TMDWFs and Soft functionsat one-loop in LaMET

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Introduction

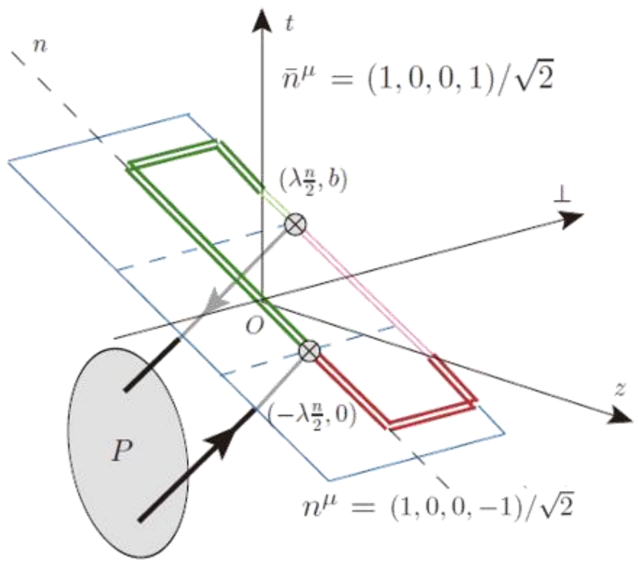
- The exploration of underlying structures of hadrons has always been one of the most important frontiers in particle and nuclear physics. Wave functions are important physical quantities describing the distributions of constituents' momentum in the hadron

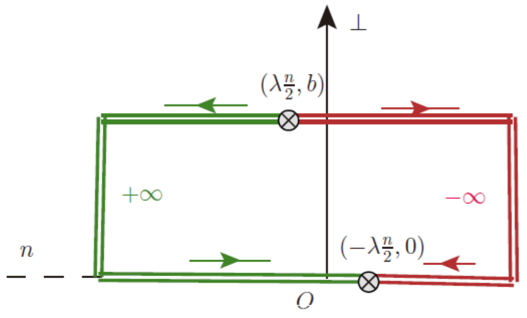
- In an exclusive process the non-perturbative LFWFs for a given Fock state are required, like the theoretical analyses of B meson weak decays. [1-3]

- In LaMET, one can construct the directly computable hadron matrix elements with non-local operators, named as quasi-distributions, on the lattice. [4-6]

[1] H. n. Li and H. L. Yu, Phys. Rev. D 53, 2480-2490 (1996)
[2] Y. Y. Keum, H. N. Li and A. I. Sanda, Phys. Rev. D 63, 054008 (2001)
[3] Y. Y. Keum, H. n. Li and A. I. Sanda, Phys. Lett. B 504, 6-14 (2001)
[4] X. Ji, Phys. Rev. Lett. 110, 262002 (2013)
[5] X. Ji, Sci. China Phys. Mech. Astron. 57, 1407-1412(2014)
[6] K. Cichy and M. Constantinou, Adv. High EnergyPhys. 2019, 3036904 (2019)

TMDWFs and Soft Function





For a pseudoscalar pion, the TMDWF is defined as:

$$\psi^{\pm}(x, b_\perp, \mu, \delta^-) = \frac{1}{-if_\pi P^+}\int \frac{d(\lambda P^+)}{2\pi} e^{-i(x-\frac{1}{2})P^+\lambda} \left\langle 0 \left| \overline{\Psi}_n^{\pm}(\lambda n/2 + b)\gamma^+\gamma^5 \Psi_n^{\pm}(-\lambda n/2) \right| P \right\rangle |_{\delta^-}$$

$$P^\mu = (P^z, 0, 0, P^z) \quad b^\mu = (0, \vec{b}_\perp, 0)$$

Field with gauge-link and delta regulator:

$$\Psi_n^{\pm}(\xi)|_{\delta^-} = \mathcal{P} e^{ig\int_0^{\pm\infty} ds n\cdot A(\xi+sn) e^{-\frac{\delta^-}{2}|s|}} \psi(\xi).$$

Normalization factor from local operator matrix:

$$\left\langle 0 \left| \overline{\psi}(0)\gamma^\mu\gamma^5\psi(0) \right| \pi \right\rangle = -if_\pi P^\mu$$

TMDWFs and Soft Function

The rapidity divergences of TMDWF can be renormalized by the on-light-cone soft function, which defined as :

$$S^{\pm}(b_\perp,\mu,\delta^+,\delta^-) = \frac{1}{N_c}\mathrm{tr}\langle 0|\mathcal{T}W_{\bar n}^{-\dagger}(b_\perp)|_{\delta^+}W_n^{\pm}(b_\perp)|_{\delta^-}W_n^{\pm\dagger}(0)|_{\delta^-}W_{\bar n}^{-}(0)|_{\delta^+}|0\rangle.$$

the physical TMDWFs amplitudes are defined as :

$$\Psi^{\pm}_{\bar q q}(x,b_\perp,\mu,\zeta) = \lim_{\delta^-\to 0}\frac{\psi^{\pm}_{\bar q q}(x,b_\perp,\mu,\delta^-)}{\sqrt{S^{\pm}(b_\perp,\mu,\delta^- e^{2y_n},\delta^-)}}$$

$$\zeta = 2(xP^+)^2 e^{2y_n} \text{ with } e^{2y_n} = \delta^+/\delta^-$$

Rapidity scale

TMDWFs and Soft Function

The four-quark form factor which could be used to obtained the soft function is defined as:

$$F(b_\perp, P_1, P_2, \mu) = \frac{\langle P_2 | (\bar{\psi}_a \Gamma \psi_b)(b)(\bar{\psi}_c \Gamma' \psi_d)(0) | P_1 \rangle}{f_\pi^2 P_1 \cdot P_2}$$

$$\Gamma = I\,,\ \gamma_5,\ \gamma_\perp \text{ or } \gamma_\perp\gamma_5 \qquad P_1^\mu = (P^z, 0, 0, P^z) \qquad P_2^\mu = (P^z, 0, 0, -P^z)$$

