pNGB Dark Matter, Cosmic Strings, and Gravitational Waves

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Based on Dan-Yang Liu, Chengfeng Cai, Xue-Min Jiang, Zhao-Huan Yu, Hong-Hao Zhang, arXiv:2208.06653, JHEP

Ze-Yu Qiu, Zhao-Huan Yu, arXiv:2304.02506, Chin. Phys. C

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Thermal Dark Matter

✨ Conventionally, **dark matter (DM)** is assumed to be a **thermal relic** remaining from the early Universe

🌙 DM relic abundance observation

👉 Particle mass $m_\chi \sim \mathcal{O}(\text{GeV}) - \mathcal{O}(\text{TeV})$

Interaction strength $\sim$ **weak strength**

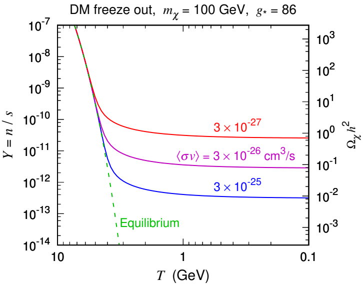
"Weakly interacting massive particles"

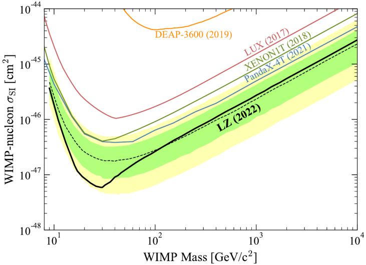
"WIMPs"

🔍 **Direct detection** for WIMPs

👉 **No robust signal found so far**

☁️ **Great challenge** to the thermal dark matter paradigm





[LZ Coll., 2207.03764]

Original pNGB Dark Matter [Gross, Lebedev, Toma, 1708.02253, PRL]

- **Standard model (SM) Higgs doublet** H, **complex scalar** S (SM singlet)
- Scalar potential respects a **softly broken global** $\mathrm{U}(1)$ **symmetry** $S \to e^{i\alpha} S$
- $\mathrm{U}(1)$ **symmetric:** $V_0 = -\frac{\mu_H^2}{2}|H|^2 - \frac{\mu_S^2}{2}|S|^2 + \frac{\lambda_H}{2}|H|^4 + \frac{\lambda_S}{2}|S|^4 + \lambda_{HS}|H|^2|S|^2$
- **Soft breaking:** $V_{\mathrm{soft}} = -\frac{\mu_S'^2}{4} S^2 + \mathrm{H.c.}$

Approximate global $\mathrm{U}(1)$

- H and S develop **vacuum expectation values (VEVs)**

⬇ v_S

$$H \to \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h \end{pmatrix}, \quad S = \frac{1}{\sqrt{2}}(v_s + s + i\chi)$$

Z_2 **symmetry**

- The **soft breaking term** V_{soft} give a mass to χ: $m_\chi = \mu_S'$
- A Z_2 **symmetry** $\chi \to -\chi$ remains after $\mathrm{U}(1)$ **spontaneous symmetry breaking**
- The **DM candidate** χ is a **stable pseudo-Nambu-Goldstone boson (pNGB)**
- Rotate **CP-even Higgs bosons** h and s to **mass eigenstates** h_1 and h_2

$$\begin{pmatrix} h \\ s \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} h_1 \\ h_2 \end{pmatrix}, \quad m_{h_1,h_2}^2 = \frac{1}{2}\left(\lambda_H v^2 + \lambda_S v_s^2 \mp \frac{\lambda_S v_s^2 - \lambda_H v^2}{\cos 2\theta}\right)$$

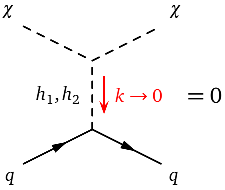
DM-nucleon Scattering [Gross, Lebedev, Toma, 1708.02253, PRL]

DM-quark interactions induce **DM-nucleon** scattering in direct detection

DM-quark scattering amplitude from Higgs portal interactions

$$\mathcal{M}(\chi q \to \chi q) \propto \frac{m_q s_\theta c_\theta}{v v_s} \left(\frac{m_{h_1}^2}{t - m_{h_1}^2} - \frac{m_{h_2}^2}{t - m_{h_2}^2} \right)$$

$$= \frac{m_q s_\theta c_\theta}{v v_s} \frac{t(m_{h_1}^2 - m_{h_2}^2)}{(t - m_{h_1}^2)(t - m_{h_2}^2)}$$



Zero momentum transfer limit $t = k^2 \to 0, \quad \mathcal{M}(\chi q \to \chi q) \to 0$

DM-nucleon scattering cross section **vanishes** at tree level

Tree-level interactions of a **pNGB** are generally **momentum-suppressed**

One-loop corrections typically lead to $\sigma_{\chi N}^{\mathrm{SI}} \lesssim \mathcal{O}(10^{-50})$ cm^2

[Azevedo *et al.*, 1810.06105, JHEP; Ishiwata & Toma, 1810.08139, JHEP]

Beyond capability of current and near future direct detection experiments

UV Completion of pNGB DM

- In the **original pNGB DM model**, the term $V_{\text{soft}} = -\frac{\mu_S'^2}{4}(S^2 + S^{\dagger 2})$, which **softly breaks** the $\mathrm{U}(1)$ **global symmetry** $S \to e^{i\alpha} S$ into a Z_2 symmetry, is ***ad hoc***

- Other soft breaking terms, such as a trilinear term $\propto S^3 + S^{\dagger 3}$, would **spoil** the **vanishing scattering amplitude**

- It demands an appropriate **ultraviolet (UV) completion** to **realize only** V_{soft}

- A possible UV completion is to **gauge the** $\mathrm{U}(1)$ **symmetry** with $B - L$ **charges**

  [Abe, Toma & Tsumura, 2001.03954, JHEP; Okada, Raut & Shafi, 2001.05910, PRD]

- We consider another option that pNGB DM arises from a **hidden** $\mathrm{U}(1)_\mathrm{X}$ **gauge symmetry**, where all the SM fields **do not** carry $\mathrm{U}(1)_\mathrm{X}$ charges

- The **gauge anomalies** are **canceled** **without** introducing **right-handed neutrinos**, so **less fields** are involved in this setup

UV Completion with a Hidden $\mathrm{U}(1)_\mathrm{X}$ Gauge Symmetry

We introduce two **complex scalar fields** S and Φ carrying $\mathrm{U}(1)_\mathrm{X}$ **charges** 1 and 2

$$D_\mu S = (\partial_\mu - \mathrm{i} g_X X_\mu) S, \quad D_\mu \Phi = (\partial_\mu - 2\mathrm{i} g_X X_\mu)\Phi$$

$$\begin{aligned}\mathcal{L} \supset\ & (D^\mu H)^\dagger (D_\mu H) + (D^\mu S)^\dagger (D_\mu S) + (D^\mu \Phi)^\dagger (D_\mu \Phi) - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} - \frac{1}{4} X^{\mu\nu} X_{\mu\nu} \\ & - \frac{s_\varepsilon}{2} B^{\mu\nu} X_{\mu\nu} + \mu_H^2 |H|^2 + \mu_S^2 |S|^2 + \mu_\Phi^2 |\Phi|^2 - \frac{\lambda_H}{2}|H|^4 - \frac{\lambda_S}{2}|S|^4 - \frac{\lambda_\Phi}{2}|\Phi|^4 \\ & - \lambda_{HS} |H|^2 |S|^2 - \lambda_{H\Phi} |H|^2 |\Phi|^2 - \lambda_{S\Phi} |S|^2 |\Phi|^2 + \frac{\mu_{S\Phi}}{\sqrt{2}} (\Phi^\dagger S^2 + \Phi S^{\dagger 2})\end{aligned}$$

