组会

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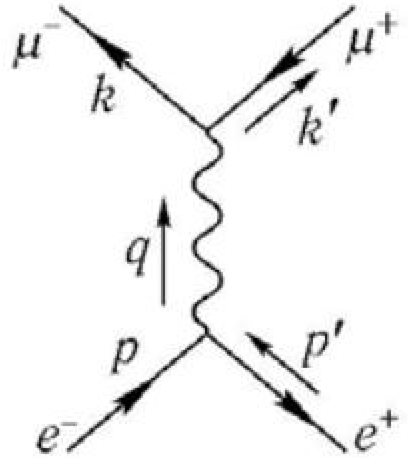
量子电动力学的基本过程

1.画出所需过程的费曼图，并用费曼规则写下振幅



$$= \bar{v}^{s'}(p')(-ie\gamma^{\mu})u^{s}(p)\left(\frac{-ig_{\mu\nu}}{q^2}\right)\bar{u}^{r}(k)(-ie\gamma^{\nu})v^{r'}(k').$$

e+e-→μ+μ-

2.将振幅平方，并使用完备性关系对自旋求和

$$i\mathcal{M}\big(e^-(p)e^+(p') \to \mu^-(k)\mu^+(k')\big) = \frac{ie^2}{q^2}\Big(\bar{v}(p')\gamma^\mu u(p)\Big)\Big(\bar{u}(k)\gamma_\mu v(k')\Big).$$

$$\big(\bar{v}\gamma^\mu u\big)^* = u^\dagger(\gamma^\mu)^\dagger(\gamma^0)^\dagger v = u^\dagger(\gamma^\mu)^\dagger\gamma^0 v = u^\dagger\gamma^0\gamma^\mu v = \bar{u}\gamma^\mu v.$$

$$|\mathcal{M}|^2 = \frac{e^4}{q^4}\Big(\bar{v}(p')\gamma^\mu u(p)\bar{u}(p)\gamma^\nu v(p')\Big)\Big(\bar{u}(k)\gamma_\mu v(k')\bar{v}(k')\gamma_\nu u(k)\Big).$$

$$\frac{1}{2}\sum_s \frac{1}{2}\sum_{s'}\sum_r\sum_{r'}\big|\mathcal{M}(s,s'\to r,r')\big|^2.$$

完备性关系: $\displaystyle\sum_s u^s(p)\bar{u}^s(p) = \not{p} + m;$ $\displaystyle\sum_s v^s(p)\bar{v}^s(p) = \not{p} - m.$

$$\begin{aligned}\sum_{s,s'}\bar{v}_a^{s'}(p')\gamma^\mu_{ab}u_b^s(p)\bar{u}_c^s(p)\gamma^\nu_{cd}v_d^{s'}(p') &= (\not{p}' - m)_{da}\gamma^\mu_{ab}(\not{p}+m)_{bc}\gamma^\nu_{cd}\\ &= \operatorname{trace}\big[(\not{p}' - m)\gamma^\mu(\not{p}+m)\gamma^\nu\big].\end{aligned}$$

$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2 = \frac{e^4}{4q^4}\operatorname{tr}\Big[(\not{p}'-m_e)\gamma^\mu(\not{p}+m_e)\gamma^\nu\Big]\operatorname{tr}\Big[(\not{k}+m_\mu)\gamma_\mu(\not{k}'-m_\mu)\gamma_\nu\Big].$$

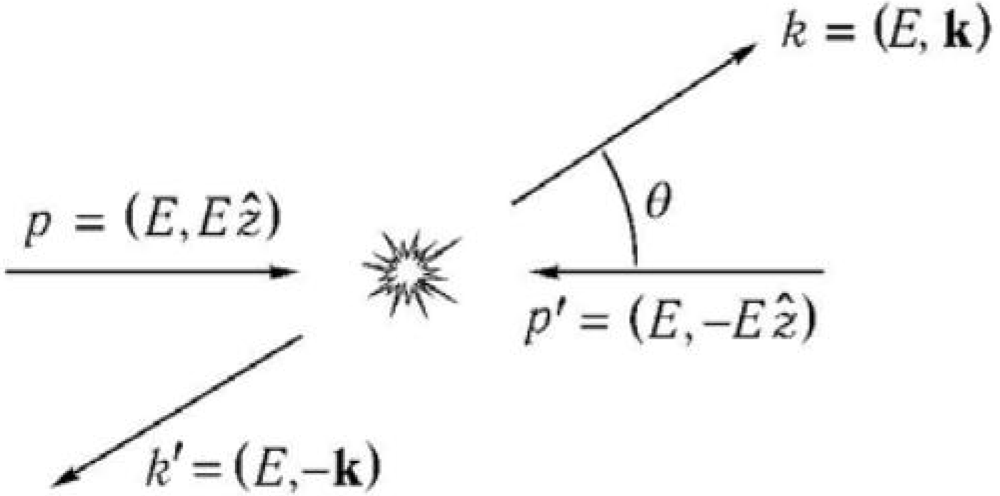
3.使用迹定理求迹，整合各项并化简答案

$$\mathrm{tr}\big[(\not{p}' - m_e)\gamma^\mu(\not{p} + m_e)\gamma^\nu\big] = 4\big[p'^\mu p^\nu + p'^\nu p^\mu - g^{\mu\nu}(p\cdot p' + m_e^2)\big].$$

$$\mathrm{tr}\big[(\not{k} + m_\mu)\gamma_\mu(\not{k}' - m_\mu)\gamma_\nu\big] = 4\big[k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu}(k\cdot k' + m_\mu^2)\big].$$

