The 29th International Workshop
on Weak Interactions and Neutrinos

The ALP explanation to muon g-2 and its test at future lepton collider

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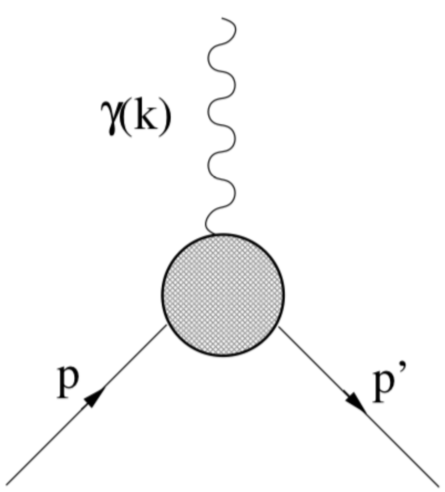
Introduction to g-2

Anomalous magnetic moment definition:

$$\vec{\mu} = g\frac{e}{2m}\vec{s} \qquad a = \frac{g-2}{2}$$

In Quantum Field Theory (with C, P invariance) $k = p' - p$



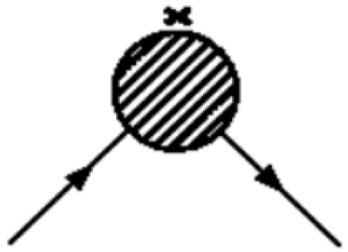
Dirac | Pauli

$$= (-ie)\bar{u}(p')\left[\gamma^\mu F_1(k^2) + \frac{i\sigma^{\mu\nu}k_\nu}{2m}F_2(k^2)\right]u(p)$$

$F_1(0) = 1$ | $F_2(0) = a$

Introduction to g-2

What's the running in the loop?



Total anomaly can be written as:

$$a_\ell = \underbrace{a_\ell^{QED} + a_\ell^{hadronic} + a_\ell^{weak}}_{\text{Standard Model}} + \underbrace{a_\ell^{BSM}}_{\text{New Physics}}$$

Introduction to g-2

The magnetic moment from such a loop enter as functions of:

$$a_\ell \sim f[\frac{m_\ell^2}{M^2}]$$

Muon is more sensitive than electron in sensing a heavy unknown particle:

$$\frac{m_\mu^2}{m_e^2} \simeq 43000$$

But τ is so short-lived (10^{-13}seconds) that no practical experiment can be designed with the current technology, current experimental bound is

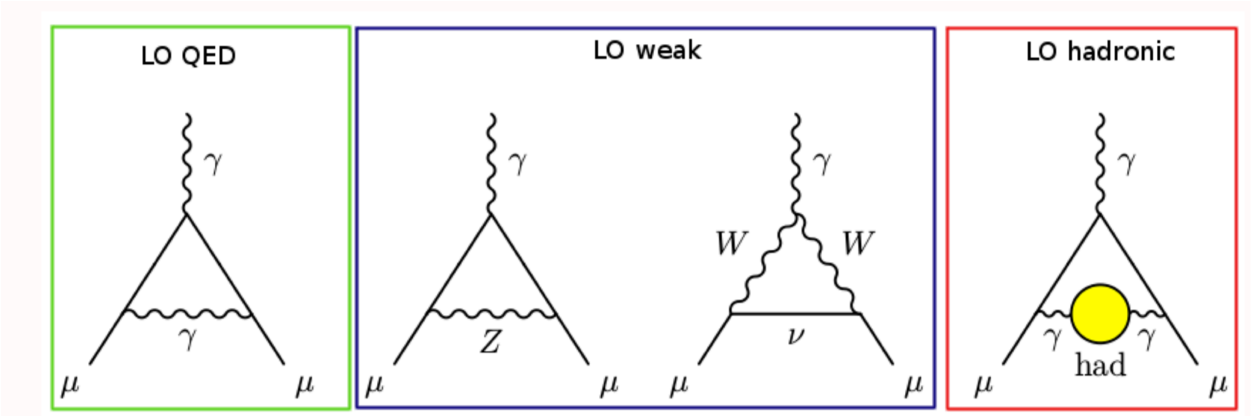
$$-0.052 < a_\tau < 0.013$$

Even the sign is not known experimentally!!!

This leaves us with the only choice μ.

The Muon g-2 anomaly

Standard Model Result (SM)



$$a_\mu^{\mathrm{SM}} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{weak}} + a_\mu^{\mathrm{had}}$$

$\times 10^{-10}$

QED Contribution	11 658 471.895 (0.015)	Aoyama et al (5-loop)
EW Contribution	15.4 (0.1)	Grendiger et al (Higgs mass)
Hadronic Contribution		
LO hadronic	694.9(4.3)	HLMNT11
NLO hadronic	-9.8(0.1)	HLMNT11
Light-by-light	10.5(2.6)	Prades, de Rafael &
Theory Total	11659182.3(4.9)	

The Muon g-2 anomaly

Positive value and a 4.2 σ (Fermilab + Brookhaven)

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{th}} = (25.1 \pm 5.9) \times 10^{-10}$$

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The Era of Anomalies

May 14, 2020 • *Physics* 13, 79

Particle physicists are faced with a growing list of "anomalies"—experimen standard model but fail to overturn it for lack of sufficient evidence.

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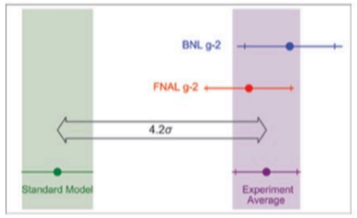


ON THE COVER

Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm

April 7, 2021

New muon magnetic moment data from a Fermilab experiment (red) combined with previous Brookhaven National Lab data (blue) is in 4.2σ tension with the value calculated by the Muon g-2 Theory Initiative (green). Selected for a Viewpoint in *Physics* and an Editors' Suggestion.

B. Abi *et al.* (Muon $g-2$ Collaboration)
Phys. Rev. Lett. **126**, 141801 (2021)

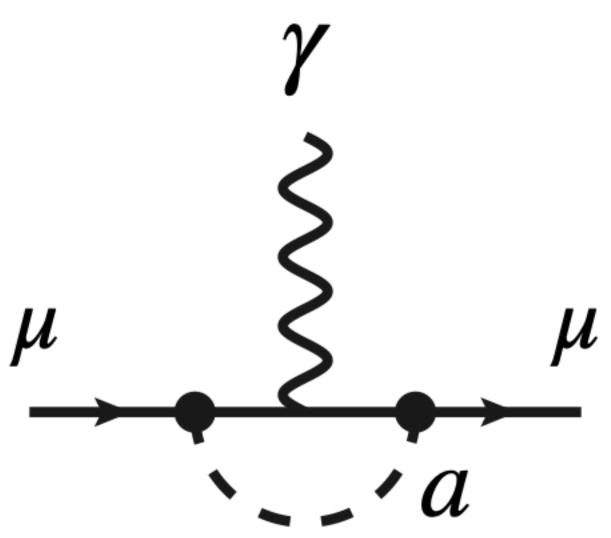
Issue 14 Table of Contents | More Covers

Yannis K. Semertzidis, IBS-CAPP and KAIST

The Scalar for Muon g-2

The (pseudo)scalar Yukawa coupling to lepton

$$\mathscr{L}_{\text{yuk}} = \phi \bar{\ell} \left(g_R + i g_I \gamma_5 \right) \ell$$



The 1-loop contribution to g-2

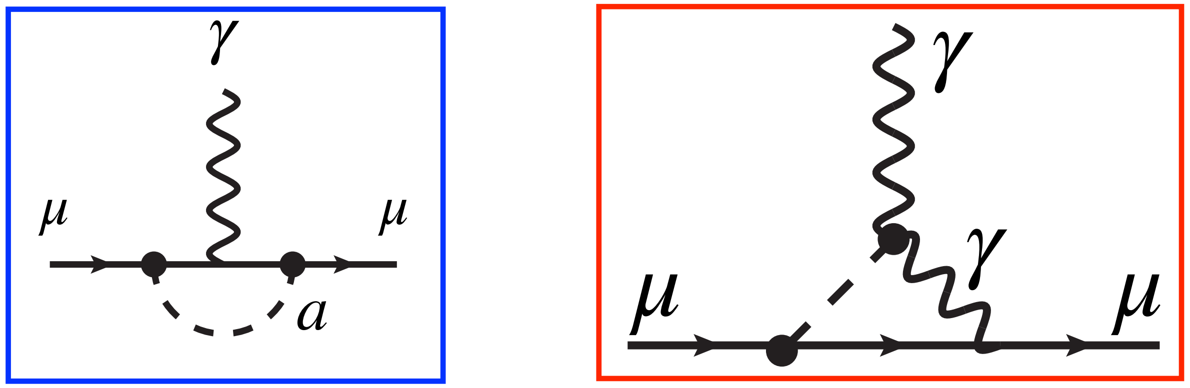
$$\Delta a_\ell = \frac{1}{8\pi^2} \int_0^1 dx \frac{(1-x)^2\left((1+x)g_R^2 - (1-x)g_I^2\right)}{(1-x)^2 + x\left(m_\phi / m_\ell\right)^2}$$