The $B^{\mu\nu} X_{\mu\nu}$ term implies a **kinetic mixing** between the $\mathrm{U}(1)_\mathrm{Y}$ gauge field B^μ and the $\mathrm{U}(1)_\mathrm{X}$ **gauge field** X^μ with a mixing parameter $s_\varepsilon \equiv \sin\varepsilon \in (-1, 1)$

S and Φ develop **nonzero VEVs** v_S and v_Φ with a **hierarchy** $v_S \sim v \ll v_\Phi$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}, \quad S = \frac{1}{\sqrt{2}} (v_S + s + \mathrm{i}\eta_S), \quad \Phi = \frac{1}{\sqrt{2}} (v_\Phi + \phi + \mathrm{i}\eta_\Phi)$$

The v_Φ **contribution** to the $\Phi^\dagger S^2$ **term** leads to the **desired soft breaking term**

$$V_\mathrm{soft} = -\frac{\mu_S'^2}{4} (S^2 + S^{\dagger 2}) \text{ with } \mu_S'^2 = 2\mu_{S\Phi} v_\Phi$$

Physical Scalars

- Rotate the scalars from the interaction bases to the mass bases

$$\begin{pmatrix} h \\ s \\ \phi \end{pmatrix} = U \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}, \quad \begin{pmatrix} \eta_S \\ \eta_\Phi \end{pmatrix} = V \begin{pmatrix} \chi \\ \tilde{\chi} \end{pmatrix}$$

- h_1 (SM-like), h_2, and h_3 are CP-**even Higgs bosons**, and $\tilde{\chi}$ is a **massless Nambu-Goldstone boson** associated with the $\mathrm{U}(1)_\mathrm{X}$ **gauge symmetry breaking**

- χ is a **pNGB DM candidate** with a mass squared of $m_\chi^2 = \frac{\mu_{S\Phi}}{2v_\Phi}(v_S^2 + 4v_\Phi^2)$

- v_Φ represents a **UV scale** that breaks the $\mathrm{U}(1)_\mathrm{X}$ **gauge symmetry** into an **approximate** $\mathrm{U}(1)_\mathrm{X}$ **global symmetry**

- Below the **lower scale** v_S, the **global** $\mathrm{U}(1)_\mathrm{X}$ is spontaneously broken, resulting in **pNGB DM**

- In the **limit** $v_\Phi \to \infty$ and $\mu_{S\Phi} \to 0$ with **finite** $\mu_S'^2$, the **original pNGB DM model** is recovered

Gauge $\mathrm{U}(1)_\mathrm{X}$

UV scale ⬇ v_Φ

Approximate global $\mathrm{U}(1)_\mathrm{X}$

Lower scale ⬇ v_S

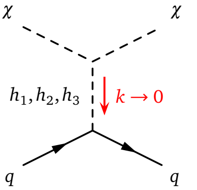
Approximate Z_2

Direct Detection

- The **UV completion** gives $\mu_S'^2$ a **dynamical origin**, but inevitably introduces the χ-χ-ϕ **coupling**, leading to a **nonvanishing** χ-nucleon scattering amplitude
- χN **scattering cross section** is **highly suppressed by** v_Φ^{-4}

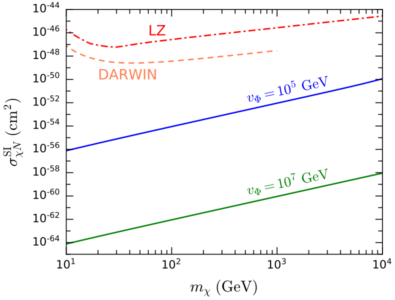
$$\sigma_{\chi N}^{\mathrm{SI}} \simeq \frac{\tilde{\lambda}^2 m_N^4 m_\chi^4 [2 + 7(f_u^N + f_d^N + f_s^N)]^2}{1296\pi (m_N + m_\chi)^2 v^4 v_\Phi^4} + \mathcal{O}(v_\Phi^{-6})$$

$$\tilde{\lambda} = \frac{\lambda_{H\Phi}\lambda_{S\Phi} - \lambda_\Phi \lambda_{HS} + 2\lambda_{HS}\lambda_{S\Phi} - 2\lambda_S \lambda_{H\Phi}}{\lambda_H \lambda_S \lambda_\Phi + 2\lambda_{HS}\lambda_{H\Phi}\lambda_{S\Phi} - \lambda_S \lambda_{H\Phi}^2 - \lambda_\Phi \lambda_{HS}^2 - \lambda_H \lambda_{S\Phi}^2}$$



- $v_\Phi = 10^5$ GeV can result in $\sigma_{\chi N}^{\mathrm{SI}}$ **much smaller** than 90% C.L. upper limits from the **LZ experiment** [2207.03764], and even **beyond the reach** of the future **DARWIN experiment** with a 200 t · yr exposure [1606.07001, JCAP]

$$v_S = 1\ \text{TeV}, \quad m_{h_2} = 300\ \text{GeV}, \quad m_{h_3} = 0.1 v_\Phi$$
$$\lambda_{HS} = 0.03, \quad \lambda_{H\Phi} = \lambda_{S\Phi} = 0.01$$



Neutral Gauge Boson Mixing

- Transform the **gauge basis** (B_μ, W^3_μ, X_μ) to the **mass basis** (A_μ, Z_μ, Z'_μ)

$$\begin{pmatrix} B_\mu \\ W^3_\mu \\ X_\mu \end{pmatrix} = V_\mathrm{K}(\varepsilon)\, R_3(\hat\theta_\mathrm{W})\, R_1(\xi) \begin{pmatrix} A_\mu \\ Z_\mu \\ Z'_\mu \end{pmatrix}$$

$$V_\mathrm{K}(\varepsilon) = \begin{pmatrix} 1 & & -t_\varepsilon \\ & 1 & \\ 0 & & 1/c_\varepsilon \end{pmatrix}, \quad R_3(\hat\theta_\mathrm{W}) = \begin{pmatrix} \hat c_\mathrm{W} & -\hat s_\mathrm{W} & \\ \hat s_\mathrm{W} & \hat c_\mathrm{W} & \\ & & 1 \end{pmatrix}, \quad R_1(\xi) = \begin{pmatrix} 1 & & \\ & c_\xi & -s_\xi \\ & s_\xi & c_\xi \end{pmatrix}$$

