Institute of Theoretical Physics
Chinese Academy of Sciences

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Effective Field Theories for

Weak Interactions and Neutrinos

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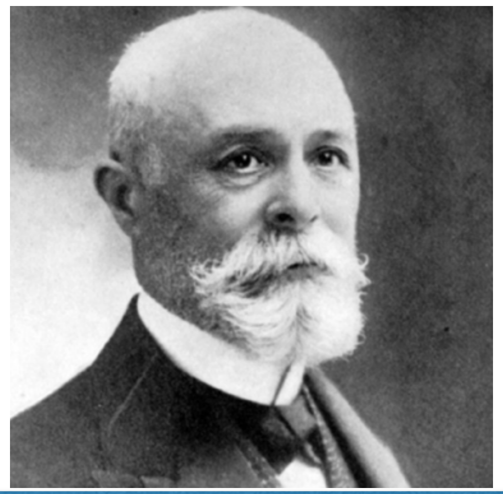
Outline

- Motivation for EFTs in weak interaction and neutrino (WIN) physics
- Standard model EFT and low energy EFT
- Chiral EFT for QCD and weak interactions
- UV Completion of EFT Operators
- Summary

Why, What and How EFT in WINs?

The first theory for weak interaction and neutrino

Four-fermion EFT theory for beta decay



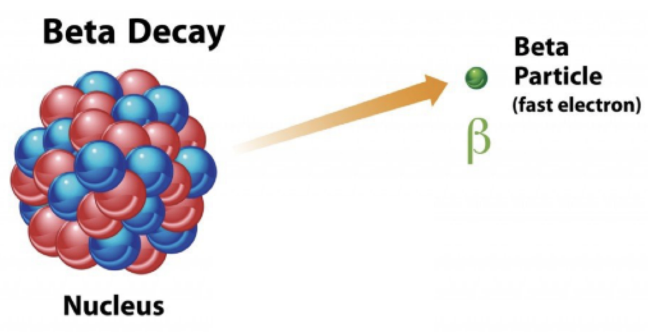
Becquerel
1896

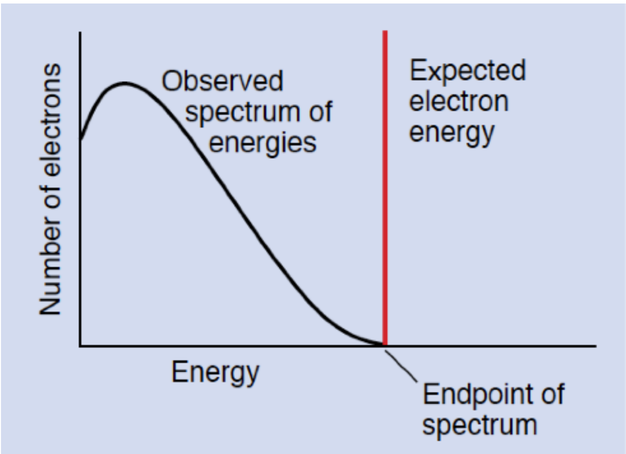
Pauli
1933

Fermi
1934

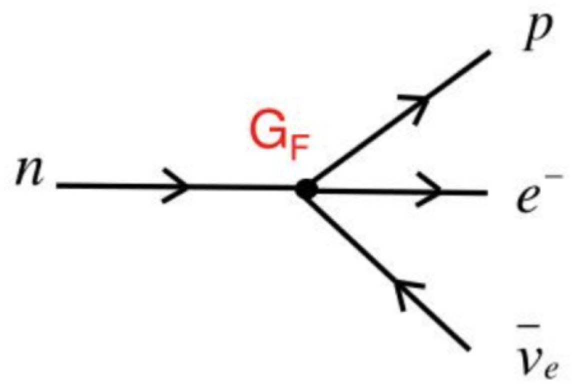
Gamov-Teller 1936
Fierz 1937

Lee-Yang 1956
Wu 1956





[see K. Heeger's talk]



$$M_{fi} = G_F \left[\overline{\psi_n}\gamma^\mu \psi_p\right] \left[\overline{\psi_e}\gamma^\mu \psi_\nu\right]$$

Four-fermi operator

First EFT

$$\mathcal{L}_i = \sum_{i=1}^{5} g_i \{\bar{\psi}_1 \mathcal{O}^i \psi_2\}\{\bar{\psi}_3 \mathcal{O}_i \psi_4\}$$

$$\mathcal{O}_i = (\ \mathbf{1},\ \gamma_\mu,\ \sigma_{\mu\nu},\ i\gamma_5\gamma_\mu,\ \text{or}\ \gamma_5\ );$$

From vector current to Fermi(V/S), GT(A/T), P

Question of Parity Conservation in Weak Interactions*

T. D. Lee, *Columbia University, New York, New York*

and

C. N. Yang,† *Brookhaven National Laboratory, Upton, New York*

(Received June 22, 1956)

If parity is not conserved in β decay, the most general form of Hamiltonian can be written as

$$\begin{aligned} H_{\text{int}} = & (\psi_p^\dagger\gamma_4\psi_n)(C_S\psi_e^\dagger\gamma_4\psi_\nu + C_S'\psi_e^\dagger\gamma_4\gamma_5\psi_\nu) \\ & + (\psi_p^\dagger\gamma_4\gamma_\mu\psi_n)(C_V\psi_e^\dagger\gamma_4\gamma_\mu\psi_\nu + C_V'\psi_e^\dagger\gamma_4\gamma_\mu\gamma_5\psi_\nu) \\ & + \tfrac{1}{2}(\psi_p^\dagger\gamma_4\sigma_{\lambda\mu}\psi_n)(C_T\psi_e^\dagger\gamma_4\sigma_{\lambda\mu}\psi_\nu \\ & + C_T'\psi_e^\dagger\gamma_4\sigma_{\lambda\mu}\gamma_5\psi_\nu) + (\psi_p^\dagger\gamma_4\gamma_\mu\gamma_5\psi_n) \\ & \times(-C_A\psi_e^\dagger\gamma_4\gamma_\mu\gamma_5\psi_\nu - C_A'\psi_e^\dagger\gamma_4\gamma_\mu\psi_\nu) \\ & + (\psi_p^\dagger\gamma_4\gamma_5\psi_n)(C_P\psi_e^\dagger\gamma_4\gamma_5\psi_\nu + C_P'\psi_e^\dagger\gamma_4\psi_\nu), \qquad \text{(A.1)} \end{aligned}$$

Low energy effective field theory (LEFT)

Four fermion EFT has been extended to describe both CC and NC weak interactions

Precision: muon decay, ve scattering, Qweak, …

Flavor physics: pion, kaon, bottom, …

[**see** X.G. He's and H.Yin's talk]

$(\overline{L}L)(\overline{L}L)$	
$\mathcal{O}_{\nu\nu}^{V,LL}$	$(\bar{\nu}_{Lp}\gamma^\mu\nu_{Lr})(\bar{\nu}_{Ls}\gamma_\mu\nu_{Lt})$
$\mathcal{O}_{ee}^{V,LL}$	$(\bar{e}_{Lp}\gamma^\mu e_{Lr})(\bar{e}_{Ls}\gamma_\mu e_{Lt})$
$\mathcal{O}_{\nu e}^{V,LL}$	$(\bar{\nu}_{Lp}\gamma^\mu\nu_{Lr})(\bar{e}_{Ls}\gamma_\mu e_{Lt})$
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$\mathcal{O}_{\nu d}^{V,LL}$	$(\bar{\nu}_{Lp}\gamma^\mu\nu_{Lr})(\bar{d}_{Ls}\gamma_\mu d_{Lt})$
$\mathcal{O}_{eu}^{V,LL}$	$(\bar{e}_{Lp}\gamma^\mu e_{Lr})(\bar{u}_{Ls}\gamma_\mu u_{Lt})$
$\mathcal{O}_{ed}^{V,LL}$	$(\bar{e}_{Lp}\gamma^\mu e_{Lr})(\bar{d}_{Ls}\gamma_\mu d_{Lt})$
$\mathcal{O}_{\nu edu}^{V,LL}$	$(\bar{\nu}_{Lp}\gamma^\mu e_{Lr})(\bar{d}_{Ls}\gamma_\mu u_{Lt})+\text{h.c.}$
$\mathcal{O}_{uu}^{V,LL}$	$(\bar{u}_{Lp}\gamma^\mu u_{Lr})(\bar{u}_{Ls}\gamma_\mu u_{Lt})$
$\mathcal{O}_{dd}^{V,LL}$	$(\bar{d}_{Lp}\gamma^\mu d_{Lr})(\bar{d}_{Ls}\gamma_\mu d_{Lt})$
$\mathcal{O}_{ud}^{V1,LL}$	$(\bar{u}_{Lp}\gamma^\mu u_{Lr})(\bar{d}_{Ls}\gamma_\mu d_{Lt})$
$\mathcal{O}_{ud}^{V8,LL}$	$(\bar{u}_{Lp}\gamma^\mu T^A u_{Lr})(\bar{d}_{Ls}\gamma_\mu T^A d_{Lt})$

$(\overline{R}R)(\overline{R}R)$	
$\mathcal{O}_{ee}^{V,RR}$	$(\bar{e}_{Rp}\gamma^\mu e_{Rr})(\bar{e}_{Rs}\gamma_\mu e_{Rt})$
$\mathcal{O}_{eu}^{V,RR}$	$(\bar{e}_{Rp}\gamma^\mu e_{Rr})(\bar{u}_{Rs}\gamma_\mu u_{Rt})$
$\mathcal{O}_{ed}^{V,RR}$	$(\bar{e}_{Rp}\gamma^\mu e_{Rr})(\bar{d}_{Rs}\gamma_\mu d_{Rt})$
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$\mathcal{O}_{\nu edu}^{V,LR}$	$(\bar{\nu}_{Lp}\gamma^\mu e_{Lr})(\bar{d}_{Rs}\gamma_\mu u_{Rt})+\text{h.c.}$
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$\mathcal{O}_{ud}^{V1,LR}$	$(\bar{u}_{Lp}\gamma^\mu u_{Lr})(\bar{d}_{Rs}\gamma_\mu d_{Rt})$
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$\mathcal{O}_{eu}^{T,RR}$	$(\bar{e}_{Lp}\sigma^{\mu\nu}e_{Rr})(\bar{u}_{Ls}\sigma_{\mu\nu}u_{Rt})$
$\mathcal{O}_{ed}^{S,RR}$	$(\bar{e}_{Lp}e_{Rr})(\bar{d}_{Ls}d_{Rt})$
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[Jenkins, Manohar, Stoffer, 2017]

[**see** K. X. Ni's and Y. Kolomensky's talk]

[**see** W. Chen's talk]

Neutrino physics: NSI, CEvNS, 0vbb, …

Rare process: cLFV, mu-e conversion, …

UV Completion of LEFT

energy

Glashow-Weinberg-Salam
1961 1967

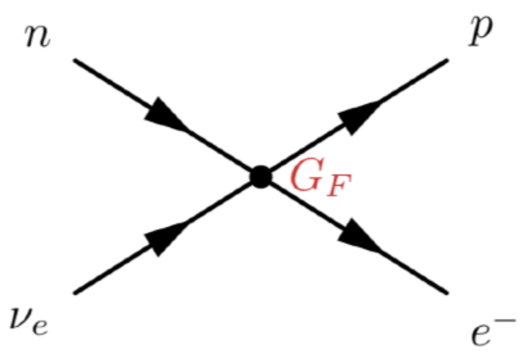
The standard model (SM) of particle physics

describe all kinds of weak interactions

V-A

Bad high energy behavior

$$\nu_e + n \to p + e^-$$



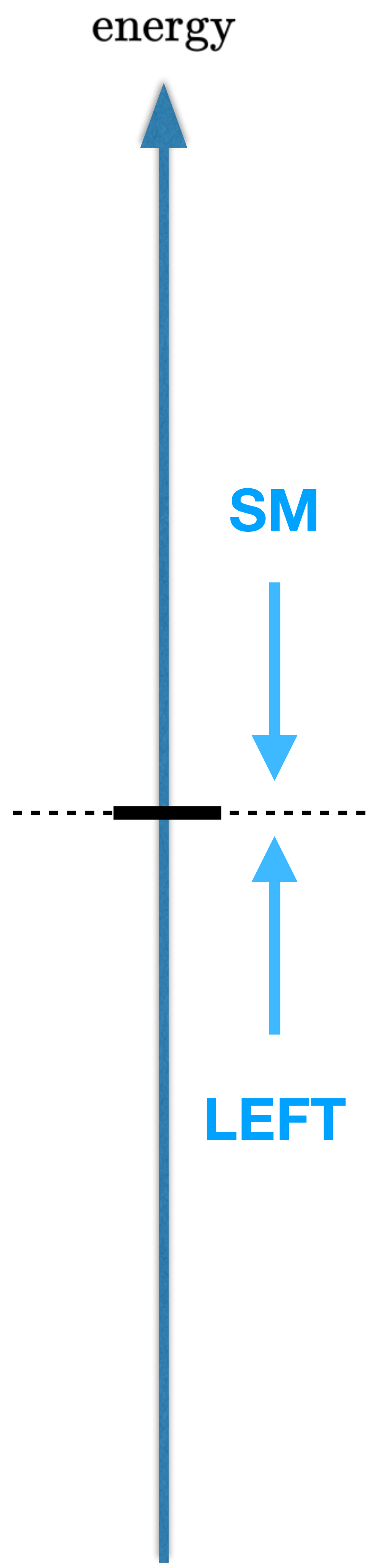
If parity is not conserved in β decay, the most general form of Hamiltonian can be written as

$$\begin{aligned} H_{\rm int} = & (\psi_p^\dagger \gamma_4 \psi_n)(C_S \psi_e^\dagger \gamma_4 \psi_\nu + C_S' \psi_e^\dagger \gamma_4 \gamma_5 \psi_\nu) \\ & + (\psi_p^\dagger \gamma_4 \gamma_\mu \psi_n)(C_V \psi_e^\dagger \gamma_4 \gamma_\mu \psi_\nu + C_V' \psi_e^\dagger \gamma_4 \gamma_\mu \gamma_5 \psi_\nu) \\ & + \tfrac{1}{2}(\psi_p^\dagger \gamma_4 \sigma_{\lambda\mu} \psi_n)(C_T \psi_e^\dagger \gamma_4 \sigma_{\lambda\mu} \psi_\nu \\ & + C_T' \psi_e^\dagger \gamma_4 \sigma_{\lambda\mu} \gamma_5 \psi_\nu) + (\psi_p^\dagger \gamma_4 \gamma_\mu \gamma_5 \psi_n) \\ & \times (-C_A \psi_e^\dagger \gamma_4 \gamma_\mu \gamma_5 \psi_\nu - C_A' \psi_e^\dagger \gamma_4 \gamma_\mu \psi_\nu) \\ & + (\psi_p^\dagger \gamma_4 \gamma_5 \psi_n)(C_P \psi_e^\dagger \gamma_4 \gamma_5 \psi_\nu + C_P' \psi_e^\dagger \gamma_4 \psi_\nu), \quad \text{(A.1)} \end{aligned}$$

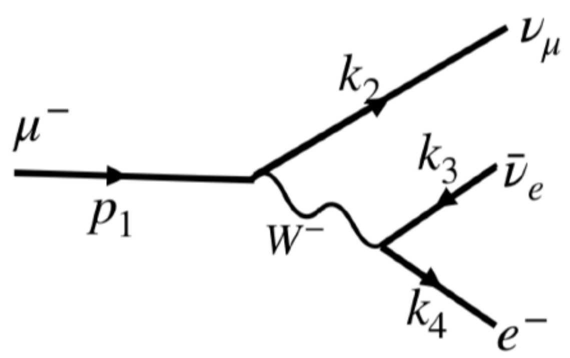
LEFT

Matching between SM and LEFT

Standard model provides a UV complete description on the weak interactions and neutrinos physics



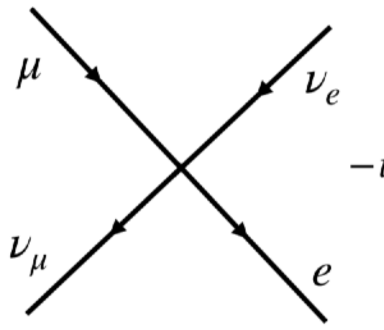
Diagrammatic approach



$$\mathcal{M} = \frac{g_L^2}{2}\bar{u}(k_2)\gamma_\rho P_L u(p_1)\frac{1}{q^2 - m_W^2}\bar{u}(k_4)\gamma_\rho P_L \mathrm{v}(k_3)$$

$$q^2 \lesssim m_\mu^2 \ll m_W^2$$

$$\frac{1}{p^2 - M_W^2} = -\frac{1}{M_W^2}\left(1 + \frac{p^2}{M_W^2} + \frac{p^4}{M_W^4} + \ldots\right)$$



$$-i\frac{g_L^2}{2m_W^2}[\gamma_\rho P_L][\gamma_\rho P_L]$$

$$\mathcal{M} = -\frac{g_L^2}{2m_W^2}\bar{u}(k_2)\gamma_\rho P_L u(p_1)\bar{u}(k_4)\gamma_\rho P_L \mathrm{v}(k_3) + \mathcal{O}(q^2/m_W^4)$$

Path Integral approach

$$\mathcal{L}_{\mathrm{UV}} \supset -W_\rho^+(\Box - m_W^2)W_\rho^- + \frac{g_L}{\sqrt{2}}\left[\bar{\nu}_e\gamma_\rho e_L + \bar{\nu}_\mu\gamma_\rho\mu_L\right]W_\rho^+ + \mathrm{h.c.}$$

$$-(\Box - m_W^2)W_\rho^- + \frac{g_L}{\sqrt{2}}\left[\bar{\nu}_e\gamma_\rho e_L + \bar{\nu}_\mu\gamma_\rho\mu_L\right] = 0$$

$$W_\rho^- = \frac{g_L}{\sqrt{2}}(\Box - m_W^2)^{-1}\left[\bar{\nu}_e\gamma_\rho e_L + \bar{\nu}_\mu\gamma_\rho\mu_L\right]$$

(Non-local) Effective Lagrangian:

$$\mathcal{L}_{\mathrm{eff}} = \frac{g_L^2}{2}\left[\bar{e}_L\gamma_\rho\nu_e + \bar{\mu}_L\gamma_\rho\nu_\mu\right](\Box - m_W^2)^{-1}\left[\bar{\nu}_e\gamma_\rho e_L + \bar{\nu}_\mu\gamma_\rho\mu_L\right]$$

$$\frac{1}{\Box - m_W^2} = -\frac{1}{m_W^2} - \frac{\Box}{m_W^4} - \frac{\Box^2}{m_W^6} - \ldots$$

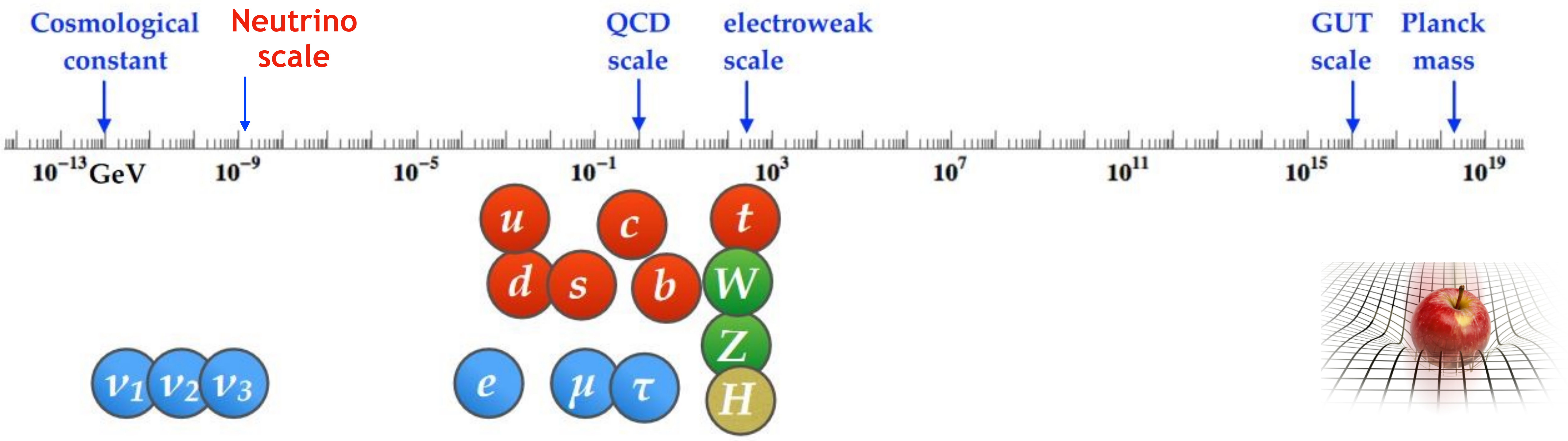
Leading (local) Effective Lagrangian:

$$\mathcal{L}_{\mathrm{eff}} = -\frac{g_L^2}{2m_W^2}\left[\bar{e}_L\gamma_\rho\nu_e + \bar{\mu}_L\gamma_\rho\nu_\mu\right]\left[\bar{\nu}_e\gamma_\rho e_L + \bar{\nu}_\mu\gamma_\rho\mu_L\right] + \mathcal{O}\left(\frac{1}{m_W^4}\right)$$

$$-\frac{g_L^2}{2m_W^4}\left[\bar{e}_L\gamma_\rho\nu_e + \bar{\mu}_L\gamma_\rho\nu_\mu\right]\Box\left[\bar{\nu}_e\gamma_\rho e_L + \bar{\nu}_\mu\gamma_\rho\mu_L\right] + \ldots$$

Beyond the standard model

The existence of neutrino masses is the first evidence of new physics beyond standard model (BSM)

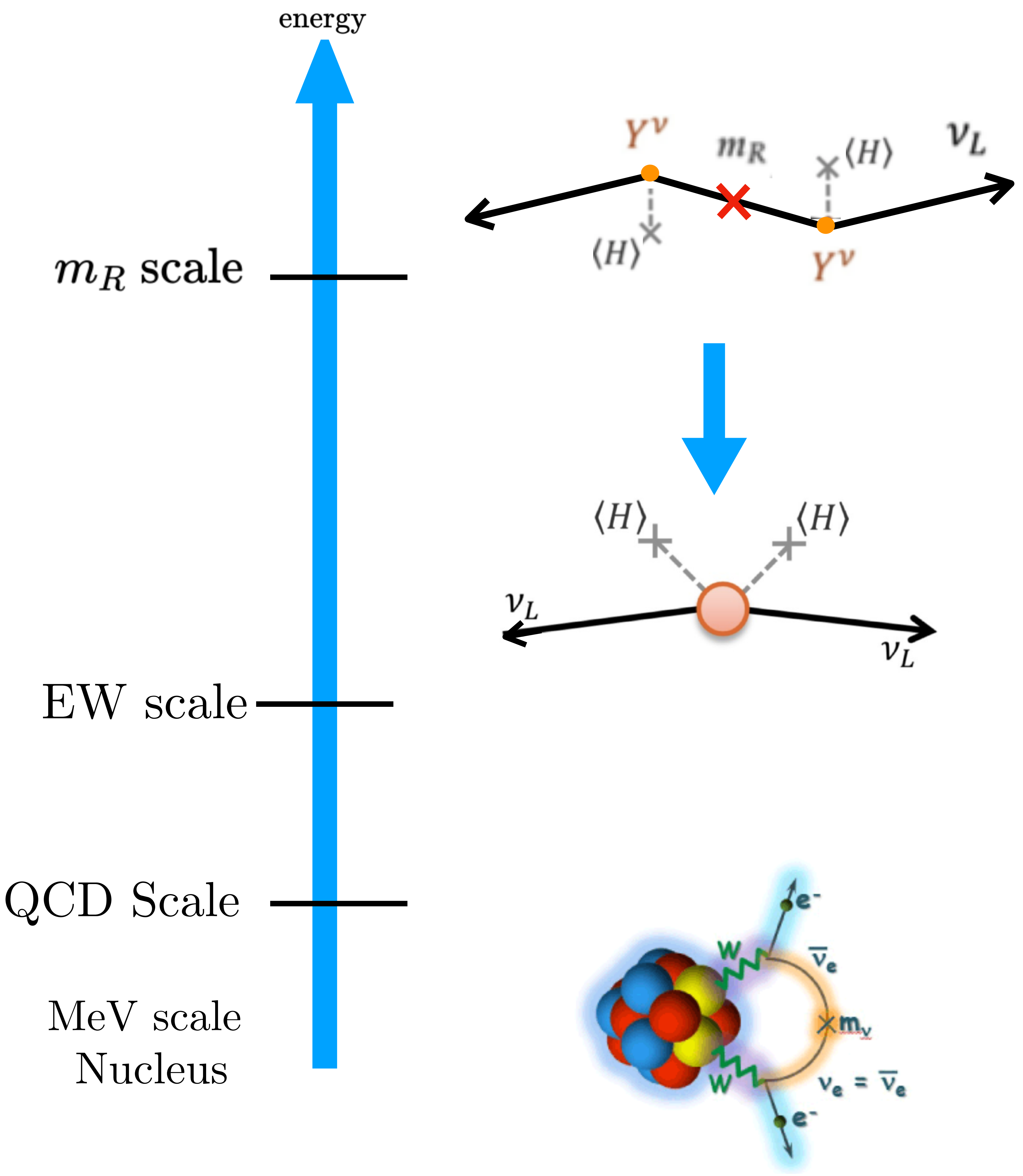


Why neutrino masses so tiny? **Why Higgs mass so light?**

[**see** L-T Wang's talk for other BSM motivations]

Majorana neutrino

The simplest way to give neutrino masses is introducing right-handed neutrinos, Majorana masses allowed



[Weinberg, 1979]

Weinberg operator

$$\mathcal{L}_{\text{Weinberg}} = \frac{c_{ij}}{\Lambda}(L_i H)(L_j H) + \text{h.c.}$$

$$\rightarrow c_{ij}\frac{v^2}{\Lambda}\nu_i\nu_j + \text{h.c.}$$

[**see also** S. Zhou's talk]

… the effective field theory point view had predicted the neutrino masses

[Weinberg, 2021]

Effective field theory

Standard model is viewed as the leading renormalizable terms of a more general effective field theory

$$\mathscr{L}_{\text{EFT}} = \mathscr{L}_{\mathscr{D}\leq 4} + \frac{\mathscr{L}_5}{\Lambda} + \frac{\mathscr{L}_6}{\Lambda^2} + \frac{\mathscr{L}_7}{\Lambda^3} + \frac{\mathscr{L}_8}{\Lambda^4} + \frac{\mathscr{L}_9}{\Lambda^5} + \dots$$

[Weinberg, 1980]

Standard Model | Weinberg Operator | Warsaw Basis

$$\frac{c_{ij}}{\Lambda}(L_i H)(L_j H) + \text{h.c.}$$

Scale separation: series expansion can be performed and truncated

Crucial difference between model and EFT

Decoupling theorem: EFT does not depend on details of UV scale

Provide modern understanding of renormalization

DOF
SM fields

Symmetry
SM gauge and Lorentz symmetry

Power Counting
Canonical dimension

Standard Model Effective Field Theory (SMEFT) provides systematic parameterization of all possible Lorentz-inv. new physics

Jiang-Hao Yu (ITP-CAS)

Standard model effective field theory

Extending the LEFT to SMEFT to describe new physics effects in weak interactions and neutrino physics

NP model

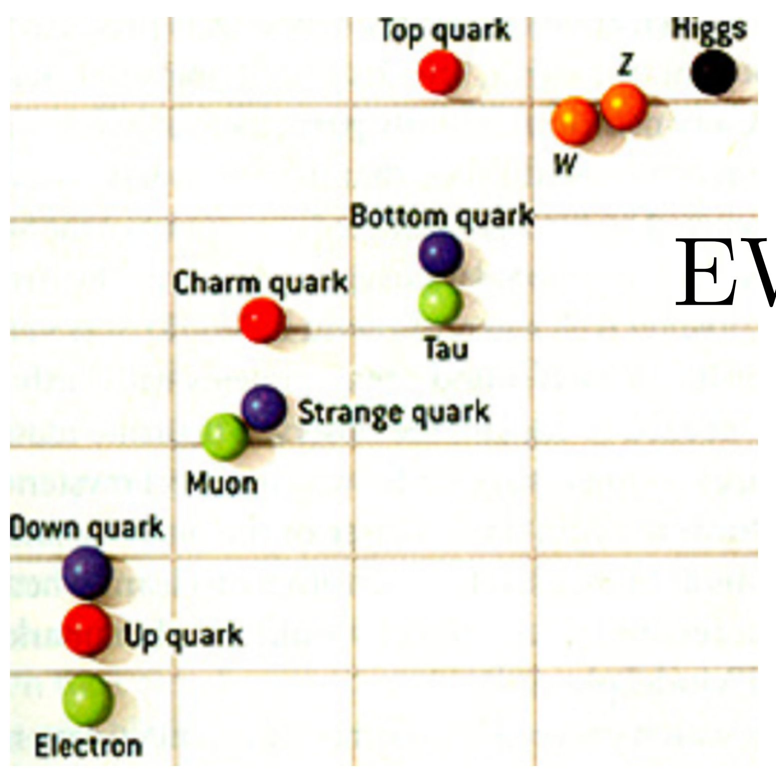
New physics scenarios, SUSY, …

NP Scale

SMEFT

Standard model effective field theory

$$\mathscr{L}_{\rm EFT} = \mathscr{L}_{\mathscr{D}\leq 4} + \frac{\mathscr{L}_5}{\Lambda} + \frac{\mathscr{L}_6}{\Lambda^2} + \ldots$$



EW scale

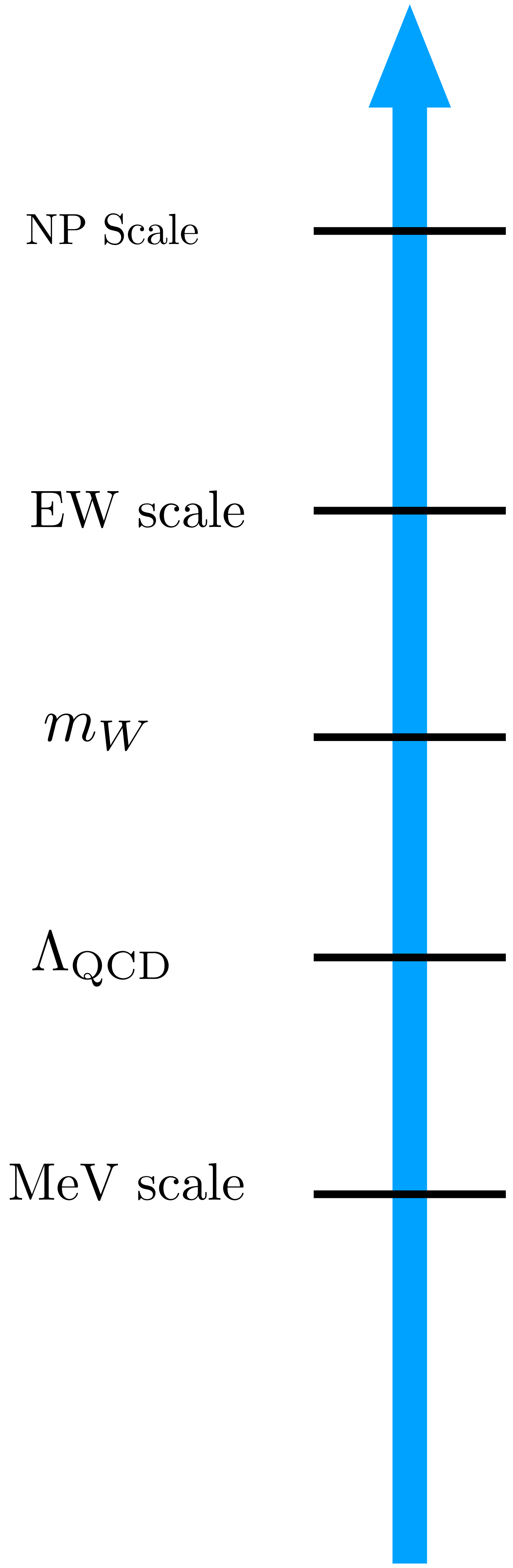
LEFT

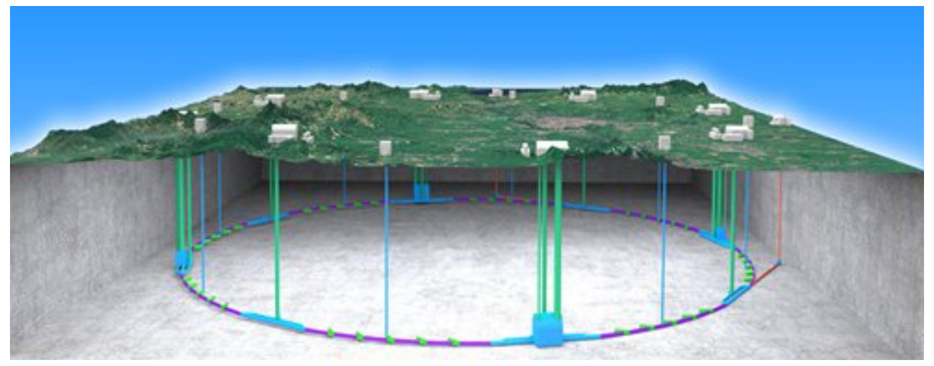
Low energy effective field theory

Matching and running among scales

Low energy probe of high energy physics

Weak interactions and neutrino processes usually involve in more scales than electroweak scale



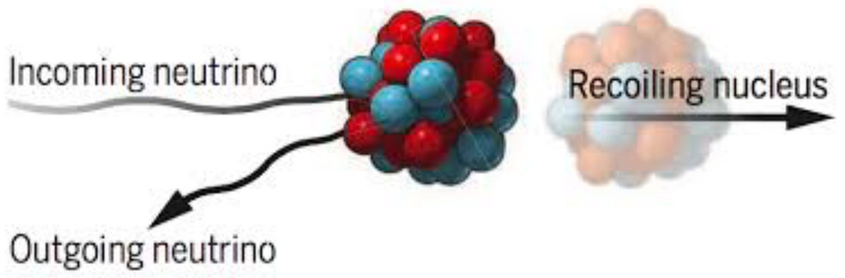


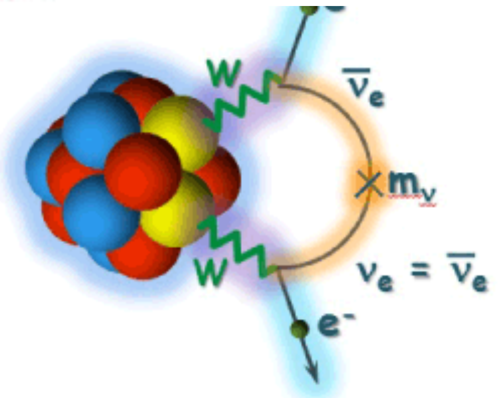
Energy frontier

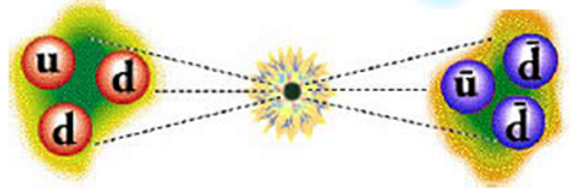
high energy, high cost!