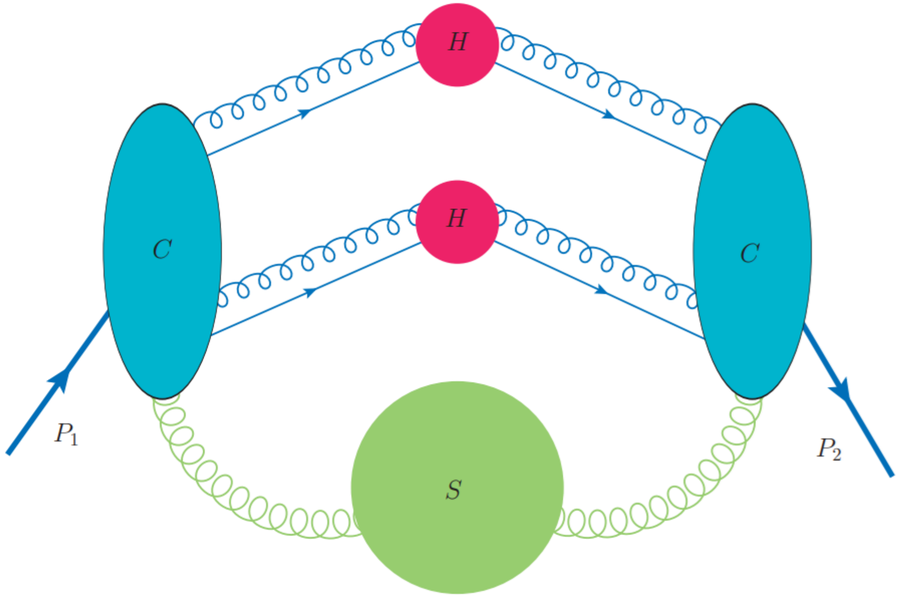
Normalization factor from local operator matrix:

$$\langle 0 | \overline{\psi}(0) \gamma^\mu \gamma^5 \psi(0) | P_1 \rangle = -i f_\pi P_1^\mu,$$
$$\langle P_2 | \overline{\psi}(0) \gamma_\mu \gamma^5 \psi(0) | 0 \rangle = i f_\pi P_{2\mu},$$

TMDWFs and Soft Function

Factorization form give the relation between TMDWF and form factor:

$$F(b_\perp, P_1, P_2, \mu) = \int dx_1 dx_2 H_F(Q^2, \bar{Q}^2, \mu^2) \left[\frac{\psi^{\pm}_{\bar{q}q}(x_2, b_\perp, \mu, \delta'^{+})}{\sqrt{S^{\pm}(b_\perp, \mu, \delta'^{+}, \delta^{-})}}\right]^{\dagger} \left[\frac{\psi^{\pm}_{\bar{q}q}(x_1, b_\perp, \mu, \delta'^{-})}{\sqrt{S^{\pm}(b_\perp, \mu, \delta^{+}, \delta'^{-})}}\right] \frac{S^{\pm}(b_\perp, \mu, \delta^{+}, \delta^{-})}{\sqrt{S^{\pm}(b_\perp, \mu, \delta'^{+}, \delta^{-}) S^{\pm}(b_\perp, \mu, \delta^{+}, \delta'^{-})}}.$$



The hard kernel is insensitive to the hadron, which could be calculated by partonic state.

In partonic state:

$$\left\langle 0 \left| \overline{\psi}_{\bar{q}}(0)\, \gamma^{+}\gamma^{5}\psi_{q}(0) \right| q\bar{q} \right\rangle |_{\text{tree}} = 2P^{+}$$

$$\psi^{\pm}_{\bar{q}q}(x, b_{\perp}, \mu, \delta^{-}) = \frac{1}{2P^{+}} \int \frac{d(\lambda P^{+})}{2\pi} e^{-i(x-\frac{1}{2})P^{+}\lambda} \left\langle 0 \left| \overline{\Psi}^{\pm}_{n}(\lambda n/2 + b)\, \gamma^{+}\gamma^{5}\Psi^{\pm}_{n}(-\lambda n/2) \right| q\bar{q} \right\rangle |_{\delta^{-}}$$

Quark pair to have the same spin and parity with pion.

$$F(b_{\perp}, P_1, P_2, \mu) = \frac{\left\langle \bar{q}_d(\bar{x}_2 P_2)\, q_a(x_2 P_2) \left| (\bar{\psi}_a \Gamma \psi_b)(b)(\bar{\psi}_c \Gamma \psi_d)(0) \right| q_b(x_1 P_1)\, \bar{q}_c(\bar{x}_1 P_1) \right\rangle}{4 P_1 \cdot P_2}$$

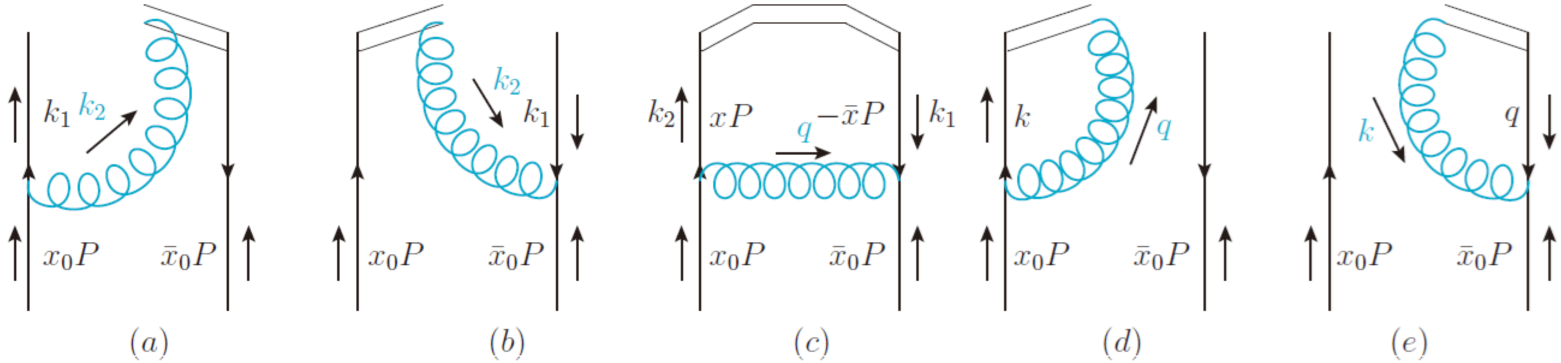
The TMDWFs:

$$\psi^{\pm}_{\bar{q}q}(x, b_\perp, \mu, \delta^-) = \delta(x - x_0) + \frac{\alpha_s C_F}{2\pi}\Big[f(x, x_0, b_\perp, \mu)\Big]_+ + \frac{\alpha_s C_F}{2\pi}\delta(x - x_0)\left[L_b\left(\frac{3}{2} + \ln\frac{-\delta^{-2} \mp i0}{4\bar{x}xP^{+2}}\right) + \frac{1}{2}\right]$$

