

Institute of High Energy Physics **Chinese Academy of Sciences**

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Charm meson decays at BESIII



OutLine

✓ Charm meson physics

✓ BESIII experiment

✓ Highlight charm meson results @ BESIII

✓ Summary



Why charm meson physics?

Pure leptonic decay



Ideal bridge to access the strong and weak effects between quarks

 \checkmark V_{cs} and V_{cd} \rightarrow Test CKM matrix unitarity

✓ Decay Constant $f_{D_{(s)}^+}$ and form factor f_+ → Calibrate LQCD

 \checkmark Branching fraction (BF) ratio \rightarrow Test lepton flavor universality





Why charm meson physics?

Hadronic decay



Probe non-perturbative QCD

- Help to understand hadron spectroscopy \checkmark
- Probe the weak decay mechanisms in \checkmark DCS decays

Amplitude analysis \rightarrow Get the information of $D \rightarrow VP, PP \dots$, where V and P denote vector and pseudoscalar, respectively.

Absolute BFs measurement \rightarrow Test theoretical calculations of these BFs and benefit the understanding of the quark SU(3) flavor symmetry and *CP* violation.



Why charm meson physics?

Quantum correlation D^0D^0 :



- \checkmark Important input in CKM γ measurement
- \checkmark Precise test of perturbative QCD calculations in charm decays, mixing and CPV

Different methods depending on the final states of D decays

- GLW : *D* decaying to *CP* eigenstates
- ADS : *D* decaying to CF/DCS eigenstates
- **GGSZ** : *D* decaying to self-conjugate eigenstates

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	Flavour	K^{\pm}
	CP-even	$K^{+}K^{-}, \pi^{+}\pi^{-}$
	CP-odd	$K^0_S \pi^0$
	Self-conjugate	

$$|\psi(3770)\rangle \rightarrow \frac{1}{\sqrt{2}}(|D^0\rangle|\overline{D}^0\rangle - |\overline{D}^0\rangle|D^0\rangle)$$

Provide direct access to the $D^0 - D^0$ strong-phase difference

$$\pi^{\mp}\pi^{\mp}\pi^{-}, K^{\pm}\pi^{\mp}\pi^{0}, K^{\pm}\pi^{\mp}, \dots$$

$$\bar{}, \pi^{0}\pi^{0}, K^{0}_{S}\pi^{0}\pi^{0}, K^{0}_{L}\pi^{0}, K^{0}_{L}\omega, \pi^{+}\pi^{-}\pi^{0^{\dagger}}, K^{0}_{S}\eta, K^{0}_{S}\omega, K^{0}_{S}\eta', K^{0}_{S}\phi, K^{0}_{L}\pi^{0}\pi^{0}$$

$$K^{0}_{S}\pi^{+}\pi^{-}, K^{0}_{S}K^{+}K^{-}, \dots$$



BESIII and Datasets





 $-D^{+(0)}$: 2.93 fb⁻¹ @ E_{cm} = 3.773 GeV. Collected in 2011 $-D_s^+$: 7.33 fb⁻¹ @ E_{cm} = 4.128 – 4.226 GeV. Collected in 2013-2017

> Single Tag (ST) : reconstruct one $D_{(s)}$ — Relative high background — Higher efficiency

 D^0 Double Tag (DT) : reconstruct both $D_{(s)}$ — Clean background for study of various decays — Systematics in the tag side almost cancel out

 N_{sig}^{DT} Absolute branching fraction via DT : $\mathcal{B}_{sig} =$ $\Sigma_{\alpha} N_{\alpha}^{\text{ST}} \epsilon_{\alpha,sig}^{\text{DT}} / \epsilon_{\alpha}^{\text{ST}}$







(Semi-)Leptonic decays

Pure leptonic decays:

 $D_{s}^{+} \rightarrow \tau^{+} \nu_{\tau}, \tau^{+} \rightarrow e^{+} \nu_{\rho} \bar{\nu}_{\tau}$, Phys. Rev. Lett. 127, 171801 (2021) $D_s^+ \to \tau^+ \nu_{\tau}, \tau^+ \to \pi^+ \bar{\nu}_{\tau} \& D_s^+ \to \mu^+ \nu_{\mu}, \text{Phys. Rev. D 104, 052009 (2021)}$ $D_{s}^{+} \rightarrow \tau^{+} \nu_{\tau}, \tau^{+} \rightarrow \pi^{+} \pi^{0} \bar{\nu}_{\tau}$, Phys. Rev. D 104, 032001 (2021) $D_s^+ \rightarrow \tau^+ \nu_{\tau}, \tau^+ \rightarrow \mu^+ \nu_{\mu} \bar{\nu}_{\tau}, \text{ arXiv:} 2303.12468$ $D_s^+ \rightarrow \tau^+ \nu_{\tau}, \tau^+ \rightarrow \pi^+ \overline{\nu}_{\tau}, \text{arXiv:2303.12600}$ $D_{c}^{*+} \rightarrow e^{+}\nu_{\rho}$, arXiv:2304.12159, first experimental result on $f_{D_{c}^{*+}}$

Semi-leptonic decays:

 $D^0 \to K_1(1270)^- e^+ \nu_{\rho}$, Phys. Rev. Lett. 127, 131801 (2021) $D_s^+ \rightarrow a_0(980)^0 e^+ \nu_{\rho}$, Phys. Rev. D 103, 092004(2021) $D^0 \to K^- e^+ \nu_{\rho} \& D^+ \to \bar{K}^0 e^+ \nu_{\rho}$, Phys. Rev. D 104, 052008 (2021) $D_s^+ \to \pi^0 \pi^0 e^+ \nu_e \& K_s^0 K_s^0 e^+ \nu_e$, Phys. Rev. D 105, L031101 (2022) $D_{s}^{+} \rightarrow \pi^{0}e^{+}\nu_{e}$, Phys. Rev. D 106, 112004 (2022) $D_{s}^{+} \rightarrow \pi^{+}\pi^{-}e^{+}\nu_{e}$, arXiv:2303.12927 $D_s^+ \rightarrow \eta e^+ \nu_e, \eta' e^+ \nu_e, \text{arXiv:2306.05194}$

CKMFitter HFLAV21	PTEP2022(2022)083C01 arXiv:2206.07501 [hep-ex]	0.97349±0.00016 0.9701±0.0081	•
CLEO CLEO CLEO BaBar Belle BESIII 0.482 fb ⁻¹ CLEO BaBar Belle BESIII 3.19 fb ⁻¹ BESIII 6.32 fb ⁻¹	PRD79(2009)052002, $\tau_e v$ PRD80(2009)112004, $\tau_\rho v$ PRD79(2009)052001, $\tau_\pi v$ PRD82(2010)091103, $\tau_{e,\mu} v$ JHEP09(2013)139, $\tau_{e,\mu,\pi} v$ PRD94(2016)072004, μv PRD79(2009)052001, μv PRD82(2010)091103, μv JHEP09(2013)139, μv JHEP09(2013)139, μv PRL122(2019)071802, μv PRD104(2021)052009, μv	$\begin{array}{c} 0.981 {\pm} 0.044 {\pm} 0.021 \\ 1.001 {\pm} 0.052 {\pm} 0.019 \\ 1.079 {\pm} 0.068 {\pm} 0.016 \\ 0.953 {\pm} 0.033 {\pm} 0.047 \\ 1.017 {\pm} 0.019 {\pm} 0.028 \\ 0.956 {\pm} 0.069 {\pm} 0.020 \\ 1.000 {\pm} 0.040 {\pm} 0.016 \\ 1.032 {\pm} 0.033 {\pm} 0.029 \\ 0.969 {\pm} 0.026 {\pm} 0.019 \\ 0.985 {\pm} 0.014 {\pm} 0.014 \\ 0.973 {\pm} 0.012 {\pm} 0.015 \end{array}$	≖≖≖⊥⊥ ⊥ ⊥
BESIII 6.32 fb ⁻¹ BESIII 6.32 fb ⁻¹ BESIII 6.32 fb ⁻¹ BESIII 7.33 fb ⁻¹ BESIII 7.33 fb ⁻¹ BESIII	PRD104(2021)052009, $\tau_{\pi}v$ PRD104(2021)032001, $\tau_{\rho}v$ PRL127(2021)171801, $\tau_{e}v$ arXiv:2303.12600 [hep-ex], $\tau_{\pi}v$ arXiv:2303.12468 [hep-ex], $\tau_{\mu}v$ τv -1 0	0.972±0.023±0.016 0.980±0.023±0.019 0.978±0.009±0.012 0.991±0.015±0.013 0.984±0.015±0.010 0.982±0.007±0.008	For the second s