- For scalar, $\Delta a_\ell > 0$
- For (psudo)scalar, $\Delta a_\ell < 0$

The pseudo-scalar solution

Further requirement for pseudo-scalar

$$\mathscr{L} = i y_{a\psi} a \bar{\psi} \gamma_5 \psi + \frac{1}{4} g_{a\gamma\gamma} \tilde{F} F$$

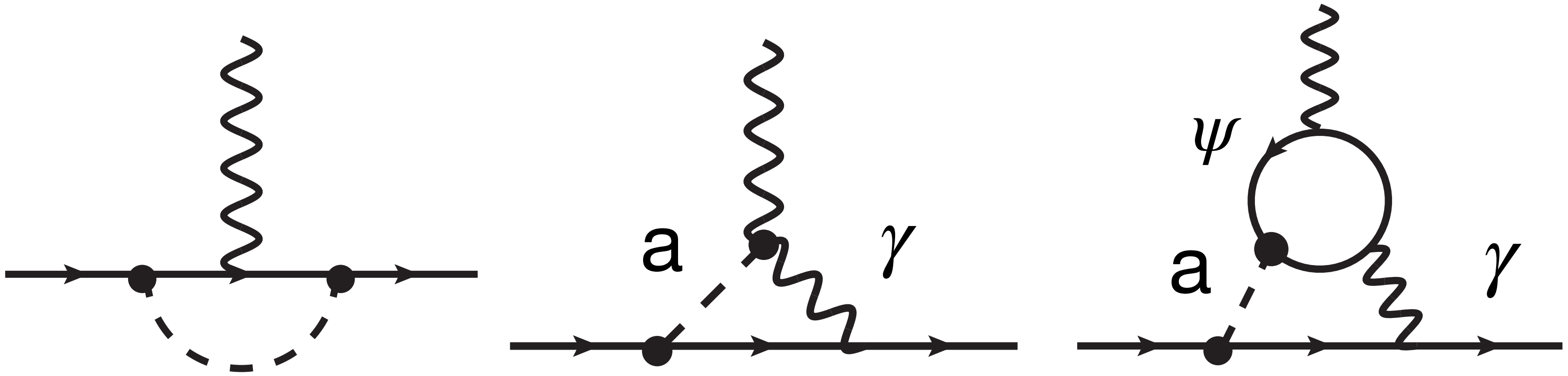


- Assumes $g_{a\gamma\gamma}$ remains essentially constant throughout the integration over virtual photon-loop momentum
- $g_{a\gamma\gamma}$ and $y_{a\ell}$ can adjust its sign to give positive result

Complete calculation for ALP

The axion-like particle Lagrangian

$$\mathscr{L}_{\rm eff}^{D\leq 5} = \sum_f \frac{C_{ff}}{2}\frac{\partial^\mu a}{f_a}\bar{f}\gamma_\mu\gamma_5 f + \frac{\alpha C_{\gamma\gamma}}{4\pi}\frac{a}{f_a}F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{\alpha C_{\gamma Z}}{2\pi s_w c_w}\frac{a}{f_a}F_{\mu\nu}\tilde{Z}^{\mu\nu} + \frac{\alpha C_{ZZ}}{4\pi s_w^2 c_w^2}\frac{a}{f_a}Z_{\mu\nu}\tilde{Z}^{\mu\nu}$$
$$+\frac{\alpha C_{WW}}{\pi s_w^2}\frac{a}{f_a}\epsilon_{\mu\nu\rho\sigma}\partial^\mu W_+^\nu \partial^\rho W_-^\sigma + \ldots$$



$\Delta a_\mu^{(1)} \propto -\frac{c_{\mu\mu}^2}{16\pi^2}$ $\Delta a_\mu^{(2)} \propto -\frac{c_{\mu\mu}c_{\gamma\gamma}\alpha}{16\pi^3}$ $\Delta a_\mu^{(3)} \propto -\frac{c_{\mu\mu}c_{ii}\alpha}{16\pi^3}$

- Different sign for $c_{\mu\mu}$ and $c_{\gamma\gamma}$ is needed
- The 3rd diagram subtlety:

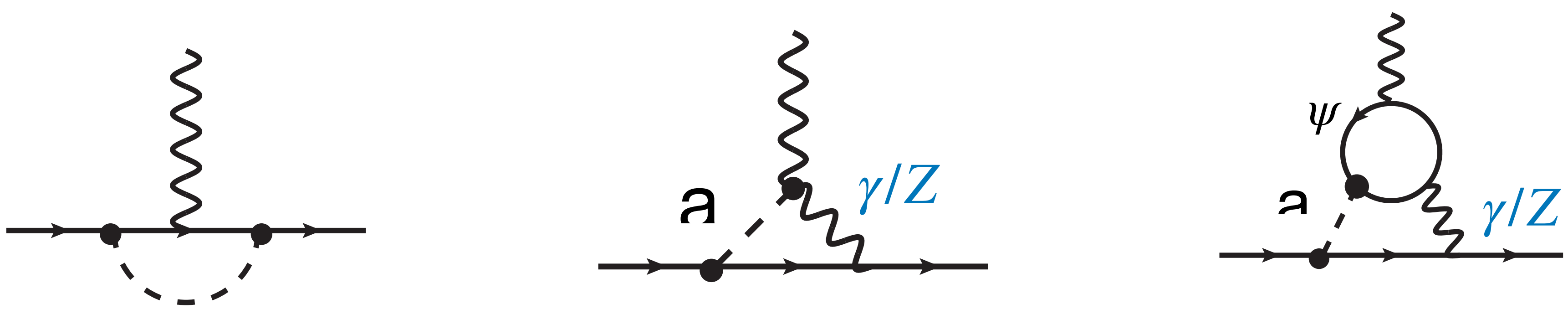
Complete calculation for ALP

The axion-like particle Lagrangian

$$\mathscr{L}_{\text{eff}}^{D\le 5} = \sum_f \frac{C_{ff}}{2}\frac{\partial^\mu a}{f_a}\bar{f}\gamma_\mu\gamma_5 f + \frac{\alpha C_{\gamma\gamma}}{4\pi}\frac{a}{f_a}F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{\alpha C_{\gamma Z}}{2\pi s_w c_w}\frac{a}{f_a}F_{\mu\nu}\tilde{Z}^{\mu\nu} + \frac{\alpha C_{ZZ}}{4\pi s_w^2 c_w^2}\frac{a}{f_a}Z_{\mu\nu}\tilde{Z}^{\mu\nu}$$

$$+\frac{\alpha C_{WW}}{\pi s_w^2}\frac{a}{f_a}\epsilon_{\mu\nu\rho\sigma}\partial^\mu W_+^\nu \partial^\rho W_-^\sigma + \dots$$

$$C_{\gamma\gamma} = C_{WW} + C_{BB} \qquad C_{\gamma Z} = c_w^2 C_{WW} - s_w^2 C_{BB} \qquad C_{ZZ} = c_w^4 C_{WW} + s_w^4 C_{BB}$$