Rapidity divergence

$$f(x, x_0, b_\perp, \mu) = \left[\left(\frac{x}{x_0(x - x_0)} - \frac{x}{x_0}\right)\left(\frac{1}{\epsilon_{\rm IR}} + L_b\right) + \frac{x}{x_0}\right]\theta(x_0 - x) + \{x \to 1 - x, x_0 \to 1 - x_0\}.$$

$$L_b = \ln\frac{\mu^2 b_\perp^2}{4e^{-2\gamma_E}} \qquad \mu = \mu_0 e^{(\ln(4\pi) - \gamma_E)/2}$$



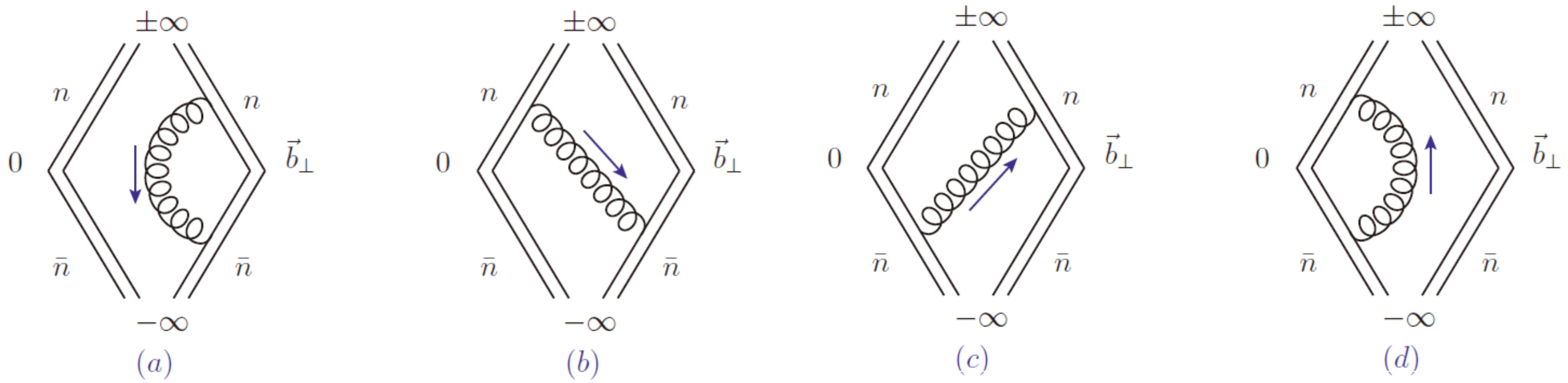
TMDWFs and Soft Function

The soft function:

$$S^{\pm}(b_\perp,\mu,\delta^+,\delta^-)=1+\frac{\alpha_s C_F}{2\pi}\left(L_b^2+2L_b\ln\frac{\mp\delta^-\delta^+-i0}{2\mu^2}+\frac{\pi^2}{6}\right)$$

The physical TMDWFs :

$$\Psi^{\pm}_{\bar{q}q}(x,b_\perp,\mu,\zeta)=\delta(x-x_0)+\frac{\alpha_s C_F}{2\pi}[f(x,x_0,b_\perp,\mu)]_+ +\frac{\alpha_s C_F}{2\pi}\delta(x-x_0)\left\{-\frac{L_b^2}{2}+L_b\left(\frac{3}{2}+\ln\frac{\mu^2}{\pm\sqrt{\zeta\bar{\zeta}}-i0}\right)+\frac{1}{2}-\frac{\pi^2}{12}\right\}$$



TMDWFs and Soft Function

The form factor:

$$F^0 = \begin{cases} \frac{1}{4N_c}, & \text{for} \quad \Gamma = I \\ -\frac{1}{4N_c}, & \text{for} \quad \Gamma = \gamma_5,\ \gamma_\perp \text{ or } \gamma_\perp\gamma_5. \end{cases}$$

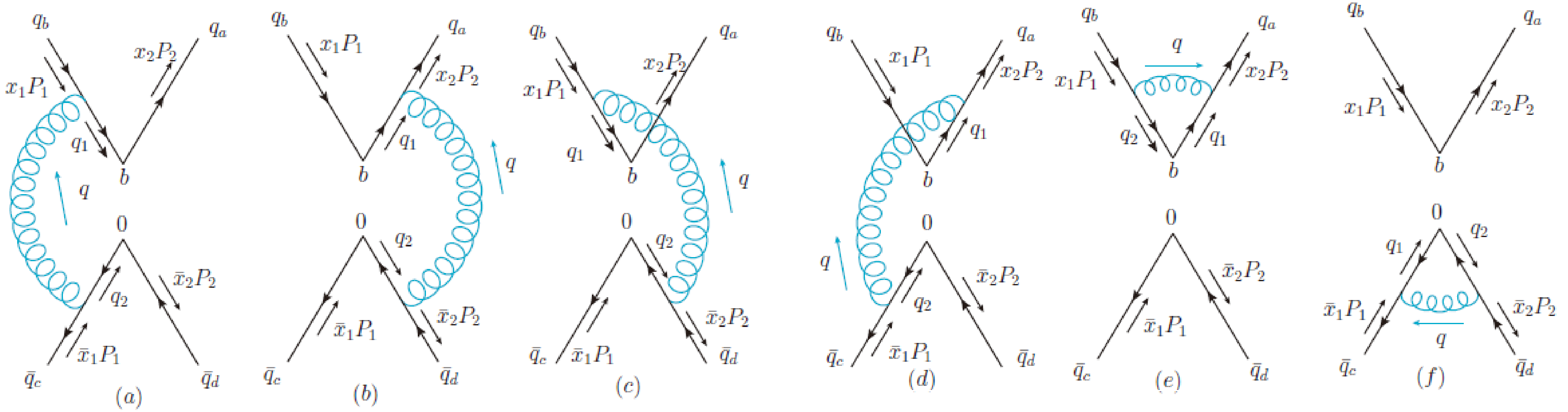
$\Gamma = I, \gamma_5$

$$F(b_\perp, P_1, P_2, \mu) = F^0\left\{1 - \frac{\alpha_s C_F}{2\pi}\left[L_b^2 + L_b\left(\ln\frac{4Q^2\bar{Q}^2}{\mu^4} - 3\right) + \frac{1}{2}\ln^2\frac{2Q^2}{\mu^2} + \frac{1}{2}\ln^2\frac{2\bar{Q}^2}{\mu^2} + 1 - \frac{3}{\epsilon_{\rm UV}}\right]\right\}.$$