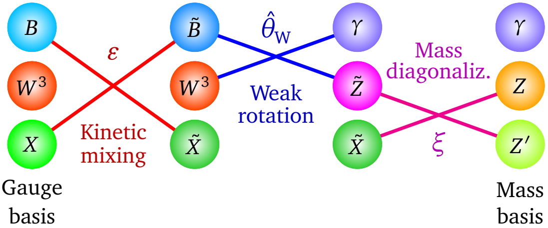
[Babu, Kolda, March-Russell, hep-ph/9710441, PRD]

$$t_\varepsilon \equiv \tan\varepsilon, \quad c_\varepsilon \equiv \cos\varepsilon$$

$$\hat s_\mathrm{W} \equiv \sin\hat\theta_\mathrm{W}, \quad \hat c_\mathrm{W} \equiv \cos\hat\theta_\mathrm{W}$$

$$\hat\theta_\mathrm{W} \equiv \tan^{-1}\frac{g'}{g}$$

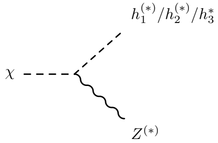
$$s_\xi \equiv \sin\xi, \quad c_\xi \equiv \cos\xi$$



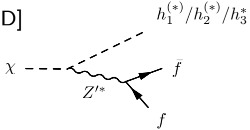
- The **hierarchy** $v \sim v_S \ll v_\Phi$ implies a **mass hierarchy** $m_{h_1} \sim m_{h_2} \ll m_{h_3} \sim m_{Z'}$

DM Lifetime

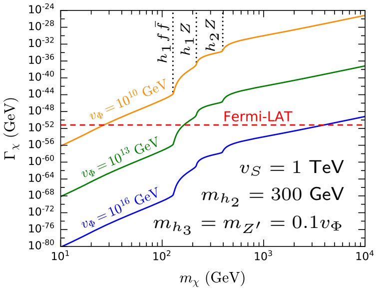
- For **finite v_Φ**, the Z-χ-h_i and Z'-χ-h_i **couplings** from gauge interactions **break** the Z_2 **symmetry** $\chi \to -\chi$, inducing χ **decay processes** $\chi \to h_i^{(*)} Z^{(*)}$ and $\chi \to h_i^{(*)} Z'^{*}$ for $m_\chi \ll m_{Z'} \sim m_{h_3}$

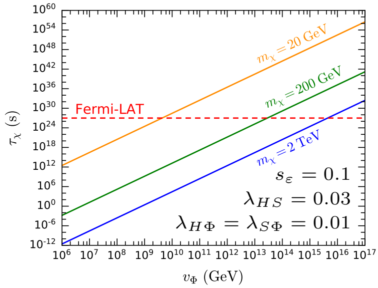


- **Fermi-LAT** γ-ray observations of dwarf galaxies imply a **bound** on the **DM lifetime**, $\tau_\chi \gtrsim 10^{27}$ s [Baring et al., 1510.00389, PRD]



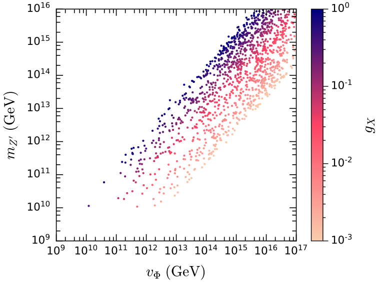
- This corresponds to $\Gamma_\chi \equiv 1/\tau_\chi \lesssim 6.6 \times 10^{-52}$ GeV, which will give a **lower bound** on the **UV scale** v_Φ

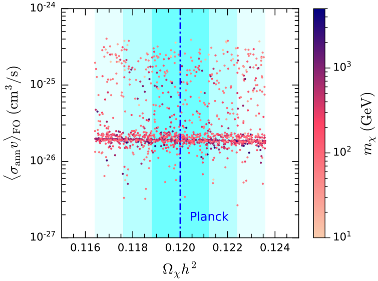


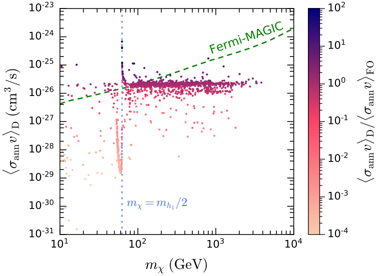


Parameter Scan

- We perform a **random scan** in 10-dimensional parameter space of $(v_S,\ v_\Phi,\ m_\chi,\ m_{h_2},\ m_{h_3},\ m_{Z'},\ \lambda_{HS},\ \lambda_{H\Phi},\ \lambda_{S\Phi},\ s_\varepsilon)$, taking into account the constraints from the **DM lifetime**, the **LHC Higgs measurements**, and the **relic abundance**
- We find that the **lower bound** on the **UV scale** v_Φ is down to $\sim 10^9$ GeV, given by the **Fermi-LAT constraint on** τ_χ

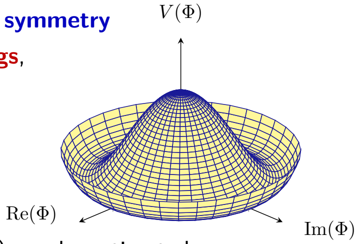






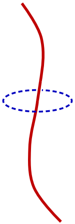
Cosmic Strings from $\mathrm{U}(1)_\mathrm{X}$ Gauge Symmetry Breaking

The **spontaneous breaking** of the $\mathrm{U}(1)_\mathrm{X}$ **gauge symmetry** at the **high UV scale** v_Φ would induce **cosmic strings**, which are **one-dimensional topological defects** concentrated with energies of scalar and gauge fields



According to the analysis on the **Abelian Higgs model** [Hill, Hodges, Turner, PRD **37**, 263 (1988)], the **cosmic string tension** μ (energy per unit length) can be estimated as

$$\mu \simeq \begin{cases} 1.19\pi v_\Phi^2 b^{-0.195}, & 0.01 < b < 100, \\ \dfrac{2.4\pi v_\Phi^2}{\ln b}, & b > 100, \end{cases} \qquad b \equiv \frac{8g_X^2}{\lambda_\Phi}$$



Degenerate vacua

$v_\Phi \mathrm{e}^{\mathrm{i}\varphi}$

$\varphi = \varphi + 2\pi n$

$n \neq 0$ leads to

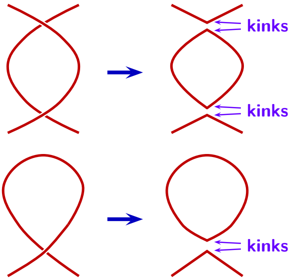
cosmic stirngs

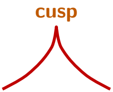
Because $\mu \propto v_\Phi^2$, a **high UV scale** v_Φ suggested by the **Fermi-LAT bound** on the χ **lifetime** would lead to cosmic strings with **high tension**

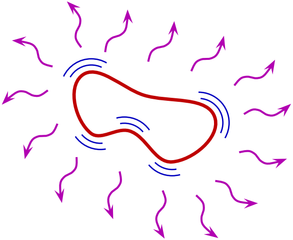
Denoting G as the **Newtonian constant of gravitation**, the **dimensionless quantity** $G\mu$ is commonly used to describe the **tension** of cosmic strings

