Enlighten Dark Photon With Kinetic Mixing

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1. Enlighten Dark Photon by Kinetic Mixing

2. Kintic Mixing and W mass anomaly

3. Non-Abelian Dark Photon Kinetic Mixing

4. CP Violating Kinetic Mixing

1. Enlighten Dark Photon by Kinetic Mixing

A photon and a pure dark photon

A theory of $U(1)_{em} x U(1)_X$ gauge group

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

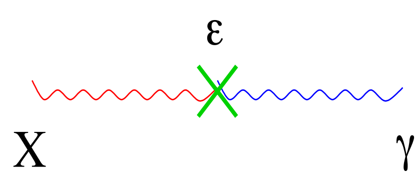
$$L = - \tfrac{1}{4} F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{4} X_{\mu\nu}X^{\mu\nu} + A_\mu j^\mu_{em} + X_\mu j^\mu_X$$

$$X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$$

A is the usual photon field and X is a new gauge field X and $j^\mu_{em,X}$ currents
X may have or not have a finite mass $m^2_A X^\mu X_\mu/2$

If j^μ_X does not involve with SM particle, X is a photon like particle which cannot be probed using laboratory probes – a **Dark Photon**

$$\epsilon X_{\mu\nu} F^{\mu\nu}$$



Possible to add the following renormalizable and gauge invariant term.

This kinetic mixing term mixes photon and Dark Photon making dark photon to interact with SM particle, Dark Photon enlightened!

Holdom 1986, Foot and He 1991,....

Some basics for Dark Photon

Work with $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$

Kinetic mixing can happen between $U(1)_Y$ and $U(1)_X$

$$\mathcal{L} = -\frac{1}{4} X_{\mu\nu} X^{\mu\nu} - \frac{\sigma}{2} X_{\mu\nu} Y^{\mu\nu} - \frac{1}{4} Y_{\mu\nu} Y^{\mu\nu} + j^{\mu}_{Y} Y_{\mu} + j^{\mu}_{X} X_{\mu}$$

Need to re-write in the canonical form to identify physics gauge bosons. (mixing term removed!)

This may generate dark photon to interact with SM J^{μ}_{Y}

How to remove the mixing term?

M He, X-G He, G. Li, arXiv: 1807.00921

Not unique! Examples

$$Case\ \ a):\quad \mathcal{L}_a = -\frac{1}{4}\hat{X}_{\mu\nu}\hat{X}^{\mu\nu} - \frac{1}{4}\hat{Y}_{\mu\nu}\hat{Y}^{\mu\nu} + j^\mu_Y \frac{1}{\sqrt{1-\sigma^2}}\hat{Y}_\mu + j^\mu_X(\hat{X}_\mu - \frac{\sigma}{\sqrt{1-\sigma^2}}\hat{Y}_\mu)\ ,$$

$$\hat{Y}_\mu = \sqrt{1-\sigma^2}Y_\mu\ ,\quad \hat{X}_\mu = \sigma Y_\mu + X_\mu\ ,$$

$$Case\ \ b):\quad \mathcal{L}_b = -\frac{1}{4}\hat{X}'_{\mu\nu}\hat{X}'^{\mu\nu} - \frac{1}{4}\hat{Y}'_{\mu\nu}\hat{Y}'^{\mu\nu} + j^\mu_Y(\hat{Y}'_\mu - \frac{\sigma}{\sqrt{1-\sigma^2}}\hat{X}'_\mu) + j^\mu_X \frac{1}{\sqrt{1-\sigma^2}}\hat{X}'_\mu\ ,$$

$$\hat{Y}'_\mu = Y_\mu + \sigma X_\mu\ ,\quad \hat{X}'_\mu = \sqrt{1-\sigma^2}X_\mu\ .$$

Case a), Redefined X does not couple to j^μ_Y is still "dark"

Case b), Redefined X does not couple to j^μ_Y is not dark any more, but Y does not couple to j^μ_X .

Which one is the correct one to choose?

Work with SM photon and dark photon

$$Y_\mu = c_W A_\mu - s_W Z_\mu \ , \quad W^3_\mu = s_W A_\mu + c_W Z_\mu \ ,$$

$$\mathcal{L}_0 = -\frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{1}{4}A_{\mu\nu}A^{\mu\nu} - \frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} - \frac{1}{2}\sigma c_W X_{\mu\nu}A^{\mu\nu} + \frac{1}{2}\sigma s_W X_{\mu\nu}Z^{\mu\nu}$$
$$+ j^\mu_{em}A_\mu + j^\mu_Z Z_\mu + j^\mu_X X_\mu + \frac{1}{2}m_Z^2 Z_\mu Z^\mu \ ,$$

Write the above into canonical form requires

$$Case \ \ a): \quad \begin{pmatrix} A \\ Z \\ X \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{1-\sigma^2 c_W^2}} & \frac{-\sigma^2 s_W c_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2 c_W^2}} & 0 \\ 0 & \frac{\sqrt{1-\sigma^2 c_W^2}}{\sqrt{1-\sigma^2}} & 0 \\ \frac{-\sigma c_W}{\sqrt{1-\sigma^2 c_W^2}} & \frac{\sigma s_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2 c_W^2}} & 1 \end{pmatrix} \begin{pmatrix} \tilde{A} \\ \tilde{Z} \\ \tilde{X} \end{pmatrix} ,$$

$$Case \ \ b): \quad \begin{pmatrix} A \\ Z \\ X \end{pmatrix} = \begin{pmatrix} 1 & \frac{-\sigma^2 s_W c_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2 c_W^2}} & \frac{-\sigma c_W}{\sqrt{1-\sigma^2 c_W^2}} \\ 0 & \frac{\sqrt{1-\sigma^2 c_W^2}}{\sqrt{1-\sigma^2}} & 0 \\ 0 & \frac{\sigma s_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2 c_W^2}} & \frac{1}{\sqrt{1-\sigma^2 c_W^2}} \end{pmatrix} \begin{pmatrix} \tilde{A}' \\ \tilde{Z}' \\ \tilde{X}' \end{pmatrix} ,$$

$$
\begin{aligned}
\mathcal{L}_a = \; & -\frac{1}{4}\tilde{X}_{\mu\nu}\tilde{X}^{\mu\nu} - \frac{1}{4}\tilde{A}_{\mu\nu}\tilde{A}^{\mu\nu} - \frac{1}{4}\tilde{Z}_{\mu\nu}\tilde{Z}^{\mu\nu} + \frac{1}{2}m_Z^2\frac{1-\sigma^2 c_W^2}{1-\sigma^2}\tilde{Z}_\mu\tilde{Z}^\mu \\
& + j_{em}^\mu\left(\frac{1}{\sqrt{1-\sigma^2 c_W^2}}\tilde{A}_\mu - \frac{\sigma^2 s_W c_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2 c_W^2}}\tilde{Z}_\mu\right) + j_Z^\mu\left(\frac{\sqrt{1-\sigma^2 c_W^2}}{\sqrt{1-\sigma^2}}\tilde{Z}_\mu\right) \\
& + j_X^\mu\left(\frac{-\sigma c_W}{\sqrt{1-\sigma^2 c_W^2}}\tilde{A}_\mu + \frac{\sigma s_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2 c_W^2}}\tilde{Z}_\mu + \tilde{X}_\mu\right), \\
\mathcal{L}_b = \; & -\frac{1}{4}\tilde{X}'_{\mu\nu}\tilde{X}'^{\mu\nu} - \frac{1}{4}\tilde{A}'_{\mu\nu}\tilde{A}'^{\mu\nu} - \frac{1}{4}\tilde{Z}'_{\mu\nu}\tilde{Z}'^{\mu\nu} + \frac{1}{2}m_Z^2\frac{1-\sigma^2 c_W^2}{1-\sigma^2}\tilde{Z}'_\mu\tilde{Z}'^\mu \\
& + j_{em}^\mu\left(\tilde{A}'_\mu - \frac{\sigma^2 s_W c_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2 c_W^2}}\tilde{Z}'_\mu - \frac{\sigma c_W}{\sqrt{1-\sigma^2 c_W^2}}\tilde{X}'_\mu\right) \\
& + j_Z^\mu\left(\frac{\sqrt{1-\sigma^2 c_W^2}}{\sqrt{1-\sigma^2}}\tilde{Z}'_\mu\right) + j_X^\mu\left(\frac{\sigma s_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2 c_W^2}}\tilde{Z}'_\mu + \frac{1}{\sqrt{1-\sigma^2 c_W^2}}\tilde{X}'_\mu\right).
\end{aligned}
$$

Which one to choose?