$\Gamma = \gamma_\perp,\ \gamma_\perp\gamma_5$

$$F(b_\perp, P_1, P_2, \mu) = F^0\left\{1 - \frac{\alpha_s C_F}{2\pi}\left[L_b^2 + L_b\left(\ln\frac{4Q^2\bar{Q}^2}{\mu^4} - 3\right) - \frac{3}{2}\ln\frac{4Q^2\bar{Q}^2}{\mu^4} + \frac{1}{2}\ln^2\frac{2Q^2}{\mu^2} + \frac{1}{2}\ln^2\frac{2\bar{Q}^2}{\mu^2} + 7\right]\right\}.$$

$$Q^2 = x_1 x_2 P_1 \cdot P_2, \quad \bar{Q}^2 = \bar{x}_1\bar{x}_2 P_1 \cdot P_2$$



TMDWFs and Soft Function

Factorization formula:

$$F(b_\perp, P_1, P_2, \mu) = \int dx_1 dx_2 H_F(Q^2, \bar{Q}^2, \mu^2) \left[\frac{\psi^{\pm}_{\bar{q}q}(x_2, b_\perp, \mu, \delta'^{+})}{\sqrt{S^{\pm}(b_\perp, \mu, \delta'^{+}, \delta^{-})}}\right]^\dagger \left[\frac{\psi^{\pm}_{\bar{q}q}(x_1, b_\perp, \mu, \delta'^{-})}{\sqrt{S^{\pm}(b_\perp, \mu, \delta^{+}, \delta'^{-})}}\right] \frac{S^{\pm}(b_\perp, \mu, \delta^{+}, \delta^{-})}{\sqrt{S^{\pm}(b_\perp, \mu, \delta'^{+}, \delta^{-}) S^{\pm}(b_\perp, \mu, \delta^{+}, \delta'^{-})}}.$$

$$H_F^{(0)} = \begin{cases} \dfrac{1}{4N_c}, & \Gamma = I \\ -\dfrac{1}{4N_c}, & \Gamma = \gamma_5,\ \gamma_\perp \text{ or } \gamma_\perp\gamma_5 \end{cases}$$

$\Gamma = I, \gamma_5$

$$H_F(Q^2, \bar{Q}^2) = H_F^{(0)}\left[1 + \frac{\alpha_s C_F}{2\pi}\left(-\frac{1}{2}\ln^2\frac{2Q^2}{\mu^2} - \frac{1}{2}\ln^2\frac{2\bar{Q}^2}{\mu^2} + \frac{\pi^2}{6} - 2\right)\right]$$

$\Gamma = \gamma_\perp,\ \gamma_\perp\gamma_5$

$$H_F(Q^2, \bar{Q}^2) = H_F^{(0)}\left[1 + \frac{\alpha_s C_F}{2\pi}\left(\frac{3}{2}\ln\frac{4Q^2\bar{Q}^2}{\mu^4} - \frac{1}{2}\ln^2\frac{2Q^2}{\mu^2} - \frac{1}{2}\ln^2\frac{2\bar{Q}^2}{\mu^2} + \frac{\pi^2}{6} - 8\right)\right]$$

Expansion by regions

Expansion by regions: three modes at leading power

hard:	$p_h^\mu \sim (Q, Q, Q),$
collinear:	$p_c^\mu \sim (Q, \Lambda, \Lambda^2/Q),$
soft:	$p_s^\mu \sim (\Lambda, \Lambda, \Lambda),$

$$p^\mu = (p^+, p_\perp, p^-),\ Q \sim P^z,\ \text{and}\ \Lambda \sim \Lambda_{\text{QCD}}$$

For four-quark form factor, there are two kinds of collinear modes.

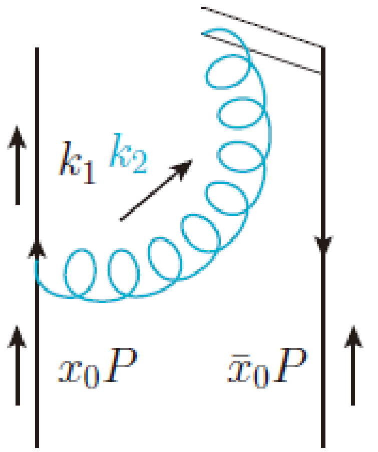
Expansion by regions

TMDWFs: example for diagram (a)

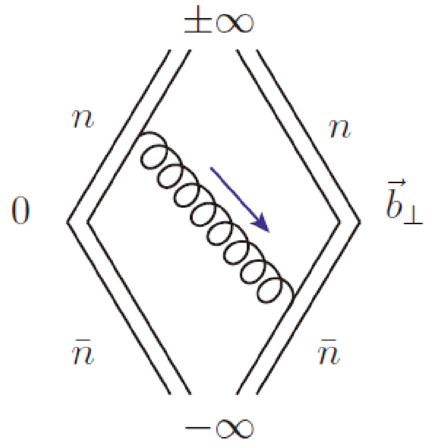
$$\begin{aligned}
\psi^{(1,a)}_{\bar{q}q}|_{\rm soft} &= \mu_0^{2\epsilon}\frac{ig^2C_F}{2}\int\frac{d^dq}{(2\pi)^d}e^{-iq\cdot b}\delta\Big[(x-x_0)P^+ + q^+\Big]\frac{\bar{v}\gamma^+\gamma^5(x_0\not{P}-\not{q})\not{n}u}{-q^+[(x_0P-q)^2+i\epsilon](q^2+i\epsilon)}|_{soft}\\
&= \delta\Big[(x-x_0)P^+\Big]\mu_0^{2\epsilon}\frac{ig^2C_F}{2}\int\frac{d^dq}{(2\pi)^d}e^{-iq\cdot b}\frac{\bar{v}\gamma^+\gamma^5x_0P^+\not{\bar{n}}\not{n}u}{-q^+(-2x_0P^+q^-)(q^2+i\epsilon)}\\
&= \delta(x-x_0)\frac{\bar{v}\gamma^+\gamma^5u}{2P^+}\mu_0^{2\epsilon}ig^2C_F\int\frac{d^dq}{(2\pi)^d}e^{-iq\cdot b}\frac{1}{q^+q^-(q^2+i\epsilon)}\\
&= \psi^{(0)}_{\bar{q}q}\times S^{(1,b)}
\end{aligned}$$

