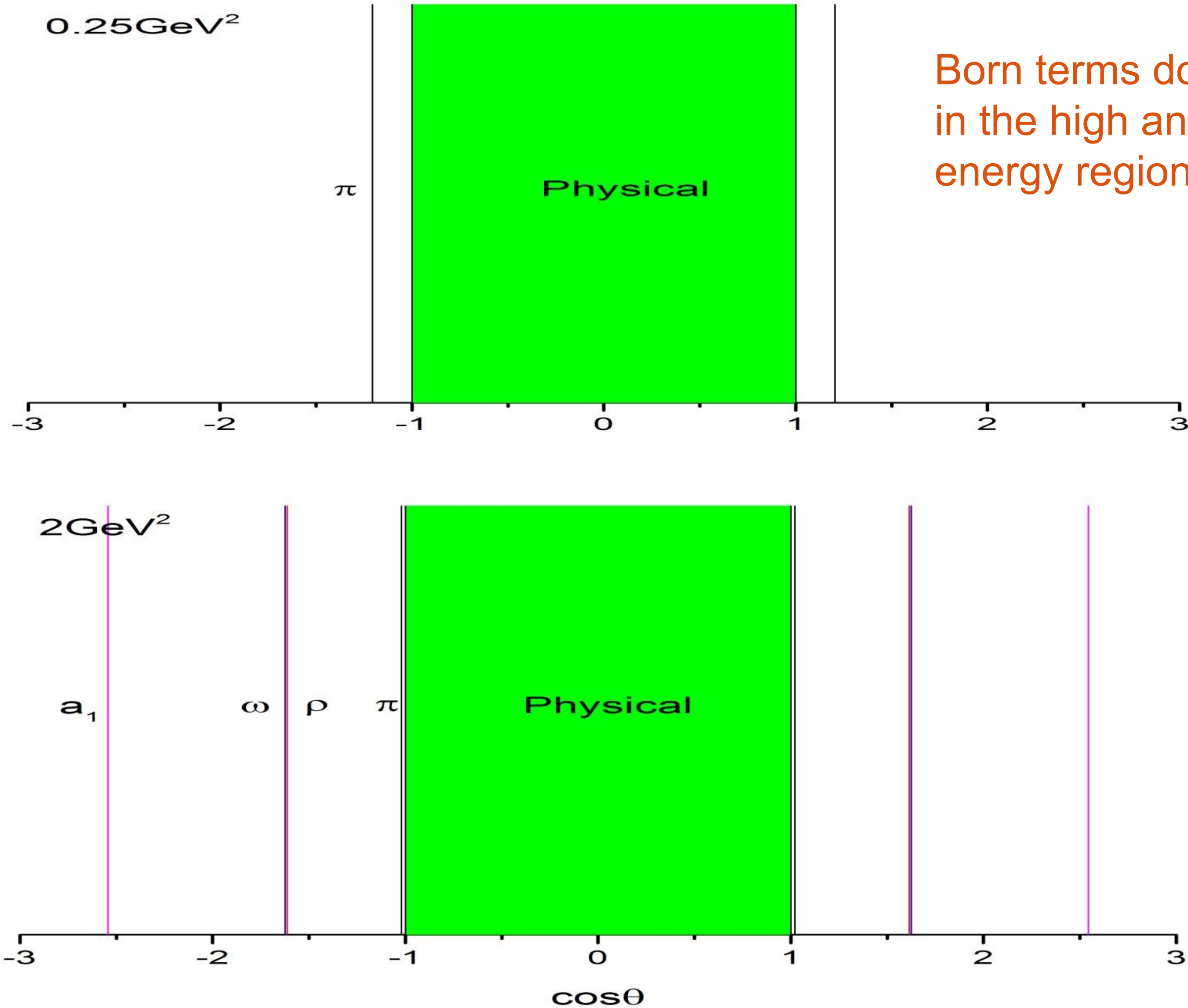
$$0 = \int_0^\infty ds \, \frac{\Delta\sigma(s)}{s},$$

$$\Delta\sigma(s) = \sigma_2(s) - \sigma_0(s)$$

$$c_1 \pm c_2 = \frac{1}{8\pi} \int_0^\infty ds \, \frac{\sigma_{||}(s) \pm \sigma_{\perp}(s)}{s^2}.$$

V.Pascalutsa & M.Vanderhaeghen, PRL105 (2010) 201603.