$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2 = \frac{8e^4}{q^4}\Big[(p\cdot k)(p'\cdot k') + (p\cdot k')(p'\cdot k) + m_\mu^2(p\cdot p')\Big].$$

4.选择一个特定的参照系（质心系），画出该参照系中的运动学变量，选择适当的一组变量来表示4动量向量



$$|\mathbf{k}| = \sqrt{E^2 - m_\mu^2}$$

$$\mathbf{k} \cdot \hat{z} = |\mathbf{k}| \cos\theta$$

$$q^2 = (p + p')^2 = 4E^2; \qquad p \cdot p' = 2E^2;$$

$$p \cdot k = p' \cdot k' = E^2 - E|\mathbf{k}| \cos\theta; \qquad p \cdot k' = p' \cdot k = E^2 + E|\mathbf{k}| \cos\theta.$$

5.用E和θ重写振幅平方，并将得到的振幅平方插入截面公式，对没有被测量的相变量进行积分

$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2 = \frac{8e^4}{16E^4}\Big[E^2(E-|\mathbf{k}|\cos\theta)^2 + E^2(E+|\mathbf{k}|\cos\theta)^2 + 2m_\mu^2 E^2\Big]$$

$$= e^4\left[\left(1+\frac{m_\mu^2}{E^2}\right) + \left(1-\frac{m_\mu^2}{E^2}\right)\cos^2\theta\right].$$

截面公式: $$\left(\frac{d\sigma}{d\Omega}\right)_{\text{CM}} = \frac{1}{2E_{\mathcal{A}}2E_{\mathcal{B}}|v_{\mathcal{A}}-v_{\mathcal{B}}|}\frac{|\mathbf{p}_1|}{(2\pi)^2\,4E_{\text{cm}}}\big|\mathcal{M}(p_{\mathcal{A}},p_{\mathcal{B}}\to p_1,p_2)\big|^2.$$

对于我们的问题 $|V_A - V_B| = 2$, $E_A = E_B = E_{cm}/2$

所以：

$$\frac{d\sigma}{d\Omega} = \frac{1}{2E_{\rm cm}^2}\frac{|\mathbf{k}|}{16\pi^2 E_{\rm cm}}\cdot\frac{1}{4}\sum_{\rm spins}|\mathcal{M}|^2$$

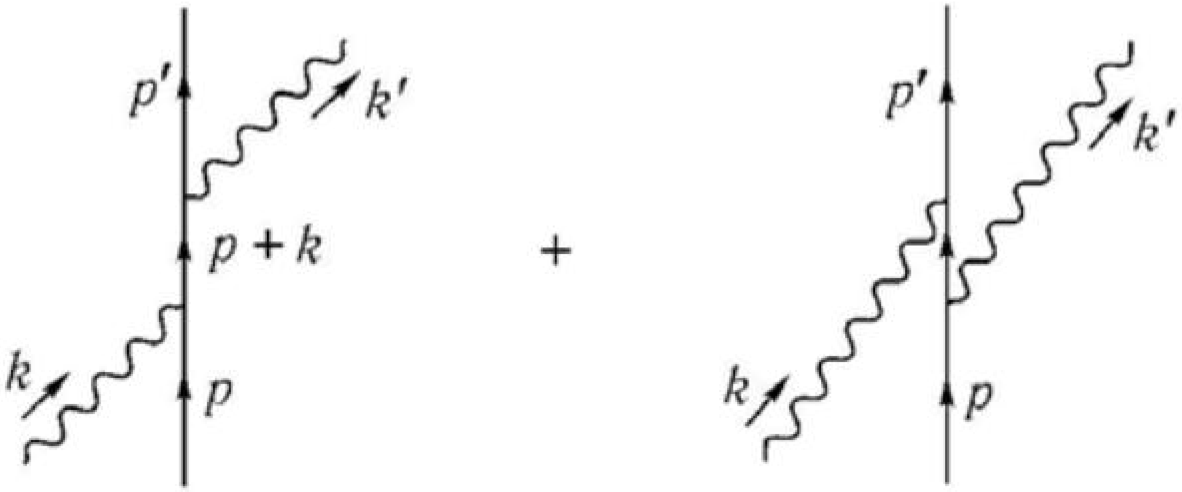
$$= \frac{\alpha^2}{4E_{\rm cm}^2}\sqrt{1-\frac{m_\mu^2}{E^2}}\left[\left(1+\frac{m_\mu^2}{E^2}\right)+\left(1-\frac{m_\mu^2}{E^2}\right)\cos^2\theta\right].$$

$$\sigma_{\rm total} = \frac{4\pi\alpha^2}{3E_{\rm cm}^2}\sqrt{1-\frac{m_\mu^2}{E^2}}\left(1+\frac{1}{2}\frac{m_\mu^2}{E^2}\right).$$

在E＞＞Mμ的高能极限下

$$\frac{d\sigma}{d\Omega} \xrightarrow[E\gg m_\mu]{} \frac{\alpha^2}{4E_{\rm cm}^2}(1+\cos^2\theta);$$

$$\sigma_{\rm total} \xrightarrow[E\gg m_\mu]{} \frac{4\pi\alpha^2}{3E_{\rm cm}^2}\left(1-\frac{3}{8}\left(\frac{m_\mu}{E}\right)^4-\cdots\right).$$