Intensity frontier

high intensity, low cost!







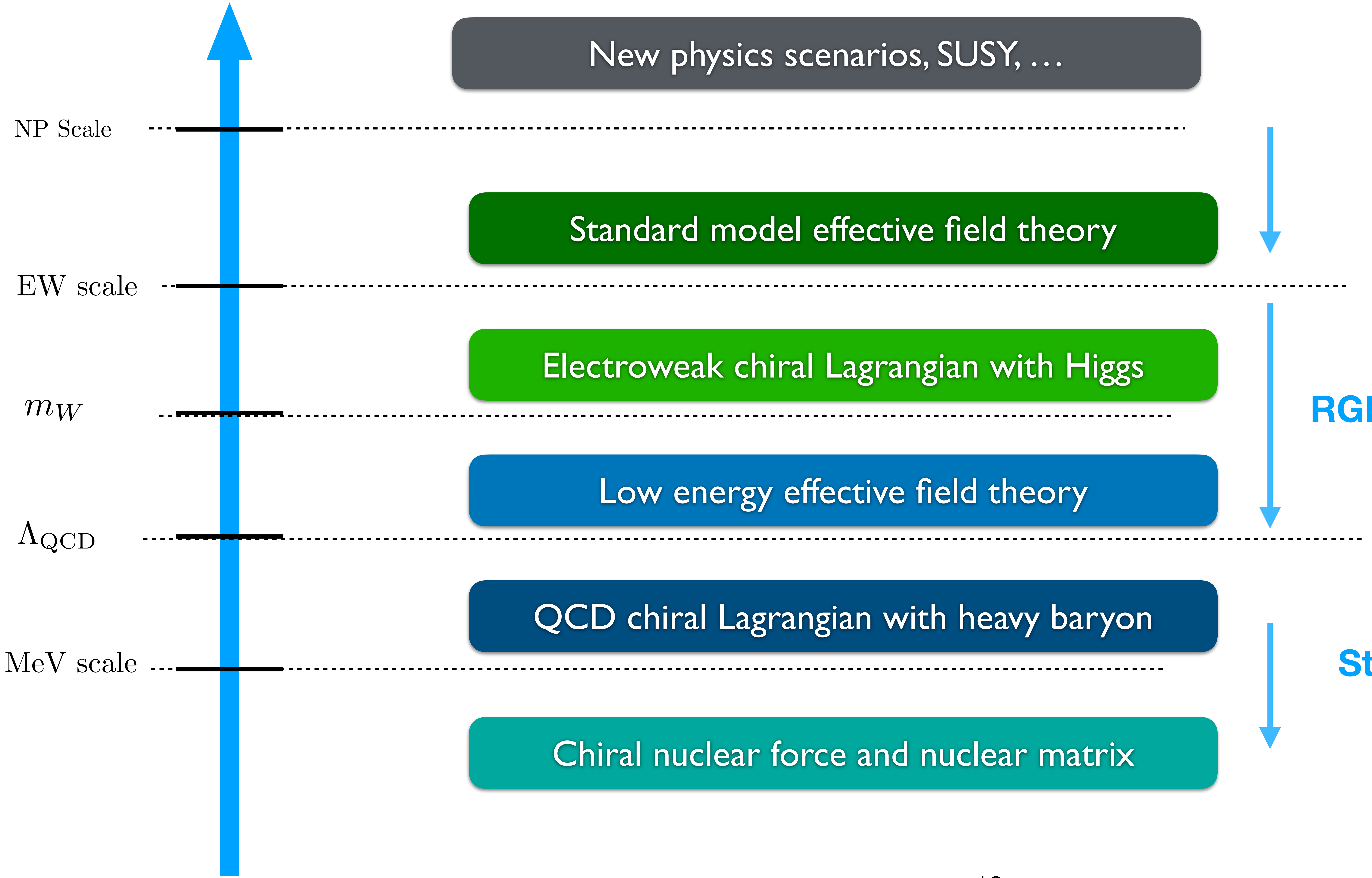
Flavor physics: pion, kaon, bottom, …

Rare process: cLFV, mu-e conversion, …

Neutrino physics: NSI, CEvNS, 0vbb, …

Tower of effective field theories

To avoid large log among scales, it is natural to consider matching and running procedures among EFTs



RGE running effects not so small

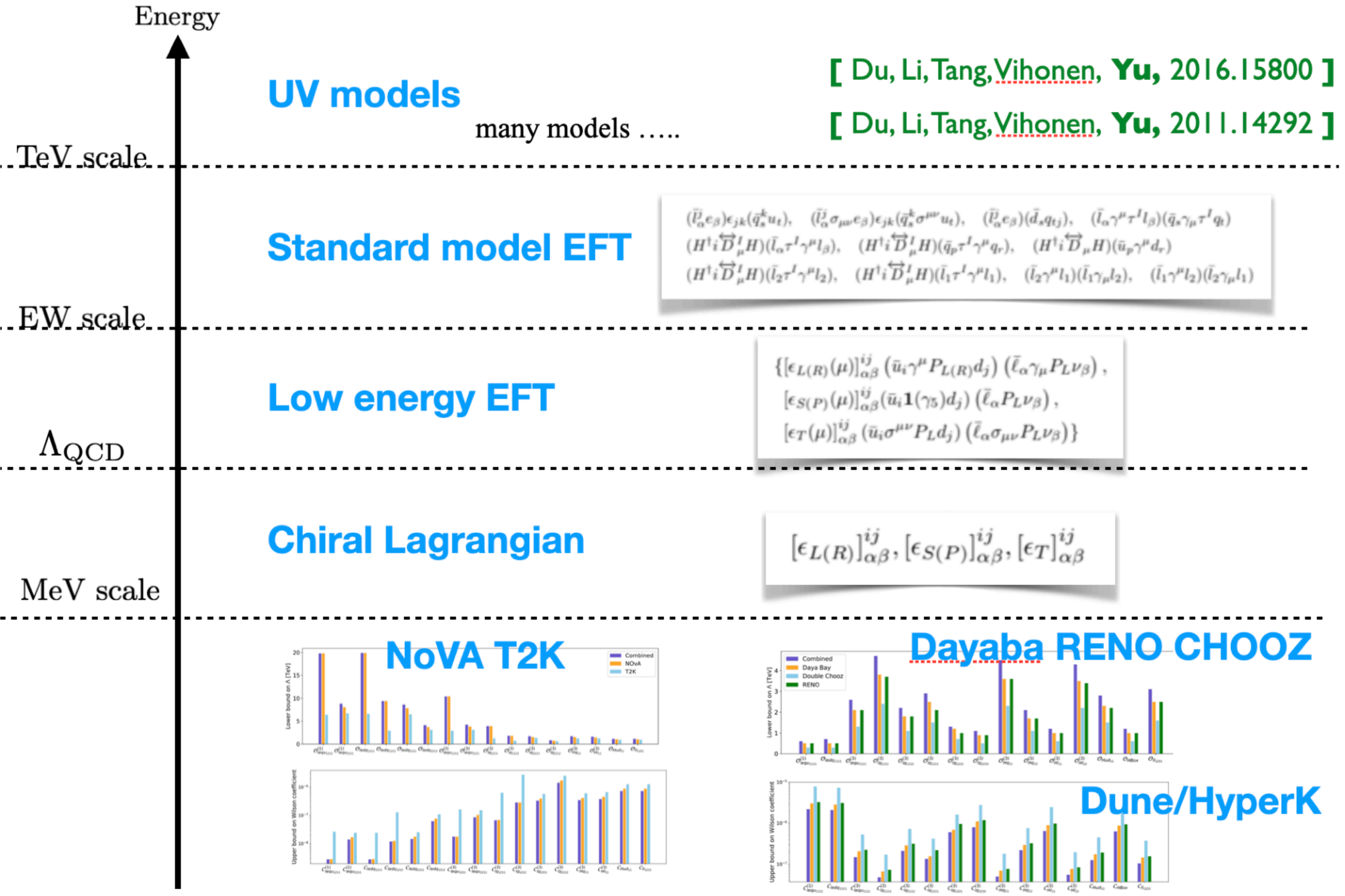
EFT framework!

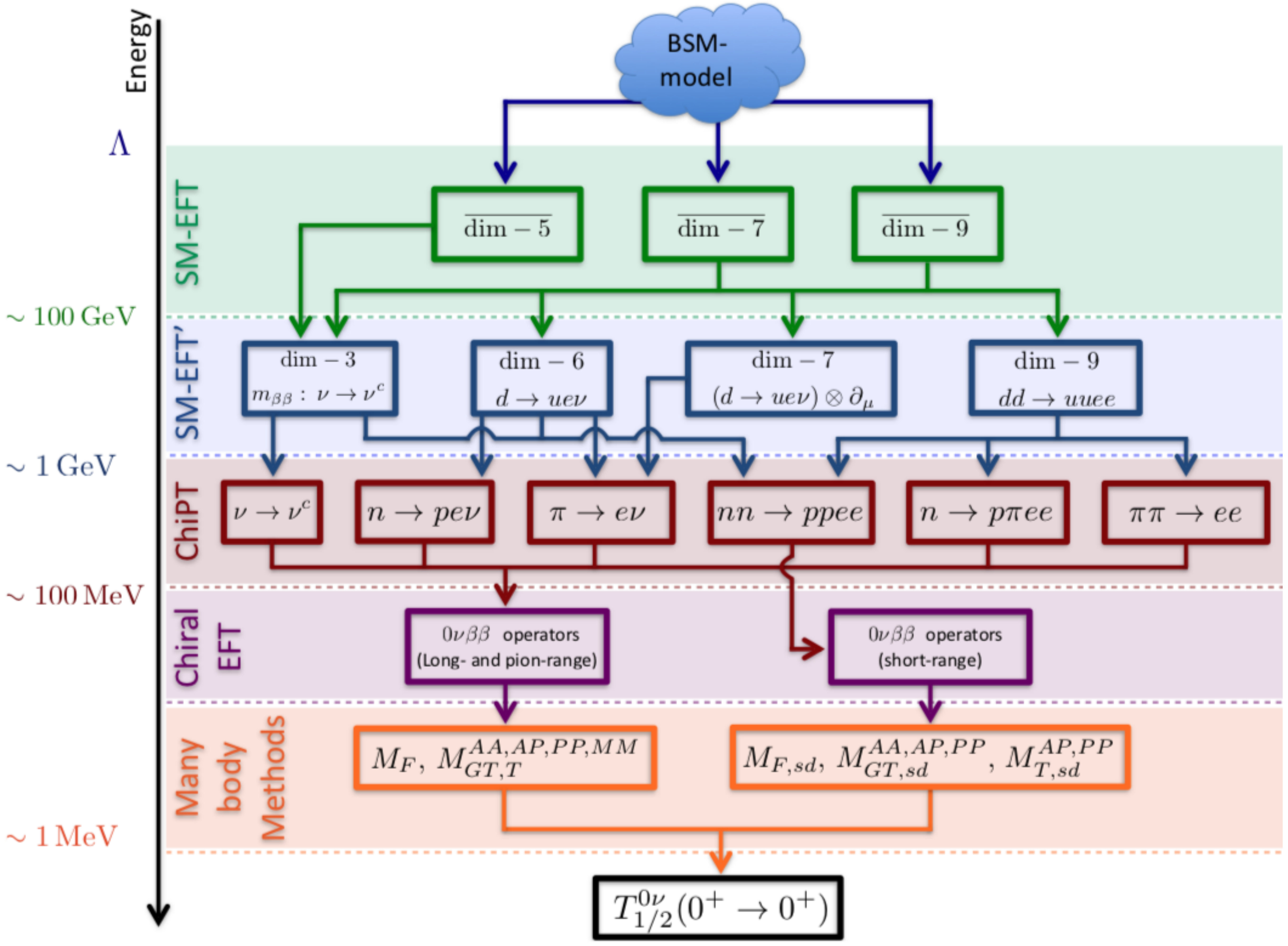
Strong coupling region

chiral expansion is perturbative!

Neutrinoless double beta decay (0vbb)

Examples of low energy probe of high energy physics: neutrino NSI, 0vbb, etc

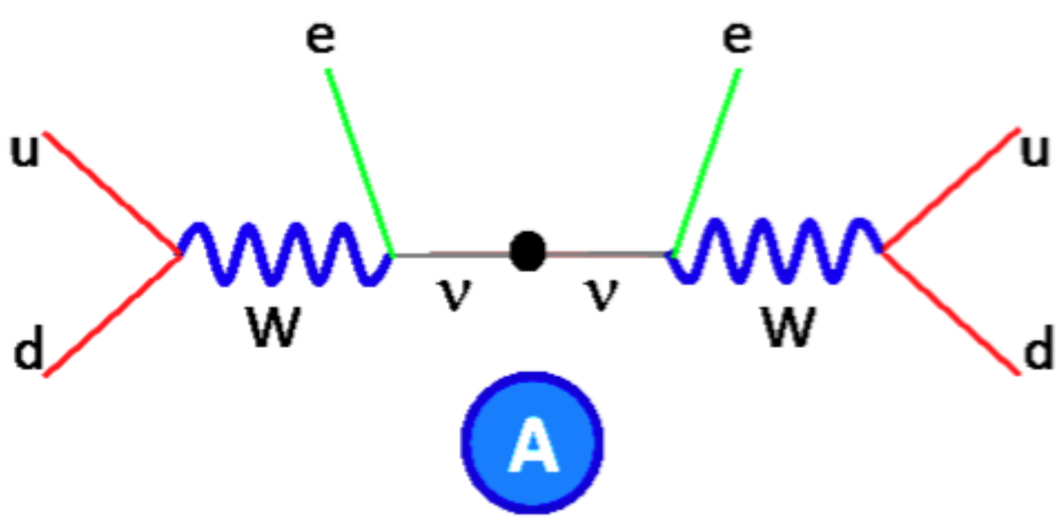




[Cirigliano, Dekens, de Vries, Graesser, Mereghetti, 2017]

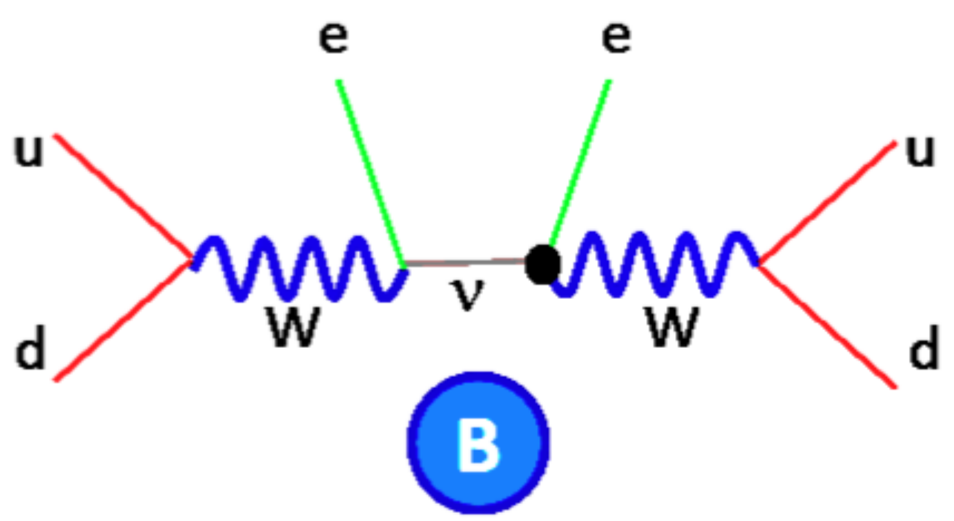
Operators for 0vbb

The higher dim operators may play the leading roles on describing 0vbb process



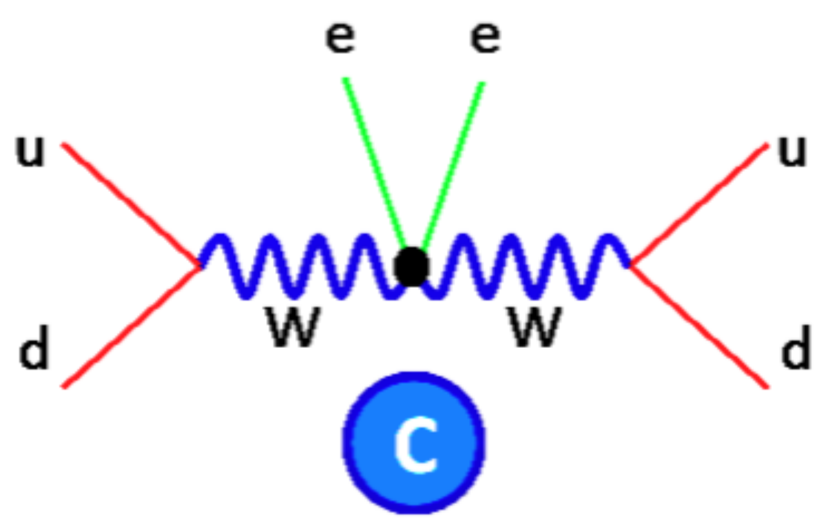
Dim-5, 7

$\mathcal{O}_5$	$\epsilon^{ik}\epsilon^{jl}(\ell_i^T C\ell_j)H_k H_l$
$\mathcal{O}_{LH}$	$\epsilon^{ik}\epsilon^{jl}(\ell_i^T C\ell_j)H_k H_l(H^\dagger H)$



Dim-7

$\mathcal{O}_{LeHD}$	$\epsilon^{ij}\epsilon^{kl}(\ell_i^T C\gamma^\mu e)H_j H_k(iD_\mu H_l)$
$\mathcal{O}_{LHW}$	$-\epsilon^{ik}(\epsilon\tau^I)^{jl}(\ell_i^T Ci\sigma^{\mu\nu}\ell_j)H_k H_l W^I_{\mu\nu}$

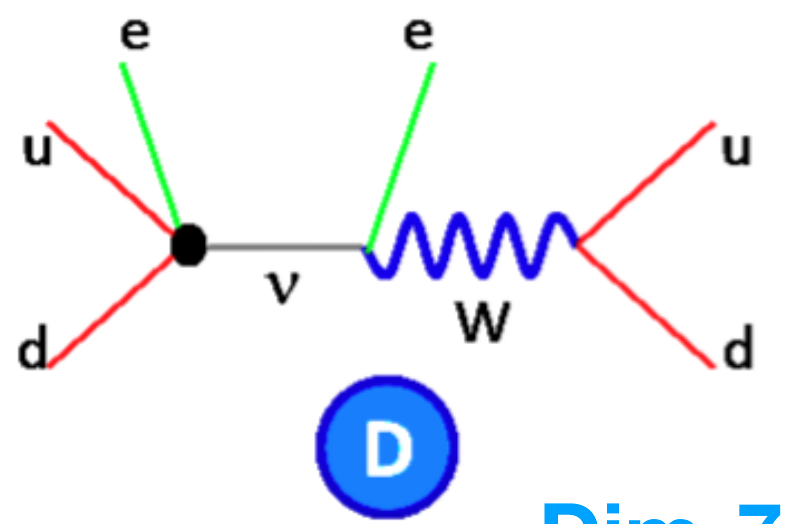


Dim-7, 9

$D^2\, H^{\dagger 2}\, L^{\dagger 2}$

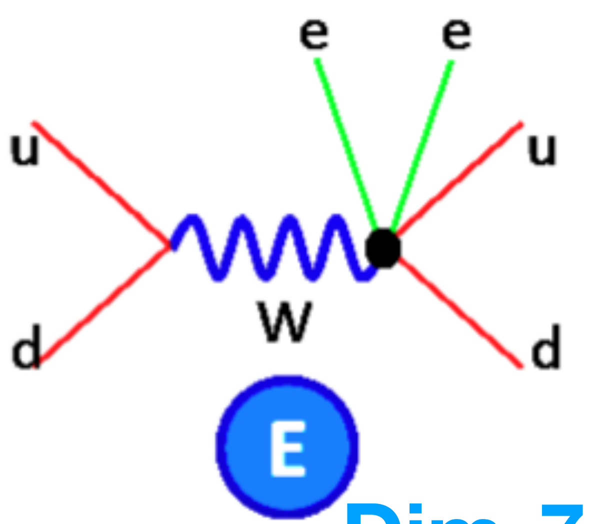
$\mathcal{O}_{LHD1}$	$\epsilon^{ij}\epsilon^{kl}(\ell_i^T CD_\mu\ell_j)(H_k D^\nu H_l)$

$D^2\, H^{\dagger 2}\, L^{\dagger 2}\, WL$



Dim-7

$\mathcal{O}_{dLQLH1}$	$\epsilon^{ij}\epsilon^{kl}(\overline{d}^a\ell_i)(q_{aj}^T C\ell_k)H_l$
$\mathcal{O}_{dLQLH2}$	$\epsilon^{ik}\epsilon^{jl}(\overline{d}^a\ell_i)(q_{aj}^T C\ell_k)H_l$
$\mathcal{O}_{dLueH}$	$\epsilon^{ij}(\overline{d}^a\ell_i)(u_a^T Ce)H_j$
$\mathcal{O}_{QuLLH}$	$\epsilon^{ij}(\overline{q}^{ak}u_a)(\ell_k^T C\ell_i)H_j$

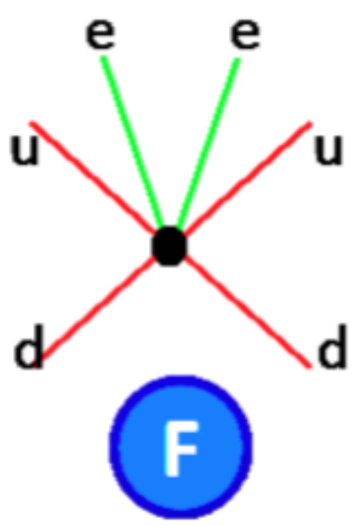


Dim-7, 9

$\mathcal{O}_{duLLD}$	$\epsilon^{ij}(\overline{d}^a\gamma^\mu u_a)(\ell_i^T CiD_\mu\ell_j)$

$D\, dc^\dagger\, L^{\dagger 2}\, uc$

$dc^\dagger\, ec\, H^\dagger\, L^\dagger\, uc\, WL$



Dim-9

$dc^2\, L^2\, Q^2$, $dc^2\, dc^\dagger\, L^2\, uc^\dagger$, $dc\, L^2\, uc\, uc^{\dagger 2}$, $dc^2\, ec^\dagger\, L\, Q\, uc^\dagger$,
$dc^{\dagger 2}\, ec^2\, uc^2$, $dc\, L^2\, Q\, Q^\dagger\, uc^\dagger$, $dc^\dagger\, ec\, L^\dagger\, Q\, uc^2$, $L^{\dagger 2}\, Q^2\, uc^2$

How to obtain complete and independent effective operators?

SMEFT and LEFT Operator Bases

[Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2201.04639]

[Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2012.09188]

[Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2007.07899]

[Hao-Lin Li, Jing Shu, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2005.00008]

[Hua-Yang Song, Hao Sun, **J.H.Yu**, 2306.05999]

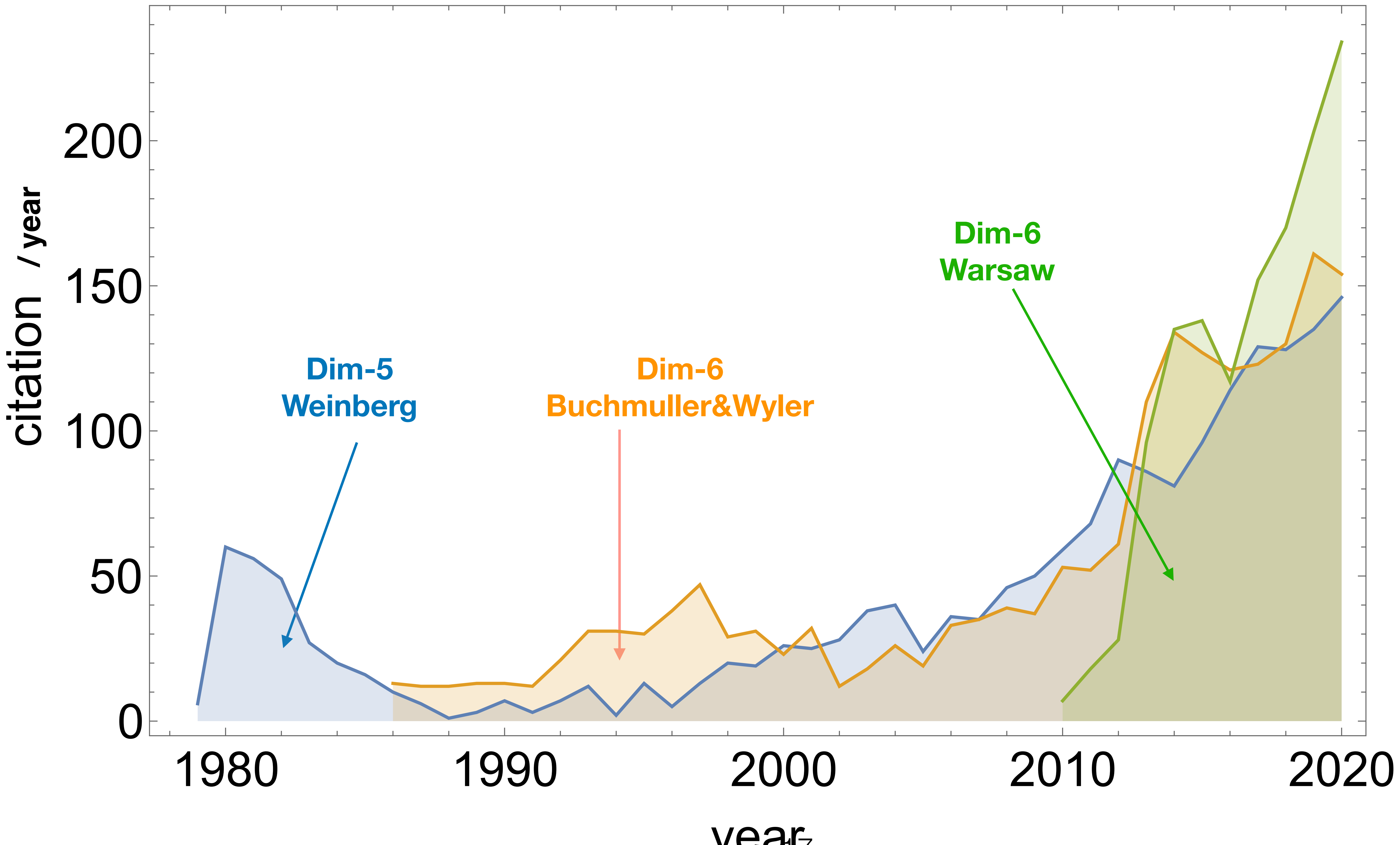
How many effective operators?

SMEFT operators parametrize new physics effects at low energy, electroweak precision, and Higgs boson physics, etc

$$\mathscr{L}_{\rm EFT} = \mathscr{L}_{\mathscr{D}\leq 4} + \frac{\mathscr{L}_5}{\Lambda} + \frac{\mathscr{L}_6}{\Lambda^2} + \frac{\mathscr{L}_7}{\Lambda^3} + \frac{\mathscr{L}_8}{\Lambda^4} + \frac{\mathscr{L}_9}{\Lambda^5} + \cdots$$

[Buchmuller and Wyler, 1986]

[Grzadkowski, Iskrzynski, Misiak, Rosiek, 2010]



SMEFT dim-6 operators

[Buchmuller and Wyler, 1986]

$O_\varphi = \frac{1}{3}(\varphi^\dagger\varphi)^3$, $O_{\partial\varphi} = \frac{1}{2}\partial_\mu(\varphi^\dagger\varphi)\,\partial^\mu(\varphi^\dagger\varphi)$.

$O_{e\varphi} = (\varphi^\dagger\varphi)(\bar{\ell}e\varphi)$, $O_{u\varphi} = (\varphi^\dagger\varphi)(\bar{q}u\tilde{\varphi})$, $O_{d\varphi} = (\varphi^\dagger\varphi)(\bar{q}d\varphi)$,

$O_G = f_{ABC}G_\mu^{A\nu}G_\nu^{B\lambda}G_\lambda^{C\mu}$, $O_{\tilde{G}} = f_{ABC}\tilde{G}_\mu^{A\nu}G_\nu^{B\lambda}G_\lambda^{C\mu}$, $O_W = \varepsilon_{IJK}W_\mu^{I\nu}W_\nu^{J\lambda}W_\lambda^{K\mu}$, $O_{\tilde{W}} = \varepsilon_{IJK}\tilde{W}_\mu^{I\nu}W_\nu^{J\lambda}W_\lambda^{K\mu}$.

$O_{\ell B} = i\bar{\ell}\gamma_\mu D_\nu \ell B^{\mu\nu}$, $O_{qB} = i\bar{q}\gamma_\mu D_\nu q B^{\mu\nu}$,

$O_{\varphi G} = \frac{1}{2}(\varphi^\dagger\varphi)G^A_{\mu\nu}G^{A\mu\nu}$, $O_{\varphi\tilde{G}} = (\varphi^\dagger\varphi)\tilde{G}^A_{\mu\nu}G^{A\mu\nu}$,

$O_{\varphi W} = \frac{1}{2}(\varphi^\dagger\varphi)W^I_{\mu\nu}W^{I\mu\nu}$, $O_{\varphi\tilde{W}} = (\varphi^\dagger\varphi)\tilde{W}^I_{\mu\nu}W^{I\mu\nu}$,

$O_{\varphi B} = \frac{1}{2}(\varphi^\dagger\varphi)B_{\mu\nu}B^{\mu\nu}$, $O_{\varphi\tilde{B}} = (\varphi^\dagger\varphi)\tilde{B}_{\mu\nu}B^{\mu\nu}$,

$O_{WB} = (\varphi^\dagger\tau^I\varphi)W^I_{\mu\nu}B^{\mu\nu}$, $O_{\tilde{W}B} = (\varphi^\dagger\tau^I\varphi)\tilde{W}^I_{\mu\nu}B^{\mu\nu}$,

$O_\varphi^{(1)} = (\varphi^\dagger\varphi)(D_\mu\varphi^\dagger D^\mu\varphi)$, $O_\varphi^{(3)} = (\varphi^\dagger D^\mu\varphi)(D_\mu\varphi^\dagger\varphi)$.

$O_{\ell W} = i\bar{\ell}\tau^I\gamma_\mu D_\nu \ell W^{I\mu\nu}$,
$O_{eB} = i\bar{e}\gamma_\mu D_\nu e B^{\mu\nu}$,
$O_{qG} = i\bar{q}\lambda^A\gamma_\mu D_\nu q G^{A\mu\nu}$,
$O_{qW} = i\bar{q}\tau^I\gamma_\mu D_\nu q W^{I\mu\nu}$,
$O_{uG} = i\bar{u}\lambda^A\gamma_\mu D_\nu u G^{A\mu\nu}$,
$O_{uB} = i\bar{u}\gamma_\mu D_\nu u B^{\mu\nu}$,
$O_{dG} = i\bar{d}\lambda^A\gamma_\mu D_\nu d G^{A\mu\nu}$,
$O_{dB} = i\bar{d}\gamma_\mu D_\nu d B^{\mu\nu}$.

$O_{\varphi\ell}^{(1)} = i(\varphi^\dagger D_\mu\varphi)(\bar{\ell}\gamma^\mu\ell)$,
$O_{\varphi\ell}^{(3)} = i(\varphi^\dagger D_\mu\tau^I\varphi)(\bar{\ell}\gamma^\mu\tau^I\ell)$,
$O_{\varphi e} = i(\varphi^\dagger D_\mu\varphi)(\bar{e}\gamma^\mu e)$,
$O_{\varphi q}^{(1)} = i(\varphi^\dagger D_\mu\varphi)(\bar{q}\gamma^\mu q)$,
$O_{\varphi q}^{(3)} = i(\varphi^\dagger D_\mu\tau^I\varphi)(\bar{q}\gamma^\mu\tau^I q)$,
$O_{\varphi_u} = i(\varphi^\dagger D_\mu\varphi)(\bar{u}\gamma^\mu u)$,
$O_{\varphi_d} = i(\varphi^\dagger D_\mu\varphi)(\bar{d}\gamma^\mu d)$,
$\varepsilon D_\mu\varphi)(\bar{u}\gamma^\mu d)$.

$O_{D_e} = (\bar{\ell}D_\mu e)D^\mu\varphi$, $O_{\bar{D}_e} = (D_\mu\bar{\ell}e)D^\mu\varphi$,
$O_{Du} = (\bar{q}D_\mu u)D^\mu\tilde{\varphi}$, $O_{\bar{D}u} = (D_\mu\bar{q}u)D^\mu\tilde{\varphi}$,
$O_{Dd} = (\bar{q}D_\mu d)D^\mu\varphi$, $O_{\bar{D}d} = (D_\mu\bar{q}d)D^\mu\varphi$,
$O_{eW} = (\bar{\ell}\sigma^{\mu\nu}\tau^I e)\varphi W^I_{\mu\nu}$, $O_{eB} = (\bar{\ell}\sigma^{\mu\nu}e)\varphi B_{\mu\nu}$,
$O_{uG} = (\bar{q}\sigma^{\mu\nu}\lambda^A u)\tilde{\varphi}G^A_{\mu\nu}$,
$O_{uW} = (\bar{q}\sigma^{\mu\nu}\tau^I u)\tilde{\varphi}W^I_{\mu\nu}$, $O_{uB} = (\bar{q}\sigma^{\mu\nu}u)\tilde{\varphi}B_{\mu\nu}$,
$O_{dG} = (\bar{q}\sigma^{\mu\nu}\lambda^A d)\varphi G^A_{\mu\nu}$,
$O_{dW} = (\bar{q}\sigma^{\mu\nu}\tau^I d)\varphi W^I_{\mu\nu}$, $O_{dW} = (\bar{q}\sigma^{\mu\nu}d)\varphi B_{\mu\nu}$.

$O_{\ell\ell}^{(1)} = \frac{1}{2}(\bar{\ell}\gamma_\mu\ell)(\bar{\ell}\gamma^\mu\ell)$, $O_{\ell\ell}^{(3)} = \frac{1}{2}(\bar{\ell}\gamma_\mu\tau^I\ell)(\bar{\ell}\gamma^\mu\tau^I\ell)$,
$O_{qq}^{(1,1)} = \frac{1}{2}(\bar{q}\gamma_\mu q)(\bar{q}\gamma^\mu q)$, $O_{qq}^{(8,1)} = \frac{1}{2}(\bar{q}\gamma_\mu\lambda^A q)(\bar{q}\gamma^\mu\lambda^A q)$,
$O_{qq}^{(1,3)} = \frac{1}{2}(\bar{q}\gamma_\mu\tau^I q)(\bar{q}\gamma^\mu\tau^I q)$, $O_{qq}^{(8,3)} = \frac{1}{2}(\bar{q}\gamma_\mu\lambda^A\tau^I q)(\bar{q}\gamma^\mu\lambda^A\tau^I q)$,
$O_{\ell q}^{(1)} = (\bar{\ell}\gamma_\mu\ell)(\bar{q}\gamma^\mu q)$, $O_{\ell q}^{(3)} = (\bar{\ell}\gamma_\mu\tau^I\ell)(\bar{q}\gamma^\mu\tau^I q)$.