Born term dominance



Born terms dominates in the high and low energy regions.

Constraints to light-by-light sumrule

- The contribution to PV sumrule is certainly not zero.
- 4π channel's contribution is significant for HLBL
- I=0:150–200 nb, I=2: 50nb

evaluation of $\Delta^I(4m_\pi^2, \infty, Z=1)$	$I=0$	$I=1$	$I=2$
$\gamma\gamma \to \pi^0$ [6] (nb)	-	-190.9±4.0	-
$\gamma\gamma \to \eta, \eta'$ [6] (nb)	-497.7±19.3	-	-
$\gamma\gamma \to a_2(1320)$ [6] (nb)	-	*135.0±12±25 †*	-
$\gamma\gamma \to \pi\pi$ (nb)	308.0±41.5	-	-44.2±6.1
$\gamma\gamma \to \overline{K}K$ (nb)	23.7±7.5	18.1±4.9	-
SUM (nb)	-166.0±46.4	-37.8±28.4	-44.2±6.1

BESIII? BelleII?

Dai&Pennington, PRD95 (2017) 056007;

Constraints to light-by-light sumrule

- 4π channel's contribution is roughly of 150–200 nb in the I = 0 mode and 50 nb in the I = 2 mode.
- We have no decomposition information about the amplitudes of multi-particles' channel.

$$\mathcal{R}(s_1, s_2; \text{channel}) = \frac{\Delta(s_1, s_2, Z = 1; \text{channel})}{\Sigma(s_1, s_2, Z_{\text{exp}}; \text{channel})}$$

contribution to PV sumrule

total cross section

Channel	Publication	E_1 (GeV)	E_2 (GeV)	Σ (nb)	$\mathcal{R}(Born)$
$\pi^+\pi^-$ $(Z = 0.6)$	[16]	2.4	4.1	0.44 ± 0.01	1.61
K^+K^- $(Z = 0.6)$	[16]	2.4	4.1	0.39 ± 0.01	1.29
$\pi^0\pi^0$ $(Z = 0.8)$	[17]	1.44	3.3	8.8 ± 0.2	1.18
$\pi^0\pi^0\pi^0$	[18]	1.525	2.425	5.8 ± 0.8	1.55
$\pi^+\pi^-\pi^0$ (non-res.)	[19]	0.8	2.1	23.0 ± 1.3	1.39
$K_sK^\pm\pi^\mp$	[20]	1.4	4.2	9.7 ± 1.6	
$\pi^+\pi^-\pi^+\pi^-$	[21]	1.1	2.5	$215 \pm 11 \pm 21$	1.49
$\pi^+\pi^-\pi^+\pi^-$	[22]	1.0	3.2	$153 \pm 5 \pm 39$	1.48
$\pi^+\pi^-\pi^0\pi^0$	[23]	0.8	3.4	$103 \pm 4 \pm 14$	1.42

Pion polarizabilities

Amplitudes

polarizability: by extrapolation at s=0

?

