Muon $g-2$ theory overview

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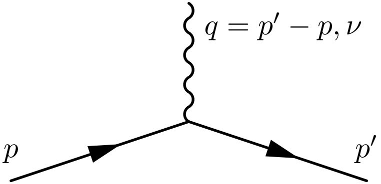
Sun Yat-sen University Zhuhai Campus

Outline

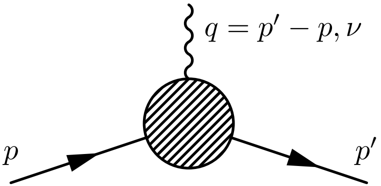
1. **Introduction**
2. Data driven approach
3. Lattice QCD
4. Hadronic light-by-light contribution (HLbL)
5. Hadronic vacuum polarization contribution (HVP)
6. Summary



Dirac equation implies:

$$\bar{u}(p')\gamma_\nu u(p)$$

$$g = 2$$



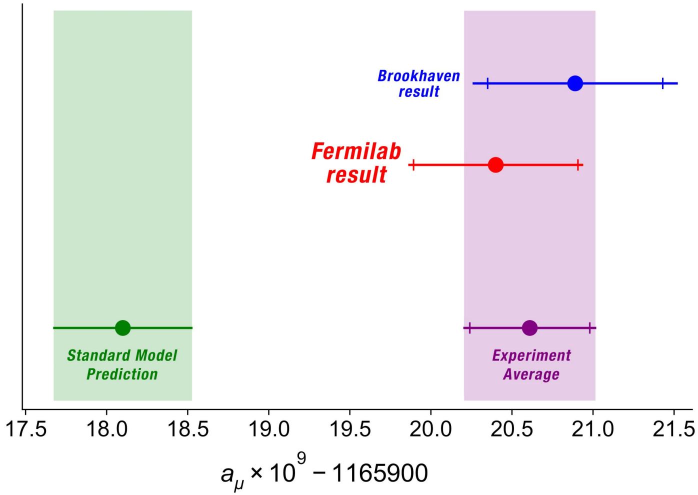
$$\bar{u}(p')\left(F_1(q^2)\gamma_\nu + i\frac{F_2(q^2)[\gamma_\nu, \gamma_\rho]q_\rho}{4m}\right)u(p)$$

(Euclidean space time)

$$a = F_2(q^2 = 0) = \frac{g-2}{2}$$

- The quantity a is called the anomalous magnetic moments.
- Its value comes from quantum correction.

Muon anomalous magnetic moment $(g-2)$



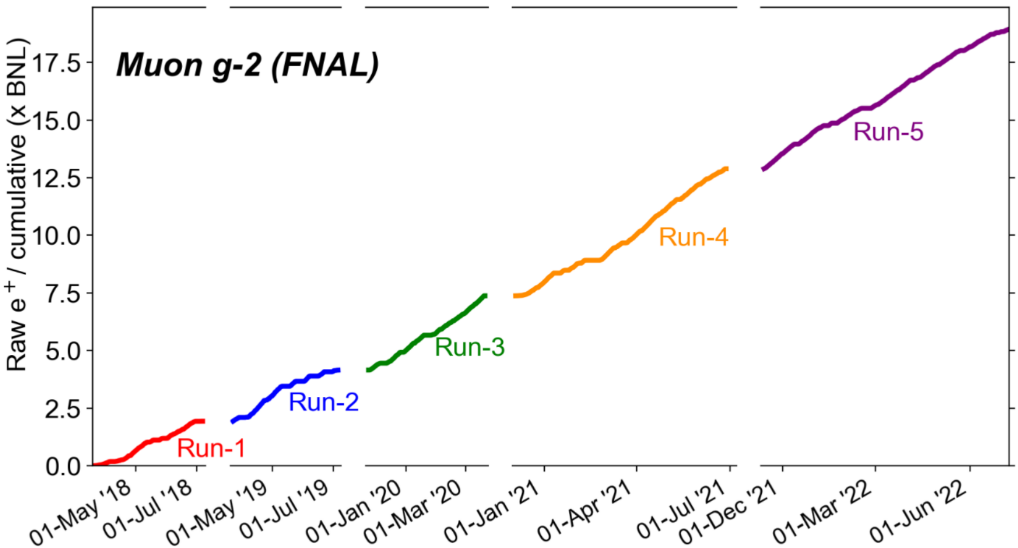
2004

Apr 7, 2021

- "So far we have analyzed less than 6% of the data that the experiment will eventually collect. Although these first results are telling us that there is an intriguing difference with the Standard Model, we will learn much more in the next couple of years." – Chris Polly, Fermilab scientist, co-spokesperson for the Fermilab muon $g-2$ experiment.

New release of Muon $g-2$ soon!

- From Alec Tewsley-Booth's talk at INT workshop, May 11, 2023.



New release of Muon $g-2$ soon!

- From David Hertzog's talk at USQCD all hands meeting, April 21, 2023.

Plans for Release of Run 2/3

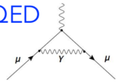
- All ω_a measurements are now *relatively* unblinded;
  - 6 pre-selected methods have the greatest sensitivity and independence; the will be averaged to provide the best and most robust result
    - 2 Recons; 2 Asymmetry Weighted Fits; 2 Ratio-Asymmetry Fits
- The magnetic field analysis is very mature and thoroughly reviewed
- The various "Beam Dynamics" corrections are nearly complete
- After documents are blessed, we will vote to unblind.
- Public release follows within a few weeks

Muon $g-2$ Theory Initiative White paper posted 10 June 2020.

132 authors from worldwide theory + experiment community. [Phys. Rept. 887 (2020) 1-166]

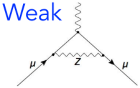
$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{Weak}) + a_\mu(\text{Hadronic})$$

QED



$+\ldots$ $116\,584\,718.9\,(1) \times 10^{-11}$ 0.001 ppm

Weak

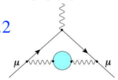


$+\ldots$ $153.6\,(1.0) \times 10^{-11}$ 0.01 ppm

Hadronic...

...Vacuum Polarization (HVP)

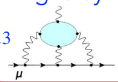
α^2



$+\ldots$ $6845\,(40) \times 10^{-11}$ [0.6%] 0.34 ppm

...Light-by-Light (HLbL)

α^3



$+\ldots$ $92\,(18) \times 10^{-11}$ [20%] 0.15 ppm

- Two methods: dispersive + data $\leftrightarrow$ lattice QCD

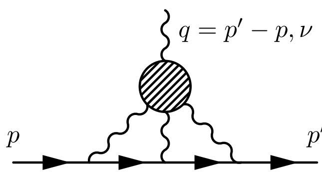
From Aida El-Khadra's theory talk during the Fermilab $g-2$ result announcement.

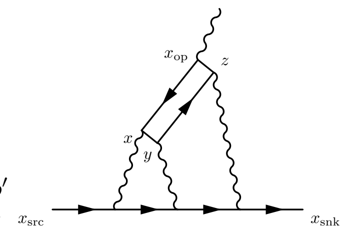
Outline

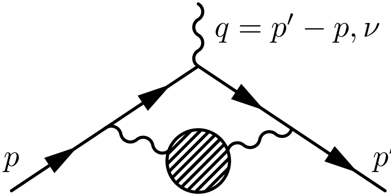
Contribution	PdRV(09) [471]	N/JN(09) [472, 573]	J(17) [27]	Our estimate
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	−19(19)	−19(13)	−20(5)	−16.4(2)
S-wave $\pi\pi$ rescattering	−7(7)	−7(2)	−5.98(1.20)	−8(1)
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	–	–	–	− 1(3) (scalars and tensors combined)
tensors	–	–	1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance	–	21(3)	20(4)	15(10)
c-loop	2.3	–	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)

Table 15: Comparison of two frequently used compilations for HLbL in units of 10^{-11} from 2009 and a recent update with our estimate. Legend: PdRV = Prades, de Rafael, Vainshtein ("Glasgow consensus"); N/JN = Nyffeler / Jegerlehner, Nyffeler; J = Jegerlehner.

