Flavor Physics and Precision Meaurements

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1.Flavor physics, where are we?

2. Anomalies, confirmed?

3. The need of going beyond SM

4. More CP violating observables

5. Theory efforts to reduce model parameters

6. Conclusions

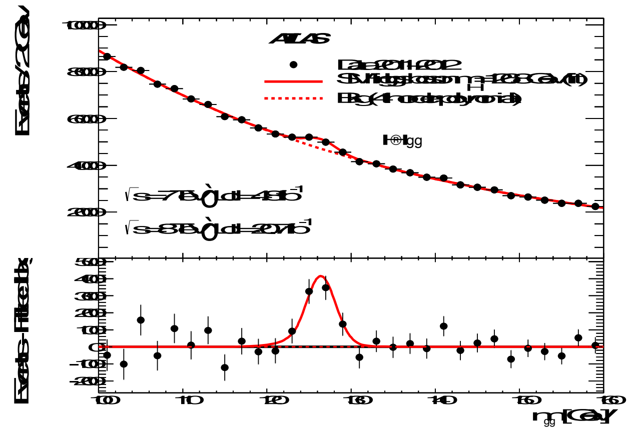
1. Favor physics, where are we?

Fundamental Interactions:

Electromagnetic Interaction, mediator: Photon

Weak Interaction, mediators: W and Z bosons

Gravitational Interaction, mediator: Graviton (?)



Particle aass generating mechanis:

Higgs Mechanism (God particle reveals it)

Quarks: The building block of Hadrons

u c t (electric charge +2/3 e) Quarks are elementary particles

d s b (electric charge -1/3 e) Three generations/families

Leptons: Particles have no strong interaction

ν_e ν_μ ν_τ (electric charge 0 e) Leptons are elementary particles

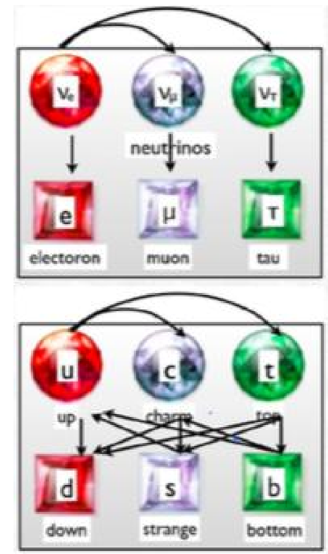
e μ τ (electric charge -1 e) Three generations/families

Flavor physics and CP symmetry

Flavor and CP violation are intimately related.

Flavors: describe several copies of the same gauge representation, namely several fields that are assigned the same quantum charges:

u, c, t; d, s, b; e, μ, τ; ν_τ ,ν_μ, ν_e;; …



Flavor physics: the study of interactions that govern flavors.

Weak interaction one type of flavor change to another type

neutral current t → c, u; b → s, d; τ → μ, e μ, e; ν_τ →ν_μ, ν_e;… ,

charged current b, s, d → t, c, u; τ, μ, e → ν_τ ,ν_μ, ν_e;…

CP symmetry: Combined symmetry of C-charge conjugation (particle and anti-particle symmetry) and P-space parity (inversion of space directions).

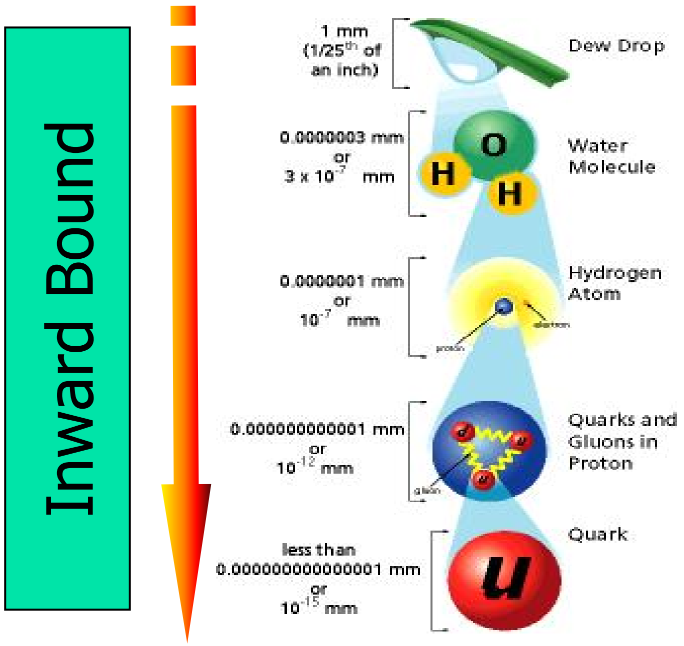
Strong and electromagnetism interactions respect these symmetries.

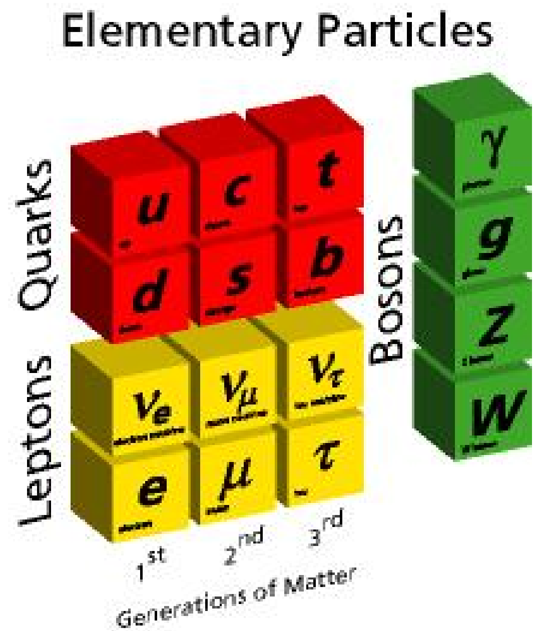
Weak interaction violates (breaks) these symmetries.

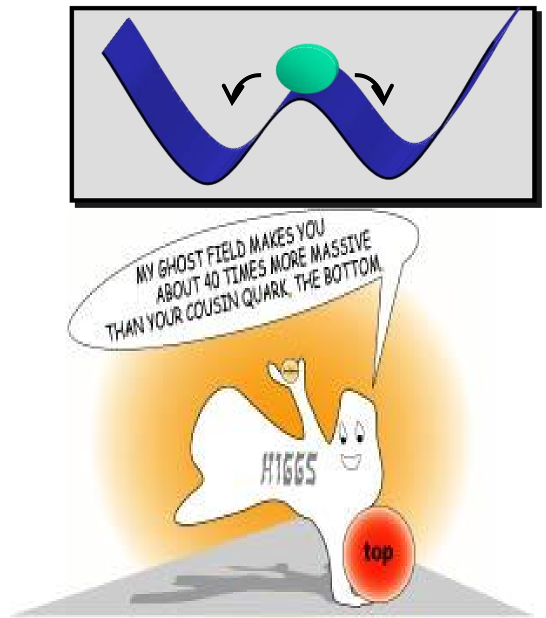
The mis-match of weak and mass eigen-state bases lead to generation mixing and CP violation!

The SM of strong and electroweak interactions

SU(3) x SU(2) x U(1) gauge theory for strong and electroweak interaction







Can one negeclects gravitation interaction when studying particle interactions?
The coulomb force between two protons: $F_c = e^2/r^2$,
And Gravitational force: $F_g = -Gm^2/r^2$ $|F_g|/|F_c| = 7\times10^{-38}$

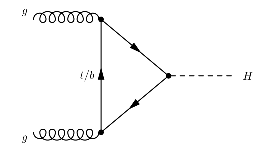
Gravitational force is much weaker than electromagnetism!

But when study cosmology › gravitational force always add up › but electromagnetism can cancel between positively and negatively charged particles!

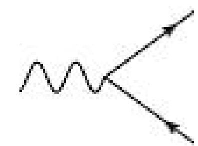
The number of genrations

In the SM, only 3 generations of quarks and leptons are allowed.

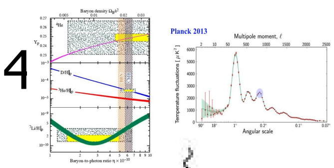
gg -> Higgs ~ (number of heavy quarks)^2, if fourth generation exist, their mass should be large, 9 times bigger production of Higgs. LHC data ruled out more than 3 generations of quarks.



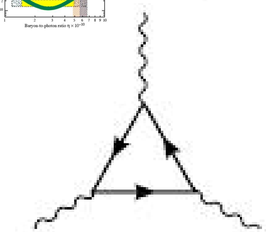
LEP already ruled out more than 3 neutrinos with mass less than $m_Z/2$.



Cosmology and astrophysics, number of light neutrinos also less than 4



SM, triangle anomaly cancellation: equal number of quarks and leptons!



There are only three generations of sequential quarks and leptons!

Why 3 generations? How do they mix with each other?

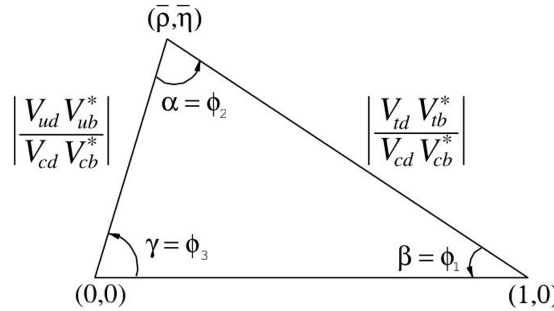
Beyond SM, conclusions may change, X-G He and G. Valencia, PPLB707 (2012)

Quark and Lepton mixing patterns

The mis-match of weak and mass eigen-state bases lead quark and lepton mix within generations.

Quark mixing the Cabibbo -Kobayashi-Maskawa (CKM) matrix $V_{\rm CKM}$,

lepton mixing the Pontecorvo -Maki-Nakawaga-Sakata (PMNS) matrix $U_{\rm PMNS}$



$$L = -\frac{g}{\sqrt{2}}\overline{U}_L\gamma^\mu V_{\rm CKM} D_L W_\mu^+ - \frac{g}{\sqrt{2}}\overline{E}_L\gamma^\mu U_{\rm PMNS} N_L W_\mu^- + H.C.\,,$$

$U_L = (u_L, c_L, t_L, ...)^T$, $D_L = (d_L, s_L, b_L, ...)^T$, $E_L = (e_L, \mu_L, \tau_L, ...)^T$, and $N_L = (\nu_1, \nu_2, \nu_3, ...)^T$

For n-generations, $V = V_{\rm CKM}$ or $U_{\rm PMNS}$ is an $n\times n$ unitary matrix.

A commonly used form of mixing matrix for three generations of fermions is given by $V_{KM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23} \end{pmatrix} \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$ are the mixing angles and δ is the CP violating phase.

If neutrinos are of Majorana type, for the PMNS matrix one should include an additional diagonal matrix with two Majorana phases $\mathrm{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1)$ multiplied to the matrix from right in the above.

Parameters in the standard model with 3 generations

Gauge boson couplings and masses: $g_1=g'$, $g_{2=g}$, $g_3=g_s$, m_γ, m_W, m_Z

Fermion Masses: m_e, m_μ, m_τ, $m_{\nu e}$, $m_{\nu\mu}$, $m_{\nu\tau}$

m_u, m_d, m_c, m_s, m_t, m_b

Higgs boson mass and couplings: m_h or λ, m_i/v to ith fermion

(Weak mixing angle θ_W: $\tan\theta_W = g_2/g_1$, $e = g_2 \sin\theta_{W)}$

$\alpha_{em} = e^2/4\pi$, $\alpha_2=g_2^2/4\pi$, $\alpha_3=\alpha_{s=}=g_s^2/4\pi$; $G_F = g^2/(4\sqrt{2}\, m_W^2)$

Mixing: quark mixing (3 mixing angles + 1 Dirac-phase)

Neutrino mixing (3 mixing angles +1 Dirac-phase + 2 Majorana-phases)

1 possible strong CP violating parameter θ

Total independent model parameters: 18 +1 without neutrino masses.

Another 9 if include neutrino masses at low energies or more.

(3 gauge couplings + 1 W or Z mass + 1 Higgs coupling or Higgs mass + (6 quark + 3 charged lepton masses)
+ 3 quark mixing angle + 1 Dirac-phase, 1 strong phase, and 3+6 neutrino masses, mixing angles and phases)

In the SM flavor physics has a lot to do with these free parameters

Flavor physics tests for SM

Discovering new phenomena, and testing various theoretical predictions

-> establishment of a theory (Determine the model parameters, looking for deviations -> modify the theory…)

Produce various particles and observe how they interact and decay

Production: e+e-, p anti-p, pp… colliders (γ, e-, ν, p..) hit on Nuclei target…

=> SM particles…

Observe various particle decays, quarks, leptons, gauge bosons, Higgs boson… t -> W + b -> l v + c light hadrons (for lighter quarks, one needs to study the hadrons containing the specific quark to see it decay properties…)

Interaction with probes: g-2 of muon (muon under know magnetic field)…

Cross sections, decay rates, production and decay asymmetries…. Obtain desired properties of a theory: coupling constants, mixing angles, parity and CP properties…

What do we know about the SM parameters?

Many are well measured

α_{em}=1/137.035999084(21) $\sin^2\theta_W$=0.23121(4) α_3=0.1179(9) (G_F = 1.1663788(6)x10^{-5} GeV^{-2})

m_Z=91.1876(21) GeV m_h=125.25(0.17) GeV

(SM: m_W=80.357(6) GeV vs. Recent CDF II data: m_W=80.4335(94) GeV 7σ away!)