cross section

Polarizabilities

Polarizabilities may also play important role on LbL sumrule

K.T.Engel et.al. PRD86 (2012) 037502

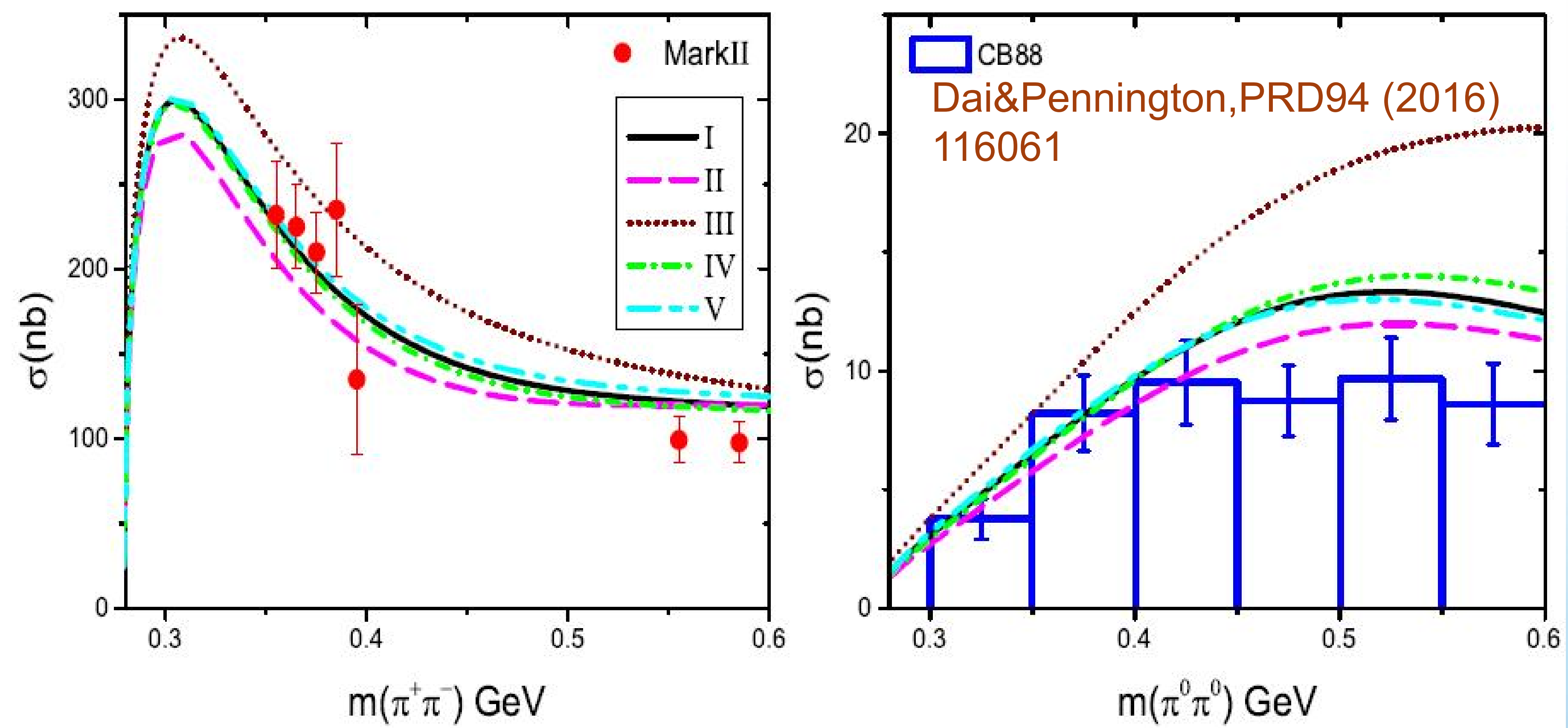
fixed by Adler zero and $(\alpha_1-\beta_1)_{\pi+}$ = 4.0