If dark photon is massive, easy to identify

X has a mass to start with: $(1/2)m_X^2 X^\mu X_\mu$

Example: get a mass from the vev of a scalar S with $U(1)_X$ charge but no SM charges.

$$Case\ \ a):\ \ \frac{1}{2}m_X^2\left(\frac{-\sigma c_W}{\sqrt{1-\sigma^2 c_W^2}}\tilde{A}_\mu + \frac{\sigma s_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2 c_W^2}}\tilde{Z}_\mu + \tilde{X}_\mu\right)^2 ,$$

$$Case\ \ b):\ \ \frac{1}{2}m_X^2\left(\frac{\sigma s_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2 c_W^2}}\tilde{Z}'_\mu + \frac{1}{\sqrt{1-\sigma^2 c_W^2}}\tilde{X}'_\mu\right)^2 .$$

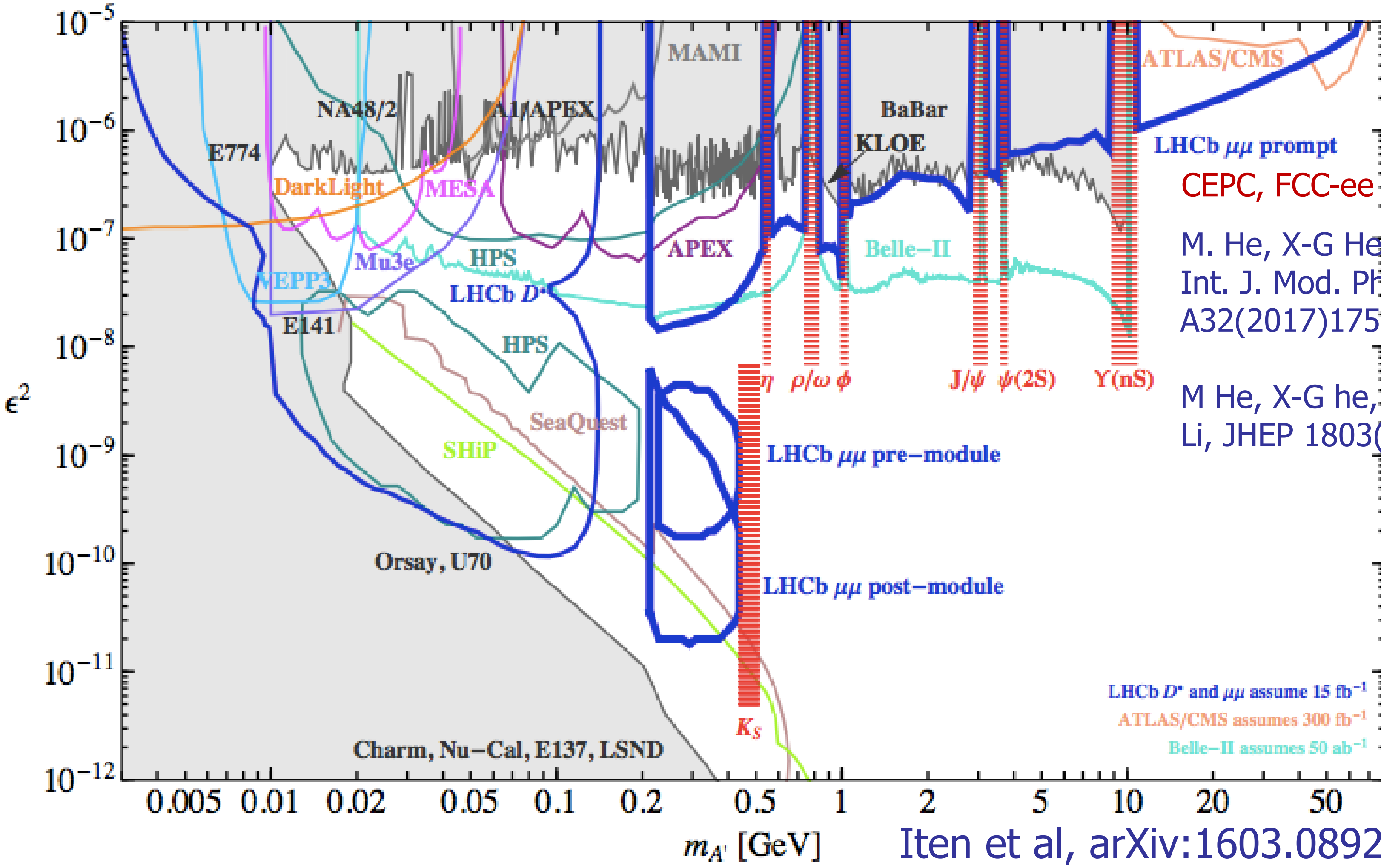
Case b) is more convenient to use, because tilde-A' already the physical massless photon, tilde-Z' and tilde X' mixing with each other

$$\begin{pmatrix} \frac{m_Z^2(1-\sigma^2 c_W^2)^2+m_X^2\sigma^2 s_W^2}{(1-\sigma^2)(1-\sigma^2 c_W^2)} & \frac{m_X^2\sigma s_W}{\sqrt{1-\sigma^2}(1-\sigma^2 c_W^2)} \\ \frac{m_X^2\sigma s_W}{\sqrt{1-\sigma^2}(1-\sigma^2 c_W^2)} & \frac{m_X^2}{1-\sigma^2 c_W^2} \end{pmatrix} , \qquad \begin{pmatrix} Z^m \\ X^m \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \tilde{Z}' \\ \tilde{X}' \end{pmatrix}$$

$$\tan(2\theta) = \frac{2m_X^2\sigma s_W\sqrt{1-\sigma^2}}{m_Z^2(1-\sigma^2 c_W^2)^2 - m_X^2[1-\sigma^2(1+s_W^2)]} .$$

Inconvenient to work with a) although finally one will reach the same interactions

Summary of constraints on the dark photon mass and coupling



CEPC, FCC-ee

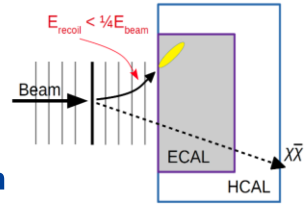
M. He, X-G He, C-K Huang, Int. J. Mod. Phys. A32(2017)1750138

M He, X-G he, C-K Huang, G Li, JHEP 1803(2018)139

Iten et al, arXiv:1603.0892

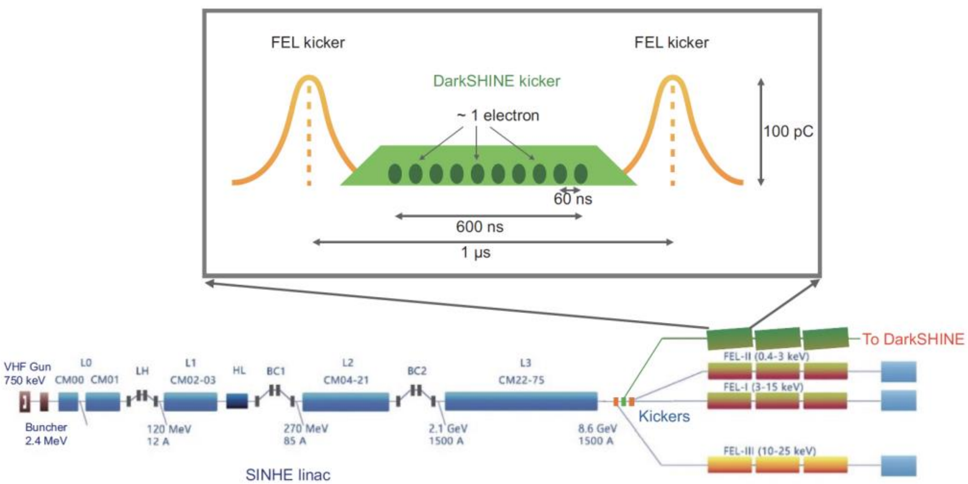
“暗光计划”(Dark SHINE) — New Initiative for Dark Photon search at SHINE facility

Innovation of Dark SHINE



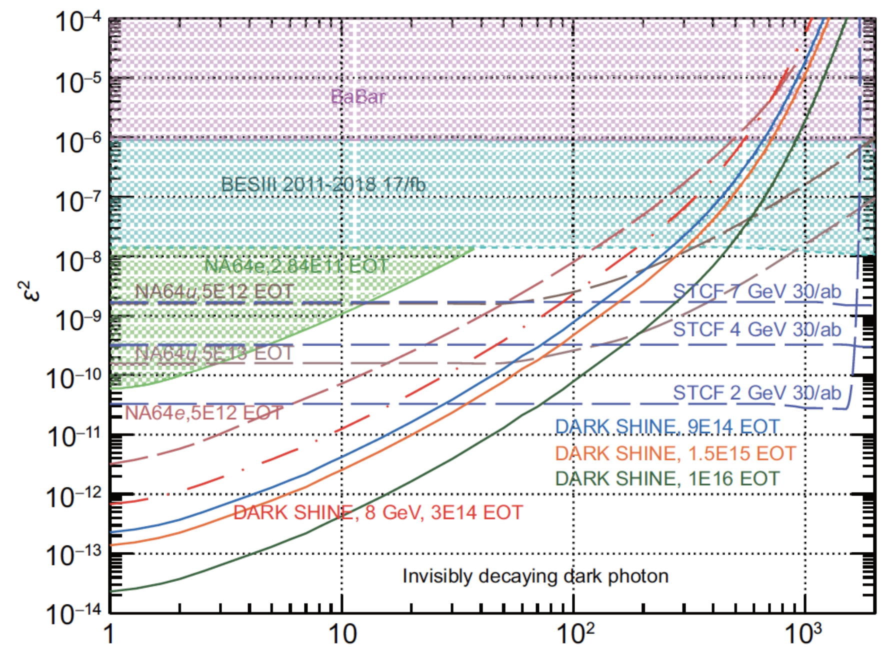
- High rate single electron beam
- “Momentum loss” method to use also angular info to enhance search sensitivity
- Being the first experiment in China of this type
- Explore the fundamental science potential of SHINE facility