Charged lepton masses:

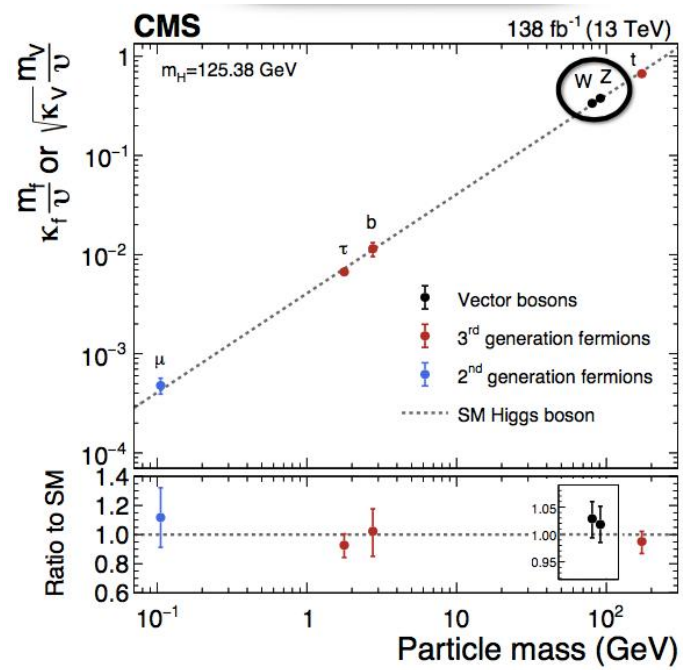
m_e=0.51099895000(15) MeV m_μ=105.6583755(23) MeV m_τ= 1776.86(12) MeV

Quark masses:

m_u =1.16(+0.49, -0.26) MeV m_d=4.67(+0.48, -0.17) MeV,

m_s=93.4(+8.6, -3.4) MeV, m_c=1.27(0.02) GeV,

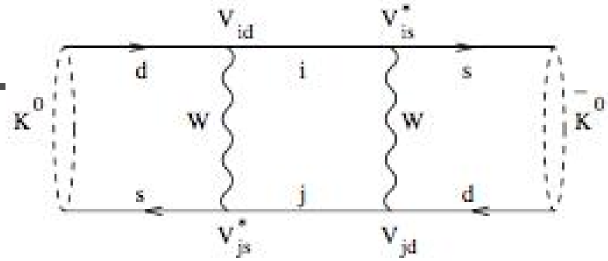
m_b=4.18(+0.03,-0.02) GeV, m_t= 172.69(0.30) GeV

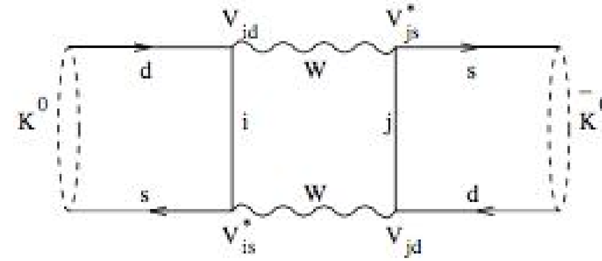


Strong CP violating phase $\theta < 10^{-9}$

What about quark and neutrino mixing angles and CP violating phases, and neutrino masses?

Loop level Meson-antiMeson mixing





$$H_{eff} = -\frac{2}{3}\frac{G_F^2 m_W^2}{\pi^2}\sum_{i,j}(V_{id}V_{is}^*)(V_{jd}V_{js}^*)B(\alpha_i,\alpha_j)\bar{s}\gamma_\mu L d\bar{s}\gamma^\mu L d\ ,$$

$$B(x,y) = (1+\frac{xy}{4})(\frac{1}{(1-x)(1-y)} + \frac{1}{x-y}[\frac{x^2 lnx}{(1-x)^2} - \frac{y^2 lny}{(1-y)^2}])$$
$$-2xy(\frac{1}{(1-x)(1-y)} + \frac{1}{x-y}[\frac{xlnx}{(1-x)^2} - \frac{ylny}{(1-y)^2}])\ ,$$

$$M_{12} \ = \ <\bar{K}^0|H_{eff}|K^0> = -\frac{1}{8}\frac{G_F^2 m_W^2}{\pi^2}\sum_{i,j}(V_{id}V_{is}^*)(V_{jd}V_{js}^*)B(\alpha_i,\alpha_j)C\ ,$$

$$C = <\bar{K}^0|\bar{s}\gamma_\mu L d\bar{s}\gamma^\mu L d|K^0>$$

Replacing (d, s) to (d,b) and (s,b), obtain $B_{d(s)}$-anti$B_{d(s)}$ mixing

Dominated by heavy top quark in the loop

For $B_{d(s)}$-anti$B_{d(s)}$ mixing, $V_{td}V_{tb}^*(V_{td}V_{tb}^*)$ term dominate! Determination of V_{td} and V_{ts} !

$2ReM_{12} = \Delta m_{b;}$ $\Delta m_B = 3.334(0.013)\times10^{-10}$ MeV; $\Delta m_{Bs}=1.1693(0.0004)\times10^{-8}$ MeV.

$\Delta m_D = 6.56(0.010)10^{-12}$ MeV. Need long distance contributions in SM.

CP violation Tests

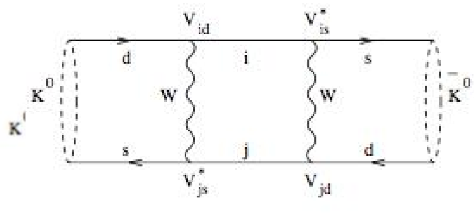
in K^0-antiK^0 mixing

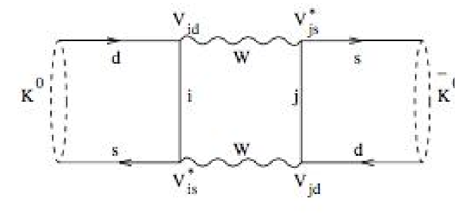
Data: $\Delta m_{L-S} \approx \Delta\Gamma_{S-L}/2 = (3.484 \pm 0.006) \times 10^{-12}$ MeV,

$\epsilon = (2.228 \pm 0.011) \times 10^{-3} exp(i\phi_\epsilon)$ with $(\phi_\epsilon = 43.52 \pm 0.05)^o$.

Assuming $Im(\Gamma_{12})$ is much smaller than $Im(M_{12})$

Theoretical estimate OK





One finally obtains

$$\epsilon \approx \frac{Im(M_{12})}{\sqrt{2}\Delta m_{L-S}} e^{i\phi_\epsilon} \, ,$$

SM is consistent with data and ϵ help to determine the phase δ

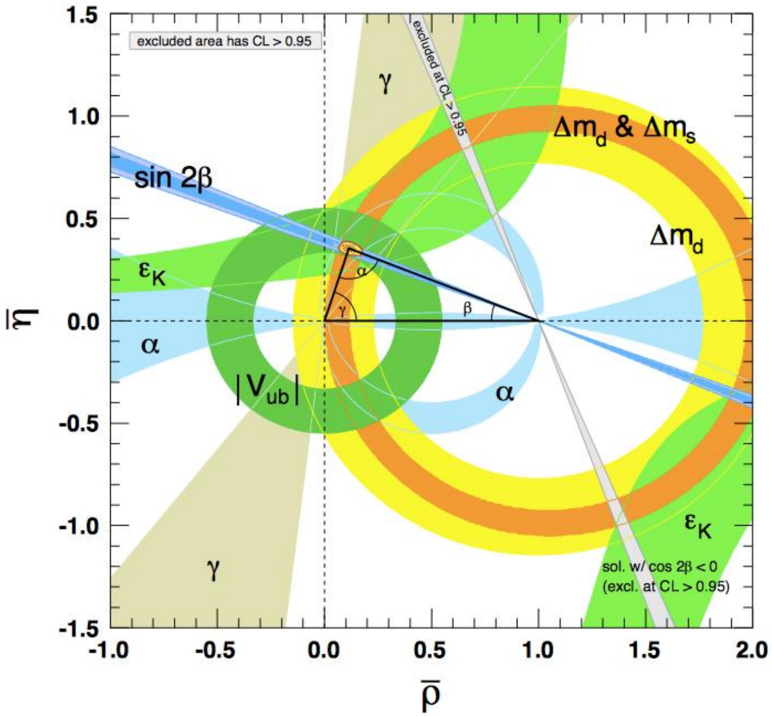
Status of Quark and Lepton Mixing

Quark Mixing

PDG

Neutrino Mixing

$$\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$



$$\lambda = 0.22500 \pm 0.00067\,, \qquad A = 0.826^{+0.018}_{-0.015}\,,$$

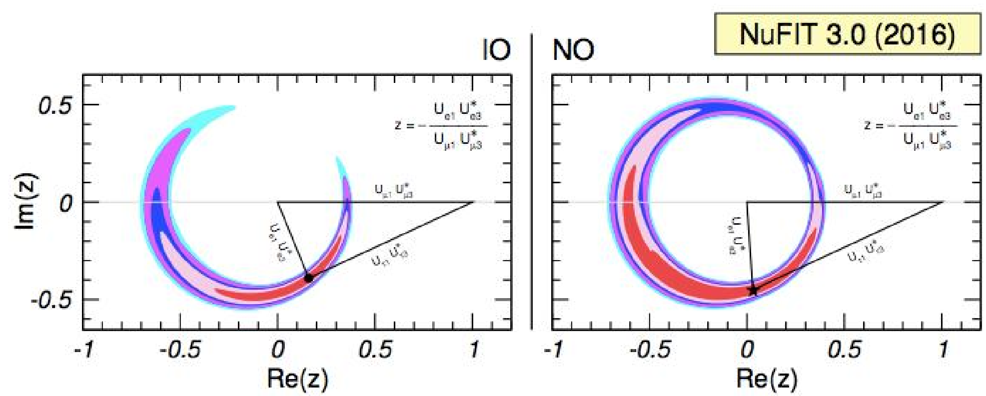
$$\bar\rho = 0.159 \pm 0.010\,, \qquad \bar\eta = 0.348 \pm 0.010\,.$$

$$\sin\theta_{12} = 0.22500 \pm 0.00067\,, \qquad \sin\theta_{13} = 0.00369 \pm 0.00011\,,$$

$$\sin\theta_{23} = 0.04182^{+0.00085}_{-0.00074}\,, \qquad \delta = 1.144 \pm 0.027\,.$$

$\Delta m^2 = m_3^2 - (m_2^2 + m_1^2)/2$. Thus, $\Delta m^2 = \Delta m_{31}^2 - \Delta m_{21}^2/2 > 0$, if $m_1 < m_2 < m_3$ and $\Delta m^2 = \Delta m_{32}^2 + \Delta m_{21}^2/2 < 0$ for $m_3 < m_1 < m_2$.

Parameter	best-fit	3σ
Δm_{21}^2 [10^{-5} eV ^{2}]	7.37	6.93 – 7.97
$\lvert\Delta m^2\rvert$ [10^{-3} eV ^{2}]	2.50 (2.46)	2.37 – 2.63 (2.33 – 2.60)
$\sin^2\theta_{12}$	0.297	0.250 – 0.354
$\sin^2\theta_{23}$, $\Delta m^2 > 0$	0.437	0.379 – 0.616
$\sin^2\theta_{23}$, $\Delta m^2 < 0$	0.569	0.383 – 0.637
$\sin^2\theta_{13}$, $\Delta m^2 > 0$	0.0214	0.0185 – 0.0246
$\sin^2\theta_{13}$, $\Delta m^2 < 0$	0.0218	0.0186 – 0.0248
δ/π	1.35 (1.32)	(0.92 – 1.99)
		((0.83 – 1.99))



Why they mix the pattern shown above?

Tests for Standard Model of CV Violation

SM can explain CPV in neutral Kaon mixing. Only doing that job is not enough to become part of a SM.

Predictions made and confirmed.

Many predictions been confirmed!

Observables: ε', time dependent A_{CP} and independent rate asymmetry S_f and C_f in K, D and B decays, and also to test unitarity triangle predicted by SM

The ϵ' in $K_{L,S} \to \pi\pi$, a measurement of direct CPV

$$\epsilon' = \frac{\eta_{+-} - \eta_{00}}{3}\ , \quad \eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)}\ , \quad \eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)}\ .$$

$$\eta_{+-} = \epsilon + i\frac{ImA_0}{ReA_0} + e^{i(\pi/2+\delta_2-\delta_0)}\frac{ReA_2}{\sqrt{2}ReA_2}\left(\frac{ImA_2}{ReA_2} - \frac{ImA_0}{ReA_0}\right)\ ,$$

$$\eta_{00} = \epsilon + i\frac{ImA_0}{ReA_0} - 2e^{i(\pi/2+\delta_2-\delta_0)}\frac{ReA_2}{\sqrt{2}ReA_0}\left(\frac{ImA_2}{ReA_2} - \frac{ImA_0}{A_0}\right)\ ,$$

$$\epsilon' = \frac{\eta_{+-} - \eta_{00}}{3} = \frac{ReA_2}{\sqrt{2}ReA_0}\left(\frac{ImA_2}{ReA_2} - \frac{ImA_0}{ReA_0}\right)e^{i(\pi/2+\delta_2-\delta_0)}\ .$$

δ_i are determined from phase shift analyses in $\pi - \pi$ scattering, and $\pi/2 + \delta_2 - \delta_0$ is found to be close to $\pi/4$.

CPT symmetry implies that this phase is equal to the phase ϕ_ϵ for ϵ.

In the literature the quantity ϵ'/ϵ is usually used.