No hard mode

$$\psi^{(1,a)}_{\bar{q}q} = \psi^{(1,a)}_{\bar{q}q}|_{\rm collinear} + \psi^{(0)}_{\bar{q}q}\times S^{(1,b)}$$



(a)



(b)

Expansion by regions

Form factor : example for diagram (a)

$$F^{(1,a)} = \mu_0^{2\epsilon}\frac{ig^2C_F}{4N_cP_1\cdot P_2}\int\frac{d^dq}{(2\pi)^d}e^{-iq\cdot b}\frac{1}{[(q+x_1P_1)^2+i\epsilon][(q-\bar{x}_1P_1)^2+i\epsilon](q^2+i\epsilon)}$$
$$\times c_\Gamma\bar{u}_a(x_2P_2)\gamma_\nu\gamma_5 v_d(\bar{x}_2P_2)\ \bar{v}_c(\bar{x}_1P_1)\gamma^\mu(\not{q}-\bar{x}_1\not{P}_1)\gamma^\nu\gamma_5(\not{q}+x_1\not{P}_1)\gamma_\mu u_b(x_1P_1)$$
$$= \mu_0^{2\epsilon}\frac{ig^2C_F}{4P_1\cdot P_2}\int\frac{d^dq}{(2\pi)^d}e^{iq\cdot b}\frac{1}{[(q-x_1P_1)^2+i\epsilon][(q+\bar{x}_1P_1)^2+i\epsilon](q^2+i\epsilon)}$$
$$\times(-H_F^{(0)})\bar{u}_a(x_2P_2)\gamma_\nu\gamma_5 v_d(\bar{x}_2P_2)\ \bar{v}_c(\bar{x}_1P_1)\gamma^\mu(\not{q}+\bar{x}_1\not{P}_1)\gamma^\nu\gamma_5(\not{q}-x_1\not{P}_1)\gamma_\mu u_b(x_1P_1)$$
$$= H_F^{(0)}\mu_0^{2\epsilon}\frac{ig^2C_F}{2P_1^+}\int\frac{d^dq}{(2\pi)^d}e^{iq\cdot b}\frac{1}{[(q-x_1P_1)^2+i\epsilon][(q+\bar{x}_1P_1)^2+i\epsilon](q^2+i\epsilon)}$$
$$\times\bar{v}_c(\bar{x}_1P_1)\gamma^\mu(\not{q}+\bar{x}_1\not{P}_1)\gamma^\nu\gamma_5(x_1\not{P}_1-\not{q})\gamma_\mu u_b(x_1P_1)$$
$$= H_F^{(0)}\times\int dx\,\psi_{\overline{q}q}^{(1,c)}(x).$$

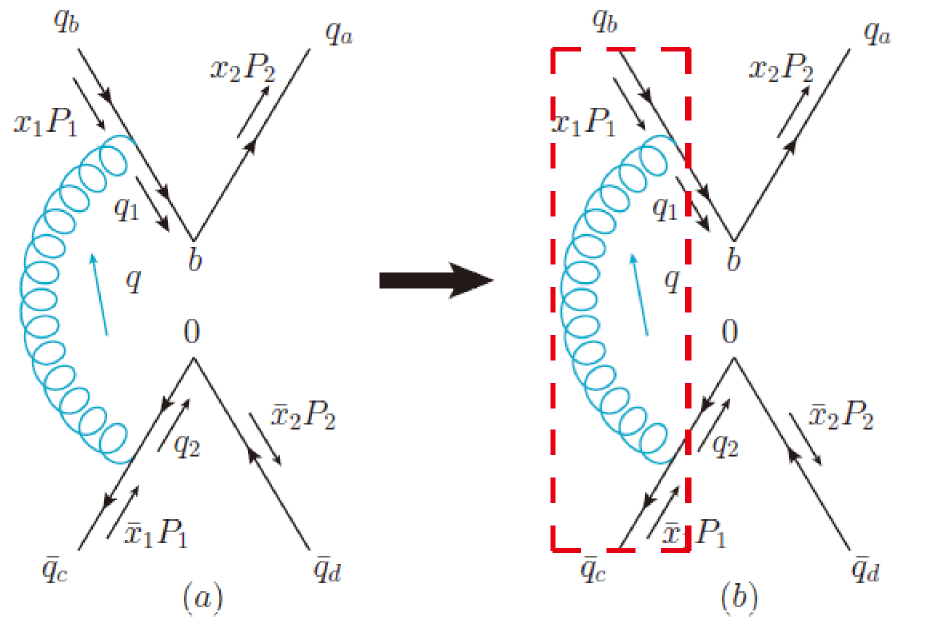


FIG. 6: Factorization of form factor shown in Fig. 5 (a). Only collinear mode contributes in this diagram, while both hard and soft contributions are power suppressed.

$$F^{(1,a)} = H_F^{(0)}\otimes\psi_{\overline{q}q}^{(1,c)}\otimes(\psi_{\overline{q}q}^{(0)})^\dagger\times\left(\frac{1}{S}\right)^{(0)}$$

Expansion by regions

Form factor : example for diagram (c)

Soft mode:

$$F^{(1,c)}|_{soft} \;=\; H_F^{(0)} \otimes \psi_{\overline{q}q}^{(0)} \otimes (\psi_{\overline{q}q}^{(0)})^\dagger \times S^{(1,b)}.$$

P_1 collinear mode :

$$F^{(1,c)}|_{collinear} = H_F^{(0)} \otimes \psi_{\overline{q}q}^{(1,a)}|_{\text{collinear}} \otimes (\psi_{\overline{q}q}^{(0)})^\dagger \times \left(\frac{1}{S}\right)^{(0)}$$