$e^-\gamma \to e^-\gamma$ （康普顿散射）

$$i\mathcal{M} = \bar{u}(p')(-ie\gamma^\mu)\epsilon_\mu^*(k')\frac{i(\not{p}+\not{k}+m)}{(p+k)^2-m^2}(-ie\gamma^\nu)\epsilon_\nu(k)u(p)$$
$$+\bar{u}(p')(-ie\gamma^\nu)\epsilon_\nu(k)\frac{i(\not{p}-\not{k}'+m)}{(p-k')^2-m^2}(-ie\gamma^\mu)\epsilon_\mu^*(k')u(p)$$

$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2 = \frac{e^4}{4}\, g_{\mu\rho}\, g_{\nu\sigma} \cdot \mathrm{tr}\Big\{(\not{p}'+m)\Big[\frac{\gamma^\mu \not{k}\gamma^\nu + 2\gamma^\mu p^\nu}{2p\cdot k} + \frac{\gamma^\nu \not{k}'\gamma^\mu - 2\gamma^\nu p^\mu}{2p\cdot k'}\Big]$$

$$\cdot\,(\not{p}+m)\Big[\frac{\gamma^\sigma \not{k}\gamma^\rho + 2\gamma^\rho p^\sigma}{2p\cdot k} + \frac{\gamma^\rho \not{k}'\gamma^\sigma - 2\gamma^\sigma p^\rho}{2p\cdot k'}\Big]\Big\}$$

$$\equiv \frac{e^4}{4}\Big[\frac{\mathbf{I}}{(2p\cdot k)^2} + \frac{\mathbf{II}}{(2p\cdot k)(2p\cdot k')} + \frac{\mathbf{III}}{(2p\cdot k')(2p\cdot k)} + \frac{\mathbf{IV}}{(2p\cdot k')^2}\Big],$$

$$\mathbf{I} = \mathrm{tr}\big[(\not{p}' + m)(\gamma^\mu \not{k}\gamma^\nu + 2\gamma^\mu p^\nu)(\not{p} + m)(\gamma_\nu \not{k}\gamma_\mu + 2\gamma_\mu p_\nu)\big].$$