$O_{ee} = \frac{1}{2}(\bar{e}\gamma_\mu e)(\bar{e}\gamma^\mu e)$,
$O_{uu}^{(1)} = \frac{1}{2}(\bar{u}\gamma_\mu u)(\bar{u}\gamma^\mu u)$, $O_{uu}^{(8)} = \frac{1}{2}(\bar{u}\gamma_\mu\lambda^A u)(\bar{u}\gamma^\mu\lambda^A u)$,
$O_{dd}^{(1)} = \frac{1}{2}(\bar{d}\gamma_\mu d)(\bar{d}\gamma^\mu d)$, $O_{dd}^{(8)} = \frac{1}{2}(\bar{d}\gamma_\mu\lambda^A d)(\bar{d}\gamma^\mu\lambda^A d)$,
$O_{eu} = (\bar{e}\gamma_\mu e)(\bar{u}\gamma^\mu u)$,
$O_{ed} = (\bar{e}\gamma_\mu e)(\bar{d}\gamma^\mu d)$,
$O_{ud}^{(1)} = (\bar{u}\gamma_\mu u)(\bar{d}\gamma^\mu d)$, $O_{ud}^{(8)} = (\bar{u}\gamma_\mu\lambda^A u)(\bar{d}\gamma^\mu\lambda^A d)$.

$O_{\ell e} = (\bar{\ell}e)(\bar{e}\ell)$,
$O_{\ell u} = (\bar{\ell}u)(\bar{u}\ell)$,
$O_{\ell d} = (\bar{\ell}d)(\bar{d}\ell)$,
$O_{qe} = (\bar{q}e)(\bar{e}q)$,
$O_{qu}^{(1)} = (\bar{q}u)(\bar{u}q)$,
$O_{qd}^{(1)} = (\bar{q}d)(\bar{d}q)$,
$O_{qde} = (\bar{\ell}e)(\bar{d}q)$.

$O_{qq}^{(1)} = (\bar{q}u)(\bar{q}d)$,
$O_{qq}^{(8)} = (\bar{q}\lambda^A u)(\bar{q}\lambda^A d)$,
$O_{\ell q} = (\bar{\ell}e)(\bar{q}u)$.
$O_{qu}^{(8)} = (\bar{q}\lambda^A u)(\bar{u}\lambda^A q)$,
$O_{qd}^{(8)} = (\bar{q}\lambda^A d)(\bar{d}\lambda^A q)$,

Equation of motion (field redefinition)

$$(D^\mu D_\mu\varphi)^j = m^2\varphi^j - \lambda(\varphi^\dagger\varphi)\varphi^j - \bar{e}\,\Gamma_e^\dagger l^j + \varepsilon_{jk}\bar{q}^k\,\Gamma_u u - \bar{d}\,\Gamma_d^\dagger q^j$$

$$i\not{D}l = \Gamma_e e\varphi, \quad i\not{D}e = \Gamma_e^\dagger\varphi^\dagger l, \quad i\not{D}q = \Gamma_u u\tilde{\varphi} + \Gamma_d d\varphi, \quad i\not{D}u = \Gamma_u^\dagger\tilde{\varphi}^\dagger q,$$

$$(D^\rho W_{\rho\mu})^I = \frac{g}{2}\left(\varphi^\dagger i\overleftrightarrow{D}_\mu^I\varphi + \bar{l}\gamma_\mu\tau^I l + \bar{q}\gamma_\mu\tau^I q\right),$$

Covariant derivative commutator

$$[D_\rho, D_\alpha] \sim X_{\rho\alpha}$$

80 → 59

Bianchi identity $D_{[\rho}X_{\mu\nu]} = 0$

Integration by part (total derivatives)

$$(D^n\varphi)^\dagger(D^m\varphi) = -(D^{n+1}\varphi)^\dagger(D^{m-1}\varphi) + \partial\left[(D^n\varphi)^\dagger(D^{m-1}\varphi)\right]$$

Fierz identity

$$T^A_{\alpha\beta}T^A_{\kappa\lambda} = \frac{1}{2}\delta_{\alpha\lambda}\delta_{\kappa\beta} - \frac{1}{6}\delta_{\alpha\beta}\delta_{\kappa\lambda}$$

$$\tau^I_{jk}\tau^I_{mn} = 2\delta_{jn}\delta_{mk} - \delta_{jk}\delta_{mn}$$

80-1-16-5+1 = 59

[Grzadkowski, Iskrzynski, Misiak, Rosiek, 2010]

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar{l}_p e_r\varphi)$
$Q_{\tilde{G}}$	$f^{ABC}\tilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p u_r\tilde{\varphi})$
Q_W	$\varepsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$Q_{\varphi D}$	$(\varphi^\dagger D^\mu\varphi)^\star(\varphi^\dagger D_\mu\varphi)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p d_r\varphi)$
$Q_{\tilde{W}}$	$\varepsilon^{IJK}\tilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$				

$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G^A_{\mu\nu}G^{A\mu\nu}$	Q_{eW}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^I\varphi W^I_{\mu\nu}$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{l}_p\gamma^\mu l_r)$
$Q_{\varphi\tilde{G}}$	$\varphi^\dagger\varphi\,\tilde{G}^A_{\mu\nu}G^{A\mu\nu}$	Q_{eB}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\varphi)(\bar{l}_p\tau^I\gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu}W^{I\mu\nu}$	Q_{uG}	$(\bar{q}_p\sigma^{\mu\nu}T^A u_r)\tilde{\varphi}\,G^A_{\mu\nu}$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{e}_p\gamma^\mu e_r)$
$Q_{\varphi\tilde{W}}$	$\varphi^\dagger\varphi\,\tilde{W}^I_{\mu\nu}W^{I\mu\nu}$	Q_{uW}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\tau^I\tilde{\varphi}\,W^I_{\mu\nu}$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{q}_p\gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu}B^{\mu\nu}$	Q_{uB}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\tilde{\varphi}\,B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\varphi)(\bar{q}_p\tau^I\gamma^\mu q_r)$
$Q_{\varphi\tilde{B}}$	$\varphi^\dagger\varphi\,\tilde{B}_{\mu\nu}B^{\mu\nu}$	Q_{dG}	$(\bar{q}_p\sigma^{\mu\nu}T^A d_r)\varphi\,G^A_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{u}_p\gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu}B^{\mu\nu}$	Q_{dW}	$(\bar{q}_p\sigma^{\mu\nu}d_r)\tau^I\varphi\,W^I_{\mu\nu}$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{d}_p\gamma^\mu d_r)$
$Q_{\varphi\tilde{W}B}$	$\varphi^\dagger\tau^I\varphi\,\tilde{W}^I_{\mu\nu}B^{\mu\nu}$	Q_{dB}	$(\bar{q}_p\sigma^{\mu\nu}d_r)\varphi\,B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\tilde{\varphi}^\dagger D_\mu\varphi)(\bar{u}_p\gamma^\mu d_r)$

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j\sigma_{\mu\nu}e_r)\varepsilon_{jk}(\bar{q}_s^k\sigma^{\mu\nu}u_t)$		

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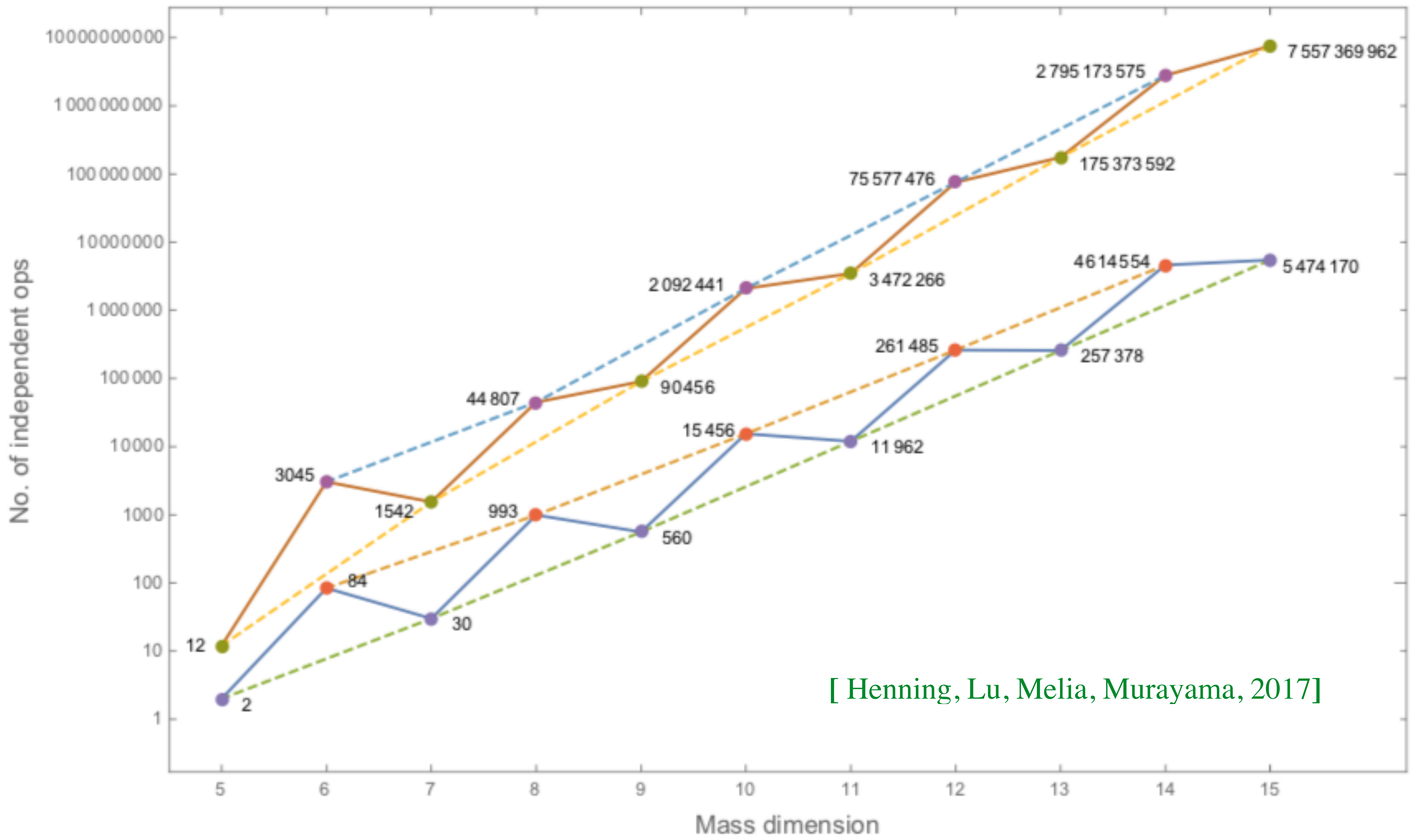
Moore's law on EFT operators

Number of EFT operators grows very fast for higher dim



Derivatives

$BWHH^\dagger D^2$

2

$$(D^2H^\dagger)HB_{L\mu\nu}W_L^{\mu\nu},\ (D^\mu D_\nu H^\dagger)HB_{L\mu\rho}W_L^{\nu\rho},\ (D_\nu D^\mu H^\dagger)HB_{L\mu\rho}W_L^{\nu\rho},\ (D_\mu H^\dagger)(D^\mu H)B_{L\nu\rho}W_L^{\nu\rho},$$
$$(D_\mu H^\dagger)(D^\nu H)B_{L\nu\rho}W_L^{\mu\rho},\ (D^\nu H^\dagger)(D_\mu H)B_{L\nu\rho}W_L^{\mu\rho},\ (D_\mu H^\dagger)H(D^\mu B_{L\nu\rho})W_L^{\nu\rho},\ (D_\mu H^\dagger)H(D^\nu B_{L\nu\rho})W_L^{\mu\rho},$$
$$(D^\nu H^\dagger)H(D_\mu B_{L\nu\rho})W_L^{\mu\rho},\ (D_\mu H^\dagger)HB_{L\nu\rho}(D^\mu W_L^{\nu\rho}),\ (D_\mu H^\dagger)HB_{L\nu\rho}(D^\nu W_L^{\mu\rho}),\ (D^\nu H^\dagger)HB_{L\nu\rho}(D_\mu W_L^{\mu\rho}),$$
$$H^\dagger(D^2H)B_{L\mu\nu}W_L^{\mu\nu},\ H^\dagger(D^\mu D_\nu H)B_{L\mu\rho}W_L^{\nu\rho},\ H^\dagger(D_\nu D^\mu H)B_{L\mu\rho}W_L^{\nu\rho},\ H^\dagger(D^\mu H)(D_\mu B_{L\nu\rho})W_L^{\nu\rho},$$
$$H^\dagger(D^\nu H)(D_\mu B_{L\nu\rho})W_L^{\mu\rho},\ H^\dagger(D_\mu H)(D^\nu B_{L\nu\rho})W_L^{\mu\rho},\ H^\dagger(D^\mu H)B_{L\nu\rho}(D_\mu W_L^{\nu\rho}),\ H^\dagger(D^\nu H)B_{L\nu\rho}(D_\mu W_L^{\mu\rho}),$$
$$H^\dagger(D_\mu H)B_{L\nu\rho}(D^\nu W_L^{\mu\rho}),\ H^\dagger H(D^2B_{L\mu\nu})W_L^{\mu\nu},\ H^\dagger H(D^\mu D_\nu B_{L\mu\rho})W_L^{\nu\rho},\ H^\dagger H(D_\nu D^\mu B_{L\mu\rho})W_L^{\nu\rho},$$
$$H^\dagger H(D^\mu B_{L\nu\rho})(D_\mu W_L^{\nu\rho}),\ H^\dagger H(D^\nu B_{L\nu\rho})(D_\mu W_L^{\mu\rho}),\ H^\dagger H(D_\mu B_{L\nu\rho})(D^\nu W_L^{\mu\rho}),\ H^\dagger HB_{L\mu\nu}(D^2W_L^{\mu\nu}),$$
$$H^\dagger HB_{L\mu\rho}(D^\mu D_\nu W_L^{\nu\rho}),\ H^\dagger HB_{L\mu\rho}(D_\nu D^\mu W_L^{\nu\rho}). \qquad (14)$$

30

Which 2 should be picked up?

Repeated fields

$QQQL$

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$$Q^{qqq\ell}_{prst} = C^{prst} \begin{array}{l} \epsilon^{abc}\epsilon^{ik}\epsilon^{jl}(L_{pi}Q_{raj})(Q_{sbk}Q_{tcl}) \\ \epsilon^{abc}\epsilon^{ik}\epsilon^{jl}(L_{pi}Q_{sbk})(Q_{raj}Q_{tcl}) \\ \epsilon^{abc}\epsilon^{ij}\epsilon^{kl}(L_{pi}Q_{sbk})(Q_{raj}Q_{tcl}) \\ \epsilon^{abc}\epsilon^{ij}\epsilon^{kl}(L_{pi}Q_{raj})(Q_{sbk}Q_{tcl}) \end{array} \qquad p,r,s,t=1,2,3$$

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What flavor relations should be imposed?

Symmetry of EFT operators

Operator has more symmetries than what we expected

Field as irreducible rep of Lorentz group

Field transforming under Little group of Poincare

SO(3,1)	SL(2,C) $SU(2)_l \times SU(2)_r$	Spinor-helicity
ϕ	$\phi \in (0,0)$	
ψ	$\psi_\alpha \in (1/2,0)$ $\quad \psi^\dagger_{\dot\alpha} \in (0,1/2)$,	λ_α
$F_{\mu\nu}$	$F_{\mathrm{L}\alpha\beta} = \frac{i}{2} F_{\mu\nu}\sigma^{\mu\nu}_{\alpha\beta} \in (1,0)$ $\quad F_{\mathrm{R}\dot\alpha\dot\beta} = -\frac{i}{2} F_{\mu\nu}\bar\sigma^{\mu\nu}_{\dot\alpha\dot\beta} \in (0,1)$.	$\lambda_\alpha\lambda_\beta$
$R_{\mu\nu\rho\sigma}$	$C_{\alpha\beta\gamma\delta} = C_{\mu\nu\rho\sigma}\sigma^{\mu\nu}_{\alpha\beta}\sigma^{\rho\sigma}_{\gamma\delta} \in (2,0)$	$\lambda_\alpha\lambda_\beta\lambda_\gamma\lambda_\delta$
D_μ	$D_{\alpha\dot\alpha} = D_\mu\sigma^\mu_{\alpha\dot\alpha} \in (1/2,1/2)$.	$\lambda_\alpha\tilde\lambda_{\dot\alpha}$

Building blocks in spinor-helicity form

$$D^{r_i}\phi_i \Leftrightarrow \lambda_i^{r_i}\tilde\lambda^{i,r_i},$$

$$D^{r_i-1/2}\psi_i^{(\dagger)} \Leftrightarrow \lambda_i^{r_i\pm 1/2}\tilde\lambda^{i,r_i\mp 1/2},$$

$$D^{r_i-1}F_{\mathrm{L/R}\,i} \Leftrightarrow \lambda_i^{r_i\pm 1}\tilde\lambda^{i,r_i\mp 1},$$

$$D^{r_i-2}C_{\mathrm{L/R}\,i} \Leftrightarrow \lambda_i^{r_i\pm 2}\tilde\lambda^{i,r_i\mp 2},$$

On-shell operator

$$\mathcal{O}_N^{(d)} = (\epsilon^{\alpha_i\alpha_j})^{\otimes n}(\tilde\epsilon_{\dot\alpha_i\dot\alpha_j})^{\otimes\tilde n}\prod_{i=1}^{N}(D^{r_i-|h_i|}\Psi_{i,a_i})^{\dot\alpha_i^{r_i+h_i}}_{\alpha_i^{r_i-h_i}}$$

$$F_{\mathrm{L}1}{}^{\alpha\beta}F_{\mathrm{L}2\alpha\beta}(D\phi_3)^\gamma{}_{\dot\alpha}(D\phi_4)_\gamma{}^{\dot\alpha}$$

$\mathcal{M} \to e^{ih_i\varphi}\mathcal{M}$

$\lambda_i \to e^{-i\varphi/2}\lambda_i, \quad \tilde\lambda^i \to e^{i\varphi/2}\tilde\lambda^i.$

$$(\epsilon^{\alpha_i\alpha_j})^{\otimes n}(\tilde\epsilon_{\dot\alpha_i\dot\alpha_j})^{\otimes\tilde n}\prod_{i=1}^{N}\lambda_i^{r_i-h_i}\tilde\lambda^{i,r_i+h_i}$$

$\langle ij\rangle = \epsilon^{\beta\alpha}\lambda_{i\alpha}\lambda_{j\beta}$

$[ij] = \tilde\lambda_{i\dot\alpha}\tilde\epsilon^{\dot\alpha\dot\beta}\tilde\lambda_{j\dot\beta}$

On-shell Brackets

$\langle 12\rangle^2\langle 34\rangle[34]$

EOM and CDC

$D^2\phi_i \Leftrightarrow p_i^2 = 0$

$[D_\mu, D_\nu] \Leftrightarrow [p_\mu, p_\nu] = 0$

Operator as spinor Young tensor

[Li, Ren, Xiao, **Yu**, Zheng, 2201.04639]
[Li, Ren, Xiao, **Yu**, Zheng, 2012.09188]
[Li, Ren, Xiao, **Yu**, Zheng, 2007.07899]
[Li, Ren, Shu, Xiao, **Yu**, Zheng, 2005.00008]

Operator

$$W_{\mu\lambda}\left(e_{cp}\sigma^{\nu\lambda}D^{\mu}L_r\right)D_\nu H^\dagger$$

↓

Spinor Tensor → **Symmetrize indices**

$$\mathcal{O}_N^{(d)} = (\epsilon^{\alpha_i\alpha_j})^{\otimes n}(\tilde{\epsilon}_{\dot{\alpha}_i\dot{\alpha}_j})^{\otimes\tilde{n}}\prod_{i=1}^{N}(D^{r_i-|h_i|}\Psi_{i,a_i})^{\dot{\alpha}_i^{r_i+h_i}}_{\alpha_i^{r_i-h_i}}$$

$$D_{[\alpha\dot{\alpha}}D_{\beta]\dot{\beta}} = D_\mu D_\nu \sigma^\mu_{[\alpha\dot{\alpha}}\sigma^\nu_{\beta]\dot{\beta}} = -D^2\epsilon_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}} + \frac{i}{2}[D_\mu, D_\nu]\epsilon_{\alpha\beta}(\bar{\sigma}^{\mu\nu})_{\dot{\alpha}\dot{\beta}},$$

$$D_{[\alpha\dot{\alpha}}\psi_{\beta]} = D_\mu\sigma^\mu_{[\alpha\dot{\alpha}}\psi_{\beta]} = -\epsilon_{\alpha\beta}(D_\mu\sigma^\mu\psi)_{\dot{\alpha}},$$

$$D_{[\alpha\dot{\alpha}}F_{L\beta]\gamma} = \frac{i}{2}D_\mu F_{L\nu\rho}\sigma^\mu_{[\alpha\dot{\alpha}}\sigma^{\nu\rho}_{\beta]\gamma} = iD^\mu F_{L\mu\nu}\epsilon_{\alpha\beta}\sigma^\nu_{\gamma\dot{\alpha}},$$

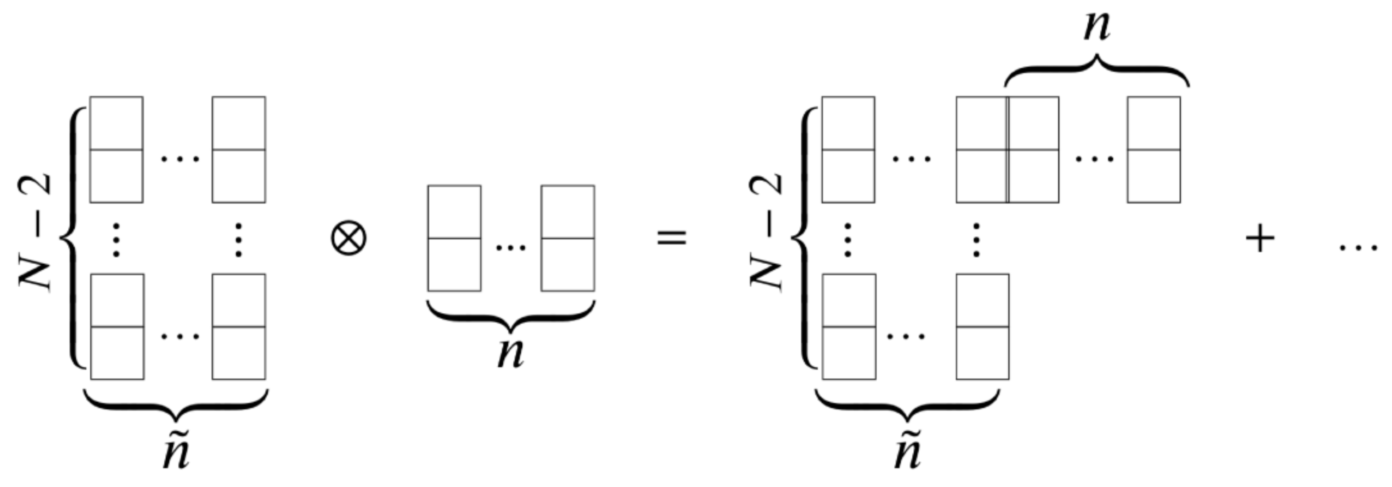
$$(D\psi)_{\alpha\beta\dot{\alpha}} = -\frac{1}{2}\epsilon_{\alpha\beta}(\not{D}\psi)_{\dot{\alpha}} + \frac{1}{2}(D\psi)_{(\alpha\beta)\dot{\alpha}}$$

↓

SL(2,C) x SU(N)

$$\lambda_i \to \sum_j U_i^j\lambda_j, \quad \tilde{\lambda}^i \to \sum_k U^{\dagger i}{}_k\tilde{\lambda}^k,$$

$$(\epsilon^{\alpha_i\alpha_j})^{\otimes n}(\tilde{\epsilon}_{\dot{\alpha}_i\dot{\alpha}_j})^{\otimes\tilde{n}}\prod_{i=1}^{N}\lambda_i^{r_i-h_i}\tilde{\lambda}^{i,r_i+h_i}$$



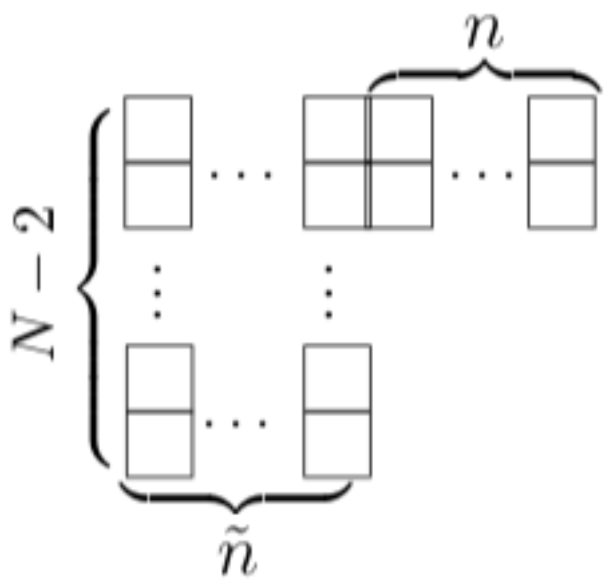
↓

Momentum conservation

$$\delta^{(4)}\left(\sum_{i=1}^{N}\lambda_i\tilde{\lambda}_i\right)$$

$$\sum_i \lambda_i\tilde{\lambda}^i \to \sum_i\sum_j\sum_k U_i^j U^{\dagger i}{}_k\lambda_j\tilde{\lambda}^k = \sum_j \lambda_j\tilde{\lambda}^j$$

SSYT



$$\begin{array}{|c|}\hline i \\ \hline j \\ \hline\end{array} \Leftrightarrow \langle ij\rangle$$

→ **On-shell Amplitude**

$$\{\overbrace{1,\ldots,1}^{\#1},\overbrace{2,\ldots,2}^{\#2},\ldots,\overbrace{N,\ldots,N}^{\#N}\}$$

$$\#i = \tilde{n} - 2h_i$$

1	1	1	2
2	3	3	4

$\langle 13\rangle\langle 13\rangle\langle 24\rangle[34]$ $\quad F_{L1}^{\alpha\beta}\psi_2^\gamma(D\psi_3)_{\alpha\beta\dot{\alpha}}(D\phi_4)_\gamma{}^{\dot{\alpha}}$

1	1	1	3
2	2	3	4

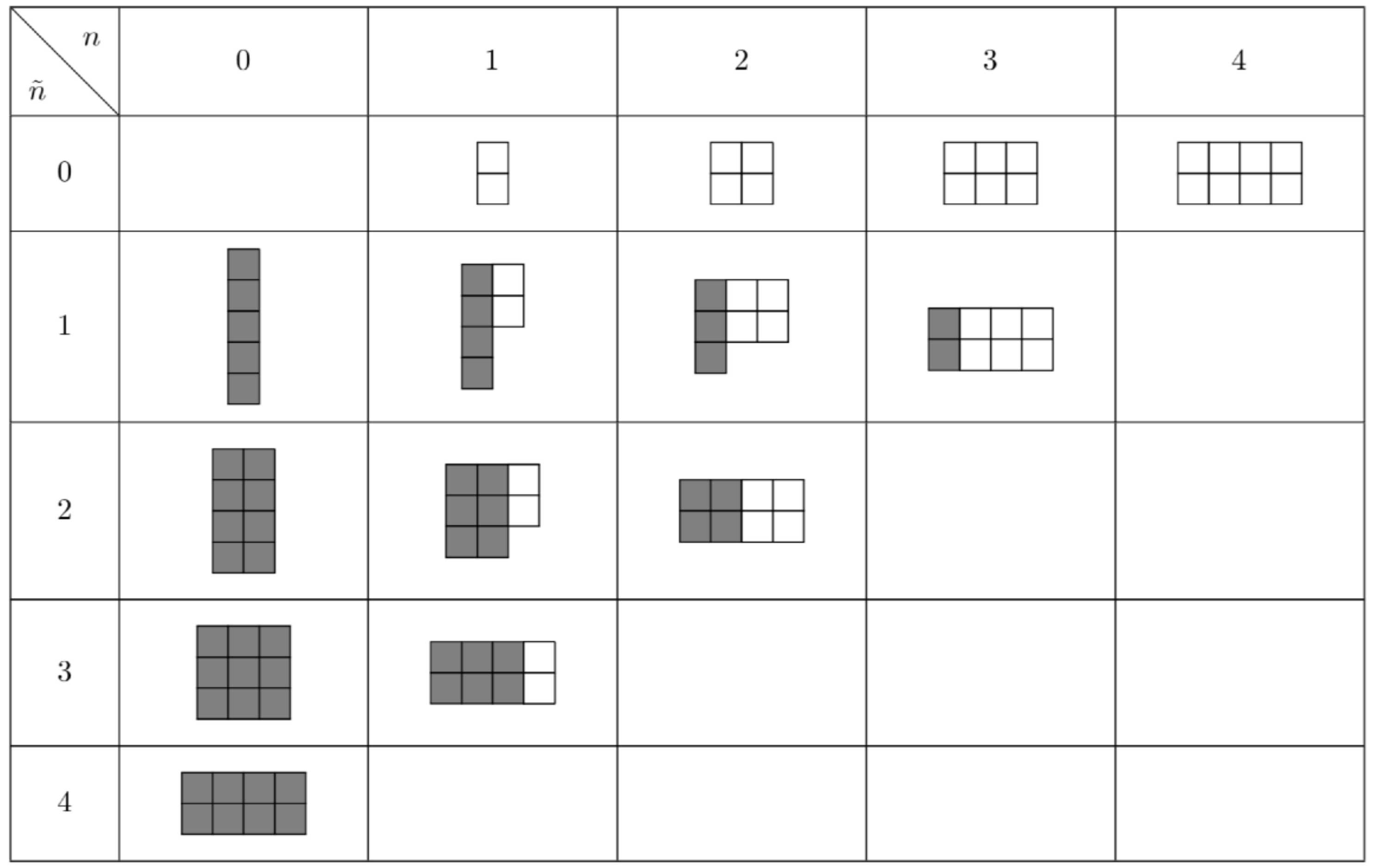
$\langle 12\rangle\langle 13\rangle\langle 34\rangle[34]$ $\quad F_{L1}^{\alpha\beta}\psi_{2\alpha}(D\psi_3)_\beta{}^\gamma{}_{\dot{\alpha}}(D\phi_4)_\gamma{}^{\dot{\alpha}}$

On-shell Amplitude correspondence

Procedure and comparison

Dim-8 operators: 993 (44807) operators for 1 (3) generations

Step-1



n \ $\bar{n}$	0	1	2	3	4
0	ϕ^8	$\psi^2\phi^5$	$\psi^4\phi^2,\ F_{\rm L}\psi^2\phi^3,\ F_{\rm L}^2\phi^4$	$F_{\rm L}\psi^4,\ F_{\rm L}^2\psi^2\phi,\ F_{\rm L}^3\phi^2$	$F_{\rm L}^4$
1	$\psi^{\dagger 2}\phi^5$	$\psi^{\dagger 2}\psi^2\phi^2,\ \psi^\dagger\psi\phi^4 D,\ \phi^6 D^2$	$F_{\rm L}\psi^{\dagger 2}\psi^2,\ F_{\rm L}^2\psi^{\dagger 2}\phi,\ \psi^\dagger\psi^3\phi D,\ F_{\rm L}\psi^\dagger\psi\phi^2 D,\ \psi^2\phi^3 D^2,\ F_{\rm L}\phi^4 D^2$	$F_{\rm L}^2\psi^\dagger\psi D,\ \psi^4 D^2,\ F_{\rm L}\psi^2\phi D^2,\ F_{\rm L}^2\phi^2 D^2$	
2	$\psi^{\dagger 4}\phi^2,\ F_{\rm R}\psi^{\dagger 2}\phi^3,\ F_{\rm R}^2\phi^4$	$F_{\rm R}\psi^{\dagger 2}\psi^2,\ F_{\rm R}^2\psi^2\phi,\ \psi^{\dagger 3}\psi\phi D,\ F_{\rm R}\psi^\dagger\psi\phi^2 D,\ \psi^{\dagger 2}\phi^3 D^2,\ F_{\rm R}\phi^4 D^2$	$F_{\rm R}^2 F_{\rm L}^2,\ F_{\rm R}F_{\rm L}\psi^\dagger\psi D,\ \psi^{\dagger 2}\psi^2 D^2,\ F_{\rm R}\psi^2\phi D^2,\ F_{\rm L}\psi^{\dagger 2}\phi D^2,\ F_{\rm R}F_{\rm L}\phi^2 D^2,\ \phi^4 D^4,\ \psi^\dagger\psi\phi^2 D^3$		
3	$F_{\rm R}\psi^{\dagger 4},\ F_{\rm R}^2\psi^{\dagger 2}\phi,\ F_{\rm R}^3\phi^2$	$F_{\rm R}^2\psi^\dagger\psi D,\ \psi^{\dagger 4} D^2,\ F_{\rm R}\psi^{\dagger 2}\phi D^2,\ F_{\rm R}^2\phi^2 D^2$			
4	$F_{\rm R}^4$				

Step-2

$BWHH^\dagger D^2$ $\quad \#1 = 3, \#2 = 3, \#3 = 1, \#4 = 1$

Step-3

1	1	1	3
2	2	2	4

1	1	1	2
2	2	3	4

2

$\langle 13\rangle\langle 13\rangle\langle 24\rangle[34]$ $\quad\langle 12\rangle\langle 13\rangle\langle 34\rangle[34]$

$B_L{}^{\alpha\beta}W_{L\alpha\beta}\,(DH^\dagger)^\gamma{}_{\dot\alpha}\,(DH)_\gamma{}^{\dot\alpha},\qquad B_L{}^{\alpha\beta}W_{L\alpha}{}^\gamma\,(DH^\dagger)_{\beta\dot\alpha}\,(DH)_\gamma{}^{\dot\alpha}$

Traditional method

[Hays, Martin, Sanz, Setford, 2018]