SHINE LINAC and Dark SHINE Kicker illustration



- SHINE Under Construction：2018 – 2026
- High rate single electron Beam：8 GeV，1 MHz，~ 3x10[14] EOT/yr
- SHINE Facility：Provide the desired beam line

Dark SHINE Expected 90% C.L. exclusion limits on ε^2 as a function of the dark photon mass



Sci. China-Phys. Mech. Astron., 66(1): 211062 (2023)

2. Dark Photon Kinetic Mixing and W Mass Anomaly

Cheng, He, Huang, Sun, Xing, PRD106(2022)05501,arxive:2004.10154; 2208.06760

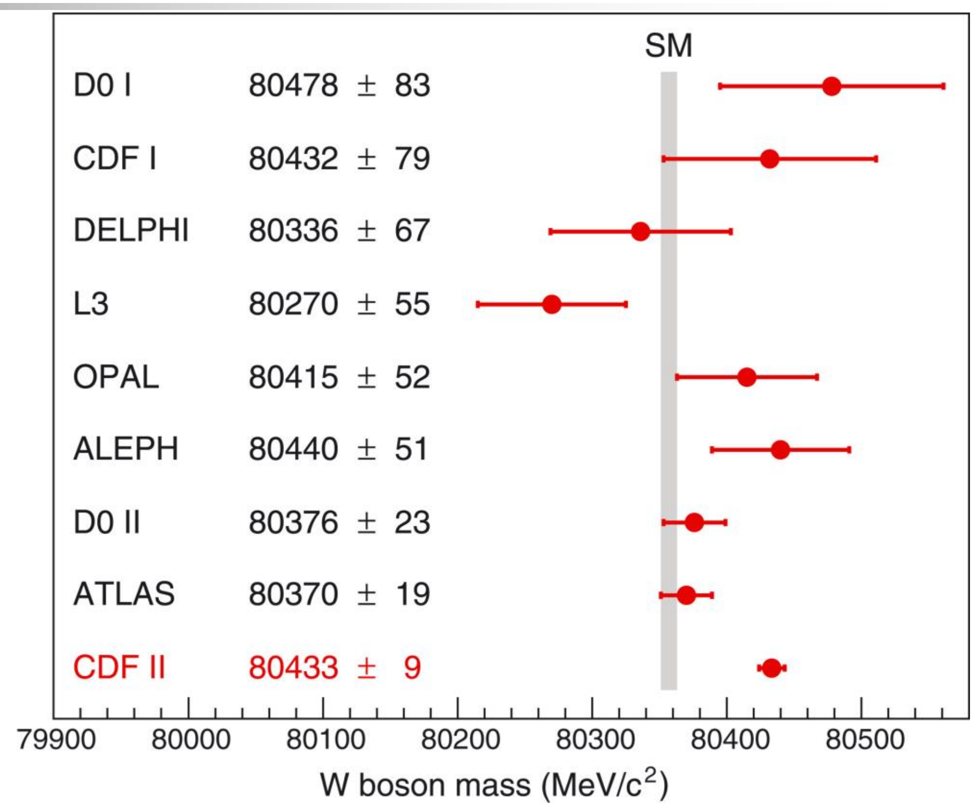
Recent CDF II analysis show that the W mass is more than SM value

(Science 376 (2022) 170)

m_W = 80,375 (6) MeV

The CDF II values is 7σ above SM value!

An anomaly!



Needs further more accurate experimental measurements.

Improved SM calculations. A hint for new physics beyond SM?

Can dark photon model help to explain the W mass anomaly.

Theoretically, the change of W mass is related to the electroweak precision oblique parameters S, T and U as

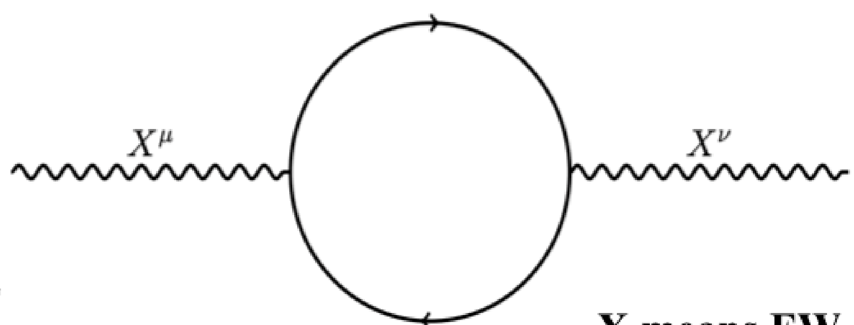
$$\Delta m_W^2 = m_Z^2 c_W^2 \left(-\frac{\alpha S}{2(c_W^2 - s_W^2)} + \frac{c_W^2 \alpha T}{(c_W^2 - s_W^2)} + \frac{\alpha U}{4 s_W^2} \right)$$

originated from tree and loop modifications beyond the SM

Tree level modification

$$\rho = \frac{m_w^2}{m_z^2 \cos^2\theta_w} = \frac{\sum_i v_i^2 \left(I_i(I_i+1) - Y_i^2\right)}{2\sum_i Y_i^2 v_i^2}$$

Loop **Vacuum polarization**



$$= i\Pi_{XX}(p^2) g^{\mu\nu}$$

X means EW gauge bosons

ρ-1= 1/(1- αT)

$$\widehat{\alpha}(M_Z)T \equiv \frac{\Pi^{\rm new}_{WW}(0)}{M_W^2} - \frac{\Pi^{\rm new}_{ZZ}(0)}{M_Z^2} \, ,$$

Neutral - charged current

$$\frac{\widehat{\alpha}(M_Z)}{4\,\widehat{s}_Z^2\widehat{c}_Z^2} S \equiv \frac{\Pi^{\rm new}_{ZZ}(M_Z^2) - \Pi^{\rm new}_{ZZ}(0)}{M_Z^2} - \frac{\widehat{c}_Z^2 - \widehat{s}_Z^2}{\widehat{c}_Z\widehat{s}_Z} \frac{\Pi^{\rm new}_{Z\gamma}(M_Z^2)}{M_Z^2} - \frac{\Pi^{\rm new}_{\gamma\gamma}(M_Z^2)}{M_Z^2} \, ,$$

Different energy scales

$$\frac{\widehat{\alpha}(M_Z)}{4\,\widehat{s}_Z^2}(S+U) \equiv \frac{\Pi^{\rm new}_{WW}(M_W^2) - \Pi^{\rm new}_{WW}(0)}{M_W^2} - \frac{\widehat{c}_Z}{\widehat{s}_Z} \frac{\Pi^{\rm new}_{Z\gamma}(M_Z^2)}{M_Z^2} - \frac{\Pi^{\rm new}_{\gamma\gamma}(M_Z^2)}{M_Z^2} \, .$$

$$
\begin{aligned}
\mathcal{L} = \quad & -\frac{1}{4}\tilde{X}'_{\mu\nu}\tilde{X}'^{\mu\nu} - \frac{1}{4}\tilde{A}'_{\mu\nu}\tilde{A}'^{\mu\nu} - \frac{1}{4}\tilde{Z}'_{\mu\nu}\tilde{Z}'^{\mu\nu} \\
& + j^{\mu}_{em}\left(\tilde{A}'_\mu - \frac{\sigma^2\tilde{s}_W\tilde{c}_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2\tilde{c}^2_W}}\tilde{Z}'_\mu - \frac{\sigma\tilde{c}_W}{\sqrt{1-\sigma^2\tilde{c}^2_W}}\tilde{X}'_\mu\right) \\
& + j^{\mu}_{Z}\left(\frac{\sqrt{1-\sigma^2\tilde{c}^2_W}}{\sqrt{1-\sigma^2}}\tilde{Z}'_\mu\right) + j^{\mu}_{X}\left(\frac{\sigma\tilde{s}_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2\tilde{c}^2_W}}\tilde{Z}'_\mu + \frac{1}{\sqrt{1-\sigma^2\tilde{c}^2_W}}\tilde{X}'_\mu\right) \\
& + \frac{1}{2}m^2_Z\frac{1-\sigma^2\tilde{c}^2_W}{1-\sigma^2}\tilde{Z}'_\mu\tilde{Z}'^\mu + \frac{1}{2}m^2_X\left(\frac{\sigma\tilde{s}_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2\tilde{c}^2_W}}\tilde{Z}'_\mu + \frac{1}{\sqrt{1-\sigma^2\tilde{c}^2_W}}\tilde{X}'_\mu\right)^2 .
\end{aligned}
$$