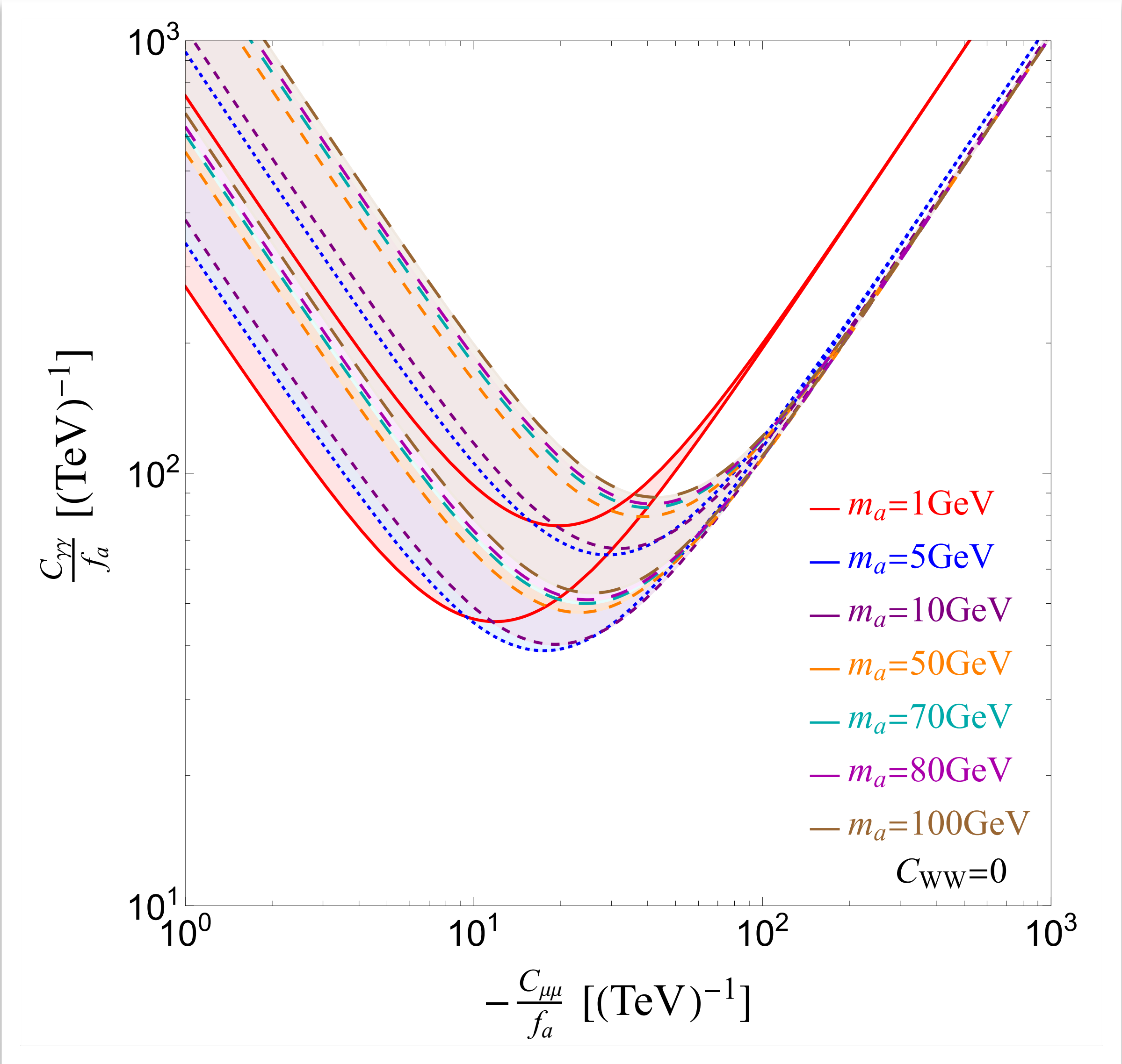


$$\Delta a_{2Z} = \frac{\alpha c_{\gamma Z} c_{\mu\mu} m_\mu^2 \left(4s_w^2 - 1\right)}{32\pi^3 c_w^2 s_w^2 f_a^2} \cdot h_Z(x, y, \mu)$$

$$\Delta a_{3\gamma} = -\frac{c_{\mu\mu}^2 \alpha}{8\pi^3}\frac{m_\mu^2}{f_a^2}\left[H_\gamma(\textbf{two loop}) + h_\gamma(\textbf{counter term})\right] \qquad m_\psi \to \infty,\ \Delta a_3 \to 0$$

$$\Delta a_{3Z} = -\frac{\alpha c_{\mu\mu}^2 m_\mu^2 \left(4s_w^2 - 1\right)^2}{128\pi^3 c_w^2 s_w^2 f_a^2}\left[H_Z(\textbf{two loop}) + h_Z(\textbf{counter term})\right]$$

Muon g-2 solution $C_{ww} = 0$



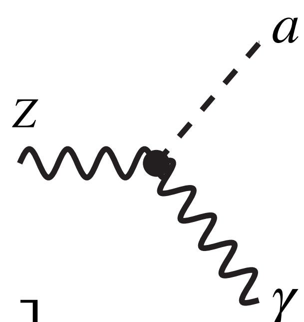
- In g-2 solution region, mostly decay to $a \to \mu^+\mu^-$
- The inclusion of Z diagram makes some difference for large m_a
- Exotic Z decay should happen

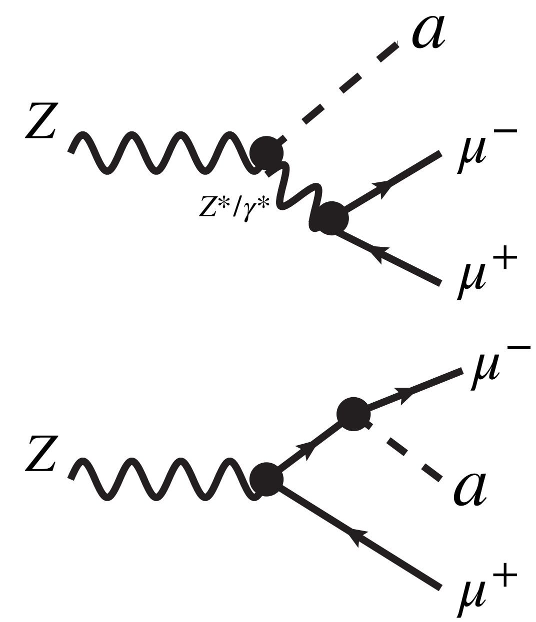
Z decay $C_{ww} = 0$

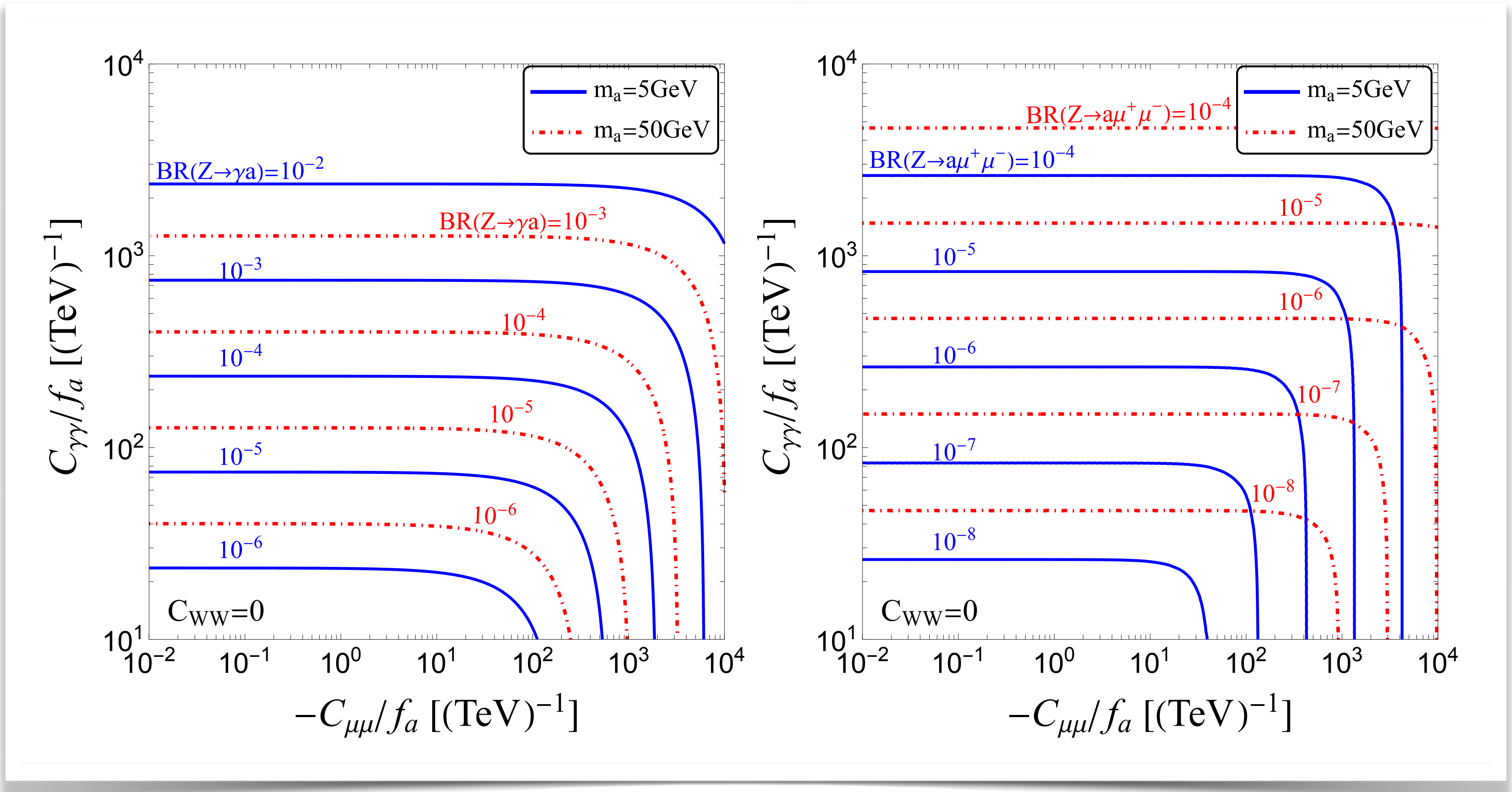
ALP production from Z decay

$$\Gamma(Z \to \gamma a) = \frac{\alpha^2(m_Z)\, m_Z^3}{96\pi^3 s_w^2 c_w^2 f_a^2} \left| C_{\gamma Z}^{\rm eff} \right|^2 \left(1 - \frac{m_a^2}{m_Z^2}\right)^3$$

$$C_{\gamma Z}^{\rm eff} = C_{\gamma Z} + C_{\mu\mu}\left(\frac{1}{4} - s_w^2\right)\left[1 + \frac{m_\mu^2}{m_a^2 - m_Z^2}\cdot\left(\mathscr{F}\left(\frac{m_a^2}{m_\mu^2}\right) - \mathscr{F}\left(\frac{m_Z^2}{m_\mu^2}\right)\right)\right]$$