easiest one to be measured by experiment

Polarizabilities $\lambda=0$	Model I	Model II	Model III	Model IV	Model V	ChPT + Resonance Model
$(\alpha_1-\beta_1)_{\pi^+}$	**4.0 ± 1.2 ± 1.4**	**0.0**	**11.6**	**4.0**	**4.0**	5.7±1.0
$(\alpha_2-\beta_2)_{\pi^+}$	15.7±1.1	13.0±1.1	20.9±1.1	13.2±3.4	18.1±2.5	16.2[21.6]
$(\alpha_1-\beta_1)_{\pi^0}$	-0.9±0.2	-0.8±0.1	-1.1±0.2	-0.8±0.2	-1.0±0.2	-1.9±0.2
$(\alpha_2-\beta_2)_{\pi^0}$	20.6±0.8	17.8±0.8	26.0±0.8	18.6±2.4	22.4±1.8	37.6±3.3
$\lambda=2$						
$(\alpha_1+\beta_1)_{\pi^+}$	0.26±0.07	0.26±0.07	0.26±0.07	0.17±0.51	0.42±0.22	0.16[0.16]
$(\alpha_2+\beta_2)_{\pi^+}$	-1.4±0.5	-1.4±0.5	-1.4±0.5	-0.9±3.5	-2.4±1.5	-0.001
$(\alpha_1+\beta_1)_{\pi^0}$	0.60±0.06	0.60±0.06	0.60±0.06	-0.04±0.52	0.90±0.17	1.1±3.3
$(\alpha_2+\beta_2)_{\pi^0}$	-3.7±0.4	-3.7±0.4	-3.7±0.4	0.4±3.4	-5.5±1.1	0.04

Polarizabilities

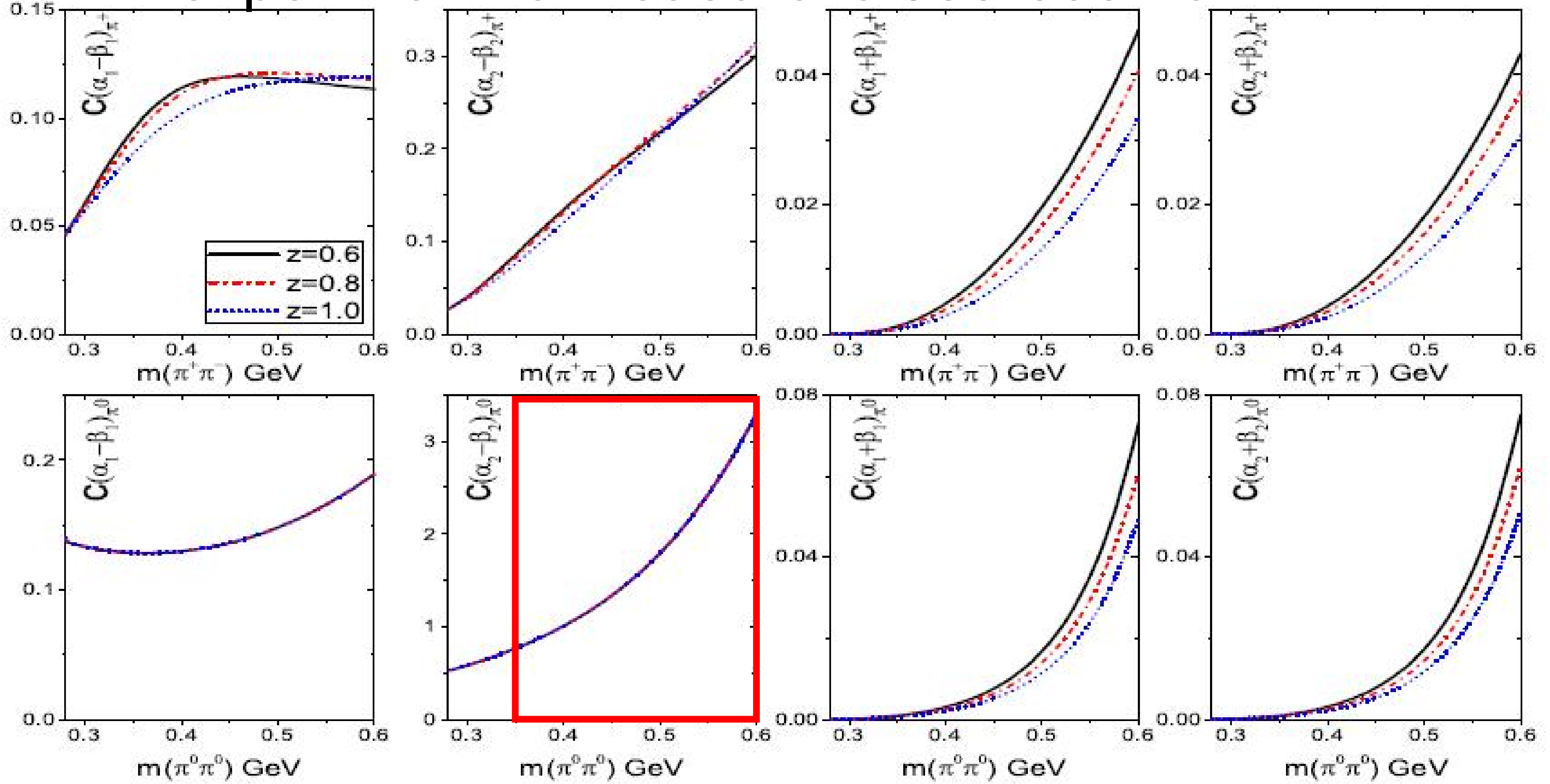
Polarizabilities plays important role on HLbL DRs



