宇称变换

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宇称变换

- 宇称变换，用符号$\hat{P}$表示，又称空间反射变换，是把空间坐标$\vec{x} \rightarrow -\vec{x}$，也叫做左右变换。宇称变换不能通过连续的洛伦兹变换来实现，是一个分立的变换。

宇称变换

- Dirac场有：

$$\psi(x)=\int\frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\mathbf{p}}}}\sum_s\left(a^s_{\mathbf{p}}u^s(p)e^{-ip\cdot x}+b^{s\dagger}_{\mathbf{p}}v^s(p)e^{ip\cdot x}\right)$$

- 其中，$Pa^s_{\mathbf{p}}P=\eta_a a^s_{-\mathbf{p}}$ ，$Pb^s_{\mathbf{p}}P=\eta_b b^s_{-\mathbf{p}}$
- 易得：

$$P\psi(x)P=\int\frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\mathbf{p}}}}\sum_s\left(\eta_a a^s_{-\mathbf{p}}u^s(p)e^{-ipx}+\eta^*_b b^{s\dagger}_{-\mathbf{p}}v^s(p)e^{ipx}\right)$$

- 引入变量：$\tilde{p}=\left(p^0,-\vec{p}\right)$

$$P\psi(x)P=\int\frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\mathbf{p}}}}\sum_s\left(\eta_a a^s_{-\mathbf{p}}u^s(p)e^{-ipx}+\eta_b^* b^{s\dagger}_{-\mathbf{p}}v^s(p)e^{ipx}\right)$$

由$\tilde{p}=\left(p^0,-\vec{p}\right)$，可以得到：

$$\tilde{p}\cdot\sigma=p\cdot\bar{\sigma}$$
$$\tilde{p}\cdot\bar{\sigma}=p\cdot\sigma$$
$$P\cdot x=\tilde{P}\cdot(t,-x)$$

这样我们可以写出：

$$u(p)=\begin{pmatrix}\sqrt{p\cdot\sigma}\,\xi\\ \sqrt{p\cdot\bar{\sigma}}\,\xi\end{pmatrix}=\begin{pmatrix}\sqrt{\tilde{p}\cdot\bar{\sigma}}\,\xi\\ \sqrt{\tilde{p}\cdot\sigma}\,\xi\end{pmatrix}=\gamma^0u(\tilde{p});$$

$$v(p)=\begin{pmatrix}\sqrt{p\cdot\sigma}\,\xi\\ -\sqrt{p\cdot\bar{\sigma}}\,\xi\end{pmatrix}=\begin{pmatrix}\sqrt{\tilde{p}\cdot\bar{\sigma}}\,\xi\\ -\sqrt{\tilde{p}\cdot\sigma}\,\xi\end{pmatrix}=-\gamma^0v(\tilde{p}).$$

改变变量得到：

$$P\psi(x)P=\int\frac{d^3\tilde{p}}{(2\pi)^3}\frac{1}{\sqrt{2E_{\tilde{p}}}}\sum_s\left(\eta_a a^s_{\tilde{p}}\gamma^0u^s(\tilde{p})e^{-i\tilde{p}(t,-\mathbf{x})}\right.$$
$$\left.-\eta_b^* b^{s\dagger}_{\tilde{p}}\gamma^0v^s(\tilde{p})e^{i\tilde{p}(t,-\mathbf{x})}\right)$$

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$$P\psi(x)P = \int \frac{d^3\tilde{p}}{(2\pi)^3}\frac{1}{\sqrt{2E_{\tilde{p}}}}\sum_s \Big(\eta_a a^s_{\tilde{p}}\gamma^0 u^s(\tilde{p})e^{-i\tilde{p}(t,-\mathbf{x})} - \eta_b^* b^{s\dagger}_{\tilde{p}}\gamma^0 v^s(\tilde{p})e^{i\tilde{p}(t,-\mathbf{x})}\Big)$$

如果使 $\eta_a = -\eta_b^*$，就有宇称变换的最终形式：

$$P\psi(t,\mathbf{x})P = \eta_a\gamma^0\psi(t,-\mathbf{x}).$$

宇称变换

各种双线性量在宇称下的变换

$$\bar{\psi}\psi, \qquad \bar{\psi}\gamma^{\mu}\psi, \qquad \bar{\psi}\gamma^{\mu}\gamma^{5}\psi, \qquad i\bar{\psi}\gamma^{5}\psi.$$

$$P\psi(t,\mathbf{x})P = \eta_a\gamma^0\psi(t,-\mathbf{x}).$$

$$P\overline{\Psi}(t,x)P = \eta_a^*\overline{\Psi}(t,x)\gamma^0$$

宇称变换

标量：

$$P\overline{\Psi}\Psi P = P\overline{\Psi}PP\Psi P$$

$$= \eta_a^* \eta_a \overline{\Psi}(t,-x)\gamma^0\gamma^0\Psi(t,-x)$$

$$= \left|\eta_a\right|^2 \overline{\Psi}\Psi(t,-x)$$

$$= \overline{\Psi}\Psi(t,-x)$$

矢量：

$$P\bar{\psi}\gamma^\mu\psi P = \bar{\psi}\gamma^0\gamma^\mu\gamma^0\psi(t,-\mathbf{x}) = \begin{cases} +\bar{\psi}\gamma^\mu\psi(t,-\mathbf{x}) & \text{for } \mu = 0, \\ -\bar{\psi}\gamma^\mu\psi(t,-\mathbf{x}) & \text{for } \mu = 1,2,3. \end{cases}$$

宇称变换

赝标量：

$$P i\bar{\psi}\gamma^5\psi P = i\bar{\psi}\gamma^0\gamma^5\gamma^0\psi(t,-\mathbf{x}) = -i\bar{\psi}\gamma^5\psi(t,-\mathbf{x});$$

$$\gamma^0\gamma^5 = -\gamma^5\gamma^0$$

赝矢量：

$$P\bar{\psi}\gamma^\mu\gamma^5\psi P = \bar{\psi}\gamma^0\gamma^\mu\gamma^5\gamma^0\psi(t,-\mathbf{x}) = \begin{cases} -\bar{\psi}\gamma^\mu\gamma^5\psi & \text{for } \mu = 0, \\ +\bar{\psi}\gamma^\mu\gamma^5\psi & \text{for } \mu = 1,2,3. \end{cases}$$

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