LHCb highlights of time-dependent CP violation in B decays

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on behalf of LHCb collaboration

Weak interaction and Neutrino Conference

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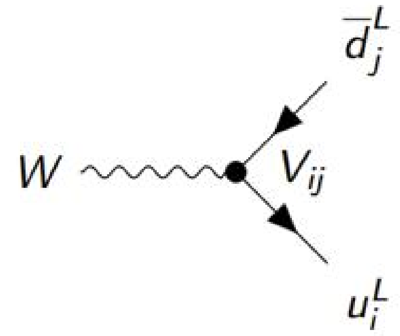
Contents

CP violation in SM

SM charged current interaction

$$\mathcal{L}_{W^\pm} = \frac{g}{\sqrt{2}} \left(\overline{u_L} \gamma^\mu W_\mu^+ V_{CKM} d_L + \overline{d_L} \gamma^\mu W_\mu^- V_{CKM}^\dagger u_L \right)$$

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



- The unitary CKM matrix V_{CKM} introduces tree-level couplings between up and down-type quarks
- 3 free parameters + CP violating phase δ
- V_{CKM} unitarity tested by over-constraining CKM parameters

In the Wolfenstein parameterization

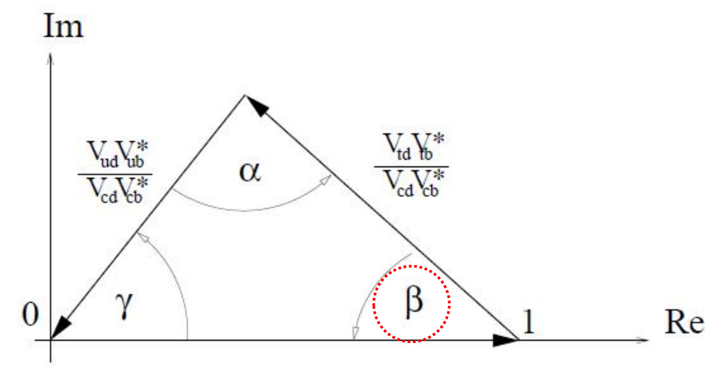
Source of CPV

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

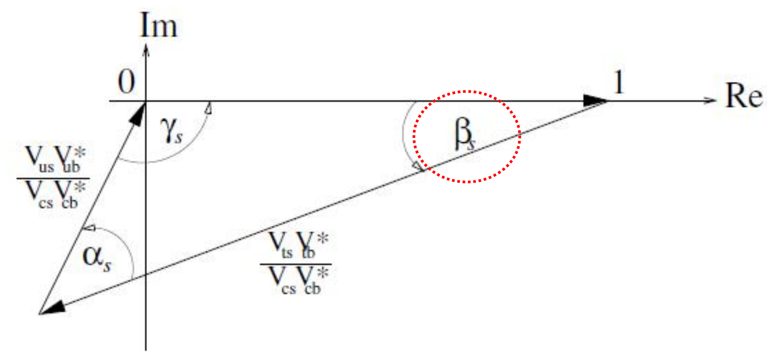
Two phenomenon triangle from unitarity

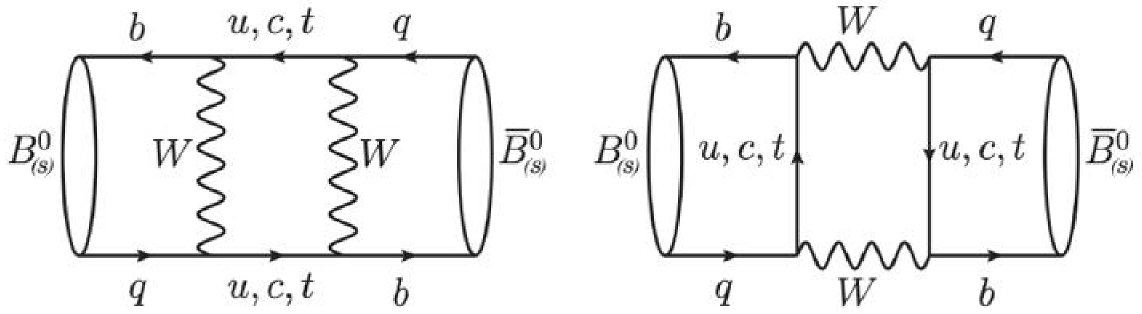
$$V_{CKM}V_{CKM}^{\dagger} = \mathbf{1},$$

$$(B_d:)V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0,$$



$$(B_s:)\,V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0.$$





- The wave function could be represented using $\psi(t) = a(t)|B_q^0 > +b(t)|\bar{B}_q^0 >$

- Mixing and decay can be described by time evolution equation

$$i\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = H\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ |M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

$$|B_{L,H}\rangle = p|B_q^0\rangle \pm q|\overline{B}_q^0\rangle$$

- Solving the equation will give us

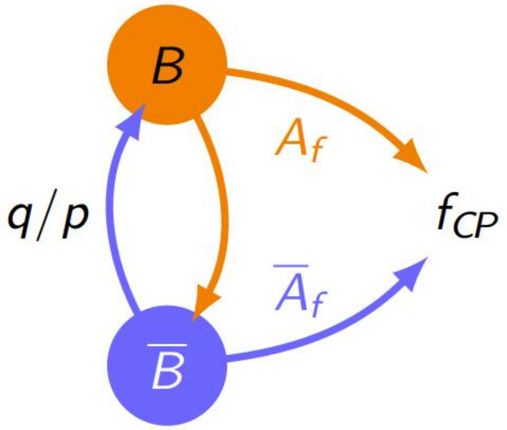
$$\begin{aligned}\Gamma_{B(\bar{B})\to f}(t) &= |A_f|^2(1+|\lambda_f|^2)e^{-\Gamma t}\times \\ &\left(\cosh\frac{\Delta\Gamma t}{2} - D_f \sinh\frac{\Delta\Gamma t}{2} \pm C_f\cos(\Delta m t) \mp S_f \sin\Delta m t\right),\end{aligned}$$

$$\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f} = \eta_f \frac{q}{p}\frac{A(\bar{B}\to\bar{f})}{A(B\to f)} \qquad D_f = \frac{2\mathrm{Re}\lambda_f}{1+|\lambda_f|^2}, C_f = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}, S_f = \frac{2\mathrm{Im}\lambda_f}{1+|\lambda_f|^2}.$$

CP violation in mixing

Unequal transition probabilities between flavour eigenstates

$$P(B \to \overline{B}) \neq P(\overline{B} \to B)$$



CP violation in decay

Unequal CP-conjugated decay rates

$$\Gamma(B \to f) \neq \Gamma(\overline{B} \to \overline{f})$$

CP violation in interference of decays with/without mixing

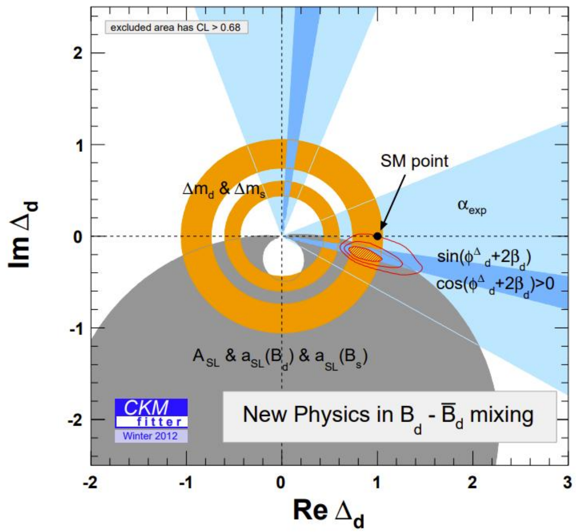
Time-dependent or time-integrated difference of decay rates of initial flavour eigenstates

$$\Gamma(B_{(\rightsquigarrow \overline{B})} \to f_{CP})(t) \neq \Gamma(\overline{B}_{(\rightsquigarrow B)} \to f_{CP})(t)$$

Opportunities for probing for new physics

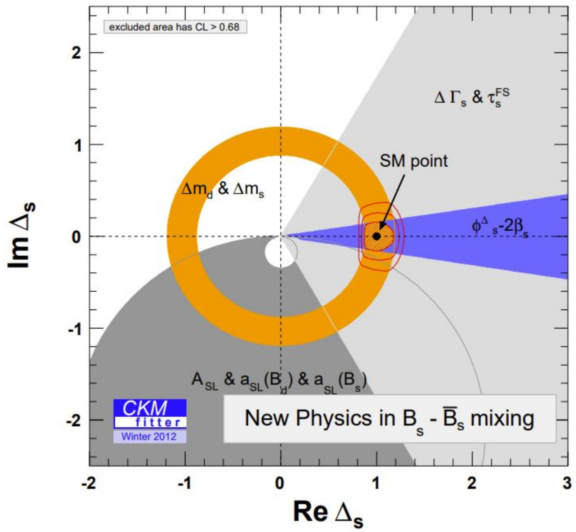
$$M_{12}^q \equiv M_{12}^{\mathrm{SM},q} \cdot \Delta_q, \quad \Delta_q \equiv |\Delta_q| e^{i\phi_q^\Delta}, \quad q = d, s,$$

- NP short-distance contributions can influence mixing $m_{12}^q = m_{12}^{\mathrm{SM},q} \cdot \Delta_q^{\mathrm{NP}}$



[PRD.86.033008]

- Through B mixing, NP energy scales of up to 20 TeV for tree level NP or 2 TeV for NP in loops can be probed

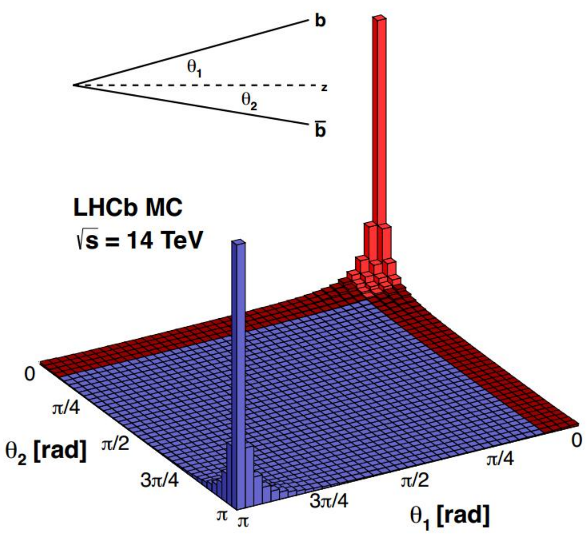


[PRD.89.033016]

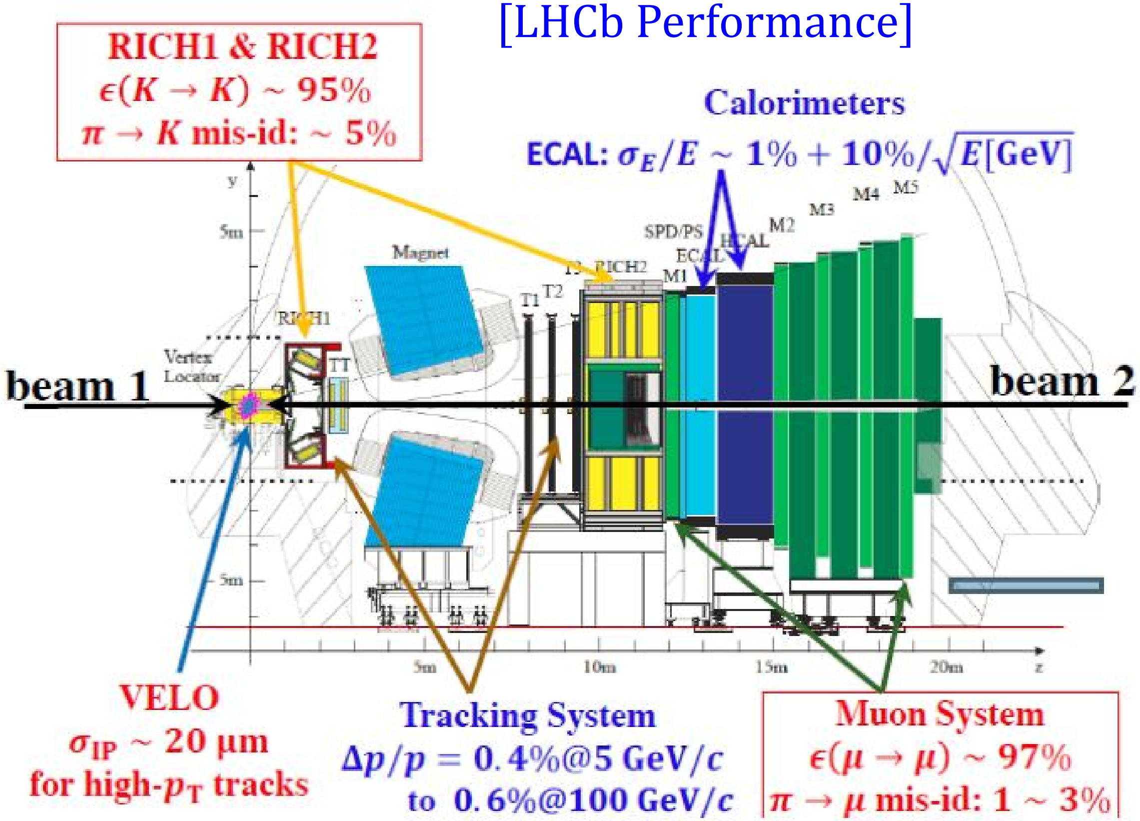
The LHCb detector

A single-arm spectrometer with a forward angular coverage $2< \eta<5$

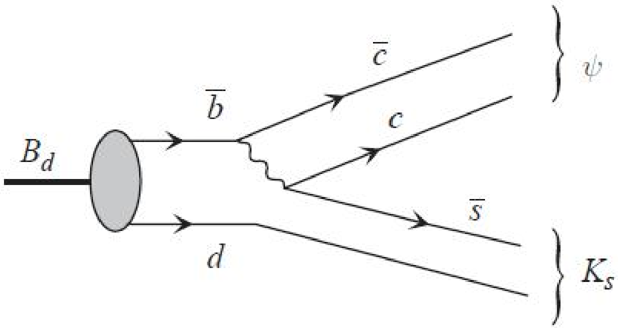
- Excellent resolution in vertex locator, decay time recostruction and Energy measurement
- Particle identification with high precision