Introduce a singlet S with vev to give the original X a mass m_X

$$
\begin{pmatrix}
\frac{m_Z^2(1-\sigma^2\tilde{c}^2_W)^2+m_X^2\sigma^2\tilde{s}^2_W}{(1-\sigma^2)(1-\sigma^2\tilde{c}^2_W)} & \frac{m_X^2\sigma\tilde{s}_W}{\sqrt{1-\sigma^2}(1-\sigma^2\tilde{c}^2_W)} \\
\frac{m_X^2\sigma\tilde{s}_W}{\sqrt{1-\sigma^2}(1-\sigma^2\tilde{c}^2_W)} & \frac{m_X^2}{1-\sigma^2\tilde{c}^2_W}
\end{pmatrix}
\qquad
\begin{pmatrix} Z \\ X \end{pmatrix} = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix}\begin{pmatrix} \tilde{Z}' \\ \tilde{X}' \end{pmatrix}
$$

$$
\tan(2\theta) = \frac{2m_X^2\sigma\tilde{s}_W\sqrt{1-\sigma^2}}{m_Z^2(1-\sigma^2\tilde{c}^2_W)^2 - m_X^2[1-\sigma^2(1+\tilde{s}^2_W)]}
$$

$$\bar{m}_Z^2 = \frac{m_Z^2(1-\sigma^2\tilde{c}_W^2)^2 + m_X^2\sigma^2\tilde{s}_W^2}{(1-\sigma^2)(1-\sigma^2\tilde{c}_W^2)} c_\theta^2 + \frac{m_X^2}{1-\sigma^2\tilde{c}_W^2} s_\theta^2 + 2 s_\theta c_\theta \frac{m_X^2 \sigma \tilde{s}_W}{\sqrt{1-\sigma^2}(1-\sigma^2\tilde{c}_W^2)} ,$$

$$\bar{m}_X^2 = \frac{m_Z^2(1-\sigma^2\tilde{c}_W^2)^2 + m_X^2\sigma^2\tilde{s}_W^2}{(1-\sigma^2)(1-\sigma^2\tilde{c}_W^2)} s_\theta^2 + \frac{m_X^2}{1-\sigma^2\tilde{c}_W^2} c_\theta^2 - 2 s_\theta c_\theta \frac{m_X^2 \sigma \tilde{s}_W}{\sqrt{1-\sigma^2}(1-\sigma^2\tilde{c}_W^2)} .$$

$$\bar{m}_Z^2 = m_Z^2(1+\tilde{z})$$

$$\begin{aligned}
\mathcal{L} = \quad & -\frac{1}{4}A_{\mu\nu}A^{\mu\nu} - \frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} + \frac{1}{2}\bar{m}_Z^2 Z^\mu Z_\mu \\
& + j_{em}^\mu A_\mu - j_{em}^\mu \left(\frac{\sigma^2 \tilde{s}_W \tilde{c}_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2\tilde{c}_W^2}} c_\theta + \frac{\sigma \tilde{c}_W}{\sqrt{1-\sigma^2\tilde{c}_W^2}} s_\theta \right) Z_\mu + j_Z^\mu \frac{\sqrt{1-\sigma^2\tilde{c}_W^2}}{\sqrt{1-\sigma^2}} c_\theta Z_\mu \\
& + j_X^\mu \left(\frac{\sigma \tilde{s}_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2\tilde{c}_W^2}} c_\theta + \frac{1}{\sqrt{1-\sigma^2\tilde{c}_W^2}} s_\theta \right) Z_\mu \\
& - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} + \frac{1}{2}\bar{m}_X^2 X^\mu X_\mu + j_X^\mu \left(-\frac{\sigma \tilde{s}_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2\tilde{c}_W^2}} s_\theta + \frac{1}{\sqrt{1-\sigma^2\tilde{c}_W^2}} c_\theta \right) X_\mu . \\
& + j_{em}^\mu \left(\frac{\sigma^2 \tilde{s}_W \tilde{c}_W}{\sqrt{1-\sigma^2}\sqrt{1-\sigma^2\tilde{c}_W^2}} s_\theta - \frac{\sigma \tilde{c}_W}{\sqrt{1-\sigma^2\tilde{c}_W^2}} c_\theta \right) X_\mu - j_Z^\mu \frac{\sqrt{1-\sigma^2\tilde{c}_W^2}}{\sqrt{1-\sigma^2}} s_\theta X_\mu . \qquad (11)
\end{aligned}$$

The above modifications can be recasted into S, T, U parameters

Follow procedure in
C-P Burgess et al., Phys. Rev. D 50(1994) 7011

$$L = \frac{1}{2}(1+z-C)m_Z^2 Z^\mu Z_\mu + (1+w-z)m_W^2 W^\mu W_\mu^\dagger$$
$$+ \left(1-\frac{A}{2}\right) j_{em}^\mu A_\mu + \left(1-\frac{C}{2}\right)(j_Z^\mu + G j_{em}^\mu) Z_\mu + \left(\left(1-\frac{B}{2}\right) j_W^\mu W_\mu^+ + h.c.\right)$$

$$j_W^\mu = -(\tilde{e}/\sqrt{2}\tilde{s}_W)\bar{f}^u \gamma^\mu L V_{KM} f^d$$

$$\alpha S = 4 s_W^2 c_W^2 (A - C) - 4 s_W c_W (c_W^2 - s_W^2) G\ , \quad \alpha T = w - z\ ,$$
$$\alpha U = 4 s_W^2 (s_W^2 A - B + c_W^2 C - 2 s_W c_W G)\ .$$

Compare our L, we have B = 0, w=0, A=0, and

$$C = 2\left(1 - \frac{\sqrt{1-\sigma^2 c_W^2}}{\sqrt{1-\sigma^2}} c_\theta\right)\ , \quad G = -\frac{\sigma^2 s_W c_W}{1-\sigma^2 c_W^2} - \frac{\sigma c_W \sqrt{1-\sigma^2}}{1-\sigma^2 c_W^2}\frac{s_\theta}{c_\theta}\ , \quad z = C + \tilde{z}$$

S, T, U parameters due to dark photon interaction

To σ^2 order, we have

$$\alpha S = \frac{4 s_W^2 c_W^2 \sigma^2}{1 - m_X^2/m_Z^2}\left(1 - \frac{s_W^2}{1 - m_X^2/m_Z^2}\right) ,$$

$$\alpha T = -\sigma^2 s_W^2 \frac{m_X^2/m_Z^2}{(1 - m_X^2/m_Z^2)^2} ,$$

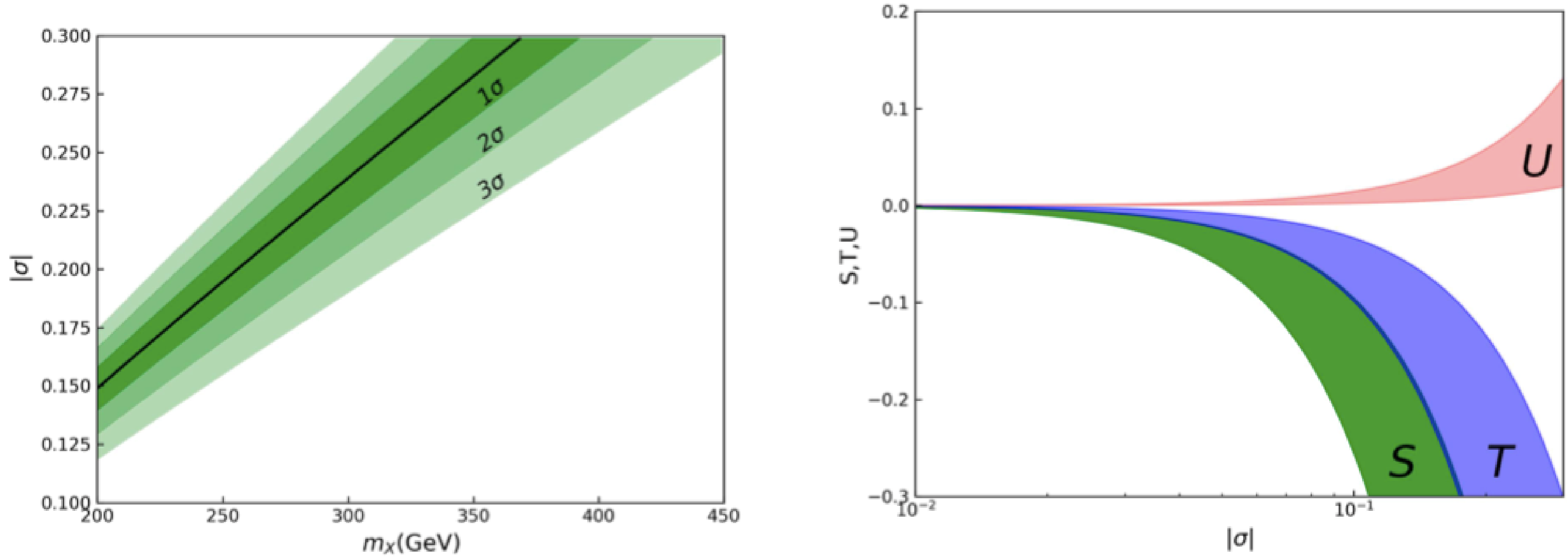
$$\alpha U = 4 s_W^4 c_W^2 \sigma^2 \left(-\frac{1 - 2m_X^2/m_Z^2}{(1 - m_X^2/m_Z^2)^2} + \frac{2}{1 - m_X^2/m_Z^2}\right) .$$