$BWHH^\dagger D^2$

$$(D^2H^\dagger)HB_{L\mu\nu}W_L^{\mu\nu},\ (D^\mu D_\nu H^\dagger)HB_{L\mu\rho}W_L^{\nu\rho},\ (D_\nu D^\mu H^\dagger)HB_{L\mu\rho}W_L^{\nu\rho},\ (D_\mu H^\dagger)(D^\mu H)B_{L\nu\rho}W_L^{\nu\rho},$$
$$(D_\mu H^\dagger)(D^\nu H)B_{L\nu\rho}W_L^{\mu\rho},\ (D^\nu H^\dagger)(D_\mu H)B_{L\nu\rho}W_L^{\mu\rho},\ (D_\mu H^\dagger)H(D^\mu B_{L\nu\rho})W_L^{\nu\rho},\ (D_\mu H^\dagger)H(D^\nu B_{L\nu\rho})W_L^{\mu\rho},$$
$$(D^\nu H^\dagger)H(D_\mu B_{L\nu\rho})W_L^{\mu\rho},\ (D_\mu H^\dagger)HB_{L\nu\rho}(D^\mu W_L^{\nu\rho}),\ (D_\mu H^\dagger)HB_{L\nu\rho}(D^\nu W_L^{\mu\rho}),\ (D^\nu H^\dagger)HB_{L\nu\rho}(D_\mu W_L^{\mu\rho}),$$
$$H^\dagger(D^2H)B_{L\mu\nu}W_L^{\mu\nu},\ H^\dagger(D^\mu D_\nu H)B_{L\mu\rho}W_L^{\nu\rho},\ H^\dagger(D_\nu D^\mu H)B_{L\mu\rho}W_L^{\nu\rho},\ H^\dagger(D^\mu H)(D_\mu B_{L\nu\rho})W_L^{\nu\rho},$$
$$H^\dagger(D^\nu H)(D_\mu B_{L\nu\rho})W_L^{\mu\rho},\ H^\dagger(D_\mu H)(D^\nu B_{L\nu\rho})W_L^{\mu\rho},\ H^\dagger(D^\mu H)B_{L\nu\rho}(D_\mu W_L^{\nu\rho}),\ H^\dagger(D^\nu H)B_{L\nu\rho}(D_\mu W_L^{\mu\rho}),$$
$$H^\dagger(D_\mu H)B_{L\nu\rho}(D^\nu W_L^{\mu\rho}),\ H^\dagger H(D^2B_{L\mu\nu})W_L^{\mu\nu},\ H^\dagger H(D^\mu D_\nu B_{L\mu\rho})W_L^{\nu\rho},\ H^\dagger H(D_\nu D^\mu B_{L\mu\rho})W_L^{\nu\rho},$$
$$H^\dagger H(D^\mu B_{L\nu\rho})(D_\mu W_L^{\nu\rho}),\ H^\dagger H(D^\nu B_{L\nu\rho})(D_\mu W_L^{\mu\rho}),\ H^\dagger H(D_\mu B_{L\nu\rho})(D^\nu W_L^{\mu\rho}),\ H^\dagger HB_{L\mu\nu}(D^2W_L^{\mu\nu}),$$
$$H^\dagger HB_{L\mu\rho}(D^\mu D_\nu W_L^{\nu\rho}),\ H^\dagger HB_{L\mu\rho}(D_\nu D^\mu W_L^{\nu\rho}). \qquad (14)$$

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EOM

$$(DH^\dagger)_{\alpha\dot\alpha}(DH)_{\beta\dot\beta}B_{L\{\gamma\delta\}}W_{L\{\xi\eta\}}\,\epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}\epsilon^{\gamma\xi}\epsilon^{\delta\eta}$$
$$(DH^\dagger)_{\alpha\dot\alpha}(DH)_{\beta\dot\beta}B_{L\{\gamma\delta\}}W_{L\{\xi\eta\}}\,\tfrac{1}{2}\epsilon^{\dot\alpha\dot\beta}\epsilon^{\delta\xi}(\epsilon^{\alpha\gamma}\epsilon^{\beta\eta}+\epsilon^{\beta\gamma}\epsilon^{\alpha\eta})$$
$$(DH^\dagger)_{\alpha\dot\alpha}\,H\,(DB_L)_{\{\beta\gamma\delta\},\dot\beta}W_{L\{\xi\eta\}}\,\epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}\epsilon^{\gamma\xi}\epsilon^{\delta\eta}$$
$$(DH^\dagger)_{\alpha\dot\alpha}\,H\,B_{L\{\xi\eta\}}(DW_L)_{\{\beta\gamma\delta\},\dot\beta}\,\epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}\epsilon^{\gamma\xi}\epsilon^{\delta\eta}$$
$$H^\dagger\,(DH)_{\alpha\dot\alpha}\,(DB_L)_{\{\beta\gamma\delta\},\dot\beta}W_{L\{\xi\eta\}}\,\epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}\epsilon^{\gamma\xi}\epsilon^{\delta\eta}$$
$$H^\dagger\,(DH)_{\alpha\dot\alpha}\,B_{L\{\xi\eta\}}(DW_L)_{\{\beta\gamma\delta\},\dot\beta}\,\epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}\epsilon^{\gamma\xi}\epsilon^{\delta\eta}$$
$$H^\dagger H\,(DB_L)_{\{\alpha\beta\gamma\},\dot\alpha}(DW_L)_{\{\xi\eta\delta\},\dot\beta}\,\epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\xi}\epsilon^{\beta\eta}\epsilon^{\gamma\delta}$$

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IBP

$$B_L{}^{\alpha\beta}W_{L\alpha\beta}\,(DH^\dagger)^\gamma{}_{\dot\alpha}\,(DH)_\gamma{}^{\dot\alpha}$$
$$B_L{}^{\alpha\beta}W_{L\alpha}{}^\gamma\,(DH^\dagger)_{\beta\dot\alpha}\,(DH)_\gamma{}^{\dot\alpha}$$

2

SMEFT operator bases up to dim-9

$$\mathscr{L}_{\text{EFT}} = \mathscr{L}_{\mathscr{D}\leq 4} + \frac{\mathscr{L}_5}{\Lambda} + \frac{\mathscr{L}_6}{\Lambda^2} + \frac{\mathscr{L}_7}{\Lambda^3} + \frac{\mathscr{L}_8}{\Lambda^4} + \frac{\mathscr{L}_9}{\Lambda^5} + \dots$$

Standard Model Effective Field Theory

SU(3) x SU(2) x U(1) gauge symmetry

- Dim-5: [Weinberg, 1979]
- Dim-6: [Buchmuller, Wyler, 1986] [Grzadkowski, Iskrzynski, Misiak, Rosiek, 2010]
- Dim-7: [Lehman, 2014] [Henning, Lu, Melia, Murayama, 2015] [Liao, Ma, 2016]
- Dim-8: [Li, Ren, Shu, Xiao, **Yu**, Zheng, **2020**] [Murphy, 2020]
- Dim-9: [Li, Ren, Xiao, **Yu**, Zheng, **2020**] [Liao, Ma, 2020]

Low Energy Effective Field Theory

SU(3) x U(1) gauge symmetry

- Dim-5: [Dirac, 1932]
- Dim-6: [Fermi, 1934] [Lee, Yang, 1956] [Jenkins, Manohar, Stoffer, 2017]
- Dim-7: [Liao, Ma, Wang, 2020] [Li, Ren, Xiao, **Yu**, Zheng, 2020]
- Dim-8: [Li, Ren, Xiao, **Yu**, Zheng, 2020] [Murphy, 2020]
- Dim-9: [Li, Ren, Xiao, **Yu**, Zheng, 2020]

Operator bases for generic EFT up to all order

Amplitude Basis Construction for Effective Field Theory [Li, Ren, Xiao, **Yu**, Zheng, 2201.04639]

https://abc4eft.hepforge.org/

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Welcome to the HEPForge Project: ABC4EFT

This is the website for the Mathematica package: Amplitude Basis Construction for Effective Field Th

Package

This package has the following features:

- It provides a general procedure to construct the independent and complete operator bases for generic L invariant effective field theory, given any kind of gauge symmetry and field content, up to any mass dim
- Various operator bases have been systematically constructed to emphasize different aspects: operator independence (y-basis), flavor relation (p-basis) and conserved quantum number (j-basis).
- It provides a systematic way to convert any operator into our on-shell amplitude basis and the basis cor can be easily done.

Authors

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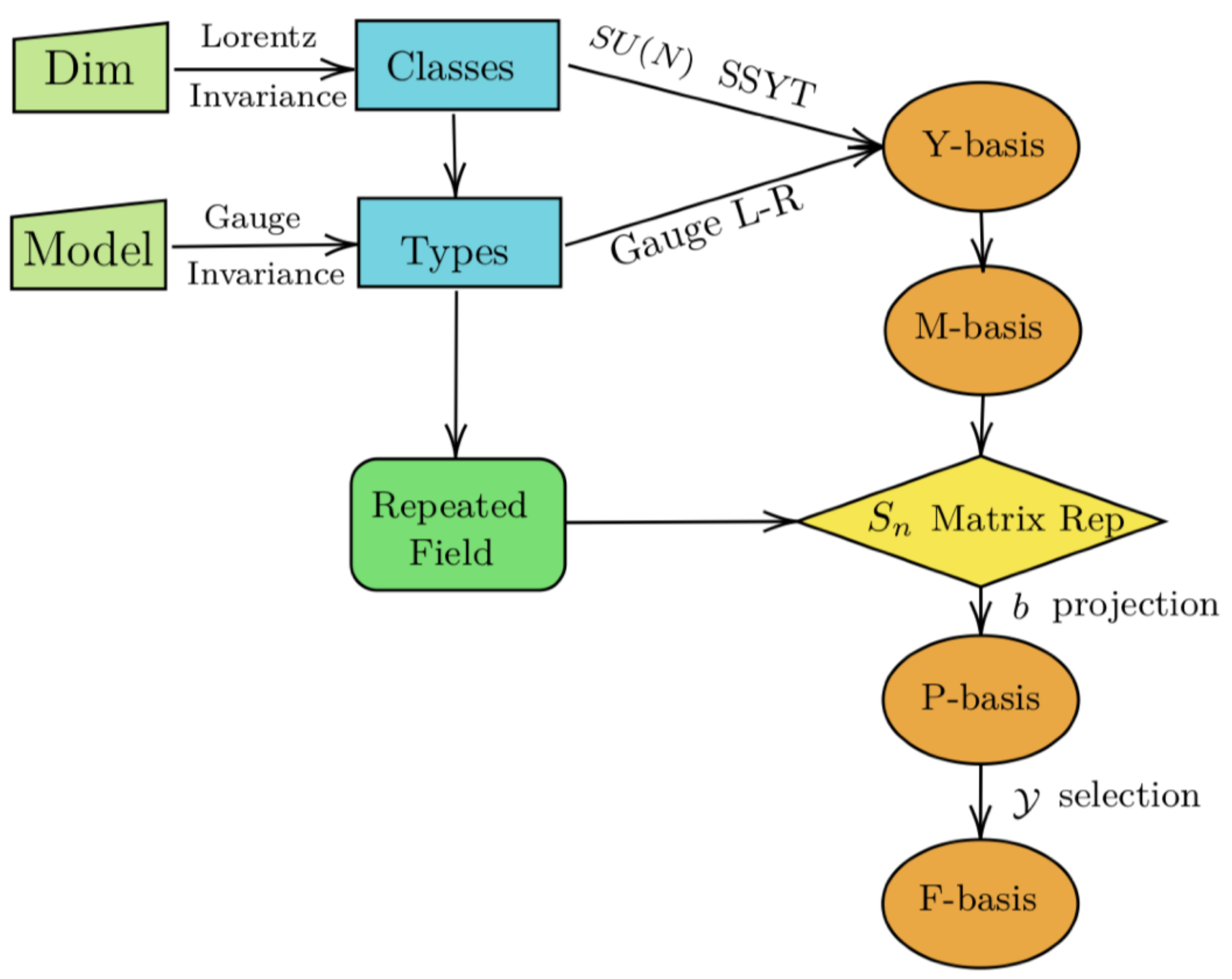

Fully Automatic

Dark matter EFT [Song, Sun, **Yu**, 2306.05999]
Sterile neutrino EFT
Gravity EFT
Axion EFT [Song, Sun, **Yu**, 2305.16770]
Dark photon EFT
...

EFTs at Broken Phase

[Hao Sun, Ming-Lei Xiao, **J.H.Yu**, 2206.07722]

[Hao Sun, Ming-Lei Xiao, **J.H.Yu**, 2206.07722]

[Hao Sun, Yi-Ning Wang, **J.H.Yu**, in préparation]

[Hua-Yang Song, Hao Sun, **J.H.Yu**, 2305.16770]

EFTs at broken phase

Standard Model Effective Field Theory

Matching

Running

Low Energy Effective Field Theory

approximate custodial symmetry
SU(2) x SU(2)

$$\Sigma \equiv (\Phi^c, \Phi) = \begin{pmatrix} \Phi^{0*} & \Phi^+ \\ -\Phi^- & \Phi^0 \end{pmatrix} \to g_L \, \Sigma \, g_R^\dagger$$

$$\langle \Sigma \rangle = \begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix} \neq 0$$

approximate chiral symmetry
SU(3) x SU(3)

$$\mathbf{q}_L \to g_L \, \mathbf{q}_L \,, \quad \mathbf{q}_R \to g_R \, \mathbf{q}_R \,,$$

$$\langle 0 | \left(\bar{\mathbf{q}}_L \mathbf{q}_R + \bar{\mathbf{q}}_R \mathbf{q}_L \right) | 0 \rangle \neq 0$$

Electroweak Chiral Lagrangian

QCD Chiral Lagrangian

$$\Phi \equiv \frac{1}{\sqrt{2}} \vec{\sigma} \cdot \vec{\varphi} = \begin{pmatrix} \frac{1}{\sqrt{2}} \varphi^0 & \varphi^+ \\ \varphi^- & -\frac{1}{\sqrt{2}} \varphi^0 \end{pmatrix}$$

SM fields and Goldstone

$$\Phi \equiv \frac{\vec{\lambda}}{\sqrt{2}} \vec{\phi} = \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}} \eta \end{pmatrix}$$

meson and baryon

SM Fermion masses from Higgs VEV

Baryon masses around cutoff scale from Trace anomaly

Goldstone EFT and power counting

Construct generic EFT for Goldstone at IR broken phase

PHYSICAL REVIEW VOLUME 166, NUMBER 5 25 FEBRUARY 1968

Nonlinear Realizations of Chiral Symmetry*

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(Received 25 September 1967)

Shift symmetry:

$$\pi \to \pi + \epsilon + \cdots$$

Goldstone mode is a fluctuation around the background in the direction of broken generator

Gapless mode

Weakly coupled at IR

Non-linear transform under G/H

No interaction at long-wave limit

$$\mathscr{L}_{\mathrm{EFT}} = \mathscr{L}_{p^2} + \mathscr{L}_{p^3} + \mathscr{L}_{p^4} + \mathscr{L}_{p^5} + \mathscr{L}_{p^6} + \ldots$$

$$\frac{f^2}{4}\langle D_\mu \mathbf{U}^\dagger D^\mu \mathbf{U} \rangle$$

Power counting: Derivative expansion

$$\Pi_{\hat{a}} \to \Pi_{\hat{a}}{}^{(g_{\mathcal{H}})} = \left(e^{i\,\alpha_a t_\pi{}^a}\right)_{\hat{a}}{}^{\hat{b}} \Pi_{\hat{b}}$$

$$\Pi_{\hat{a}} \to \Pi^{(g_{\mathcal{G}/\mathcal{H}})}{}_{\hat{a}} = \Pi_{\hat{a}} + \frac{f}{\sqrt{2}}\alpha_{\hat{\alpha}} + \mathcal{O}\left(\alpha\frac{\Pi^2}{f} + \alpha\frac{\Pi^3}{f^2}\ldots\right)$$

$$U[\Pi] = e^{i\frac{\sqrt{2}}{f}\Pi_{\hat{a}}(x)\widehat{T}^{\hat{a}}}$$

$$g \cdot U[\Pi] = U[\Pi^{(g)}] \cdot h[\Pi;\, g] \qquad h[\Pi;\, g] = e^{i\,\zeta_a[\Pi;g]T^a}$$

Coset Construction

[Callan, Coleman, Wess, Zumino, 1969]

CCWZ chiral Lagrangian

Define the nonlinear Goldstone matrix

[Callan, Coleman, Wess, Zumino, 1969]

$$\Omega(\Pi) \equiv \exp\left[\frac{i}{2f}\Pi(x)\right] \to \Omega(\Pi^{(\mathfrak{g})}) = \mathfrak{g}\Omega(\Pi)\mathfrak{h}^{-1}(\Pi;\mathfrak{g})$$

CCWZ Coset

$$-i\Omega^\dagger\partial_\mu\Omega = d_\mu^{\hat{a}}T^{\hat{a}} + E_\mu^a T^a \equiv d_\mu + E_\mu$$

$$d_\mu \to \mathfrak{h} d_\mu \mathfrak{h}^{-1}, \quad E_\mu \to \mathfrak{h} E_\mu \mathfrak{h}^{-1} - ih\partial_\mu \mathfrak{h}^{-1}$$

$$\partial_\mu \to D_\mu = \partial_\mu + iA_\mu$$
$$A_\mu = A_\mu^{\hat{a}}T^{\hat{a}} + A_\mu^a T^a$$

Building block

$$d_\mu(\Pi), \quad E_{\mu\nu}(\Pi) \quad \nabla_\mu \equiv \partial_\mu + iE_\mu$$
$$f_{\mu\nu} = \Omega^\dagger F_{\mu\nu}\Omega = f_{\mu\nu}^{-\hat{a}}T^{\hat{a}} + f_{\mu\nu}^{+a}T^a$$

$$E_{\mu\nu} = -i[u_\mu, u_\nu] + f_{\mu\nu}^+$$
$$d_\mu = u_\mu$$

Symmetric Coset

$$[T_{\hat{a}}, T_{\hat{b}}] \propto T_c$$

$$\Omega \to \mathfrak{g}\Omega\mathfrak{h}^{-1}, \quad \Omega \to \mathfrak{h}\Omega\mathfrak{g}_{\mathcal{R}}^{-1}$$

$$U \equiv \Omega^2 \to \mathfrak{g}\, U \mathfrak{g}_{\mathcal{R}}^{-1}$$

$$D_\mu U \equiv \partial_u U + iA_u U - iUA_\mu^{(R)}$$
$$A_\mu^{(R)} \equiv A_\mu^a T^a - A_\mu^{\hat{a}}T^{\hat{a}}$$

Building block

$$u_\mu = i\Omega(D_\mu U)^\dagger\Omega \qquad D_\mu$$
$$f_{\mu\nu}^{\pm} = \frac{1}{2}\left(f_{\mu\nu} \pm f_{\mu\nu}^{(R)}\right) = \Omega^\dagger F_{\mu\nu}\Omega \pm F_{\mu\nu}^{(R)}$$

$$\Omega(\Pi) \equiv \begin{bmatrix} u(\Pi) & 0 \\ 0 & u^\dagger(\Pi) \end{bmatrix} \qquad u \to \sqrt{\mathfrak{g}_L U \mathfrak{g}_R^\dagger} = \mathfrak{g}_L\, u\mathfrak{h}^{-1} = \mathfrak{h}^{-1} u\mathfrak{g}_R$$

$$U(\Pi) \equiv u^2(\Pi) \longrightarrow \mathfrak{g}_L \mathbf{U}(\Pi)\mathfrak{g}_R^\dagger$$

QCD Chiral Lag

$$u_\mu \to \mathfrak{h}\, u_\mu\, \mathfrak{h}^{-1}$$

$$u_\mu = \mathrm{i}\left[u^\dagger(\partial_\mu - \mathrm{i}r_\mu)u - u(\partial_\mu - \mathrm{i}l_\mu)u^\dagger\right]$$

$$\chi_\pm = u^\dagger\chi u^\dagger \pm u\chi^\dagger u\,,$$
$$f_{\mu\nu}^\pm = uf_{\mu\nu}^{\mathrm{L}}u^\dagger \pm u^\dagger f_{\mu\nu}^{\mathrm{R}}u\,,$$

EW Chiral Lag

$$\mathbf{V}_\mu \to \mathfrak{g}_L \mathbf{V}\mathfrak{g}_L^{-1}$$

$$\mathbf{V}_\mu(x) = i\mathbf{U}(x)D_\mu\mathbf{U}(x)^\dagger$$

$$\mathbf{T} = \mathbf{U}\mathcal{T}_R\mathbf{U}^\dagger \longrightarrow \mathfrak{g}_L\mathbf{T}\mathfrak{g}_L^\dagger \qquad \hat{W}_{\mu\nu} \longrightarrow \mathfrak{g}_L\hat{W}_{\mu\nu}\mathfrak{g}_L^\dagger$$
$$\mathbf{Y} = \mathbf{U}\mathcal{Y}_R\mathbf{U}^\dagger \longrightarrow \mathfrak{g}_L\mathbf{Y}\mathfrak{g}_L^\dagger \qquad \hat{B}_{\mu\nu} \longrightarrow \mathfrak{g}_R\hat{B}_{\mu\nu}\mathfrak{g}_R^\dagger$$

Adler zero condition for Goldstone boson

Chiral symmetry (PCAC)

$$\alpha \to \beta \quad \xleftrightarrow{\text{At low energy}} \quad \alpha + n_1\pi \to \beta + n_2\pi$$

Goldberger-Trieman, Callan-Trieman, Adler-Weisberger, etc

Adler Zero condition [Adler, 1965]

$$T(\alpha + \phi(p), \beta) = -\frac{p_\mu}{F} R^\mu(p) \xrightarrow{p \to 0} 0$$

Amplitude (soft limit of external leg s)

$$\mathcal{A}(1,\ldots,N,s) \xrightarrow{p_s \to 0} \begin{cases} \left(S^{(0)}(s) + S^{(\text{sub})}(s)\right)\mathcal{A}(1,\ldots,N) \\ \mathcal{O}(p_s^\sigma) \quad \text{for Goldstone Boson} \end{cases}$$

$$\{-1/2, -1/2, 1, 0, 0\} \qquad \begin{array}{|c|c|c|c|}\hline 1&1&1&4\\\hline 2&2&2&5\\\hline 4&5\\\cline{1-2}\end{array}, \quad \begin{array}{|c|c|c|c|}\hline 1&1&1&2\\\hline 2&2&5&5\\\hline 4&4\\\cline{1-2}\end{array}, \quad \begin{array}{|c|c|c|c|}\hline 1&1&1&2\\\hline 2&2&4&4\\\hline 5&5\\\cline{1-2}\end{array}, \quad \begin{array}{|c|c|c|c|}\hline 1&1&1&2\\\hline 2&2&4&5\\\hline 4&5\\\cline{1-2}\end{array},$$

Expand the soft-limit amplitude into the SSYT basis

Put constraints on the SSYT basis

$$\mathcal{B}_i^{(N)}(p_\pi \to 0) = \sum_{l=1}^{d_N} \mathcal{K}_{il}\mathcal{B}_l^{(N)}$$

$$\begin{array}{|c|c|c|c|}\hline 1&1&1&4\\\hline 2&2&2&5\\\hline 4&5\\\cline{1-2}\end{array}, \quad \begin{array}{|c|c|c|c|}\hline 1&1&1&2\\\hline 2&2&4&5\\\hline 4&5\\\cline{1-2}\end{array},$$

[Sun, Xiao, **Yu**, 2210.14939]
[Sun, Xiao, **Yu**, 2206.07722]
[Low, Shu, Xiao, Zheng, 2022]

custodial/chiral symmetry breaking: spurion

Chiral Lagrangian for QCD and EW theories

$$\mathscr{L}_{\text{EFT}} = \mathscr{L}_{p^2} + \mathscr{L}_{p^3} + \mathscr{L}_{p^4} + \mathscr{L}_{p^5} + \mathscr{L}_{p^6} + \dots$$

ChPT and Chiral EFT

LO Lagrangian [Weinberg, 1979]

Pure mesonic
[Gasser, Leutwyler, 1984, 1985]
[Fearing, Scherer 1994]
[Bijnens, Colangelo, Ecker, 1999]
[Jiang, Ge, Wang, 2014]
[Bijnens, Hermansson, Wang, 2018]

nucleon-meson
[Krause, 1990]
[Ecker, 1994]
[Fettes, Meisner, Mojzis, Steininger, 2000]
[Oller, Verbeni, Prades, 2006]
[Frink, Meisner, 2006]
[Jiang, Chen, Liu, 2017]

nucleon-nucleon
[Weinberg 1990]
[van Kolck, Ordonez, 1992]
[Petschauer, Kaiser, 2013]
[Petschauer, Haidenbauer, Kaiser, Meisner, Weise, 2020]
[Sun, Wang, **Yu**, in préparation]

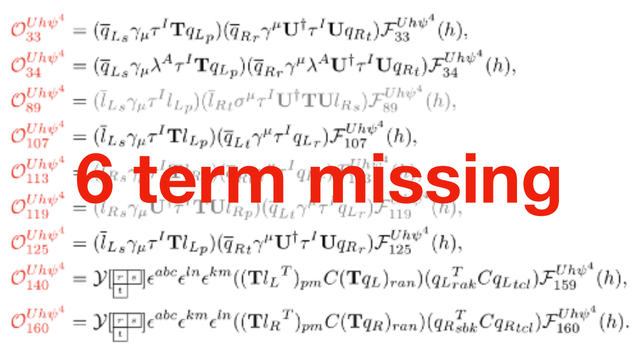
EW Chiral Lagrangian = HEFT

LO Lagrangian [Weinberg, 1979]

NLO bosonic
[Appelquist, Bernard, 1980]
[Longhitano, 1980, 1981]
[Feruglio, 1993]

NLO 2-fermion [Buchalla, Cata, Krause, 2014]

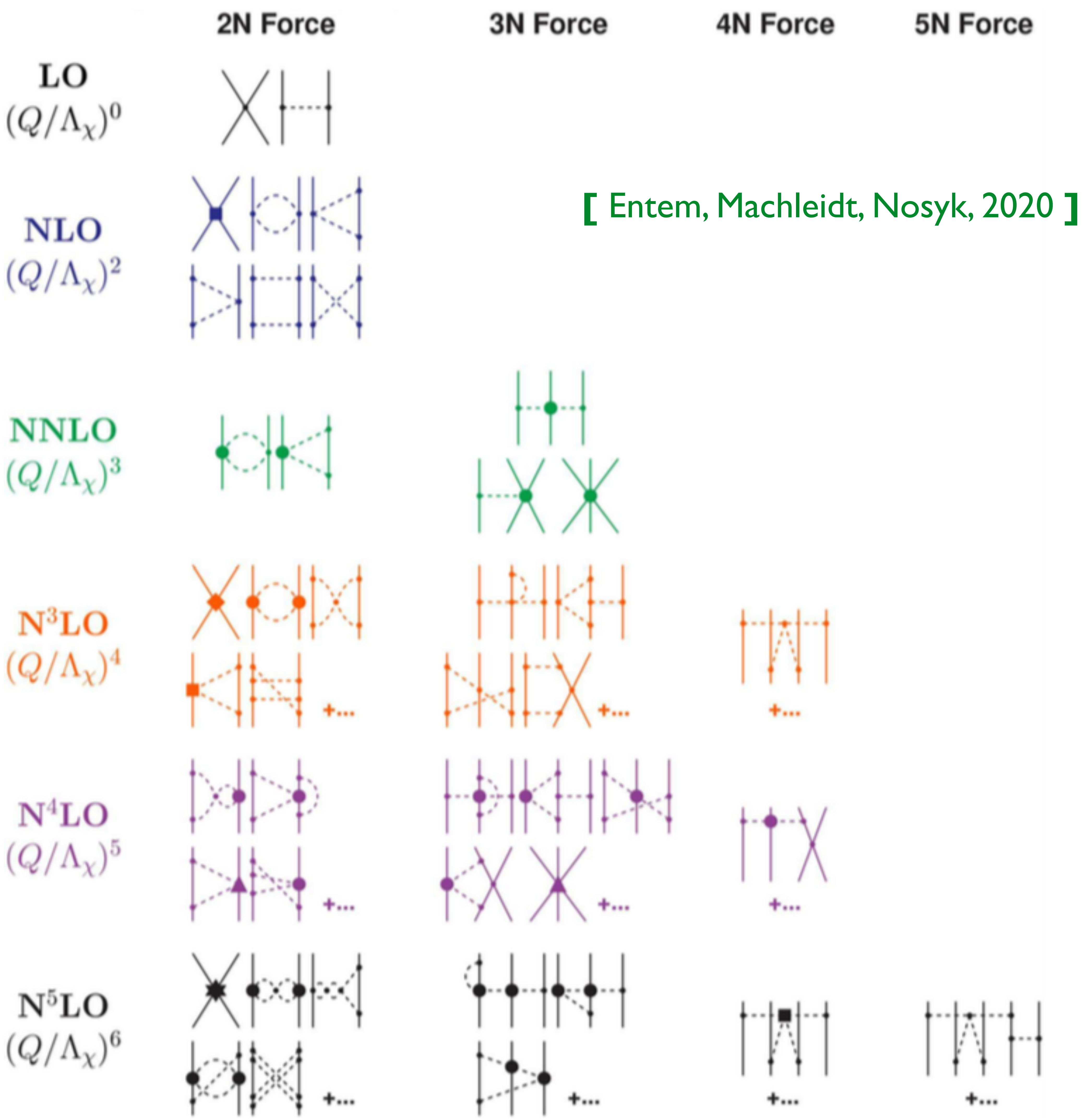
NLO 4-fermion
[Buchalla, Cata, Krause, 2014]
[Pich, Rosell, Santos, Sanz-Cillero, 2015, 2018]
[Sun, Xiao, **Yu**, 2206.07722]



NNLO Basis [Sun, Xiao, **Yu**, 2210.14939]

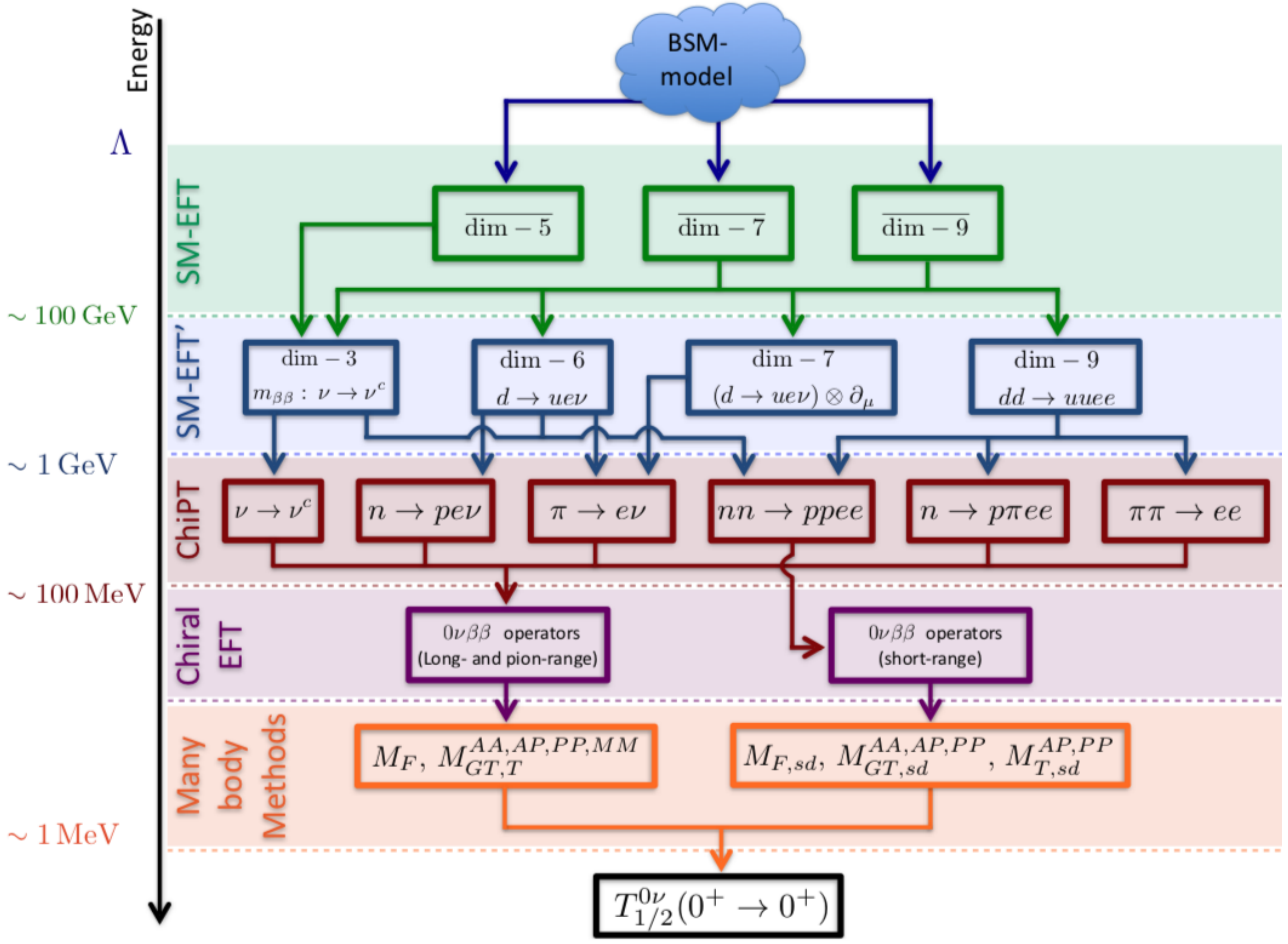
Why higher order chiral Lagrangian?