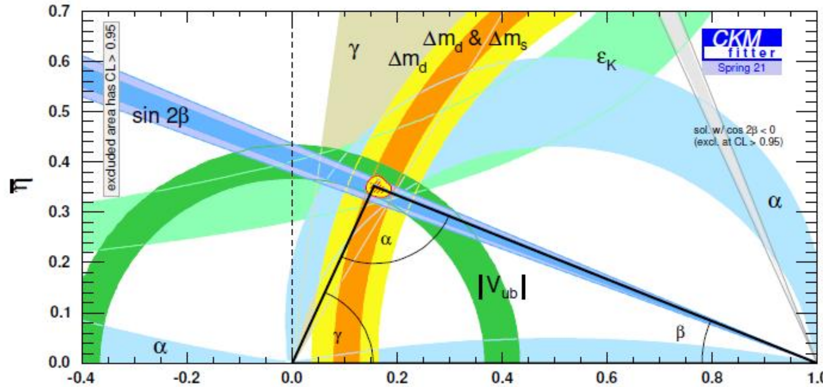


[LHCb Performance]



CP violation in $B^0 \to \psi K_S$ decays



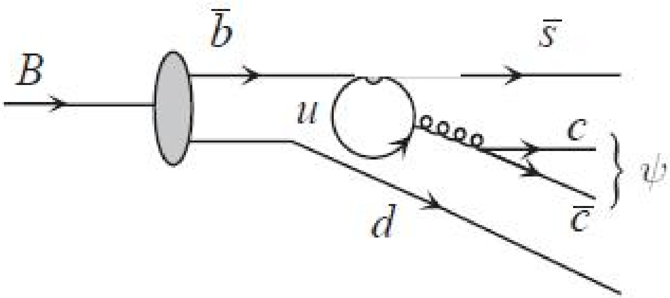


$\sin(2\beta) \neq 0 \Rightarrow$ *CP* violation [CKMFitter]

In the SM prediction

$$\lambda_{J/\psi K_S^0} = \eta_f \frac{q}{p}\frac{\bar{A}_{\bar{f}}}{A_f} = -\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} = -\, e^{-2i\beta}$$

Less than 1% penguin contribution in sin2β



$$\mathrm{Im}\lambda(\psi K_S^0) \simeq \sin 2\beta,$$

$$\beta = \arg\left(-\frac{V_{cb}^* V_{cd}}{V_{tb}^* V_{td}}\right)$$

CP violation in $B^0 \to \psi K_S$ decays

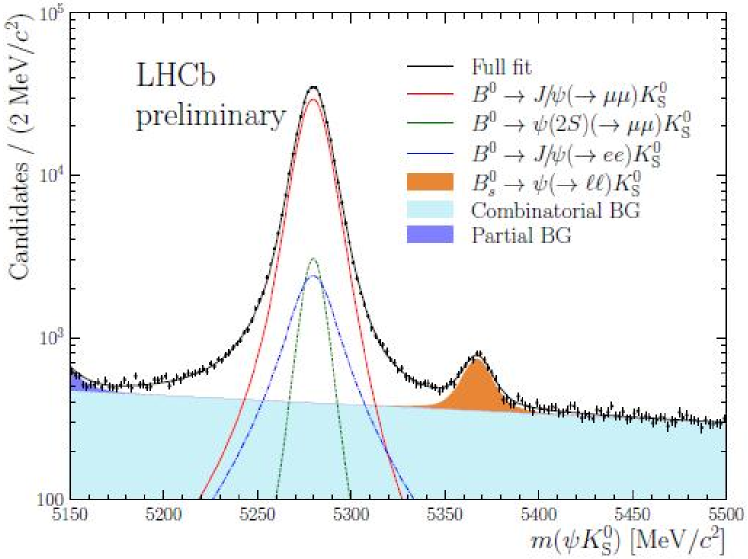
The decay channel $B^0 \to \psi K_S$ offers a theoretically clean access to the CKM angle β.

$$\mathcal{A}^{CP}(t) = \frac{\Gamma(\overline{B}^0(t) \to \psi K^0_S) - \Gamma(B^0(t) \to \psi K^0_S)}{\Gamma(\overline{B}^0(t) \to \psi K^0_S) + \Gamma(B^0(t) \to \psi K^0_S)} \approx \underbrace{D_{\Delta t} D_{FT}}_{\text{Experimental dilution factors}} S \sin(\Delta m_d t)$$

Three main channels are used to determine $\mathcal{A}^{CP}(t)$

[LHCb-Paper-2023-013 In preparation]

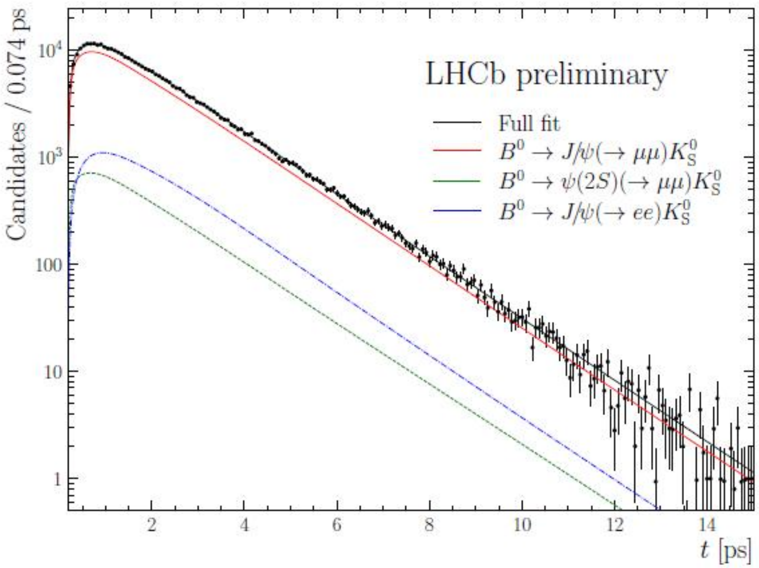
- $B^0 \to J/\psi(\to \mu\mu) K^0_S(\to \pi^+\pi^-)(82\%)$
- $B^0 \to J/\psi(\to ee) K^0_S(\to \pi^+\pi^-)(12\%)$
- $B^0 \to \psi(2S)(\to \mu\mu) K^0_S(\to \pi^+\pi^-)(6\%)$

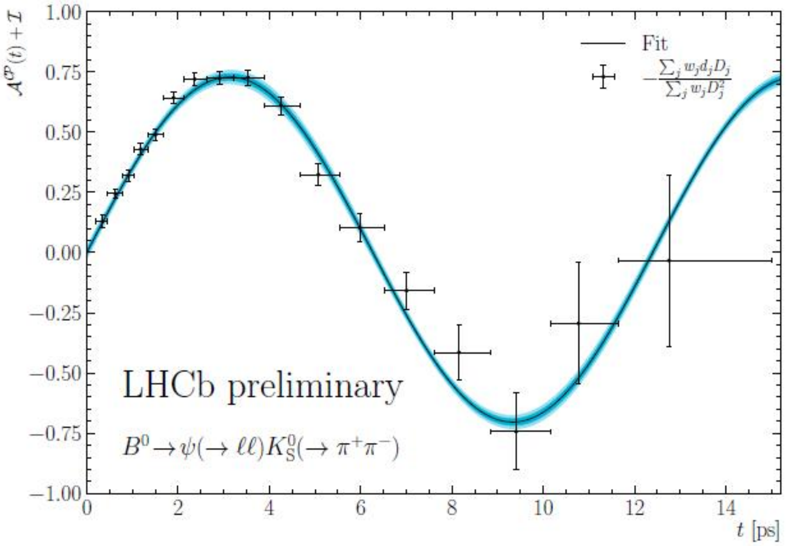


From mass fits, sWeights are obtained for effective background subtraction in CP fit [sFit]

CP violation in $B^0 \to \psi K_S$ decays

Decay time fit to the Run2 data sample, and the time dependent asymmetry obtained in Run2 data

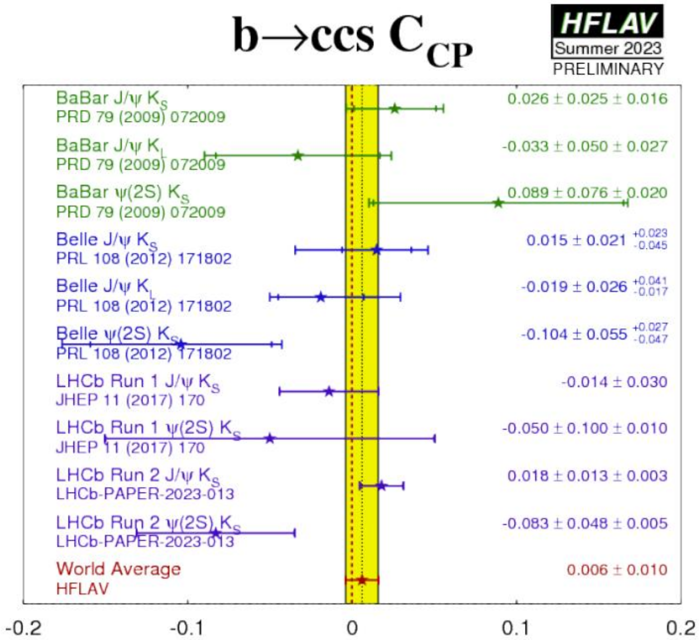




[LHCb-Paper-2023-013 In preparation]

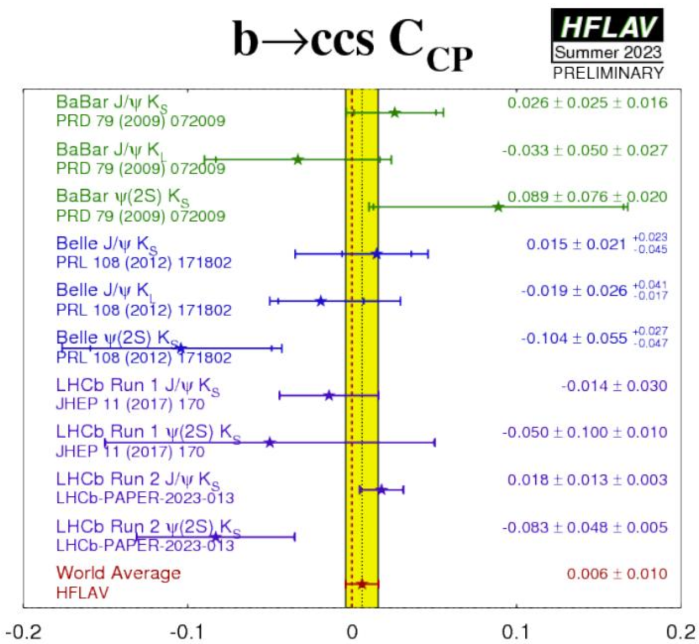
$$S^{\text{Run 2}}_{\psi K^0_S} = 0.716 \pm 0.013\,(\text{stat}) \pm 0.008\,(\text{syst})$$
$$C^{\text{Run 2}}_{\psi K^0_S} = 0.012 \pm 0.012\,(\text{stat}) \pm 0.003\,(\text{syst})$$