ETM(2+1+1) FMILC(2+1+1) FLAG21(2+1+1)	PRD91(2015)054507 PRD98(2018)074512 arXiv:2111.09849 [hep-lat]	247.2±4.1 249.9±0.4 249.9±0.5	
HFLAV21 CLEO CLEO BaBar Belle BESIII 0.482 fb ⁻¹ CLEO BaBar Belle BESIII 3.19 fb ⁻¹ BESIII 6.32 fb ⁻¹	arXiv:2206.07501 [hep-ex] PRD79(2009)052002, $\tau_e v$ PRD80(2009)112004, $\tau_\rho v$ PRD79(2009)052001, $\tau_\pi v$ PRD82(2010)091103, $\tau_{e,\mu} v$ JHEP09(2013)139, $\tau_{e,\mu} v$ PRD94(2016)072004, μv PRD79(2009)052001, μv PRD79(2009)052001, μv PRD82(2010)091103, μv JHEP09(2013)139, μv PRL122(2019)071802, μv PRD104(2021)052009, μv	$\begin{array}{c} 252.2\pm2.5\\ 251.8\pm11.2\pm5.3\\ 257.0\pm13.3\pm5.0\\ 277.1\pm17.5\pm4.0\\ 244.6\pm8.6\pm12.0\\ 261.1\pm4.8\pm7.2\\ 245.5\pm17.8\pm5.1\\ 256.7\pm10.2\pm4.0\\ 264.9\pm8.4\pm7.6\\ 248.8\pm6.6\pm4.8\\ 253.0\pm3.7\pm3.6\\ 249.8\pm3.0\pm3.9\\ \end{array}$	╪╪┶┰╻╴╴╴╴╴╴╴
BESIII 6.32 fb ⁻¹ BESIII 6.32 fb ⁻¹ BESIII 6.32 fb ⁻¹ BESIII 7.33 fb ⁻¹ BESIII 7.33 fb ⁻¹ BESIII BESIII	$\begin{array}{c} \textbf{PRD104(2021)052009, } \tau_{\pi}\nu \\ \textbf{PRD104(2021)032001, } \tau_{\rho}\nu \\ \textbf{PRL127(2021)171801, } \tau_{e}\nu \\ \textbf{arXiv:2303.12600 [hep-ex], } \tau_{\pi}\nu \\ \textbf{arXiv:2303.12468 [hep-ex], } \tau_{\mu}\nu \\ \textbf{tv} \\ \textbf{100} \\ \textbf{f}_{D^{+}} (\textbf{MeV}) \end{array}$	249.7±6.0±4.2 251.6±5.9±4.9 251.1±2.4±3.0 254.3±4.0±3.3 252.7±3.8±2.6 252.1±1.7±2.0	HH H-H H-H H-H F-1 Combined 300



Amplitude analysis of $D_{s}^{+} \rightarrow K_{s}^{0}K_{s}^{0}\pi^{+}$



about 97% purity



S(1710): mixture of $f_0(1710)$ and $a_0(1710)$:

- Destructive interference in $D_s^+ \to K^+ K^- \pi^+$ PRD 104, 012016 (2021)
- Constructive interference in $D_s^+ \to K_s^0 K_s^0 \pi^+$



First amplitude analysis

Phys. Rev. D 105, L051103 (2022)



Observation of isospin-one $a_0(1710)$

Consistent with the $K^*\bar{K}^*$ molecule hypothesis of $f_0(1710)$ $\mathscr{B}(D_s^+ \to K_S^0 K_S^0 \pi^+) = (0.68 \pm 0.04_{\text{stat.}} \pm 0.01_{\text{syst.}})\%$









Amplitude analysis of $D_{s}^{+} \rightarrow K_{s}^{0}K^{+}\pi^{0}$



	FF (%)	BF (10 ⁻³)	
$(892)^0 K^+$	$32.7 \pm 2.2 \pm 1.9$	$4.77 \pm 0.38 \pm 0.32$	>
$(892)^+ K_S^0$	$13.9\pm1.7\pm1.3$	$2.03 \pm 0.26 \pm 0.20$	>
$(80)^{+}\pi^{0}$	$7.7\pm1.7\pm1.8$	$1.12 \pm 0.25 \pm 0.27$	(
$(410)^0 K^+$	$6.0\pm1.4\pm1.3$	$0.88 \pm 0.21 \pm 0.19$,
$(817)^{+}\pi^{0}$	$23.6\pm3.4\pm2.0$	$3.44 \pm 0.52 \pm 0.32$	>