Experiment value from NA48 and KTeV: $\varepsilon'/\varepsilon = 16.6(2.3) \times 10^{-4}$

SM calculation for ε'/ε

Tree and penguin contributions

$$\mathcal{H}_{\rm eff}(\Delta S=1)=\frac{G_F}{\sqrt{2}}V^*_{us}V_{ud}\sum_{i=1}^{10}[z_i(\mu)+\tau\, y_i(\mu)]Q_i(\mu),$$

$$\tau=-\frac{V^*_{ts}V_{td}}{V^*_{us}V_{ud}}.$$

$$Q_1=(\bar{s}_i u_j)_{V-A}(\bar{u}_j d_i)_{V-A},$$

$$Q_2=(\bar{s}u)_{V-A}(\bar{u}d)_{V-A},$$

$$Q_3=(\bar{s}d)_{V-A}\sum_q(\bar{q}q)_{V-A},$$

$$Q_4=(\bar{s}_i d_j)_{V-A}\sum_q(\bar{q}_j q_i)_{V-A},$$

$$Q_5=(\bar{s}d)_{V-A}\sum_q(\bar{q}q)_{V+A},$$

$$Q_6=(\bar{s}_i d_j)_{V-A}\sum_q(\bar{q}_j q_i)_{V+A}.$$

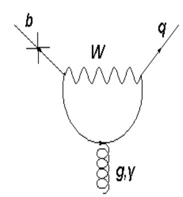
$$Q_7=\frac{3}{2}(\bar{s}d)_{V-A}\sum_q e_q(\bar{q}q)_{V+A},$$

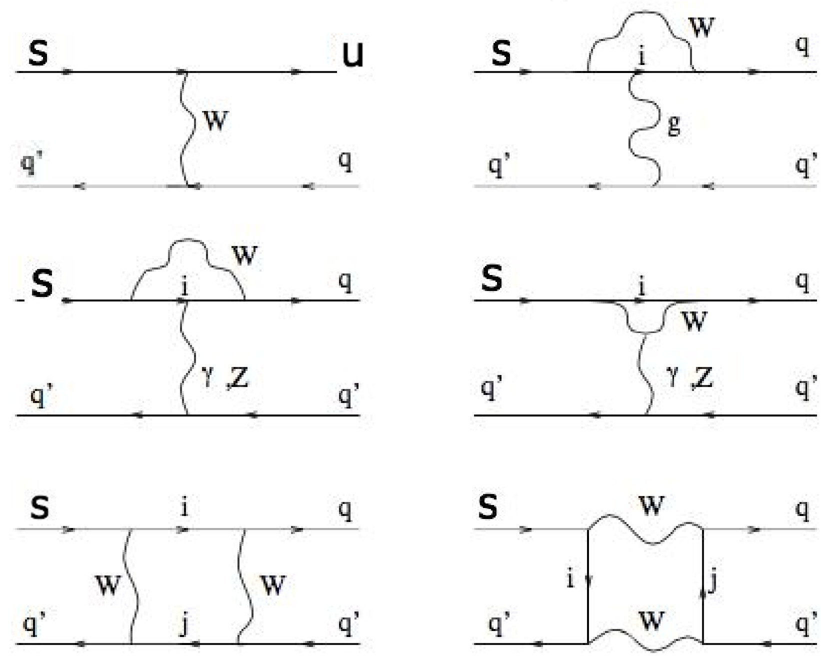
$$Q_8=\frac{3}{2}(\bar{s}_i d_j)_{V-A}\sum_q e_q(\bar{q}_j q_i)_{V+A},$$

$$Q_9=\frac{3}{2}(\bar{s}d)_{V-A}\sum_q e_q(\bar{q}q)_{V-A},$$

$$Q_{10}=\frac{3}{2}(\bar{s}_i d_j)_{V-A}\sum_q e_q(\bar{q}_j q_i)_{V-A}.$$

s->u $\bar{q}'$ q, s -> d $\bar{q}'$q'





Replacing s to b, q to d, or s, apply to b -> u $\bar{q}''$ q, b -> q $\bar{q}''$ q' decays.

Experimental measurement of ϵ'/ϵ

1993 NA31 at CERN, $\epsilon'/\epsilon = (2.3 \pm 0.7) \times 10^{-3}$

1993 E731 at Fermilab, $\epsilon'/\epsilon = (0.74 \pm 0.59) \times 10^{-3}$.

1999 KTeV at Fermlab, $\epsilon'/\epsilon = (2.8 \pm 0.41) \times 10^{-3}$

1999 NA48 at CERN, $Re(\epsilon'/\epsilon) = (1.85 \pm 0.45 \pm 0.58) \times 10^{-3}$

Experiment value from NA48 and KTeV: $\varepsilon'/\varepsilon = 16.6(2.3) \times 10^{-4}$

Lattice calculation: $21.7(8.4) \times 10^{-4}$ (PRD 102 (2020) 505459)

Chiral perturbation calculation: $14(5) \times 10^{-4}$ (Conf. Ser. 1562(2020) 012011)

SM is consistent with data

There are rooms for new physics beyond SM…Keep an eye on this

Conditions for CP asymmetry: $|A_f| \neq |\bar{A}_{\bar{f}}|$

Parametrized

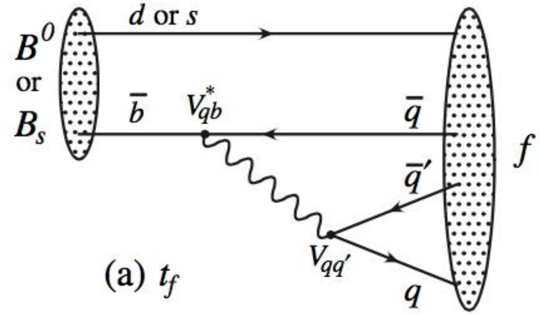
$$A_f = A_1 e^{i(\delta_1^s + \delta_1^w)} + A_2 e^{i(\delta_2^s + \delta_2^w)} ,$$
$$\bar{A}_{\bar{f}} = \eta^{CP}(A_1 e^{i(\delta_1^s - \delta_1^w)} + A_2 e^{i(\delta_2^s - \delta_2^w)}) ,$$

δ_i^s are the strong phases and δ_i^w are the CP violating weak phases. $|\eta^{CP}| = 1$

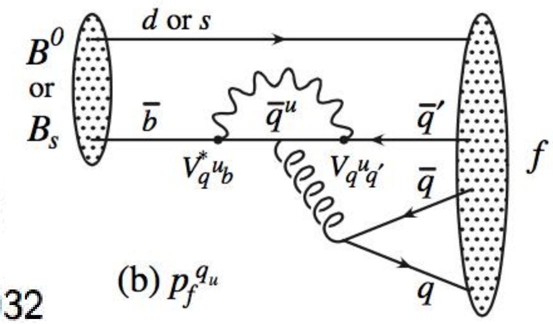
$$A_{CP} = \frac{-2A_1A_2 sin(\delta_1^w - \delta_2^w) sin(\delta_1^s - \delta_2^s)}{A_1^2 + A_2^2 + 2A_1A_2 cos(\delta_1^w - \delta_2^w) cos(\delta_1^w - \delta_w^2)} .$$

There must more than one amplitudes with different strong and weak phases!

$A_{CP}(B_s^0 \to \pi^+ K^-$ 0.224 ± 0.0124 and $A_{CP}(B^0 \to \pi^- K^+) = -0.0834 \pm 0.0032$



(a) t_f



(b) $p_f^{q_u}$

These measurements are in consistent

With SM predictions!

$\bar{b} \to \bar{q}q\bar{q}'$	$B^0 \to f$	$B_s^0 \to f$	CKM dependence of A_f	Suppression
$\bar{b} \to \bar{c}c\bar{s}$	ψK_S	$\psi\phi$	$(V_{cb}^*V_{cs})T + (V_{ub}^*V_{us})P^u$	loop $\times \lambda^2$
$\bar{b} \to \bar{s}s\bar{s}$	ϕK_S	$\phi\phi$	$(V_{cb}^*V_{cs})P^c + (V_{ub}^*V_{us})P^u$	λ^2
$\bar{b} \to \bar{u}u\bar{s}$	$\pi^0 K_S$	K^+K^-	$(V_{cb}^*V_{cs})P^c + (V_{ub}^*V_{us})T$	λ^2/loop
$\bar{b} \to \bar{c}c\bar{d}$	D^+D^-	ψK_S	$(V_{cb}^*V_{cd})T + (V_{tb}^*V_{td})P^t$	loop
$\bar{b} \to \bar{s}s\bar{d}$	K_SK_S	ϕK_S	$(V_{tb}^*V_{td})P^t + (V_{cb}^*V_{cd})P^c$	$\lesssim 1$
$\bar{b} \to \bar{u}u\bar{d}$	$\pi^+\pi^-$	$\rho^0 K_S$	$(V_{ub}^*V_{ud})T + (V_{tb}^*V_{td})P^t$	loop
$\bar{b} \to \bar{c}u\bar{d}$	$D_{CP}\pi^0$	$D_{CP}K_S$	$(V_{cb}^*V_{ud})T + (V_{ub}^*V_{cd})T'$	λ^2
$\bar{b} \to \bar{c}u\bar{s}$	$D_{CP}K_S$	$D_{CP}\phi$	$(V_{cb}^*V_{us})T + (V_{ub}^*V_{cs})T'$	$\lesssim 1$

Time dependent asymmetry

$$A(t)_{CP} = \frac{\Gamma(\bar{M}(t) \to f_{CP}) - \Gamma(M(t) \to f_{CP})}{\Gamma(\bar{M}(t) \to f_{CP}) + \Gamma(M(t) \to f_{CP})} .$$

In the limit $|q/p| = 1$, one obtains

$$A(t)_{CP} = \frac{-C_f cos(\Delta m t) + S_f sin(\Delta m t)}{cosh(\Delta\Gamma t/2) + A_f^{\Delta\Gamma} sinh(\Delta\Gamma t/2)} ,$$

$$C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} , \quad S_f = \frac{2\Im(\lambda_f)}{1 + |\lambda_f|^2} , \quad A_f^{\Delta\Gamma|} = \frac{2\Re(\lambda_f)}{1 + |\lambda_f|^2} , \quad \lambda_f = \frac{q}{p}\frac{\bar{A}_{CP}}{A_{CP}} .$$

CPT sum rule: $|C_f|^2 + |S_f|^2 + |A_f^{\Delta\Gamma}|^2 = 1$.

In the SM, for $B_s^0 - \bar{B}_s^0$ system, good approximation $q/p = V_{tb}^* V_{ts} / V_{tb} V_{ts}^*$,
For $B_0 - \bar{B}^0$ system, $q/p = V_{tb}^* V_{td} / V_{tb} V_{td}^*$. $|q/p| = 1$.

Measurements of S_f and C_f in B decays played an important role in verifying the standard model for CP violation.

C_f type: D -> K^+K^-, $\pi^+\pi^-$; S_f type: B^0 -> J/ψ K_S^0 , $\pi^+\pi^-$.

A large number of CP violating observables measured

$$\Delta A_{CP} = A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) = (-0.154 \pm 0.029)\%$$

$A_{CP}(B^0 \to K^+\pi^-) = -0.0834 \pm 0.0032$

$A_{CP}(B^0 \to K^*(892)^+\pi^-) = -0.27 \pm 0.04$

$S_{D^{*-}D^+}(B^0 \to D^*(2010)^- D^+) = -0.83 \pm 0.09$

$S_{D^{*+}D^-}(B^0 \to D^*(2010)^+ D^-) = -0.80 \pm 0.09$

$S_+(B^0 \to D^{*+}D^{*-}) = -0.73 \pm 0.09$

$S_{D^+D^-}(B^0 \to D^+D^-) = -0.76^{+0.15}_{-0.13}$ (S = 1.2)

$S(B^0 \to J/\psi(1S)\rho^0) = -0.66^{+0.16}_{-0.12}$

$S_{D^{(*)}_{CP}h^0}(B^0 \to D^{(*)}_{CP}h^0) = -0.66 \pm 0.12$

$S_{\eta' K^0}(B^0 \to \eta' K^0) = 0.63 \pm 0.06$

$S_{K^+K^-K^0_S}(B^0 \to K^+K^-K^0_S \text{ nonresonant}) = -0.66 \pm 0.11$

$S_{K^+K^-K^0_S}(B^0 \to K^+K^-K^0_S \text{ inclusive}) = -0.65 \pm 0.12$

$S_{\phi K^0_S}(B^0 \to \phi K^0_S) = 0.59 \pm 0.14$

$C_{\pi\pi}(B^0 \to \pi^+\pi^-) = -0.314 \pm 0.030$

$S_{\pi\pi}(B^0 \to \pi^+\pi^-) = -0.670 \pm 0.030$

$\Delta C_{\rho\pi}(B^0 \to \rho^+\pi^-) = 0.27 \pm 0.06$

$S_{\eta_c K^0_S}(B^0 \to \eta_c K^0_S) = 0.93 \pm 0.17$

$\sin(2\beta) = 0.699 \pm 0.017$

$S_{J/\psi(nS)K^0}(B^0 \to J/\psi(nS)K^0) = 0.701 \pm 0.017$

$S_{\chi_{c1}K^0_S}(B^0 \to \chi_{c1}K^0_S) = 0.63 \pm 0.10$

$\sin(2\beta_{\text{eff}})(B^0 \to K^+K^-K^0_S) = 0.77^{+0.13}_{-0.12}$

$\alpha = (85.2^{+4.8}_{-4.3})^\circ$

$A_{CP}(B_s \to \pi^+K^-) = 0.224 \pm 0.012$

$A_{CP}(B^+ \to D_{CP(+1)}K^+) = 0.132 \pm 0.015$ (S = 1.8)

$A_{ADS}(B^+ \to DK^+) = -0.451 \pm 0.026$

$A_{ADS}(B^+ \to D\pi^+) = 0.129 \pm 0.014$

$A_{ADS}(B^+ \to D^*(D\gamma)K^+) = -0.6 \pm 1.3$

$A_{ADS}(B^+ \to D^*(D\pi^0)K^+) = 0.72 \pm 0.29$

$A_{CP}(B^+ \to \pi^+\pi^-\pi^+) = 0.057 \pm 0.013$

$A_{CP}(B^+ \to \rho^0K^+) = 0.37 \pm 0.10$

$A_{CP}(B^+ \to K^+K^-\pi^+) = -0.122 \pm 0.021$

$A_{CP}(B^+ \to K^+K^-K^+) = -0.033 \pm 0.008$

$\gamma = (65.9^{+3.3}_{-3.5})^\circ$

$r_B(B^+ \to D^0K^+) = 0.0994 \pm 0.0026$

$\delta_B(B^+ \to D^0K^+) = (127.7^{+3.6}_{-3.9})^\circ$

$r_B(B^+ \to D^0K^{*+}) = 0.101^{+0.016}_{-0.034}$

$\delta_B(B^+ \to D^0K^{*+}) = (48^{+59}_{-16})^\circ$

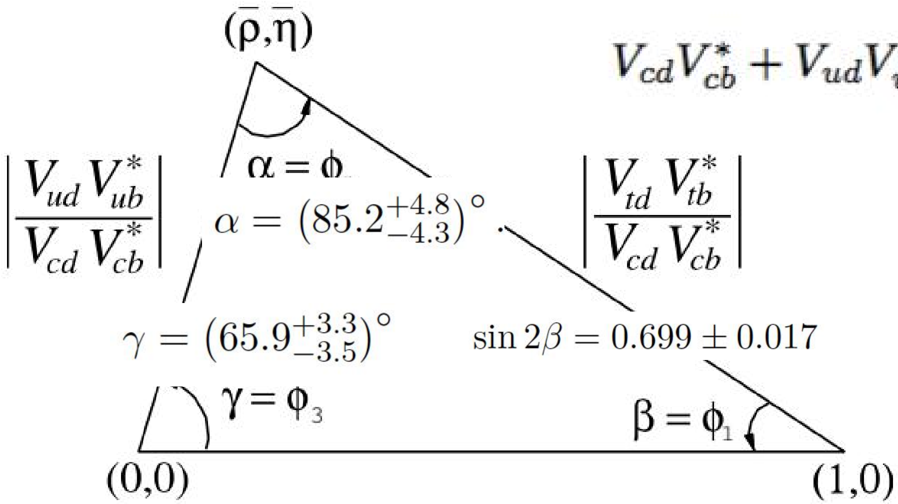
$r_B(B^+ \to D^{*0}K^+) = 0.104^{+0.013}_{-0.014}$