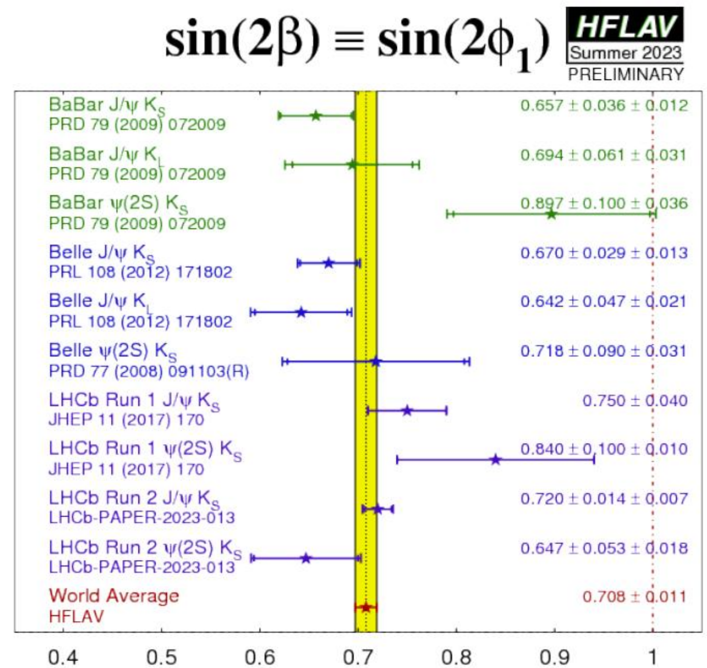
CP violation in $B^0 \to \psi K_S$ decays



[HFLAV]

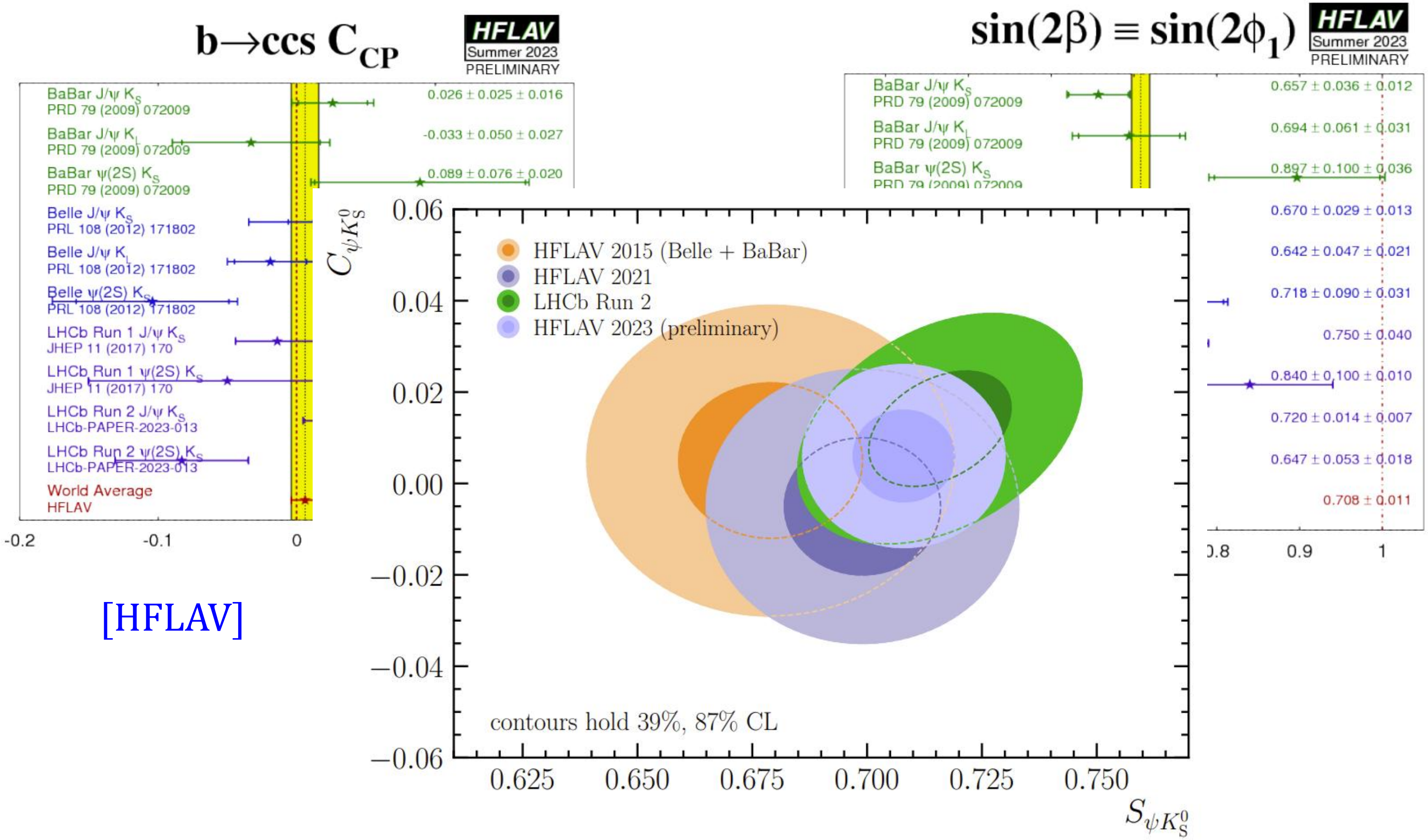
CP violation in $B^0 \to \psi K_S$ decays





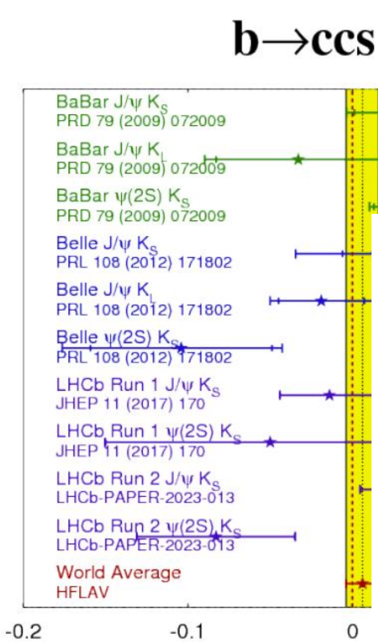
[HFLAV]

CP violation in $B^0 \to \psi K_S$ decays

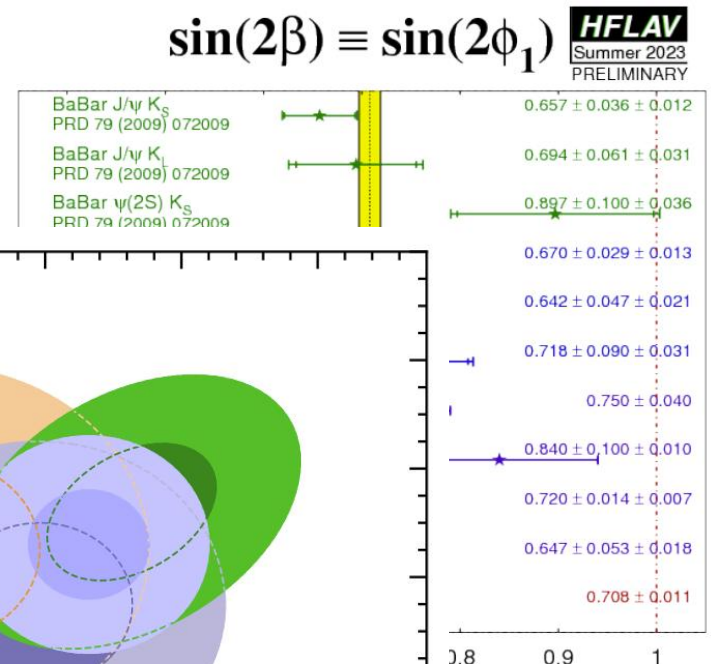


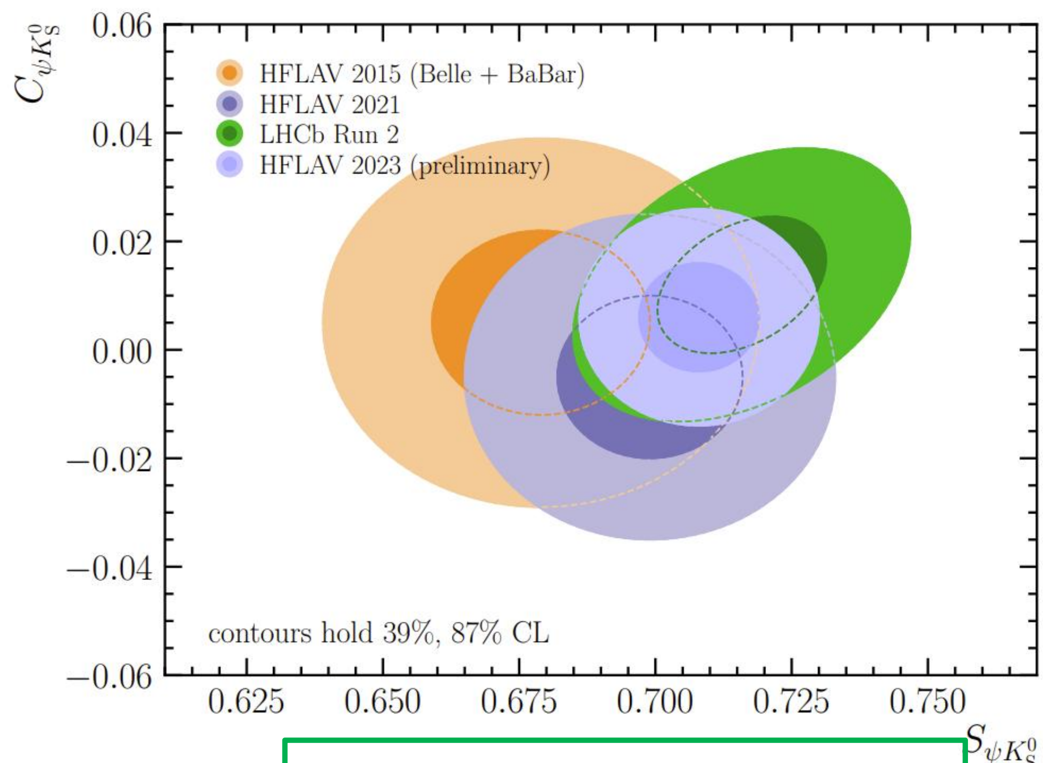
[HFLAV]

CP violation in $B^0 \to \psi K_S$ decays



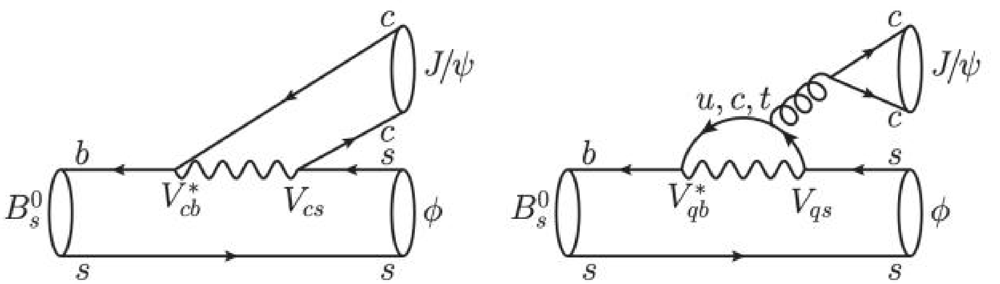
[HFLAV]





This measurement is the most precise single measurement of sin(2β) to date

CP violation in $B_s \to J/\psi KK$ decays

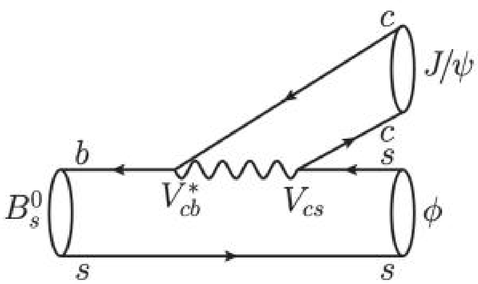


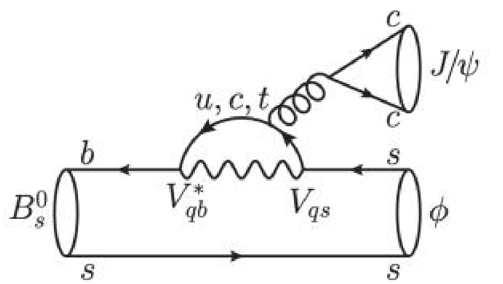
In the SM prediction

$$\lambda_{J/\psi\phi} = \left(\frac{q}{p}\right)_{B_s} \frac{\bar{A}_{J/\psi\phi}}{A_{J/\psi\phi}} = \eta \left(\frac{V_{tb}^* V_{ts}}{V_{tb} V_{ts}^*}\right) \left(\frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}}\right) = \eta e^{+2i\beta_s}.$$

$$\phi_s^{\mathrm{SM}} = -2\beta_s + \delta\phi_s^{\mathrm{peng}} \approx -2\beta_s = -2\arg\left(-\frac{V_{cb}V_{cs}^*}{V_{tb}V_{ts}^*}\right)$$

CP violation in $B_s \to J/\psi KK$ decays





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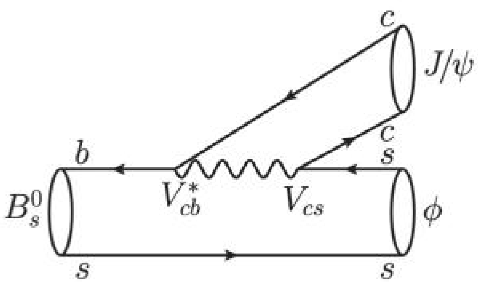
$$\phi_s^{\text{SM}} = -2\beta_s + \delta\phi_s^{\text{peng}} \approx -2\beta_s = -2\arg\left(-\frac{V_{cb}V_{cs}^*}{V_{tb}V_{ts}^*}\right)$$