- Values in the table is in unit of 10^{-11}.
- Uncertainty of the analytically approach mostly come from the short distance part.







At leading order (LO), i.e., $\mathcal{O}(\alpha^2)$, the dispersion integral reads

$$a_\mu^{\text{HVP LO}} = \frac{\alpha^2}{3\pi^2} \int_{m_\pi^2}^{\infty} \frac{K(s)}{s} R(s)\, ds\,, \tag{1}$$

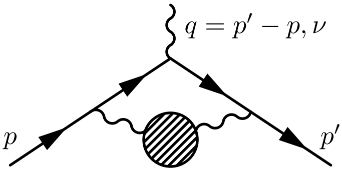
$R(s)$ is the so-called (hadronic) R-ratio defined by

$$R(s) = \frac{\sigma^0(e^+e^- \to \text{hadrons}(+\gamma))}{\sigma_{\text{pt}}}\,, \quad \sigma_{\text{pt}} = \frac{4\pi\alpha^2}{3s}\,. \tag{2}$$

$K(s)$ is an analytically known kernel function

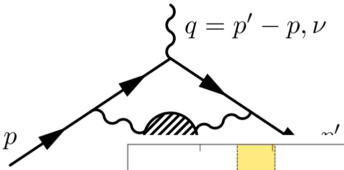
$$K(s) = \frac{x^2}{2}(2-x^2) + \frac{(1+x^2)(1+x)^2}{x^2}\left(\log(1+x) - x + \frac{x^2}{2}\right) + \frac{1+x}{1-x}x^2\log x\,, \tag{3}$$

where $x = \frac{1-\beta_\mu}{1+\beta_\mu}$, $\beta_\mu = \sqrt{1-4m_\mu^2/s}$.



$$a_\mu^{\text{HVP LO}} = 693.1(2.8)_{\text{exp}}(2.8)_{\text{sys}}(0.7)_{\text{DV+QCD}} \times 10^{-10}$$
$$= 693.1(4.0) \times 10^{-10}\,.$$

- Based on analysis of DHMZ, KNT, G. Colangelo et al, M. Hoferichter et al using experimental inputs from CLEO, SND, BESIII, CMD-2, BABAR, KLOE, etc.
- The final error is evaluated as the quadratic sum of difference sources.
- The first error 2.8×10^{-10} refers to the experimental uncertainties, where 2π channel (1.9×10^{-10}), 3π channel (1.5×10^{-10}).
- The last systematic error are from the high energy region estimated from Quark-hadron duality violation (DV) or perturbative QCD (pQCD).
- The middle systematic error 2.8×10^{-10} mostly from the 2π channel. Estimated as half the difference between evaluations without BABAR and KLOE. This difference exceeds the difference between the DHMZ and KNT evaluations (1.8×10^{-10})



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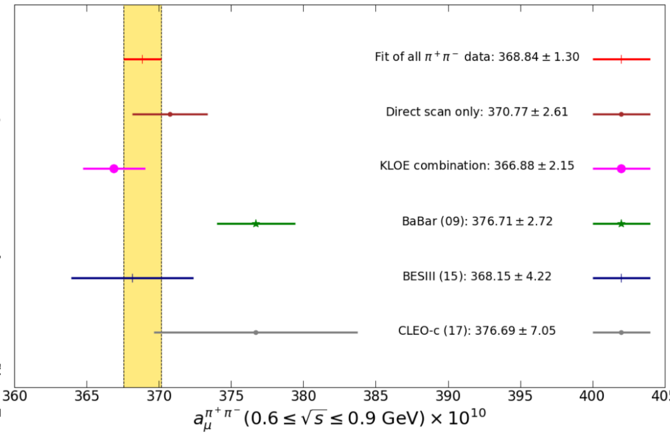

- Based on ana… …t al using experimental … …, etc.
- The final erro…
- The first erro… …re 2π channel (1.9×10^{-10})
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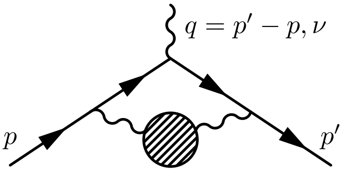

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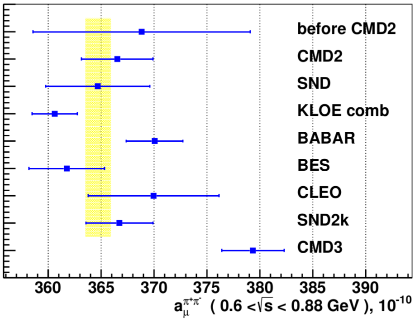
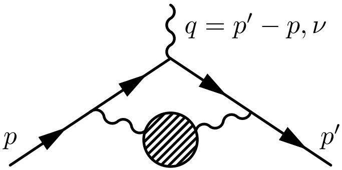


Figure 36: The $\pi^+\pi^-(\gamma)$ contribution to $a_\mu^{had,LO}$ from energy range $0.6 < \sqrt{s} < 0.88$ GeV obtained from this and other experiments.

Experiment	$a_\mu^{\pi^+\pi^-,LO}$, 10^{-10}
before CMD2	368.8 ± 10.3
CMD2	366.5 ± 3.4
SND	364.7 ± 4.9
KLOE	360.6 ± 2.1
BABAR	370.1 ± 2.7
BES	361.8 ± 3.6
CLEO	370.0 ± 6.2
SND2k	366.7 ± 3.2
CMD3	379.3 ± 3.0

Table 4: The $\pi^+\pi^-(\gamma)$ contribution to $a_\mu^{had,LO}$ from energy range $0.6 < \sqrt{s} < 0.88$ GeV obtained from this and other experiments.

arXiv:2302.08834



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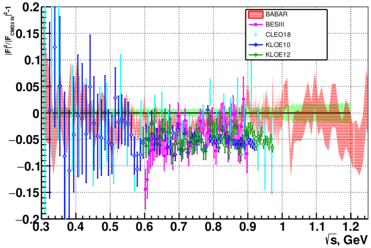


Figure 34: The relative difference of the ISR measurements to the CMD-3 fit.

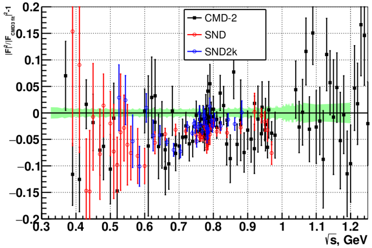
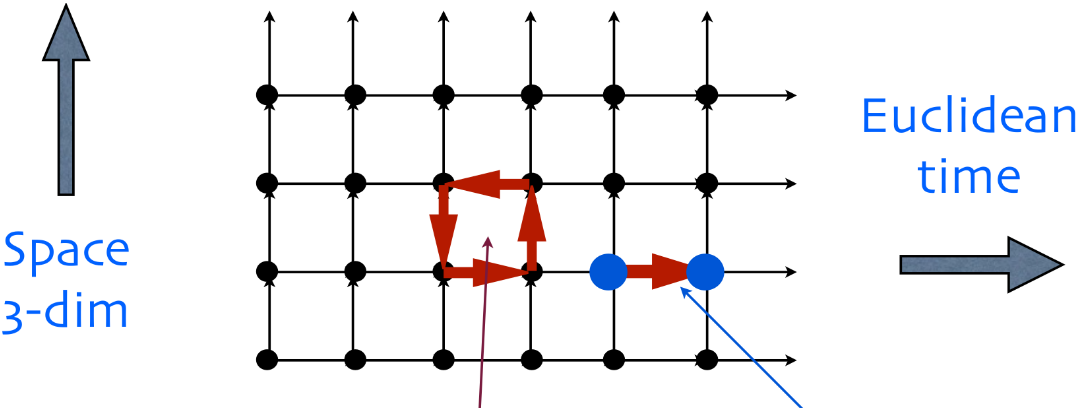