$$\begin{aligned} \Delta m_W^2 &= m_Z^2 c_W^2 \left(-\frac{\alpha S}{2(c_W^2 - s_W^2)} + \frac{c_W^2 \alpha T}{(c_W^2 - s_W^2)} + \frac{\alpha U}{4 s_W^2}\right) \\ &= -m_Z^2 c_W^2 \frac{m_Z^2(1 - s_W^2)\sigma^2 s_W^2}{(m_X^2 - m_Z^2)(-1 + 2 s_W^2)} . \end{aligned}$$

$m_X > m_Z$ will help to explain the W mass anomaly.

The non-abelian mixing part β is very small and is neglected

Numerical Results



(a) (b)

FIG. 1: (a) The CDF allowed regions in $m_X - |\sigma|$ plane. The allowed parameter space is shown in black line for central value, the 1σ, 2σ and 3σ ranges are also shown. (b) The S, T and U parameters as functions of $|\sigma|$ for m_X in the range of $200 - 300$ GeV. The size of observables decrease when m_X increases.

3. Non-Abelian Dark Photon Kinetic Mixing

Naively, not possible to have Abelian-No-Abelian Kinetic mixing, $W^a_{\mu\nu} X^{\mu\nu}$, is not gauge invariant!

Assuming that there is a field Δ^a transforming as 3 under $SU(2)_W$, then one can make gauge singlet: $W^a_{\mu\nu} X^{\mu\nu} \Delta^a$

If the VEV of $\langle\Delta^a\rangle = v_3/\mathrm{sqrt}(2)$ along a particular direction in group space is not zero, one can generate kinetic mixing term

$$W^3_{\mu\nu} X^{\mu\nu} v_3/\mathrm{sqrt}(2)$$

Problem: not renormalizable.

If one gives up renormalizability one can write higher order operators to generate abelian and non-abelian gauge fields mixing!

In fact in the SM, one can generate such a mixing between $SU(2)_L$ and $U(1)_Y$

$$W^a_{\mu\nu} X^{\mu\nu} (H^+ \tau^a H)$$

Here H is the usual SM doublet!

Possible to have kinetic mixing between ablian and non-abelian gauge fields.

J. Cline and A. Frey, arXiv: 1408.0233; G. Barello abd s. Chang, PRD94(2016)055018

C. Arguelles, X-G He, G. Ovanesyan, T Peng, M Ramsey-Musolf, PLB770(2017)101

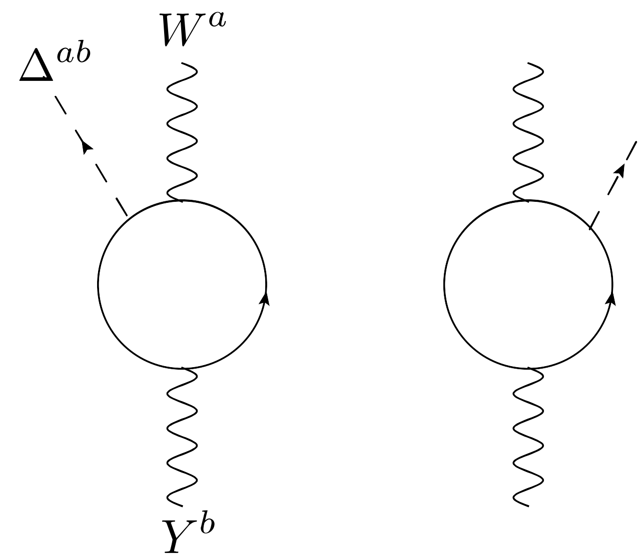
UV completion of kinetic mixing of Abeliand-NonAbelian gauge field?

Yes, they can be generated at loop level starting from a renormalizable theory.

The particle in the loop carry both abelian and non-abelian charges.

One can even talking about

SU(N) and SU(m) kinetic mixing



$$W^a_{\mu\nu} Y^{b\mu\nu} \Delta_{ab}$$

Kinetic mixing between an Abelian and a non-Abelian fields should be very common when going beyond SM.

- Kinetic mixing can also be induced for abelian and non-abelian gauge particles
- Effects can be searched at colliders

4. CP Violating Kinetic Mixing

CP violating kinetic mixing allowed?

K Fuyuto, X-G He, G. Li, M Ramsey-Musolf Phys. Rev.D101 (2020) 075016

For Abelian kinetic mixing

$Y^{\mu\nu}X_{\mu\nu}$, CP conserving

$Y^{\mu\nu}\tilde{X}_{\mu\nu}$, with $\tilde{X}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}X^{\alpha\beta}$, CP violating

But

$$Y^{\mu\nu}\tilde{X}_{\mu\nu} = -\epsilon_{\mu\nu\alpha\beta}\partial^\alpha(Y^{\mu\nu}X^\beta)$$

It is a total derivative, can be dropped off. No physical effects.

For Non-Abelian kinetic mixing

$$\mathrm{Tr}(W_{\mu\nu}\Sigma)\tilde{X}^{\mu\nu}/\Lambda \qquad \Sigma = \frac{1}{2}\begin{pmatrix} \Sigma^0 & \sqrt{2}\Sigma^+ \\ \sqrt{2}\Sigma^- & -\Sigma^0 \end{pmatrix}$$

$$(x_0 + \Sigma^0)\tilde{X}^{\mu\nu}W^+_\mu W^-_\nu$$

x_0 vev of Σ_0

Allowed! There are physical effects.

A model study

$$\mathcal{L}^{(d=5)} = -\frac{\beta}{\Lambda}\mathrm{Tr}\left[W_{\mu\nu}\Sigma\right] X^{\mu\nu} - \frac{\tilde{\beta}}{\Lambda}\mathrm{Tr}\left[W_{\mu\nu}\Sigma\right]\tilde{X}^{\mu\nu}$$

$$\mathcal{L}^{(d=5)} \supset -\frac{1}{2}\left(\alpha_{ZX} Z_{\mu\nu} X^{\mu\nu} + \alpha_{AX} F_{\mu\nu} X^{\mu\nu}\right)$$

$$-\frac{\tilde{\beta}}{2\Lambda}\tilde{X}^{\mu\nu}\left[s_W F_{\mu\nu}\Sigma^0 - i g_2 (x_0 + \Sigma^0)\left(W_\mu^- W_\nu^+ - W_\mu^+ W_\nu^-\right)\right]$$

$$V(H,\Sigma) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2 - \frac{M_\Sigma^2}{2} F + \frac{b_4}{4} F^2 + a_1 H^\dagger \Sigma H + \frac{a_2}{2} H^\dagger H F,$$

$$\alpha_{ZX(AX)} = \beta x_0 c_W (s_W)/\Lambda$$

$F = \mathrm{Tr}(\Sigma^+\Sigma)$.

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} h \\ \Sigma^0 \end{pmatrix}$$

$$H = (\phi^+,\ (h + i\phi^0)/\sqrt{2})$$

a_1 breaks CP explicitly, x_0 breaks CP spontaneously

Physical effects

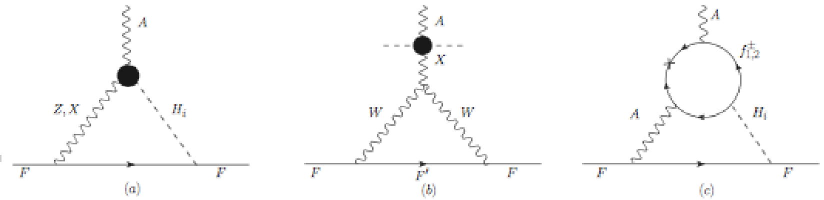
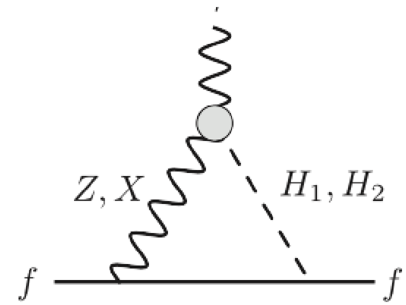


FIG. 2: Different contributions to fermion EDMs.