ALP decay $C_{ww} = 0$

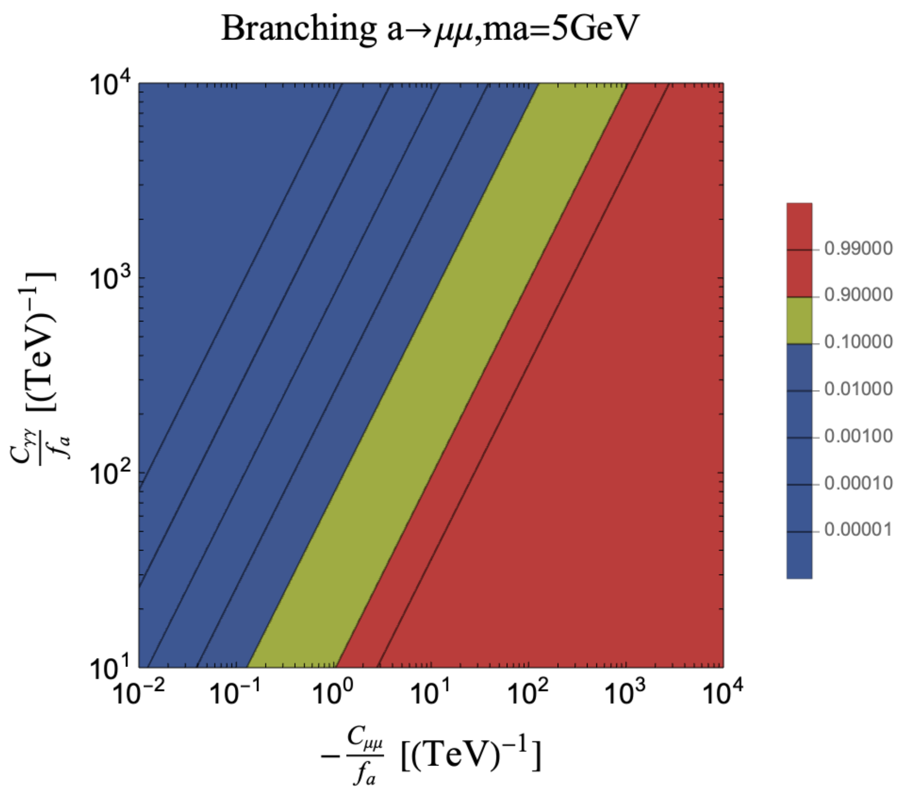
ALP decay

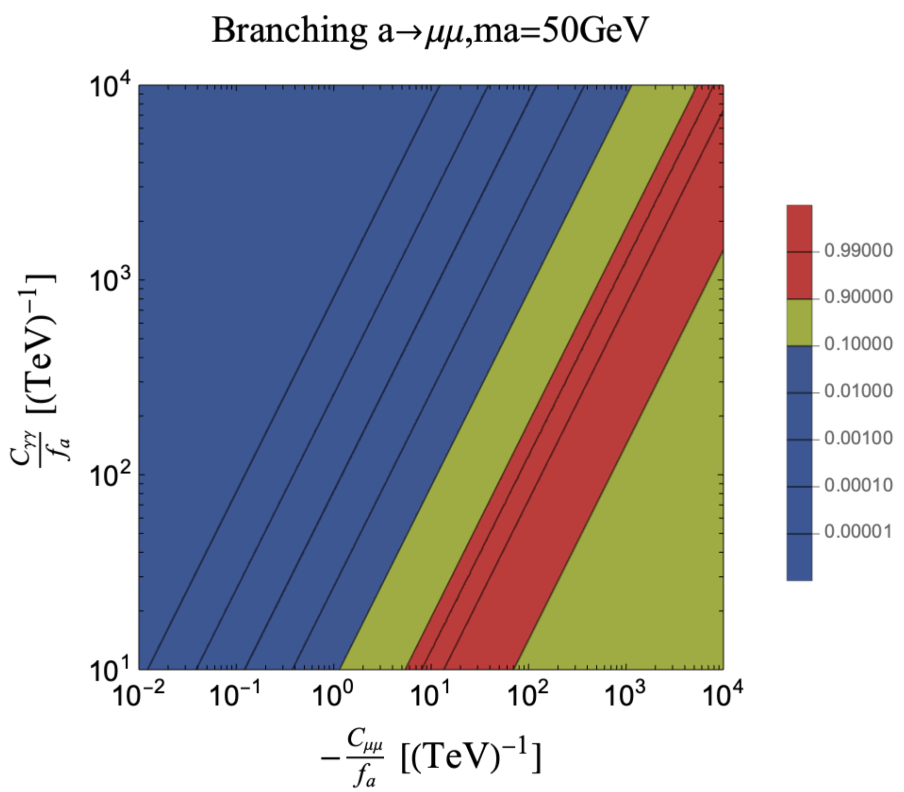
$$\Gamma\left(a \to \mu^+\mu^-\right) = \frac{m_a m_\mu^2}{8\pi f_a^2}\left|C_{\mu\mu}^{\mathrm{eff}}\right|^2 \sqrt{1-\frac{4m_\mu^2}{m_a^2}}$$

$$\Gamma(a \to \gamma\gamma) = \frac{\alpha^2 m_a^3}{64\pi^3 f_a^2}\left|C_{\gamma\gamma}^{\mathrm{eff}}\right|^2$$

$$C_{\gamma\gamma}^{\mathrm{eff}} = C_{\gamma\gamma} + C_{\mu\mu}\left[1+\frac{m_\mu^2}{m_a^2}\cdot\mathscr{F}\left(\frac{m_a^2}{m_\mu^2}\right)\right] + \mathcal{O}(\alpha)$$

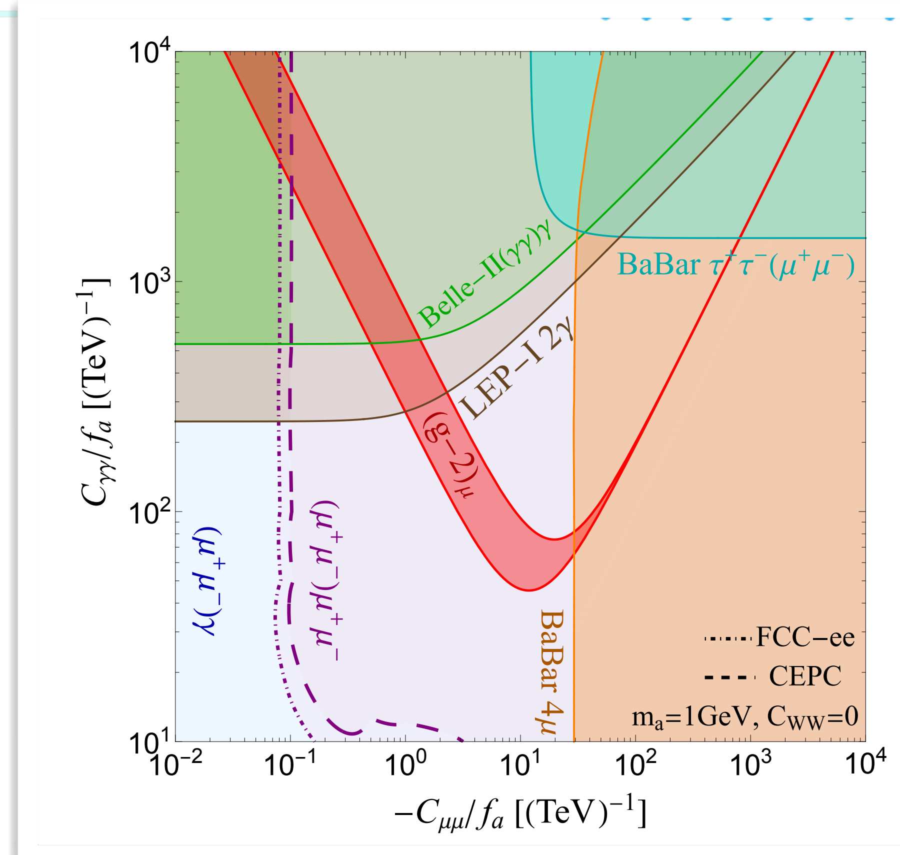
$$C_{\mu\mu}^{\mathrm{eff}} = C_{\mu\mu} + \mathcal{O}\left(\alpha^2\right)$$