Gravitational Waves of Cosmic Strings

- A **network** of **cosmic strings** would be formed in the early universe after the spontaneous breaking of the $\mathrm{U}(1)_{\mathrm{X}}$ gauge symmetry
- According to the analysis of string dynamics, the **intersections** of **long strings** could produce **closed loops**, whose size is smaller than the Hubble radius
- **Cosmic string loops** could further fragment into smaller loops or reconnect to **long strings**
- Loops typically have localized features called **"cusps"** and **"kinks"**
- The **relativistic oscillations** of the **loops** due to their **tension** emit **Gravitational Waves (GWs)**, and the loops would **shrink** because of **energy loss**
- Moreover, the **cusps** and **kinks** propagating along the loops could produce **GW bursts** [Damour & Vilenkin, gr-qc/0004075, PRL]





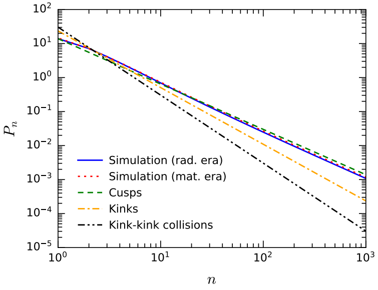


Power of Gravitational Radiation

At the **emission time** $t_{\rm e}$, a **cosmic string loop** of **length** L emits GWs with **frequencies** $f_{\rm e} = \frac{2n}{L}$

$n = 1, 2, 3, \cdots$ denotes the **harmonic modes** of the loop oscillation

Denoting P_n as the **power** of **gravitational radiation** for the harmonic mode n in units of $G\mu^2$, the total power is given by $P = G\mu^2 \sum_n P_n$



According to the **simulation** of **smoothed cosmic string loops** [Blanco-Pillado & Olum, 1709.02693, PRD], P_n for loops in the **radiation** and **matter** eras are obtained

The **total dimensionless power** $\Gamma = \sum_n P_n$ is estimated to be ~ 50

For comparison, analytic studies show that $P_n \simeq \frac{\Gamma}{\zeta(q) n^q}$ with $q = \frac{4}{3}, \frac{5}{3}, 2$ for **cusps**, **kinks**, and **kink-kink collisions**

Stochastic GW Background Induced by Cosmic Strings

The **energy** of **cosmic strings** is converted into the **energy** of **GWs**, and an **stochastic GW background (SGWB)** is formed due to **incoherent superposition**

The **SGWB energy density** $\rho_{\rm GW}$ per unit frequency at the present is

$$\frac{\mathrm{d}\rho_{\rm GW}}{\mathrm{d}f} = G\mu^2 \int_0^{z_*} \frac{1}{H(z)(1+z)^6} \sum_n \frac{2nP_n}{f^2}\, \mathsf{n}\left(\frac{2n}{f(1+z)}, t(z)\right) \mathrm{d}z$$

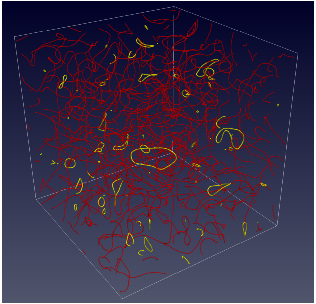
$\mathsf{n}(L,t)\,\mathrm{d}L$ is the **number density** of **cosmic string loops** at cosmic time t in length interval $\mathrm{d}L$

$H(z)$ is the Hubble rate and z_* is the redshift where the GW emissions start

The **SGWB spectrum** is often represented by

$$\Omega_{\rm GW}(f) = \frac{1}{\rho_c}\frac{\mathrm{d}\rho_{\rm GW}}{\mathrm{d}\ln f} = \frac{f}{\rho_c}\frac{\mathrm{d}\rho_{\rm GW}}{\mathrm{d}f}$$

$\rho_c = \dfrac{3H_0^2}{8\pi G}$ is the critical density



[Kitajima, Nakayama, 2212.13573]

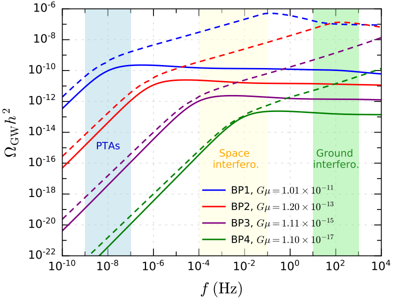
Loop Number Density: BOS model

- There are various approaches for modeling the **loop number density** $\mathsf{n}(L,t)$
- The **BOS model** [Blanco-Pillado, Olum & Shlaer, 1309.6637, PRD] extrapolates the loop production function found in simulations of Nambu-Goto strings
- The loop number densities produced in the **radiation** and **matter** era, and that **produced in the radiation era and still surviving in the matter era** are given by

$$\mathsf{n}_\mathrm{r}(L,t) \simeq \frac{0.18\,\theta(0.1t-L)}{t^4(\gamma+\gamma_\mathrm{d})^{5/2}}$$

$$\mathsf{n}_\mathrm{m}(L,t) \simeq \frac{(0.27-0.45\gamma^{0.31})\,\theta(0.18t-L)}{t^4(\gamma+\gamma_\mathrm{d})^2}$$

$$\mathsf{n}_{\mathrm{r}\to\mathrm{m}}(L,t) \simeq \frac{0.18 t_\mathrm{eq}^{1/2}\,\theta(0.09t_\mathrm{eq}-\gamma_\mathrm{d}t-L)}{t^{9/2}(\gamma+\gamma_\mathrm{d})^{5/2}}$$



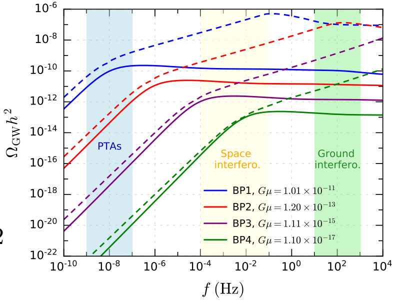
BOS model: solid lines

- $\gamma \equiv \dfrac{L}{t}$ is a **dimensionless variable**
- $\gamma_\mathrm{d} = -\dfrac{\mathrm{d}L}{\mathrm{d}t} \simeq \Gamma G\mu$ is the **loop shrinking rate**
- $t_\mathrm{eq} = 51.1 \pm 0.8$ kyr is the cosmic time at the **matter-radiation equality**

Loop Number Density: LRS model

The **LRS model** [Lorenz, Ringeval & Sakellariadou, 1006.0931, JCAP] takes into account the **gravitational backreaction effect**, which prevents loop production below a certain scale $\gamma_c \simeq 20(G\mu)^{1+2\chi}$ [Polchinski & Rocha, gr-qc/0702055, PRD]

$$\mathsf{n}(L,t) \simeq \begin{cases} \dfrac{C}{t^4(\gamma+\gamma_d)^{3-2\chi}}, & \gamma_d < \gamma \\ \dfrac{(3\nu-2\chi-1)C}{2t^4(1-\chi)\gamma_d\gamma^{2(1-\chi)}}, & \gamma_c < \gamma < \gamma_d \\ \dfrac{(3\nu-2\chi-1)C}{2t^4(1-\chi)\gamma_d\gamma_c^{2(1-\chi)}}, & \gamma < \gamma_c \end{cases}$$