Amplitude analysis of $D_{s}^{+} \rightarrow \pi^{+}\pi^{0}\eta'$

Decay		$\mathcal{B}(\%)$		
Theory	$D_s^+ o ho^+ \eta'$	3.0 ± 0.5 [1]	1.7 [2]	1.6 [2]
	$D_s^+ \to \pi^+ \pi^0 \eta'$	$5.6\pm0.5\pm0.6$	CLE	0
Experiment	$D_s^+ o ho^+ \eta'$	$5.8\pm1.4\pm0.4$		
	$D_s^+ \to \pi^+ \pi^0 \eta'$	< 5.1	BESI	II
	(nonresonant)	(90% confidence level)		

Large deviation between theoretical predictions and experimental measurements [1] Phys. Rev. D 84 (2011) 074019 [2] Phys. Rev. D 89 (2014) 054006

Branching fraction measurement with best precision : $\mathscr{B}(D_s^+ \to \pi^+ \pi^0 \eta') = (6.15 \pm 0.25_{\text{stat.}} \pm 0.18_{\text{syst.}})\%$

 $\mathscr{B}(D_s^+ \to (\pi^+ \pi^0)_S \eta') < 0.1 \% @ 90\% \text{ CL}$ $\mathscr{B}(D_{c}^{+} \to (\pi^{+}\pi^{0})_{P}\eta') < 0.74\% @ 90\% \text{ CL}$

JHEP04(2022)058

Amplitude analysis of $D^0 \to K_{S,L}^0 \pi^+ \pi^-$

16490 events for $K_S^0 \pi^+ \pi^-$, 39085 for $K_I^0 \pi^+ \pi^-$

arXiv:2212.09048

The U-spin parameters are assumed to be unity in the model-independent strong phase measurement of $D^0 \to K^0_{SL} \pi^+ \pi^-$.

Determine the complex U-spin breaking parameters by simultaneous fit of $D^0 \to K^0_{SI} \pi^+ \pi^-$

 \rightarrow Reduce the main uncertainty from predicted $\Delta c_i(s_i)$

Resonance	$K_{ m L}^0 \pi^+ \pi^- \ F F_R \ [\%]$	$K_{ m S}^0 \pi^+ \pi^- \; F F_R \; [\%]$
ho(770)	$18.16^{+0.53}_{-0.45}\pm2.50$	$18.90 \pm 0.42 \pm 2.12$
$\omega(782)$	$0.06^{+0.03}_{-0.02}\pm0.04$	$0.54 \pm 0.09 \pm 0.14$
$f_2(1270)$	$0.40 \pm 0.08 \pm 0.37$	$0.61^{+0.13}_{-0.11}\pm 0.29$
ho(1450)	$0.42 \pm 0.08 \pm 0.53$	$0.21 \pm 0.10 \pm 0.40$
$\pi\pi$ S-wave	$10.12^{+0.32}_{-0.33}\pm0.96$	$10.24 \pm 0.23 \pm 1.62$

Resonance	$ \hat{ ho} $	$\arg(\hat{ ho})$ [°]	$ 1-2{ m tan}^2 heta_C\hat ho ^2$
ho(770)	$1.93 \pm 0.27 \pm 0.42$	$-90.6 \pm 5.8 \pm 7.6$	$1.05 \pm 0.04 \pm 0.06$
$\omega(782)$	$6.13 \pm 0.75 \pm 0.53$	$2.2\pm7.0\pm4.8$	$0.12 \pm 0.05 \pm 0.04$
$f_2(1270)$	$3.75 \pm 0.90 \pm 0.81$	$-56.5 \pm 16.8 \pm 12.9$	$0.72 \pm 0.20 \pm 0.15$
ho(1450)	$12.12 \pm 2.92 \pm 1.88$	$78.4 \pm 14.4 \pm 15.6$	$2.19 \pm 0.95 \pm 0.83$
$\pi\pi$ S-wave	$0.37 \pm 0.21 \pm 0.37$	$-164.4 \pm 15.7 \pm 13.4$	$1.08 \pm 0.04 \pm 0.08$