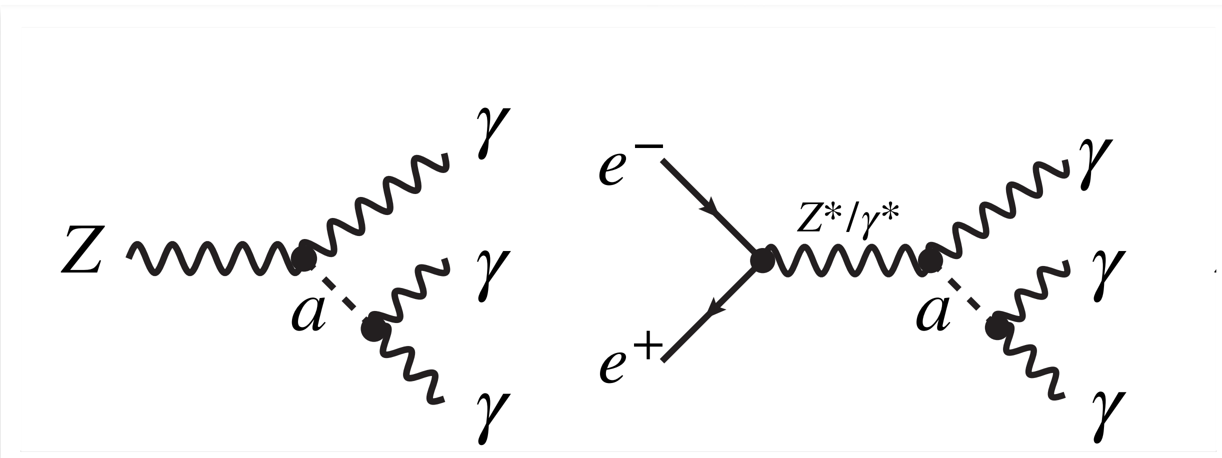


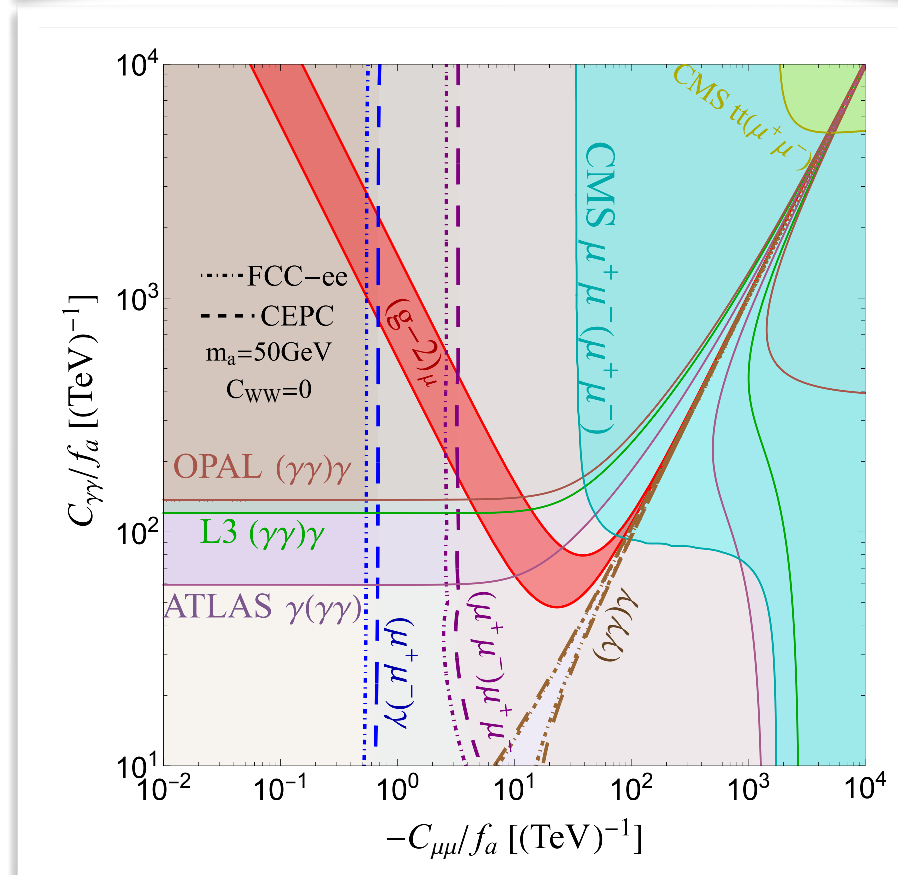


Existing constraints $C_{ww} = 0$

- Constraining a-γ coupling only:
  - Belle-II, LEP:
    $e^+e^- \to a\gamma \to (\gamma\gamma)\gamma$
  - LHC: $pp \to a\gamma \to (\gamma\gamma)\gamma$

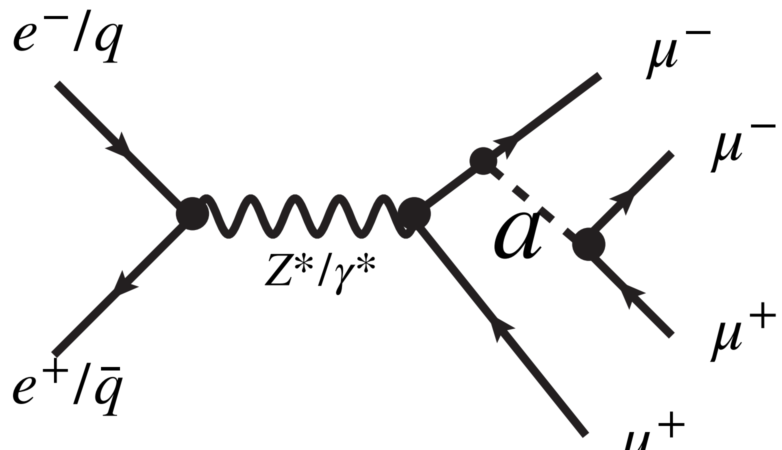






Existing constraints $C_{ww} = 0$

- Constraining a-μ coupling only:
  - BaBar: recast $e^+e^- \to \mu^+\mu^- Z'$
  - CMS(4μ): $pp \to \mu^+\mu^-\phi$