$$\begin{aligned}
\mathrm{tr}\big[\not{p}'\gamma^\mu \not{k}\gamma^\nu \not{p}\gamma_\nu \not{k}\gamma_\mu\big] &= \mathrm{tr}\big[(-2\not{p}')\not{k}(-2\not{p})\not{k}\big] \\
&= \mathrm{tr}\big[4\not{p}'\not{k}(2p\cdot k - \not{k}\not{p})\big] \\
&= 8p\cdot k\ \ \mathrm{tr}[\not{p}'\not{k}] \\
&= 32(p\cdot k)(p'\cdot k).
\end{aligned}$$

```
e⁻ γ -> e⁻ γ

onShell = {k.k -> 0, p1.p1 -> m^2, p2.p2 -> m^2};

In[37]:= Contract[Spur[γ.p2, γ_μ, γ.k, γ_ν, γ.p1, γ_ν, γ.k, γ_μ]] /. onShell // LoopRefine

Out[37]= 32 k.p1 k.p2

In[39]:= Contract[Spur[2 γ.p2, γ_μ, γ.k, γ_ν, γ.p1, γ_μ, p1_ν 1]] /. onShell // LoopRefine

Out[39]= -16 m^2 k.p2

In[35]:= Contract[Spur[2 γ.p2, γ_μ, p1_ν 1, γ.p1, γ_ν, γ.k, γ_μ]] /. onShell // LoopRefine

Out[35]= -16 m^2 k.p2

In[36]:= Contract[Spur[4 γ.p2, γ_μ, p1_ν 1, γ.p1, γ_μ, p1_ν 1]] /. onShell // LoopRefine

Out[36]= -32 m^2 p1.p2

In[40]:= Contract[Spur[m^2 1, γ_μ, γ.k, γ_ν, γ_ν, γ.k, γ_μ]] /. onShell // LoopRefine

Out[40]= 0

In[41]:= Contract[Spur[2 m^2 1, γ_μ, γ.k, γ_ν, γ_μ, p1_ν 1]] /. onShell // LoopRefine

Out[41]= 32 m^2 k.p1

In[42]:= Contract[Spur[2 m^2 1, γ_μ, p1_ν 1, γ_ν, γ.k, γ_μ]] /. onShell // LoopRefine

Out[42]= 32 m^2 k.p1

In[43]:= Contract[Spur[4 m^2 1, γ_μ, p1_ν 1, γ_μ, p1_ν 1]] /. onShell // LoopRefine

Out[43]= 64 m^4
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$$\mathbf{I} = 16\big(4m^4 - 2m^2 p\cdot p' + 4m^2 p\cdot k - 2m^2 p'\cdot k + 2(p\cdot k)(p'\cdot k)\big).$$

引入Mandelstam变量：