Ab initio nuclear force



[Entem, Machleidt, Nosyk, 2020]

Higher order chiral perturbation

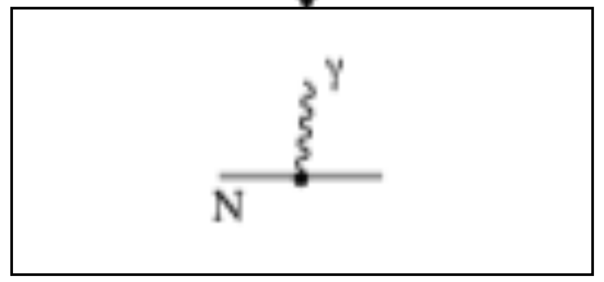


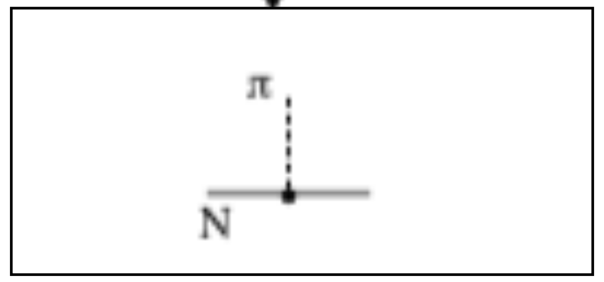
Jiang-Hao Yu (ITP-CAS)

Chiral nuclear force

Meson theory

$$\mathcal{L}_\sigma = \overline{N_L}\, i\not{D}\, N_L + \overline{N_R}\, i\not{D}\, N_R - g\overline{N_R}\,\Sigma\, N_L - g\overline{N_L}\,\Sigma^\dagger\, N_R$$



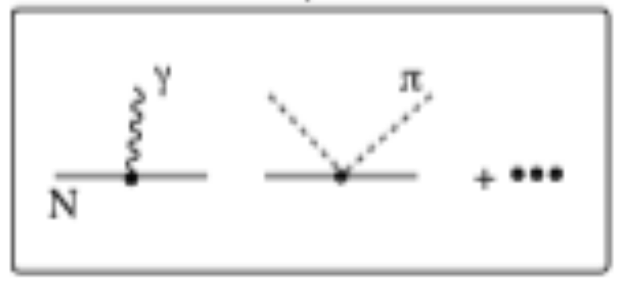


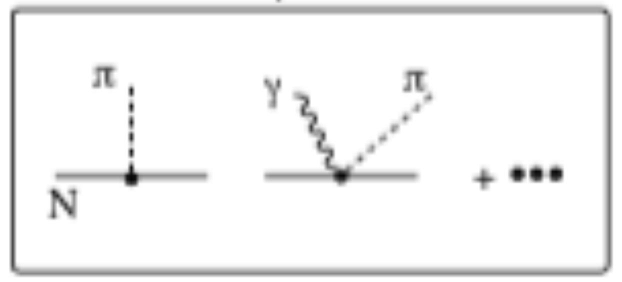
Chiral EFT

$$\mathcal{L}^{(1)}_{\pi N} = \bar{\psi}\left(i\not{D} - m_N + \tfrac{1}{2} g_A \gamma_\mu \gamma_5 u^\mu\right)\psi$$

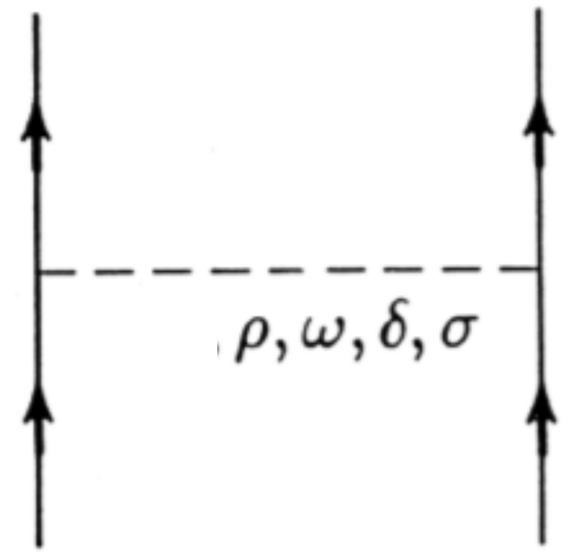
$$M_N g_A(0) = F_\pi\, g_{\pi NN}$$

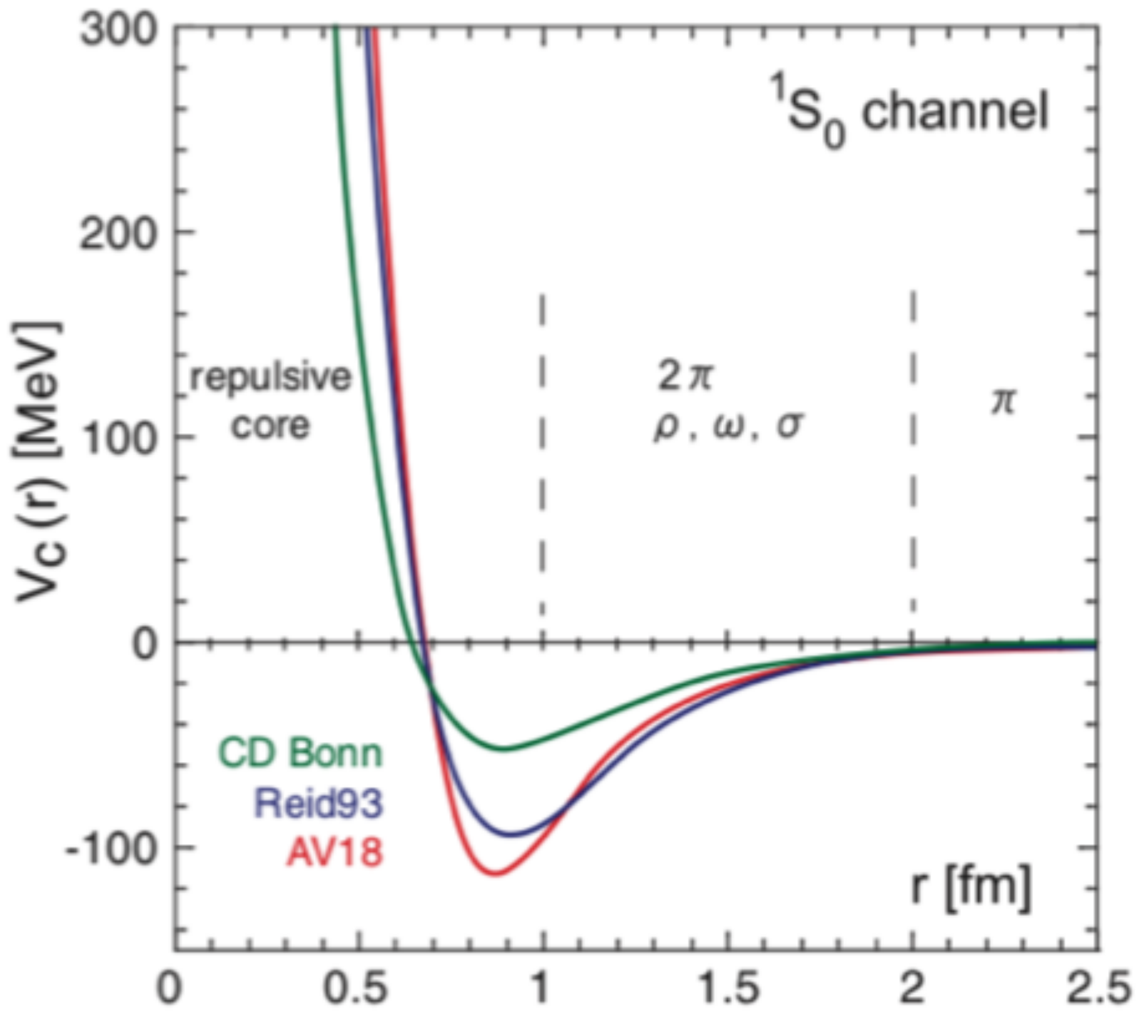
$$g_A \simeq 1.27,\ g_{\pi NN} \simeq 13.40$$

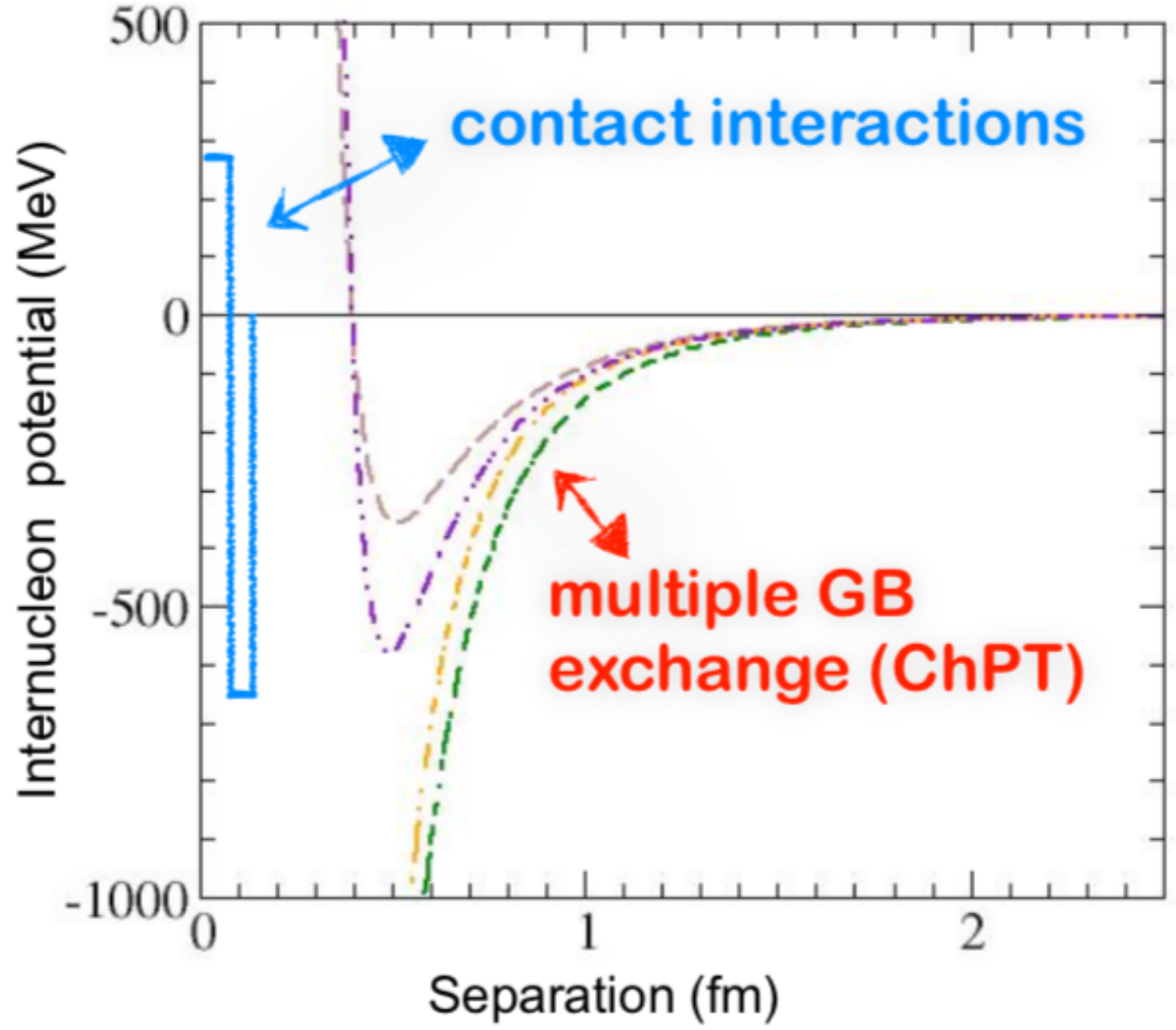




Goldberger-Treiman Relation





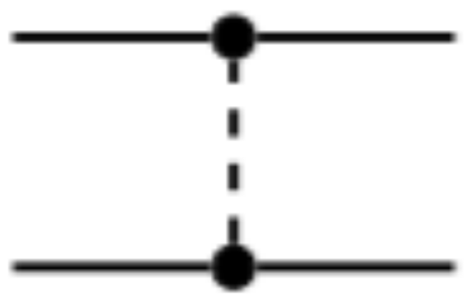


$$\mathcal{L} = N^\dagger\left(i\partial_0 + \frac{\vec{\nabla}^2}{2m_N}\right)N - \frac{1}{2}C_S(N^\dagger N)^2 - \frac{1}{2}C_T(N^\dagger\vec{\sigma}N)^2 + \underbrace{\ldots}_{\text{terms with} \geq 2 \text{ derivatives}}$$

Chiral effective field theory

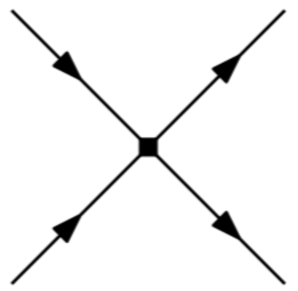
Weinberg power counting $\mu = 2 + 2\ell - r + \sum_i V_i \left(d_i + \frac{1}{2} n_i - 2 \right)$

[Weinberg, 1990]



Dim = 2(1-2+2/2) = 0

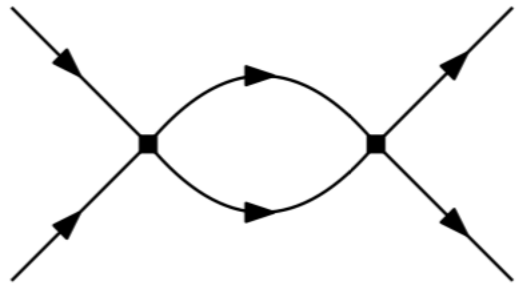
$$V_{1\pi} = -\left(\frac{g_A}{2F_\pi}\right)^2 \frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{q^2 + M_\pi^2} \vec{\tau}_1 \cdot \vec{\tau}_2 \sim \mathcal{O}(1)$$

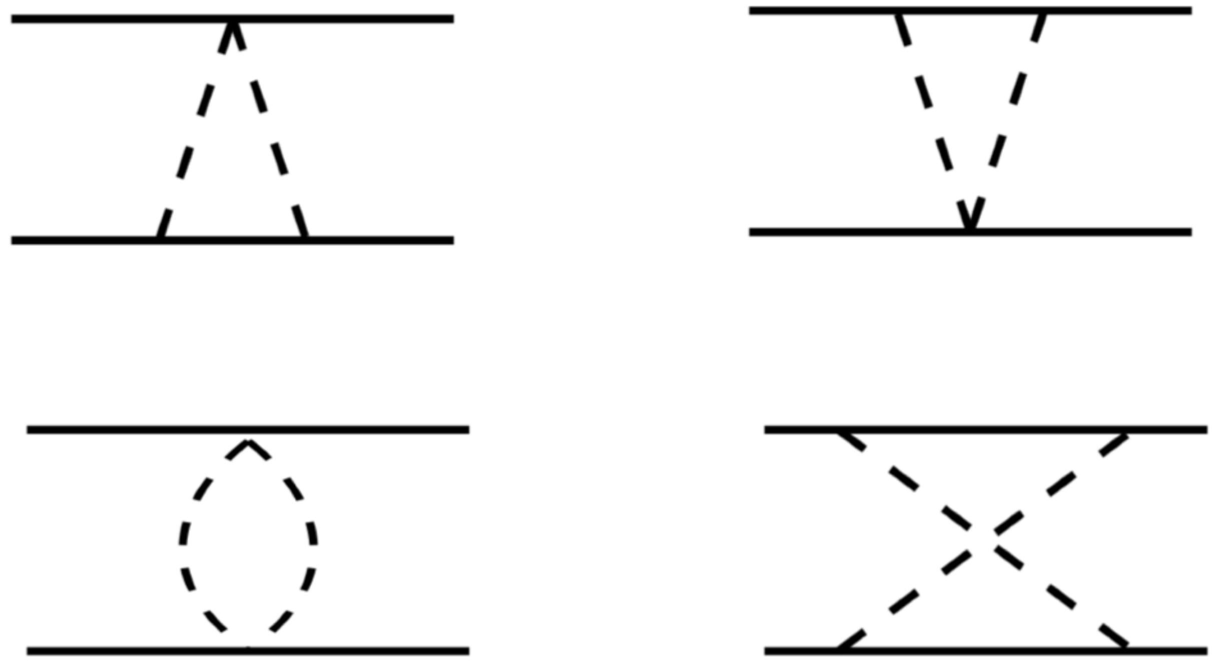


= (0-2+4/2) = 0

$-C_0$

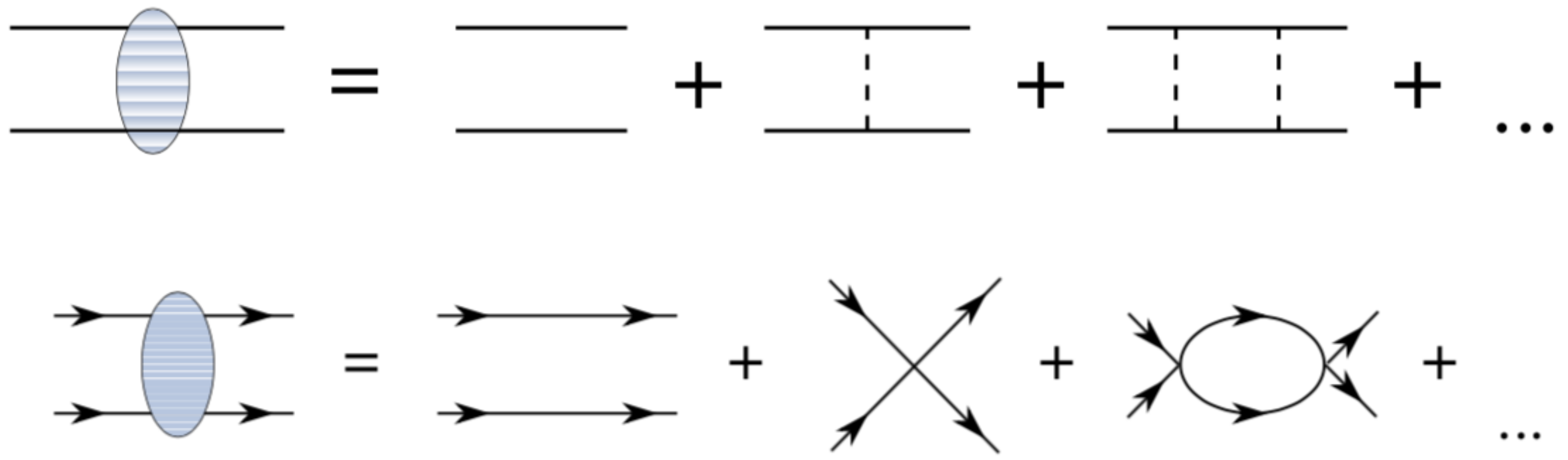
Irreducible Dim = 2+2-2+2(1-2+2/2) = 2



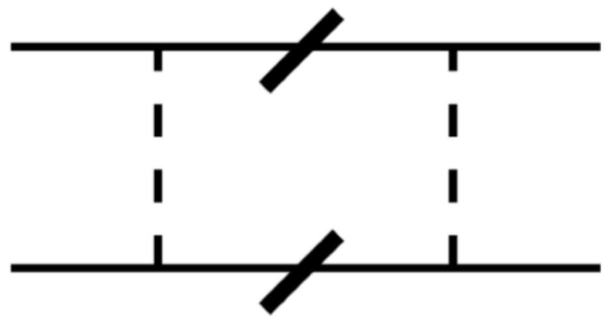


Pinch singularity

$$\frac{Q^5}{4\pi M} \times C_0 \left(\frac{M}{Q^2}\right)\left(\frac{M}{Q^2}\right) C_0 \sim C_0^2 \frac{MQ}{4\pi}$$



Reducible 2PI



$$\sim \left(\frac{g_A}{F_\pi}\right)^2 \frac{Q}{\Lambda_{NN}} \qquad \Lambda_{NN} = \frac{4\pi F_\pi^2}{g_A^2 m_N} \sim f_\pi$$

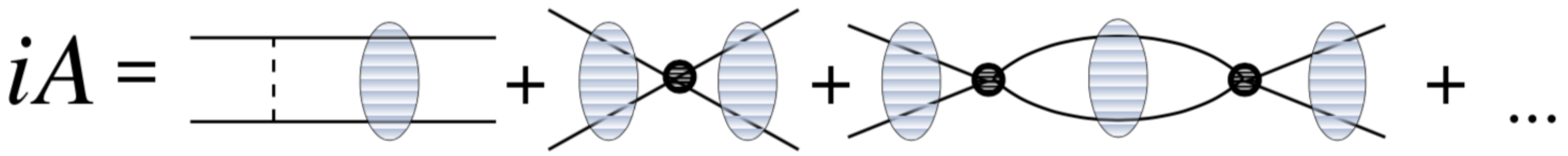
Chiral EFT operators

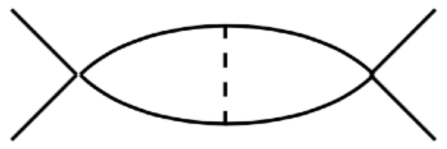
- **Weinberg:** $C_0^R \sim \mathcal{O}(1)$ $V_{\rm Weinberg}^{\rm LO} \sim \mathcal{O}(1)$, $V_{\rm Weinberg}^{\rm NLO} \sim \mathcal{O}(p^2)$
  $\mu \sim \mathcal{O}(1)$ $C_2^R \sim \mathcal{O}(1)$ [i.e. scaling of C_{2n} according to NDA (~ O(1))]

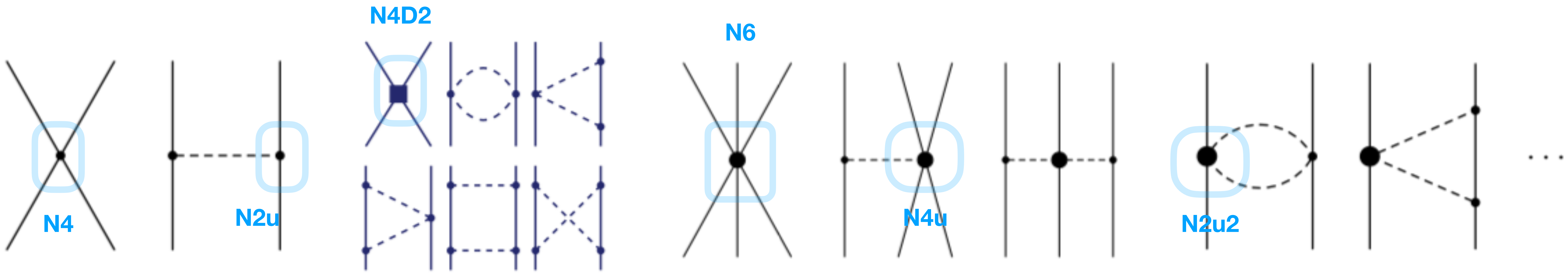
  [Weinberg, 1990]

- **KSW:** $C_0^R \sim \mathcal{O}(p^{-1})$ $V_{\rm KSW}^{\rm LO} \sim \mathcal{O}(p^{-1})$, $V_{\rm KSW}^{\rm NLO} \sim \mathcal{O}(1)$
  $\mu \sim \mathcal{O}(p)$ $C_2^R \sim \mathcal{O}(p^{-2})$ [i.e. scaling of C_{2n} as $C_{2n} \sim O(p^{-1-n})$]

  [Kaplan, Savage, Wise, 1998]







[Petschauer, Haidenbauer, Kaiser, Meisner, Weise, 2020]

$$\langle \bar{B}B\bar{B}B \rangle,\ \langle \bar{B}\bar{B}BB \rangle,\ \langle \bar{B}B \rangle \langle \bar{B}B \rangle,\ \langle \bar{B}\bar{B} \rangle \langle BB \rangle,$$
$$\langle \bar{B}\chi B\bar{B}B \rangle,\ \langle \bar{B}B\chi\bar{B}B \rangle,\ \langle \bar{B}\chi B \rangle \langle \bar{B}B \rangle,\ \ldots$$

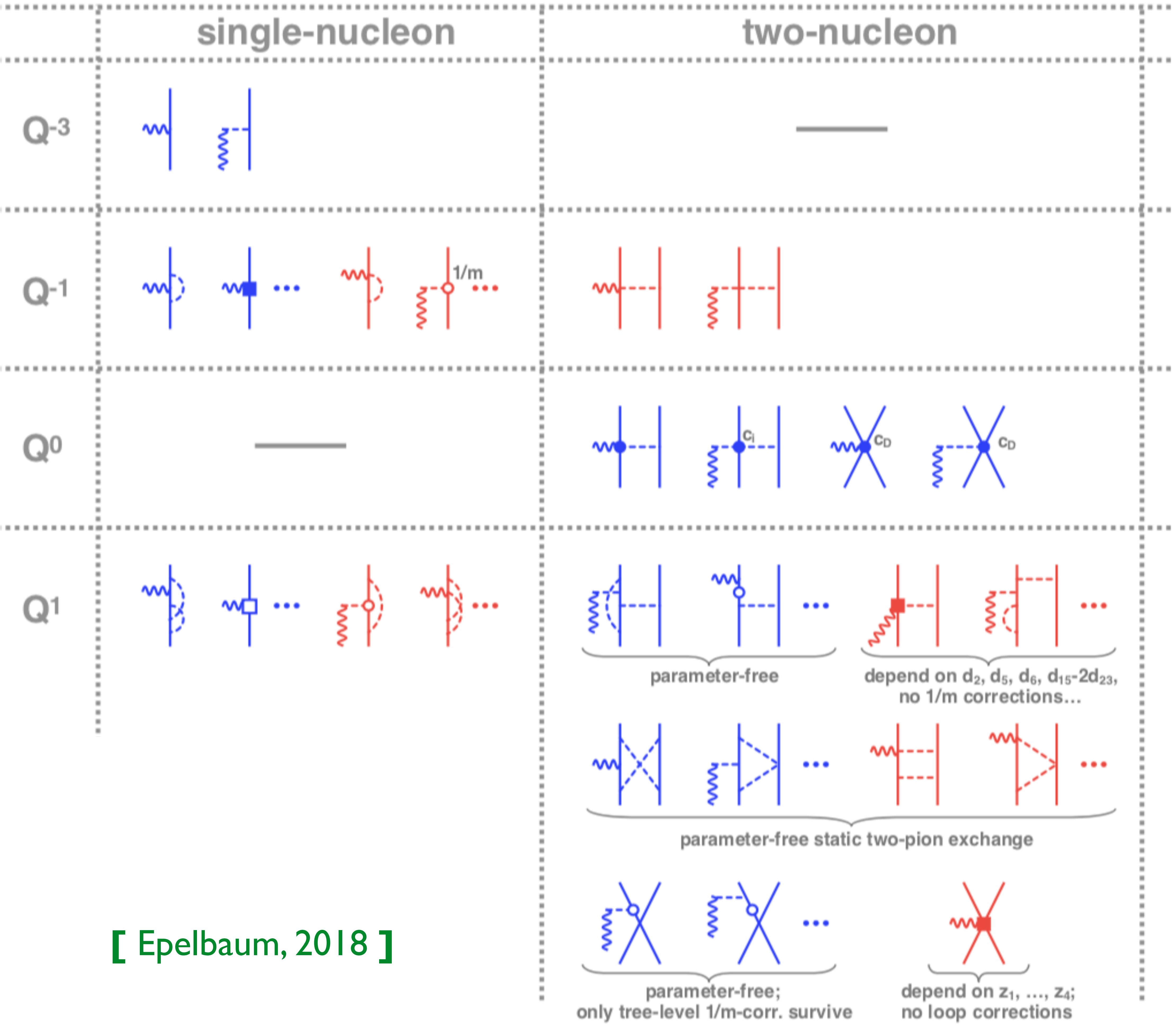
[Petschauer, Kaiser, 2013]

$$\langle \bar{B}\bar{B}\bar{B}BBB \rangle, \langle \bar{B}\bar{B}B\bar{B}BB \rangle, \langle \bar{B}\bar{B}BBB\bar{B} \rangle,$$
$$\langle \bar{B}B\bar{B}B\bar{B}B \rangle, \langle \bar{B}\bar{B}BB \rangle \langle \bar{B}B \rangle, \langle \bar{B}B\bar{B}B \rangle \langle \bar{B}B \rangle,$$
$$\langle \bar{B}\bar{B}\bar{B}B \rangle \langle BB \rangle, \langle \bar{B}\bar{B}\bar{B} \rangle \langle BBB \rangle, \langle \bar{B}\bar{B}B \rangle \langle B\bar{B}B \rangle,$$
$$\langle \bar{B}B \rangle \langle \bar{B}B \rangle \langle \bar{B}B \rangle, \langle \bar{B}\bar{B} \rangle \langle \bar{B}B \rangle \langle BB \rangle,$$

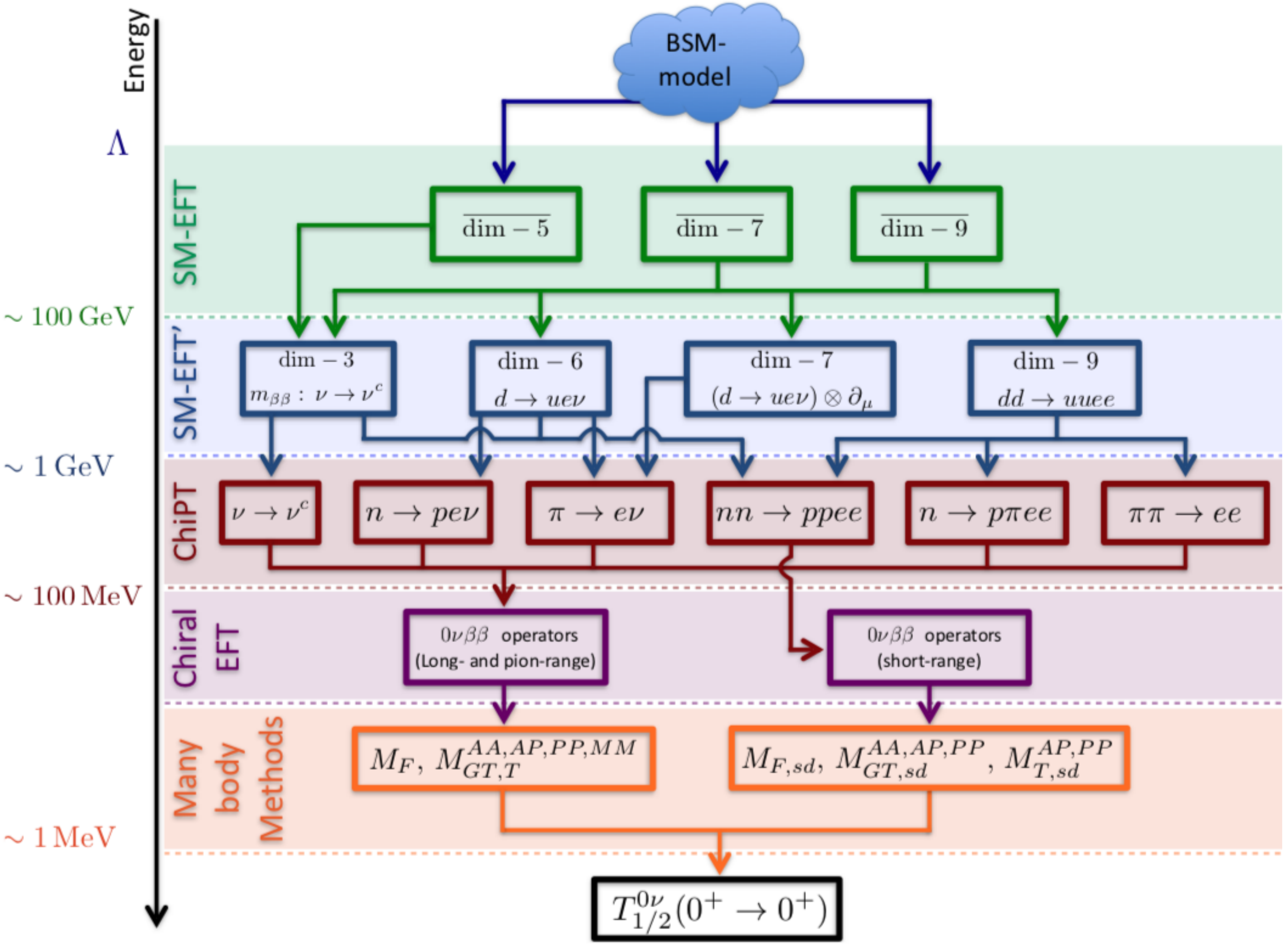
[Sun, Wang, **Yu**, in préparation]

Nuclear weak currents

Explore the nuclear weak currents (EDM, 0vbb, etc) in chiral EFT



[Epelbaum, 2018]



UV Completion of EFT Operators

[Hao-Lin Li, Yu-Han Ni, Ming-Lei Xiao, **J.H.Yu**, 2204.03660]

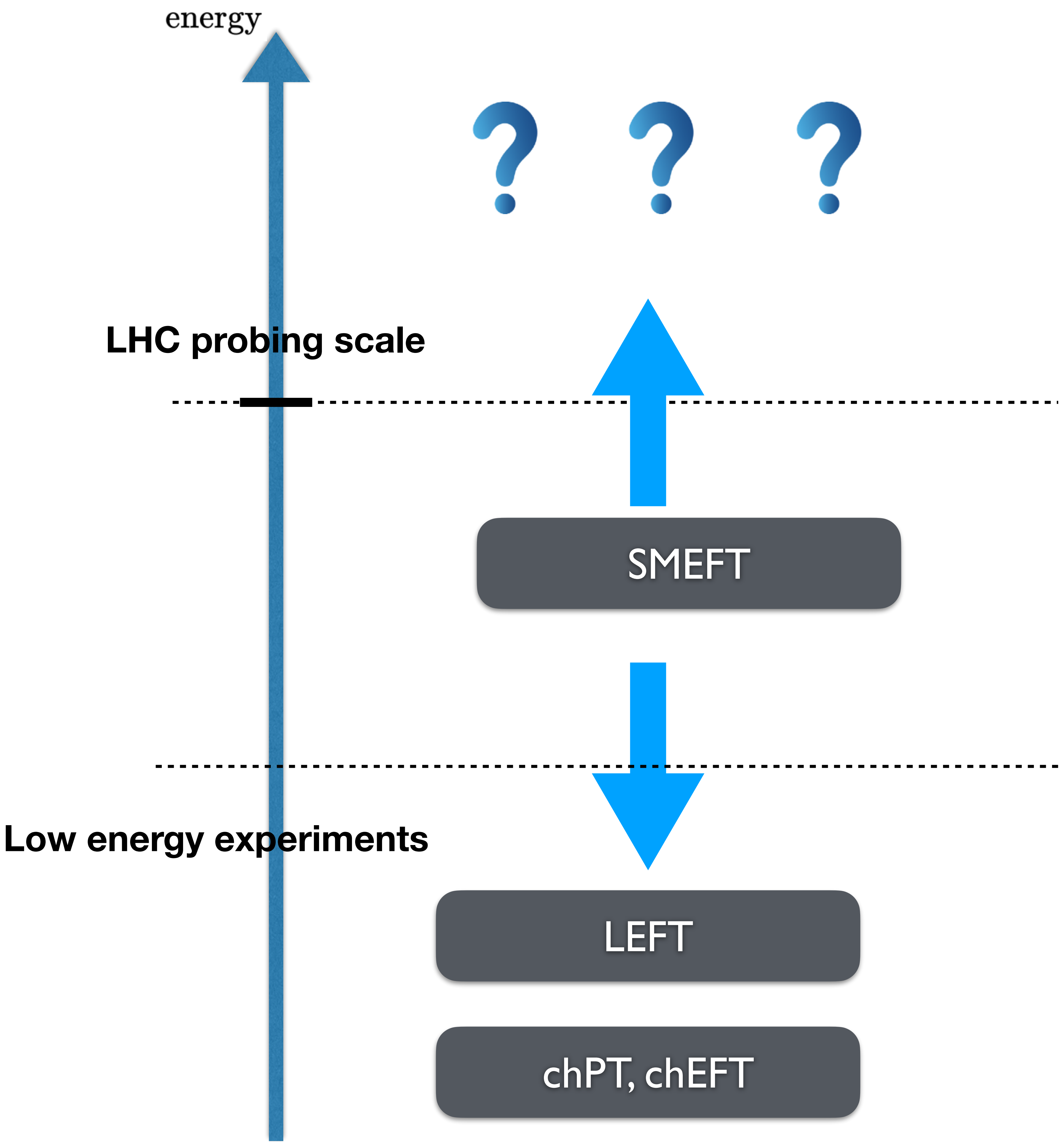
[Xu-Xiang Li, Zhe Ren, **J.H.Yu,** in preparation]

[Hao-Lin Li, Yu-Han Ni, Ming-Lei Xiao, **J.H.Yu**, in preparation]

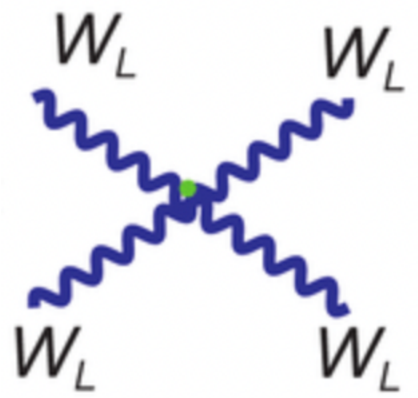
[Gang Li, **J.H.Yu**, Xiang Zhao, in progress]

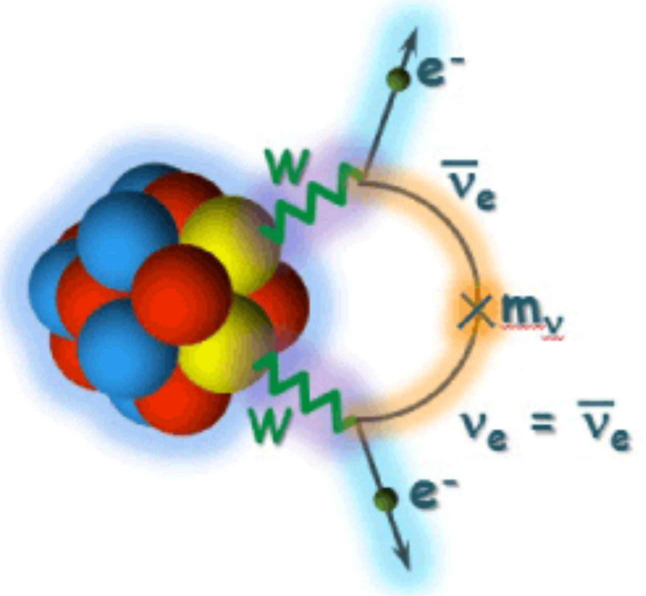
EFT inverse problem

After writing down the effective operators, what is the next step?