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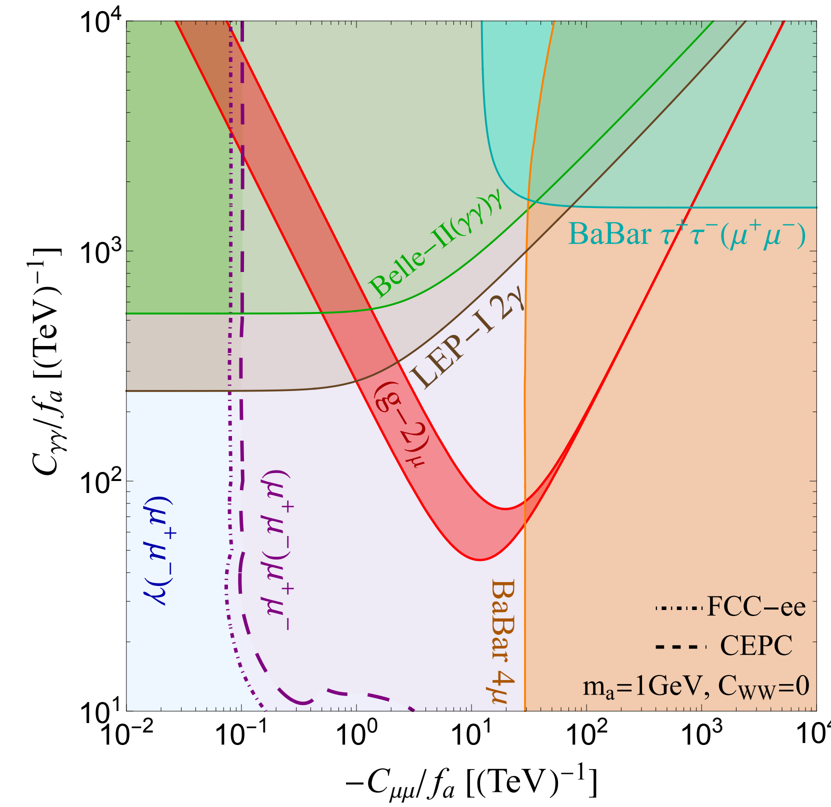
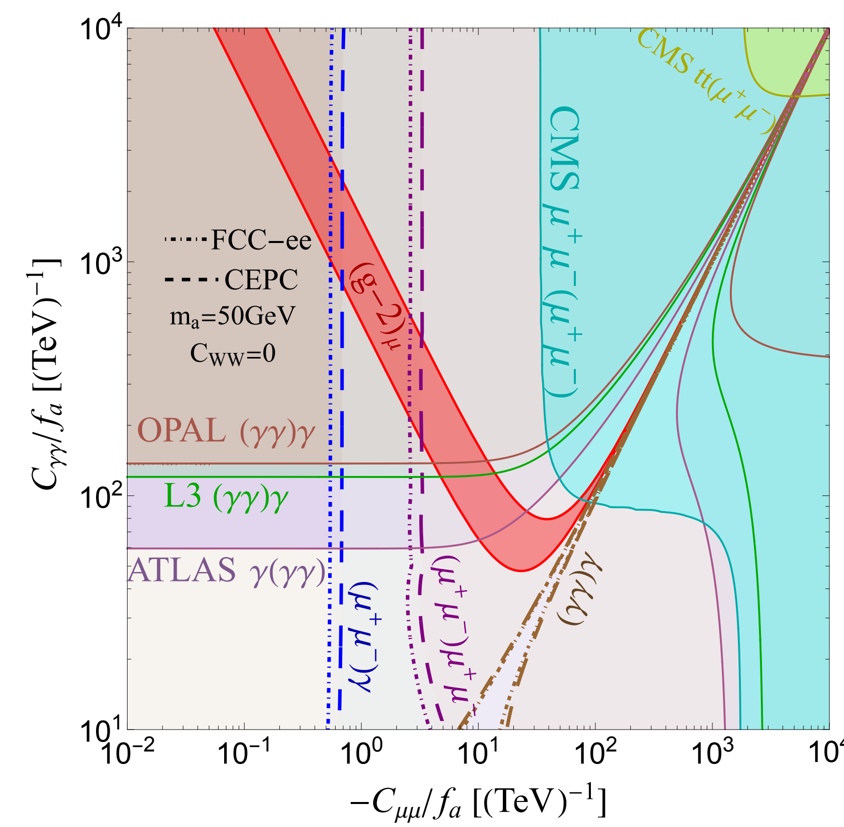
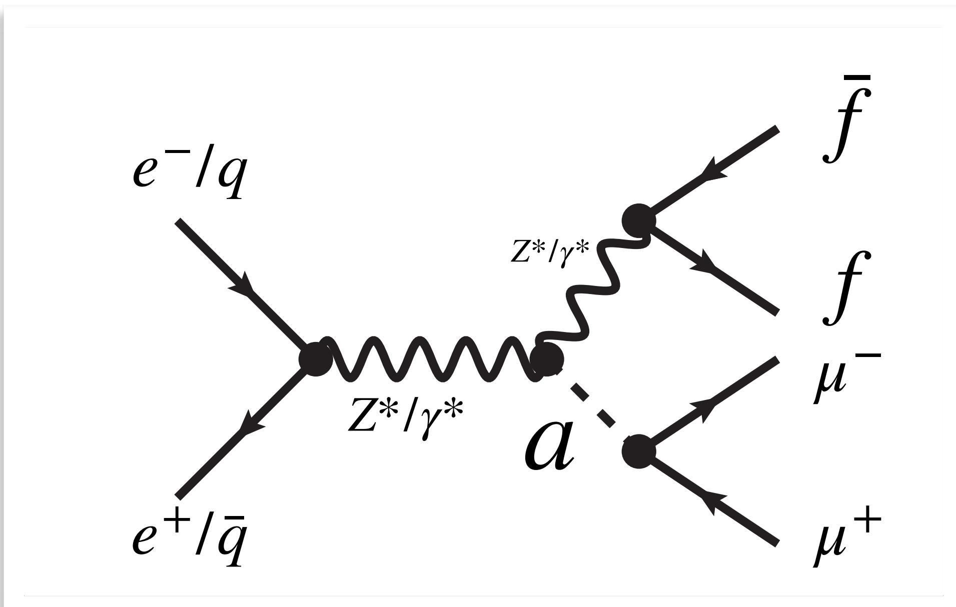
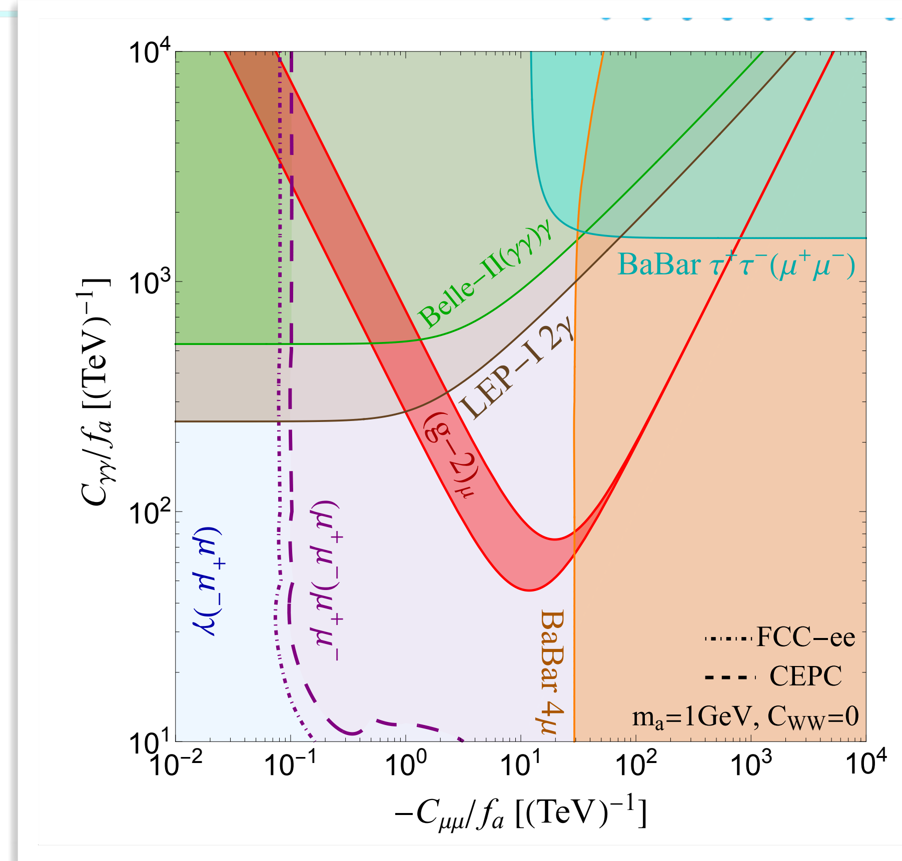