LRS model: dashed lines

Radiation era: $\nu = 1/2,\ C \simeq 0.0796,\ \chi \simeq 0.2$

Matter era: $\nu = 3/2,\ C \simeq 0.0157,\ \chi \simeq 0.295$

Smaller $G\mu$ means smaller GW emission power, and loops could survive longer, leading to **more smaller loops** radiating at **higher f**

The **LRS model** gives a **very high number density** of **small loops** in the $\gamma < \gamma_c$ regime, which significantly contribute to **high frequency GWs**

GW Experiments

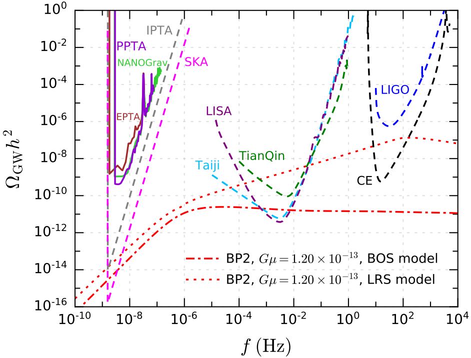
The **SGWB** originating from **cosmic strings** covers an **extremely broad range** of **GW frequencies**

It is an interesting target for various types of **GW experiments**

Pulsar timing arrays (PTAs) in 10^{-9}–10^{-7} Hz: **NANOGrav**, **PPTA**, **EPTA**, **CPTA**, **IPTA**, **SKA**, $\cdots$

Ground-based interferometers in 10–10^{3} Hz: **LIGO**, **Virgo**, **KAGRA**, **CE**, **ET**, $\cdots$

Space-borne interferometers in 10^{-4}–10^{-1} Hz: **LISA**, **TianQin**, **Taiji**, **BBO**, **DECIGO**, $\cdots$



Constraints and Sensitivity of GW Experiments

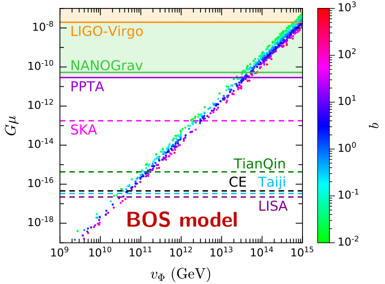
Constraints from **LIGO-Virgo**, **NANOGrav**, and **PPTA** have excluded the parameter points with $v_\Phi \gtrsim 5 \times 10^{13}$ (7×10^{11}) GeV assuming the **BOS** (**LRS**) model for loop production

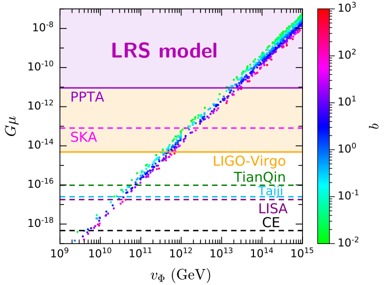
According to the **sensitive curves** $\Omega_n h^2$ of future GW experiments, the **signal-to-noise ratio (SNR)** can be estimated as

$$\rho = \sqrt{\mathcal{T} \int_{f_{\min}}^{f_{\max}} \left[\frac{\Omega_{\mathrm{GW}}(f)}{\Omega_n(f)}\right]^2 \mathrm{d}f}$$

$\mathcal{T}$ is the practical **observation time**

We take $\rho_{\mathrm{thr}} = 10$ as the threshold for detecting a GW signal, and find that **LISA** (**CE**) can probe v_Φ down to $\sim 2 \times 10^{10}$ (5×10^{9}) GeV for the **BOS** (**LRS**) model

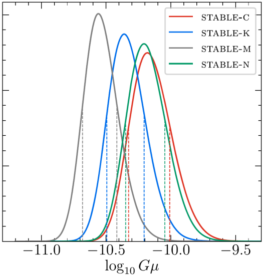


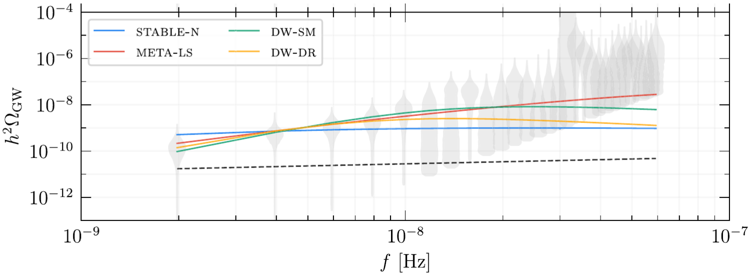


Positive Evidence for an SGWB from PTAs!

💥 On June 29, PTA experiments **NANOGrav** [2306.16219, ApJL], **CPTA** [2306.16216, RAA], **PPTA** [2306.16215, ApJL], and **EPTA** [2306.16214, 2306.16227] simultaneously reported **strong evidence of an SGWB!**

✨ According to the **NANOGrav analysis**, however, the GW spectrum from **stable cosmic strings** seems either **too weak** or **too flat** to explain the data





Summary

- We propose an **UV-complete model** for **pNGB DM** with a **hidden** $\mathrm{U}(1)_\mathrm{X}$ **gauge symmetry**
- DM scattering off nucleons is highly suppressed by the UV scale v_Φ and **direct detection constraints** can be **easily evaded**
- The **bound** on the **DM lifetime** implies that the **UV scale** v_Φ should be **higher than** 10^9 GeV
- The **spontaneous breaking** of the $\mathrm{U}(1)_\mathrm{X}$ **gauge symmetry** at **such a high scale** would induce **cosmic strings** with **high tension**, resulting in a **stochastic GW background** with a **high energy density**
- We find that **most viable parameter points** can be well studied in future GW experiments

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Thanks for your attention!

Scalar Boson Masses

After the scalar fields obtain the nonzero VEVs, the **mass terms** for the **CP-even scalars** (h, s, ϕ) and the **CP-odd scalars** (η_S, η_Φ) become

$$\mathcal{L}_{\text{mass}} \supset -\frac{1}{2}\begin{pmatrix} h & s & \phi \end{pmatrix} M_{\text{E}}^2 \begin{pmatrix} h \\ s \\ \phi \end{pmatrix} - \frac{1}{2}\begin{pmatrix} \eta_S & \eta_\Phi \end{pmatrix} M_{\text{O}}^2 \begin{pmatrix} \eta_S \\ \eta_\Phi \end{pmatrix}$$

$$M_{\text{E}}^2 = \begin{pmatrix} \lambda_H v^2 & \lambda_{HS} v v_S & \lambda_{H\Phi} v v_\Phi \\ \lambda_{HS} v v_S & \lambda_S v_S^2 & \lambda_{S\Phi} v_S v_\Phi - \mu_{S\Phi} v_S \\ \lambda_{H\Phi} v v_\Phi & \lambda_{S\Phi} v_S v_\Phi - \mu_{S\Phi} v_S & \lambda_\Phi v_\Phi^2 + \dfrac{\mu_{S\Phi} v_S^2}{2 v_\Phi} \end{pmatrix}, \quad M_{\text{O}}^2 = \mu_{S\Phi} \begin{pmatrix} 2 v_\Phi & -v_S \\ -v_S & \dfrac{v_S^2}{2 v_\Phi} \end{pmatrix}$$