Figure 35: The relative difference of the previous energy-scan measurements to the CMD-3 fit.

generally shows larger pion form factor in the whole energy range under discussion. The most significant difference to other energy scan measurements, including previous CMD-2 measurement, is observed at the left side of ρ-meson ($\sqrt{s} = 0.6 - 0.75$ GeV), where it reach up to 5%, well beyond the combined systematic and statistical errors of the new and previous results. The source of this difference is unknown at the moment.

arXiv:2302.08834

Outline



$$S_E^{\mathrm{latt}} = -\sum_{\Box} \frac{6}{g^2} \,\mathrm{Re}\,\mathrm{tr}_N(U_{\Box,\mu\nu}) - \sum_q \overline{q}(D_\mu^{\mathrm{lat}}\gamma_\mu + am_q)q$$

Wilson gauge action   Lattice fermion action

Figure credit: Stephen R. Sharpe.

$$\langle \mathcal{O}(U,q,\bar{q})\rangle = \frac{\int [\mathcal{D}U] \prod_q [\mathcal{D}q_q][\mathcal{D}\bar{q}_q] e^{-S_E^{\text{latt}}} \mathcal{O}(U,q,\bar{q})}{\int [\mathcal{D}U] \prod_q [\mathcal{D}q_q][\mathcal{D}\bar{q}_q] e^{-S_E^{\text{latt}}}}$$

$$= \frac{\int [\mathcal{D}U] e^{-S_{\text{gauge}}^{\text{latt}}} \prod_q \det\left(D_\mu^{\text{latt}}\gamma_\mu + am_q\right) \tilde{\mathcal{O}}(U)}{\int [\mathcal{D}U] e^{-S_{\text{gauge}}^{\text{latt}}} \prod_q \det\left(D_\mu^{\text{latt}}\gamma_\mu + am_q\right)}$$

Monte Carlo:

- The integration is performed for all the link variables: U. Dimension is $L^3 \times T \times 4 \times 8$.
- Sample points the following distribution:

$$e^{-S_{\text{gauge}}^{\text{latt}}(U)} \prod_q \det\left(D_\mu^{\text{latt}}(U)\gamma_\mu + am_q\right)$$

- Therefore:

$$\langle \mathcal{O}(U,q,\bar{q})\rangle = \frac{1}{N_{\text{conf}}} \sum_{k=1}^{N_{\text{conf}}} \tilde{\mathcal{O}}(U^{(k)})$$

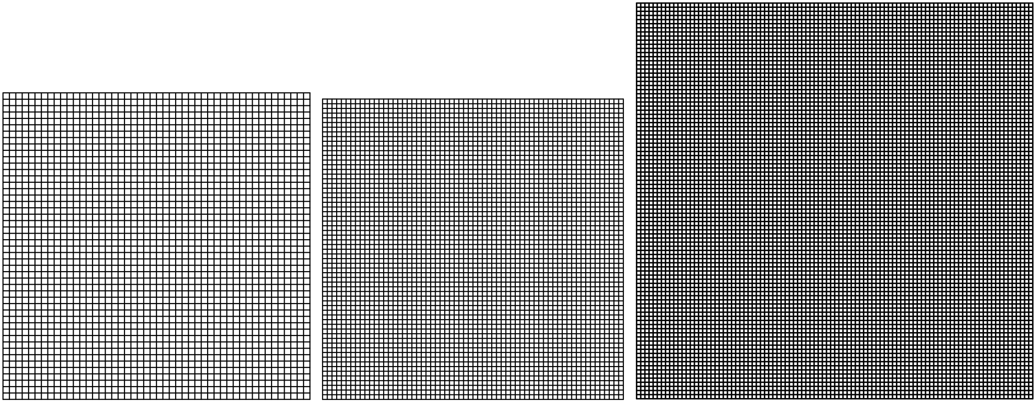
- Parameters in lattice QCD calculations (e.g. isospin symmetric ($m_u = m_d = m_l$) and three flavor u, d, s theory):

$$g \qquad am_l \qquad am_s$$

  Note that lattice spacing a is determined by g via the renormalization group equation.

- The experimental inputs needed to determine these parameters can be: m_π/m_Ω, m_K/m_Ω.

- RBC-UKQCD Domain wall fermion action and Iwasaki gauge action ensembles.
- At physical pion mass (almost).
- 48I, 64I, 96I with $a^{-1} = 1.73,\ 2.36,\ 2.68$ GeV, $L = 5.47,\ 5.36,\ 7.06$ fm.



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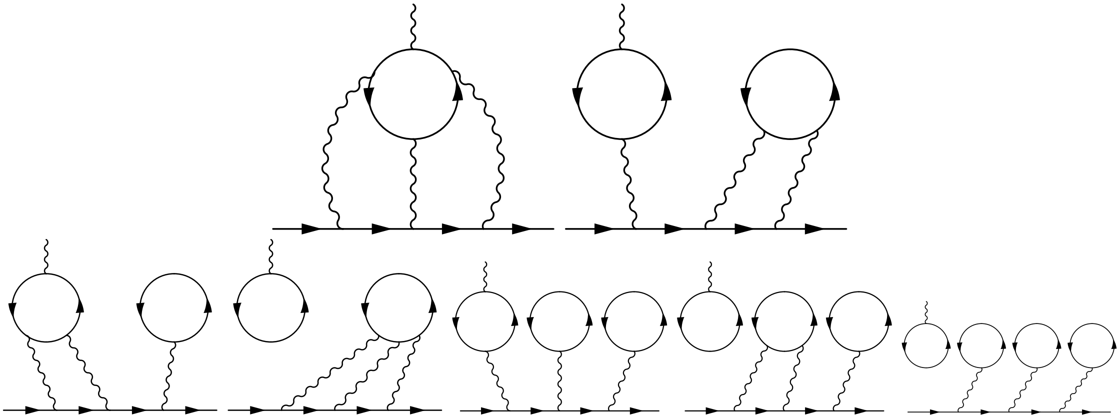
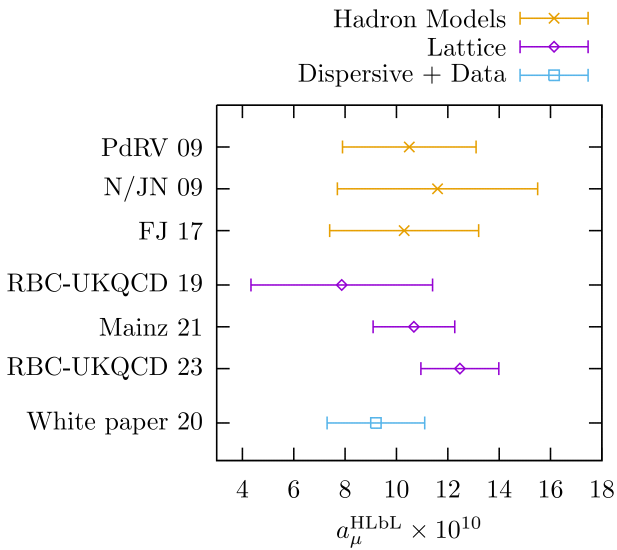


- Gluons and sea quark loops (not directly connected to photons) are included automatically to all orders!
- There are additional different permutations of photons not shown.
- The second row diagrams are suppressed by flavor SU(3) symmetry (and small charge factors, $1/N_c$, etc). The contributions are numerically very small.