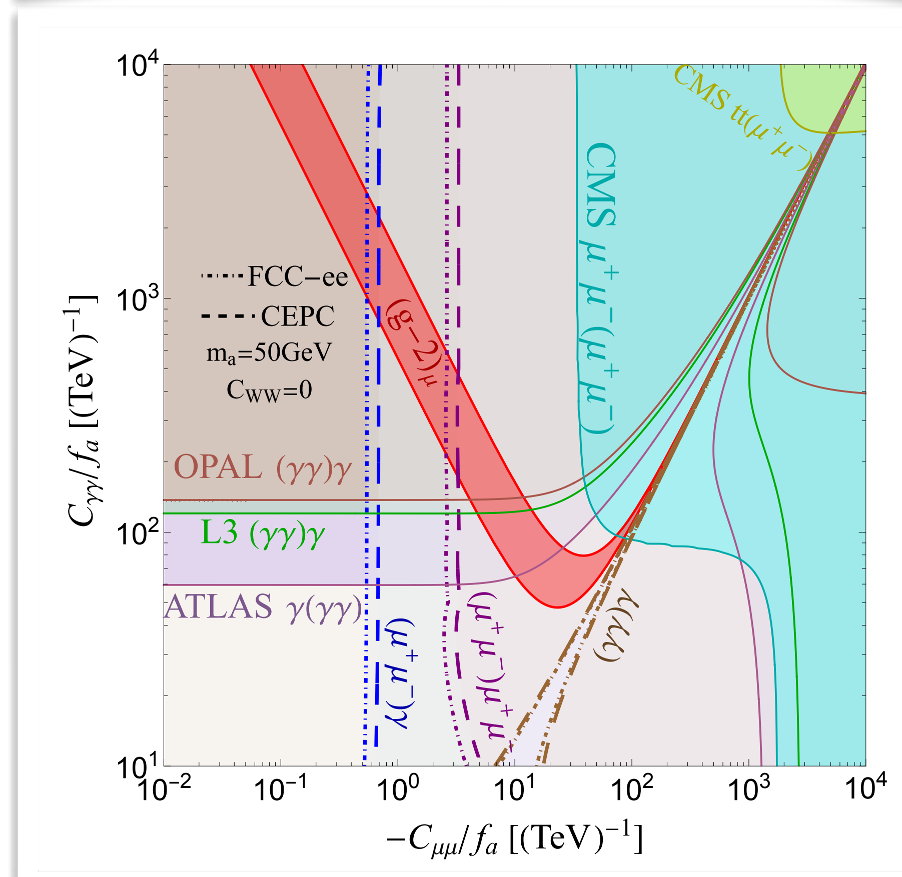


Existing constraints $C_{ww} = 0$

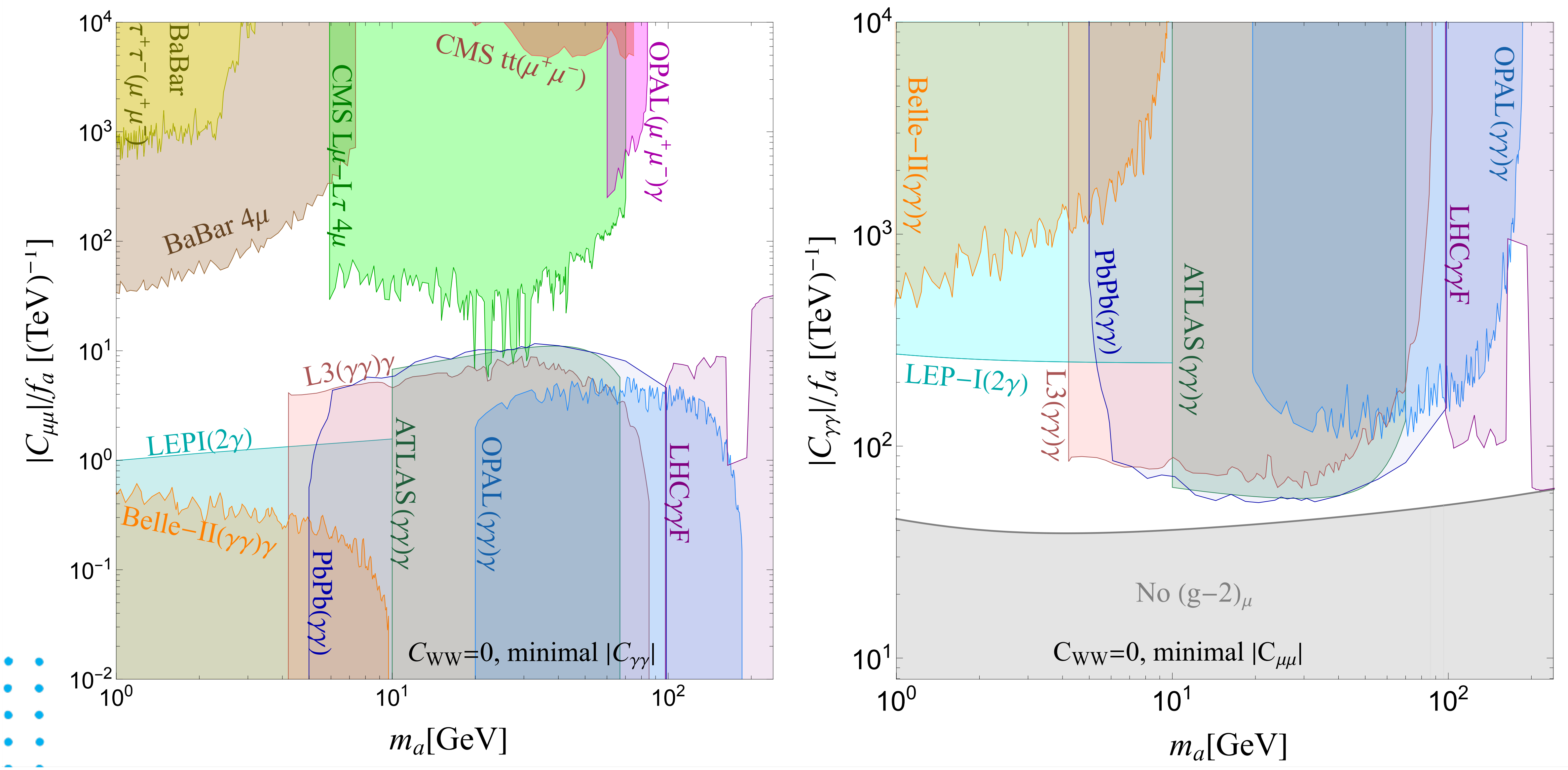
- Constraining both coupling
  - CMS($\bar{t}t + 2\mu$): $pp \to \bar{t}t\phi \to \bar{t}t(\mu^+\mu^-)$