$$s = (p+k)^2 = 2p\cdot k + m^2 = 2p'\cdot k' + m^2;$$

$$t = (p'-p)^2 = -2p\cdot p' + 2m^2 = -2k\cdot k';$$

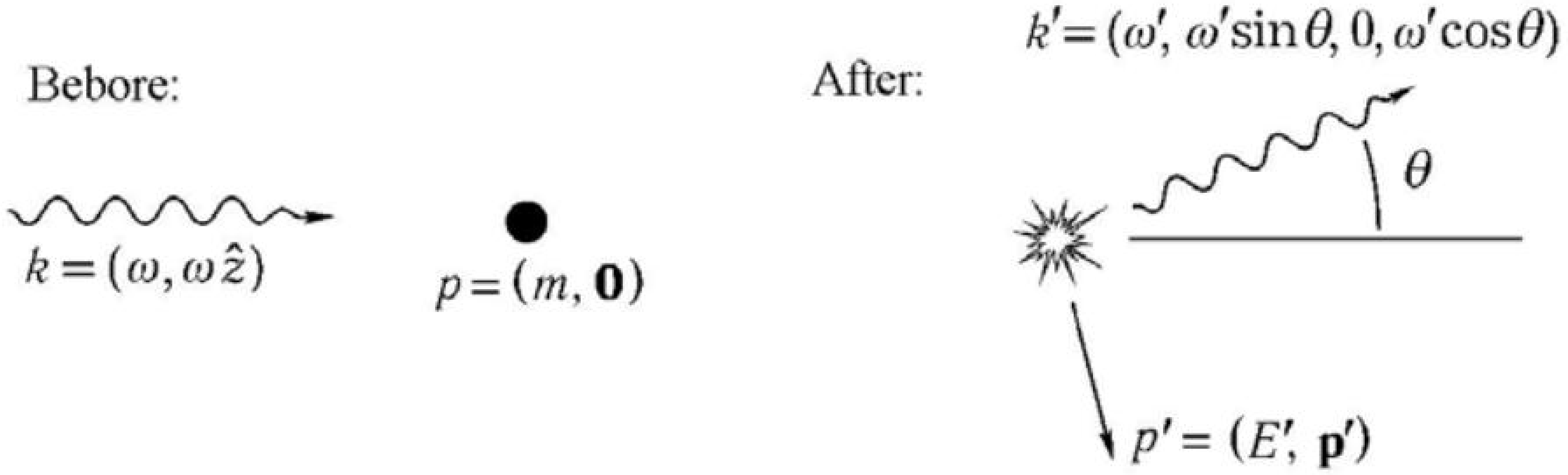
$$u = (k'-p)^2 = -2k'\cdot p + m^2 = -2k\cdot p' + m^2.$$

$$\mathbf{I} = 16\big(2m^4 + m^2(s-m^2) - \tfrac{1}{2}(s-m^2)(u-m^2)\big).$$

$$\mathbf{IV} = 16\big(2m^4 + m^2(u-m^2) - \tfrac{1}{2}(s-m^2)(u-m^2)\big).$$

$$\mathbf{II} = \mathbf{III} = -8\big(4m^4 + m^2(s-m^2) + m^2(u-m^2)\big).$$

$$\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2 = 2e^4\left[\frac{p\cdot k'}{p\cdot k} + \frac{p\cdot k}{p\cdot k'} + 2m^2\left(\frac{1}{p\cdot k} - \frac{1}{p\cdot k'}\right) + m^4\left(\frac{1}{p\cdot k} - \frac{1}{p\cdot k'}\right)^2\right].$$



$$\frac{1}{4}\sum_{\text{spins}} |M|^2 = 2e^4\left[\frac{w'}{w}+\frac{w}{w}+2m\left(\frac{1}{w}-\frac{1}{w'}\right)+m^2\left(\frac{1}{w}-\frac{1}{w'}\right)^2\right]$$

$$\frac{d\sigma}{d\cos\theta}=\frac{1}{2\omega}\frac{1}{2m}\cdot\frac{1}{8\pi}\frac{(\omega')^2}{\omega m}\cdot\left(\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2\right).$$

$$\frac{d\sigma}{d\cos\theta}=\frac{\pi\alpha^2}{m^2}\left(\frac{\omega'}{\omega}\right)^2\left[\frac{\omega'}{\omega}+\frac{\omega}{\omega'}-\sin^2\theta\right],$$

在ω→0的极限下： $\dfrac{d\sigma}{d\cos\theta}=\dfrac{\pi\alpha^2}{m^2}(1+\cos^2\theta);\qquad \sigma_{\text{total}}=\dfrac{8\pi\alpha^2}{3m^2}.$

文献分享

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The right top coupling in the aligned two-Higgs-doublet model