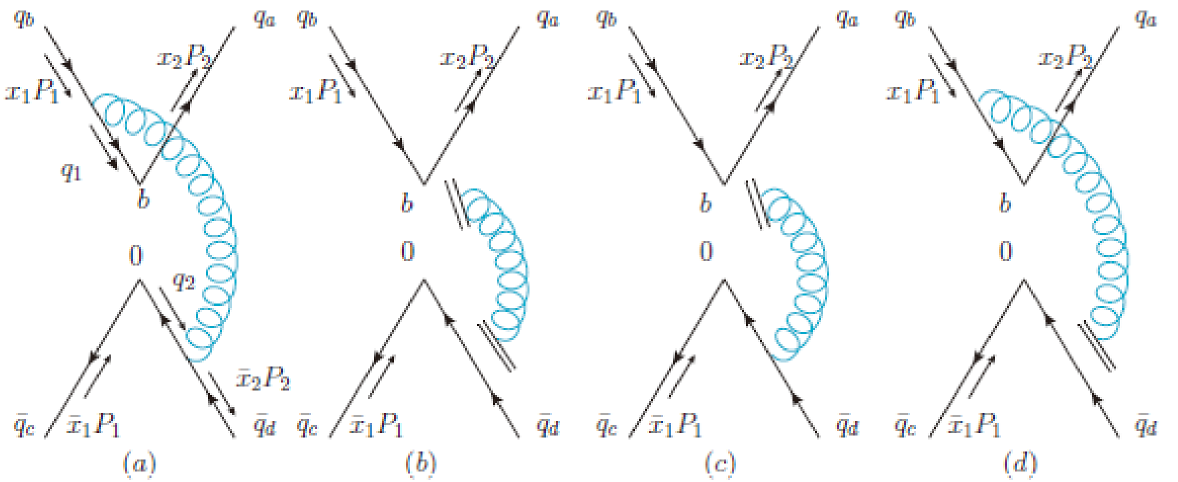


FIG. 7: Factorization of form factor shown in Fig. 5 (c). The collinear, and soft modes contribute in this diagram, while the hard mode's contribution is power suppressed.

No hard mode

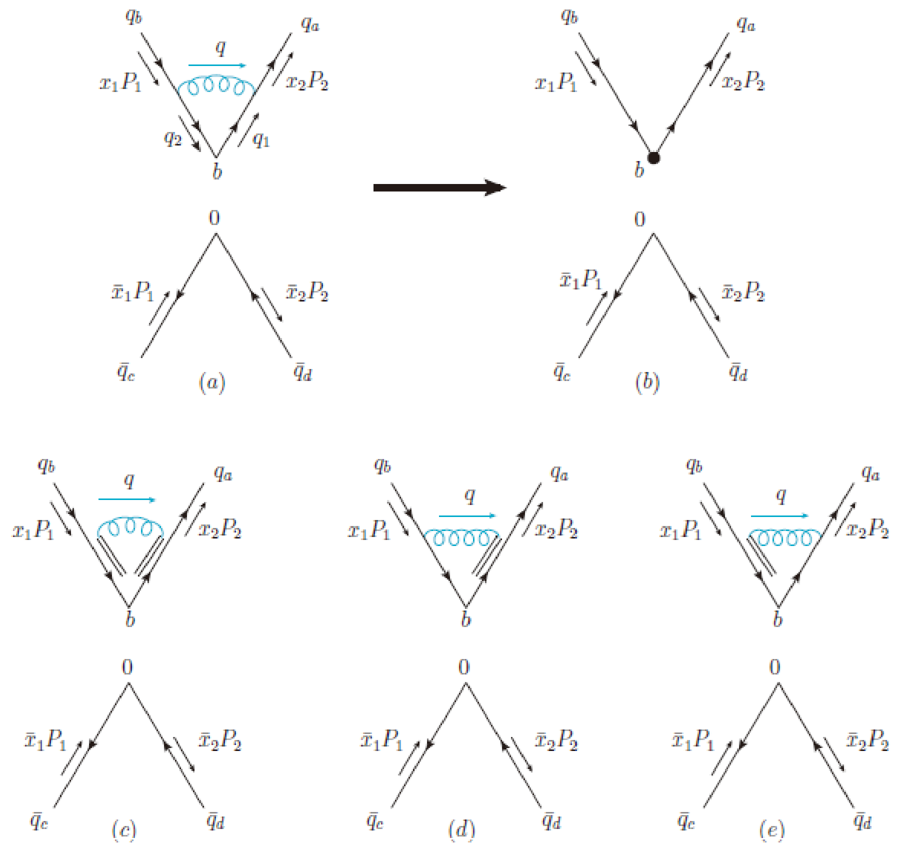
Sum:

$$F^{(1,c)} \;=\; H_F^{(0)} \otimes \psi_{\overline{q}q}^{(1,a)} \otimes (\psi_{\overline{q}q}^{(0)})^\dagger \times \left(\frac{1}{S}\right)^{(0)} + H_F^{(0)} \otimes \psi_{\overline{q}q}^{(0)} \otimes (\psi_{\overline{q}q}^{(1,a)})^\dagger \times \left(\frac{1}{S}\right)^{(0)} + H_F^{(0)} \otimes \psi_{\overline{q}q}^{(0)} \otimes (\psi_{\overline{q}q}^{(0)})^\dagger \times \left(\frac{1}{S}\right)^{(1,b)}$$

Expansion by regions

Form factor : example for diagram (e)

$$
\begin{aligned}
F^{(1,e)} &= H_F^{(1,e)} \otimes \psi_{\bar{q}q}^{(0)} \otimes (\psi_{\bar{q}q}^{(0)})^\dagger \times \left(\frac{1}{S}\right)^{(0)} \\
&+ H_F^{(0)} \otimes \psi_{\bar{q}q}^{(1,d)} \otimes (\psi_{\bar{q}q}^{(0)})^\dagger \times \left(\frac{1}{S}\right)^{(0)} \\
&+ H_F^{(0)} \otimes \psi_{\bar{q}q}^{(0)} \otimes (\psi_{\bar{q}q}^{(1,d)})^\dagger \times \left(\frac{1}{S}\right)^{(0)} \\
&+ H_F^{(0)} \otimes \psi_{\bar{q}q}^{(0)} \otimes (\psi_{\bar{q}q}^{0)})^\dagger \times \left(\frac{1}{S}\right)^{(1,d)} .
\end{aligned}
$$



$$F = H_F \otimes \psi_{\bar{q}q} \otimes (\psi_{\bar{q}q})^\dagger \times \frac{1}{S},$$