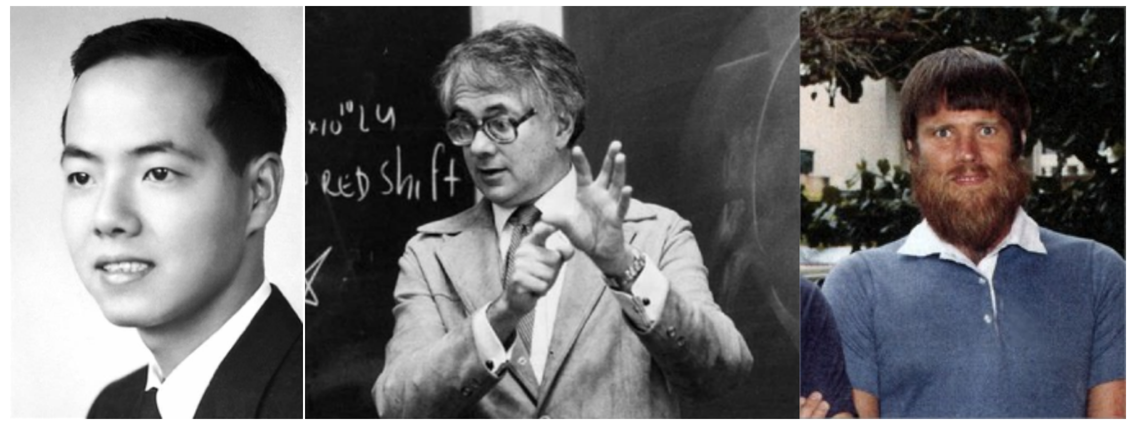
Bad high energy behavior
Of EFT operators





Lesson from four-fermion EFT

Glashow-Weinberg-Salam
1961 1967

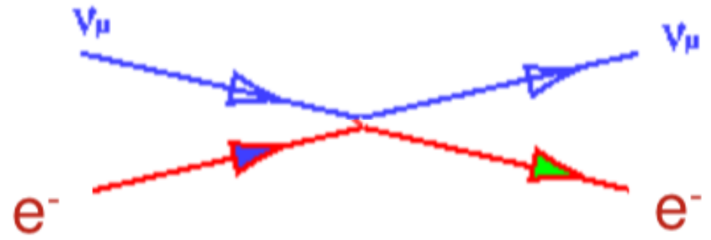


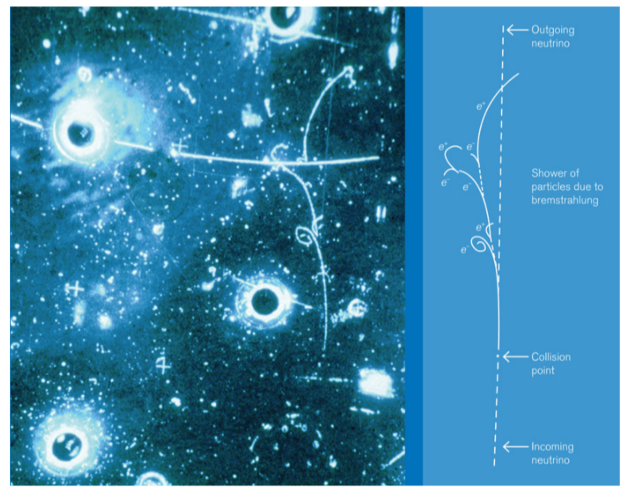
Lee-Georgi-Glashow
1960 1972

Nobel prize
1979

Nobel Prize before W/Z discovery

$\nu_\mu\, e^- \to \nu_\mu\, e^-$





CERN Bubble Chamber
1973

Effective interaction detected!

V-A

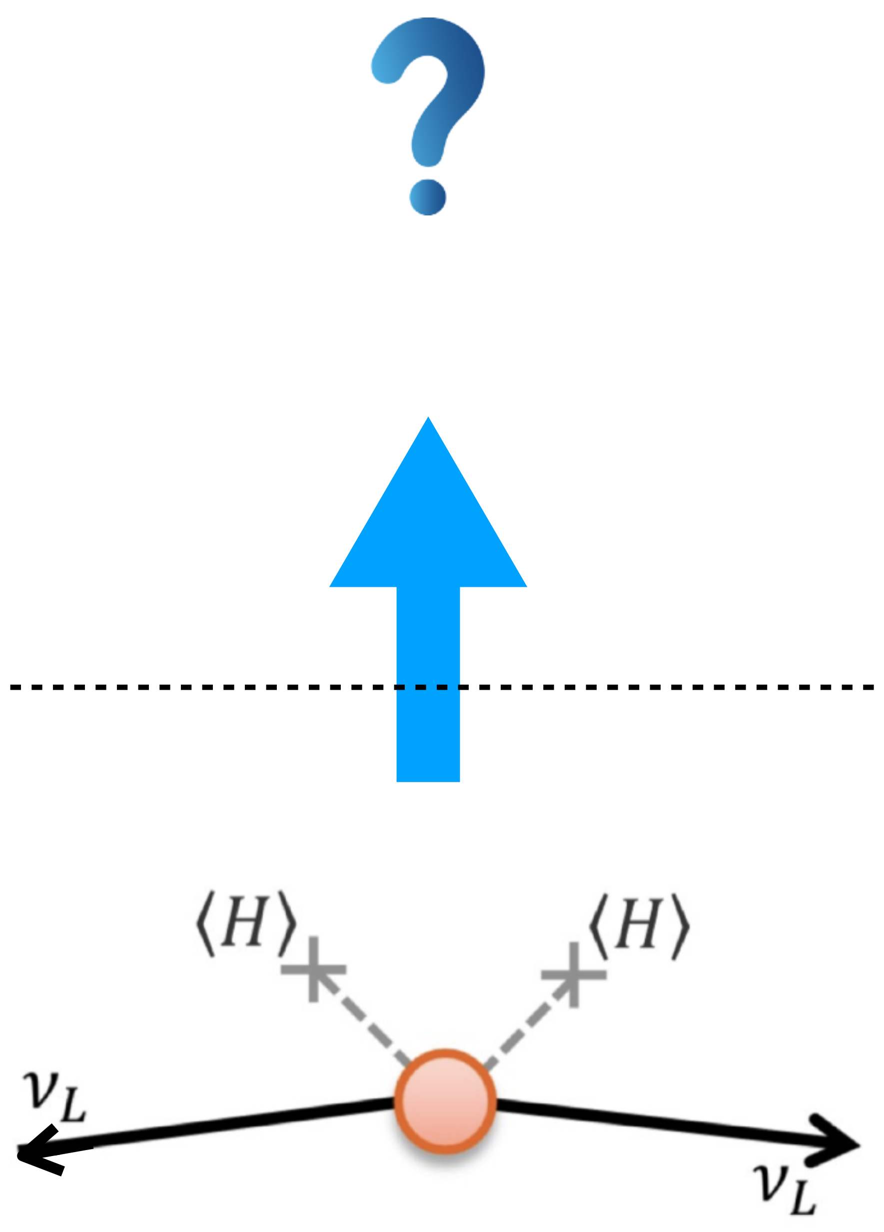
If parity is not conserved in β decay, the most general form of Hamiltonian can be written as

$$\begin{aligned} H_{\text{int}} = & (\psi_p^\dagger\gamma_4\psi_n)(C_S\psi_e^\dagger\gamma_4\psi_\nu + C_S'\psi_e^\dagger\gamma_4\gamma_5\psi_\nu) \\ & +(\psi_p^\dagger\gamma_4\gamma_\mu\psi_n)(C_V\psi_e^\dagger\gamma_4\gamma_\mu\psi_\nu + C_V'\psi_e^\dagger\gamma_4\gamma_\mu\gamma_5\psi_\nu) \\ & +\tfrac{1}{2}(\psi_p^\dagger\gamma_4\sigma_{\lambda\mu}\psi_n)(C_T\psi_e^\dagger\gamma_4\sigma_{\lambda\mu}\psi_\nu \\ & +C_T'\psi_e^\dagger\gamma_4\sigma_{\lambda\mu}\gamma_5\psi_\nu) + (\psi_p^\dagger\gamma_4\gamma_\mu\gamma_5\psi_n) \\ & \times(-C_A\psi_e^\dagger\gamma_4\gamma_\mu\gamma_5\psi_\nu - C_A'\psi_e^\dagger\gamma_4\gamma_\mu\psi_\nu) \\ & +(\psi_p^\dagger\gamma_4\gamma_5\psi_n)(C_P\psi_e^\dagger\gamma_4\gamma_5\psi_\nu + C_P'\psi_e^\dagger\gamma_4\psi_\nu), \qquad \text{(A.1)} \end{aligned}$$

LEFT

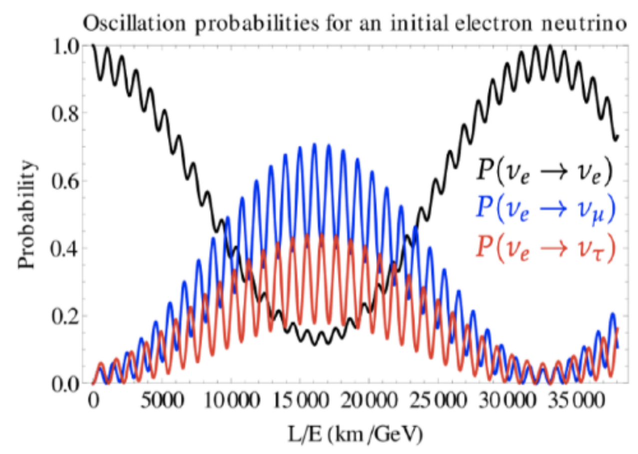
Similar story: neutrino masses

The existence of neutrino masses is the first evidence of new physics beyond standard model



SMEFT

$$\frac{1}{2} m_\nu \, \overline{\nu_L^c} \nu_L$$



LEFT

Seesaw tree UVs

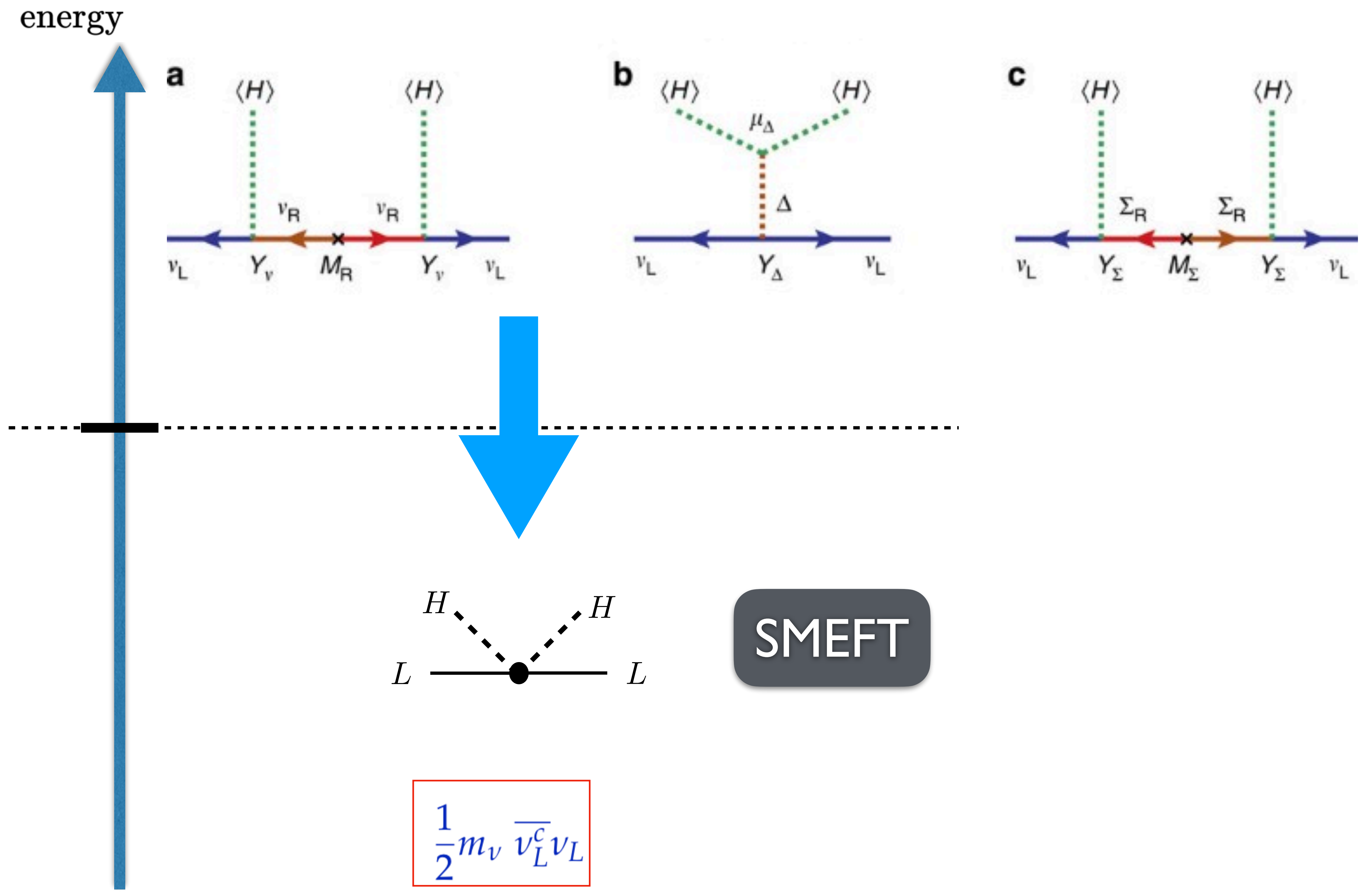
Top-down Approach

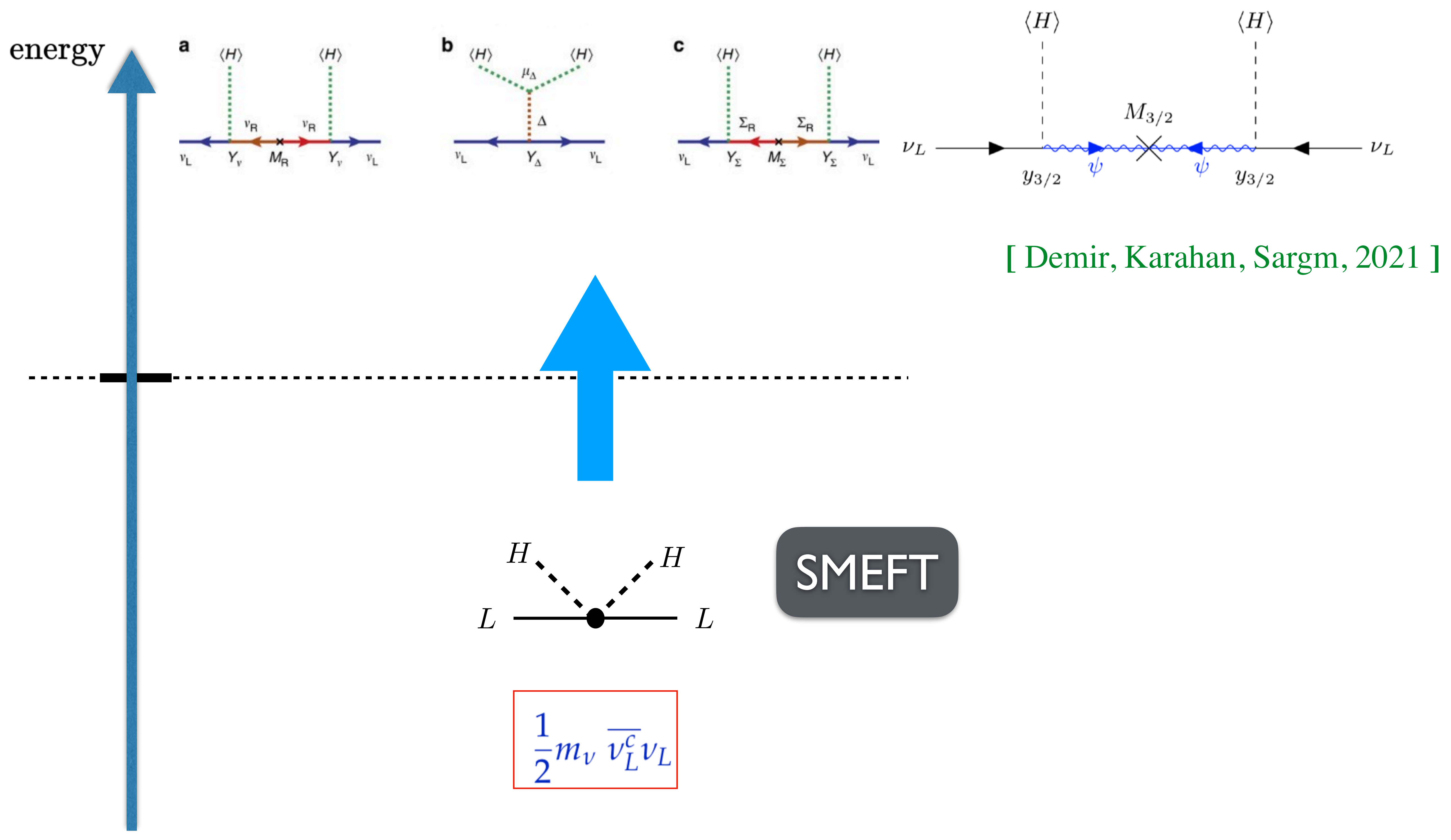
Bottom-up Approach

[Yanagida 1979, Gell-Mann, Ramond, Slansky1979, Mahapatra, Senjanovic, 1980]

[Schecheter, Walle, 1980, Cheng, Li 1980, Magg and Wetterich 1980]

[Foot, Lew, He, Joshi 1989]





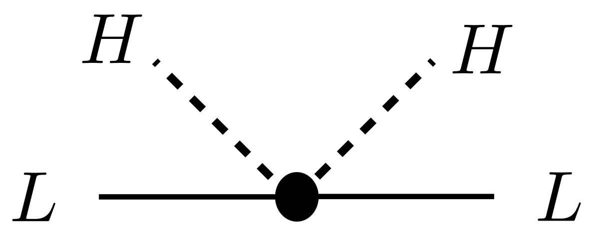
Consider Angular momentum conservation

Pauli-Lubanski Casimir

Weinberg operator as on-shell amplitude

$$\mathcal{O}^S = (HL)(HL)$$

$$\mathcal{B}^y = \langle 12 \rangle$$



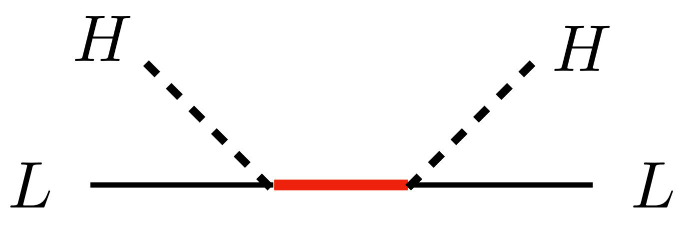
$$\langle p_L, h_L; p_H, h_H | p'_L, h'_L; p'_H, h'_H \rangle$$

[Li, Ni, Xiao, **Yu**, 2204.03660]

Acting on the Pauli-Lubanski Casimir, obtain the eigenvalues on spin!

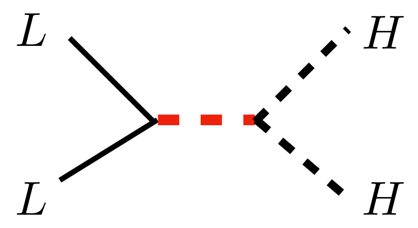
$$\mathbf{W}^2 \langle p_L, h_L; p_H, h_H | P, J, J_z \rangle = -P^2 J(J+1) \langle p_L, h_L; p_H, h_H | P, J, J_z \rangle \quad = -s \sum_J J(J+1) \mathcal{O}^{\mathrm{J}}$$

$$W^2_{\{1,3\}} \mathcal{B}^y = -\frac{3}{4} s_{13} \langle 12 \rangle$$



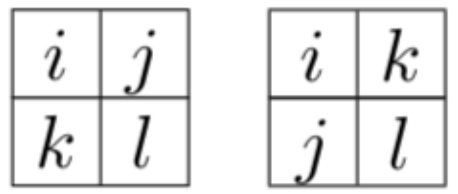
$$J = \frac{1}{2}$$

$$W^2_{\{1,2\}} \mathcal{B}^y = 0$$



$$J = 0$$

Acting on the SU(2) Casimir, obtain the eigenvalues on gauge!



$$\mathcal{B}^R_1 = \epsilon^{ik} \epsilon^{jl}$$
$$\mathcal{B}^R_2 = \epsilon^{ij} \epsilon^{kl}$$

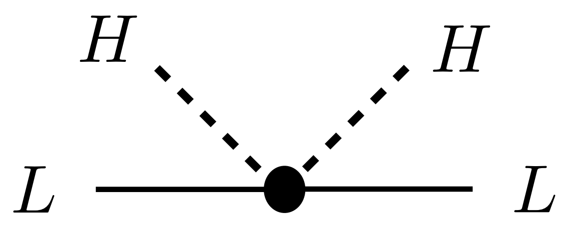
$$\mathbf{C}^2 \mathcal{B}^R = r(r+1) \mathcal{B}^R$$

$$\mathcal{B}^{\mathbf{R}} = \begin{cases} \epsilon^{ik} \epsilon^{jl} & \mathbf{R} = 1 \\ \epsilon^{ik} \epsilon^{jl} - 2\epsilon^{ij} \epsilon^{kl} & \mathbf{R} = 3 \end{cases}$$

Only 3 types of seesaw at dim-5

Generalized partial wave analysis for Poincare/Gauge Casimir

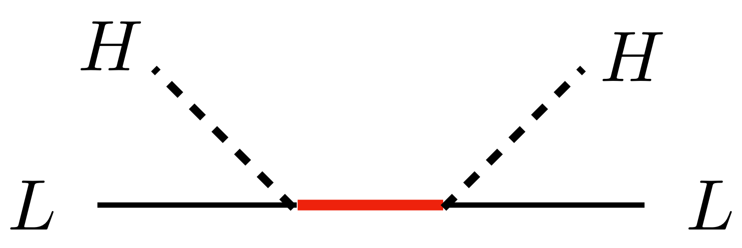
[Li, Ni, Xiao, **Yu**, 2204.03660]



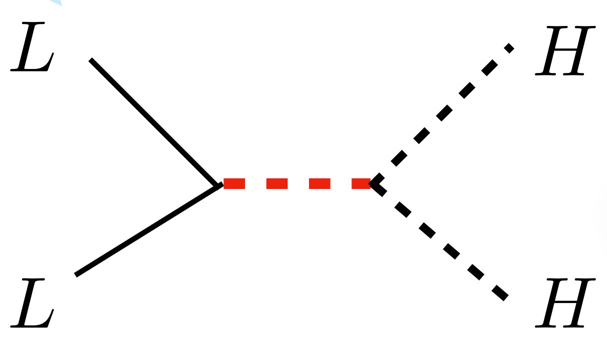
$$\mathbf{W}^2 \mathcal{B}^J = -sJ(J+1)\mathcal{B}^J$$

$W^2_{\{1,3\}}\mathcal{B}^y = -\frac{3}{4}s_{13}\langle 12\rangle$ $\quad LH \to LH$ channel

$LL \to HH$ channel $\quad W^2_{\{1,2\}}\mathcal{B}^y = 0$



$J = \frac{1}{2}$



$J = 0$

Type-I and III: SU(2) **single and triplet**

j-basis	Model
$\mathcal{O}^{(1/2,\mathbf{1})}_{HL\to HL} = \mathcal{O}^S + \mathcal{O}^A$	type I
$\mathcal{O}^{(1/2,\mathbf{3})}_{HL\to HL} = \mathcal{O}^S - 3\mathcal{O}^A$	type III

$\mathcal{O}^S = (HL)(HL), \quad \mathcal{O}^A = (HH)(LL)$

Type-II: SU(2) **triplet**, or singlet (excluded by repeated field)

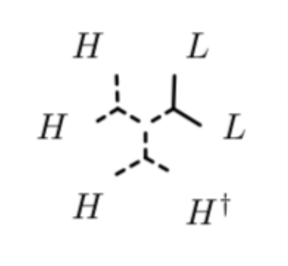
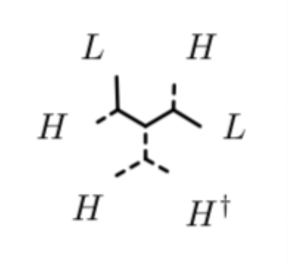
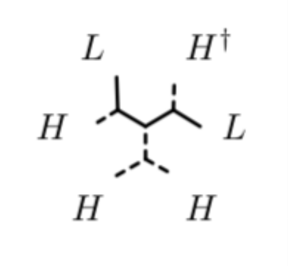
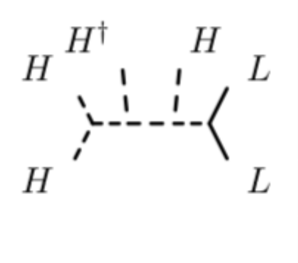
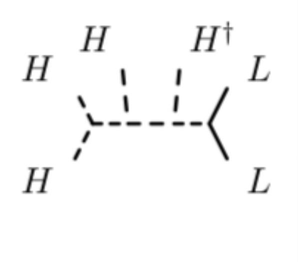
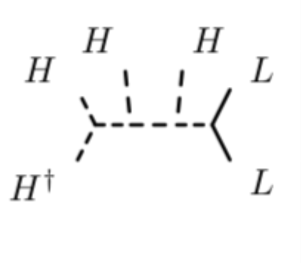
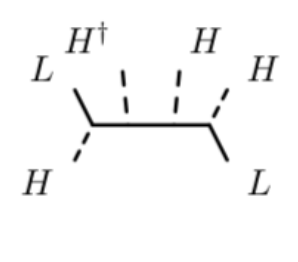
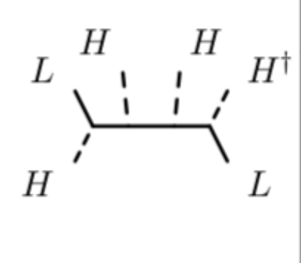
j-basis	Model
$\mathcal{O}^{(0,\mathbf{1})}_{HH\to LL} = \mathcal{O}^A$	N/A
$\mathcal{O}^{(0,\mathbf{3})}_{HH\to LL} = \mathcal{O}^S$	type II

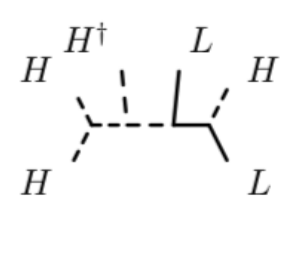
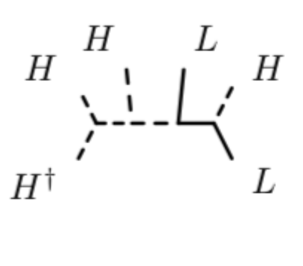
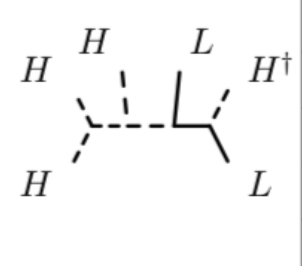
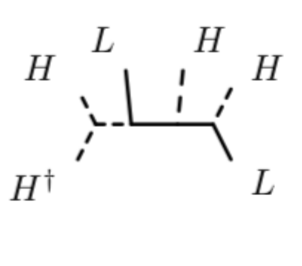
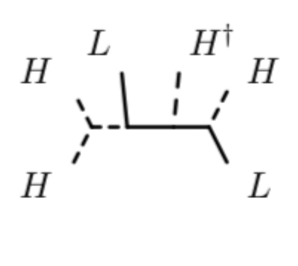
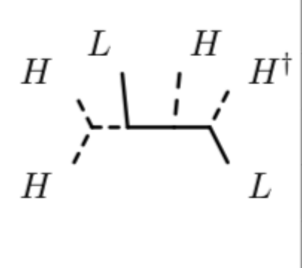
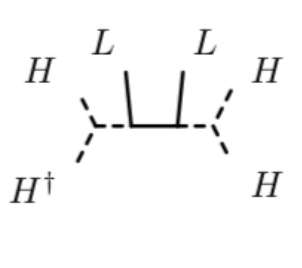
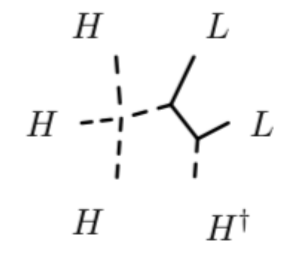
Dim-7 Operators

$$\mathcal{B}^y_{\psi^2\phi^2D^2} = \begin{pmatrix} s_{34}\langle 12\rangle \\ [34]\langle 13\rangle\langle 24\rangle \end{pmatrix} \qquad W^2_{\{1,3\}}\mathcal{B}^y = s_{24}\begin{pmatrix} -\frac{15}{4} & 2 \\ 0 & -\frac{3}{4} \end{pmatrix}\mathcal{B}^y \qquad \Rightarrow \mathcal{B}^j = \begin{cases} 3s_{34}\langle 12\rangle + 2[34]\langle 13\rangle\langle 24\rangle & J=\frac{3}{2} \\ \langle 13\rangle\langle 24\rangle & J=\frac{1}{2} \end{cases}$$