Diagonalization: $U^{\text{T}} M_{\text{E}}^2 U = \text{diag}(m_{h_1}^2, m_{h_2}^2, m_{h_3}^2), \quad V^{\text{T}} M_{\text{O}}^2 V = \text{diag}(m_\chi^2, 0)$

$$V = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix}, \quad s_\beta = \frac{v_S}{\sqrt{v_S^2 + 4 v_\Phi^2}}$$

Shorthand notations: $s_\beta \equiv \sin\beta$, $c_\beta \equiv \cos\beta$, and $t_\beta \equiv \tan\beta$

Scalar Interactions

In the basis of the **mass eigenstates** χ and h_i, the **scalar trilinear couplings** are

$$\mathcal{L}_{\rm tri} = -\frac{1}{2}\sum_{i=1}^{3} g_{h_i\chi^2} h_i \chi^2 - \sum_{i,j,k=1}^{3} g_{h_ih_jh_k} h_i h_j h_k$$

$$\begin{aligned} g_{h_i\chi^2} &= (\lambda_{HS}c_\beta^2 + \lambda_{H\Phi}s_\beta^2)vU_{1i} + (\lambda_S v_S c_\beta^2 + \lambda_{S\Phi}v_S s_\beta^2 + 2\mu_{S\Phi}s_\beta c_\beta)U_{2i} \\ &\quad + [\lambda_\Phi v_\Phi s_\beta^2 + (\lambda_{S\Phi}v_\Phi + \mu_{S\Phi})c_\beta^2]U_{3i} \\ g_{h_ih_jh_k} &= \frac{1}{2}(\lambda_H v U_{1i} + \lambda_{HS}v_S U_{2i} + \lambda_{H\Phi}v_\Phi U_{3i})U_{1j}U_{1k} \\ &\quad + \frac{1}{2}[\lambda_{HS}vU_{1i} + \lambda_S v_S U_{2i} + (\lambda_{S\Phi}v_\Phi - \mu_{S\Phi})U_{3i}]U_{2j}U_{2k} \\ &\quad + \frac{1}{2}(\lambda_{H\Phi}vU_{1i} + \lambda_{S\Phi}v_S U_{2i} + \lambda_\Phi v_\Phi U_{3i})U_{3j}U_{3k} \end{aligned}$$

The **Yukawa couplings** are $\mathcal{L}_{h_iff} = -\sum_f \sum_{i=1}^{3} \frac{m_f U_{1i}}{v} h_i \bar{f} f$

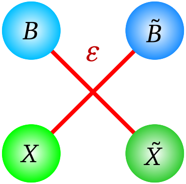
f denotes any **SM fermion**

Kinetic Mixing

- For the $\mathrm{U}(1)_\mathrm{Y}$ and $\mathrm{U}(1)_\mathrm{X}$ gauge fields B_μ and X_μ, the gauge invariant kinetic terms in the Lagrangian reads

$$\mathcal{L}_\mathrm{K} = -\frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{4}X^{\mu\nu}X_{\mu\nu} - \frac{s_\varepsilon}{2}B^{\mu\nu}X_{\mu\nu} = -\frac{1}{4}\begin{pmatrix} B^{\mu\nu} & X^{\mu\nu}\end{pmatrix}\begin{pmatrix} 1 & s_\varepsilon \\ s_\varepsilon & 1\end{pmatrix}\begin{pmatrix} B_{\mu\nu} \\ X_{\mu\nu}\end{pmatrix}$$

- Field strengths $B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu$ and $X_{\mu\nu} \equiv \partial_\mu X_\nu - \partial_\nu X_\mu$
- The **kinetic mixing** term is parametrized by $s_\varepsilon \in (-1, 1)$, beyond which the canonical kinetic terms have **wrong signs**
- Introduce $\varepsilon \in (-\pi/2, \pi/2)$ to express $s_\varepsilon = \sin\epsilon$
- $\mathcal{L}_\mathrm{K}$ can be made canonical via a $\mathrm{GL}(2,\mathbb{R})$ transformation



$$\begin{pmatrix} B_\mu \\ X_\mu\end{pmatrix} = V_\mathrm{k}\begin{pmatrix} \tilde{B}_\mu \\ \tilde{Z}_\mu\end{pmatrix}, \quad V_\mathrm{k} \equiv \begin{pmatrix} 1 & -t_\varepsilon \\ 0 & 1/c_\varepsilon\end{pmatrix}, \quad \begin{aligned} t_\varepsilon &\equiv \tan\varepsilon \\ c_\varepsilon &\equiv \cos\varepsilon \end{aligned}$$

$$V_\mathrm{k}^\mathrm{T}\begin{pmatrix} 1 & s_\varepsilon \\ s_\varepsilon & 1\end{pmatrix}V_\mathrm{k} = \begin{pmatrix} 1 & \\ & 1\end{pmatrix} \quad \Rightarrow \quad \mathcal{L}_\mathrm{K} = -\frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{4}\tilde{Z}'^{\mu\nu}\tilde{Z}'_{\mu\nu}$$

Gauge Boson Masses

- Mass-squared matrix for (B_μ, W_μ^3, X_μ) generated by the VEVs v, v_S, and v_Φ

$$M_{\mathrm{N}}^2 = \begin{pmatrix} g'^2 v^2/4 & -gg'v^2/4 & \\ -gg'v^2/4 & g^2v^2/4 & \\ & & g_X^2(v_S^2 + 4v_\Phi^2) \end{pmatrix}, \quad W^\pm \textbf{ boson mass } m_W = \frac{1}{2}gv$$

- Taking into account the **kinetic mixing** s_ε and the diagonalization of the mass-squared matrix, the **photon** γ remain **massless**, while the masses of the Z **boson** and a new **massive neutral vector boson** Z' are given by

$$m_Z^2 = \hat{m}_Z^2(1 + \hat{s}_{\mathrm{W}} t_\varepsilon t_\xi), \quad m_{Z'}^2 = \frac{\hat{m}_{Z'}^2}{c_\varepsilon^2(1 + \hat{s}_{\mathrm{W}} t_\varepsilon t_\xi)}$$

- Direct contributions from the VEVs: $\hat{m}_Z^2 \equiv (g^2 + g'^2)v^2/4, \quad \hat{m}_{Z'}^2 \equiv g_X^2(v_S^2 + 4v_\Phi^2)$
- **Weak mixing angle** $\hat{\theta}_{\mathrm{W}}$ satisfies $\hat{s}_{\mathrm{W}} \equiv \sin\hat{\theta}_{\mathrm{W}} = \dfrac{g'}{\sqrt{g^2 + g'^2}}, \quad \hat{c}_{\mathrm{W}} \equiv \cos\hat{\theta}_{\mathrm{W}}$
- **Rotation angle** ξ is given by $\tan 2\xi = \dfrac{s_{2\varepsilon}\hat{s}_{\mathrm{W}} v^2(g^2 + g'^2)}{c_\varepsilon^2 v^2(g^2 + g'^2)(1 - \hat{s}_{\mathrm{W}}^2 t_\varepsilon^2) - 4g_X^2(v_S^2 + 4v_\Phi^2)}$