$\delta_B(B^+ \to D^{*0}K^+) = (314.8^{+7.9}_{-9.9})^\circ$

The Unitarity Triangle

$$\sum_i V_{ij} V_{ik}^* = \delta_{jk}\ , \quad \sum_i V_{ji} V_{ki}^* = \delta_{jk}\ ,$$

$$V_{cd}V_{cb}^* + V_{ud}V_{ub}^* + V_{td}V_{tb}^* = 0 \;\to\; 1 + \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0\,.$$



$$\alpha + \beta + \gamma = 180^o$$

PDG

$$\alpha + \beta + \gamma = (173 \pm 6)^\circ$$

The Jarlskog parameter J (1985)

$$\mathrm{Im}\left[V_{ij}V_{kl}V_{il}^*V_{kj}^*\right] = J \sum_{m,n} \varepsilon_{ikm}\varepsilon_{jln}$$

$$J = s_{12}c_{12}s_{23}c_{23}s_{13}c_{13}^2 \sin\delta$$

$$= (3.08^{+0.15}_{-0.13})\text{x}10^{-5}$$

The area of the triangle = J/2

CPV in SM is always proportional to J

$$\alpha = \mathrm{Arg}(-V_{td}V_{tb}^*/V_{ub}^*V_{ud}),\ \beta = \mathrm{Arg}(-V_{cd}V_{cb}^*/V_{tb}^*V_{td}),\ \text{and}\ \gamma = \mathrm{Arg}(-V_{ud}V_{ub}^*/V_{cb}^*V_{cd})$$

Some interesting results

SU(3) symmetry predicts

$$A(\bar{B}^0 \to K^-\pi^+) = V_{ub}V_{us}^*T + V_{tb}V_{ts}^*P\ , \quad A(B^0 \to K^+\pi^-) = V_{ub}^*V_{us}T + V_{tb}^*V_{ts}P\ ,$$
$$A(\bar{B}_s^0 \to K^+\pi^-) = V_{ub}V_{ud}^*T + V_{tb}V_{td}^*P\ , \quad A(B_s^0 \to K^-\pi^+) = V_{ub}^*V_{ud}T + V_{tb}^*V_{td}P\ .$$

$$\frac{A_{CP}(B^0 \to K^+\pi^-)}{A_{CP}(B_s^0 \to K^-\pi^+)} + r_c\frac{Br(B_s^0 \to K^-\pi^+)\tau_{B^0}}{Br(B^0 \to K^+\pi^-)\tau_{B_s^0}} = 0\ .$$

In SU(3) limit, r_c =1. Data gives: r_c=1.26+-0.18

SU(3) is a good approximate symmetry. Deshpande and He, PRL75(1995)1703; He, EPJC9(1999)443; He, Li, Lin, JHEP08(2013)065.

C_f type: D –> K^+K^-, $\pi^+\pi^-$ $\Delta A_{CP} = A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) = (-0.154 \pm 0.029)\%$

Unexpected! Short distance contributions are small

Long distance strong interaction effects important at Charm scale.

SM ~ $2*10^{-4}$ need new physics (Chala, Lenz, Rusov, Scholtz, JHEP07(2019) 161)

Global fit for D -> PP decays, can accommodate

C.W. Chiang and H.Y. Cheng, PRD86(2012) 034036; HN Li, CD Lu, FS Yu, PRD86 (2012)036012.

Cannot be sure if SM is in conflict with data. Room for new physics.

Flavor changing hadronic decays

Tree and penguin contributions

$$\mathcal{H}_{\rm eff}(\Delta S=1)=\frac{G_F}{\sqrt{2}}V_{us}^*V_{ud}\sum_{i=1}^{10}[z_i(\mu)+\tau\, y_i(\mu)]Q_i(\mu)$$

$$Q_7=\frac{3}{2}(\bar s d)_{V-A}\sum_q e_q(\bar q q)_{V+A},$$

$$Q_8=\frac{3}{2}(\bar s_i d_j)_{V-A}\sum_q e_q(\bar q_j q_i)_{V+A},$$

$$Q_9=\frac{3}{2}(\bar s d)_{V-A}\sum_q e_q(\bar q q)_{V-A},$$

$$Q_{10}=\frac{3}{2}(\bar s_i d_j)_{V-A}\sum_q e_q(\bar q_j q_i)_{V-A}.$$

$$Q_1=(\bar s_i u_j)_{V-A}(\bar u_j d_i)_{V-A},$$

$$Q_2=(\bar s u)_{V-A}(\bar u d)_{V-A},$$

$$Q_3=(\bar s d)_{V-A}\sum_q(\bar q q)_{V-A},$$

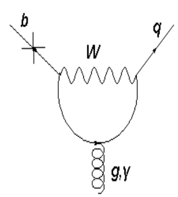
$$Q_4=(\bar s_i d_j)_{V-A}\sum_q(\bar q_j q_i)_{V-A}.$$

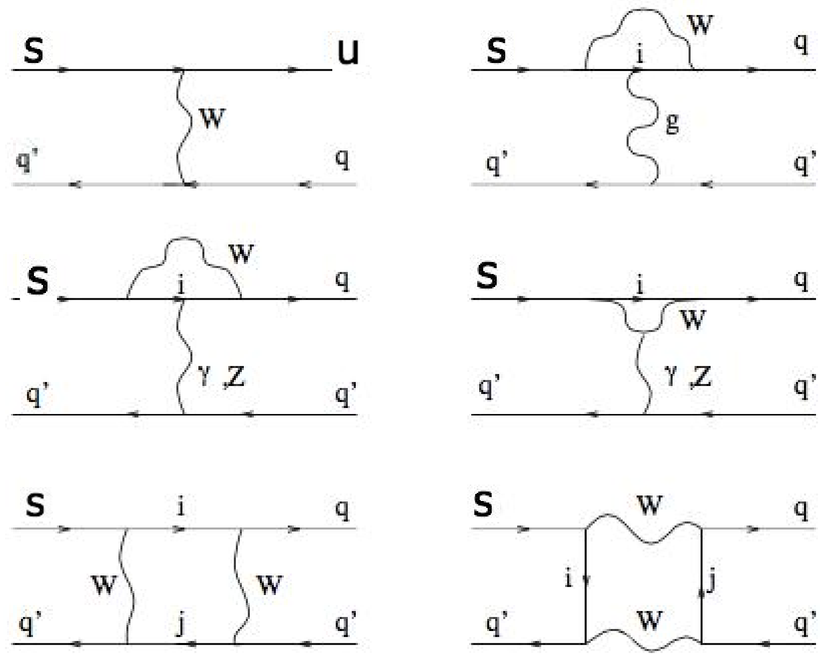
$$Q_5=(\bar s d)_{V-A}\sum_q(\bar q q)_{V+A},$$

$$Q_6=(\bar s_i d_j)_{V-A}\sum_q(\bar q_j q_i)_{V+A}.$$

$$\tau=-\frac{V_{ts}^*V_{td}}{V_{us}^*V_{ud}}.$$

s->u $\bar{q}'$ q, s -> d $\bar{q}'$q'





Replacing s to b, q to d, or s, apply to b -> u q'' $\bar{q}$, b -> q q'' $\bar{q}$ decays.
Similarly for, c -> s u d, udd, uss. Tree dominate, Penguin very small...

Theoretical calculations

Naive factorization (M. Wirbel etal, A. Ali, ...) QCD factorization (M. Benek et al., Lu, Xiao,...) and PQCD caculations (HN Li et al, , Lu, Yang, Xiao...et al), for hadronic B-meson, b-baryon decays better than D-meson, c-baryon decays. Resonable results for branching ratios, and CP violation for B to PP.

SU(3) flavor symmetry approaches (Chau, Cheng etal., Savage etal, Gronau et al, XG He et al, Chiang, HN Li, et al, Geng, liu et al, Hsian et al, Wang, Shi, He...). Fitting for B-meson to Octet meson P, B -> PP.

New measurement enable do some detailed analysis for anti-triplet c-baryon T_{c3} to Octet baryon T_8+P well. minimal-chi-square/degree ~ 1.

But large SU(3) breaking for seemi-leptonic T_{c3} -> T_8+l v badly (He, Huang, Wang, Xing; Geng et al, Wang et al...).

A puzzle!

Charm baryon decays: T_{c3} -> T_8+P and SU(3)

Huang, Xing,He, JHEP05(2022)191; Xing, He, Huang, Yang, arXiv: 2305.1854

$$T_{c\bar 3} = \begin{pmatrix} 0 & \Lambda_c^+ & \Xi_c^+ \\ -\Lambda_c^+ & 0 & \Xi_c^0 \\ -\Xi_c^+ & -\Xi_c^0 & 0 \end{pmatrix}, P = \begin{pmatrix} \frac{\pi^0+\eta_q}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{-\pi^0+\eta_q}{\sqrt{2}} & K^0 \\ K^- & \bar K^0 & \eta_s \end{pmatrix}$$

$$\phi = (39.3 \pm 1.0)^\circ$$

$$T_8 = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda^0}{\sqrt{6}} \end{pmatrix} \quad \begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}$$

$$(H_{\bar 6})^{31}_3 = -(H_{\bar 6})^{13}_3 = (H_{\bar 6})^{12}_2 = -(H_{\bar 6})^{21}_2 = \sin\theta,$$
$$(H_{15})^{31}_3 = (H_{15})^{13}_3 = -(H_{15})^{21}_2 = -(H_{15})^{12}_2 = \sin\theta.$$
$$(H_{\bar 6})^{21}_3 = -(H_{\bar 6})^{12}_3 = \sin^2\theta, \quad (H_{15})^{21}_3 = (H_{15})^{12}_3 = \sin^2\theta.$$

$$\begin{aligned} \mathcal{M} = \; & a_{15} \times (T_{c\bar 3})_i (H_{\overline{15}})^{\{ik\}}_j (\overline{T_8})^j_k P^l_l \\ + \; & b_{15} \times (T_{c\bar 3})_i (H_{\overline{15}})^{\{ik\}}_j (\overline{T_8})^l_k P^j_l \\ + \; & c_{15} \times (T_{c\bar 3})_i (H_{\overline{15}})^{\{ik\}}_j (\overline{T_8})^j_l P^l_k \\ + \; & d_{15} \times (T_{c\bar 3})_i (H_{\overline{15}})^{\{jk\}}_l (\overline{T_8})^l_j P^i_k \\ + \; & e_{15} \times (T_{c\bar 3})_i (H_{\overline{15}})^{\{jk\}}_l (\overline{T_8})^i_j P^l_k \\ + \; & a_6 \times (T_{c\bar 3})^{[ik]} (H_{\bar 6})_{\{ij\}} (\overline{T_8})^j_k P^l_l \\ + \; & b_6 \times (T_{c\bar 3})^{[ik]} (H_{\bar 6})_{\{ij\}} (\overline{T_8})^l_k P^j_l \\ + \; & c_6 \times (T_{c\bar 3})^{[kl]} (H_{\bar 6})_{\{ij\}} (\overline{T_8})^j_l P^l_k \\ + \; & d_6 \times (T_{c\bar 3})^{[ik]} (H_{\bar 6})_{\{ij\}} (\overline{T_8})^i_k P^j_l . \end{aligned}$$

$$\frac{d\Gamma}{d\cos\theta_M} = \frac{G_F^2 |\vec p_{B_n}| (E_{B_n} + M_{B_n})}{8\pi M_{B_c}} (|F|^2 + \kappa^2 |G|^2) \times (1 + \alpha \hat\omega_i \cdot \hat p_{B_n}),$$

$$\alpha = 2\mathrm{Re}(\mathrm{F} * \mathrm{G})\kappa / (|\mathrm{F}|^2 + \kappa^2 |\mathrm{G}|^2),$$
$$\kappa = |\vec p_{B_n}| / (E_{B_n} + M_{B_n}).$$

$$q_6 = G_F \bar u (f_6^q - g_6^q \gamma_5) u, \quad q = a, b, c, d,$$
$$q_{15} = G_F \bar u (f_{15}^q - g_{15}^q \gamma_5) u, \quad q = a, b, c, d, e.$$

$$a = a_6 - a_{15}, \quad a' = a_6 + a_{15},$$
$$f^a = f_6^a - f_{15}^a, \quad g^a = g_6^a - g_{15}^a,$$
$$f^{a'} = f_6^a + f_{15}^a, \quad g^{a'} = g_6^a + g_{15}^a.$$

F&G known functions of qi, currently no measured decay parameters is related to a’

TABLE I: Experimental data and fitting results of anti-triplet charmed baryons two-body decays.