Dim-7 tree-level seesaw

59 UV tree-level models, 19 topologies, one genuine dim-7 seesaw **[Li, Ni, Xiao, Yu, 2204.03660]**

Topology	j-basis	Quantum numbers $\{J, \mathbf{R}, Y\}$
	$\mathcal{O}_{\{12\vert 34\vert 56\},1} = 2\mathcal{O}^p_1 - 4\mathcal{O}^p_2,$	$\{0,3,-1\},\{0,3,1\},\{0,3,0\}$
	$\mathcal{O}_{\{12\vert 34\vert 56\},2} = 2\mathcal{O}^p_2 + 4\mathcal{O}^p_3,$	$\{0,3,-1\},\{0,1,1\},\{0,3,0\}$
	$\mathcal{O}_{\{12\vert 34\vert 56\},3} = 12\mathcal{O}^p_4,$	$\{0,1,-1\},\{0,3,1\},\{0,3,0\}$
	$\mathcal{O}_{\{12\vert 34\vert 56\},4} = 4\mathcal{O}^p_1 + 4\mathcal{O}^p_2,$	$\{0,3,-1\},\{0,3,1\},\{0,1,0\}$
	$\mathcal{O}_{\{12\vert 34\vert 56\},5} = 2\mathcal{O}^p_4 + 4\mathcal{O}^p_5.$	$\{0,1,-1\},\{0,1,1\},\{0,1,0\}$
	$\mathcal{O}_{\{13\vert 24\vert 56\},1} = -\mathcal{O}^p_2 - 2\mathcal{O}^p_3 + 3\mathcal{O}^p_4,$	$\{\frac{1}{2},3,0\},\{\frac{1}{2},3,0\},\{0,3,0\}$
	$\mathcal{O}_{\{13\vert 24\vert 56\},2} = -\mathcal{O}^p_1 + 3\mathcal{O}^p_2 + 2\mathcal{O}^p_3 + 3\mathcal{O}^p_4,$	$\{\frac{1}{2},3,0\},\{\frac{1}{2},1,0\},\{0,3,0\}$
	$\mathcal{O}_{\{13\vert 24\vert 56\},3} = -\mathcal{O}^p_1 + \mathcal{O}^p_2 - 2\mathcal{O}^p_3 - 3\mathcal{O}^p_4,$	$\{\frac{1}{2},1,0\},\{\frac{1}{2},3,0\},\{0,3,0\}$
	$\mathcal{O}_{\{13\vert 24\vert 56\},4} = -\mathcal{O}^p_1 - \mathcal{O}^p_2 + 3\mathcal{O}^p_4 + 6\mathcal{O}^p_5,$	$\{\frac{1}{2},3,0\},\{\frac{1}{2},3,0\},\{0,1,0\}$
	$\mathcal{O}_{\{13\vert 24\vert 56\},5} = \mathcal{O}^p_1 + \mathcal{O}^p_2 + \mathcal{O}^p_4 + 2\mathcal{O}^p_5.$	$\{\frac{1}{2},1,0\},\{\frac{1}{2},1,0\},\{0,1,0\}$
	$\mathcal{O}_{\{16\vert 23\vert 45\},1} = 2\mathcal{O}^p_1 - 2\mathcal{O}^p_2 - 2\mathcal{O}^p_3 + 6\mathcal{O}^p_4 + 6\mathcal{O}^p_5,$	$\{\frac{1}{2},3,-1\},\{\frac{1}{2},3,0\},\{0,3,1\}$
	$\mathcal{O}_{\{16\vert 23\vert 45\},2} = -2\mathcal{O}^p_1 - \mathcal{O}^p_2 - \mathcal{O}^p_3 + 3\mathcal{O}^p_4 + 3\mathcal{O}^p_5,$	$\{\frac{1}{2},3,-1\},\{\frac{1}{2},1,0\},\{0,3,1\}$
	$\mathcal{O}_{\{16\vert 23\vert 45\},3} = 3\mathcal{O}^p_2 + 3\mathcal{O}^p_3 + 3\mathcal{O}^p_4 + 3\mathcal{O}^p_5,$	$\{\frac{1}{2},1,-1\},\{\frac{1}{2},3,0\},\{0,3,1\}$
	$\mathcal{O}_{\{16\vert 23\vert 45\},4} = \mathcal{O}^p_2 - \mathcal{O}^p_3 - 3\mathcal{O}^p_4 + 3\mathcal{O}^p_5,$	$\{\frac{1}{2},3,-1\},\{\frac{1}{2},3,0\},\{0,1,1\}$
	$\mathcal{O}_{\{16\vert 23\vert 45\},5} = \mathcal{O}^p_2 - \mathcal{O}^p_3 + \mathcal{O}^p_4 - \mathcal{O}^p_5.$	$\{\frac{1}{2},1,-1\},\{\frac{1}{2},1,0\},\{0,1,1\}$
	$\mathcal{O}_{\{12\vert 125\vert 34\},1} = \mathcal{O}^p_1 + 4\mathcal{O}^p_2,$	$\{0,3,-1\},\{0,4,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{12\vert 125\vert 34\},2} = -8\mathcal{O}^p_1 + 4\mathcal{O}^p_2,$	$\{0,3,-1\},\{0,2,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{12\vert 125\vert 34\},3} = -12\mathcal{O}^p_4,$	$\{0,1,-1\},\{0,2,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{12\vert 125\vert 34\},4} = -2\mathcal{O}^p_2 - 4\mathcal{O}^p_3,$	$\{0,3,-1\},\{0,2,-\frac{1}{2}\},\{0,1,1\}$
	$\mathcal{O}_{\{12\vert 125\vert 34\},5} = -2\mathcal{O}^p_4 - 4\mathcal{O}^p_5.$	$\{0,1,-1\},\{0,2,-\frac{1}{2}\},\{0,1,1\}$
	$\mathcal{O}_{\{12\vert 126\vert 34\},1} = 3\mathcal{O}^p_1,$	$\{0,3,-1\},\{0,4,-\frac{3}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{12\vert 126\vert 34\},2} = 12\mathcal{O}^p_2,$	$\{0,3,-1\},\{0,2,-\frac{3}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{12\vert 126\vert 34\},3} = -12\mathcal{O}^p_4,$	$\{0,1,-1\},\{0,2,-\frac{3}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{12\vert 126\vert 34\},4} = -2\mathcal{O}^p_2 - 4\mathcal{O}^p_3,$	$\{0,3,-1\},\{0,2,-\frac{3}{2}\},\{0,1,1\}$
	$\mathcal{O}_{\{12\vert 126\vert 34\},5} = -2\mathcal{O}^p_4 - 4\mathcal{O}^p_5.$	$\{0,1,-1\},\{0,2,-\frac{3}{2}\},\{0,1,1\}$
	$\mathcal{O}_{\{12\vert 124\vert 36\},1} = -\mathcal{O}^p_1 - 4\mathcal{O}^p_3,$	$\{0,3,-1\},\{0,4,-\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{12\vert 124\vert 36\},2} = 2\mathcal{O}^p_1 + 6\mathcal{O}^p_2 + 2\mathcal{O}^p_3,$	$\{0,3,-1\},\{0,2,-\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{12\vert 124\vert 36\},3} = -6\mathcal{O}^p_4 - 6\mathcal{O}^p_5,$	$\{0,1,-1\},\{0,2,-\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{12\vert 124\vert 36\},4} = -2\mathcal{O}^p_1 + 2\mathcal{O}^p_2 + 2\mathcal{O}^p_3,$	$\{0,3,-1\},\{0,2,-\frac{1}{2}\},\{0,1,0\}$
	$\mathcal{O}_{\{12\vert 124\vert 36\},5} = -2\mathcal{O}^p_4 + 2\mathcal{O}^p_5.$	$\{0,1,-1\},\{0,2,-\frac{1}{2}\},\{0,1,0\}$
	$\mathcal{O}_{\{13\vert 135\vert 24\},1} = \mathcal{O}^p_1 - 2\mathcal{O}^p_3 - 6\mathcal{O}^p_5,$	$\{\frac{1}{2},3,0\},\{\frac{1}{2},4,\frac{1}{2}\},\{\frac{1}{2},3,0\}$
	$\mathcal{O}_{\{13\vert 135\vert 24\},2} = -\mathcal{O}^p_1 - 3\mathcal{O}^p_2 - 4\mathcal{O}^p_3 + 9\mathcal{O}^p_4 + 6\mathcal{O}^p_5,$	$\{\frac{1}{2},3,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{\frac{1}{2},3,0\}$
	$\mathcal{O}_{\{13\vert 135\vert 24\},3} = \mathcal{O}^p_1 - \mathcal{O}^p_2 + 2\mathcal{O}^p_3 + 3\mathcal{O}^p_4,$	$\{\frac{1}{2},1,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{\frac{1}{2},3,0\}$
	$\mathcal{O}_{\{13\vert 135\vert 24\},4} = \mathcal{O}^p_1 - 3\mathcal{O}^p_2 - 2\mathcal{O}^p_3 - 3\mathcal{O}^p_4,$	$\{\frac{1}{2},3,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{\frac{1}{2},1,0\}$
	$\mathcal{O}_{\{13\vert 135\vert 24\},5} = \mathcal{O}^p_1 + \mathcal{O}^p_2 + \mathcal{O}^p_4 + 2\mathcal{O}^p_5.$	$\{\frac{1}{2},1,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{\frac{1}{2},1,0\}$
	$\mathcal{O}_{\{16\vert 146\vert 23\},1} = -2\mathcal{O}^p_1 + 4\mathcal{O}^p_3 - 12\mathcal{O}^p_5,$	$\{\frac{1}{2},3,-1\},\{\frac{1}{2},4,-\frac{1}{2}\},\{\frac{1}{2},3,0\}$
	$\mathcal{O}_{\{16\vert 146\vert 23\},2} = 2\mathcal{O}^p_1 - 3\mathcal{O}^p_2 - \mathcal{O}^p_3 + 9\mathcal{O}^p_4 + 3\mathcal{O}^p_5,$	$\{\frac{1}{2},3,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{\frac{1}{2},3,0\}$
	$\mathcal{O}_{\{16\vert 146\vert 23\},3} = 3\mathcal{O}^p_2 + 3\mathcal{O}^p_3 + 3\mathcal{O}^p_4 + 3\mathcal{O}^p_5,$	$\{\frac{1}{2},1,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{\frac{1}{2},3,0\}$
	$\mathcal{O}_{\{16\vert 146\vert 23\},4} = 2\mathcal{O}^p_1 + \mathcal{O}^p_2 + \mathcal{O}^p_3 - 3\mathcal{O}^p_4 - 3\mathcal{O}^p_5,$	$\{\frac{1}{2},3,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{\frac{1}{2},1,0\}$
	$\mathcal{O}_{\{16\vert 146\vert 23\},5} = -\mathcal{O}^p_2 + \mathcal{O}^p_3 - \mathcal{O}^p_4 + \mathcal{O}^p_5,$	$\{\frac{1}{2},1,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{\frac{1}{2},1,0\}$

	$\mathcal{O}_{\{13\vert 123\vert 45\},1} = \mathcal{O}^p_1 - 4\mathcal{O}^p_2 - 4\mathcal{O}^p_3,$	$\{\frac{1}{2},3,0\},\{0,4,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{13\vert 123\vert 45\},2} = 2\mathcal{O}^p_1 + \mathcal{O}^p_2 + \mathcal{O}^p_3 - 9\mathcal{O}^p_4 - 9\mathcal{O}^p_5,$	$\{\frac{1}{2},3,0\},\{0,2,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{13\vert 123\vert 45\},3} = 2\mathcal{O}^p_1 + \mathcal{O}^p_2 + \mathcal{O}^p_3 + 3\mathcal{O}^p_4 + 3\mathcal{O}^p_5,$	$\{\frac{1}{2},1,0\},\{0,2,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{13\vert 123\vert 45\},4} = \mathcal{O}^p_2 - \mathcal{O}^p_3 + 3\mathcal{O}^p_4 - 3\mathcal{O}^p_5,$	$\{\frac{1}{2},3,0\},\{0,2,-\frac{1}{2}\},\{0,1,1\}$
	$\mathcal{O}_{\{13\vert 123\vert 45\},5} = -\mathcal{O}^p_2 + \mathcal{O}^p_3 + \mathcal{O}^p_4 - \mathcal{O}^p_5.$	$\{\frac{1}{2},1,0\},\{0,2,-\frac{1}{2}\},\{0,1,1\}$
	$\mathcal{O}_{\{13\vert 123\vert 46\},1} = \mathcal{O}^p_1 - 4\mathcal{O}^p_2 - 4\mathcal{O}^p_3,$	$\{\frac{1}{2},3,0\},\{0,4,-\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{13\vert 123\vert 46\},2} = \mathcal{O}^p_1 + 2\mathcal{O}^p_2 - \mathcal{O}^p_3 - 9\mathcal{O}^p_5,$	$\{\frac{1}{2},3,0\},\{0,2,-\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{13\vert 123\vert 46\},3} = -\mathcal{O}^p_1 - 2\mathcal{O}^p_2 + \mathcal{O}^p_3 - 3\mathcal{O}^p_5,$	$\{\frac{1}{2},1,0\},\{0,2,-\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{13\vert 123\vert 46\},4} = -\mathcal{O}^p_1 - \mathcal{O}^p_3 + 6\mathcal{O}^p_4 + 3\mathcal{O}^p_5,$	$\{\frac{1}{2},3,0\},\{0,4,-\frac{1}{2}\},\{0,1,0\}$
	$\mathcal{O}_{\{13\vert 123\vert 46\},5} = -\mathcal{O}^p_1 - \mathcal{O}^p_3 - 2\mathcal{O}^p_4 - \mathcal{O}^p_5.$	$\{\frac{1}{2},1,0\},\{0,2,-\frac{1}{2}\},\{0,1,0\}$
	$\mathcal{O}_{\{16\vert 126\vert 34\},1} = 6\mathcal{O}^p_1,$	$\{\frac{1}{2},3,-1\},\{0,4,-\frac{3}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{16\vert 126\vert 34\},2} = -3\mathcal{O}^p_2 + 9\mathcal{O}^p_4,$	$\{\frac{1}{2},3,-1\},\{0,4,-\frac{3}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{16\vert 126\vert 34\},3} = -3\mathcal{O}^p_2 - 3\mathcal{O}^p_4,$	$\{\frac{1}{2},3,-1\},\{0,4,-\frac{3}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{16\vert 126\vert 34\},4} = -\mathcal{O}^p_2 - 2\mathcal{O}^p_3 + 3\mathcal{O}^p_4 + 6\mathcal{O}^p_5,$	$\{\frac{1}{2},3,-1\},\{0,4,-\frac{3}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{16\vert 126\vert 34\},5} = \mathcal{O}^p_2 + 2\mathcal{O}^p_3 + \mathcal{O}^p_4 + 2\mathcal{O}^p_5.$	$\{\frac{1}{2},3,-1\},\{0,4,-\frac{3}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{23\vert 235\vert 46\},1} = \mathcal{O}^p_1 - 2\mathcal{O}^p_3 + 6\mathcal{O}^p_5,$	$\{\frac{1}{2},3,0\},\{\frac{1}{2},4,\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{23\vert 235\vert 46\},2} = \mathcal{O}^p_1 - 6\mathcal{O}^p_2 - 5\mathcal{O}^p_3 - 3\mathcal{O}^p_5,$	$\{\frac{1}{2},3,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{23\vert 235\vert 46\},3} = \mathcal{O}^p_1 + 2\mathcal{O}^p_2 - \mathcal{O}^p_3 - 3\mathcal{O}^p_5,$	$\{\frac{1}{2},1,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{23\vert 235\vert 46\},4} = -\mathcal{O}^p_1 - \mathcal{O}^p_3 - 6\mathcal{O}^p_4 - 3\mathcal{O}^p_5,$	$\{\frac{1}{2},3,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,1,0\}$
	$\mathcal{O}_{\{23\vert 235\vert 46\},5} = -\mathcal{O}^p_1 - \mathcal{O}^p_3 + 2\mathcal{O}^p_4 + \mathcal{O}^p_5.$	$\{\frac{1}{2},1,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,1,0\}$
	$\mathcal{O}_{\{13\vert 136\vert 45\},1} = \mathcal{O}^p_1 + 2\mathcal{O}^p_2 + 2\mathcal{O}^p_3 - 6\mathcal{O}^p_4 - 6\mathcal{O}^p_5,$	$\{\frac{1}{2},3,0\},\{\frac{1}{2},4,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{13\vert 136\vert 45\},2} = 2\mathcal{O}^p_1 - 5\mathcal{O}^p_2 - 5\mathcal{O}^p_3 - 3\mathcal{O}^p_4 - 3\mathcal{O}^p_5,$	$\{\frac{1}{2},3,0\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{13\vert 136\vert 45\},3} = 2\mathcal{O}^p_1 + \mathcal{O}^p_2 + \mathcal{O}^p_3 + 3\mathcal{O}^p_4 + 3\mathcal{O}^p_5,$	$\{\frac{1}{2},1,0\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{13\vert 136\vert 45\},4} = \mathcal{O}^p_2 - \mathcal{O}^p_3 + 3\mathcal{O}^p_4 - 3\mathcal{O}^p_5,$	$\{\frac{1}{2},3,0\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,1,1\}$
	$\mathcal{O}_{\{13\vert 136\vert 45\},5} = -\mathcal{O}^p_2 + \mathcal{O}^p_3 + \mathcal{O}^p_4 - \mathcal{O}^p_5.$	$\{\frac{1}{2},1,0\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,1,1\}$
	$\mathcal{O}_{\{16\vert 156\vert 34\},1} = 2\mathcal{O}^p_1 - 4\mathcal{O}^p_2 + 12\mathcal{O}^p_4,$	$\{\frac{1}{2},3,-1\},\{\frac{1}{2},4,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{16\vert 156\vert 34\},2} = 4\mathcal{O}^p_1 + \mathcal{O}^p_2 - 3\mathcal{O}^p_4,$	$\{\frac{1}{2},3,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{16\vert 156\vert 34\},3} = -3\mathcal{O}^p_2 - 3\mathcal{O}^p_4,$	$\{\frac{1}{2},1,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{16\vert 156\vert 34\},4} = -\mathcal{O}^p_2 - 2\mathcal{O}^p_3 + 3\mathcal{O}^p_4 + 6\mathcal{O}^p_5,$	$\{\frac{1}{2},3,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,1,1\}$
	$\mathcal{O}_{\{16\vert 156\vert 34\},5} = \mathcal{O}^p_2 + 2\mathcal{O}^p_3 + \mathcal{O}^p_4 + 2\mathcal{O}^p_5.$	$\{\frac{1}{2},1,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,1,1\}$
	$\mathcal{O}_{\{34\vert 134\vert 56\},1} = -\mathcal{O}^p_1 + 2\mathcal{O}^p_2 + 6\mathcal{O}^p_4,$	$\{0,3,1\},\{\frac{1}{2},4,\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{34\vert 134\vert 56\},2} = 4\mathcal{O}^p_1 - 8\mathcal{O}^p_2 + 12\mathcal{O}^p_4,$	$\{0,3,1\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{34\vert 134\vert 56\},3} = 2\mathcal{O}^p_2 + 4\mathcal{O}^p_3,$	$\{0,1,1\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{34\vert 134\vert 56\},4} = -4\mathcal{O}^p_1 - 4\mathcal{O}^p_2,$	$\{0,3,1\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,1,0\}$
	$\mathcal{O}_{\{34\vert 134\vert 56\},5} = 2\mathcal{O}^p_4 + 4\mathcal{O}^p_5.$	$\{0,1,1\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,1,0\}$
	$\mathcal{O}_{\{16\vert 126\},1} = 6\mathcal{O}^p_1,$	$\{\frac{1}{2},3,-1\},\{0,4,-\frac{3}{2}\}$
	$\mathcal{O}_{\{16\vert 126\},2,3} = \begin{pmatrix} \mathcal{O}^p_2 - \mathcal{O}^p_3 - 3\mathcal{O}^p_4 + 3\mathcal{O}^p_5 \\ -2\mathcal{O}^p_2 - \mathcal{O}^p_3 + 6\mathcal{O}^p_4 + 3\mathcal{O}^p_5 \end{pmatrix},$	$\{\frac{1}{2},3,-1\},\{0,2,-\frac{3}{2}\}$
	$\mathcal{O}_{\{16\vert 126\},4,5} = \begin{pmatrix} -\mathcal{O}^p_2 - 2\mathcal{O}^p_3 - \mathcal{O}^p_4 - 2\mathcal{O}^p_5 \\ -2\mathcal{O}^p_2 - \mathcal{O}^p_3 - 2\mathcal{O}^p_4 - \mathcal{O}^p_5 \end{pmatrix}.$	$\{\frac{1}{2},1,-1\},\{0,2,-\frac{3}{2}\}$

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[Bonnet, Hernandez, Ota, Winter, 2009]

Complete dim-7 tree UVs

[Li, Ni, Xiao, **Yu**, 2204.03660]

$(\mathrm{SU}(3)_c, \mathrm{SU}(2)_2, \mathrm{U}(1)_y)$	Scalar
$S1\ (\mathbf{1},\mathbf{1},0)$	$H^3H^\dagger L^2[(S6),(F5),(F1),(S4,S6),$ $(S4,F5),(S4,F1),(F3,F5),(F1,F3),(S6,F3)]$
$S2\ (\mathbf{1},\mathbf{1},1)$	$e_{\mathbb{C}}HL^3[(S4),(F4),(F1)]$ $d_{\mathbb{C}}HL^2Q[(S4),(F10),(F9)]$ $HL^2Q^\dagger u_{\mathbb{C}}^\dagger[(S4),(F8),(F12)]$ $De_{\mathbb{C}}H^{\dagger 3}L^\dagger[(F1),(F3),(V3)]$
$S4\ \left(\mathbf{1},\mathbf{2},\frac{1}{2}\right)$	$e_{\mathbb{C}}HL^3[(S6),(S2),(F5),(F1)]$ $d_{\mathbb{C}}HL^2Q[(S6),(S2),(F5),(F1)]$ $HL^2Q^\dagger u_{\mathbb{C}}^\dagger[(S6),(S2),(F5),(F1)]$ $H^3H^\dagger L^2[(S6),(F5),(F1),(S5,S6),(S1,S6),$ $(S6,F5),(S6,F1),(S5,F5),(S5,F1),(S1,F5),(S1,F1)]$
$S5\ (\mathbf{1},\mathbf{3},0)$	$H^3H^\dagger L^2[(S6),(F1,F5),(S6,S7),(S4,S6),(S7,F5),$ $(S4,F5),(S4,F1),(F5,F7),(F3,F5),(F1,F3),(S6,F7),(S6,F3)]$
$S6\ (\mathbf{1},\mathbf{3},1)$	$D^2H^2L^2$ $e_{\mathbb{C}}HL^3[(S4),(F4),(F5)]$ $d_{\mathbb{C}}HL^2Q[(S4),(F10),(F14)]$ $HL^2Q^\dagger u_{\mathbb{C}}^\dagger[(S4),(F13),(F12)]$ $De_{\mathbb{C}}H^{\dagger 3}L^\dagger[(F5),(F3),(V3)]$ $H^2L^2W_L[(F7)]$ $H^3H^\dagger L^2[(S4),(S8),(S7),(S5),(S1),(F5,F6),(F1,F6),$ $(S5,S7),(S4,S5),(S1,S4),(S7,F5),(S4,F5),(S4,F1),(F5,F7),(F3,F5),$ $(F1,F3),(S8,F6),(F6,F7),(F3,F6),(S5,F7),(S5,F3),(S1,F3)]$ $H^2L^2W_L$ $B_LH^2L^2$ $e_{\mathbb{C}}HL^3$ $HL^2Q^\dagger u_{\mathbb{C}}^\dagger$ $d_{\mathbb{C}}HL^2Q$ $De_{\mathbb{C}}^\dagger H^3L$
$S7\ \left(\mathbf{1},\mathbf{4},\frac{1}{2}\right)$	$H^3H^\dagger L^2[(S6),(F5),(S5,S6),(S6,F5),(S5,F5)]$
$S8\ \left(\mathbf{1},\mathbf{4},\frac{3}{2}\right)$	$H^3H^\dagger L^2[(S6),(F6),(S6,F6)]$
$S10\ \left(\mathbf{3},\mathbf{1},-\frac{1}{3}\right)$	${d_{\mathbb{C}}}^2HLu_{\mathbb{C}}[(S12),(F10),(F1)]$ $d_{\mathbb{C}}HL^2Q[(S12),(F10),(F1)]$ $d_{\mathbb{C}}e_{\mathbb{C}}^\dagger HLu_{\mathbb{C}}^\dagger[(S12),(F10),(F1)]$ $d_{\mathbb{C}}HLQ^{\dagger 2}[(S12),(F10),(F1)]$
$S11\ \left(\mathbf{3},\mathbf{1},\frac{2}{3}\right)$	${d_{\mathbb{C}}}^3H^\dagger L[(S12),(F11),(F2)]$ ${d_{\mathbb{C}}}^2HLu_{\mathbb{C}}[(F11),(S13),(F1)]$ ${d_{\mathbb{C}}}^2e_{\mathbb{C}}^\dagger HQ^\dagger[(S13),(F3),(F8)]$
$S12\ \left(\mathbf{3},\mathbf{2},\frac{1}{6}\right)$	${d_{\mathbb{C}}}^3H^\dagger L[(S11),(F11)]$ ${d_{\mathbb{C}}}^2HLu_{\mathbb{C}}[(F11),(S10),(F10)]$ $d_{\mathbb{C}}HL^2Q[(S10),(S14),(F5),(F1),(F14),(F9)]$ $d_{\mathbb{C}}e_{\mathbb{C}}^\dagger HLu_{\mathbb{C}}^\dagger[(S10),(F3),(F12)]$
$S13\ \left(\mathbf{3},\mathbf{2},\frac{7}{6}\right)$	${d_{\mathbb{C}}}^2HLu_{\mathbb{C}}[(S11),(F10)]$ ${d_{\mathbb{C}}}^2e_{\mathbb{C}}^\dagger HQ^\dagger[(S11),(F10)]$
$S14\ \left(\mathbf{3},\mathbf{3},-\frac{1}{3}\right)$	$d_{\mathbb{C}}HL^2Q[(S12),(F10),(F5)]$ $d_{\mathbb{C}}HLQ^{\dagger 2}[(S12),(F10),(F5)]$

$(\mathrm{SU}(3)_c, \mathrm{SU}(2)_2, \mathrm{U}(1)_y)$	Fermion
$F1\ (\mathbf{1},\mathbf{1},0)$	$D^2H^2L^2$ $e_{\mathbb{C}}HL^3[(S4),(S2)]$ $d_{\mathbb{C}}HL^2Q[(S4),(S10),(S12)]$ $HL^2Q^\dagger u_{\mathbb{C}}^\dagger[(S4),(V5),(V8)]$ $De_{\mathbb{C}}H^{\dagger 3}L^\dagger[(F3),(V2)]$ ${d_{\mathbb{C}}}^2HLu_{\mathbb{C}}[(S11),(S10)]$ $d_{\mathbb{C}}e_{\mathbb{C}}^\dagger HLu_{\mathbb{C}}^\dagger[(S10),(V5)]$ $d_{\mathbb{C}}HLQ^{\dagger 2}[(S10),(V8)]$ $H^2L^2W_L[(F5)]$ $H^3H^\dagger L^2[(S4),(S5,F5),(S1),(S6,F6),(F3,F5),(F3),(F3,F6),$ $(S4,S6),(S6,F3),(S4,S5),(S1,S4),(S5,F3),(S1,F3)]$ $H^2L^2W_L$ $B_LH^2L^2$ $e_{\mathbb{C}}HL^3$ $HL^2Q^\dagger u_{\mathbb{C}}^\dagger$ $d_{\mathbb{C}}HL^2Q$ $De_{\mathbb{C}}^\dagger H^3L$
$F2\ (\mathbf{1},\mathbf{1},1)$	${d_{\mathbb{C}}}^3H^\dagger L[(S11)]$
$F3\ \left(\mathbf{1},\mathbf{2},\frac{1}{2}\right)$	$De_{\mathbb{C}}H^{\dagger 3}L^\dagger[(F5),(F1),(S6),(V2)]$ $d_{\mathbb{C}}e_{\mathbb{C}}^\dagger HLu_{\mathbb{C}}^\dagger[(S12),(V8)]$ ${d_{\mathbb{C}}}^2e_{\mathbb{C}}^\dagger HQ^\dagger[(V8),(S11)]$ $H^3H^\dagger L^2[(F5),(F1,F5),(F1),(F5,F6),(F1,F6),(S6,F5),$ $(S6,F1),(S5,F5),(S5,F1),(S1,F5),(S1,F1),(S6,F6),(S5,S6),(S1,S6)]$
$F4\ \left(\mathbf{1},\mathbf{2},\frac{3}{2}\right)$	$e_{\mathbb{C}}HL^3[(S6),(S2)]$
$F5\ (\mathbf{1},\mathbf{3},0)$	$e_{\mathbb{C}}HL^3[(S4),(S6)]$ $d_{\mathbb{C}}HL^2Q[(S4),(S12),(S14)]$ $HL^2Q^\dagger u_{\mathbb{C}}^\dagger[(S4),(V9),(V8)]$ $D^2H^2L^2$ $De_{\mathbb{C}}H^{\dagger 3}L^\dagger[(S6),(F3),(V5)]$ $d_{\mathbb{C}}HLQ^{\dagger 2}[(S14),(V8)]$ $H^2L^2W_L[(F7),(F1)]$ $H^3H^\dagger L^2[(S4),(S7),(S5,F1),(S1),(S6,F6),(F7),(F3),(F1,F3),$ $(F6,F7),(F3,F6),(S6,S7),(S4,S6),(S6,F7),(S6,F3),(S5,S7),$ $(S4,S5),(S1,S4),(S5,F7),(S5,F3),(S1,F3)]$ $H^2L^2W_L$ $B_LH^2L^2$ $e_{\mathbb{C}}HL^3$ $HL^2Q^\dagger u_{\mathbb{C}}^\dagger$ $d_{\mathbb{C}}HL^2Q$ $De_{\mathbb{C}}^\dagger H^3L$
$F6\ (\mathbf{1},\mathbf{3},1)$	$H^3H^\dagger L^2[(S8),(S6,F5),(S6,F1),(F5,F7),(F3,F5),(F1,F3),(S6,S8),(S6,F7),(S6,F3)]$
$F7\ \left(\mathbf{1},\mathbf{4},\frac{1}{2}\right)$	$H^2L^2W_L[(F5),(S6)]$ $H^3H^\dagger L^2[(F5),(S6,F5),(F5,F6),(S6,F6),(S5,F5),(S5,S6)]$
$F8\ \left(\mathbf{3},\mathbf{1},-\frac{1}{3}\right)$	$HL^2Q^\dagger u_{\mathbb{C}}^\dagger[(S2),(V8)]$ $d_{\mathbb{C}}HLQ^{\dagger 2}[(V8),(S12),(V5)]$ ${d_{\mathbb{C}}}^2e_{\mathbb{C}}^\dagger HQ^\dagger[(V5),(S11)]$
$F9\ \left(\mathbf{3},\mathbf{1},\frac{2}{3}\right)$	$d_{\mathbb{C}}HL^2Q[(S12),(S2)]$
$F10\ \left(\mathbf{3},\mathbf{2},-\frac{5}{6}\right)$	${d_{\mathbb{C}}}^2HLu_{\mathbb{C}}[(S12),(S10),(S13)]$ $d_{\mathbb{C}}HL^2Q[(S10),(S6),(S2),(S14)]$ $d_{\mathbb{C}}e_{\mathbb{C}}^\dagger HLu_{\mathbb{C}}^\dagger[(S10),(V3),(V8)]$ $d_{\mathbb{C}}HLQ^{\dagger 2}[(S10),(S14),(V9),(V5)]$
$F11\ \left(\mathbf{3},\mathbf{2},\frac{1}{6}\right)$	${d_{\mathbb{C}}}^3H^\dagger L[(S11),(S12)]$ ${d_{\mathbb{C}}}^2HLu_{\mathbb{C}}[(S11),(S12)]$
$F12\ \left(\mathbf{3},\mathbf{2},\frac{7}{6}\right)$	$HL^2Q^\dagger u_{\mathbb{C}}^\dagger[(S6),(S2),(V9),(V5)]$ $d_{\mathbb{C}}e_{\mathbb{C}}^\dagger HLu_{\mathbb{C}}^\dagger[(V5),(S12),(V3)]$
$F13\ \left(\mathbf{3},\mathbf{3},-\frac{1}{3}\right)$	$HL^2Q^\dagger u_{\mathbb{C}}^\dagger[(S6),(V8)]$ $d_{\mathbb{C}}HLQ^{\dagger 2}[(V8),(S12),(V9)]$
$F14\ \left(\mathbf{3},\mathbf{3},\frac{2}{3}\right)$	$d_{\mathbb{C}}HL^2Q[(S12),(S6)]$

Complete dim-6 tree UVs

[Li, Ni, Xiao, **Yu**, 2204.03660]

Scalar	
$(SU(3)_c, SU(2)_2, U(1)_y)$	
$S1\ (\mathbf{1},\mathbf{1},0)$	$B_L{}^2HH^\dagger$ $D^2H^2H^{\dagger 2}$ $d_{\mathbb{C}}HH^{\dagger 2}Q[(F11),(F8)]$ $e_{\mathbb{C}}HH^{\dagger 2}L[(F3),(F2)]$ $G_L{}^2HH^\dagger$ $H^2H^\dagger Qu_{\mathbb{C}}[(S4),(F11),(F9)]$ $HH^\dagger W_L{}^2$ $H^3H^{\dagger 3}[(S6),(S2),(S5),(S4,S6),(S2,S4),(S4,S5),(S4)]$ $e_{\mathbb{C}}HH^{\dagger 2}L$ $d_{\mathbb{C}}HH^{\dagger 2}Q$ $H^2H^\dagger Qu_{\mathbb{C}}$
$S2\ (\mathbf{1},\mathbf{1},1)$	$d_{\mathbb{C}}HH^{\dagger 2}Q[(S4),(F10),(F9)]$ $e_{\mathbb{C}}HH^{\dagger 2}L[(S4),(F4),(F1)]$ $H^2H^\dagger Qu_{\mathbb{C}}[(F8),(F12)]$ $L^2L^{\dagger 2}$ $H^3H^{\dagger 3}[(S4),(S5),(S5,S6),(S1),(S4,S5),(S1,S4),(S5,S6),(S4,S6)]$
$S3\ (\mathbf{1},\mathbf{1},2)$	$e_{\mathbb{C}}{}^2e_{\mathbb{C}}^{\dagger 2}$
$S4\ (\mathbf{1},\mathbf{2},\frac{1}{2})$	$d_{\mathbb{C}}^\dagger e_{\mathbb{C}}LQ^\dagger$ $d_{\mathbb{C}}HH^{\dagger 2}Q[(S6),(S2)]$ $e_{\mathbb{C}}HH^{\dagger 2}L[(S6),(S2)]$ $H^2H^\dagger Qu_{\mathbb{C}}$ $H^2H^\dagger Qu_{\mathbb{C}}[(S5),(S1)]$ $QQ^\dagger u_{\mathbb{C}}u_{\mathbb{C}}^\dagger$ $H^3H^{\dagger 3}[(S6),(S2),(S5,S6),(S2,S5),$ $(S1,S6),(S1,S2),(S2,S6),(S5),(S1,S5),(S1)]$
$S5\ (\mathbf{1},\mathbf{3},0)$	$B_LHH^\dagger W_L$ $D^2H^2H^{\dagger 2}$ $d_{\mathbb{C}}HH^{\dagger 2}Q[(F11),(F13)]$ $e_{\mathbb{C}}HH^{\dagger 2}L[(F3),(F6)]$ $H^2H^\dagger Qu_{\mathbb{C}}[(S4),(F11),(F14)]$ $HH^\dagger W_L{}^2$ $H^3H^{\dagger 3}[(S7),(S6),(S2,S6),(S1),(S6,S7),(S4,S6),(S2,S4),(S4),(S1,S4)]$ $e_{\mathbb{C}}HH^{\dagger 2}L$ $d_{\mathbb{C}}HH^{\dagger 2}Q$ $H^2H^\dagger Qu_{\mathbb{C}}$
$S6\ (\mathbf{1},\mathbf{3},1)$	$d_{\mathbb{C}}HH^{\dagger 2}Q[(S4),(F10),(F14)]$ $e_{\mathbb{C}}HH^{\dagger 2}L[(S4),(F4),(F5)]$ $H^2H^\dagger Qu_{\mathbb{C}}[(F13),(F12)]$ $L^2L^{\dagger 2}$ $H^3H^{\dagger 3}[(S7),(S4),(S8),(S5),(S5),(S1),(S2,S5)$ $,(S5,S7),(S4,S5),(S1,S4),(S2,S5),(S2,S4)]$
$S7\ (\mathbf{1},\mathbf{4},\frac{1}{2})$	$H^3H^{\dagger 3}$ $H^3H^{\dagger 3}[(S6),(S5),(S5,S6)]$
$S8\ (\mathbf{1},\mathbf{4},\frac{3}{2})$	$H^3H^{\dagger 3}$ $H^3H^{\dagger 3}[(S6)]$
$S9\ (\mathbf{3},\mathbf{1},-\frac{4}{3})$	$u_{\mathbb{C}}{}^2u_{\mathbb{C}}^{\dagger 2}$
$S10\ (\mathbf{3},\mathbf{1},-\frac{1}{3})$	$Q^2Q^{\dagger 2}$ $e_{\mathbb{C}}LQu_{\mathbb{C}}$ $e_{\mathbb{C}}Q^{\dagger 2}u_{\mathbb{C}}$ $e_{\mathbb{C}}e_{\mathbb{C}}^\dagger u_{\mathbb{C}}u_{\mathbb{C}}^\dagger$
$S11\ (\mathbf{3},\mathbf{1},\frac{2}{3})$	$d_{\mathbb{C}}{}^2d_{\mathbb{C}}^{\dagger 2}$
$S12\ (\mathbf{3},\mathbf{2},\frac{1}{6})$	$d_{\mathbb{C}}d_{\mathbb{C}}^\dagger LL^\dagger$
$S13\ (\mathbf{3},\mathbf{2},\frac{7}{6})$	$LL^\dagger u_{\mathbb{C}}u_{\mathbb{C}}^\dagger$
$S14\ (\mathbf{3},\mathbf{3},-\frac{1}{3})$	$Q^2Q^{\dagger 2}$
$S15\ (\mathbf{6},\mathbf{1},-\frac{2}{3})$	$d_{\mathbb{C}}{}^2d_{\mathbb{C}}^{\dagger 2}$
$S16\ (\mathbf{6},\mathbf{1},\frac{1}{3})$	$d_{\mathbb{C}}Q^2u_{\mathbb{C}}$ $d_{\mathbb{C}}d_{\mathbb{C}}^\dagger u_{\mathbb{C}}u_{\mathbb{C}}^\dagger$
$S17\ (\mathbf{6},\mathbf{1},\frac{4}{3})$	$u_{\mathbb{C}}{}^2u_{\mathbb{C}}^{\dagger 2}$
$S18\ (\mathbf{6},\mathbf{3},\frac{1}{3})$	$Q^2Q^{\dagger 2}$
$S19\ (\mathbf{8},\mathbf{2},\frac{1}{2})$	$QQ^\dagger u_{\mathbb{C}}u_{\mathbb{C}}^\dagger$