Muon $g-2$ HLbL: Lattice QCD results

- RBC-UKQCD 19:
  Physical m_π and QED$_L$.
  Extrapolate to infinite volume via $1/L^2$.

- Mainz 21:
  Heavy m_π and QED$_\infty$.
  Calculated all the sub-leading disconnected diagrams.
  Fit the contribution in the long distance region to improve statistics.
  Extrapolate to the physical pion mass and infinite volume via $(e^{-m_\pi L/2})$.

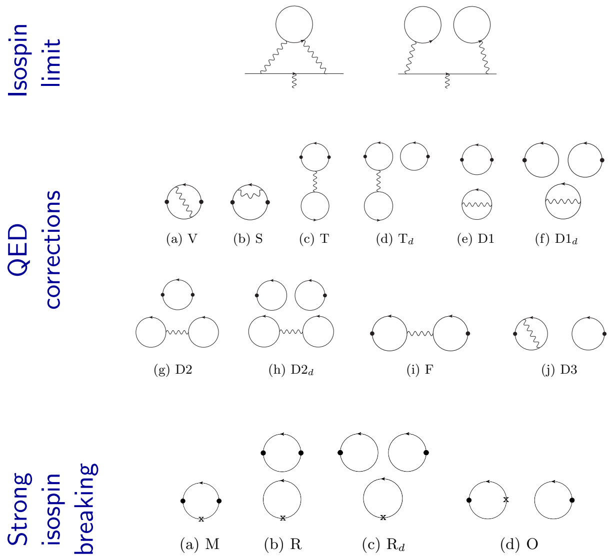


- RBC-UKQCD 23: Physical m_π and QED$_\infty$.
  Mainly based on the "48I" ensemble. Several new error reduction techniques are developed to reduce the statistical noise in the long distance region.

- Required precision for HLbL: 10% to match Fermilab's final results. (Very close now.)

Outline

Muon $g-2$ HVP: diagrams

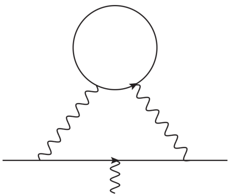


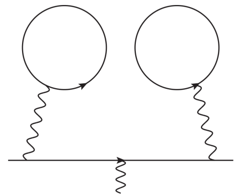
- Gluons and sea quark loops (not directly connected to photons) are included automatically to all orders!
- Need to calculate and cross check all the contributions.

T. Blum 2003; D. Bernecker, H. Meyer 2011.

$$C(t) = \frac{1}{3}\sum_{\vec{x}}\sum_{j=0,1,2}\langle J_j^{em}(\vec{x},t)J_j^{em}(0)\rangle_{\mathrm{QCD}}$$

$$a_\mu^{\mathrm{HVP\ LO}} = \left(\frac{\alpha}{\pi}\right)^2\int_0^\infty dK^2 f(K^2)\hat{\Pi}(K^2) = \sum_{t=0}^{+\infty} w(t)C(t)$$





QED

and

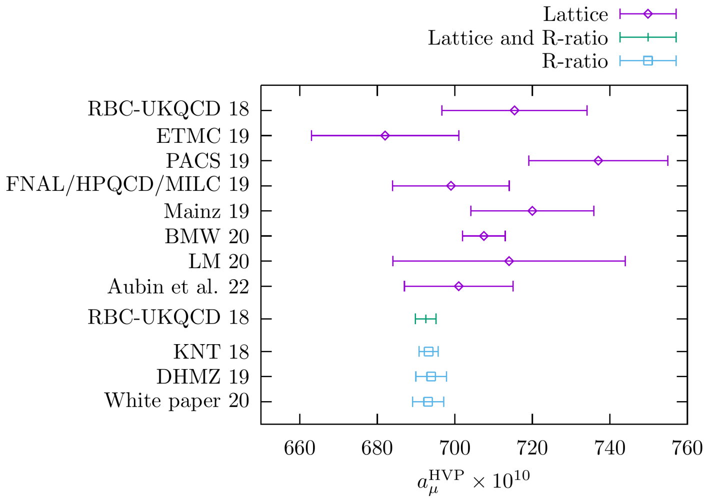
strong isospin

breaking

$a_\mu^{\mathrm{HVP,\,LO}}(ud)$	$a_\mu^{\mathrm{HVP,\,LO}}(s)$	$a_\mu^{\mathrm{HVP,\,LO}}(c)$	$a_{\mu,\mathrm{disc}}^{\mathrm{HVP,\,LO}}$	$\delta a_\mu^{\mathrm{HVP}}$
650.2(11.6)	53.2(0.3)	14.6(0.1)	−13.7(2.9)	7.2(3.4)

- From muon $g-2$ theory initiative white paper (2020). Value in unit of 10^{-10}
- Light quark connected diagram has the largest contribution and largest uncertainty.

- Dispersive method via R-ratio (red points) is mature and reproducible.
- Lattice (blue points) errors are limited by statistics. Except for BMW, which beats down the statistical error, result is limited by systematic error: BMW 20: $707.5(2.3)_{\text{stat}}(5.0)_{\text{sys}}$
- Lattice-QCD calculations of comparable precision needed.
- Consistency is needed to claim new physics.



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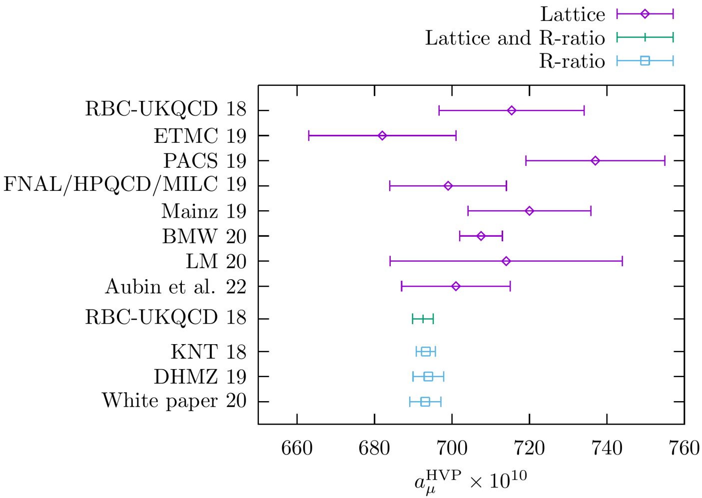
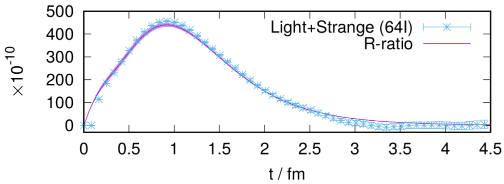


Muon $g-2$ HVP: uncertainties

T. Blum 2003; D. Bernecker, H. Meyer 2011.

$$C(t) = \frac{1}{3}\sum_{\vec{x}}\sum_{j=0,1,2}\langle J_j^{em}(\vec{x},t)J_j^{em}(0)\rangle_{\rm QCD}$$

$$a_\mu^{\rm HVP\ LO} = \sum_{t=0}^{+\infty} w(t)C(t)$$

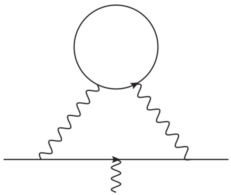


- Statistical error is mostly from:

  Light quark connected diagram at $t \gtrsim 1.5$ fm

  - More configurations (BMW 20 used $\sim 20,000$).
  - Use low modes averaging to gain full volume average. ✓
  - Bounding method on the long distance tail. ✓
  - Study the $\pi\pi$ system spectrum to calculate $C(t)$ large t.
    - Not used in any published work yet!
    - On-going efforts with promising initial results.