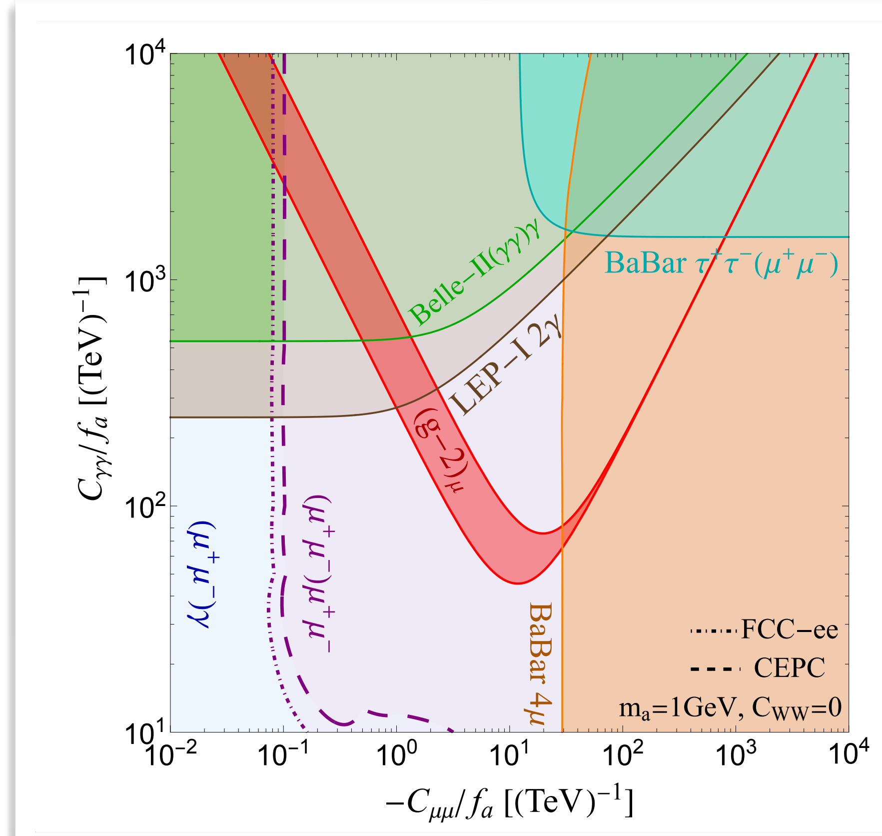


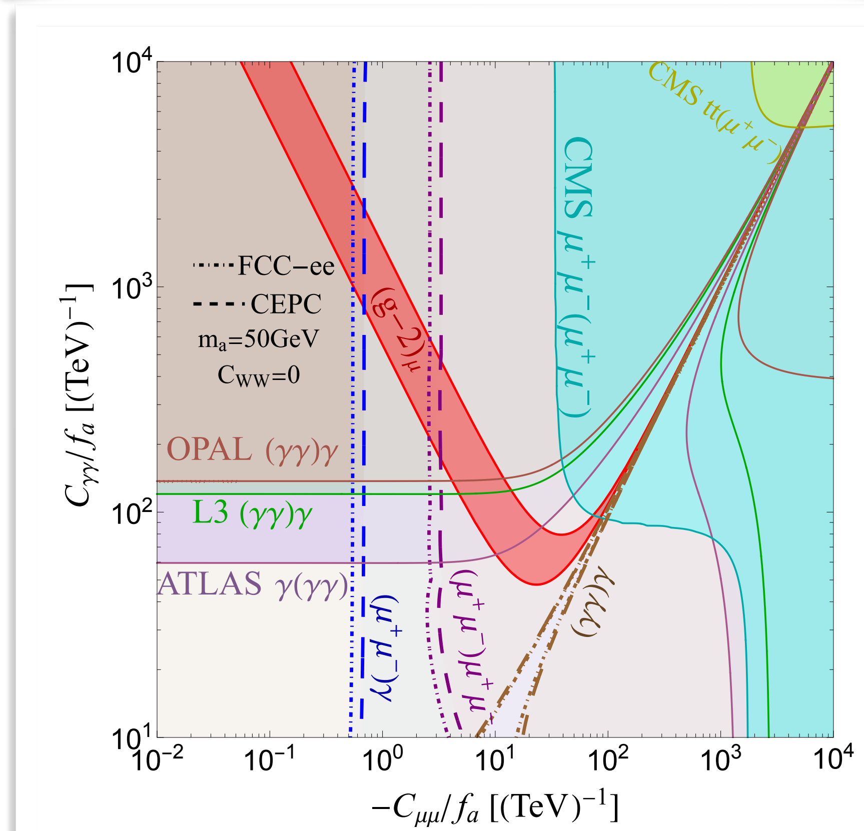
Combined Constraints $C_{ww} = 0$



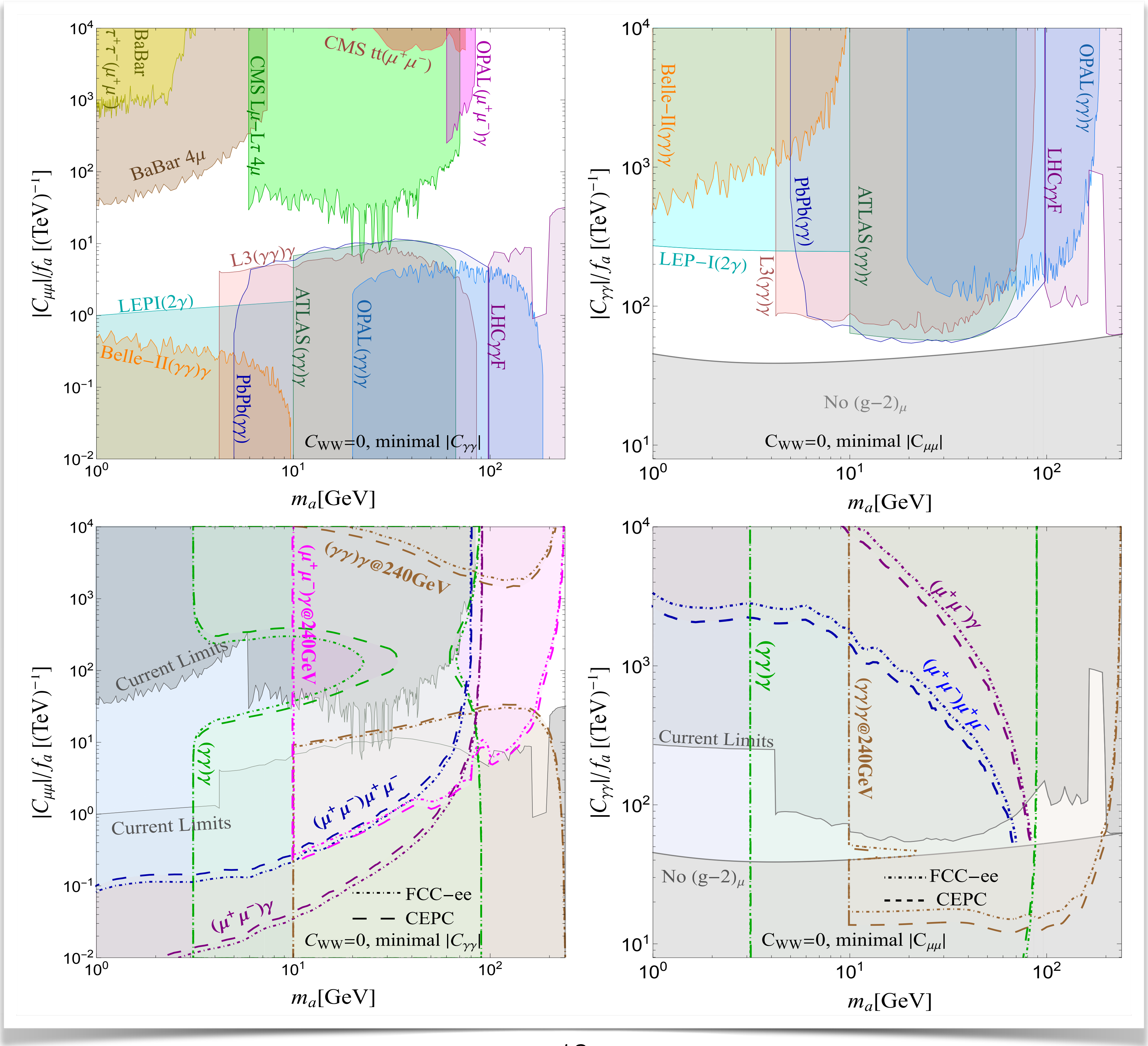
Searching at CEPC $C_{ww} = 0$

- The exotic Z decay: $Z \to a + \gamma$ and $Z \to a + \mu^+ + \mu^-$
- With ALP decay: $a \to \mu^+\mu^-/(\gamma\gamma)$
- Relevant SM background: $\mu^+\mu^-\gamma$ and 4μ
    - Cuts: $E_\gamma > 2\text{GeV}$, $p_T^\mu > 5\text{GeV}$, $|\eta| < 3$, $\Delta R_{\mu\mu} > 0.1$, dimuon resolution: $\left|\frac{m_{\mu\mu}}{m_a} - 1\right| < 0.19\,\%$





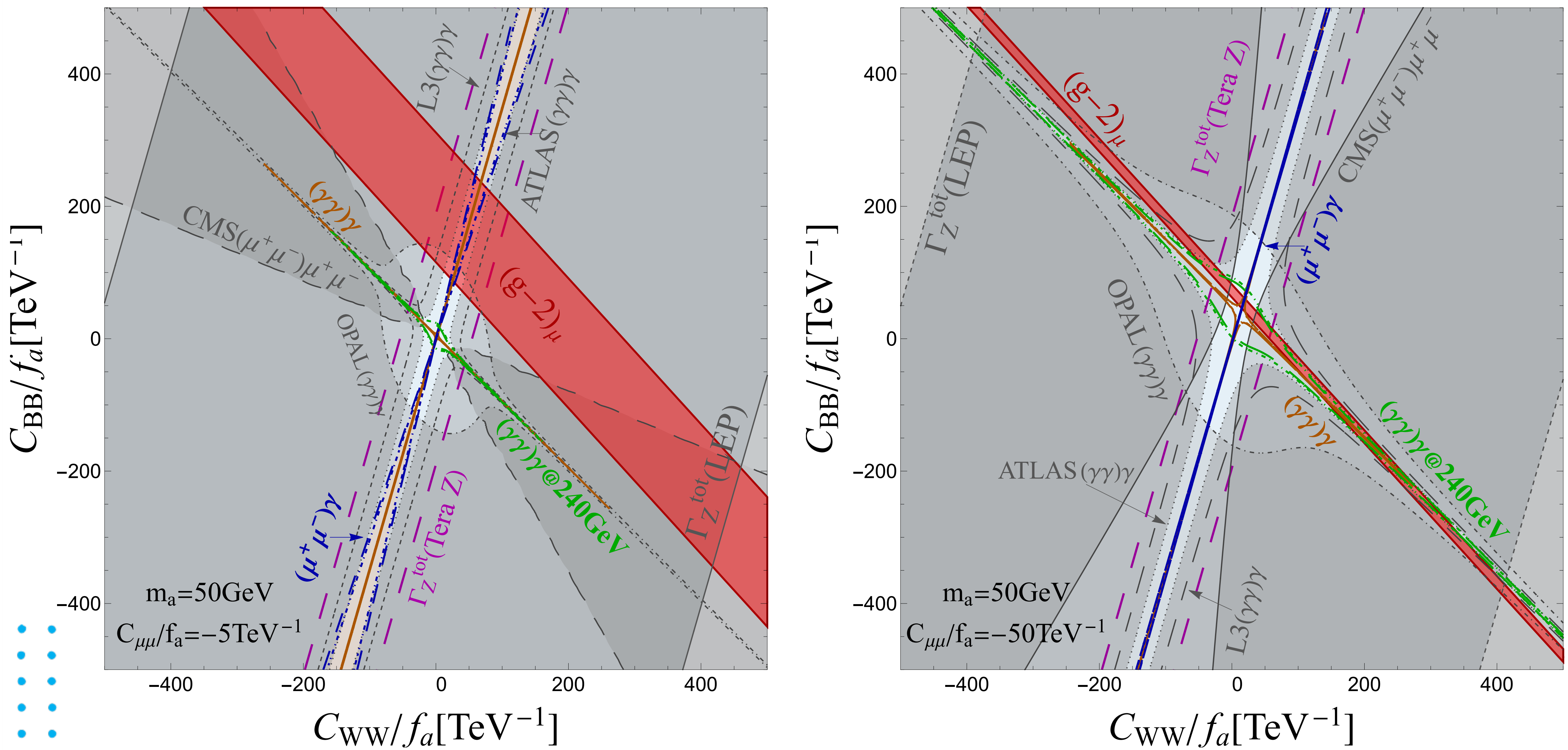
Searching at CEPC $C_{ww} = 0$



Searching at CEPC $C_{ww} \neq 0$

The free parameters are

$$\{m_a, C_{\mu\mu}, C_{WW}, C_{BB}\}$$



Summary

- Muon g-2 experiments show 4.2σ discrepancy with SM
- ALP can provide a solution with couplings $C_{\mu\mu}$ and $C_{\gamma\gamma}$
- In UV model, $C_{\gamma\gamma}$ comes from C_{WW} and C_{BB}, leads to $C_{\gamma Z}$
- At Z-factory, it leads to exotic Z decay:
  - $Z \to a\gamma, a\mu^+\mu^-$
  - Future Z factory can provide sensitivity covers most of the g-2 region for $m_a \lesssim m_Z$

Thank you!