Channel	Branching ratio			
	Lastest measurement in 2022(%)	Experimental data(%)	Previous work(%) [14]	This work(%)
$\Lambda_c^+ \to pK_S^0$	–	1.59 ± 0.08 [39]	1.587 ± 0.077	1.606 ± 0.077
$\Lambda_c^+ \to p\eta$	–	0.142 ± 0.012[39]	0.127 ± 0.024	0.141 ± 0.011
$\Lambda_c^+ \to p\eta'$	$0.0562^{+0.0246}_{-0.0204} \pm 0.0026$[30] $0.0473 \pm 0.0082 \pm 0.0046 \pm 0.0024$[34]	0.0484 ± 0.0091[30, 34]	0.27 ± 0.38	0.0468 ± 0.0066
$\Lambda_c^+ \to \Lambda\pi^+$	$1.31 \pm 0.08 \pm 0.05$[33]	1.30 ± 0.06[33, 39]	1.307 ± 0.069	1.328 ± 0.055
$\Lambda_c^+ \to \Sigma^0\pi^+$	$1.22 \pm 0.08 \pm 0.07$[33]	1.27 ± 0.06[33, 39]	1.272 ± 0.056	1.260 ± 0.046
$\Lambda_c^+ \to \Sigma^+\pi^0$	–	1.25 ± 0.10[39]	1.283 ± 0.057	1.274 ± 0.047
$\Lambda_c^+ \to \Xi^0 K^+$	–	0.55 ± 0.07[39]	0.548 ± 0.068	0.430 ± 0.030
$\Lambda_c^+ \to \Lambda^0 K^+$	$0.0621 \pm 0.0044 \pm 0.0026 \pm 0.0034$[31] $0.0657 \pm 0.0017 \pm 0.0011 \pm 0.0035$[35]	0.064 ± 0.003[31, 35, 39]	0.064 ± 0.010	0.0646 ± 0.0028
$\Lambda_c^+ \to \Sigma^+\eta$	$0.416 \pm 0.075 \pm 0.021 \pm 0.033$[36]	0.32 ± 0.043[36, 39]	0.45 ± 0.19	0.329 ± 0.042
$\Lambda_c^+ \to \Sigma^+\eta'$	$0.314 \pm 0.035 \pm 0.011 \pm 0.025$[36]	0.437 ± 0.084[36, 39]	1.5 ± 0.6	0.444 ± 0.070
$\Lambda_c^+ \to \Sigma^0 K^+$	$0.047 \pm 0.009 \pm 0.001 \pm 0.003$[32] $0.0358 \pm 0.0019 \pm 0.0006 \pm 0.0019$[35]	0.0382 ± 0.0025[32, 35, 39]	0.0504 ± 0.0056	0.0381 ± 0.0017
$\Lambda_c^+ \to n\pi^+$	$0.066 \pm 0.012 \pm 0.004$[33]	0.066 ± 0.0126[33]	0.035 ± 0.011	0.0651 ± 0.0026
$\Lambda_c^+ \to \Sigma^+ K_s^0$	$0.048 \pm 0.014 \pm 0.002 \pm 0.003$[32]	0.048 ± 0.0145[32]	0.0103 ± 0.0042	0.0327 ± 0.0029
$\Xi_c^+ \to \Xi^0\pi^+$	–	1.6 ± 0.8[39]	0.54 ± 0.18	0.887 ± 0.080
$\Xi_c^0 \to \Lambda K_S^0$	–	0.32 ± 0.07[39]	0.334 ± 0.065	0.261 ± 0.043
$\Xi_c^0 \to \Xi^-\pi^+$	–	1.43 ± 0.32[39]	1.21 ± 0.21	1.06 ± 0.20
$\Xi_c^0 \to \Xi^- K^+$	–	0.039 ± 0.012[39]	0.047 ± 0.0083	0.0474 ± 0.0090
$\Xi_c^0 \to \Sigma^0 K_S^0$	–	0.054 ± 0.016[39]	0.069 ± 0.024	0.054 ± 0.016
$\Xi_c^0 \to \Sigma^+ K^-$	–	0.18 ± 0.04[39]	0.221 ± 0.068	0.188 ± 0.039

Channel	Asymmetry parameter α			
	Lastest measurement in 2022	Experimental data	Previous work[14]	This work
$\alpha(\Lambda_c^+ \to pK_S^0)$	–	0.18 ± 0.45[39]	0.19 ± 0.41	0.49 ± 0.20
$\alpha(\Lambda_c^+ \to \Lambda\pi^+)$	$-0.755 \pm 0.005 \pm 0.003$[35]	-0.755 ± 0.0058[35, 39]	-0.841 ± 0.083	-0.7542 ± 0.0058
$\alpha(\Lambda_c^+ \to \Sigma^0\pi^+)$	$-0.463 \pm 0.016 \pm 0.008$[35]	-0.466 ± 0.0178[35, 39]	-0.605 ± 0.088	-0.471 ± 0.015
$\alpha(\Lambda_c^+ \to \Sigma^+\pi^0)$	$-0.48 \pm 0.02 \pm 0.02$[36]	-0.48 ± 0.03[36, 39]	-0.603 ± 0.088	-0.468 ± 0.015
$\alpha(\Xi_c^0 \to \Xi^-\pi^+)$	–	-0.64 ± 0.051[39]	-0.56 ± 0.32	-0.654 ± 0.050
$\alpha(\Lambda_c^+ \to \Sigma^0 K^+)$	$-0.54 \pm 0.18 \pm 0.09$[35]	-0.54 ± 0.20[35]	-0.953 ± 0.040	-0.9958 ± 0.0045
$\alpha(\Lambda_c^+ \to \Lambda K^+)$	$-0.585 \pm 0.049 \pm 0.018$[35]	-0.585 ± 0.052[35]	-0.24 ± 0.15	-0.545 ± 0.046
$\alpha(\Lambda_c^+ \to \Sigma^+\eta)$	$-0.99 \pm 0.03 \pm 0.05$[36]	-0.99 ± 0.058[36]	0.3 ± 3.8	-0.970 ± 0.046
$\alpha(\Lambda_c^+ \to \Sigma^+\eta')$	$-0.46 \pm 0.06 \pm 0.03$[36]	-0.46 ± 0.067[36]	0.8 ± 1.9	-0.455 ± 0.064

SU(3) symmetry parameters from fitting (χ^2/d.o.f.=1.21)				
Vector(f)	$f^a = 0.0155 \pm 0.0040$	$f_6^b = 0.0215 \pm 0.0092$	$f_6^c = 0.0356 \pm 0.0071$	$f_6^d = -0.0138 \pm 0.0080$
	$f_{15}^b = -0.0161 \pm 0.0042$	$f_{15}^c = 0.0149 \pm 0.0080$	$f_{15}^d = -0.0253 \pm 0.0031$	$f_{15}^e = 0.0798 \pm 0.0087$
Axial-vector(g)	$g^a = -0.039 \pm 0.012$	$g_6^b = -0.240 \pm 0.011$	$g_6^c = 0.121 \pm 0.019$	$g_6^d = -0.067 \pm 0.014$
	$g_{15}^b = 0.1134 \pm 0.0074$	$g_{15}^c = 0.014 \pm 0.018$	$g_{15}^d = -0.0387 \pm 0.0085$	$g_{15}^e = 0.0209 \pm 0.0092$

TABLE II: SU(3) amplitudes and predicted branching fractions (the third column) and polarization parameters (the fourth column) of anti-triplet charmed baryons decays into an octet baryon and an octet meson.

Channel	SU(3) amplitude	Branching ratio(10^{-2})	α
$\Lambda_c^+ \to \Sigma^0\pi^+$	$(-b_6+b_{15}+c_6-c_{15}+d_6)/\sqrt{2}$	1.260 ± 0.046	-0.470 ± 0.015
$\Lambda_c^+ \to \Lambda\pi^+$	$-(b_6-b_{15}+c_6-c_{15}+d_6+2e_{15})/\sqrt{6}$	1.328 ± 0.055	-0.7542 ± 0.0058
$\Lambda_c^+ \to \Sigma^+\pi^0$	$(b_6-b_{15}-c_6+c_{15}-d_6)/\sqrt{2}$	1.274 ± 0.047	-0.468 ± 0.015
$\Lambda_c^+ \to pK_S^0$	$(\sin^2\theta\,(-d_6+d_{15}+e_{15})+b_6-b_{15}-e_{15})/\sqrt{2}$	1.606 ± 0.077	0.49 ± 0.20
$\Lambda_c^+ \to \Xi^0K^+$	$-c_6+c_{15}+d_{15}$	0.430 ± 0.030	0.955 ± 0.018
$\Xi_c^+ \to \Sigma^+K_S^0$	$(\sin^2\theta\,(b_6-b_{15}-e_{15})-d_6+d_{15}+e_{15})/\sqrt{2}$	0.77 ± 0.32	0.29 ± 0.29
$\Xi_c^+ \to \Xi^0\pi^+$	$-d_6-d_{15}-e_{15}$	0.887 ± 0.080	-0.902 ± 0.039
$\Xi_c^0 \to \Sigma^0K_S^0$	$(-\sin^2\theta\,(b_6+b_{15}-e_{15})+(c_6+c_{15}+d_6-e_{15}))/2$	0.054 ± 0.016	-0.75 ± 0.24
$\Xi_c^0 \to \Lambda K_S^0$	$\sqrt{3}\sin^2\theta\,(b_6+b_{15}-2c_6-2c_{15}-2d_6+e_{15})/6$ $+\sqrt{3}(2b_6+2b_{15}-c_6-c_{15}-d_6-e_{15})/6$	0.261 ± 0.043	0.984 ± 0.084
$\Xi_c^0 \to \Sigma^+K^-$	$c_6+c_{15}+d_{15}$	0.188 ± 0.039	0.98 ± 0.20
$\Xi_c^0 \to \Xi^-\pi^+$	$b_6+b_{15}+e_{15}$	1.06 ± 0.20	-0.654 ± 0.050
$\Xi_c^0 \to \Xi^0\pi^0$	$(-b_6-b_{15}+d_6+d_{15})/\sqrt{2}$	0.130 ± 0.051	-0.28 ± 0.18
$\Lambda_c^+ \to \Sigma^0K^+$	$\sin\theta\,(-b_6+b_{15}+d_6+d_{15})/\sqrt{2}$	0.0381 ± 0.0017	-0.9959 ± 0.0044
$\Lambda_c^+ \to \Lambda K^+$	$-\sin\theta\,(b_6-b_{15}-2c_6+2c_{15}+d_6+3d_{15}+2e_{15})/\sqrt{6}$	0.0646 ± 0.0028	-0.545 ± 0.046
$\Lambda_c^+ \to \Sigma^+K_S^0/K_L^0$	$\sin\theta\,(-b_6+b_{15}+d_6-d_{15})/\sqrt{2}$	0.0327 ± 0.0029	-0.52 ± 0.11
$\Lambda_c^+ \to p\pi^0$	$\sin\theta\,(-c_6+c_{15}-d_6+e_{15})/\sqrt{2}$	0.021 ± 0.010	-0.21 ± 0.18
$\Lambda_c^+ \to n\pi^+$	$-\sin\theta\,(c_6-c_{15}+d_6+e_{15})$	0.0651 ± 0.0026	0.533 ± 0.047
$\Xi_c^+ \to \Sigma^0\pi^+$	$-\sin\theta\,(b_6-b_{15}-c_6+c_{15}+d_{15}+e_{15})/\sqrt{2}$	0.3194 ± 0.0088	-0.728 ± 0.018
$\Xi_c^+ \to \Lambda\pi^+$	$\sin\theta\,(-b_6+b_{15}-c_6+c_{15}+2d_6+3d_{15}+e_{15})/\sqrt{6}$	0.0222 ± 0.0032	-0.16 ± 0.17
$\Xi_c^+ \to \Sigma^+\pi^0$	$\sin\theta\,(b_6-b_{15}-c_6+c_{15}-d_{15}-e_{15})/\sqrt{2}$	0.247 ± 0.020	0.46 ± 0.19
$\Xi_c^+ \to pK_S^0/K_L^0$	$\sin\theta\,(-b_6+b_{15}+d_6-d_{15})$	0.177 ± 0.016	-0.361 ± 0.081
$\Xi_c^+ \to \Xi^0K^+$	$-\sin\theta\,(c_6-c_{15}+d_6+e_{15})$	0.1361 ± 0.0063	0.371 ± 0.036
$\Xi_c^0 \to \Sigma^0\pi^0$	$-\frac{1}{2}\sin\theta\,(b_6+b_{15}+c_6+c_{15}-d_{15}-e_{15})$	0.00014 ± 0.00030	0.3 ± 2.3
$\Xi_c^0 \to \Lambda\pi^0$	$\sin\theta\,(b_6+b_{15}+c_6+c_{15}-2d_6-3d_{15}+e_{15})/2\sqrt{3}$	0.0375 ± 0.0076	0.74 ± 0.16
$\Xi_c^0 \to \Sigma^+\pi^-$	$-\sin\theta\,(c_6+c_{15}+d_{15})$	0.0116 ± 0.0026	0.96 ± 0.25
$\Xi_c^0 \to pK^-$	$\sin\theta\,(c_6+c_{15}+d_{15})$	0.0138 ± 0.0045	0.89 ± 0.38
$\Xi_c^0 \to \Sigma^-\pi^+$	$-\sin\theta\,(b_6+b_{15}+e_{15})$	0.057 ± 0.011	-0.723 ± 0.050
$\Xi_c^0 \to nK_S^0/K_L^0$	$\sin\theta\,(-b_6-b_{15}+c_6+c_{15}+d_6)$	0.0234 ± 0.0060	0.66 ± 0.34
$\Xi_c^0 \to \Xi^-K^+$	$\sin\theta\,(b_6+b_{15}+e_{15})$	0.0474 ± 0.0090	-0.610 ± 0.048
$\Xi_c^0 \to \Xi^0K_S^0/K_L^0$	$\sin\theta\,(b_6+b_{15}-c_6-c_{15}-d_6)$	0.0114 ± 0.0023	0.87 ± 0.30
$\Lambda_c^+ \to pK_L^0$	$(\sin^2\theta\,(-d_6+d_{15}+e_{15})-b_6+b_{15}+e_{15})/\sqrt{2}$	1.688 ± 0.080	0.56 ± 0.20
$\Lambda_c^+ \to nK^+$	$\sin^2\theta\,(d_6+d_{15}+e_{15})$	0.001022 ± 0.000091	-0.980 ± 0.019
$\Xi_c^+ \to \Sigma^0K^+$	$\sin^2\theta\,(b_6-b_{15}+e_{15})/\sqrt{2}$	0.01156 ± 0.00033	-0.9961 ± 0.0014
$\Xi_c^+ \to \Lambda K^+$	$\sin^2\theta\,(b_6-b_{15}-2c_6+2c_{15}-2d_6-e_{15})/\sqrt{6}$	0.00441 ± 0.00019	0.624 ± 0.033
$\Xi_c^+ \to \Sigma^+K_L^0$	$(\sin^2\theta\,(b_6-b_{15}-e_{15})+d_6-d_{15}-e_{15})/\sqrt{2}$	0.95 ± 0.35	0.57 ± 0.28
$\Xi_c^+ \to p\pi^0$	$\sin^2\theta\,(c_6-c_{15}+d_{15})/\sqrt{2}$	0.00046 ± 0.00021	-0.29 ± 0.38
$\Xi_c^+ \to n\pi^+$	$\sin^2\theta\,(c_6-c_{15}-d_{15})$	0.00619 ± 0.00040	0.945 ± 0.020
$\Xi_c^0 \to \Sigma^0K_L^0$	$(-\sin^2\theta\,(b_6+b_{15}-e_{15})-(c_6+c_{15}+d_6-e_{15}))/2$	0.069 ± 0.019	-0.51 ± 0.29
$\Xi_c^0 \to \Lambda K_L^0$	$\sqrt{3}\sin^2\theta\,(b_6+b_{15}-2c_6-2c_{15}-2d_6+e_{15})/6$ $-\sqrt{3}(2b_6+2b_{15}-c_6-c_{15}-d_6-e_{15})/6$	0.243 ± 0.043	0.996 ± 0.043
$\Xi_c^0 \to p\pi^-$	$\sin^2\theta\,(c_6+c_{15}+d_{15})$	0.00082 ± 0.00029	0.87 ± 0.40
$\Xi_c^0 \to \Sigma^-K^+$	$\sin^2\theta\,(b_6+b_{15}+e_{15})$	0.00258 ± 0.00049	-0.689 ± 0.050
$\Xi_c^0 \to n\pi^0$	$-\sin^2\theta\,(c_6+c_{15}-d_{15})/\sqrt{2}$	0.00194 ± 0.00031	0.9997 ± 0.0091

TABLE III: SU(3) amplitudes and predicted branching fractions (the third column) and polarization parameters (the fourth column) of anti-triplet charmed baryons decays into an octet baryon and η or η'. In this table "–" represent the channel can not be prediction due to the limit of experimental data.