Electroweak (EW) Current Interactions

At tree level, the **charge current interactions** of SM fermions are not affected by the kinetic mixing, remaining a form of

$$\mathcal{L}_{\mathrm{CC}} = \frac{1}{\sqrt{2}}(W_\mu^+ J_W^{+,\mu} + \mathrm{H.c.}), \quad J_W^{+,\mu} = g(\bar{u}_{i\mathrm{L}}\gamma^\mu V_{ij} d_{j\mathrm{L}} + \bar{\nu}_{i\mathrm{L}}\gamma^\mu \ell_{i\mathrm{L}})$$

v is still directly related to the Fermi constant $G_{\mathrm{F}} = \dfrac{g^2}{4\sqrt{2}m_W^2} = \dfrac{1}{\sqrt{2}v^2}$

Neutral current interactions become $\mathcal{L}_{\mathrm{NC}} = j^\mu_{\mathrm{EM}} A_\mu + j^\mu_Z Z_\mu + j^\mu_{Z'} Z'_\mu$

Electromagnetic current $j^\mu_{\mathrm{EM}} = \sum_f Q_f e \bar{f}\gamma^\mu f$ with $e = gg'/\sqrt{g^2 + g'^2}$

Z **current** $j^\mu_Z = \dfrac{ec_\xi(1+\hat{s}_{\mathrm{W}}t_\varepsilon t_\xi)}{2\hat{s}_{\mathrm{W}}\hat{c}_{\mathrm{W}}} \sum_f \bar{f}\gamma^\mu(T^3_f - 2Q_f s_*^2 - T^3_f\gamma_5)f + \dfrac{s_\xi}{c_\varepsilon} j^\mu_X$

Z' **current**
$$j^\mu_{Z'} = \frac{e(\hat{s}_{\mathrm{W}}t_\varepsilon c_\xi - s_\xi)}{2\hat{s}_{\mathrm{W}}\hat{c}_{\mathrm{W}}} \sum_f \bar{f}\gamma^\mu(T^3_f - 2Q_f \hat{s}^2_{\mathrm{W}} - T^3_f\gamma_5)f - \hat{c}_{\mathrm{W}} t_\varepsilon c_\xi j^\mu_{\mathrm{EM}} + \frac{c_\xi}{c_\varepsilon} j^\mu_X$$

$\mathrm{U}(1)_{\mathrm{X}}$ **current** $j^\mu_X = \mathrm{i}g_X(S^\dagger \overleftrightarrow{\partial^\mu} S + 2\Phi^\dagger \overleftrightarrow{\partial^\mu}\Phi)$, $\quad s_*^2 \equiv \hat{s}^2_{\mathrm{W}} + \hat{c}^2_{\mathrm{W}} \dfrac{\hat{s}_{\mathrm{W}} t_\varepsilon t_\xi}{1+\hat{s}_{\mathrm{W}} t_\varepsilon t_\xi}$

Independent Parameters

✏️ In the **SM**, the weak mixing angle obeys $s_{\mathrm{W}}^2 c_{\mathrm{W}}^2 = \dfrac{\pi\alpha}{\sqrt{2} G_{\mathrm{F}} m_Z^2}$ at tree level

💡 Use this relation to define a **"physical" weak mixing angle** θ_{W} via the best measured parameters α, G_{F}, and m_Z [Burgess *et al.*, hep-ph/9312291, PRD]

🔮 Similar relation in the **hidden** $\mathrm{U}(1)_{\mathrm{X}}$ **gauge theory**: $\hat{s}_{\mathrm{W}}^2 \hat{c}_{\mathrm{W}}^2 = \dfrac{\pi\alpha}{\sqrt{2} G_{\mathrm{F}} \hat{m}_Z^2}$

👉 $\hat{s}_{\mathrm{W}} \hat{c}_{\mathrm{W}} \hat{m}_Z = s_{\mathrm{W}} c_{\mathrm{W}} m_Z$ 👉 $s_{\mathrm{W}}^2 c_{\mathrm{W}}^2 = \dfrac{\hat{s}_{\mathrm{W}}^2 \hat{c}_{\mathrm{W}}^2}{1 + \hat{s}_{\mathrm{W}} t_\varepsilon t_\xi}$

🎈 The angle ξ satisfies $t_\xi = \dfrac{2\hat{s}_{\mathrm{W}} t_\varepsilon}{1-r} \left[1 + \sqrt{1 - r\left(\dfrac{2\hat{s}_{\mathrm{W}} t_\varepsilon}{1-r}\right)^2} \right]^{-1}$ with $r \equiv \dfrac{m_{Z'}^2}{m_Z^2}$

🎯 Utilizing these relations, we obtain $\hat{s}_{\mathrm{W}}$ and t_ξ as functions of s_ε and $m_{Z'}$

🚩 The related **independent parameters** can be chosen as $\{m_{Z'},\ s_\varepsilon,\ v_S,\ v_\Phi\}$

🖋️ EW gauge couplings $g = \dfrac{e}{\hat{s}_{\mathrm{W}}}$ and $g' = \dfrac{e}{\hat{c}_{\mathrm{W}}}$ with $e = \sqrt{4\pi\alpha}$

Z_2-violating Couplings

The Z-χ-h_i and Z'-χ-h_i couplings from $Z_\mu j_Z^\mu + Z'_\mu j_{Z'}^\mu$ are

$$\mathcal{L}_{\chi h_i} = \sum_{i=1}^{3} (g_{Z\chi h_i} Z_\mu \chi \overleftrightarrow{\partial^\mu} h_i + g_{Z'\chi h_i} Z'_\mu \chi \overleftrightarrow{\partial^\mu} h_i)$$

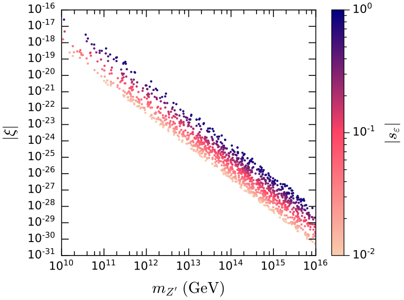
$$g_{Z\chi h_i} = \frac{g_X s_\xi}{c_\varepsilon}(c_\beta U_{2i} - 2 s_\beta U_{3i})$$

$$g_{Z'\chi h_i} = \frac{g_X c_\xi}{c_\varepsilon}(c_\beta U_{2i} - 2 s_\beta U_{3i})$$

These couplings **break** the Z_2 **symmetry** $\chi \to -\chi$, inducing **decay processes** of χ

In order to be a **viable DM candidate**, χ should have a **sufficiently long lifetime**

$g_{Z\chi h_i}$ would be **greatly suppressed** by $m_{Z'}$ (or the **UV scale** v_Φ) because of the approximate relation $\xi \simeq -\dfrac{s_W t_\varepsilon m_Z^2}{m_{Z'}^2}$



Higgs Physics

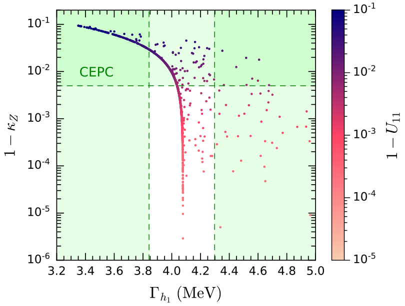
- Couplings of the **SM-like Higgs boson** h_1 to SM particles can be parametrized as