Fermion	
$(SU(3)_c, SU(2)_2, U(1)_y)$	
$F1\ (\mathbf{1},\mathbf{1},0)$	$DHH^\dagger LL^\dagger$ $e_{\mathbb{C}}HH^{\dagger 2}L[(F3),(S2)]$ $e_{\mathbb{C}}HH^{\dagger 2}L$
$F2\ (\mathbf{1},\mathbf{1},1)$	$B_Le_{\mathbb{C}}H^\dagger L$ $DHH^\dagger LL^\dagger$ $e_{\mathbb{C}}HH^{\dagger 2}L[(F4),(F3),(S1)]$ $e_{\mathbb{C}}HH^{\dagger 2}L$
$F3\ (\mathbf{1},\mathbf{2},\frac{1}{2})$	$B_Le_{\mathbb{C}}H^\dagger L$ $e_{\mathbb{C}}HH^{\dagger 2}L[(F5),(F1),(F6),(F2),(S5),(S1)]$
$F4\ (\mathbf{1},\mathbf{2},\frac{3}{2})$	$De_{\mathbb{C}}e_{\mathbb{C}}^\dagger HH^\dagger$ $e_{\mathbb{C}}HH^{\dagger 2}L[(F6),(F2),(S6),(S2)]$ $e_{\mathbb{C}}HH^{\dagger 2}L$
$F5\ (\mathbf{1},\mathbf{3},0)$	$DHH^\dagger LL^\dagger$ $e_{\mathbb{C}}HH^{\dagger 2}L[(F3),(S6)]$ $e_{\mathbb{C}}HH^{\dagger 2}L$
$F6\ (\mathbf{1},\mathbf{3},1)$	$e_{\mathbb{C}}H^\dagger LW_L$ $e_{\mathbb{C}}HH^{\dagger 2}L[(F4),(F3),(S5)]$
$F8\ (\mathbf{3},\mathbf{1},-\frac{1}{3})$	$B_Ld_{\mathbb{C}}H^\dagger Q$ $d_{\mathbb{C}}G_LH^\dagger Q$ $DHH^\dagger QQ^\dagger$ $d_{\mathbb{C}}HH^{\dagger 2}Q[(F10),(F11),(S1)]$
$F9\ (\mathbf{3},\mathbf{1},\frac{2}{3})$	$DHH^\dagger QQ^\dagger$ $B_LHQu_{\mathbb{C}}$ $G_LHQu_{\mathbb{C}}$ $d_{\mathbb{C}}HH^{\dagger 2}Q[(F11),(S2)]$ $H^2H^\dagger Qu_{\mathbb{C}}$
$F10\ (\mathbf{3},\mathbf{2},-\frac{5}{6})$	$Dd_{\mathbb{C}}d_{\mathbb{C}}^\dagger HH^\dagger$ $d_{\mathbb{C}}HH^{\dagger 2}Q[(F13),(F8),(S6),(S2)]$ $d_{\mathbb{C}}HH^{\dagger 2}Q$
$F11\ (\mathbf{3},\mathbf{2},\frac{1}{6})$	$B_Ld_{\mathbb{C}}H^\dagger Q$ $B_LHQu_{\mathbb{C}}$ $G_LHQu_{\mathbb{C}}$ $DHH^\dagger u_{\mathbb{C}}u_{\mathbb{C}}^\dagger$ $d_{\mathbb{C}}HH^{\dagger 2}Q[(F14),(F9),(F13),(F8),(S5),(S1)]$ $H^2H^\dagger Qu_{\mathbb{C}}[(F14),(F9),(F13),(F8),(S5),(S1)]$
$F12\ (\mathbf{3},\mathbf{2},\frac{7}{6})$	$DHH^\dagger u_{\mathbb{C}}u_{\mathbb{C}}^\dagger$ $H^2H^\dagger Qu_{\mathbb{C}}[(F14),(F9),(S6),(S2)]$ $H^2H^\dagger Qu_{\mathbb{C}}$
$F13\ (\mathbf{3},\mathbf{3},-\frac{1}{3})$	$d_{\mathbb{C}}H^\dagger QW_L$ $d_{\mathbb{C}}HH^{\dagger 2}Q[(F10),(F11),(S5)]$ $H^2H^\dagger Qu_{\mathbb{C}}[(F11),(S6)]$
$F14\ (\mathbf{3},\mathbf{3},\frac{2}{3})$	$HQu_{\mathbb{C}}W_L$ $d_{\mathbb{C}}HH^{\dagger 2}Q[(F11),(S6)]$ $H^2H^\dagger Qu_{\mathbb{C}}[(F11),(F12),(S5)]$

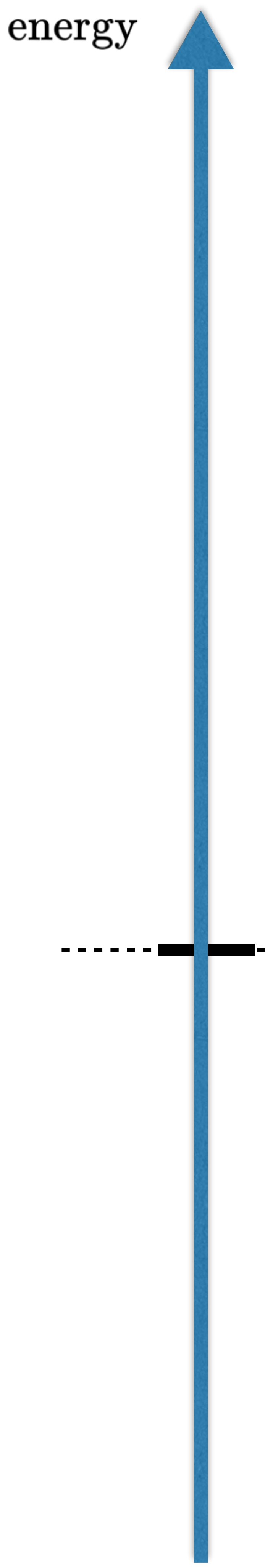
New LHC searches!

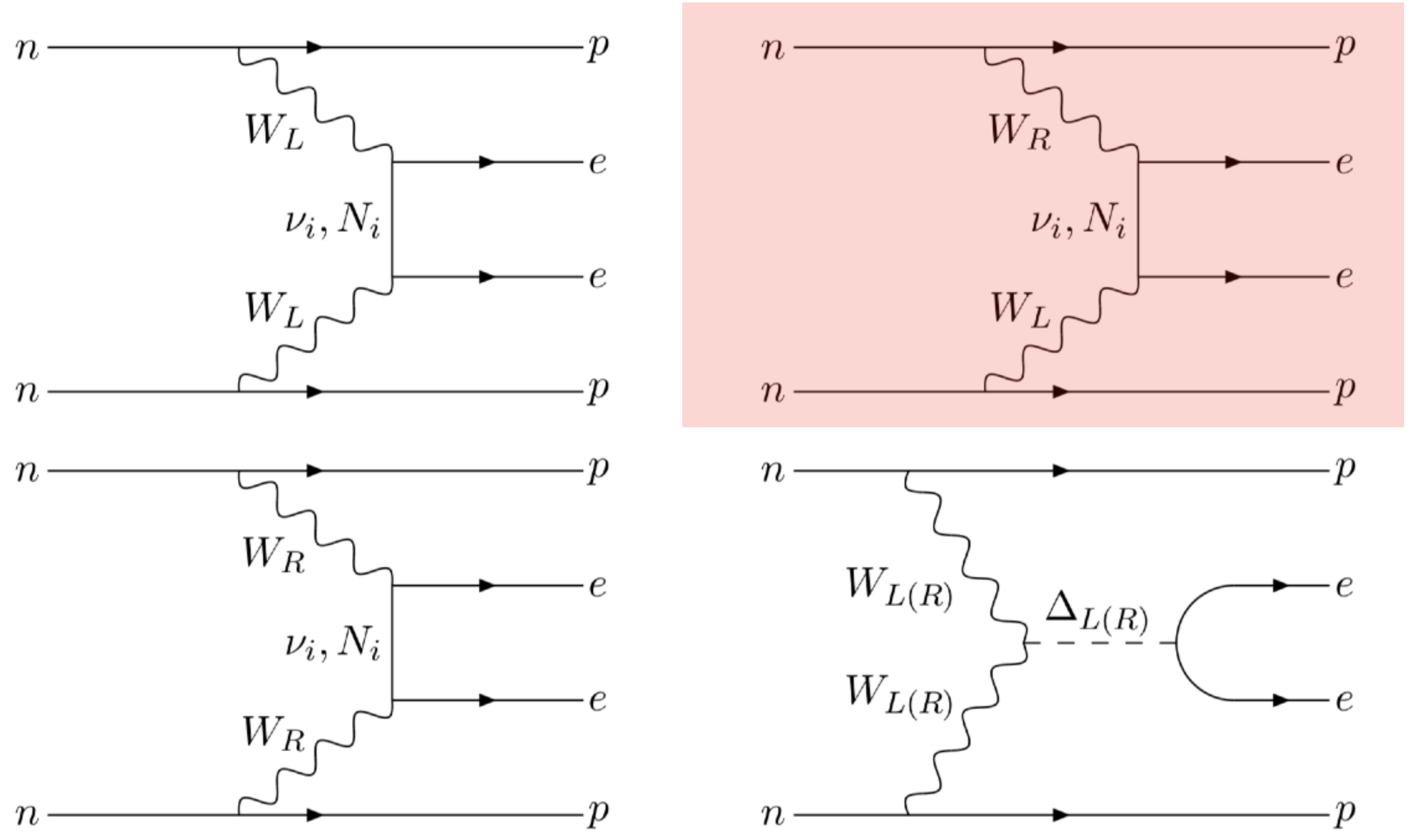
Vector	
$(SU(3)_c, SU(2)_2, U(1)_y)$	
$V1\ (\mathbf{1},\mathbf{1},0)$	$d_{\mathbb{C}}{}^2d_{\mathbb{C}}^{\dagger 2}$ $d_{\mathbb{C}}d_{\mathbb{C}}^\dagger e_{\mathbb{C}}e_{\mathbb{C}}^\dagger$ $e_{\mathbb{C}}{}^2e_{\mathbb{C}}^{\dagger 2}$ $Dd_{\mathbb{C}}d_{\mathbb{C}}^\dagger HH^\dagger$ $De_{\mathbb{C}}e_{\mathbb{C}}^\dagger HH^\dagger$ $D^2H^2H^{\dagger 2}$ $d_{\mathbb{C}}d_{\mathbb{C}}^\dagger LL^\dagger$ $e_{\mathbb{C}}e_{\mathbb{C}}^\dagger LL^\dagger$ $DHH^\dagger LL^\dagger$ $L^2L^{\dagger 2}$ $d_{\mathbb{C}}d_{\mathbb{C}}^\dagger QQ^\dagger$ $e_{\mathbb{C}}e_{\mathbb{C}}^\dagger QQ^\dagger$ $DHH^\dagger QQ^\dagger$ $LL^\dagger QQ^\dagger$ $Q^2Q^{\dagger 2}$ $d_{\mathbb{C}}d_{\mathbb{C}}^\dagger u_{\mathbb{C}}u_{\mathbb{C}}^\dagger$ $e_{\mathbb{C}}e_{\mathbb{C}}^\dagger u_{\mathbb{C}}u_{\mathbb{C}}^\dagger$ $DHH^\dagger u_{\mathbb{C}}u_{\mathbb{C}}^\dagger$ $LL^\dagger u_{\mathbb{C}}u_{\mathbb{C}}^\dagger$ $QQ^\dagger u_{\mathbb{C}}u_{\mathbb{C}}^\dagger$ $d_{\mathbb{C}}HH^{\dagger 2}Q$ $e_{\mathbb{C}}HH^{\dagger 2}L$ $H^2H^\dagger Qu_{\mathbb{C}}$ $e_{\mathbb{C}}HH^{\dagger 2}L$ $d_{\mathbb{C}}HH^{\dagger 2}Q$ $H^2H^\dagger Qu_{\mathbb{C}}$
$V2\ (\mathbf{1},\mathbf{1},1)$	$D^2H^2H^{\dagger 2}$ $Dd_{\mathbb{C}}H^{\dagger 2}u_{\mathbb{C}}^\dagger$ $d_{\mathbb{C}}d_{\mathbb{C}}^\dagger u_{\mathbb{C}}u_{\mathbb{C}}^\dagger$ $e_{\mathbb{C}}HH^{\dagger 2}L$ $d_{\mathbb{C}}HH^{\dagger 2}Q$ $H^2H^\dagger Qu_{\mathbb{C}}$ $d_{\mathbb{C}}HH^{\dagger 2}Q$
$V3\ (\mathbf{1},\mathbf{2},\frac{3}{2})$	$e_{\mathbb{C}}e_{\mathbb{C}}^\dagger LL^\dagger$
$V4\ (\mathbf{1},\mathbf{3},0)$	$D^2H^2H^{\dagger 2}$ $DHH^\dagger LL^\dagger$ $L^2L^{\dagger 2}$ $DHH^\dagger QQ^\dagger$ $LL^\dagger QQ^\dagger$ $Q^2Q^{\dagger 2}$ $e_{\mathbb{C}}HH^{\dagger 2}L$ $d_{\mathbb{C}}HH^{\dagger 2}Q$ $H^2H^\dagger Qu_{\mathbb{C}}$ $e_{\mathbb{C}}HH^{\dagger 2}L$
$V5\ (\mathbf{3},\mathbf{1},\frac{2}{3})$	$d_{\mathbb{C}}^\dagger e_{\mathbb{C}}LQ^\dagger$
$V6\ (\mathbf{3},\mathbf{1},\frac{5}{3})$	$e_{\mathbb{C}}e_{\mathbb{C}}^\dagger u_{\mathbb{C}}u_{\mathbb{C}}^\dagger$
$V7\ (\mathbf{3},\mathbf{2},-\frac{5}{6})$	$d_{\mathbb{C}}d_{\mathbb{C}}^\dagger LL^\dagger$ $d_{\mathbb{C}}^\dagger e_{\mathbb{C}}LQ^\dagger$ $e_{\mathbb{C}}e_{\mathbb{C}}^\dagger QQ^\dagger$ $d_{\mathbb{C}}L^\dagger Q^\dagger u_{\mathbb{C}}$ $e_{\mathbb{C}}Q^{\dagger 2}u_{\mathbb{C}}$ $QQ^\dagger u_{\mathbb{C}}u_{\mathbb{C}}^\dagger$
$V8\ (\mathbf{3},\mathbf{2},\frac{1}{6})$	$d_{\mathbb{C}}d_{\mathbb{C}}^\dagger QQ^\dagger$ $d_{\mathbb{C}}L^\dagger Q^\dagger u_{\mathbb{C}}$ $LL^\dagger u_{\mathbb{C}}u_{\mathbb{C}}^\dagger$
$V9\ (\mathbf{3},\mathbf{3},\frac{2}{3})$	$LL^\dagger QQ^\dagger$
$V10\ (\mathbf{6},\mathbf{2},-\frac{1}{6})$	$d_{\mathbb{C}}d_{\mathbb{C}}^\dagger QQ^\dagger$
$V11\ (\mathbf{6},\mathbf{2},\frac{5}{6})$	$QQ^\dagger u_{\mathbb{C}}u_{\mathbb{C}}^\dagger$
$V12\ (\mathbf{8},\mathbf{1},0)$	$d_{\mathbb{C}}{}^2d_{\mathbb{C}}^{\dagger 2}$ $d_{\mathbb{C}}d_{\mathbb{C}}^\dagger QQ^\dagger$ $Q^2Q^{\dagger 2}$ $d_{\mathbb{C}}d_{\mathbb{C}}^\dagger u_{\mathbb{C}}u_{\mathbb{C}}^\dagger$ $QQ^\dagger u_{\mathbb{C}}u_{\mathbb{C}}^\dagger$ $u_{\mathbb{C}}{}^2u_{\mathbb{C}}^{\dagger 2}$
$V13\ (\mathbf{8},\mathbf{1},1)$	$d_{\mathbb{C}}d_{\mathbb{C}}^\dagger u_{\mathbb{C}}u_{\mathbb{C}}^\dagger$
$V14\ (\mathbf{8},\mathbf{3},0)$	$Q^2Q^{\dagger 2}$

[de Blas, Criado, Perez-Victoria, Santiago, 2017]

Complete dim-7 UV for 0vbb

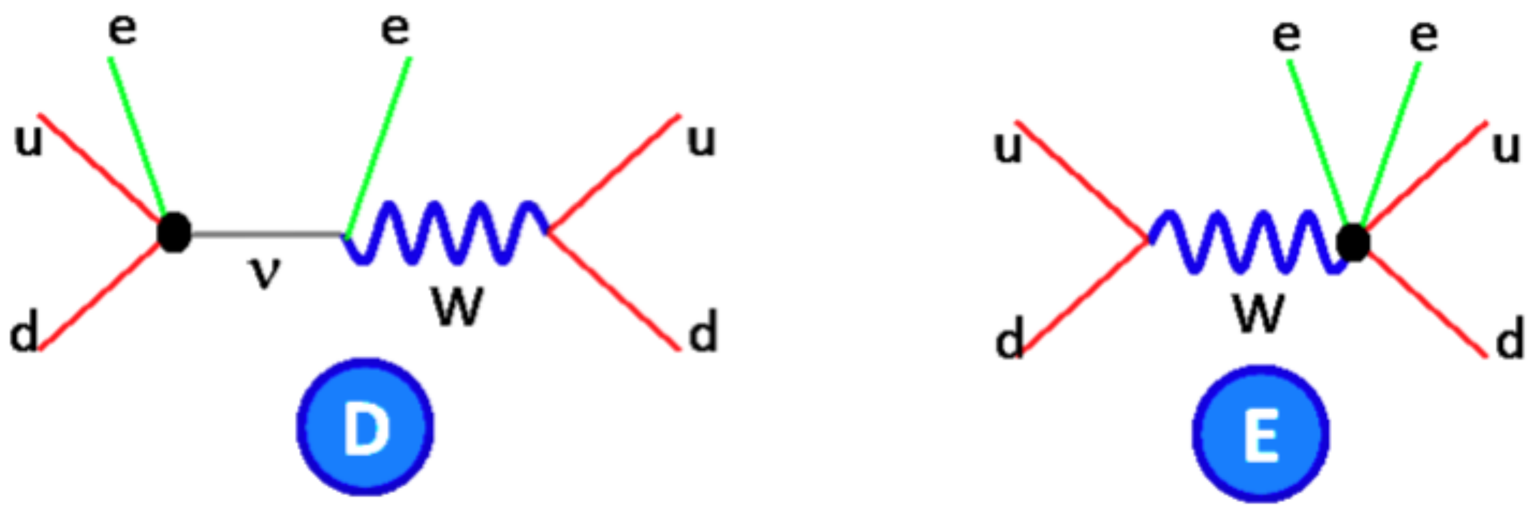
[Li, Ren, **Yu**, in preparation]





B preserving		B preserving		B violating	
$(S,1,1,1)$	$(S,1,2,1/2)$	$(S,3,2,1/6)$	$(F,3,2,7/6)$	$(S,3,1,-1/3)$	$(S,3,2,1/6)$
$(S,3,2,1/6)$	$(S,3,3,-1/3)$	$(S,3,2,1/6)$	$(F,3,3,2/3)$	$(S,3,1,-1/3)$	$(F,3,2,-5/6)$
$(S,1,1,1)$	$(F,3,1,-1/3)$	$(S,3,3,-1/3)$	$(F,3,2,-5/6)$	$(V,3,2,1/6)$	$(F,1,2,1/2)$
$(S,1,1,1)$	$(F,3,1,2/3)$	$(V,1,1,1)$	$(F,1,2,1/2)$	$(V,3,2,1/6)$	$(F,3,1,-1/3)$
$(S,1,1,1)$	$(F,3,2,-5/6)$	$(V,1,2,3/2)$	$(F,3,2,-5/6)$	$(V,3,2,1/6)$	$(F,3,2,-5/6)$
$(S,1,1,1)$	$(F,3,2,7/6)$	$(V,1,2,3/2)$	$(F,3,2,7/6)$	$(V,3,2,1/6)$	$(F,3,3,-1/3)$
$(S,1,2,1/2)$	$(F,1,3,0)$	$(V,3,1,2/3)$	$(F,3,2,7/6)$	$(V,3,1,2/3)$	$(V,3,2,1/6)$
$(S,3,2,1/6)$	$(F,1,2,1/2)$	$(V,3,3,3/2)$	$(F,3,2,7/6)$	$(V,3,2,1/6)$	$(V,3,3,2/3)$
$(S,3,2,1/6)$	$(F,3,1,2/3)$	$(V,1,1,1)$	$(V,1,2,3/2)$		

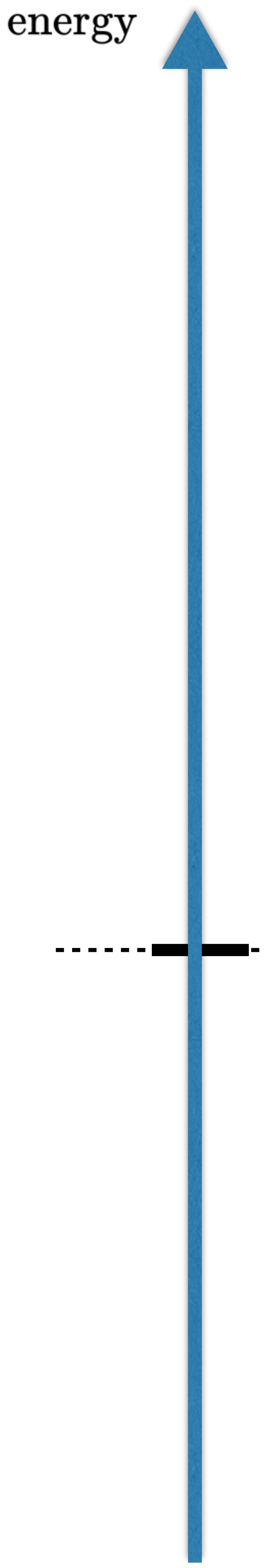
	$\mathcal{O}_{LeHD}$	$\mathcal{O}_{eLLLH}$	$\mathcal{O}_{dLQLH1}$	$\mathcal{O}_{dLQLH2}$	$\mathcal{O}_{dLueH}$	$\mathcal{O}_{QuLLH}$
S_2		$S_4/F_1/F_4$		$S_4/F_9/F_{10}$		$S_4/F_8/F_{12}$
S_4		$S_2/S_6/F_5$	$S_6/F_1/F_5$	$S_2/S_6/F_1/F_5$		$S_2/S_6/F_1/F_5$
S_6	F_3/F_5	$S_4/F_4/F_5$	$S_4/F_{10}/F_{14}$	$S_4/F_{10}/F_{14}$		$S_4/F_{12}/F_{13}$
S_{12}			$F_1/F_5/F_{14}$	$F_5/F_9/F_{14}$	F_3/F_{12}	
F_1	F_3/V_2	S_2	S_4/S_{12}	S_4	V_2/V_5	S_4/V_5
F_3	$S_6/F_1/F_5/V_2$				S_{12}/V_2	
F_4		S_2/S_6				
F_5	S_6/F_3	S_4/S_6	S_4/S_{12}	S_4/S_{12}		S_4/V_9
F_8						S_2
F_9				S_2/S_{12}		
F_{10}			S_6	S_2/S_6	V_3	
F_{12}					$S_{12}/V_3/V_5$	$S_2/S_6/V_5/V_9$
F_{13}						S_6
F_{14}			S_6/S_{12}	S_6/S_{12}		
V_2	$F_1/F_3/V_3$				$F_1/F_3/V_3$	
V_3	V_2				$F_{10}/F_{12}/V_2$	
V_5					F_1/F_{12}	F_1/F_{12}
V_9						F_5/F_{12}

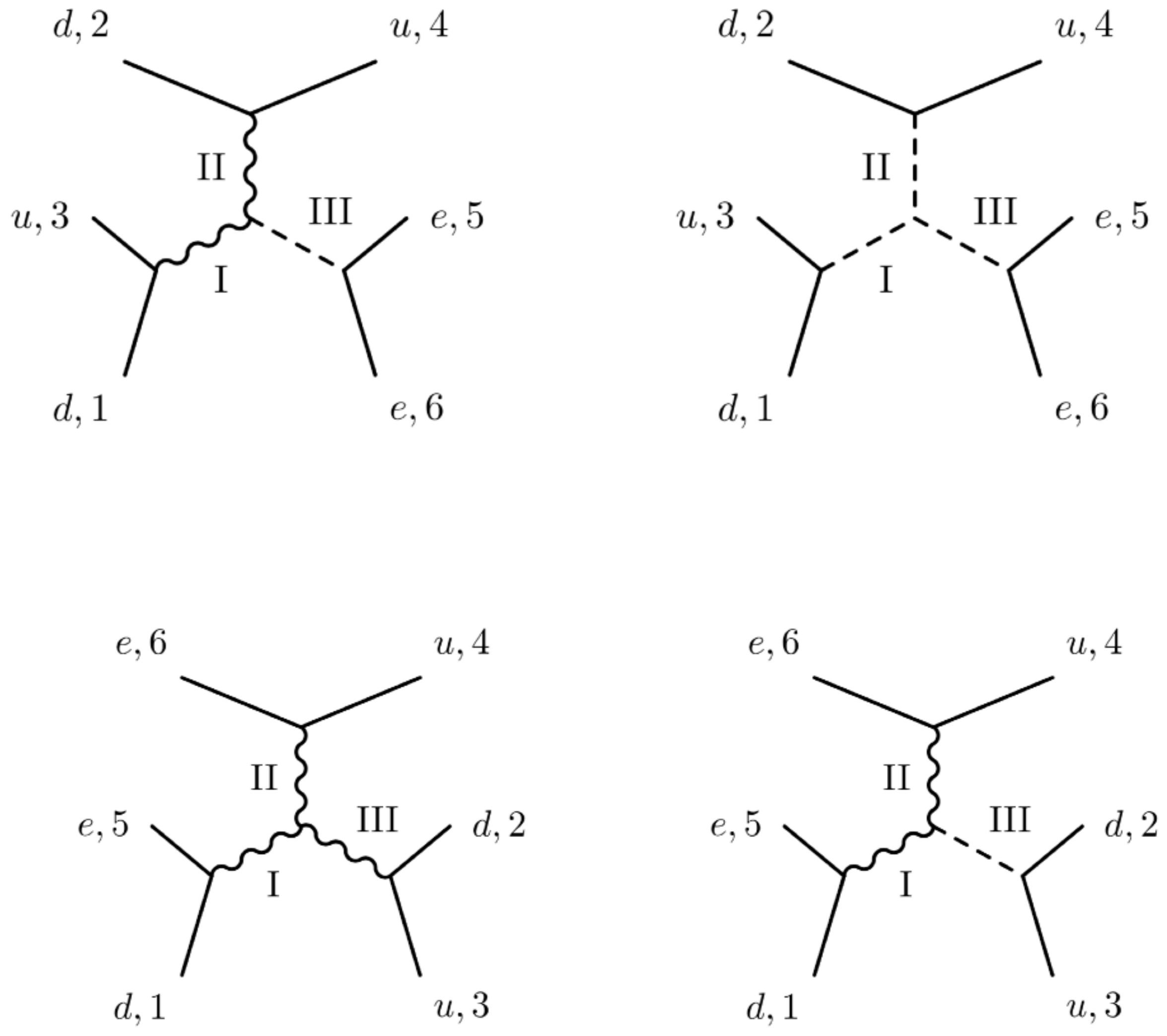


Complete dim-9 UV for 0vbb

[Li, Ni, Xiao, **Yu**, in preparation]

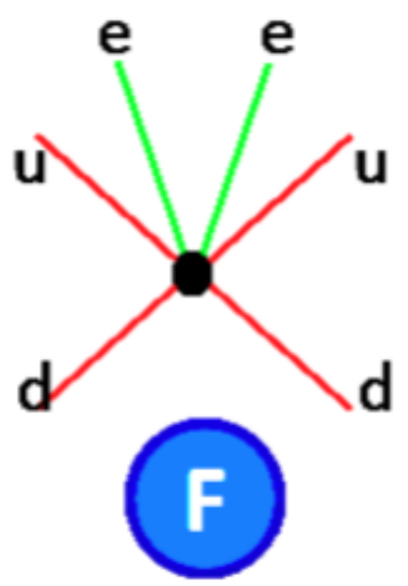
[Li, **Yu**, Zhao, in progress]





$(\mathbf{r}_i, J_i)$	$(1,1,0)$	$(0,0,0)$
$(\mathbf{8}_2, \mathbf{8}_2, \mathbf{1}_3)$	$\frac{4}{3}\mathcal{O}_1 + 2\mathcal{O}_2$	$-\frac{2}{3}\mathcal{O}_2$
$(\mathbf{8}_2, \mathbf{8}_2, \mathbf{1}_1)$	$-2\mathcal{O}_2$	$\frac{4}{3}\mathcal{O}_1 + \frac{2}{3}\mathcal{O}_2$
$(\mathbf{1}_2, \mathbf{1}_2, \mathbf{1}_3)$	$-4\mathcal{O}_1 - 6\mathcal{O}_2$	$2\mathcal{O}_2$
$(\mathbf{1}_2, \mathbf{1}_2, \mathbf{1}_1)$	$6\mathcal{O}_2$	$-4\mathcal{O}_1 - 2\mathcal{O}_2$

$(\mathbf{r}_i, J_i)$	$(1,1,1)$	$(1,1,0)$
$(\mathbf{3}_3, \mathbf{8}_2, \bar{\mathbf{3}}_2)$	$-\frac{8}{3}\mathcal{O}_1$	$\frac{4}{3}\mathcal{O}_1 + \frac{8}{3}\mathcal{O}_2$
$(\mathbf{3}_1, \mathbf{8}_2, \bar{\mathbf{3}}_2)$	$-\frac{8}{3}\mathcal{O}_1$	0
$(\mathbf{3}_3, \mathbf{1}_2, \bar{\mathbf{3}}_2)$	$-\frac{8}{3}\mathcal{O}_1$	$\frac{4}{3}\mathcal{O}_1 + \frac{8}{3}\mathcal{O}_2$
$(\mathbf{3}_1, \mathbf{1}_2, \bar{\mathbf{3}}_2)$	$-\frac{8}{3}\mathcal{O}_1$	0

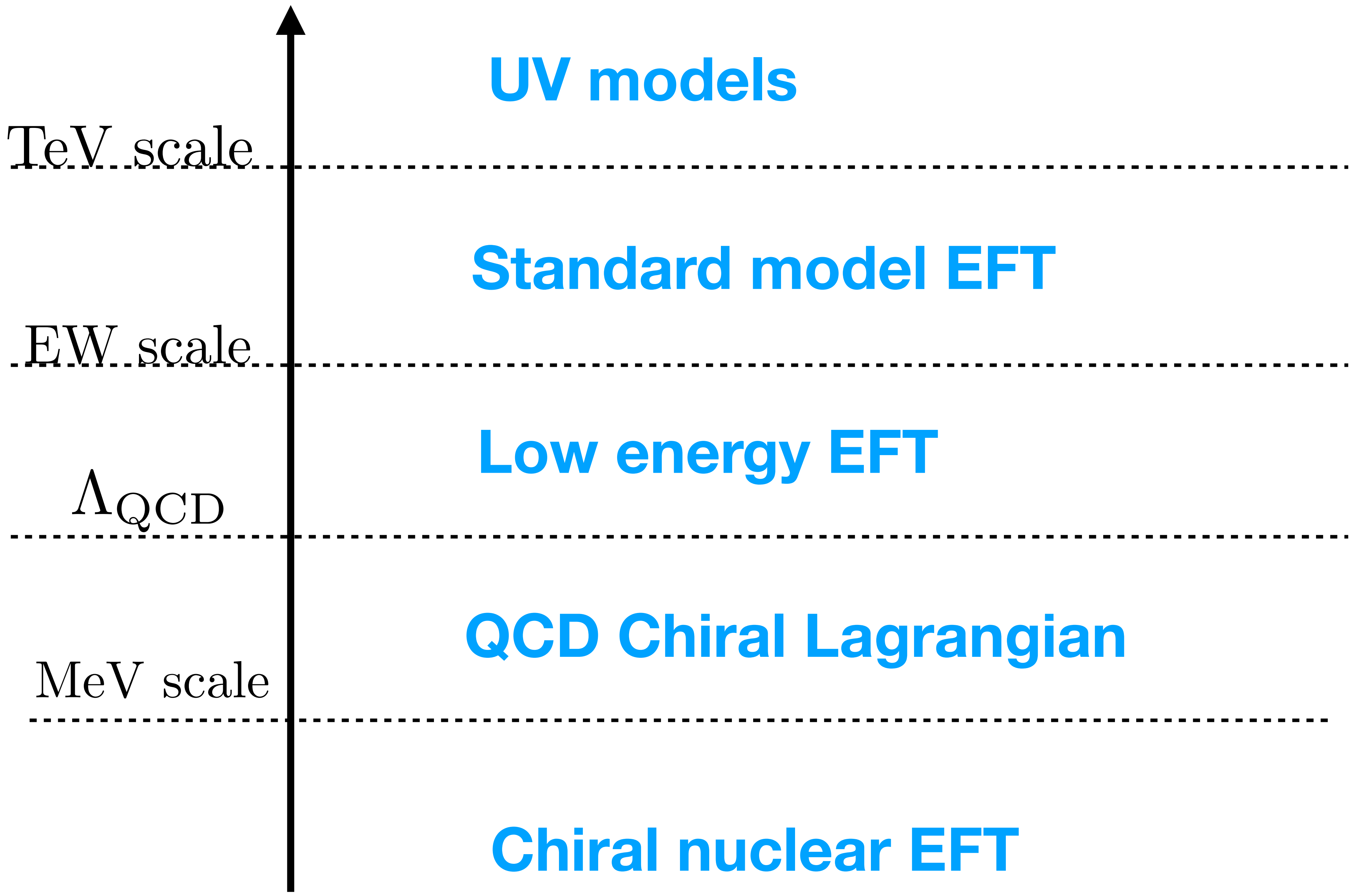


$$\mathcal{O}_1 = \tfrac{1}{4}({L^\dagger}_u{}^j {L^\dagger}_v{}^i)(Q_{pai}Q_{rbj})(u_{cs}{}^b u_{ct}{}^a)$$

$$\mathcal{O}_2 = -\tfrac{1}{4}({L^\dagger}_u{}^j {L^\dagger}_v{}^i)(Q_{pai}u_{cs}{}^b)(Q_{rbj}uc_t{}^a).$$

Summary

- The EFT framework provides most general description on weak interactions and neutrinos



Low energy probe of high energy physics

(Intensity frontier)

Construct operator bases in each levels of EFTs using spinor Young tensor

Complete UVs can be explored via Casimir projection

- With the whole EFT framework, we are ready to investigate WIN pheno in a systematic way

(dark matter, axion, dark photon EFT not discussed here)

Thanks for your attention!