EDM: $$\mathcal{L}^{\rm EDM} = -\frac{i}{2} d_f \bar{f} \sigma^{\mu\nu} \gamma_5 f F_{\mu\nu}.$$

$$d_f = \frac{e}{8\pi^2} \frac{m_f}{v} c_\theta s_\theta \Big[C_Z V_Z^f f\left(r_{ZH_1},\ r_{ZH_2}\right) + C_X V_X^f f\left(r_{XH_1},\ r_{XH_2}\right) \Big]$$

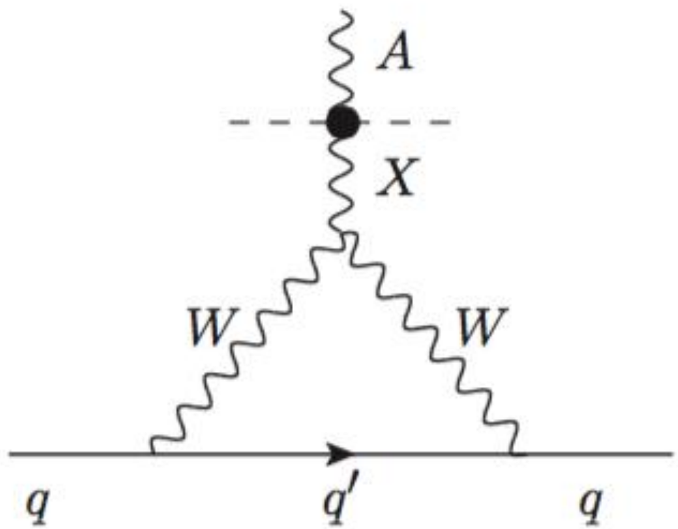


$$C_Z = \frac{\tilde{\beta}}{\Lambda} s_W s_\xi, \qquad C_X = \frac{\tilde{\beta}}{\Lambda} s_W c_\xi, \qquad r_{Z(X)H} = m_{Z(X)}^2 / m_H^2,$$

$$V_Z^f = \left(c_\xi - s_\xi \alpha_{ZX}\right) \frac{g_Z^f}{c_W s_W} - Q_f \alpha_{AX} s_\xi, \qquad g_Z^f = I/2 - s_W^2 Q_f$$

$$V_X^f = -\left(s_\xi + c_\xi \alpha_{ZX}\right) \frac{g_Z^f}{c_W s_W} - Q_f \alpha_{AX} c_\xi, \qquad \tan 2\xi = -\frac{2 m_Z^2 \alpha_{ZX}}{m_Z^2 - m_X^2}$$

$$f(x, y) = \frac{1}{2} \log\left(\frac{m_{H_1}^2}{m_{H_2}^2}\right) - \frac{1}{2}\left(\frac{x \log x}{1 - x} - \frac{y \log y}{1 - y}\right)$$



$$d_q^X \approx \frac{1}{32\pi^2}\frac{m_q}{2m_W^2}g^3\frac{\tilde{\beta}_X v_\Sigma}{\Lambda}$$

$$d_q \approx 4q^2/(q^2 - m_X^2)\epsilon_{AX} d_d^X$$

edm $q^2 = 0$, so no edm in the usual sense.

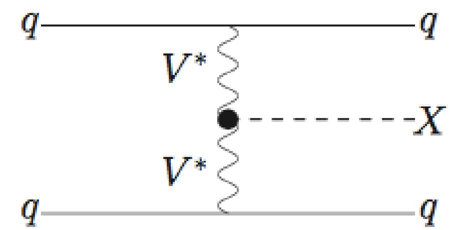
Mixing between X and Z,
generates Z weak dipole

$$d_q^Z \approx \sin\xi d_q^X$$

Challenging to measure it. Maybe at super-Z factory (FCCee, CEPC..)

We also tried to study collider signature:

Jet correlations probe effects of $(x_0/\Lambda)\tilde{X}^{\mu\nu}W_\mu^+W_\nu^-$

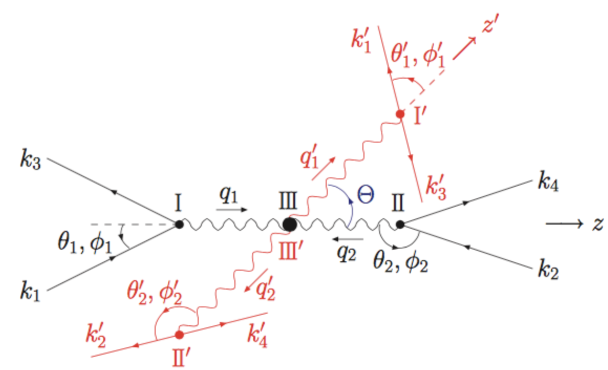


$$\mathcal{A} = \frac{\sigma(\sin\Delta\phi_{jj} > 0) - \sigma(\sin\Delta\phi_{jj} < 0)}{\sigma(\sin\Delta\phi_{jj} > 0) + \sigma(\sin\Delta\phi_{jj} < 0)}$$

$\Delta\phi_{jj} = \phi_{j_1} - \phi_{j_2}$,

ϕ_{j_1} and ϕ_{j_2} are the azimuthal angles of the jets in the forward and backward regions of the detector,

$$0 \leq \Delta\phi_{jj} \leq \pi \text{ and } -\pi \leq \Delta\phi_{jj} \leq 0$$



Unfortunately, although at parton level, the asymmetry can be large, but the back ground is to large, very chllenging to observe it experimentally.

A renormalizable model

Yu Cheng, X-G He, M. Ramsey-Musolf, Jin Sun, PRD 105(2022)095010, arXiv:2104.11563

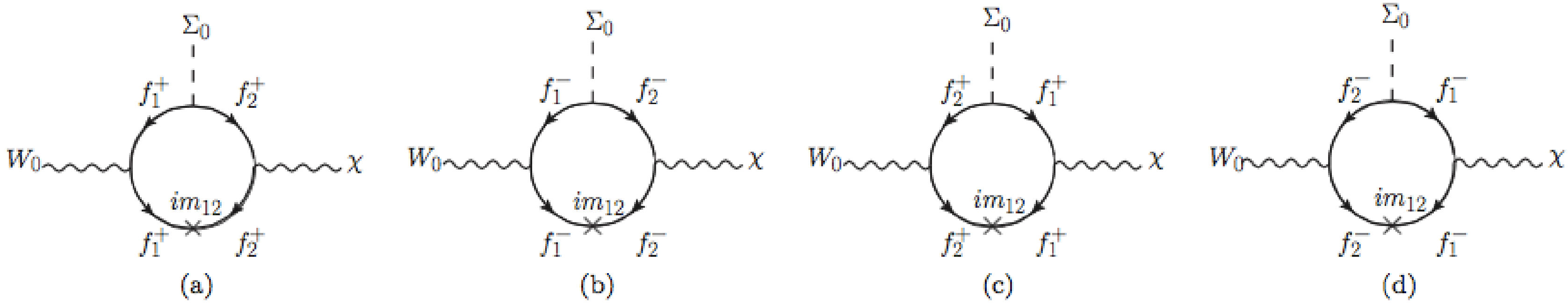
Generate $$\mathcal{L}^{(d=5)} = -\frac{\beta}{\Lambda}\mathrm{Tr}\left[W_{\mu\nu}\Sigma\right] X^{\mu\nu} - \frac{\tilde{\beta}}{\Lambda}\mathrm{Tr}\left[W_{\mu\nu}\Sigma\right] \tilde{X}^{\mu\nu}$$

by loop to make the model renormalizable!

Type-III Seesaw come to be a natural extension

Right-handed neutrino f in the loop, non-trivial SU(2) representation needed, type-III seesaw,

f = (3,0) (x) -- new need x non-zero to connect X!



New particles: Σ:(3,0)(0)

$$\Sigma = \frac{\tau^a}{2}\Sigma^a = \frac{1}{2}\begin{pmatrix} \Sigma^3 & \sqrt{2}\Sigma^+ \\ \sqrt{2}\Sigma^- & -\Sigma^3 \end{pmatrix}$$

f: (3,0)(x)

$$f_R = \frac{1}{2}\sigma^a f_R^a = \frac{1}{2}\begin{pmatrix} f_R^0 & \sqrt{2}f_R^+ \\ \sqrt{2}f_R^- & -f_R^0 \end{pmatrix}, \quad f_L = f_R^c = \frac{1}{2}\sigma^a f_L^a = \frac{1}{2}\begin{pmatrix} f_L^0 = (f_R^0)^c & \sqrt{2}f_L^+ = \sqrt{2}(f_R^-)^c \\ \sqrt{2}f_L^- = \sqrt{2}(f_R^+)^c & -f_L^0 = -(f_R^0)^c \end{pmatrix}$$

Anomaly cancellation: f_1 (3,0)(x_f), f_2: (3,0)($-x_f$), f3: (3,0)(0)

Also for generate a non-zero kinetic mixing.