Polarizations in $D^0 \rightarrow \omega \phi$

Single tag method — only one D^0 meson is reconstructed

Phys. Rev. Lett. 128, 011803 (2022)

- Black dots: data
- Green: longitudinal
- Cyan: PHSP

- ω and ϕ are transversely polarized
- Black curves: fit results
 Contradict existing model

predictions

Phys. Rev. D 81, 114020 (2010); J. High Energy Phys. 03 (2014) 042

BF measurements of DCS decays

• $D^+ \to K^+ \pi^- \pi^0$ Phys. Rev. Lett. 125, 141802 (2020) • $D^0 \to K^+ \pi^- \pi^0$, $K^+ \pi^- \pi^0 \pi^0$ $\mathscr{B}(D^+ \to K^+ \pi^+ \pi^- \pi^0) = (1.13 \pm 0.08(\text{stat}) \pm 0.03(\text{syst})) \times 10^{-3} \ \mathscr{B}(D^0 \to K^+ \pi^- \pi^0) = [3.13^{+0.60}_{-0.56}(\text{stat}) \pm 0.15(\text{syst})] \times 10^{-4}$ with removing the contribution of the known decays $\mathscr{B}(D^0 \to K^+ \pi^- \pi^0 \pi^0) < 3.6 \times 10^{-4} @ 90 \% \text{ CL}.$ $D^+ \to K^+ \eta, K^+ \omega, K^+ \phi$ Phys. Rev. D 105, 112001 (2022) $\mathscr{B}(D^+ \to K^+ \pi^+ \pi^- \pi^0) / \mathscr{B}(D^+ \to K^- \pi^+ \pi^+ \pi^0) = (1.81 \pm 0.15) \%$ • $D^+ \to K^+ \pi^0 \pi^0, K^+ \pi^0 \eta$ JHEP09(2022)107 \rightarrow Significantly larger than the values (0.21–0.58)% measured for the other DCS decays

Decay mode	$N_{ m DT}$	$\epsilon_{ m sig}(\%)$	$\mathcal{B}_{\mathrm{sig}}$ (×10)
$D^+ \to K^+ \pi^0 \pi^0$	42.8 ± 7.2	18.08 ± 0.03	2.1 ± 0.4 =
$D^+ \to K^+ \pi^0 \eta$	19.2 ± 5.0	20.50 ± 0.03	2.1 ± 0.5 =
$D^+ \to K^{*+} \pi^0$	$16.6\substack{+6.6 \\ -6.2}$	13.02 ± 0.03	$3.4^{+1.4}_{-1.3}\pm$
$D^+ \to K^{*+} \eta$	$10.9\substack{+4.4\\-3.8}$	16.60 ± 0.04	$4.4^{+1.8}_{-1.5}\pm$

Strong phase measurements

• $D^0 \to K^- \pi^+$ Eur. Phys. J. C 82, 1009 (2022)

 $\delta_D^{K\pi} = (187.6^{+8.9+5.4}_{-9.7-6.4})^\circ$, most precise measurement

$$r_D^{K\pi} exp(-i\delta_D^{K\pi}) = \frac{\langle K^+\pi^- | D^0 \rangle}{\langle K^+\pi^- | \bar{D}^0 \rangle},$$

where $r_D^{K\pi}$ are $\delta_D^{K\pi}$ the ratio of amplitudes and phase difference, respectively, between the DCS and CF decays. $A_{K\pi} = 0.132 \pm 0.001 \pm 0.007, 30\%$ more precision $A_{K\pi}^{\pi\pi\pi^0} = 0.130 \pm 0.012 \pm 0.008$ $\mathscr{B}(D^0 \to K_L^0 \pi^0) = (0.97 \pm 0.03 \pm 0.02) \times 10^{-2}$ $\mathscr{B}(D^0 \to K_L^0 \omega) = (1.09 \pm 0.06 \pm 0.03) \times 10^{-2}$ $\mathscr{B}(D^0 \to K_L^0 \pi^0 \pi^0) = (1.26 \pm 0.05 \pm 0.03) \times 10^{-2}.$

• $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ Phys. Rev. D 106, 092004 (2022) $F_{+} = 0.735 \pm 0.015 \pm 0.005,$ \rightarrow predominantly *CP* – even most precise determination • $D^0 \to K^+ K^- \pi^+ \pi^-$ Phys. Rev. D 107, 032009 (2023) $F_{+} = 0.730 \pm 0.037 \pm 0.021,$ \rightarrow predominantly *CP* – even first model-independent measurement of F_{+} of this decay • $D^0 \rightarrow K^0_{\rm S} \pi^+ \pi^- \pi^0$ arXiv:2305.03975 $F_{+} = 0.235 \pm 0.010 \pm 0.002,$