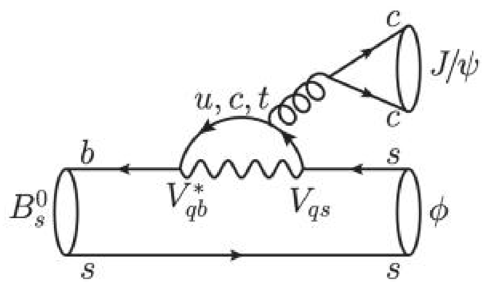
- Golden channel: $B_s^0 \to J/\psi\phi$

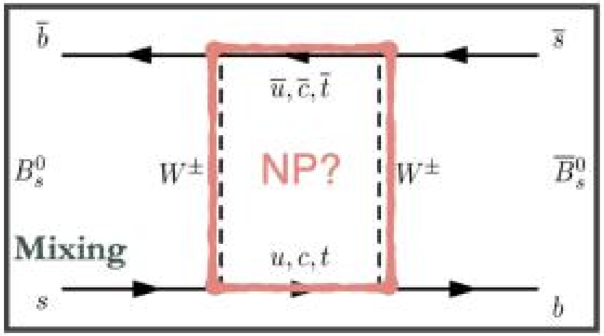
$$A_{CP}(t) = \frac{\Gamma(\bar{B}_s^0 \to J/\psi KK) - \Gamma(B_s^0 \to J/\psi KK)}{\Gamma(\bar{B}_s^0 \to J/\psi KK) + \Gamma(B_s^0 \to J/\psi KK)} = \eta_f \cdot \sin\phi_s^{obs} \cdot \sin(\Delta m_s t)$$

- CP eigenvalue of the final state $\eta_f = (-1)^L$
- A mixture of CP-even & CP-odd components → angular analysis

CP violation in $B_s \to J/\psi KK$ decays







In the SM prediction

$$\lambda_{J/\psi\phi} = \left(\frac{q}{p}\right)_{B_s} \frac{\bar{A}_{J/\psi\phi}}{A_{J/\psi\phi}} = \eta \left(\frac{V_{tb}^* V_{ts}}{V_{tb} V_{ts}^*}\right) \left(\frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}}\right) = \eta e^{+2i\beta_s}.$$

$$\phi_s^{\rm SM} = -2\beta_s + \delta\phi_s^{\rm peng} \approx -2\beta_s = -2\arg\left(-\frac{V_{cb} V_{cs}^*}{V_{tb} V_{ts}^*}\right)$$

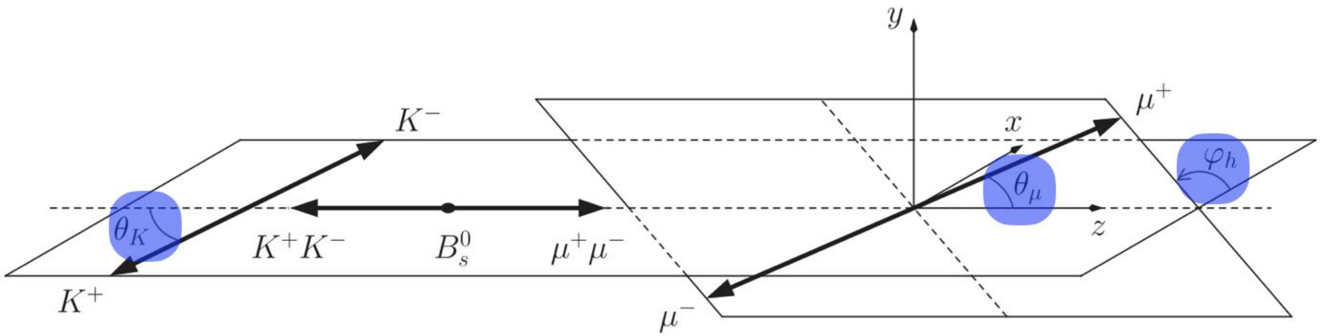
$$\phi_s = \phi_s^{\rm tree} + \delta\phi_s^{\rm penguin} + \delta\phi_s^{\rm NP}$$

sizable modification to Φ_s in BSM

- Golden channel: $B_s^0 \to J/\psi\phi$

$$A_{CP}(t) = \frac{\Gamma(\bar{B}_s^0 \to J/\psi KK) - \Gamma(B_s^0 \to J/\psi KK)}{\Gamma(\bar{B}_s^0 \to J/\psi KK) + \Gamma(B_s^0 \to J/\psi KK)} = \eta_f \cdot \sin\phi_s^{obs} \cdot \sin(\Delta m_s t)$$

- CP eigenvalue of the final state $\eta_f = (-1)^L$
- A mixture of CP-even & CP-odd components → angular analysis



- $B_s^0 \to J/\psi KK$

$$\frac{d^4\Gamma(t)}{dt\,d\cos\theta_K\,d\cos\theta_l\,d\phi} = \sum_{k=1}^{10} N_k h_k(t) f_k(\theta_K, \theta_l, \phi),$$

[arxiv.1306.1911]

$$h_k(t) = \frac{3}{4\pi} e^{-\Gamma_s t}\left\{a_k \cosh\frac{\Delta\Gamma_s t}{2} + b_k \sinh\frac{\Delta\Gamma_s t}{2} + c_k \cos(\Delta m_s t) + d_k \sin(\Delta m_s t)\right\}.$$

very sinsitive to ϕ_s, need FT

k	$f_k(\theta,\psi,\varphi)$	N_k	a_k	b_k	c_k	d_k
1	$2\cos^2\psi\,(1-\sin^2\theta\cos^2\phi)$	$\lvert A_0(0)\rvert^2$	1	$-\cos\phi_s$	0	$\sin\phi_s$
2	$\sin^2\psi\,(1-\sin^2\theta\sin^2\phi)$	$\lvert A_\parallel(0)\rvert^2$	1	$-\cos\phi_s$	0	$\sin\phi_s$
3	$\sin^2\psi\sin^2\theta$	$\lvert A_\perp(0)\rvert^2$	1	$\cos\phi_s$	0	$-\sin\phi_s$
7	$\frac{2}{3}(1-\sin^2\theta\cos^2\phi)$	$\lvert A_S(0)\rvert^2$	1	$\cos\phi_s$	0	$-\sin\phi_s$

$A_0(0) \to$ CP even
$A_\parallel(0) \to$ CP even
$A_\perp(0) \to$ CP odd

......

Limited sinsitivity on ϕ_s, no FT

- $\overline{B}_s^0 \to J/\psi KK$ **contrary sign on $\sin\Delta m_s t$ & $\cos\Delta m_s t$**

Experimental:

$$\mathcal{PDF}(\mathrm{t}) \propto \epsilon(t) \cdot \epsilon(\Omega) \cdot \mathrm{e}^{-\frac{1}{2}\Delta m_s^2 \sigma_t^2} \cdot (1-2\omega) \cdot \Gamma(t,\Omega)$$

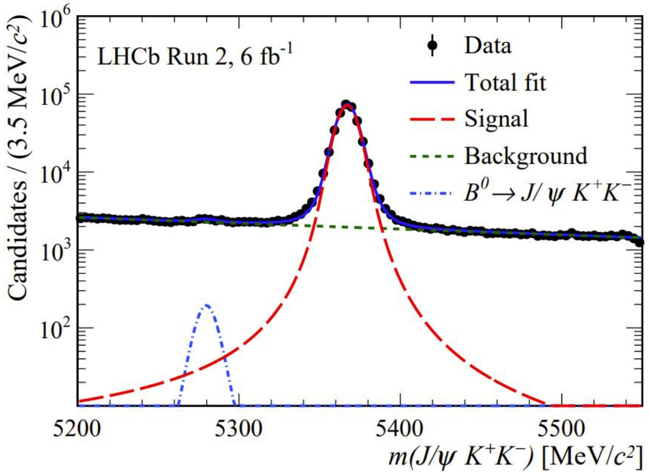
- Key steps to extract Φ_s
  - Modelling of angular acceptance and decay time acceptance
  - Flavour tagging and time resolution calibration

(The σ_t in $B_s \rightarrow J/\psi KK$ decay is found to be ~ 42 fs, and $\varepsilon_{tag}(1-2w)^2$ is found to be ~4.2%)

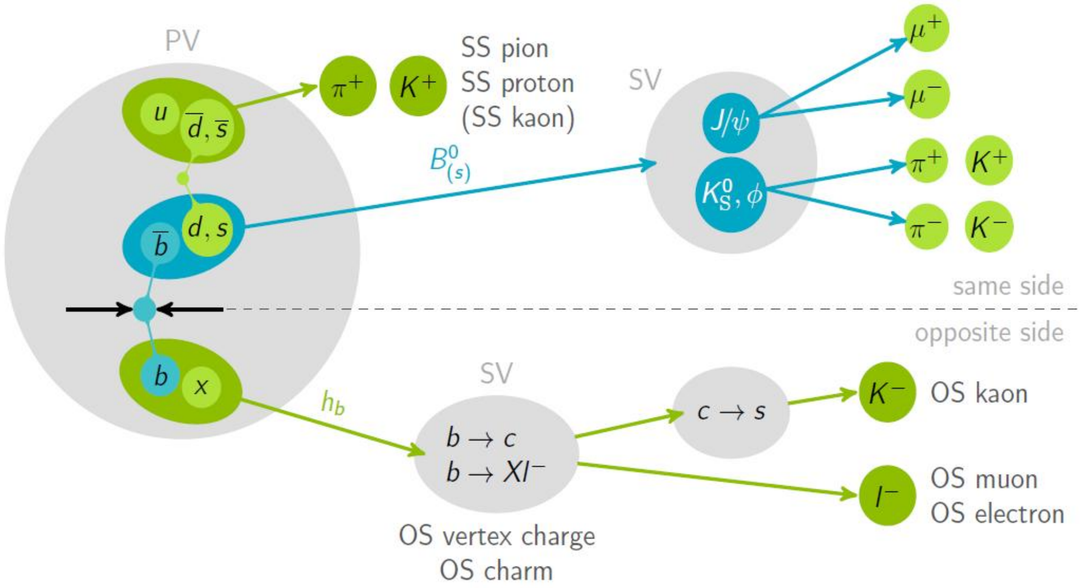
[LHCb-Paper-2023-016 In preparation]

signal candidates: 349000

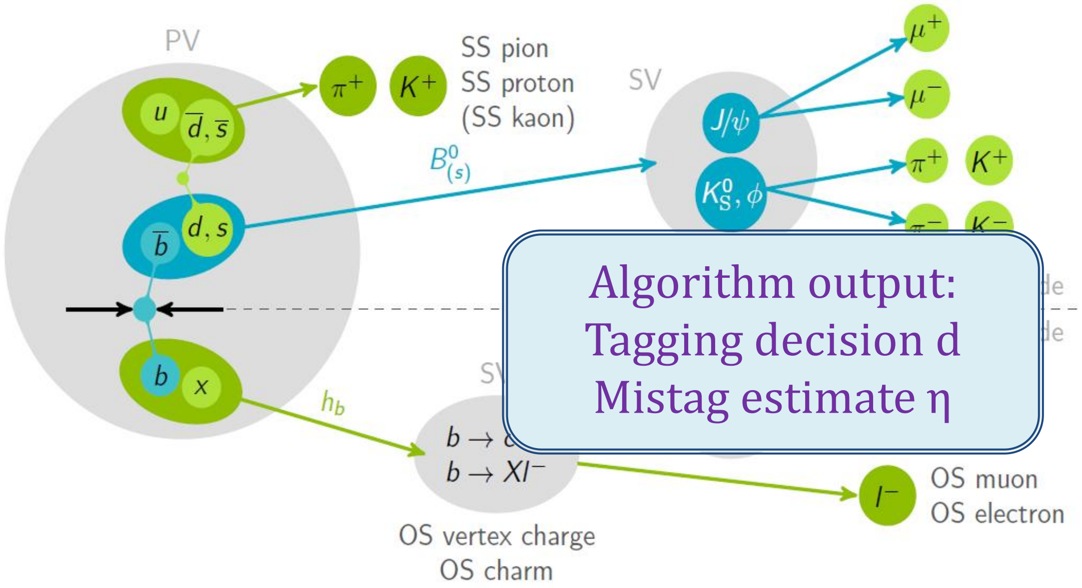
Splot technique is used for effective background subtraction [sFit]