Cesar Ayala*[1,2], Gabriel A. González-Sprinberg†[3], R. Martinez‡[4] and Jordi Vidal§[1]

研究动机：

*顶夸克作为 SM 中最重的粒子，对理解电弱对称破缺机制起着非常重要的作用；

**在该模型中，耦合的虚部相比于SM大一个数量级，实部相比于SM大三个数量级，可能会存在新物理。

left $\begin{cases} B(t \to u\gamma) < 0.85 \times 10^{-5} \\ B(t \to c\gamma) < 4.2 \times 10^{-5} \end{cases}$

right $\begin{cases} B(t \to u\gamma) < 1.2 \times 10^{-5} \\ B(t \to c\gamma) < 4.5 \times 10^{-5} \end{cases}$

$\sqrt{s} = 13\,TeV$，积分亮度 $= 139\,fb^{-1}$

2205.02557

实验那篇文章

ATLAS

1011.[illegible]的参考文献[15]

left $\begin{cases} B(t \to u\gamma) < 2.8 \times 10^{-5} \\ B(t \to c\gamma) = 2.2 \times 10^{-5} \end{cases}$

right $\begin{cases} B(t \to u\gamma) = 6.1 \times 10^{-5} \\ B(t \to c\gamma) = 18 \times 10^{-5} \end{cases}$

2014年ATLAS[illegible]

$\sqrt{s} = 8\,TeV$，$\mathcal{L} = \cancel{139}\ 81\,fb^{-1}$

积分亮度

理论框架：the aligned two-Higgs-doublet model

Higgs 基 $\Phi_1 = \begin{bmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + S_1 + iG^0) \end{bmatrix}, \qquad \Phi_2 = \begin{bmatrix} H^+ \\ \frac{1}{\sqrt{2}}(S_2 + iS_3) \end{bmatrix}$

Higgs 基下

$$\mathcal{L}_{Y,\text{Higgs}} = -\frac{\sqrt{2}}{v}\left[\bar{Q}'_L(M'_d\Phi_1 + Y'_d\Phi_2)d'_R + \bar{Q}'_L(M'_u\tilde{\Phi}_1 + Y'_u\tilde{\Phi}_2)u'_R + \bar{L}'_L(M'_\ell\Phi_1 + Y'_\ell\Phi_2)\ell'_R\right] + \text{h.c.}$$

质量基下

$$\begin{aligned}
\mathcal{L}_{Y,\text{mass}} = &-i\frac{G^0}{v}\left\{\bar{d}_L M_d d_R + \bar{u}_R M_u^\dagger u_L + \bar{\ell}_L M_\ell \ell_R\right\} \\