$(\alpha_1-\beta_1)_{\pi+} = 11.6$, has been exclude by CB's data, JLAB's new measurement?

Correlation functions

- The correlations between pion polarizabilities and $\gamma\gamma\rightarrow\pi\pi$ cross sections: the best region for experiment to measure is 350-600MeV.



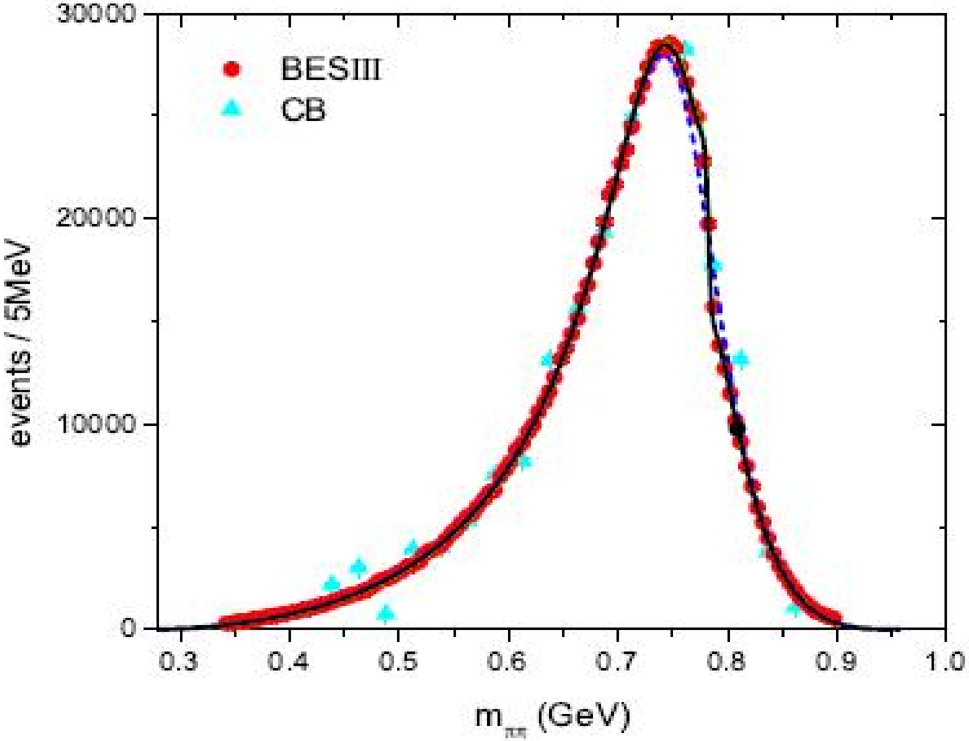
LbL

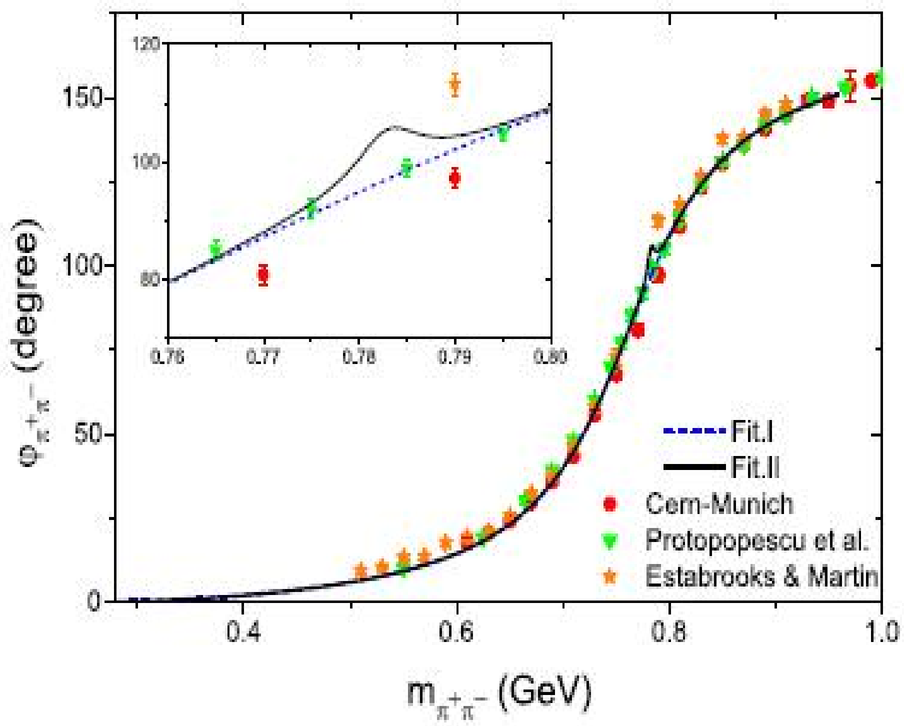
- $\pi^+\pi^-$ P-wave phase-shift should take into consideration of isospin violation