 \rightarrow predominantly *CP* – odd

Summary

✓ Charm is charming

- (Semi-)leptonic decays access to CKM matrix elements and calibrate LQCD
- \bullet model-independent determination of γ and charm mixing/CPV

✓ BESIII makes great achievements

- Precise measurements of $D_s^+ \to \tau^+ \nu_{\tau}$
- Observation of new isospin-one particle $a_0(1817)$
- Puzzle of $P \rightarrow VV$ polarization

✓ Bright future of charm meson decays at BESIII

- Lots of results are ready to be published
- 20 fb⁻¹ ψ (3770) data at BESIII by next year CPC 44, 040001 (2020)

Hadronic decays are key labs to understand non-perturbative QCD; provide crucial inputs to

Thanks for your attention!

Back up

Amplitude formalization in $D^0 \rightarrow K_{SI}^0 \pi^+ \pi^-$

In simultaneous fit of $D^0 \to K_S^0 \pi^+ \pi^-$ and $D^0 \to K_L^0 \pi^+ \pi^-$, they share the same model, magnitudes and phases, while the total amplitude are:

$$A(D^0 \to K_L^0 \pi^+ \pi^-) = \sum_r A^r$$

where θ_C is Cabibbo missing angle (sin $\theta_C = 0.22650 \pm 0.00048$), $\hat{\rho}_{kCP}$ is U-spin breaking parameter.

Standalone results of $D^0 \rightarrow K_{S,L}^0 \pi^+ \pi^-$

Resonance	$K_{\rm L}^0 \pi^+ \pi^-$ FF(%) (Simultaneous fit)	$K_{\rm L}^0 \pi^+ \pi^-$ FF(%) (Standalone $K_{\rm L}^0 \pi^+ \pi^-$)
ho(770)	$18.16\substack{+0.53 \\ -0.45}$	18.94 ± 1.20
$\omega(782)$	$0.06\substack{+0.03\\-0.02}$	0.06 ± 0.03
$f_2(1270)$	0.40 ± 0.08	0.36 ± 0.08
ho(1450)	0.42 ± 0.08	0.43 ± 0.10
$K^{*}(892)^{-}$	$56.98\substack{+0.58\\-0.56}$	57.1 ± 1.65
$K_2^*(1430)^-$	$1.64\substack{+0.10 \\ -0.09}$	1.58 ± 0.15
$K^{*}(1680)^{-}$	$0.25\substack{+0.06 \\ -0.05}$	0.22 ± 0.11
$K^{*}(1410)^{-}$	0.19 ± 0.06	0.11 ± 0.06
$K^{*}(892)^{+}$	0.45 ± 0.05	0.37 ± 0.06
$K_2^*(1430)^+$	0.05 ± 0.02	0.02 ± 0.02
$K^{*}(1410)^{+}$	0.04 ± 0.02	0.02 ± 0.02
$K_0^*(1430)^-$	$6.84\substack{+0.24 \\ -0.25}$	5.80 ± 0.38
$\pi\pi$ S-wave	$10.12\substack{+0.32 \\ -0.33}$	10.39 ± 1.72
Total FF	$95.59\substack{+2.16 \\ -2.07}$	95.40 ± 5.58

Definitions in $D^0 \rightarrow \omega \phi$

 θ_{ω} is the angle between $\mathbf{p}_{\pi^+}^{\omega} \times \mathbf{p}_{\pi^-}^{\omega}$ and $-\mathbf{p}_{D^0}^{\omega}$ in the ω rest frame, and θ_K is the angle between $\mathbf{p}_{K^-}^{\phi}$ and $-\mathbf{p}_{D^0}^{\phi}$ in the ϕ rest frame. Here, $\mathbf{p}_{\pi^+}^{\omega}$, $\mathbf{p}_{\pi^-}^{\omega}$, $\mathbf{p}_{K^-}^{\phi}$, and $\mathbf{p}_{D^0}^{\omega/\phi}$ are the momenta of the π^+ , π^- , K^- , and D^0 in the rest frame of either the ω or ϕ meson, respectively.