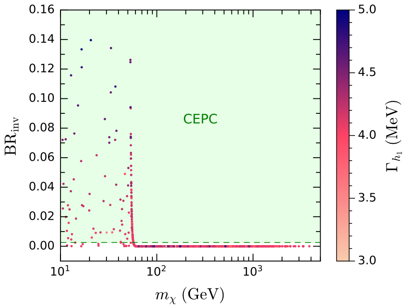
$$\mathcal{L}_{h_1} = \kappa_W \frac{2m_W^2}{v} h_1 W_\mu^+ W^{-,\mu} + \kappa_Z \frac{m_Z^2}{v} h_1 Z_\mu Z^\mu - \sum_f \kappa_f \frac{m_f}{v} h_1 \bar{f} f$$

- The **SM** corresponds to $\kappa_W = \kappa_Z = \kappa_f = 1$, while this model gives

$$\kappa_W = \kappa_f = U_{11},, \quad \kappa_Z = U_{11} c_\xi^2 (1 + \hat{s}_W t_\varepsilon t_\xi) + \frac{s_\xi^2 g_X^2 v}{c_\varepsilon^2 m_Z^2} (U_{21} v_S + 4 U_{31} v_\Phi)$$

- **Exotic h_1 decay channels** may include $h_1 \to \chi\chi$, $h_1 \to \chi Z$, and $h_1 \to h_2 h_2$





Parameter Point Selection

The following criteria are used to select the parameter points

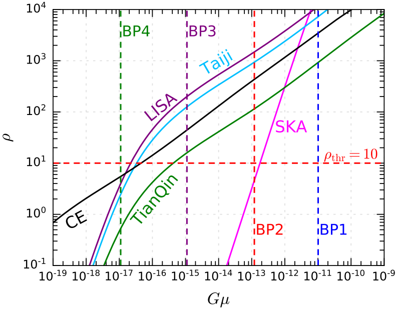
1. In order to guarantee the **vacuum stability**, the scalar potential should satisfy the **copositivity criteria**
2. The **lifetime** of the **pNGB DM particle** χ should satisfy the **Fermi-LAT bound** $\tau_\chi \gtrsim 10^{27}$ s
3. The **DM relic abundance** $\Omega_\chi h^2$ calculated by `micrOMEGAs` should be in the 3σ range of the **Planck value** $\Omega_{\mathrm{DM}} h^2 = 0.1200 \pm 0.0012$
4. The **total** $\chi\chi$ **annihilation cross section** $\langle \sigma_{\mathrm{ann}} v \rangle$ should not be excluded by the upper limits at the 95% C.L. given by the combined **Fermi-LAT** and **MAGIC** γ**-ray observations** of dwarf spheroidal galaxies in the $b\bar{b}$ channel
5. The signal strengths of the **SM-like Higgs boson** h_1 should be consistent with the **LHC Higgs measurements** at 95% C.L. based on the `HiggsSignals` calculation
6. The **exotic Higgs boson** h_2 should not be excluded at 95% C.L. by the **direct searches** at the **LHC** and the **Tevatron** according to `HiggsBounds`

Benchmark Points

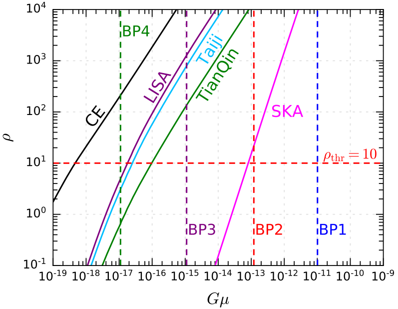
	BP1	BP2	BP3	BP4
v_S (GeV)	1953	2101	548.5	1388
v_Φ (GeV)	1.335×10^{13}	1.939×10^{12}	1.969×10^{11}	3.179×10^{10}
m_χ (GeV)	199.8	56.26	98.16	123.1
m_{h_2} (GeV)	986.7	627.7	484.3	362.6
m_{h_3} (GeV)	8.403×10^{12}	1.469×10^{12}	1.893×10^{11}	8.312×10^{9}
$m_{Z'}$ (GeV)	7.255×10^{11}	5.929×10^{11}	9.661×10^{10}	4.979×10^{10}
$\lambda_{H\Phi}$	-6.330×10^{-2}	-3.786×10^{-1}	-1.278×10^{-2}	-6.114×10^{-2}
$\lambda_{S\Phi}$	-2.870×10^{-1}	-5.416×10^{-2}	2.813×10^{-1}	3.188×10^{-2}
λ_{HS}	3.259×10^{-1}	1.189×10^{-1}	-1.750×10^{-1}	1.819×10^{-2}
s_ε	4.840×10^{-3}	3.222×10^{-1}	7.161×10^{-2}	1.929×10^{-3}
$G\mu$	1.01×10^{-11}	1.20×10^{-13}	1.11×10^{-15}	1.10×10^{-17}
$\Omega_\chi h^2$	0.118	0.121	0.120	0.119
$\sigma^{\rm SI}_{\chi N}$ (cm^2)	1.38×10^{-86}	1.62×10^{-86}	1.59×10^{-82}	8.45×10^{-77}
$\langle \sigma_{\rm ann} v \rangle$ (cm^3/s)	2.00×10^{-26}	2.87×10^{-29}	2.01×10^{-26}	1.71×10^{-26}
$\rho_{\rm LISA}$ (BOS)	1.15×10^{4}	1.48×10^{3}	2.00×10^{2}	3.97
$\rho_{\rm Taiji}$ (BOS)	7.26×10^{3}	9.37×10^{2}	1.26×10^{2}	2.45
$\rho_{\rm TianQin}$ (BOS)	9.25×10^{2}	1.15×10^{2}	1.59×10^{1}	5.28×10^{-1}
$\rho_{\rm CE}$ (BOS)	3.49×10^{3}	4.33×10^{2}	4.42×10^{1}	5.48
$\rho_{\rm LISA}$ (LRS)	1.15×10^{7}	1.38×10^{5}	1.28×10^{3}	4.93
$\rho_{\rm Taiji}$ (LRS)	7.19×10^{6}	8.57×10^{4}	7.95×10^{2}	3.05
$\rho_{\rm TianQin}$ (LRS)	1.20×10^{6}	1.42×10^{4}	1.36×10^{2}	6.48×10^{-1}
$\rho_{\rm CE}$ (LRS)	4.36×10^{6}	2.18×10^{6}	2.02×10^{4}	2.11×10^{2}

Sensitivity of Future GW Experiments

BOS model



LRS model



Expected upper limits on $G\mu$ corresponding to $\rho_{\rm thr} = 10$

	LISA	Taiji	TianQin	CE	SKA
BOS	2.21×10^{-17}	3.34×10^{-17}	4.28×10^{-16}	4.54×10^{-17}	1.77×10^{-13}
LRS	1.79×10^{-17}	2.51×10^{-17}	9.67×10^{-17}	4.66×10^{-19}	8.09×10^{-14}