Channel	SU(3) amplitude	Branching fraction(10^{-2})	α
$\Lambda_c^+ \to \Sigma^+\eta$	$\cos\phi(-2a_6+2a_{15}-b_6+b_{15}-c_6+c_{15}+d_6)/\sqrt{2}-\sin\phi(-a_6+a_{15}+d_{15})$	0.329 ± 0.042	-0.970 ± 0.046
$\Lambda_c^+ \to \Sigma^+\eta'$	$\sin\phi(-2a_6+2a_{15}-b_6+b_{15}-c_6+c_{15}+d_6)/\sqrt{2}+\cos\phi(-a_6+a_{15}+d_{15})$	0.444 ± 0.070	-0.455 ± 0.064
$\Lambda_c^+ \to p\eta$	$\sin\theta\big(\cos\phi\,(-2a_6+2a_{15}-c_6+c_{15}+d_6-e_{15})/\sqrt{2}$ $-\sin\phi\,(-a_6+a_{15}-b_6+b_{15}+d_{15}+e_{15})\big)$	$0.141+0.011$	$0.93+0.11$
$\Lambda_c^+ \to p\eta'$	$\sin\theta\big(\sin\phi\,(-2a_6+2a_{15}-c_6+c_{15}+d_6-e_{15})/\sqrt{2}$ $+\cos\phi\,(-a_6+a_{15}-b_6+b_{15}+d_{15}+e_{15})\big)$	0.0468 ± 0.0066	-0.990 ± 0.018
$\Xi_c^+ \to \Sigma^+\eta$	$\sin\theta\big(\cos\phi\,(-2a_6+2a_{15}-b_6+b_{15}-c_6+c_{15}+d_{15}+e_{15})/\sqrt{2}$ $-\sin\phi\,(-a_6+a_{15}+d_6-e_{15})\big)$	0.114 ± 0.022	0.97 ± 0.11
$\Xi_c^+ \to \Sigma^+\eta'$	$\sin\theta\big(\sin\phi\,(-2a_6+2a_{15}-b_6+b_{15}-c_6+c_{15}+d_{15}+e_{15})/\sqrt{2}$ $+\cos\phi\,(-a_6+a_{15}+d_6-e_{15})\big)$	0.125 ± 0.022	-0.456 ± 0.070
$\Xi_c^+ \to p\eta$	$\sin^2\theta\big(\cos\phi\,(2a_6-2a_{15}+c_6-c_{15}-d_{15})/\sqrt{2}$ $-\sin\phi\,(a_6-a_{15}+b_6-b_{15}-d_6)\big)$	0.00938 ± 0.00071	-0.003 ± 0.61
$\Xi_c^+ \to p\eta'$	$\sin^2\theta\big(\sin\phi\,(2a_6-2a_{15}+c_6-c_{15}-d_{15})/\sqrt{2}$ $+\cos\phi\,(a_6-a_{15}+b_6-b_{15}-d_6)\big)$	0.0095 ± 0.0011	-0.9981 ± 0.0058
$\Xi_c^0 \to \Xi^0\eta$	$\cos\phi(2a_6+2a_{15}+b_6+b_{15}-d_6+d_{15})/\sqrt{2}-\sin\phi(a_6+a_{15}+c_6+c_{15})$	-	-
$\Xi_c^0 \to \Xi^0\eta'$	$\sin\phi(2a_6+2a_{15}+b_6+b_{15}-d_6+d_{15})/\sqrt{2}+\cos\phi(a_6+a_{15}+c_6+c_{15})$	-	-
$\Xi_c^0 \to \Sigma^0\eta$	$\sin\theta\big(\,cos\phi\,(2a_6+2a_{15}+b_6+b_{15}+c_6+c_{15}+d_{15}-e_{15})/2$ $-\sin\phi\,(a_6+a_{15}-d_6+e_{15})/\sqrt{2}\big)$	-	-
$\Xi_c^0 \to \Sigma^0\eta'$	$\sin\theta(\sin\phi\,(2a_6+2a_{15}+b_6+b_{15}+c_6+c_{15}+d_{15}-e_{15})/2$ $-\cos\phi\,(a_6+a_{15}-d_6+e_{15})/\sqrt{2})$	-	-
$\Xi_c^0 \to \Lambda\eta$	$\big(-\cos\phi\,(6a_6+6a_{15}+b_6+b_{15}+c_6+c_{15}-2d_6+3d_{15}+e_{15})/(2\sqrt{3})$ $-\sin\phi\,(-3a_6-3a_{15}-2b_6-2b_{15}-2c_6-2c_{15}+d_6+e_{15})/\sqrt{6}\big)\sin\theta$	-	-
$\Xi_c^0 \to \Lambda\eta'$	$\big(-\sin\phi\,(6a_6+6a_{15}+b_6+b_{15}+c_6+c_{15}-2d_6+3d_{15}+e_{15})/(2\sqrt{3})$ $+\cos\phi\,(-3a_6-3a_{15}-2b_6-2b_{15}-2c_6-2c_{15}+d_6+e_{15})/\sqrt{6}\big)\sin\theta$	-	-
$\Xi_c^0 \to n\eta$	$\sin^2\theta(\cos\phi\,(2a_6+2a_{15}+c_6+c_{15}+d_{15})/\sqrt{2}$ $-\sin\phi\,(a_6+a_{15}+b_6+b_{15}-d_6)\big)$	-	-
$\Xi_c^0 \to n\eta$	$\sin^2\theta(\sin\phi\,(2a_6+2a_{15}+c_6+c_{15}+d_{15})/\sqrt{2}$ $+\cos\phi\,(a_6+a_{15}+b_6+b_{15}-d_6)\big)$	-	-

Predcit the undetermined branching ratios

$$r^f = f^{a\prime}/f^a, \quad r^g = g^{a\prime}/g^a.$$

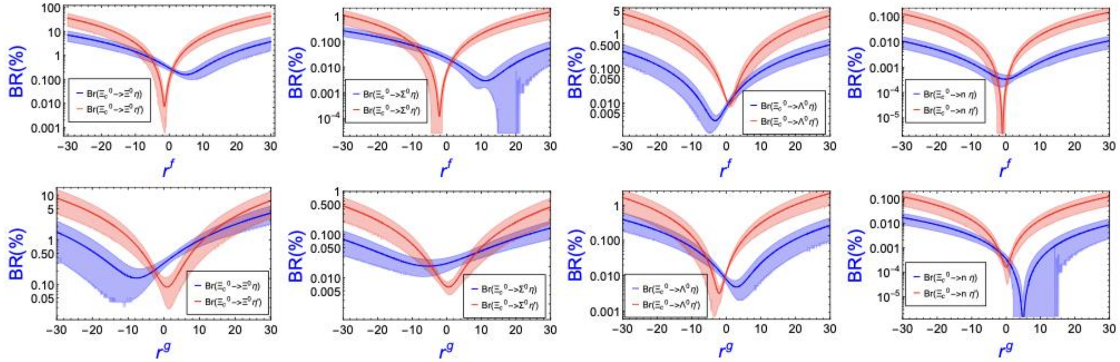


FIG. 1: The branching ratios which depend on the r^f (first line) and r^g (second line) for the 8 undetermined decays: $\Xi_c^0 \to \Xi^0\eta^{(\prime)}, \Xi_c^0 \to \Sigma^0\eta^{(\prime)}, \Xi_c^0 \to \Lambda^0\eta^{(\prime)}, \Xi_c^0 \to \Xi^0\eta^{(\prime)}$. In these figures, we set another parameter $r^{f(g)} = 0$.

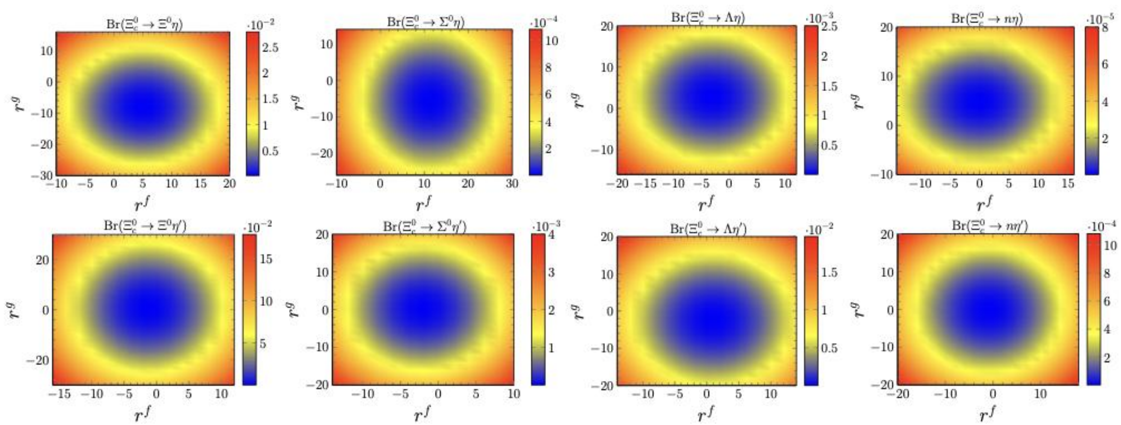


FIG. 2: The branching ratios of the 8 undetermined decays: $\Xi_c^0 \to \Xi^0\eta^{(\prime)}, \Xi_c^0 \to \Sigma^0\eta^{(\prime)}, \Xi_c^0 \to \Lambda^0\eta^{(\prime)}, \Xi_c^0 \to \Xi^0\eta^{(\prime)}$ on the $r^f - r^g$ plane

$$Br(\Xi_c^0 \to \Xi^0\eta') \geq 0.002\%,$$
$$Br(\Xi_c^0 \to \Sigma^0\eta') \geq 9 \times 10^{-7},$$
$$Br(\Xi_c^0 \to \Lambda^0\eta') \geq 4.8 \times 10^{-6},$$
$$Br(\Xi_c^0 \to n\eta') \geq 6 \times 10^{-8}.$$

Large SU(3) breaking in T_{c3} -> T_8+l v

He, Huang, Wang, Xing, PLB823, (2021) 136765.

channel	branching ratio(%)	
	experimental data	SU(3) symmetry
$\Lambda_c^+ \to \Lambda^0 e^+ \nu_e$	3.6 ± 0.4 [33]	3.6 ± 0.4 (input)
$\Lambda_c^+ \to \Lambda^0 \mu^+ \nu_\mu$	3.5 ± 0.5 [33]	3.5 ± 0.5 (input)
$\Xi_c^+ \to \Xi^0 e^+ \nu_e$	2.3 ± 1.5 [33]	12.17 ± 1.35
$\Xi_c^0 \to \Xi^- e^+ \nu_e$	1.54 ± 0.35 [4, 5]	4.10 ± 0.46
$\Xi_c^0 \to \Xi^- \mu^+ \nu_\mu$	1.27 ± 0.44 [4]	3.98 ± 0.57

$$\mathcal{H}_{c\to d/s} = \frac{G_F}{\sqrt{2}}\left[V_{cq}^* \bar{q}\gamma^\mu(1-\gamma_5)c\ \bar{\nu}_\ell\gamma_\mu(1-\gamma_5)\ell\right] + h.c.,$$

$$(H_3)^1 = 0, \quad (H_3)^2 = V_{cd}^*, \quad (H_3)^3 = V_{cs}^*.$$

$$H_{\lambda,\lambda_w} = a_1^{\lambda,\lambda_w} \times (T_{c\bar{3}})^{[ij]}(H_3)^k \epsilon_{ikm}(T_8)_j^m$$

$$a_1^{\lambda,\lambda_w} = \bar{u}(\lambda)\left[f_1\gamma^\mu + f_2\frac{i\sigma^{\nu\mu}}{M_i}q^\nu + f_3\frac{q^\mu}{M_i}\right]u(\lambda_i)\epsilon_\mu^*(\lambda_w) - \bar{u}(\lambda)\left[f_1'\gamma^\mu + f_2'\frac{i\sigma^{\nu\mu}}{M_i}q^\nu + f_3'\frac{q^\mu}{M_i}\right]\gamma_5 u(\lambda_i)\epsilon_\mu^*(\lambda_w).$$

channel	amplitude
$\Lambda_c^+ \to \Lambda^0 \ell^+ \nu_\ell$	$-\sqrt{\frac{2}{3}}a_1^{\lambda,\lambda_w} V_{cs}^*$
$\Lambda_c^+ \to n \ell^+ \nu_\ell$	$a_1^{\lambda,\lambda_w} V_{cd}^*$
$\Xi_c^+ \to \Sigma^0 \ell^+ \nu_\ell$	$\frac{a_1^{\lambda,\lambda_w} V_{cd}^*}{\sqrt{2}}$
$\Xi_c^+ \to \Lambda^0 \ell^+ \nu_\ell$	$-\frac{a_1^{\lambda,\lambda_w} V_{cd}^*}{\sqrt{6}}$
$\Xi_c^+ \to \Xi^0 \ell^+ \nu_\ell$	$-a_1^{\lambda,\lambda_w} V_{cs}^*$
$\Xi_c^0 \to \Sigma^- \ell^+ \nu_\ell$	$a_1^{\lambda,\lambda_w} V_{cd}^*$
$\Xi_c^0 \to \Xi^- \ell^+ \nu_\ell$	$a_1^{\lambda,\lambda_w} V_{cs}^*$