Flavor Tagging in LHCb

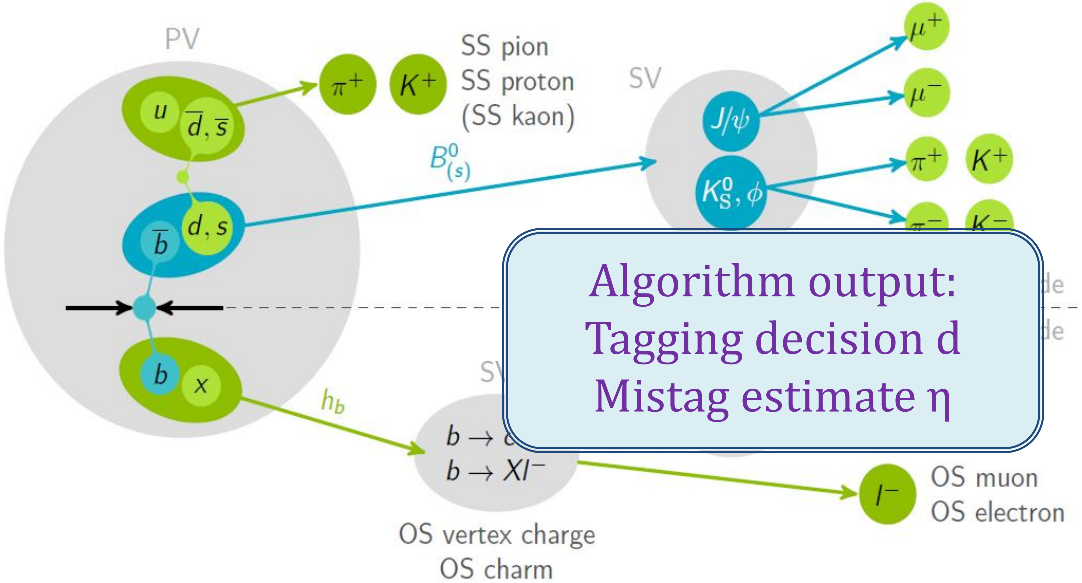


Flavor Tagging in LHCb



Algorithm output:
Tagging decision d
Mistag estimate η

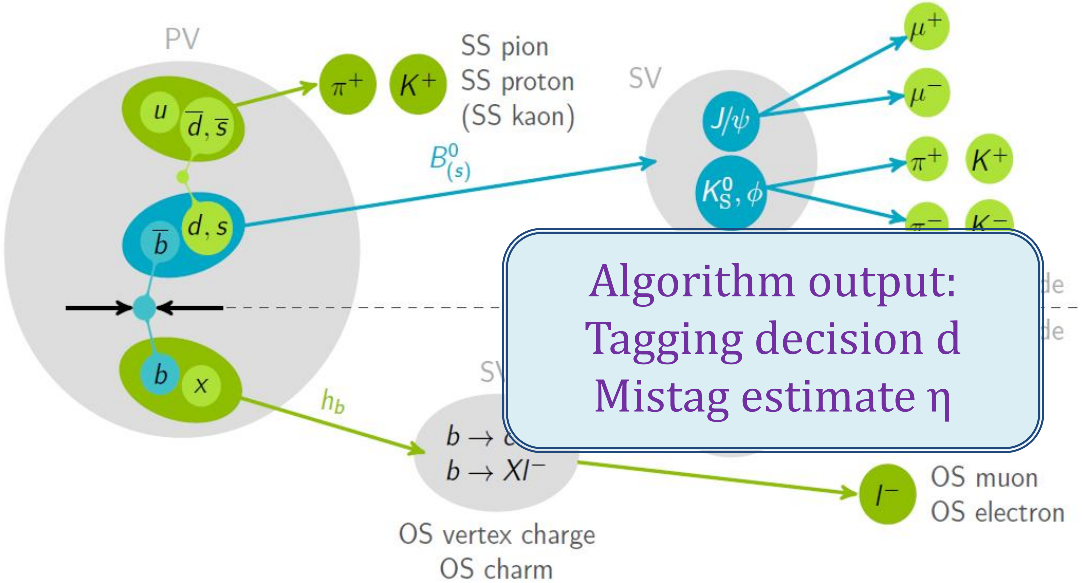
Flavor Tagging in LHCb



Algorithm output:
Tagging decision d
Mistag estimate η

Control Channel for calibration:
SS tagging: $B_s \to D_s^- \pi^+$
OS tagging: $B^+ \to J/\psi K^+$

Flavor Tagging in LHCb

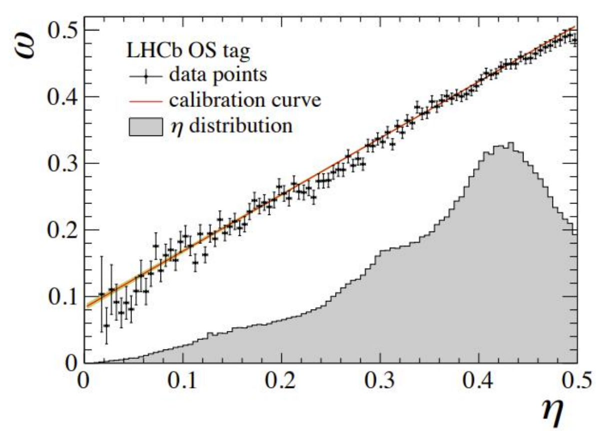


Algorithm output:
Tagging decision d
Mistag estimate η

Control Channel for calibration:
SS tagging: $B_s \to D_s^- \pi^+$
OS tagging: $B^+ \to J/\psi K^+$

The estimated mistag is calibrated using linear equation:

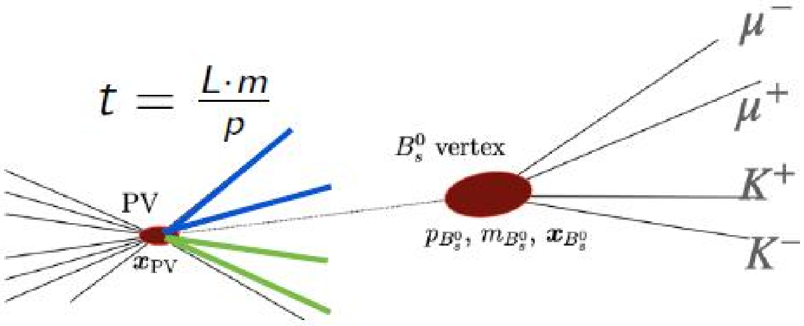
$$\omega = p_0 + p_1(\eta - \langle\eta\rangle)$$



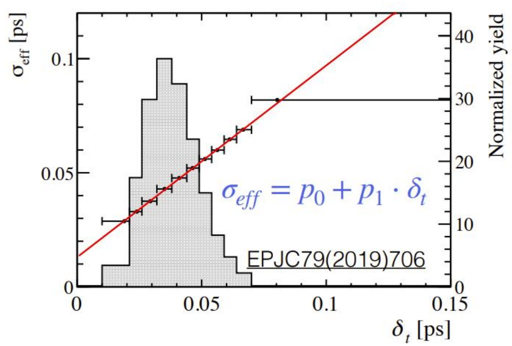
[LHCb-Paper-2023-016 In preparation]

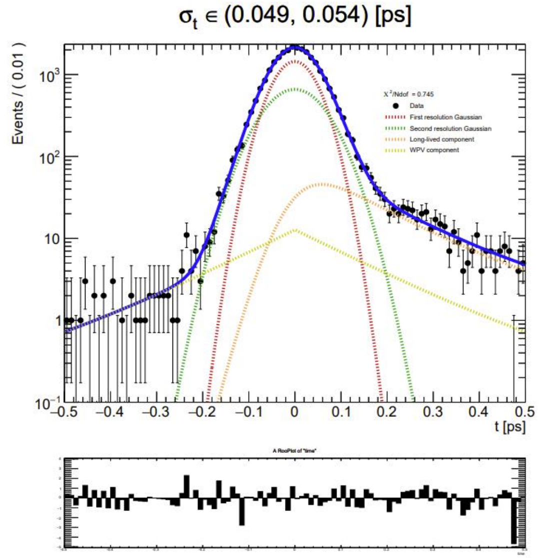
Decay-time resolution

$$\delta_t^2 \approx (\frac{m}{\rho})^2 \sigma_L^2 + (\frac{t}{\rho})^2 \sigma_p^2$$



•Calibrated using prompt J/ψKK events with all tracks coming from pp collision (PV)





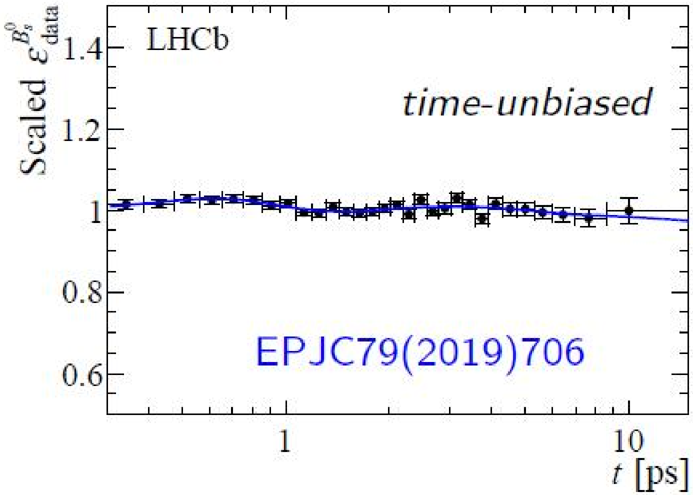
- Effective Gaussian resolution, with width parameterised as

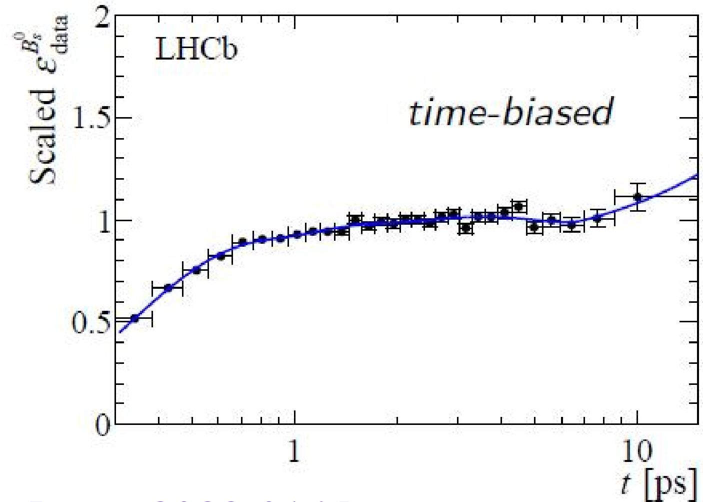
$$\sigma_{\rm eff} = p_0 + p_1 \delta_t \rightarrow 42 \text{ fs in average}, \mathcal{D} \sim 0.75$$

- Small bias (~5 fs) in VELO is corrected by adding as mean μ of the Gaussian resolution model

CP violation in $B_s \to J/\psi KK$ decays

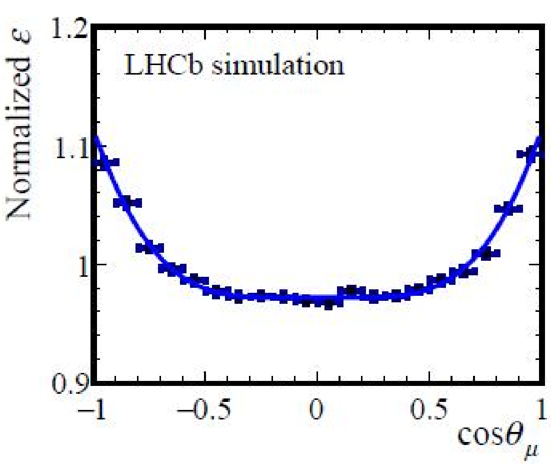
Decay-time efficiency obtained in data

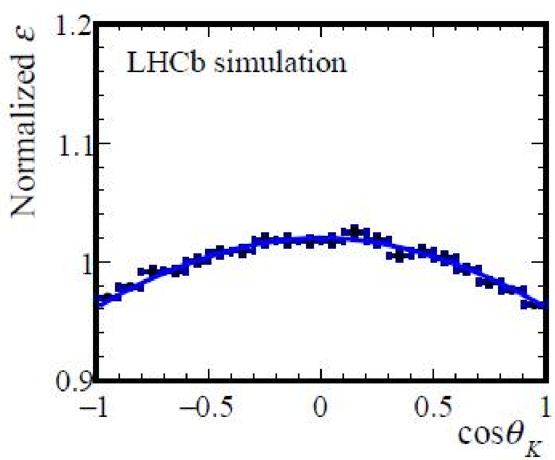


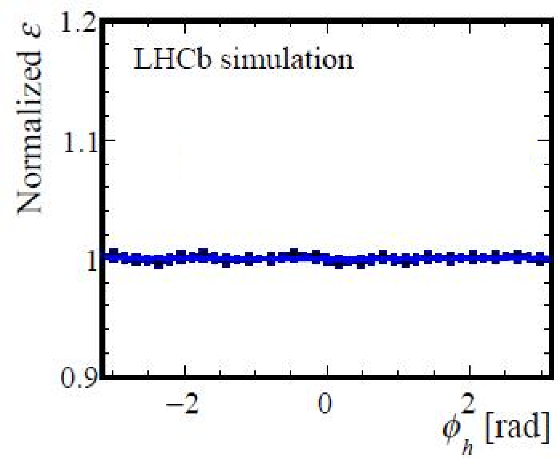


[LHCb-Paper-2023-016 In preparation]