S_X: (1,0) ($-2x_f$) $\quad < S_X >= v_s/\sqrt{2}$ generate a dark photon mass $m_X^2 = x_f^2 g_X^2 v_s^2$.

Seesaw neutrino masses

Need two new Higgs doublet

$$L_m = -\frac{1}{2}(\bar{\nu}_L, \bar{\nu}_R^c)\begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}(\nu_L^c, \nu_R) - (\bar{E}_L, \bar{f}_L)\begin{pmatrix} m_e & \sqrt{2}M_D \\ 0 & M_R \end{pmatrix}(E_L, f_R),$$

$$M_D = \begin{pmatrix} \frac{Y_{fL11}v_1'}{\sqrt{2}} & \frac{Y_{fL12}v_2'}{\sqrt{2}} & \frac{Y_{fL13}v}{\sqrt{2}} \\ \frac{Y_{fL21}v_1'}{\sqrt{2}} & \frac{Y_{fL22}v_2'}{\sqrt{2}} & \frac{Y_{fL23}v}{\sqrt{2}} \\ \frac{Y_{fL31}v_1'}{\sqrt{2}} & \frac{Y_{fL32}v_2'}{\sqrt{2}} & \frac{Y_{fL33}v}{\sqrt{2}} \end{pmatrix}, \quad M_R = \begin{pmatrix} \frac{Y_{fs1}v_s}{\sqrt{2}} & m_{12} & 0 \\ m_{12} & \frac{Y_{fs2}v_s}{\sqrt{2}} & 0 \\ 0 & 0 & m_{33} \end{pmatrix},$$

$$H_1' : (1,2)(-1/2, -x_f),\ H_2' : (1,2)(-1/2, x_f)$$

$$-\bar{L}_L Y_{fL1} H_1' f_{R\,1} - \bar{L}_L Y_{fL2} H_2' f_{R\,2}$$

Kinetic mixing generated

$$\frac{\tilde{\beta}_X}{2\Lambda} = \frac{\beta_X}{2\Lambda} = \frac{1}{8\pi^2} g g_X x_f Re(m_{12} Y_{f\sigma}) \left[f(m_1, m_2, p_W, p_X) + f(m_2, m_1, p_W, p_X) \right] .$$

$$f(m_1, m_2, p_W, p_X) = \int_0^1 dx \int_0^{1-x} dy \left(\frac{m_1^2}{m_1^2 - m_2^2} \frac{1-x-y}{m_1^2 - y(m_1^2 - m_2^2) - x p_W^2 - y p_X^2 + (x p_W - y p_X)^2} - \frac{m_2^2}{m_1^2 - m_2^2} \frac{1-x-y}{m_2^2 - y(m_2^2 - m_1^2) - y p_W^2 - x p_X^2 + (y p_W - x p_X)^2} \right)$$

Degenerate $m_1 = m_2 = m_{12}$ = m limit: $\tilde{\beta}_X / \Lambda \approx g g_X x_f |Y^*_{f\sigma}| \sin\delta / 6\pi^2 m.$

LHC search, m >900 GeV $5 \sin\delta \times 10^{-4}$

Replacing sinδ by cosδ, one obtains β_X/Λ

Other fermions in the loop. Scalars in the loop does not work!

Including $-(1/2)\alpha_{XY} X^{\mu\nu} B_{\mu\nu}$.

$$\begin{pmatrix} A \\ Z \\ X \end{pmatrix} = \begin{pmatrix} 1 & 0 & -\epsilon_{AX} \\ 0 & 1 & -\xi - \epsilon_{ZX} \\ 0 & \xi & 1 \end{pmatrix} \begin{pmatrix} A^m \\ Z^m \\ X^m \end{pmatrix}$$

$$\epsilon_{AX} = \alpha_{XY} c_W + \beta_X s_W v_\Sigma/\Lambda \qquad \epsilon_{ZX} = -\alpha_{XY} s_W + \beta_X c_W v_\Sigma/\Lambda.$$

$$\xi \approx -m_Z^2 \epsilon_{ZX}/(m_Z^2 - m_X^2).$$

Consider two cases for illustration

Case I: $\alpha_{XY} = 0$. CPV determined ~ $\tilde{\beta}_X \beta_X$ ~ $\sin\delta \cos\delta$ largest ½

Case II: $\alpha_{XY} = 10^{-2}$ allowed for m_X a few tens of GeV

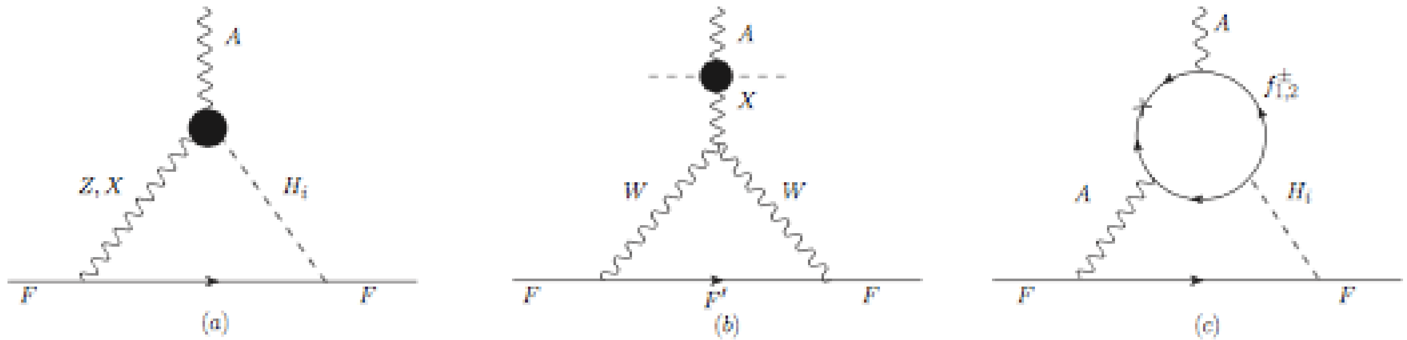


FIG. 2: Different contributions to fermion EDMs.

The 3rd one vanishes in this model!

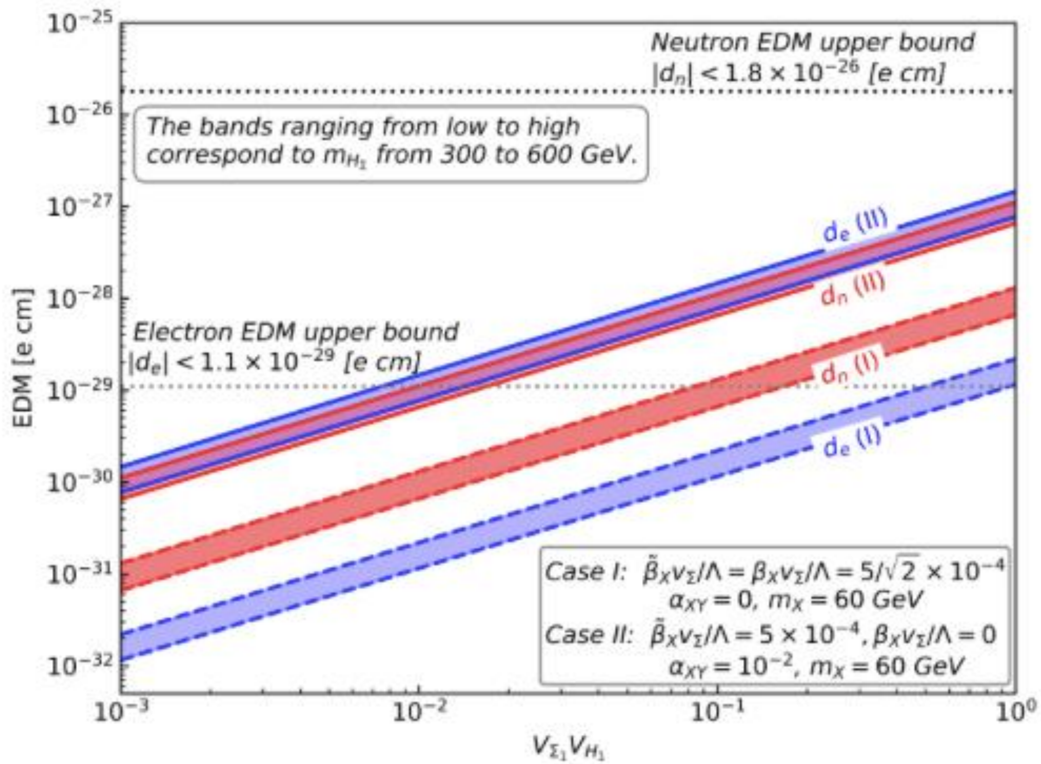


FIG. 3: The ranges for d_n and d_e for *Cases I* and *II*. The 90% C.L. upper limits for electron and neutron EDMs are from Ref. [17] and Ref. [18], respectively.