SU(3) fit: a very bad one !

channel	branching ratio(%)	
	experimental data	fit data
$\Lambda_c^+ \to \Lambda^0 e^+ \nu_e$	3.60 ± 0.40	1.94 ± 0.18
$\Lambda_c^+ \to \Lambda^0 \mu^+ \nu_\mu$	3.5 ± 0.5	1.87 ± 0.176
$\Xi_c^+ \to \Xi^0 e^+ \nu_e$	2.3 ± 1.5	6.53 ± 0.60
$\Xi_c^0 \to \Xi^- e^+ \nu_e$	1.54 ± 0.35	2.17 ± 0.20
$\Xi_c^0 \to \Xi^- \mu^+ \nu_\mu$	1.27 ± 0.44	2.09 ± 0.19
$\chi^2/d.o.f = 14.3$	$f_1 = 1.05 \pm 0.30$	$f_1' = 0.11 \pm 0.95$

SU(3) breaking effects

$$M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} \sim m_s \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = m_s \times \omega.$$

$$H_{\lambda,\lambda_w}(T_{c\bar{3}}\omega \to T_{c6}) = d^{\lambda,\lambda_w} \times (T_{c\bar{3}})^{[ij]}\omega^k_j (T_{c6})_{\{ki\}}.$$

$$\Xi_c^{0/+mass} = \cos\theta \times \Xi_c^{0/+} + \sin\theta \times \Xi_c^{0/+\prime},$$

$$H^{mass}_{\lambda,\lambda_w} \propto V^*_{cs}(a_1^{\lambda,\lambda_w} + a_2^{\lambda,\lambda_w} - a_4^{\lambda,\lambda_w} + a_5^{\lambda,\lambda_w} + \frac{c_1^{\lambda,\lambda_w}}{\sqrt{2}}\theta),$$

$$\begin{aligned} H_{\lambda,\lambda_W} = \; & a_1^{\lambda,\lambda_w} \times (T_{c\bar{3}})^{[ij]}(H_3)^k \epsilon_{ikm}(T_8)^m_j + a_2^{\lambda,\lambda_w} \times (T_{c\bar{3}})^{[in]}(H_3)^k \epsilon_{ikm}(T_8)^m_j \omega^j_n \\ + \; & a_3^{\lambda,\lambda_w} \times (T_{c\bar{3}})^{[in]}(H_3)^k \epsilon_{kjm}(T_8)^m_i \omega^j_n + a_4^{\lambda,\lambda_w} \times (T_{c\bar{3}})^{[in]}(H_3)^k \epsilon_{jim}(T_8)^m_k \omega^j_n \\ + \; & a_5^{\lambda,\lambda_w} \times (T_{c\bar{3}})^{[ij]}(H_3)^k \epsilon_{inm}(T_8)^m_j \omega^n_k. \end{aligned}$$

$$a_4^{\prime\lambda,\lambda_w} = a_4^{\lambda,\lambda_w} + c_1^{\lambda,\lambda_w}\theta/\sqrt{2} \text{ and } a_2^{\prime\lambda,\lambda_w} = a_2^{\lambda,\lambda_w} + \sqrt{2}c_1^{\lambda,\lambda_w}\theta,$$

channel	amplitude I	amplitude II
$\Lambda_c^+ \to \Lambda^0 l^+ \nu$	$-\sqrt{\frac{2}{3}}(a_1^{\lambda,\lambda_w} + a_5^{\lambda,\lambda_w})V^*_{cs}$	$-\sqrt{\frac{2}{3}}(a_1^{\lambda,\lambda_w} + a_5^{\lambda,\lambda_w})V^*_{cs}$
$\Lambda_c^+ \to n l^+ \nu$	$a_1 V^*_{cd}$	$a_1 V^*_{cd}$
$\Xi_c^+ \to \Sigma^0 \ell^+ \nu_\ell$	$\frac{(a_1^{\lambda,\lambda_w} + a_3^{\lambda,\lambda_w} - a_4^{\lambda,\lambda_w} - \frac{c_1^{\lambda,\lambda_w}}{\sqrt{2}}\theta)V^*_{cd}}{\sqrt{2}}$	$\frac{(a_1^{\lambda,\lambda_w} + a_3^{\lambda,\lambda_w} - a_4^{\prime\lambda,\lambda_w})V^*_{cd}}{\sqrt{2}}$
$\Xi_c^+ \to \Lambda^0 \ell^+ \nu_\ell$	$-\frac{(a_1^{\lambda,\lambda_w} + 2a_2^{\lambda,\lambda_w} - a_3^{\lambda,\lambda_w} - a_4^{\lambda,\lambda_w} + \frac{3c_1^{\lambda,\lambda_w}}{\sqrt{2}}\theta)V^*_{cd}}{\sqrt{6}}$	$-\frac{(a_1^{\lambda,\lambda_w} + 2a_2^{\prime\lambda,\lambda_w} - a_3^{\lambda,\lambda_w} - a_4^{\prime\lambda,\lambda_w})V^*_{cd}}{\sqrt{6}}$
$\Xi_c^+ \to \Xi^0 \ell^+ \nu_\ell$	$-(a_1^{\lambda,\lambda_w} + a_2^{\lambda,\lambda_w} - a_4^{\lambda,\lambda_w} + a_5^{\lambda,\lambda_w} + \frac{c_1^{\lambda,\lambda_w}}{\sqrt{2}}\theta)V^*_{cs}$	$-(a_1^{\lambda,\lambda_w} + a_2^{\prime\lambda,\lambda_w} - a_4^{\prime\lambda,\lambda_w} + a_5^{\lambda,\lambda_w})V^*_{cs}$
$\Xi_c^0 \to \Sigma^- \ell^+ \nu_\ell$	$(a_1^{\lambda,\lambda_w} + a_3^{\lambda,\lambda_w} - a_4^{\lambda,\lambda_w} - \frac{c_1^{\lambda,\lambda_w}}{\sqrt{2}}\theta)V^*_{cd}$	$(a_1^{\lambda,\lambda_w} + a_3^{\lambda,\lambda_w} - a_4^{\prime\lambda,\lambda_w})V^*_{cd}$
$\Xi_c^0 \to \Xi^- \ell^+ \nu_\ell$	$(a_1^{\lambda,\lambda_w} + a_2^{\lambda,\lambda_w} - a_4^{\lambda,\lambda_w} + a_5^{\lambda,\lambda_w} + \frac{c_1^{\lambda,\lambda_w}}{\sqrt{2}}\theta)V^*_{cs}$	$(a_1^{\lambda,\lambda_w} + a_2^{\prime\lambda,\lambda_w} - a_4^{\prime\lambda,\lambda_w} + a_5^{\lambda,\lambda_w})V^*_{cs}$

channel	branching ratio(%)		
	experimental data	fit data(pole model)	fit data(constant).
$\Lambda_c^+ \to \Lambda^0 e^+ \nu_e$	3.6 ± 0.4	3.61 ± 0.32	3.62 ± 0.32
$\Lambda_c^+ \to \Lambda^0 \mu^+ \nu_\mu$	3.5 ± 0.5	3.48 ± 0.30	3.45 ± 0.30
$\Xi_c^+ \to \Xi^0 e^+ \nu_e$	2.3 ± 1.5	3.89 ± 0.73	3.92 ± 0.73
$\Xi_c^0 \to \Xi^- e^+ \nu_e$	1.54 ± 0.35	1.29 ± 0.24	1.31 ± 0.24
$\Xi_c^0 \to \Xi^- \mu^+ \nu_\mu$	1.27 ± 0.44	1.24 ± 0.23	1.24 ± 0.23
fit parameter (pole model)	$f_1 = 1.01 \pm 0.87,\ \delta f_1 = -0.51 \pm 0.92$ $f_1' = 0.60 \pm 0.49,\ \delta f_1' = -0.23 \pm 0.41$		$\chi^2/d.o.f = 1.6$
fit parameter (constant)	$f_1 = 0.86 \pm 0.92,\ \delta f_1 = -0.25 \pm 0.88$ $f_1' = 0.85 \pm 0.36,\ \delta f_1' = -0.43 \pm 0.50$		$\chi^2/d.o.f = 1.9$

δf_1 and δf_1' breaking effects, as large as the symmetric effects!

2. Anomalies, confirmed?

CKM unitarity anomaly?

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1,$$

$|V_{ub}|^2 \sim 10^{-5}$ negligible, so usually study

$\Delta = |V_{ud}|^2 + |V_{us}|^2 - 1$

Zoom in superallowed 0+ -> 0+ nuclei transition and K -> π l ν show about 3σ level deviation

aeXiv:2208.11707

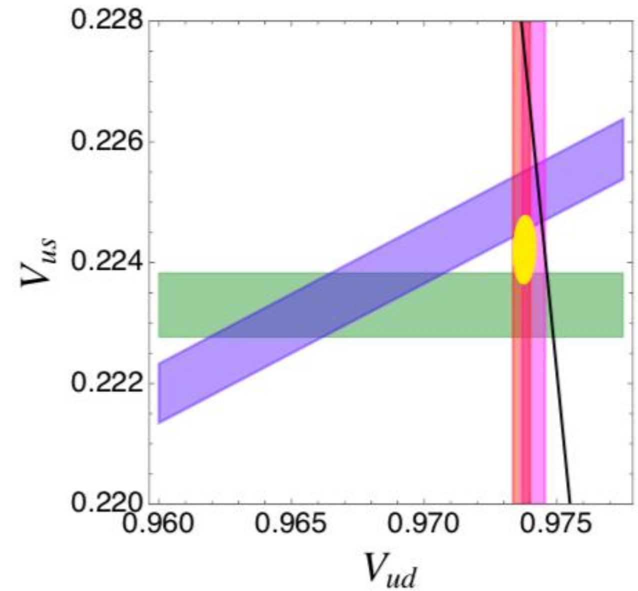
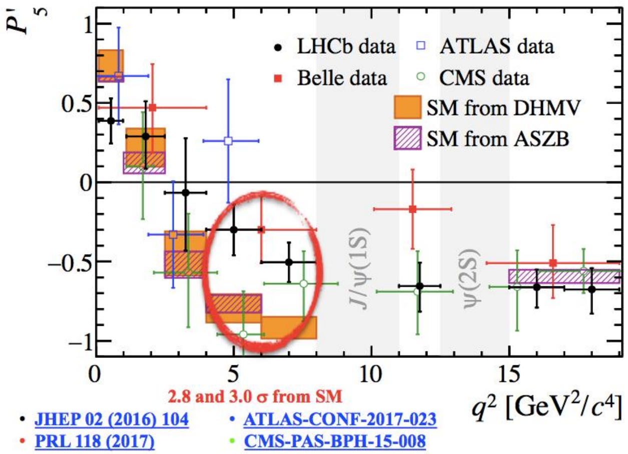


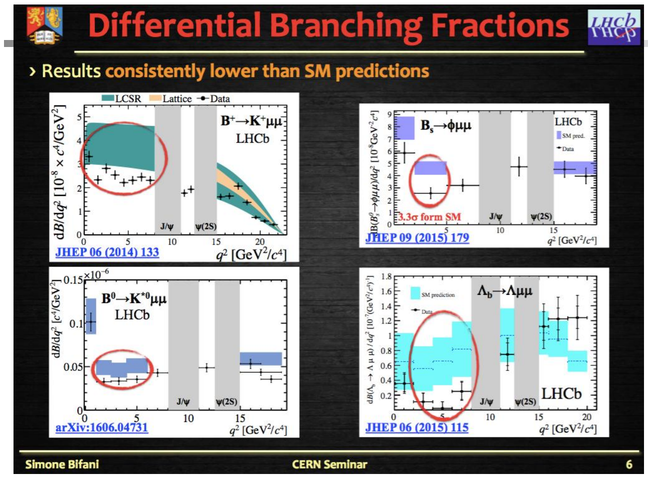
Figure 1: Constraints in the V_{ud}–V_{us} plane. The partially overlapping vertical bands correspond to $V_{ud}^{0^+\to 0^+}$ (leftmost, red) and $V_{ud}^{n,\,best}$ (rightmost, violet). The horizontal band (green) corresponds to $V_{us}^{K_{\ell 3}}$. The diagonal band (blue) corresponds to $(V_{us}/V_{ud})_{K_{\ell 2}/\pi_{\ell 2}}$. The unitarity circle is denoted by the black solid line. The 68% C.L. ellipse from a fit to all four constraints is depicted in yellow (V_{ud} = 0.97378(26), V_{us} = 0.22422(36), χ^2/dof = 6.4/2, p-value 4.1%), it deviates from the unitarity line by 2.8σ. Note that the significance tends to increase in case τ decays are included.