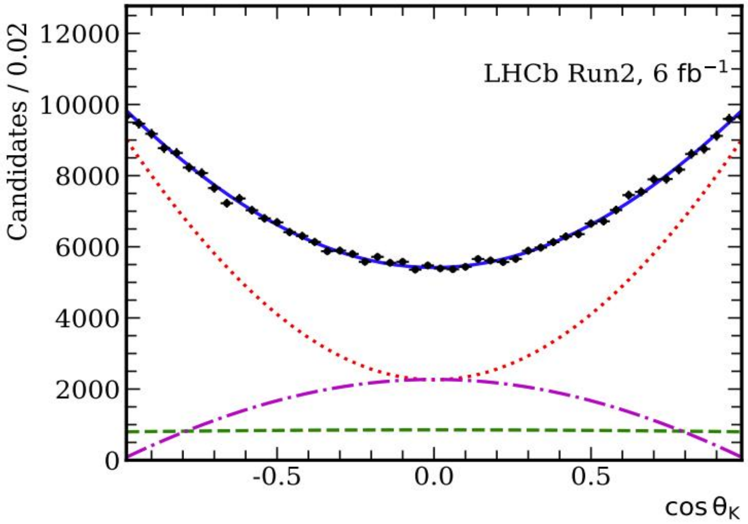
Angular efficiency obtained in data



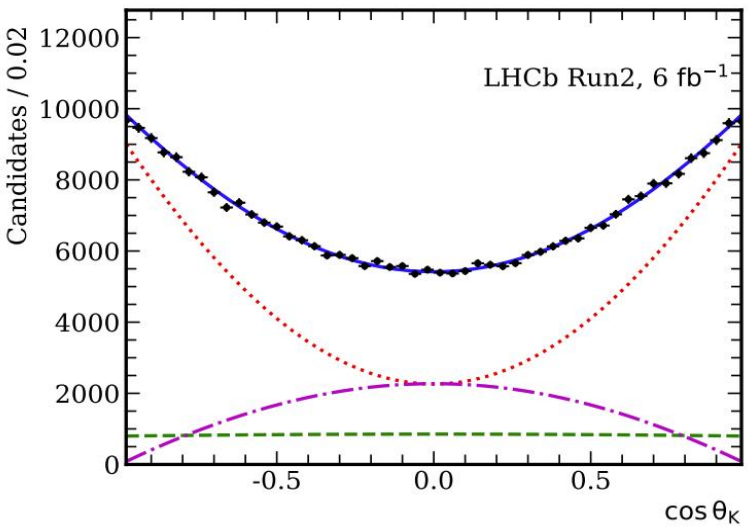


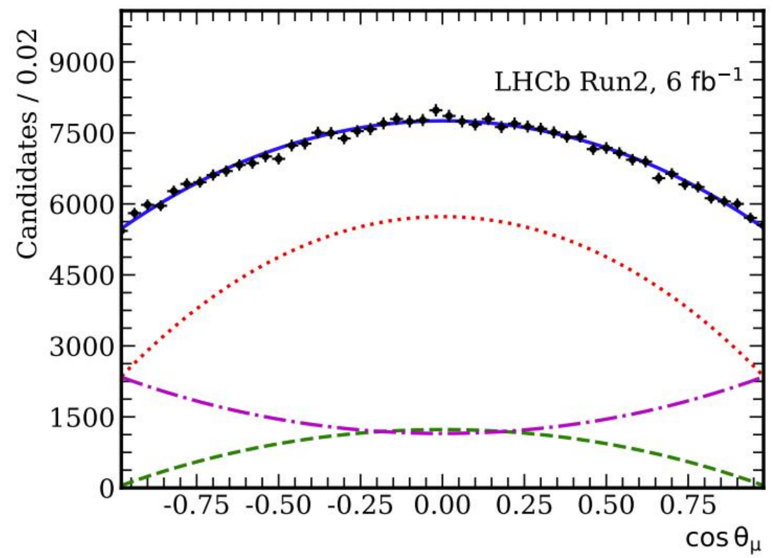


CP violation in $B_s \rightarrow J/\psi KK$ decays

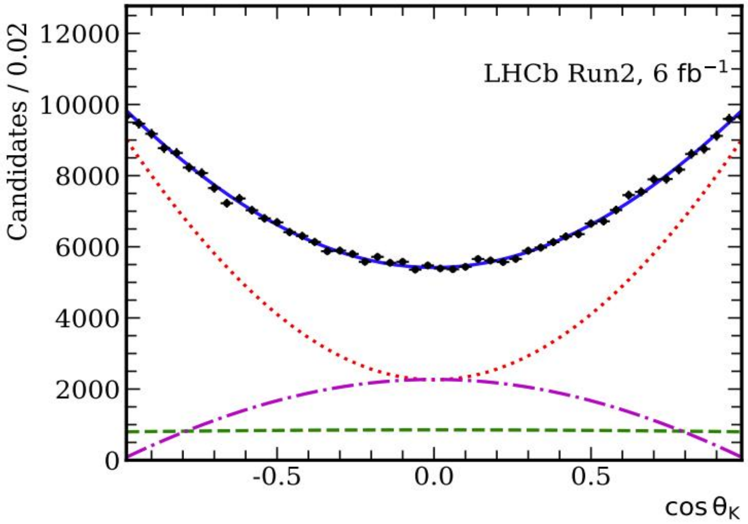


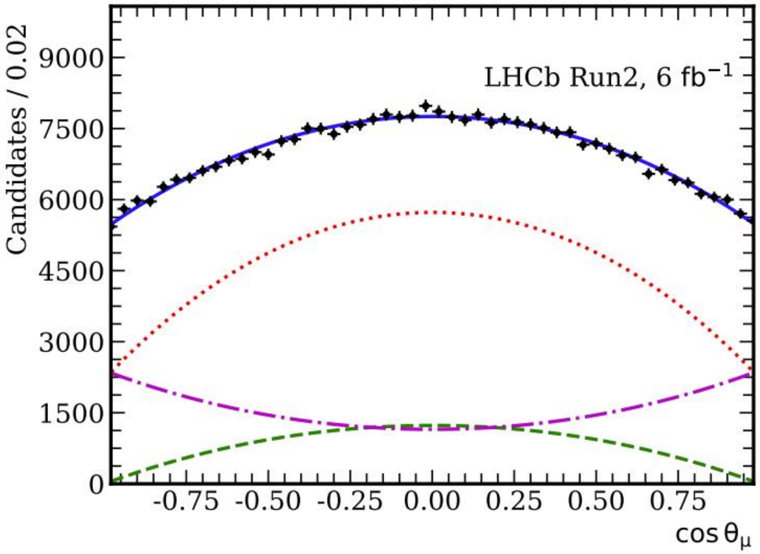
CP violation in $B_s \rightarrow J/\psi KK$ decays

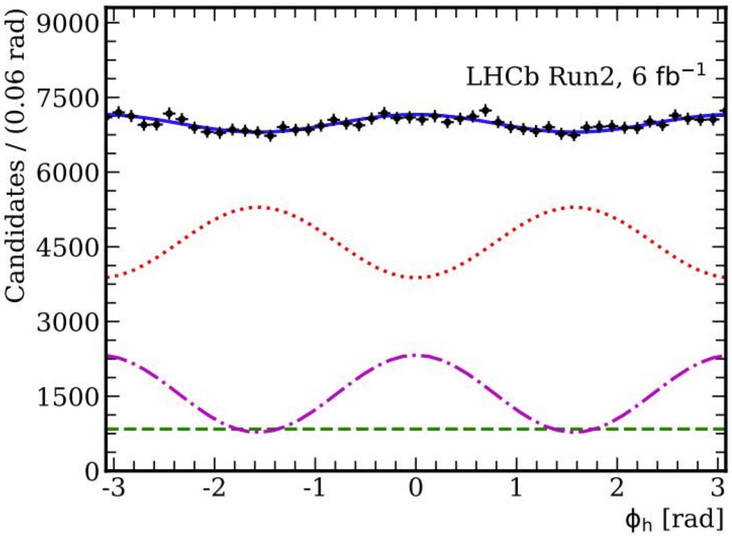




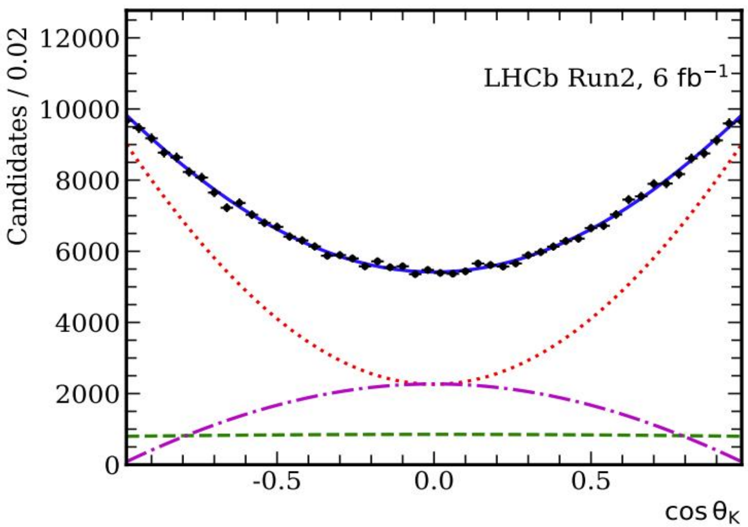
CP violation in $B_s \to J/\psi KK$ decays