W mass anomaly again

Cheng, He, Huang, Sun, Xing, PRD106(2022)05501,arxive:2004.10154; 2208.06760

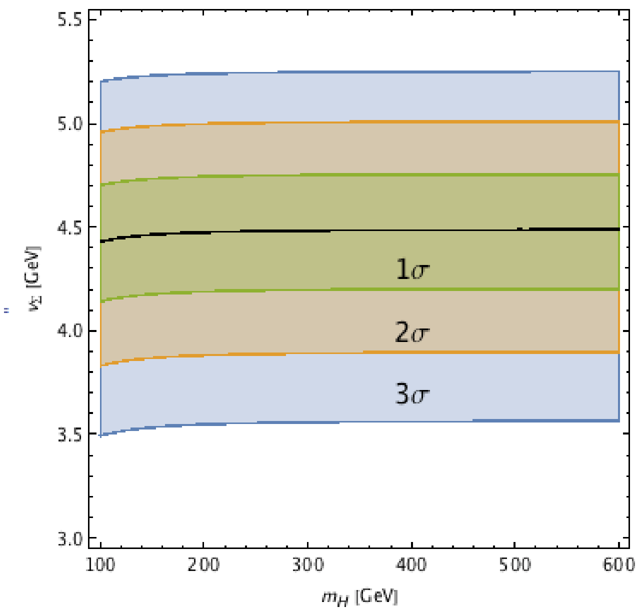
In CP violating Dark photon kinetic mixing, there is also a scalar triple contribution to W mass. Tree and Loop contribution from Y=0 triple Σ

$$V(\phi,\Sigma)=-m_\phi^2(\phi^+\phi)+\lambda_0(\phi^+\phi)^2-M_\Sigma^2 Tr(\Sigma^2)+\lambda_1 Tr(\Sigma^4)+\lambda_2(Tr(\Sigma^2))^2$$
$$+\alpha(\phi^+\phi)Tr(\Sigma^2)+\beta\phi^+\Sigma^2\phi+a_1\phi^+\Sigma\phi\,.$$

$$S=-\frac{2}{\pi}\sum_{I_3=-1}^{1}(I_3c_W^2)^2\xi\left(\frac{m_{I_3}^2}{m_Z^2},\frac{m_{I_3}^2}{m_Z^2}\right)=-\frac{4c_W^4}{\pi}\xi\left(\frac{m_{H^+}^2}{m_Z^2},\frac{m_{H^+}^2}{m_Z^2}\right)\approx-\frac{c_W^4}{15\pi}\frac{m_Z^2}{m_{H^+}^2}\,,$$

$$T=\frac{1}{16\pi c_W^2 s_W^2}\sum_{I_3=-1}^{1}(2-I_3^2+I_3)\eta\left(\frac{m_{I_3}^2}{m_Z^2},\frac{m_{I_3-1}^2}{m_Z^2}\right)=\frac{1}{12\pi c_W^2 s_W^2 m_Z^2}\frac{(m_{H^+}^2-m_{H^0}^2)^2}{m_{H^+}^2}\approx 0\,,$$

$$U=\frac{1}{6\pi}\sum_{I_3=-1}^{1}(2-3I_3^2)\ln\frac{m_{I_3}^2}{\mu^2}+\frac{1}{\pi}\sum_{I_3=-1}^{1}\left[2(I_3c_W^2)^2\xi\left(\frac{m_{I_3}^2}{m_Z^2},\frac{m_{I_3}^2}{m_Z^2}\right)-(2-I_3^2+I_3)\xi\left(\frac{m_{I_3}^2}{m_W^2},\frac{m_{I_3-1}^2}{m_W^2}\right)\right]$$
$$=\frac{1}{6\pi}\ln\frac{m_{H^0}^4}{m_{H^+}^4}+\frac{4}{\pi}\left[c_W^4\xi\left(\frac{m_{H^+}^2}{m_Z^2},\frac{m_{H^+}^2}{m_Z^2}\right)-\xi\left(\frac{m_{H^+}^2}{m_W^2},\frac{m_{H^0}^2}{m_W^2}\right)\right]\approx-\frac{c_W^2 s_W^2}{15\pi}\frac{m_Z^2}{m_{H^+}^2}\,.$$



Tree: $\Delta m_W^2=m_Z^2c_W^2\frac{c_W^2}{c_W^2-s_W^2}\frac{4v_\Sigma^2}{v^2}$

Loop: $\Delta m_W^2=m_Z^2c_W^2\frac{\alpha}{60\pi}\frac{c_W^2}{c_W^2-s_W^2}\frac{m_Z^2}{m_{H^+}^2}\,.$

Both tree and loop can have significant contributions to W mass. Can explain W mass anomaly!

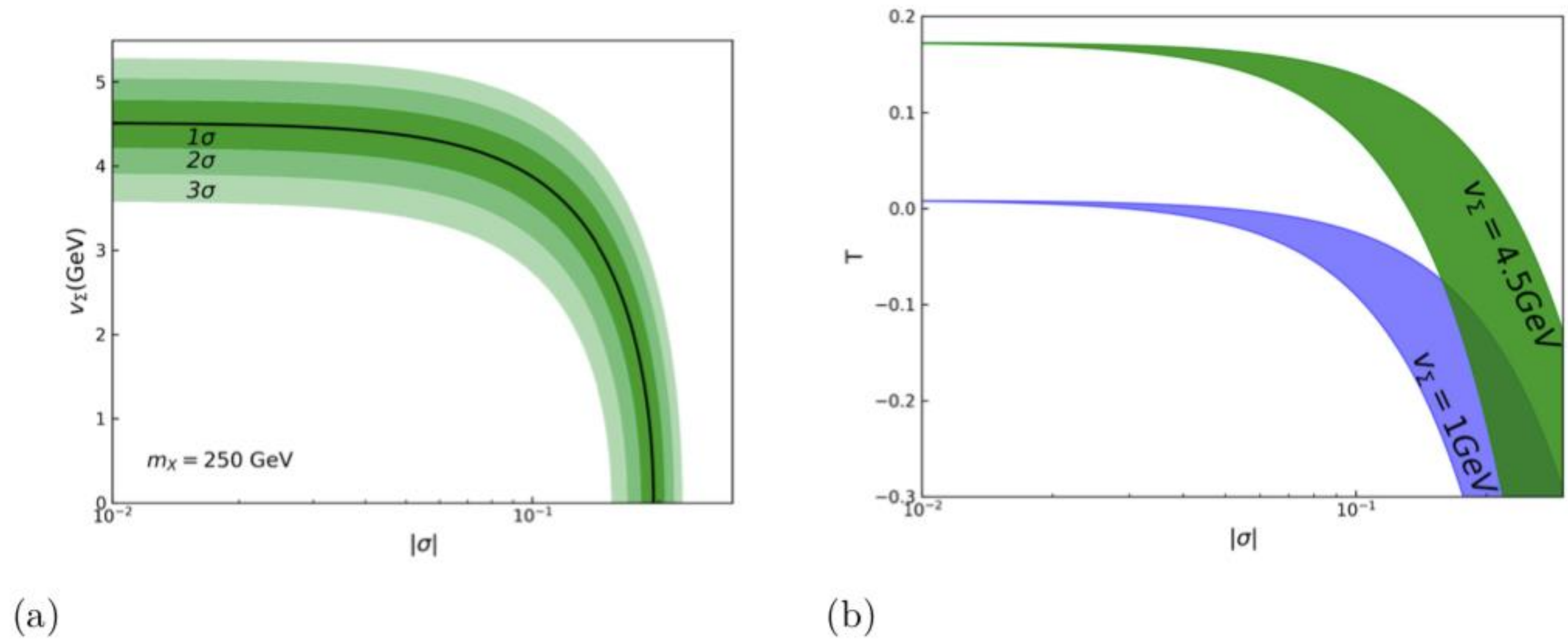


FIG. 2: (a) The CDF allowed regions in $|\sigma| - v_\Sigma$ plane for some given values for m_X. (b) The T parameter for two different values of v_Σ.

- CP violating kinetic mixing can be induced for abelian and non-abelian gauge particles.
- Effects can be searched by studying EDM of fundamental particles.
- Study of the CP violation effects are challenging at colliders.
- Can help to easy the W mass anomaly problem