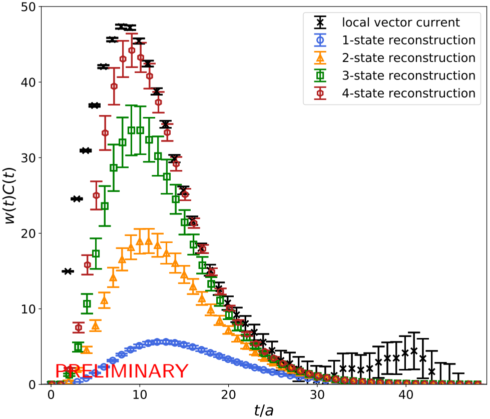
- Systematic error is mostly from the **continuum extrapolation**.

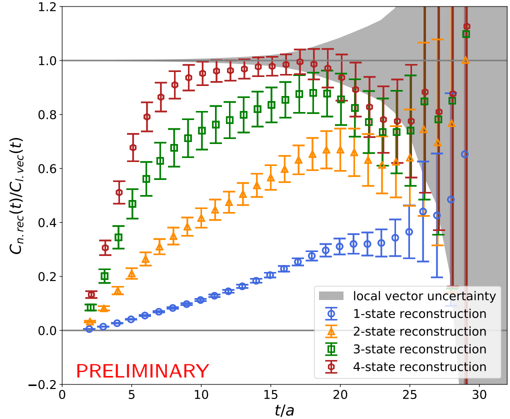


- Main idea is that: one does not have to calculate the long distance part of the correlation function directly.

$$C(t) = \frac{1}{3}\sum_{\vec{x}}\sum_{j=0,1,2}\langle J_j(\vec{x},t)J_j(0)\rangle$$
$$= \sum_{n}\frac{V}{3}\sum_{j=0,1,2}\langle 0|J_j(0)|n\rangle\langle n|J_j(0)|0\rangle e^{-E_n t}$$

- The summation over n is limited to zero momentum states and states are normalized to "1".
- At large t, only lowest few states contribute. We only need the matrix elements $\langle n|J_j(0)|0\rangle$ and the corresponding energy E_n.
- Need to study the spectrum of the $\pi\pi$ system!
- Can reduce the statistical error beyond the gauge noise limit!





GEVP results to reconstruct long-distance behavior of local vector correlation function needed to compute connected HVP

Explicit reconstruction good estimate of correlation function at long-distance, missing excited states at short-distance

More states $\implies$ better reconstruction, can replace $C(t)$ at shorter distances

RBC-UKQCD by Aaron Meyer and Christoph Lehner

Preliminary

RBC-UKQCD PRL 121, 022003 (2018)

Window contribution allows a high precision study of the continuum extrapolation.

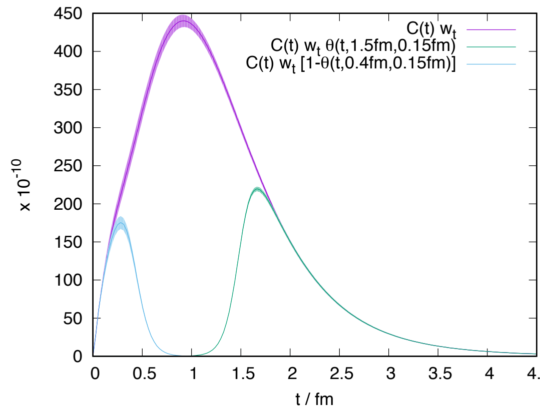
$$a_\mu^{\text{HVP LO}} = \sum_{t=0}^{+\infty} w(t) C(t)$$

$$w(t) = w^{\text{SD}}(t) + w^{\text{W}}(t) + w^{\text{LD}}(t)$$

- Splitting sum into three parts allows crosschecks:
  - short distance $\Leftarrow$ discretization effects
  - long distance $\Leftarrow$ noisy $\pi\pi$ tail
  - intermediate (Window): sweet spot
- Can form windows from $R(e^+e^-)$ dispersive analysis too.

  Combine "window" from lattice with dispersive analysis.

- Compare "window" among lattice-QCD calculations



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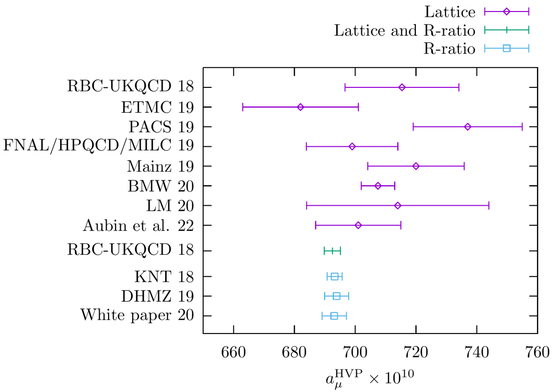

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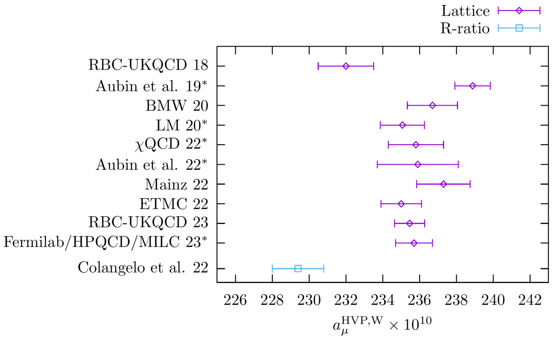

- Lattice QCD community has already reached consensus on the window contribution!
- The consensus has a noticeable tension with the dispersive results.

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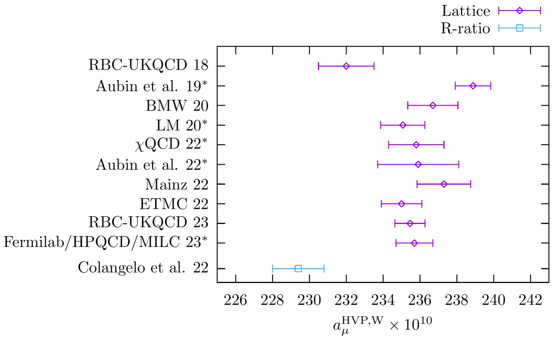
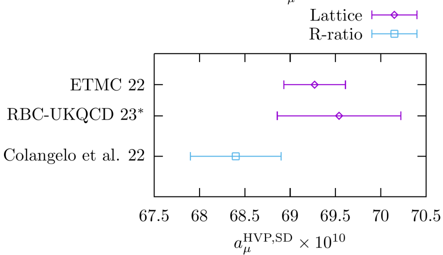




- From Martin Hoferichter's talk at INT workshop, May 11, 2023.