&-\left(1 + \frac{S_1}{v}\right)\left\{\bar{u}_L M_u u_R + \bar{d}_L M_d d_R + \bar{\ell}_L M_\ell \ell_R\right\} \\
&-\frac{1}{v}(S_2 + iS_3)\left\{\bar{d}_L Y_d d_R + \bar{u}_R Y_u^\dagger u_L + \bar{\ell}_L Y_\ell \ell_R\right\} \\
&-\frac{\sqrt{2}}{v}G^+\left\{\bar{u}_L V_{\text{CKM}} M_d d_R - \bar{u}_R M_u^\dagger V_{\text{CKM}} d_L + \bar{\nu}_L M_\ell \ell_R\right\} \\
&-\frac{\sqrt{2}}{v}H^+\left\{\varsigma_d\, \bar{u}_L V_{\text{CKM}} M_d d_R - \varsigma_u\, \bar{u}_R M_u^\dagger V_{\text{CKM}} d_L + \varsigma_\ell\, \bar{\nu}_L M_\ell \ell_R\right\} + \text{h.c.}\,,
\end{aligned}$$

$$\mathcal{M}_{tbW} = -\frac{e}{\sin\theta_w\sqrt{2}}\epsilon^{\mu *} \times$$

$$\overline{u}_b(p')\left[\gamma_\mu(V_L P_L + V_R P_R) + \frac{i\sigma_{\mu\nu}q^\nu}{m_W}(g_L P_L + g_R P_R)\right] u_t(p),$$

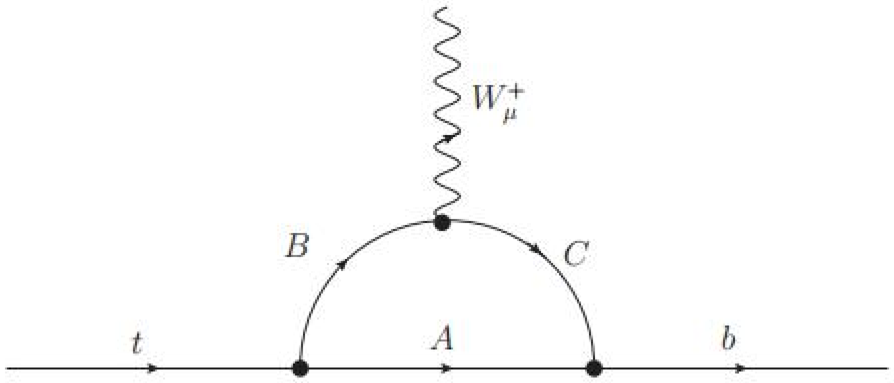


Figure 1: One-loop contributions to the V_R coupling in the $t \to bW^+$ vertex.

手性规定，所有的贡献都与底部质量成正比，可以写成：

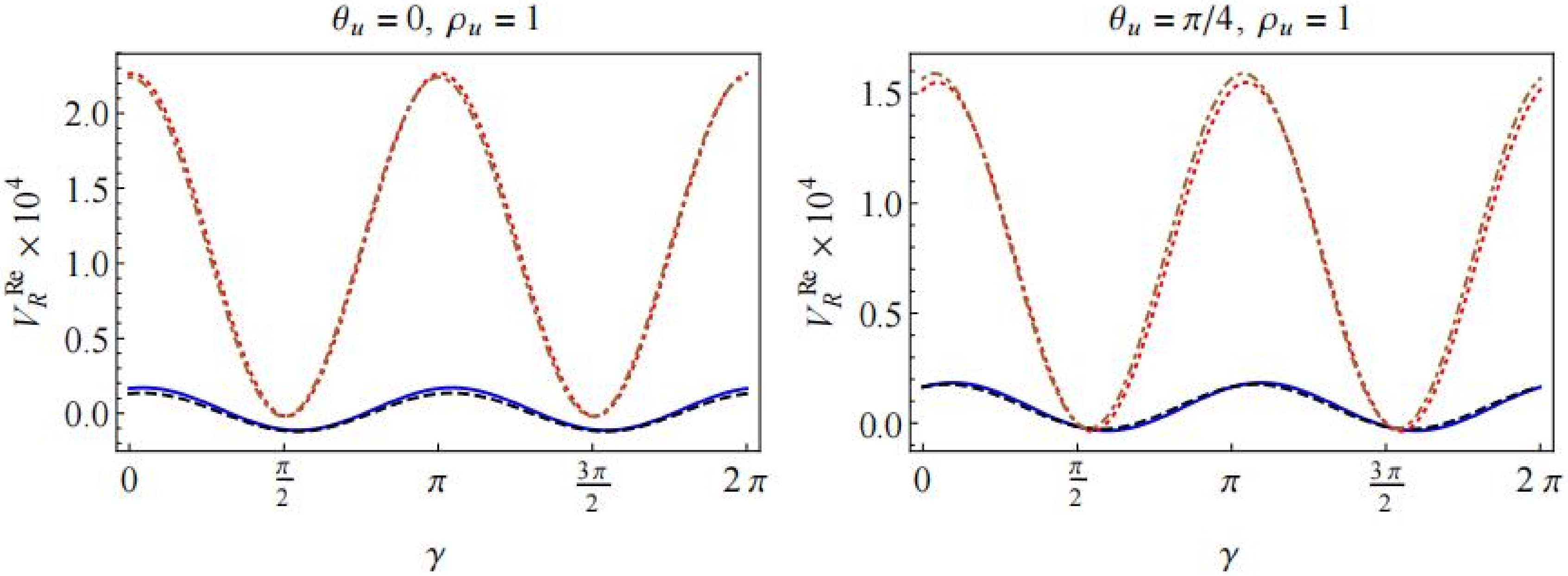
$$V_R^{ABC} = \alpha\, V_{tb}\, r_b\, I^{ABC},$$

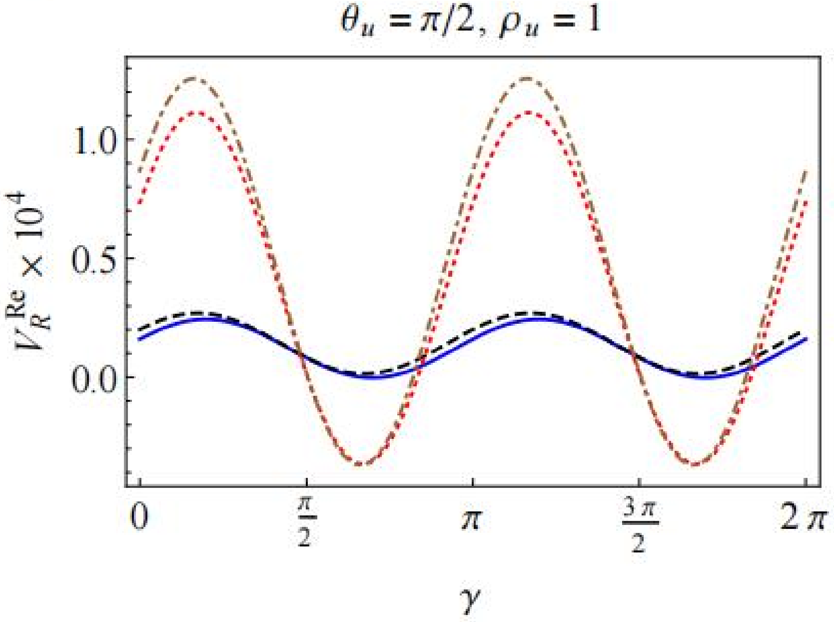
Type	Particles in the loop ABC	
(1)	$t\,\varphi_i\,H^-$	$\varphi_i = h, H, A$
(2)	$b\,H^+\,\varphi_i$	
(3)	$\varphi_i\,t\,b$	
(4)	$t\,\varphi_i\,W^-$	$\varphi_i = h, H$
(5)	$b\,W^+\,\varphi_i$	
(6)	$t\,\varphi_i\,G^-$	
(7)	$b\,G^+\,\varphi_i$	

Table 1: Classification of the new the Feynman diagrams by the particles circulating in the loop.