Dai et.al.,PRD97 (2018) 036012

$\gamma^* \to \pi^+\pi^-$

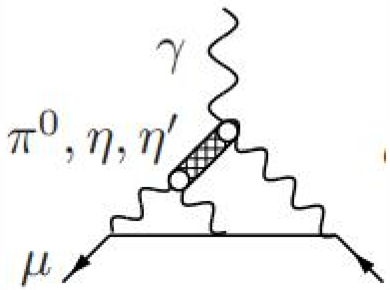
$\gamma^*\gamma \to \pi^+\pi^-$



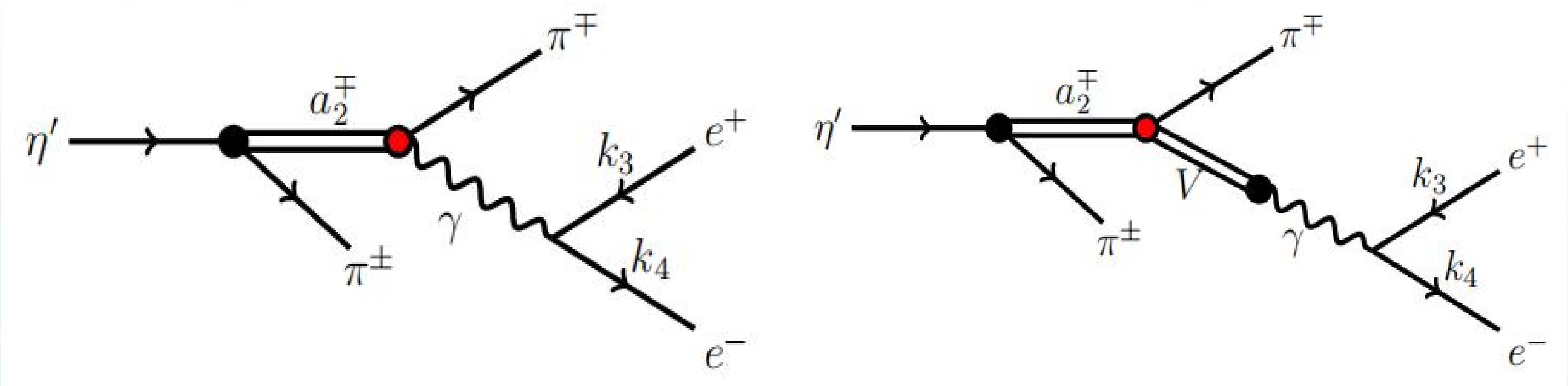


TFFs

- TFFs



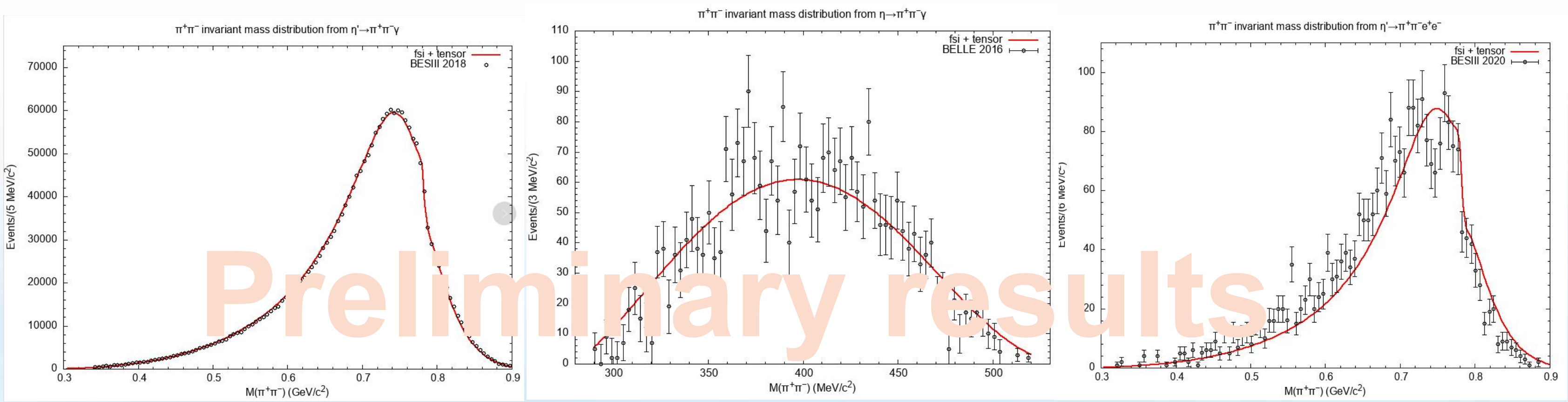
Jegerlehner&Nyffeler, Phys.Rept. 477 (2009) 1-110



- Tensors are included in RChT.
- High energy constraints to reduce unknown couplings

TFFs

- TFFs



Ye, *et.al.*, in preparation

- HLbL contribution from pseudoscalar poles

$$a_\mu^{\mathrm{LbL};\pi^0} = -\frac{2\alpha^3}{3\pi^2}\int_0^\infty \mathrm{d}Q_1\mathrm{d}Q_2 \int_{-1}^{+1} \mathrm{d}t\,\sqrt{1-t^2}\,Q_1^3 Q_2^3\,\left[F_1\,P_6\,I_1(Q_1,Q_2,t) + F_2\,P_7\,I_2(Q_1,Q_2,t)\right]$$

4、Summary

FSI

Amplitude analsysis connects QFT principles and Exp. FSI needs to be considered when performing amplitude analysis.

HVP

Ours has a significant discrepancy with the latest FNAL's. Processes of multi-body channels needs to be studied.

HLBL

We have strong constraints to HLBL amplitudes. 4π's can not be ignored. $\pi\pi\pi\pi$, $\pi\pi KK$?

Next?

Further study of light hadrons is neccessary to give a more reliable answer to muon g-2; Discrepancy between LQCD v.s. data driven+ChEFT+FSI?

Thank You For your patience !