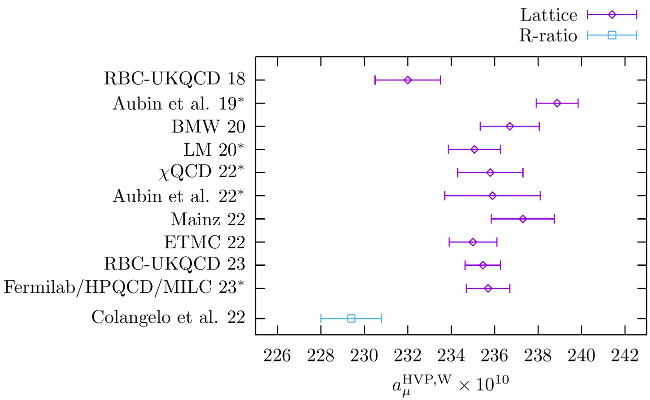
Role of isospin breaking: phenomenological estimates

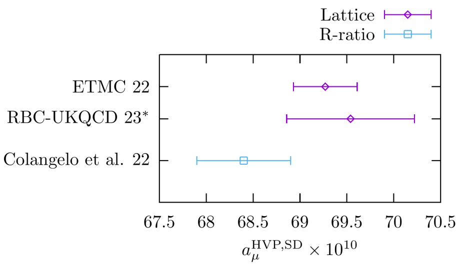
	SD window		int window		LD window		full HVP	
	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$
$\pi^0\gamma$	0.16(0)	–	1.52(2)	–	2.70(4)	–	4.38(6)	–
$\eta\gamma$	0.05(0)	–	0.34(1)	–	0.31(1)	–	0.70(2)	–
ρ–ω mixing	–	0.05(0)	–	0.83(6)	–	2.79(11)	–	3.68(17)
FSR (2π)	0.11(0)	–	1.17(1)	–	3.14(3)	–	4.42(4)	–
M_{π^0} vs. $M_{\pi^\pm}$ (2π)	0.04(1)	–	$-0.09(7)$	–	$-7.62(14)$	–	$-7.67(22)$	–
FSR (K^+K^-)	0.07(0)	–	0.39(2)	–	0.29(2)	–	0.75(4)	–
kaon mass (K^+K^-)	$-0.29(1)$	0.44(2)	$-1.71(9)$	2.63(14)	$-1.24(6)$	1.91(10)	$-3.24(17)$	4.98(26)
kaon mass ($\bar{K}^0K^0$)	0.00(0)	$-0.41(2)$	$-0.01(0)$	$-2.44(12)$	$-0.01(0)$	$-1.78(9)$	$-0.02(0)$	$-4.62(23)$
total	0.14(1)	0.08(3)	1.61(12)	1.02(20)	$-2.44(16)$	2.92(17)	$-0.68(29)$	4.04(39)
BMWc 2020	–	–	$-0.09(6)$	0.52(4)	–	–	$-1.5(6)$	1.9(1.2)
RBC/UKQCD 2018	–	–	0.0(2)	0.1(3)	–	–	$-1.0(6.6)$	10.6(8.0)
JLM 2021	–	–	–	–	–	–	–	3.32(89)

- Reasonable agreement with BMWc 2020, RBC/UKQCD 2018, and James, Lewis, Maltman 2021

- The current white paper result for the HVP is a community-vetted method average for the data-driven approach. It accounts for spreads in sub-contributions between individual results (KNT/DHMZ) that may not be visible in the agreement of looking at the final results for the HVP. Its error estimate accounts for the tension between BaBar and KLOE experimental inputs. New CMD3 results have larger tension.
- We are now in the fortunate situation that we have a first lattice result with sub-percent precision (BMW). It is clear that to safely assess systematic uncertainties, most notably the one related to the choice of the lattice regulator, calculations by other lattice groups with a similar precision will be essential. The importance of having more than one lattice calculations of the same quantity and obtained with different lattice discretizations is well understood inside the lattice community.
- On the way to a lattice QCD average for HVP, it is prudent to also look at individual sub-contributions and their agreement, similar to what was done for the data driven approach. For example, individual QED corrections should be cross checked (currently there are some tensions).

- The previous tension in the standard iso-symmetric window results has already been resolved among lattice QCD calculations!





$*$: Use iso-symmetric, quark connected, light quark contribution from this work and remaining contributions from RBC-UKQCD 18 (W) or ETMC 22 (SD).

Outline

Summary

- Hadronic light-by-light (HLbL) contribution:
  We have reliable and consistent calculations using both lattice QCD and data driven dispersive approach. The results have good precision.

- Hadronic vacuum polarization (HVP) contribution:
  - Short distance (SD) part: Results are consistent and precise.
  - Middle window (W) part: Consensus is reached among lattice QCD calculations. However, there is more than 3σ tension between:
    - lattice QCD consensus and previous data driven results,
    - new CMD-3 and previous data driven results.

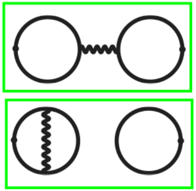
    Lattice QCD consensus appears to be consistent with the new CMD-3 results.
  - Long distance (LD) part and its QED corrections: Some tension ($\sim 2\sigma$) between BMW 20 and previous data driven results. BMW 20 appears to be consistent with the new CMD-3 results. Results from other lattice collaborations are coming.

More information is needed to draw firm conclusion right now.

Thank You!

HVP: Compare BMW 20 and RBC-UKQCD 18

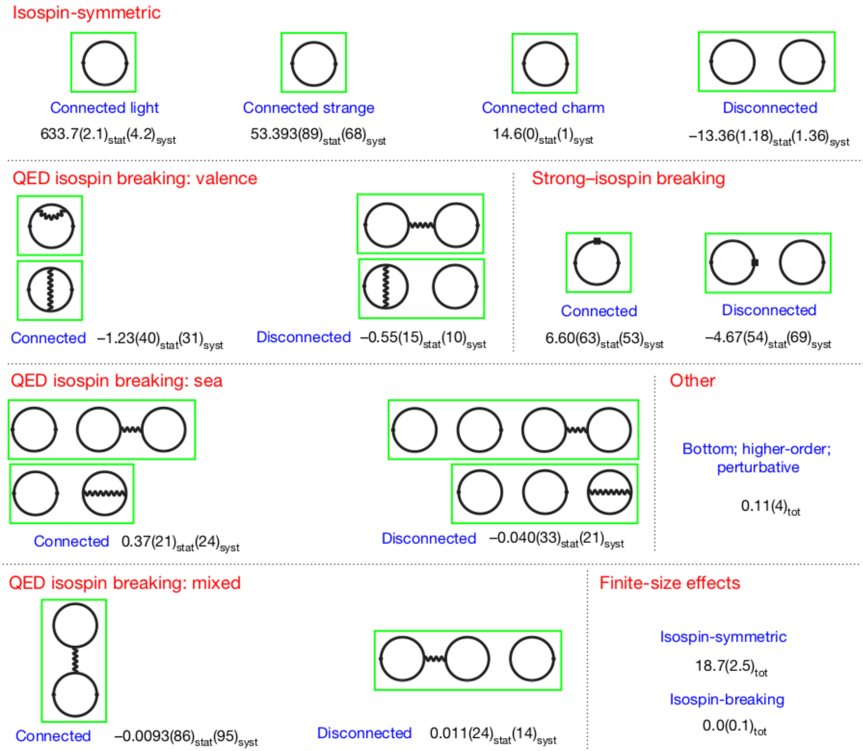
$a_\mu^{\text{ud, conn, isospin}}$	$649.7(14.2)_\text{S}(2.8)_\text{C}(3.7)_\text{V}(1.5)_\text{A}(0.4)_\text{Z}(0.1)_\text{E48}(0.1)_\text{E64}$
$a_\mu^{\text{s, conn, isospin}}$	$53.2(0.4)_\text{S}(0.0)_\text{C}(0.3)_\text{A}(0.0)_\text{Z}$
$a_\mu^{\text{c, conn, isospin}}$	$14.3(0.0)_\text{S}(0.7)_\text{C}(0.1)_\text{Z}(0.0)_\text{M}$
$a_\mu^{\text{uds, disc, isospin}}$	$-11.2(3.3)_\text{S}(0.4)_\text{V}(2.3)_\text{L}$
$a_\mu^{\text{QED, conn}}$	$5.9(5.7)_\text{S}(0.3)_\text{C}(1.2)_\text{V}(0.0)_\text{A}(0.0)_\text{Z}(1.1)_\text{E}$
$a_\mu^{\text{QED, disc}}$	$-6.9(2.1)_\text{S}(0.4)_\text{C}(1.4)_\text{V}(0.0)_\text{A}(0.0)_\text{Z}(1.3)_\text{E}$
a_μ^{SIB}	$10.6(4.3)_\text{S}(0.6)_\text{C}(6.6)_\text{V}(0.1)_\text{A}(0.0)_\text{Z}(1.3)_\text{E48}$
$a_\mu^{\text{udsc, isospin}}$	$705.9(14.6)_\text{S}(2.9)_\text{C}(3.7)_\text{V}(1.8)_\text{A}(0.4)_\text{Z}(2.3)_\text{L}(0.1)_\text{E48}$ $(0.1)_\text{E64}(0.0)_\text{M}$
$a_\mu^{\text{QED, SIB}}$	$9.5(7.4)_\text{S}(0.7)_\text{C}(6.9)_\text{V}(0.1)_\text{A}(0.0)_\text{Z}(1.7)_\text{E}(1.3)_\text{E48}$
$a_\mu^{\text{R-ratio}}$	
a_μ	$715.4(16.3)_\text{S}(3.0)_\text{C}(7.8)_\text{V}(1.9)_\text{A}(0.4)_\text{Z}(1.7)_\text{E}(2.3)_\text{L}$ $(1.5)_\text{E48}(0.1)_\text{E64}(0.3)_\text{b}(0.2)_\text{c}(1.1)_{\overline{\text{S}}}(0.3)_{\overline{\text{Q}}}(0.0)_\text{M}$