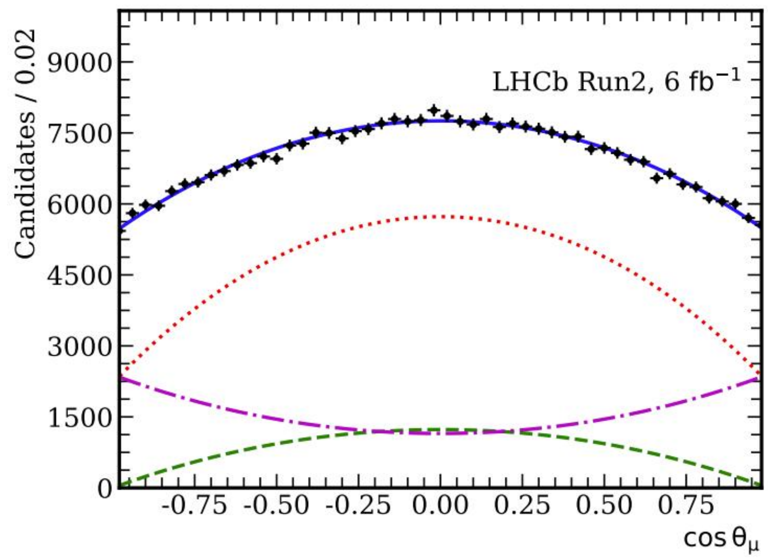


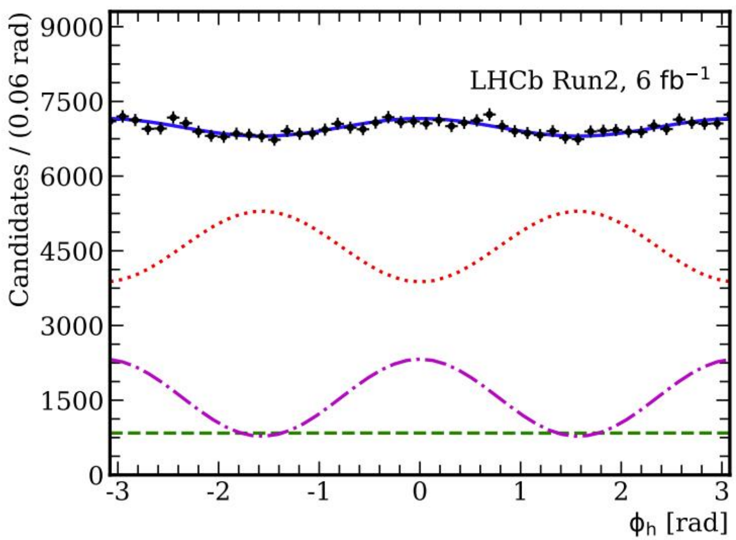


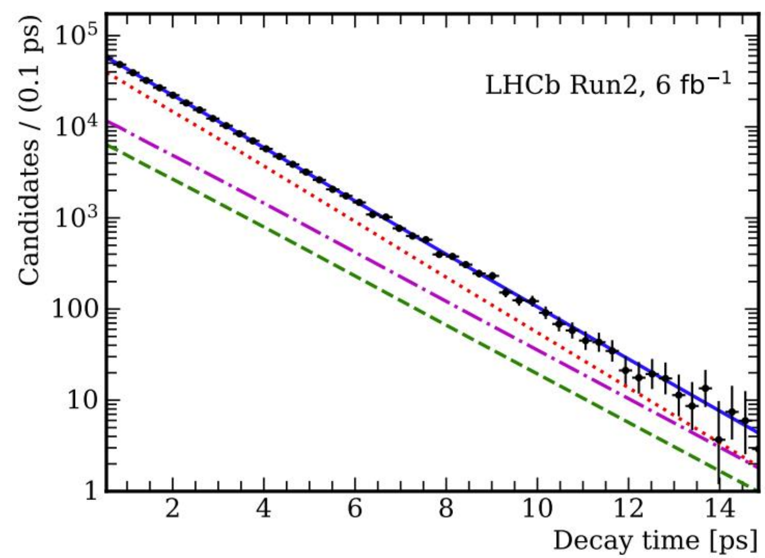


CP violation in $B_s \rightarrow J/\psi KK$ decays



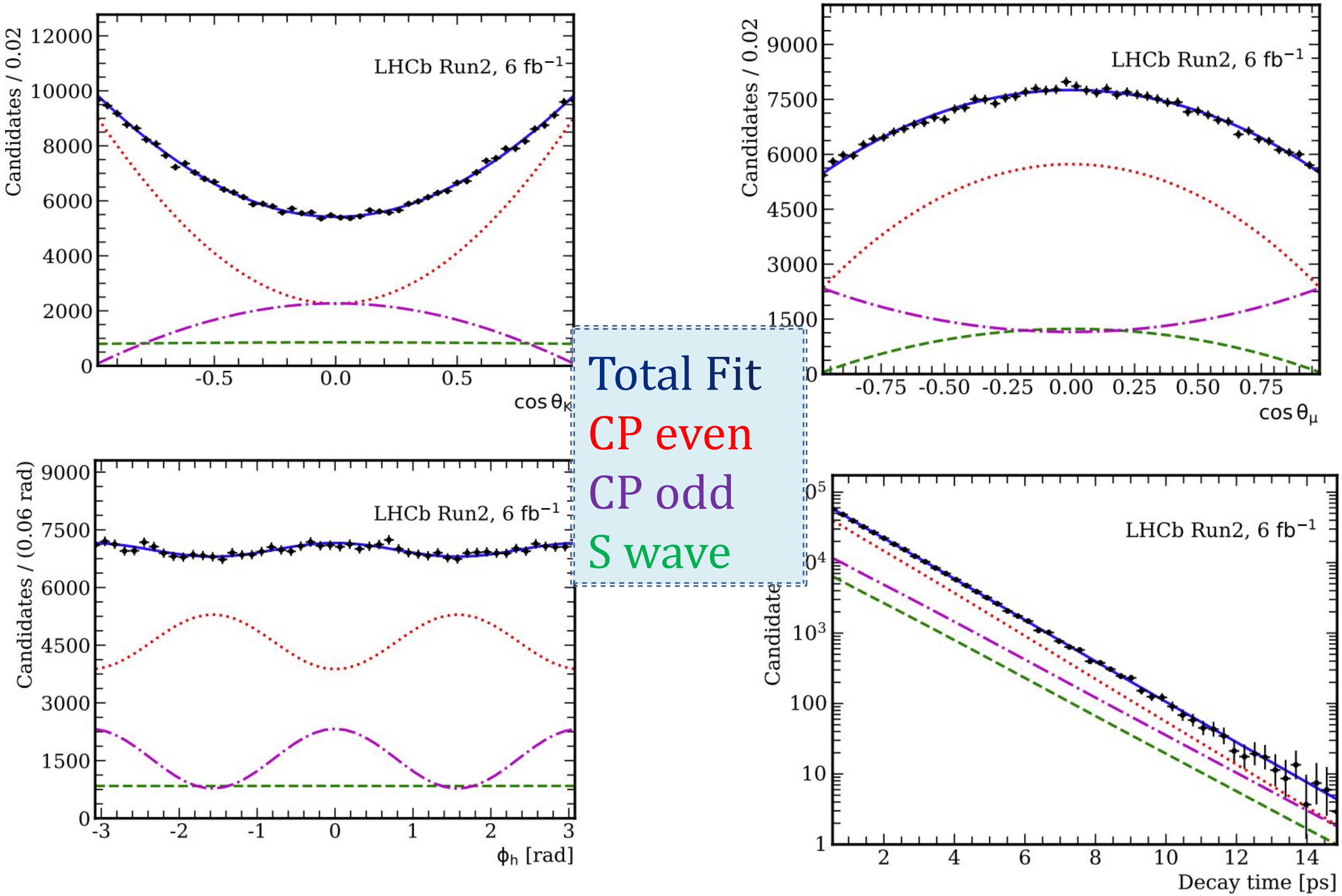




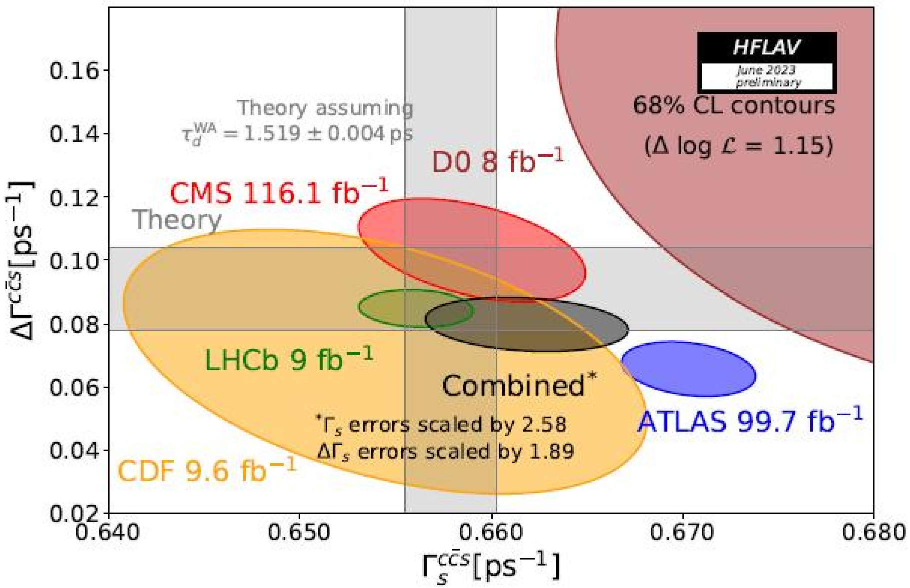


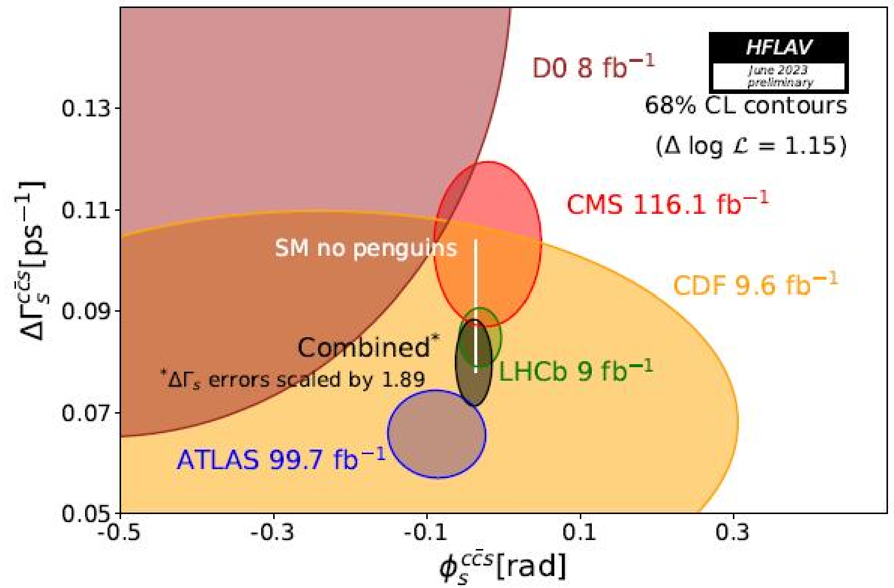
CP violation in $B_s \rightarrow J/\psi KK$ decays

[LHCb-Paper-2023-016 In preparation]



CP violation in $B_s \to J/\psi KK$ decays





[LHCb-Paper-2023-016 In preparation]

$$\phi_s[\text{rad}] = -0.039 + 0.022 \pm 0.006$$
$$|\lambda| = 1.001 \pm 0.011 \pm 0.005$$

- Consistent with the prediction of Global fits assuming SM:

$$\phi_s^{\text{CKMfitter}} \approx (-0.0368^{+0.0006}_{-0.0009})\ \text{rad},\ \phi_s^{\text{UTfitter}} = -0.0370 \pm 0.0010\ \text{rad}$$

Polarisation-dependent fit

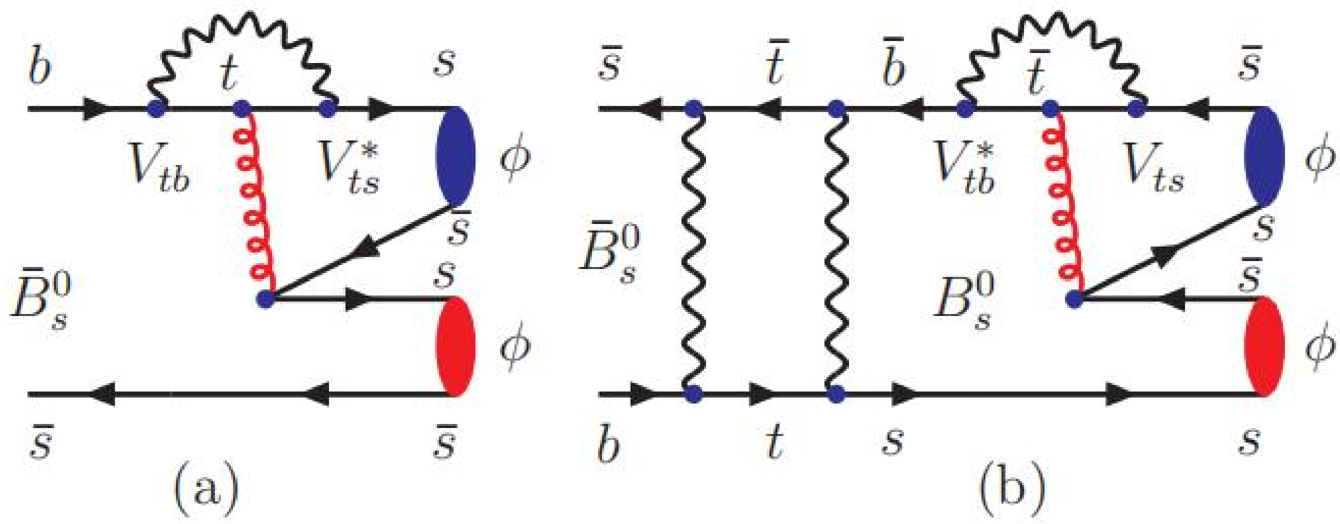
New physics effects can vary in different polarisation states

- Allow $|\lambda|$ and ϕ_s differ in polarisation states
- Shows no evidence for any polarisation dependence

Parameters	Values (stat. unc. only)
ϕ_s^0 [rad]	-0.034 ± 0.023
$\phi_s^{\parallel} - \phi_s^0$ [rad]	-0.002 ± 0.021
$\phi_s^{\perp} - \phi_s^0$ [rad]	$-0.001 ^{+0.020}_{-0.021}$
$\phi_s^{S} - \phi_s^0$ [rad]	$0.022 ^{+0.027}_{-0.026}$
$\lvert\lambda^0\rvert$	$0.969 ^{+0.025}_{-0.024}$
$\lvert\lambda^{\parallel}/\lambda^0\rvert$	$0.982 ^{+0.055}_{-0.052}$
$\lvert\lambda^{\perp}/\lambda^0\rvert$	$1.107 ^{+0.082}_{-0.076}$
$\lvert\lambda^{S}/\lambda^0\rvert$	$1.121 ^{+0.084}_{-0.078}$

[LHCb-Paper-2023-016 In preparation]

CP violation in $B_s \to \Phi\Phi$ decays



- $B_s^0 \to \phi\phi$ is a golden channel to study CP violation in $b \to s$ decays
  - ➢ Tiny CP violation expected in the SM (uplimit: 0.02 rad)
  - ➢ Sensitive to NP in B_s^0 mixing and $b \to s$ decay

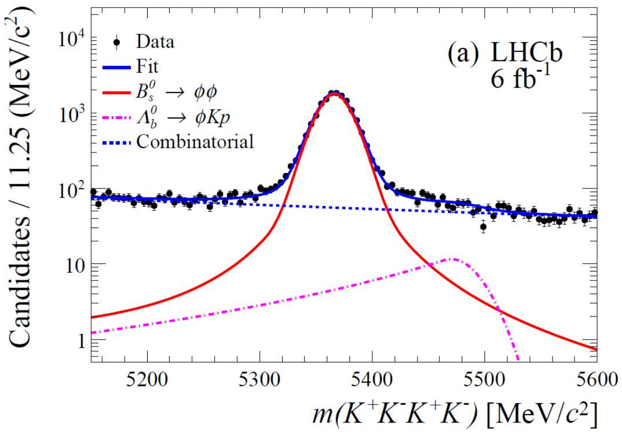
- The polarization dependence in $B_s^0 \to \phi\phi$ is tested for the first time.