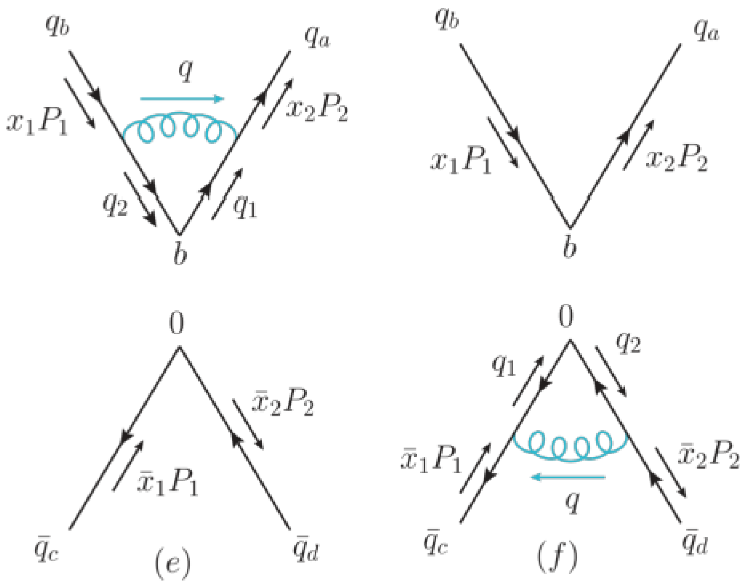
Expansion by regions

The total result could be written as the TMD factorization of the form factor at one-loop level:

$$F = H_F \otimes \psi_{\bar{q}q} \otimes (\psi_{\bar{q}q})^\dagger \times \frac{1}{S}$$

Where the pertubative hard kernal:

$$H_F = H_F^{(0)} + H_F^{(1,e)} + H_F^{(1,f)}$$



Lattice results

Similarly defined the quasi-TMDWF in partonic state:

$$\tilde{\Psi}^{\pm}_{q\bar{q}}\left(x, b_\perp, \mu, \zeta^z\right) = \lim_{L\to\infty} \int \frac{d\lambda}{4\pi} e^{-i\left(x-\frac{1}{2}\right)\left(-P^z\right)\lambda} \frac{\left\langle 0 \left| \overline{\Psi}_{\mp n_z}\left(\frac{\lambda n_z}{2} + b\right) \gamma^z \gamma^5 \Psi_{\mp n_z}\left(-\frac{\lambda n_z}{2}\right) \right| q\bar{q} \right\rangle}{\sqrt{Z_E\left(2L, b_\perp, \mu\right)}}$$

$$\Psi_{\mp n_z}(\xi) = \mathcal{P} e^{ig\int_0^{\mp L+\xi\cdot n_z} ds n_z\cdot A(\xi+sn_z)} \psi(\xi)$$

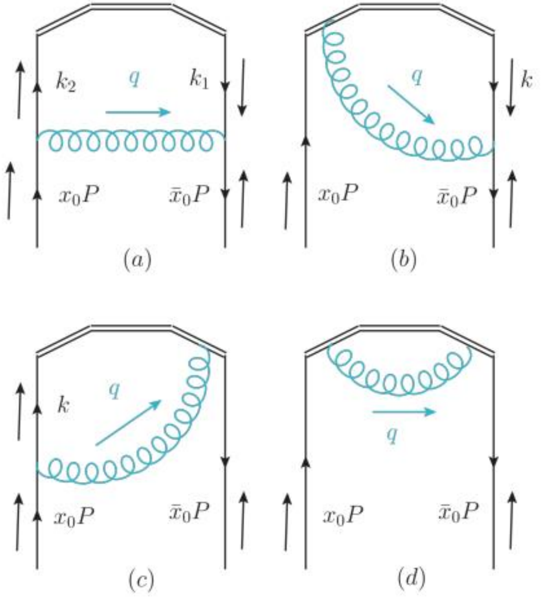
The Wilson loop used to renormalized the rapidity divergence:

$$Z_E\left(2L, b_\perp, \mu\right) = \frac{1}{N_c} \operatorname{tr}\langle 0|\mathcal{T} W(\mathcal{C})|0\rangle$$

Lattice results

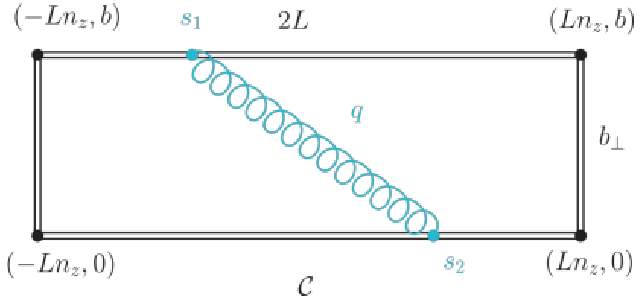
Similar defined the quasi-TMDWF in partonic state:

$$\tilde{\Psi}^{\pm}_{q\bar{q}}(x, b_\perp, \mu, \zeta^z) = \delta(x - x_0) + \frac{\alpha_s C_F}{2\pi}[f(x, x_0, b_\perp, \mu)]_+ + \frac{\alpha_s C_F}{2\pi}\delta(x - x_0)A^{\pm}\left(x, \mu, \zeta^z, \bar{\zeta}^z\right)$$



where:

$$A^{\pm}\left(x, \mu, \zeta^z, \bar{\zeta}^z\right) = -\frac{L_b^2}{2} + \frac{5}{2}L_b - \frac{3}{2} - \frac{\pi^2}{2} + \left[-\frac{1}{4}\ln^2\frac{-\zeta^z \pm i0}{\mu^2} + \frac{1}{2}(1 - L_b)\ln\frac{-\zeta^z \pm i0}{\mu^2} + \{\zeta^z \to \bar{\zeta}^z\}\right]$$



Lattice results

The matching between LC and quasi-TMDWF give:

$$\widetilde{\Psi}^{\pm}_{\bar{q}q}\left(x, b_\perp, \mu, \zeta^z\right) S_r^{\frac{1}{2}}\left(b_\perp, \mu\right) = H_1^{\pm}\left(\zeta^z, \bar{\zeta}^z, \mu\right) e^{\frac{1}{2}\ln\frac{\mp\zeta^z + i0}{\zeta} K_1(b_\perp, \mu)} \Psi^{\pm}_{\bar{q}q}\left(x, b_\perp, \mu, \zeta\right)$$

Where the hard kernel could be given as:

$$H_1^{\pm}\left(\zeta^z, \bar{\zeta}^z, \mu\right) = 1 + \frac{\alpha_s C_F}{2\pi}\left\{-\frac{5\pi^2}{12} - 2 + \frac{1}{2}\left[\ln\frac{-\zeta^z \pm i0}{\mu^2} - \frac{1}{2}\ln^2\frac{-\zeta^z \pm i0}{\mu^2} + \{\zeta^z \to \bar{\zeta}^z\}\right]\right\}.$$