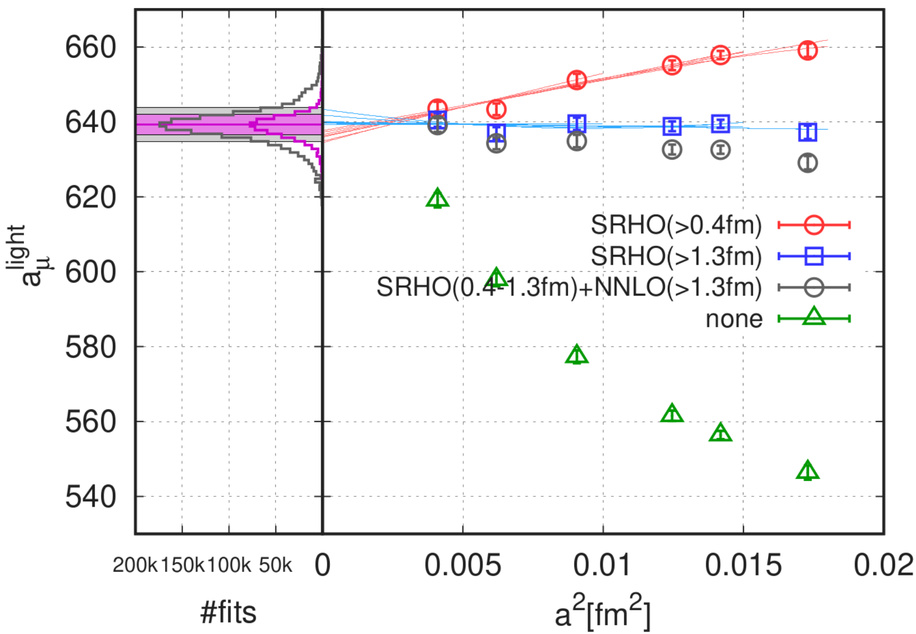
Disconnected $-0.55(15)_{\text{stat}}(10)_{\text{syst}}$

- The left table shows result from RBC-UKQCD 18. The right figure shows the result from BMW 20.
- This discrepancy needs further study and more cross checks.

HVP: BMW 2021 - all channels

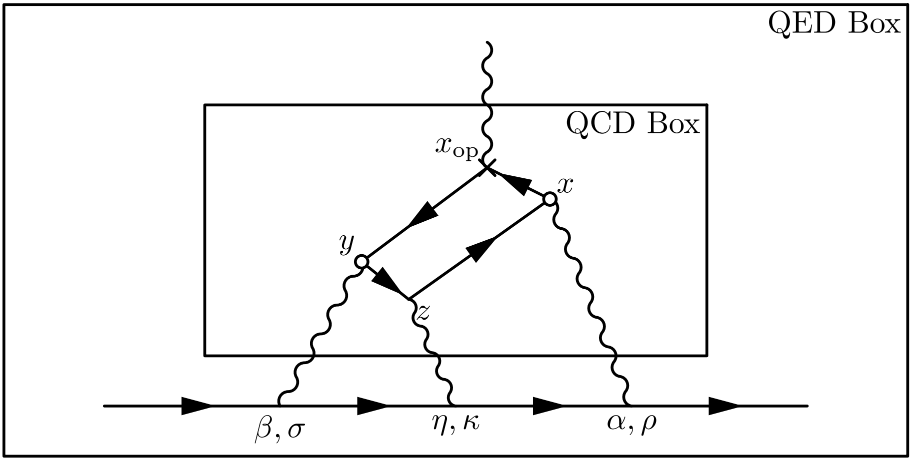


$$a_\mu^{\text{LO-HVP}}\ (\times 10^{10}) = 707.5(2.3)_{\text{stat}}(5.0)_{\text{syst}}(5.5)_{\text{tot}}$$



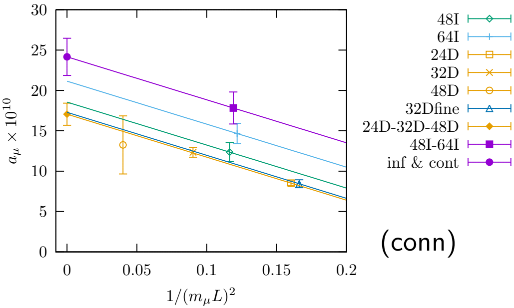
- Staggered fermion has a special lattice artifacts: taste breaking effects.
- Curves show different treatments of correcting the taste breaking effects.

- The basic idea is to right the Feynman diagram in coordinate space, and then it is possible and natural to use infinite volume for QED and finite (by necessity) for QCD.

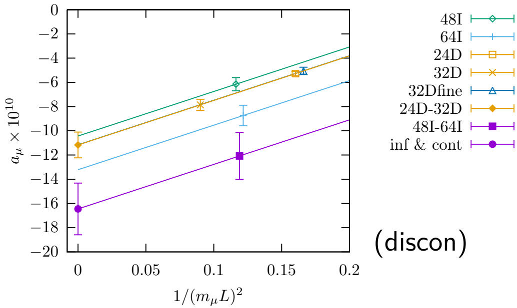


- In a way, this is the HLbL version of the Bernecker-Meyer formula: coordinate space (sum over t), infinite-volume QED kernel, finite volume (by necessity) calculation of the correlator.