Scalar mass scenarios (in GeV)					Type of line and color
	m_h	m_H	m_A	$m_{H^\pm}$	
I	125.09	173.21	150	320	
Ii	125.09	173.21	150	150	
II	125.09	866.05	866.05	320	
IIi	125.09	866.05	866.05	150	

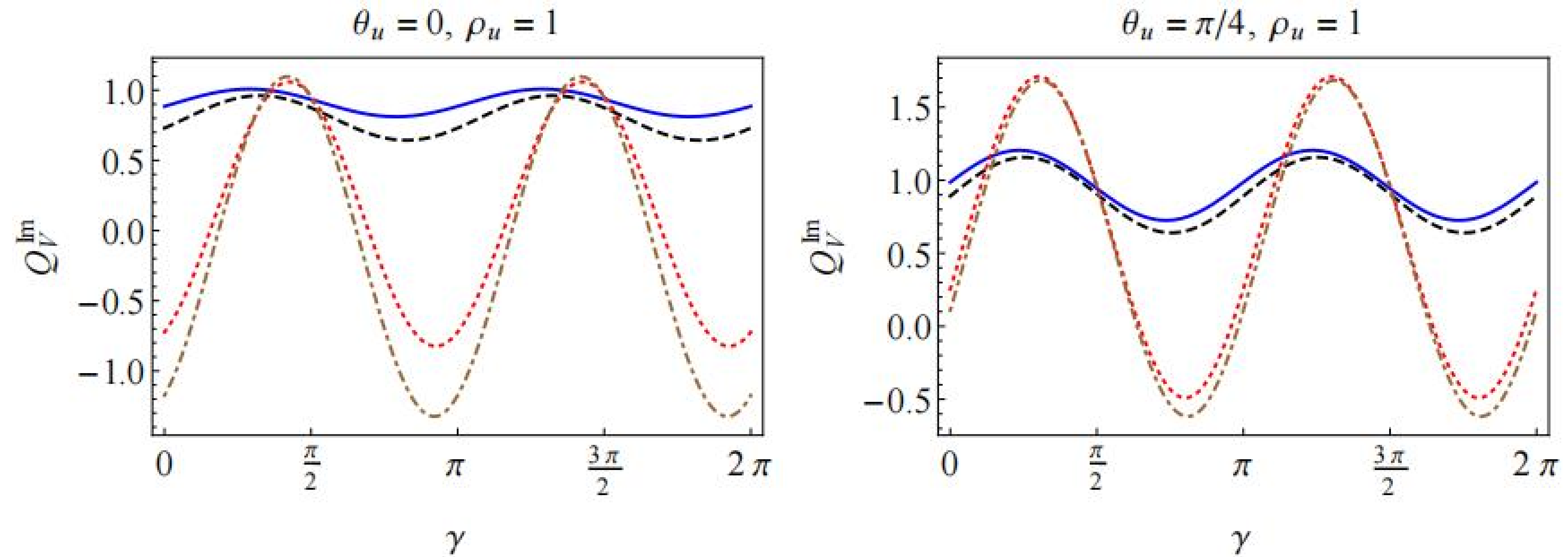
Table 2: Different scalar mass scenarios taken for the analysis. As specified in the table, each scenario is identified by a different color and type of line in the plots.

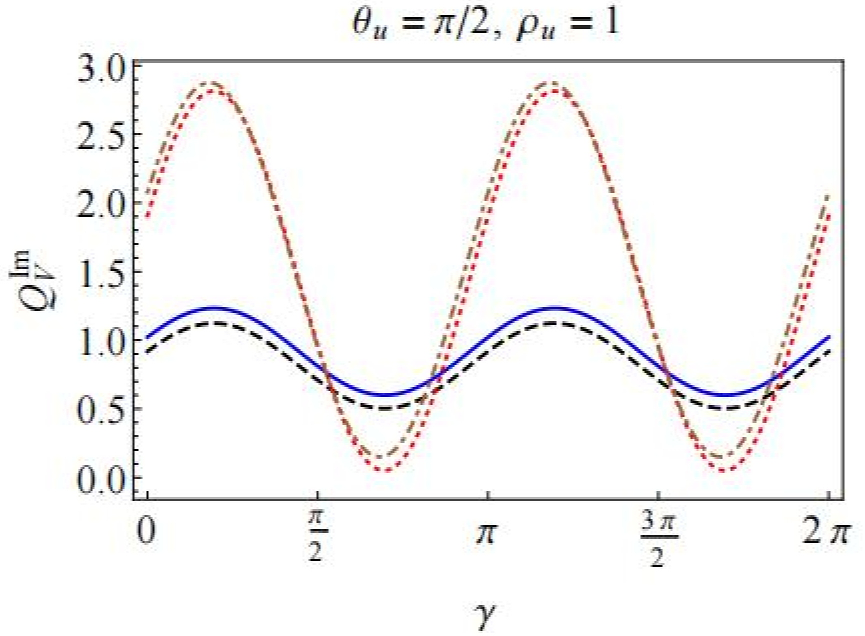




$$V_R(EW) = (-0.015 + 8.92\,i) \times 10^{-5}.$$

Figure 2: $V_R^{\mathrm{Re}} = Re\left(V_R^{A2HDM}\right)$, as a function of the γ scalar mixing angle, for different θ_u values and $p_{u,d} = 1$, $\theta_d = \pi/4$.





$$Q_V^{\mathrm{Im}} \equiv \frac{Im\left(V_R^{A2HDM}\right)}{Im\left(V_R^{EW}\right)}.$$

Figure 3: Q_V^{Im}, eq. (13) dependence with the γ scalar mixing angle, for different θ_u values and for $\rho_{u,d} = 1$, $\theta_d = \pi/4$.

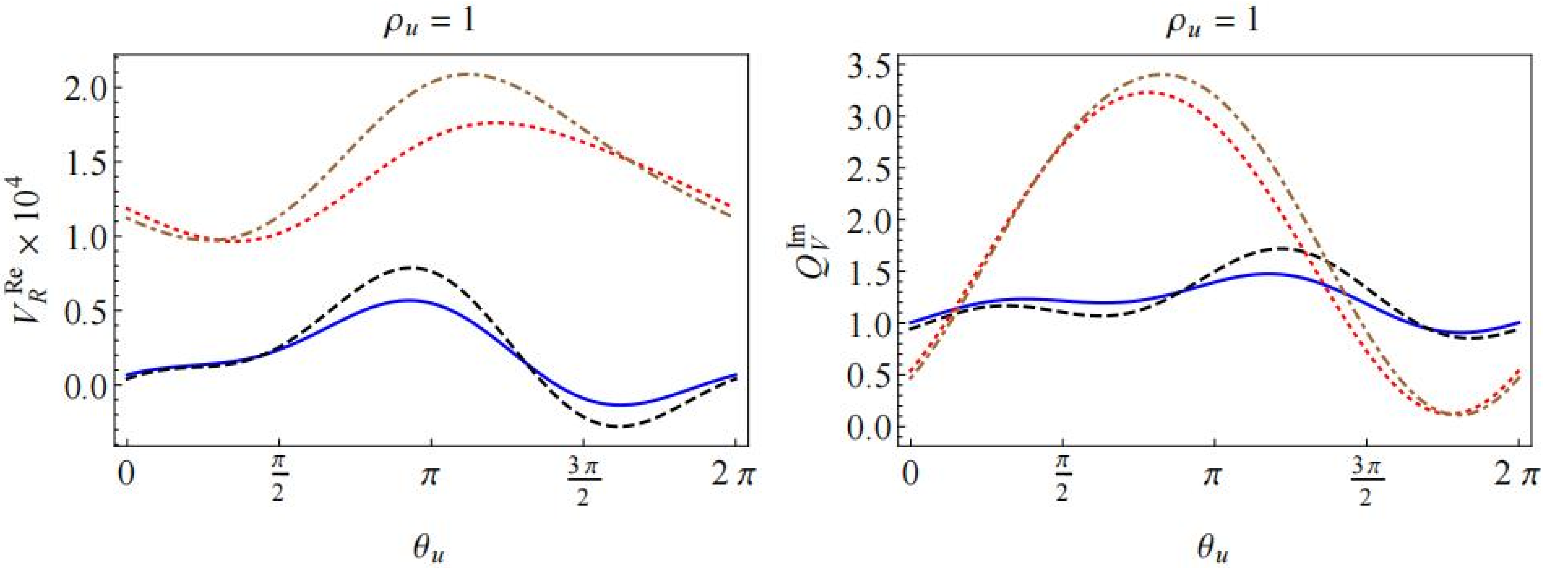


Figure 4: $V_R^{\text{Re}} = Re(V_R^{A2HDM})$ and Q_V^{Im} (eq. (13)) as function of the θ_u parameter, with $\gamma = \pi/4$, $\rho_d = 1$ and $\theta_d = \pi/4$.

$$V_R(EW) = (-0.015 + 8.92\, i) \times 10^{-5}.$$

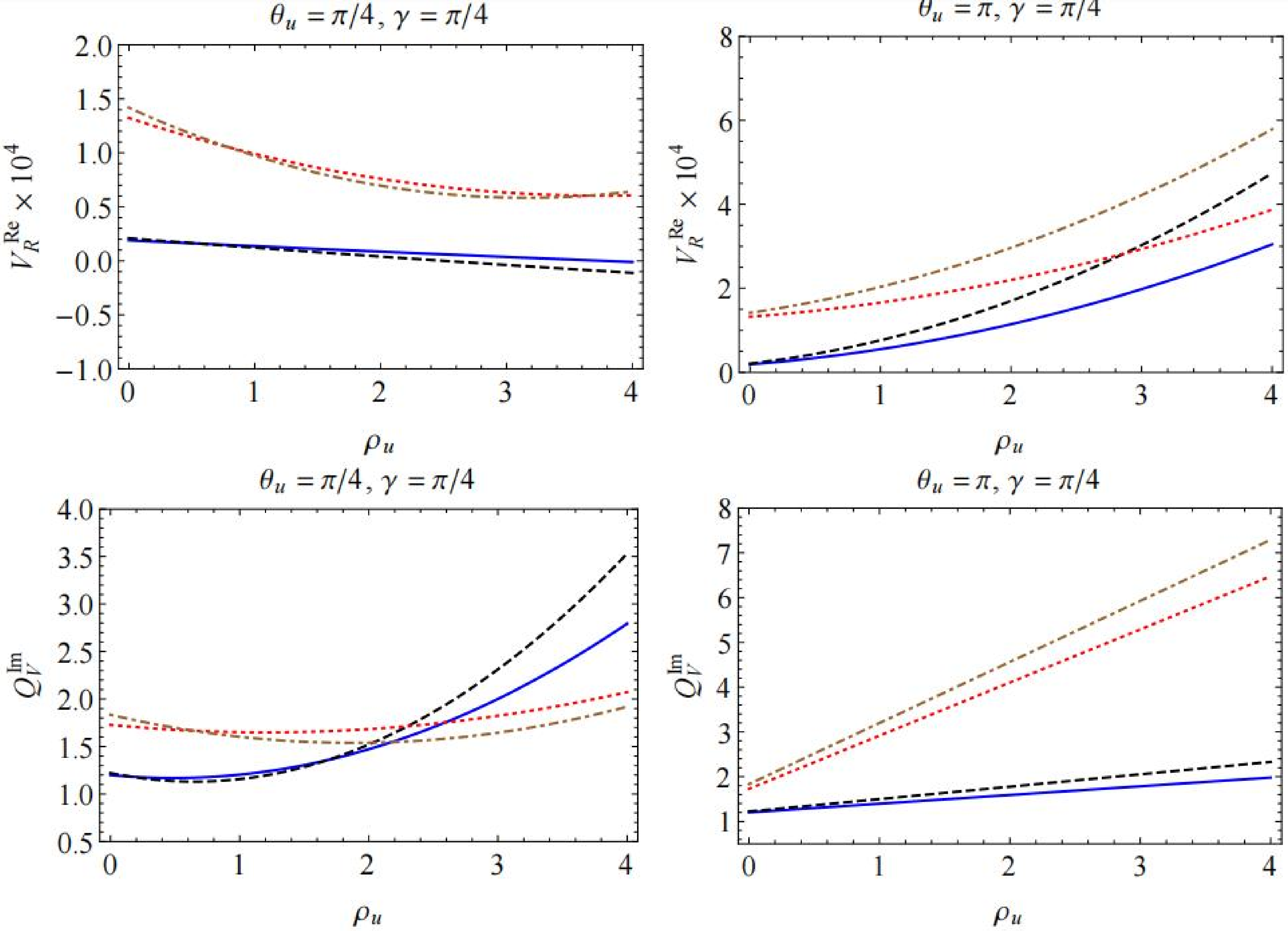


Figure 5: $V_R^{\mathrm{Re}} = Re(V_R^{A2HDM})$ and Q_V^{Im}, eq. (13), as a function of ρ_u for values of θ_u given in the plots and for fixed values $\gamma = \pi/4$, $\rho_d = 1$ and $\theta_d = \pi/4$.

小结：

*在该模型中，耦合的虚部相比于SM大一个数量级，实部相比于SM大三个数量级

谢谢！