Lattice results

Substituting the factorization formular:

$$F(b_\perp, P_1, P_2, \mu) = \int dx_1 dx_2 H(x_1, x_2) S_r(b_\perp, \mu)\ \tilde{\Psi}^\dagger_{q\overline{q}}(x_2, b_\perp, \mu, \zeta_2^z) \tilde{\Psi}_{q\overline{q}}(x_1, b_\perp, \mu, \zeta_1^z)$$

Where the hard kernel:

$$H(x_1, x_2) = \frac{H_F(Q^2, \bar{Q}^2, \mu^2)}{\left[H_1^\pm\left(\zeta_2^z, \bar{\zeta}_2^z, \mu\right)\right]^\dagger \left[H_1^\pm\left(\zeta_1^z, \bar{\zeta}_1^z, \mu\right)\right]}$$

Lattice results

$\Gamma = I, \gamma_5$

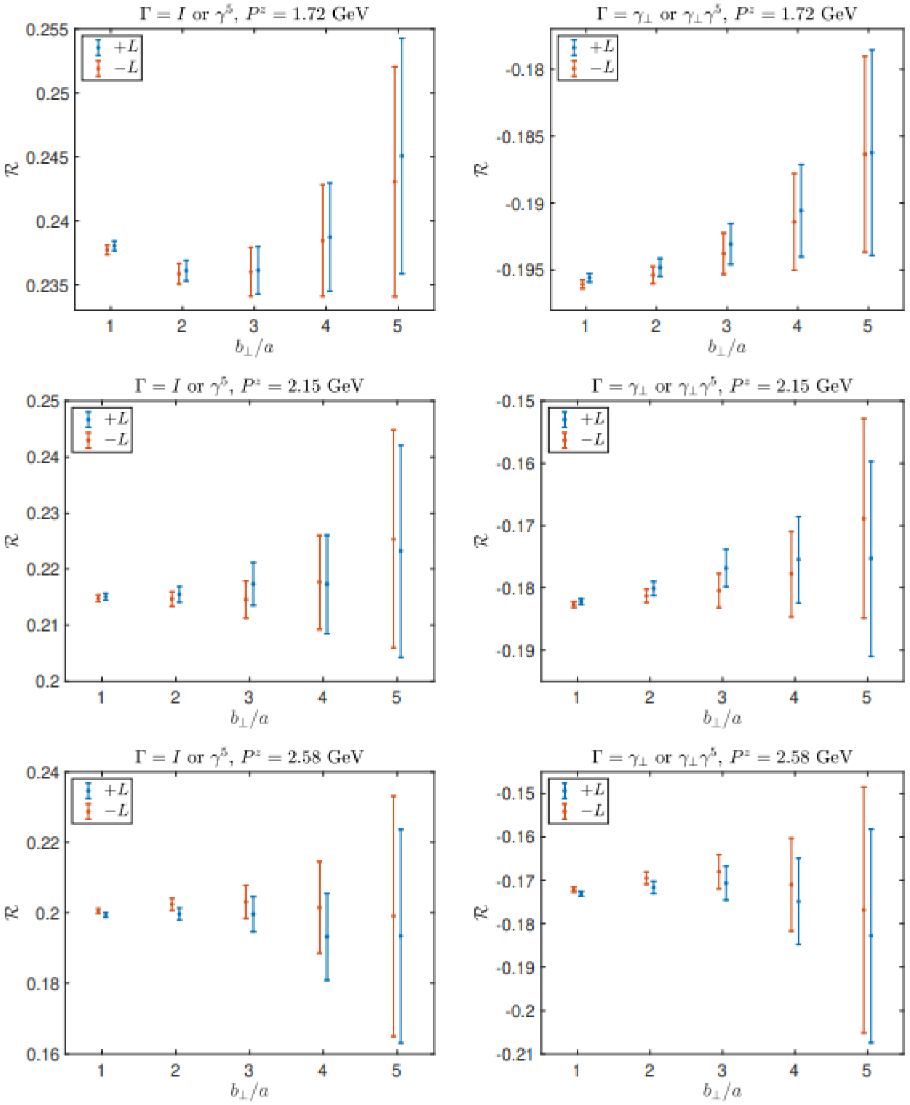
$$H(x_1,x_2) = H^{(0)}\left\{1+\frac{\alpha_s C_F}{2\pi}\left[2+\pi^2+\frac{1}{2}\ln^2\left(-\frac{x_2}{x_1}\mp i0\right)+\frac{1}{2}\ln^2\left(-\frac{\bar{x}_2}{\bar{x}_1}\mp i0\right)-\ln\frac{16x_1x_2\bar{x}_1\bar{x}_2P^{z4}}{\mu^4}\right]\right\}$$

$\Gamma = \gamma_\perp, \ \gamma_\perp\gamma_5$

$$H(x_1,x_2) = H^{(0)}\left\{1+\frac{\alpha_s C_F}{2\pi}\left[\pi^2-4+\frac{1}{2}\ln^2\left(-\frac{x_2}{x_1}\mp i0\right)+\frac{1}{2}\ln^2\left(-\frac{\bar{x}_2}{\bar{x}_1}\mp i0\right)+\frac{1}{2}\ln\frac{16x_1\bar{x}_1x_2\bar{x}_2P^{z4}}{\mu^4}\right]\right\}$$

These results could be used in Lattice calculation to get the reduced soft function.

Lattice results



Lattice data on quasi-TMDWFs from LPC

LPC Collabration, Phys. Rev. D 106(2022) 034509

$$S_r(b_\perp, \mu) = \frac{F(b_\perp, P_1, P_2, \mu)}{\mathcal{H}}$$

$$\mathcal{H} = \int dx_1 dx_2 H(x_1, x_2) \tilde{\Psi}^\dagger(x_2, b_\perp, P^z, \zeta_2^z) \tilde{\Psi}(x_1, b_\perp, P^z, \zeta_1^z)$$

The ratio:

$$\mathcal{R} = \frac{\mathcal{H}_1 - \mathcal{H}_0}{\mathcal{H}_0}$$

Summation

- In LaMET, the TMDWFs can be extracted from the first-principle simulation of a four-quark form factor.

- The way of expansion by regions could proof the TMD factorization of the form factor at one-loop level.

- These results are helpful to precisely extract the soft functions and TMD wave functions from the first-principle in future.

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THANKS FOR WATCHING

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