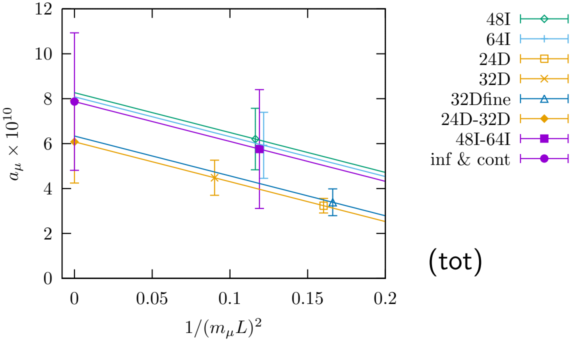
$$a_\mu(L, a^{\mathrm{I}}, a^{\mathrm{D}}) = a_\mu\left(1 - \frac{b_2}{(m_\mu L)^2} - c_1^{\mathrm{I}}(a^{\mathrm{I}}\ \mathrm{GeV})^2 - c_1^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^2 + c_2^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^4\right)$$



(conn)

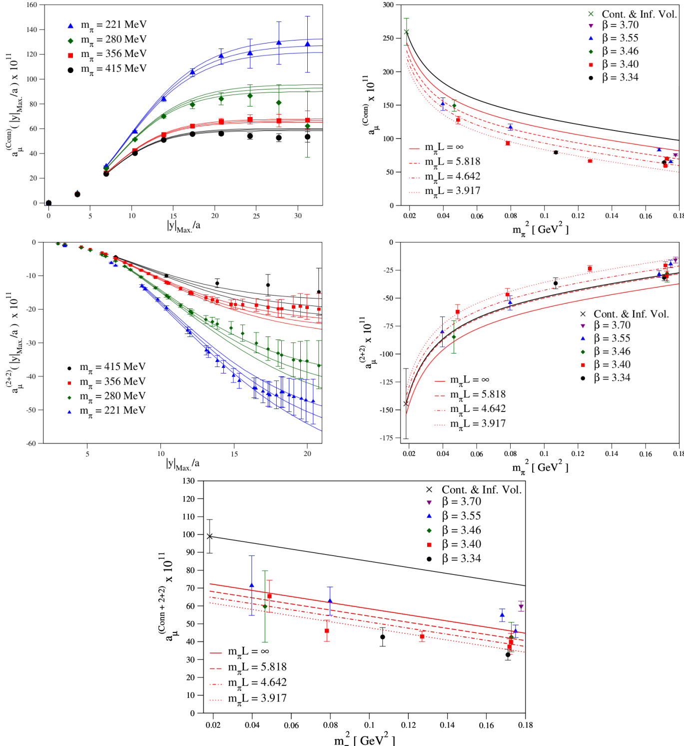


(discon)

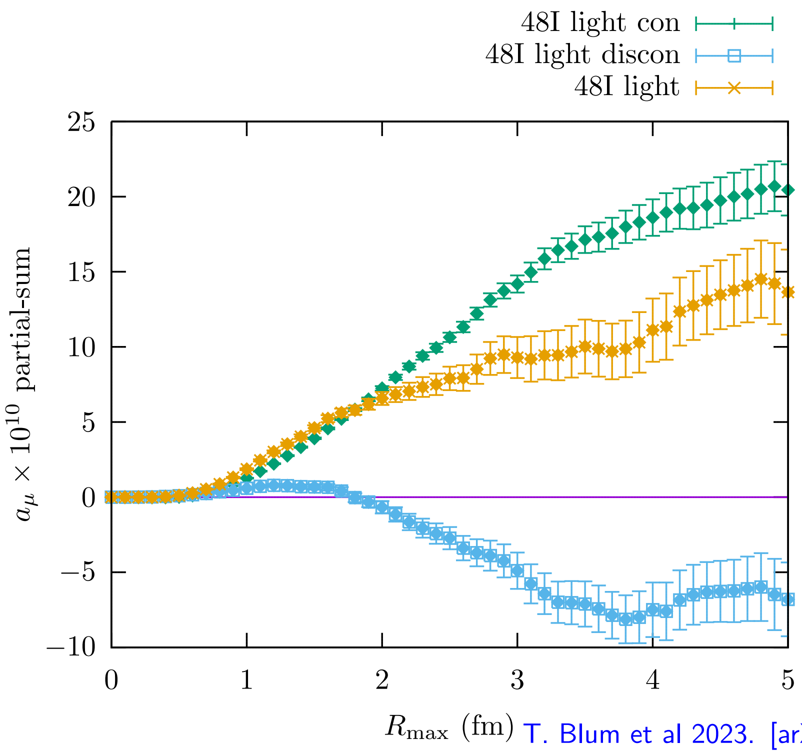


(tot)

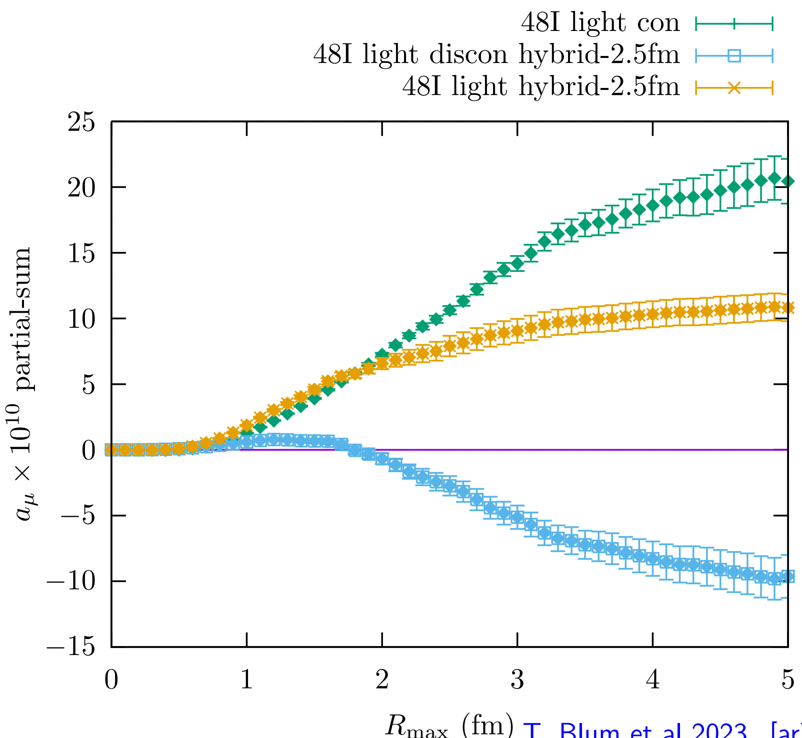
T. Blum et al 2020. (PRL 124, 13, 132002)



E.H. Chao et al 2021. (EPJC 81, 7, 651)

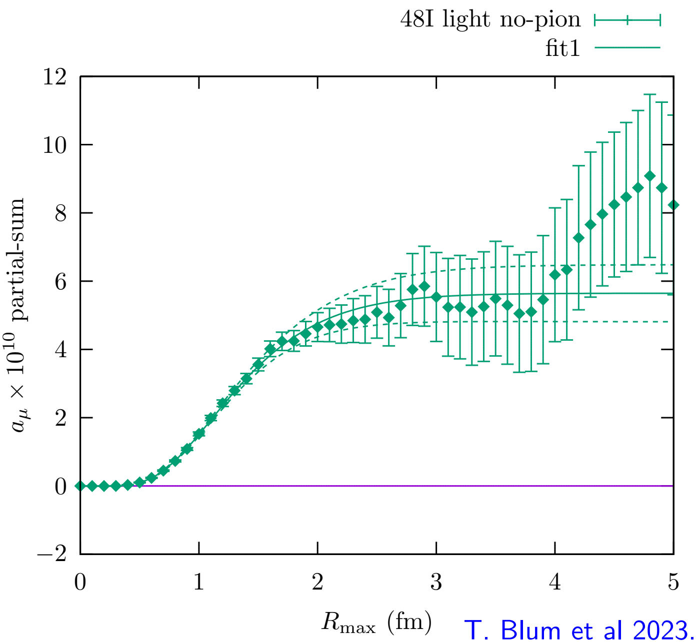


T. Blum et al 2023. [arXiv:2304.04423]



T. Blum et al 2023. [arXiv:2304.04423]

$$f(R_{\max}) = A\frac{R_{\max}^6}{R_{\max}^3 + C^3}e^{-BR_{\max}}$$



T. Blum et al 2023. [arXiv:2304.04423]