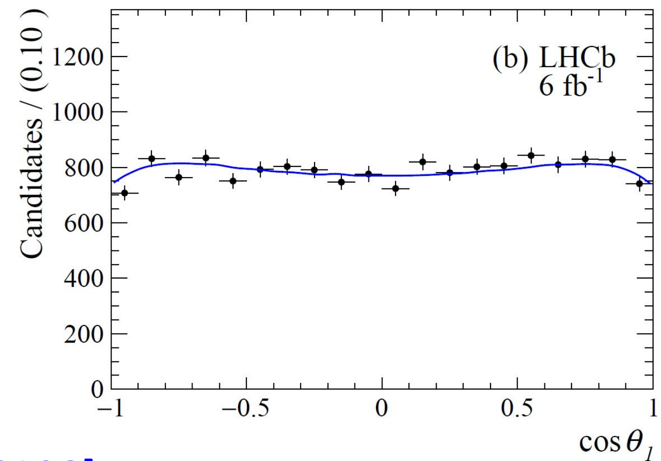
CP violation in $B_s \to \Phi\Phi$ decays

Very similar analysis strategy as $B_s^0 \to J/\psi K^+K^-$

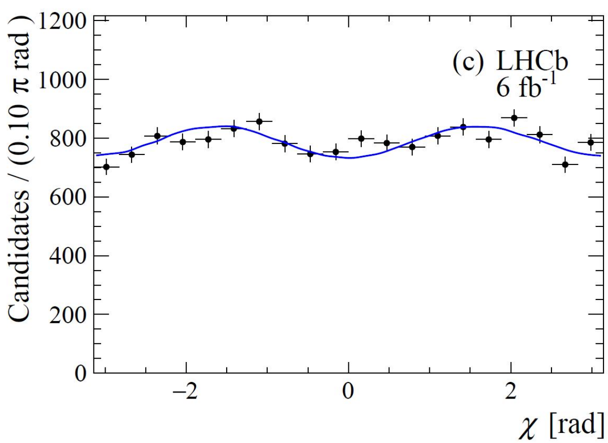
→ 1. Flavor-tagged time-dependent angular analysis

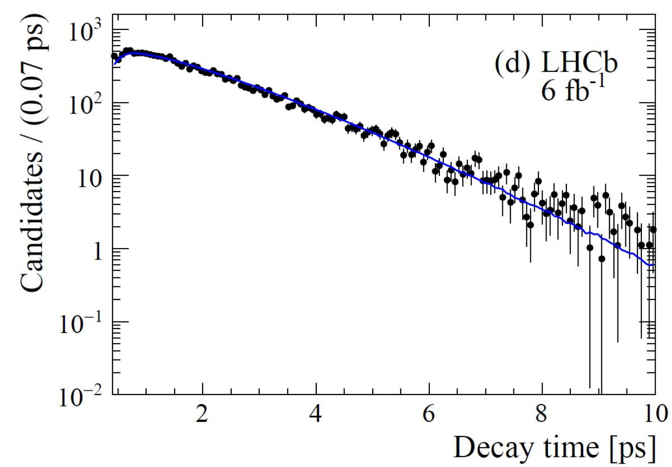
2.Very small S-wave contribution thus negaligible





[arxiv.2304.06198]

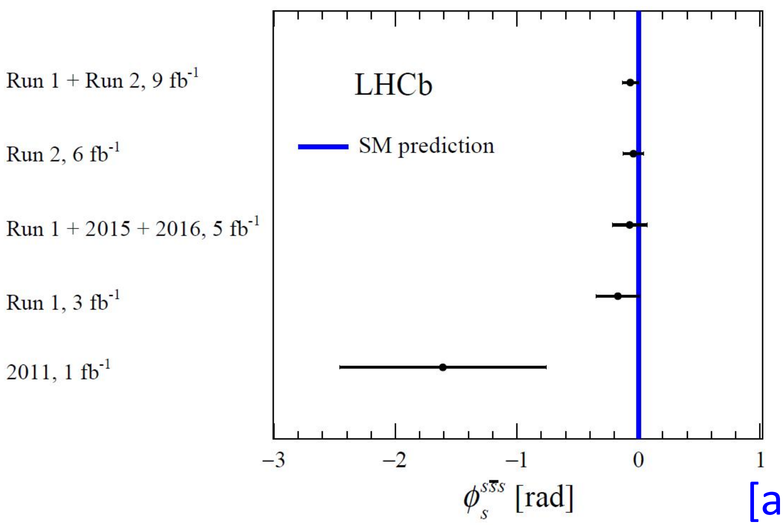


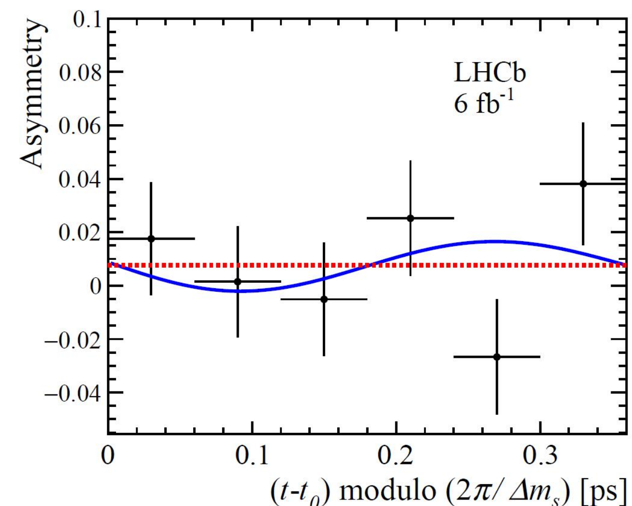


Fitting Result

$$\phi_s^{s\bar{s}s} = -0.042 \pm 0.077 \pm 0.010 \text{ rad}$$
$$|\lambda| = 1.004 \pm 0.030 \pm 0.009$$

- Consistent with SM expectation
- No sign of polarisation-dependence observed

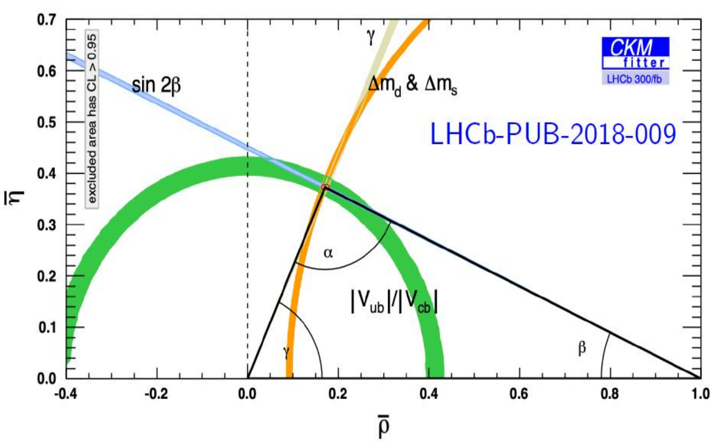


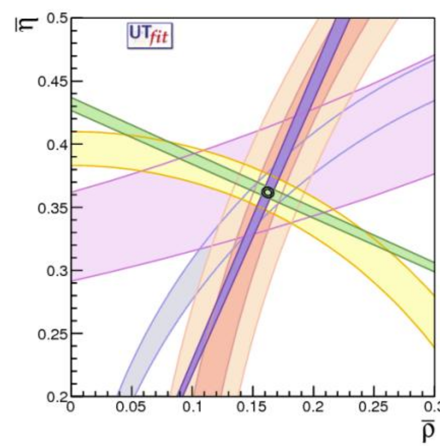


[arxiv.2304.06198]

Summary

- Time-dependent measurements of CP violation are peformed with the full LHCb Run2 data sample
  - $B^0 \to \psi K_S^0$: $\sin(2\beta) = 0.716 \pm 0.013 \pm 0.008$
  - $B_s \to J/\psi KK$: $\phi_s = -0.039 \pm 0.022 \pm 0.006$ rad
  - $B_s \to \Phi\Phi$: $\phi_s^{s\bar{s}s} = -0.042 \pm 0.075 \pm 0.009$ rad
- LHCb dominates the world average of many CPV measurements
- Still statistics limited, Upgrade I and II needed to further test the SM and search for NP indirectly





arxiv.1812.07638

Time-dependent angular fit

EPJC79(2019)706

$$\mathscr{P}(t,\theta_K,\theta_\mu,\phi_h\,|\,\delta_t) \propto \sum_{k=1}^{10} N_k h_k(t) f_k(\theta_K,\theta_\mu,\phi_h) \quad \rightarrow \phi_s,\ \Delta m_s,\ \Delta\Gamma_s,\ \Gamma_s-\Gamma_d$$

$$\mathcal{P}\left(t,\Omega|\mathrm{q}^{\mathrm{OS}},\mathrm{q}^{\mathrm{SSK}},\eta^{\mathrm{OS}},\eta^{\mathrm{SSK}},\delta_t\right) \propto \sum_{k=1}^{10} C_{\mathrm{SP}}^k N_k f_k(\Omega)\varepsilon_{\mathrm{data}}^{B_s^0}(t) \cdot \left\{\left[\mathcal{Q}\left(\mathrm{q}^{\mathrm{OS}},\mathrm{q}^{\mathrm{SSK}},\eta^{\mathrm{OS}},\eta^{\mathrm{SSK}}\right) h_k\left(t|B_s^0\right) + \bar{\mathcal{Q}}\left(\mathrm{q}^{\mathrm{OS}},\mathrm{q}^{\mathrm{SSK}},\eta^{\mathrm{OS}},\eta^{\mathrm{SSK}}\right) h_k\left(t|\bar{B}_s^0\right)\right] \otimes \mathcal{R}\left(t-t'|\delta_t\right)\right\}$$

Angular amplitudes

C_{SP}^k account for the interference between P- and S- wave

flavor tagging

time-dependent oscillation

decay-time efficiency

decay-time resolution

$$h_k(t|B_s^0) = \frac{3}{4\pi}e^{-\Gamma t}\left(a_k\cosh\frac{\Delta\Gamma t}{2} + b_k\sinh\frac{\Delta\Gamma t}{2} + c_k\cos(\Delta m t) + d_k\sin(\Delta m t)\right),$$

$$h_k(t|\bar{B}_s^0) = \frac{3}{4\pi}e^{-\Gamma t}\left(a_k\cosh\frac{\Delta\Gamma t}{2} + b_k\sinh\frac{\Delta\Gamma t}{2} - c_k\cos(\Delta m t) - d_k\sin(\Delta m t)\right),$$

a_k, b_k, c_k, d_k involve strong and weak phases (δ, ϕ_s) of each component

k	A_k	$f_k(\theta_\mu,\theta_K,\varphi_h)$
1	$\|A_0\|^2$	$2\cos^2\theta_K\sin^2\theta_\mu$
2	$\|A_\parallel\|^2$	$\sin^2\theta_k(1-\sin^2\theta_\mu\cos^2\varphi_h)$
3	$\|A_\perp\|^2$	$\sin^2\theta_k(1-\sin^2\theta_\mu\sin^2\varphi_h)$
4	$\|A_\parallel A_\perp\|$	$\sin^2\theta_k\sin^2\theta_\mu\sin 2\varphi_h$
5	$\|A_0 A_\parallel\|$	$\frac{1}{2}\sqrt{2}\sin 2\theta_k\sin 2\theta_\mu\cos\varphi_h$
6	$\|A_0 A_\perp\|$	$-\frac{1}{2}\sqrt{2}\sin 2\theta_k\sin 2\theta_\mu\sin\varphi_h$
7	$\|A_S\|^2$	$\frac{2}{3}\sin^2\theta_\mu$
8	$\|A_S A_\parallel\|$	$\frac{1}{3}\sqrt{6}\sin\theta_k\sin 2\theta_\mu\cos\varphi_h$
9	$\|A_S A_\perp\|$	$-\frac{1}{3}\sqrt{6}\sin\theta_k\sin 2\theta_\mu\sin\varphi_h$
10	$\|A_S A_0\|$	$\frac{4}{3}\sqrt{3}\cos\theta_K\sin^2\theta_\mu$

2.Polarization dependent test of $B_s \rightarrow \Phi\Phi$

$$\phi_{s,0} = -0.18 \pm 0.09 \text{ rad}, \qquad |\lambda_0| = 1.02 \pm 0.17,$$
$$\phi_{s,\parallel} - \phi_{s,0} = 0.12 \pm 0.09 \text{ rad}, \qquad |\lambda_\perp/\lambda_0| = 0.97 \pm 0.22,$$
$$\phi_{s,\perp} - \phi_{s,0} = 0.17 \pm 0.09 \text{ rad}, \qquad |\lambda_\parallel/\lambda_0| = 0.78 \pm 0.21,$$

3.CKM matrix representation in phase convention.

$$V_{CKM} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{+i\beta_s} & |V_{tb}| \end{pmatrix} + \mathcal{O}(\lambda^5).$$