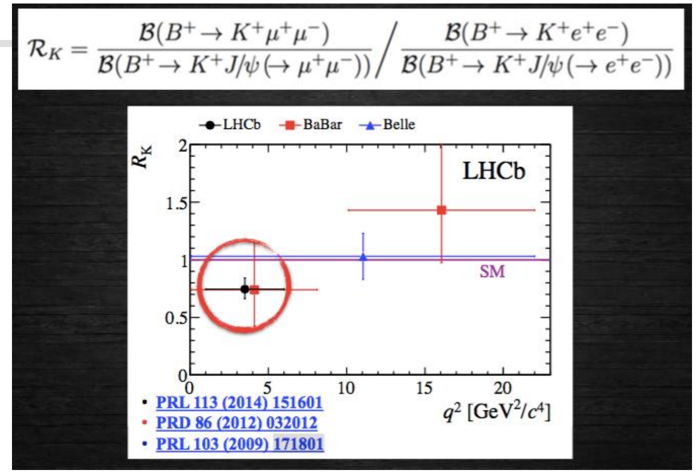
$\Delta^{(1)}_{\rm CKM}$	−0.00176(56)	−0.00173(55)	−0.00162(56)	−0.00185(56)	−0.00171(55)	−0.00151(56)	−0.00195(56)
	−3.1σ	−3.1σ	−2.9σ	−3.3σ	−3.1σ	−2.7σ	−3.5σ

The $R_{K(*)}$ anomalies

Deviation used to be about 4σ







$$R_{K,K^*}(q_a^2,q_b^2) = \frac{\displaystyle\int_{q_a^2}^{q_b^2} \frac{d\Gamma(B^{(+,0)}\to K^{(+,*0)}\mu^+\mu^-)}{dq^2}dq^2}{\displaystyle\int_{q_a^2}^{q_b^2} \frac{d\Gamma(B^{(+,0)}\to K^{(+,*0)}e^+e^-)}{dq^2}dq^2}$$

$$\text{low-}q^2 \begin{cases} R_K = 0.994\ ^{+0.090}_{-0.082}\,(\text{stat})\ ^{+0.029}_{-0.027}\,(\text{syst}), \\ R_{K^*} = 0.927\ ^{+0.093}_{-0.087}\,(\text{stat})\ ^{+0.036}_{-0.035}\,(\text{syst}), \end{cases}$$

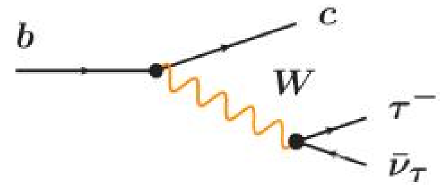
$$\text{central-}q^2 \begin{cases} R_K = 0.949\ ^{+0.042}_{-0.041}\,(\text{stat})\ ^{+0.022}_{-0.022}\,(\text{syst}), \\ R_{K^*} = 1.027\ ^{+0.072}_{-0.068}\,(\text{stat})\ ^{+0.027}_{-0.026}\,(\text{syst}), \end{cases}$$

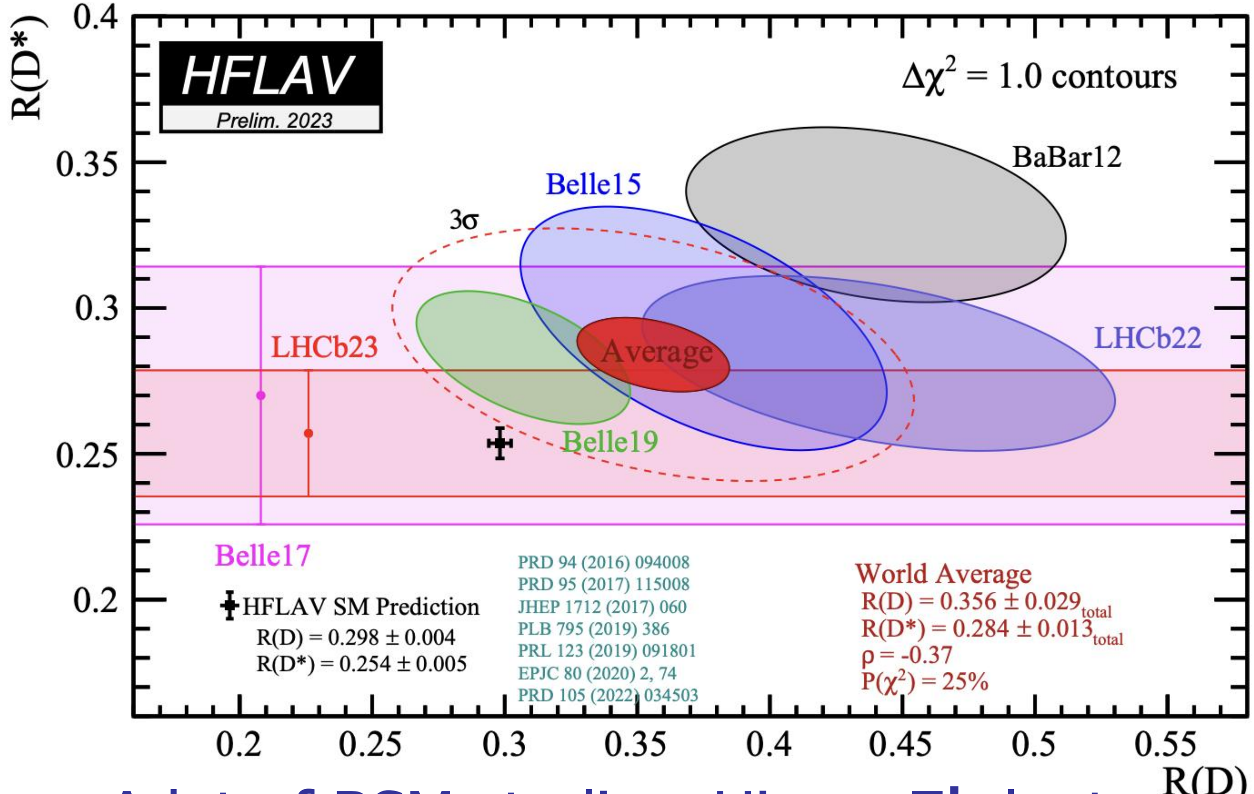
LHCb last year Christmas gift, e-Print: 2212.09153 [hep-ex] Now 1σ

	R_K low-q^2	R_K central-q^2	R_{K^*} low-q^2	R_{K^*} central-q^2
SM prediction	0.9936	1.0007	0.9832	0.9964
SM uncertainty	0.0003	0.0003	0.0014	0.0006

The $R_{D(*)}$ anomalies lowered to 3σ

$$R(D^{(*)}) = \frac{Br(D^{(*)} \to \tau\bar{\nu}_\tau)}{Br(D^{(*)} \to l\bar{\nu})}$$

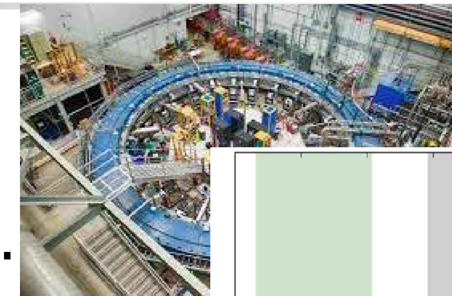




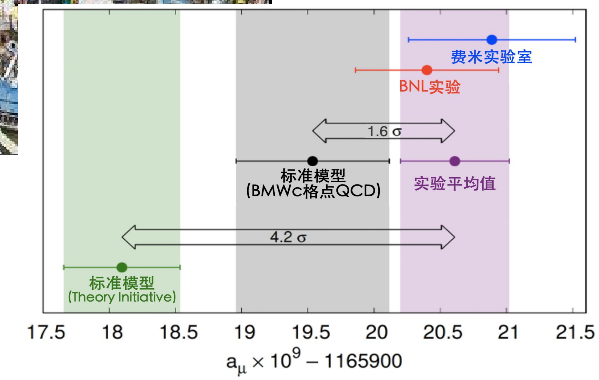
A lot of BSM studies, Higgs. Z’, leptoquark...

Muon a_μ has also been measured to high precision.

BNL experiment (1997 – 2001) final result
for $\Delta a_\mu = a_\mu(exp) - a_\mu(SM)$ at 2.7σ larger than zero.



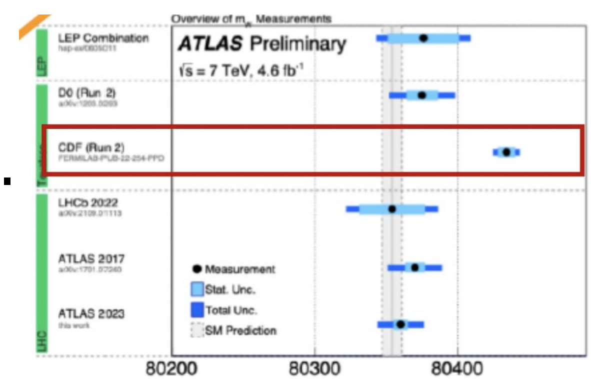
FNL experiment first result announce in April, 2021, confirm BNL result but with a high confidence level at 3.3σ.



Combining BNL and FNL results, Δa_μ =251(59)x10^{-11} .
The deviation away from SM is at 4.2σ level!

W mass anomaly

Recent Lattice calculation indicate the deviation is only at one σ level. More accurate theory calculations and Experimental measurement needed to confirm this anomaly.



m_W = 80360±5(stat.) ±15(syst.) = 80360 ±16 MeV

A lot of efforts have been made to explain this anomaly
Z', leptoquark, higgs…. New data to be released soon by Fermilab!?

CP violation anomaly in $\tau^- \to K_S^0 \pi^- \nu_\tau$

SM prediction is as follow due to neutrak Kaon mixing

$$A_Q = \frac{B(\tau^+ \to K_S^0 \pi^+ \bar{\nu}_\tau) - B(\tau^- \to K_S^0 \pi^- \nu_\tau)}{B(\tau^+ \to K_S^0 \pi^+ \bar{\nu}_\tau) + B(\tau^- \to K_S^0 \pi^- \nu_\tau)} = (+0.36 \pm 0.01)\%$$

Experimental measurement differnt by a sign!

$$A_Q = (-0.36 \pm 0.23 \pm 0.11)\%$$

Difficult to produce such a large CP violation even with new physics BSM

Need careful experimental checking!

What Anomalies tell us?

anomaly – Cambridge Dictionary

noun [C or U] · UK /əˈnɒm.ə.li/ US /əˈnɑː.mə.li/ FORMAL

a person or thing that is different from what is usual, or not in agreement with something else and therefore not satisfactory:

Statistical anomalies can make it difficult to compare economic data from one year to the next.

The anomaly of the social security system is that you sometimes have more money without a job.

Unitarity, B decays and muon g-2 that are different from SM predictions and therefore not satisfactory.

These anomalies might be some hints of something more that just SM.

Will these anomalies stand with time??? More Data!!!

Flavor violation in leptonic processes

M. Blennow et al., arXiv:2306.010040

Observable	Experimental bound
$\mu \to e\gamma$	$4.2 \cdot 10^{-13}$ [98]
$\tau \to e\gamma$	$3.3 \cdot 10^{-8}$ [99]
$\tau \to \mu\gamma$	$4.2 \cdot 10^{-8}$ [100]
$\mu \to eee$	$1.0 \cdot 10^{-12}$ [101]
$\tau \to eee$	$2.7 \cdot 10^{-8}$ [102]
$\tau \to \mu\mu\mu$	$2.1 \cdot 10^{-8}$ [102]
$\mu \to e$ (Ti)	$4.3 \cdot 10^{-12}$ [103]
$\mu \to e$ (Au)	$7.0 \cdot 10^{-13}$ [104]

3N-SS	Normal Ordering		Inverted Ordering	
	68%CL	95%CL	68%CL	95%CL
$\eta_{ee} = \frac{\lvert\theta_e\rvert^2}{2}$	$[0.28, 0.99] \cdot 10^{-3}$	$1.3 \cdot 10^{-3}$	$[0.31, 1.0] \cdot 10^{-3}$	$1.4 \cdot 10^{-3}$
$\eta_{\mu\mu} = \frac{\lvert\theta_\mu\rvert^2}{2}$	$1.3 \cdot 10^{-7}$	$1.1 \cdot 10^{-5}$	$1.2 \cdot 10^{-7}$	$1.0 \cdot 10^{-5}$
$\eta_{\tau\tau} = \frac{\lvert\theta_\tau\rvert^2}{2}$	$[0.3, 3.9] \cdot 10^{-4}$	$1.0 \cdot 10^{-3}$	$1.7 \cdot 10^{-4}$	$8.1 \cdot 10^{-4}$
$\mathrm{Tr}\,[\eta] = \frac{\lvert\theta\rvert^2}{2}$	$[0.35, 1.3] \cdot 10^{-3}$	$1.9 \cdot 10^{-3}$	$[0.33, 1.0] \cdot 10^{-3}$	$1.5 \cdot 10^{-3}$
$\lvert\eta_{e\mu}\rvert = \frac{\lvert\theta_e\theta_\mu^*\rvert}{2}$	$8.5 \cdot 10^{-6}$	$1.2 \cdot 10^{-5}$	$8.5 \cdot 10^{-6}$	$1.2 \cdot 10^{-5}$
$\lvert\eta_{e\tau}\rvert = \frac{\lvert\theta_e\theta_\tau^*\rvert}{2}$	$[1.3, 5.1] \cdot 10^{-4}$	$9.0 \cdot 10^{-4}$	$3.3 \cdot 10^{-4}$	$8.0 \cdot 10^{-4}$
$\lvert\eta_{\mu\tau}\rvert = \frac{\lvert\theta_\mu\theta_\tau^*\rvert}{2}$	$5.0 \cdot 10^{-6}$	$5.7 \cdot 10^{-5}$	$3.8 \cdot 10^{-6}$	$1.8 \cdot 10^{-5}$

$$\theta_\alpha = \frac{\theta}{\sqrt{2}}\left(\sqrt{1+\rho}\,U_{\alpha3}^* + \sqrt{1-\rho}\,U_{\alpha2}^*\right) \quad \text{for Normal Ordering (NO),}$$

$$\theta_\alpha = \frac{\theta}{\sqrt{2}}\left(\sqrt{1+\rho}\,U_{\alpha2}^* + \sqrt{1-\rho}\,U_{\alpha1}^*\right) \quad \text{for Inverted Ordering (IO),}$$

$$\rho = \frac{\sqrt{\Delta m_{31}^2} - \sqrt{\Delta m_{21}^2}}{\sqrt{\Delta m_{31}^2} + \sqrt{\Delta m_{21}^2}} \quad \text{for NO,}$$

$$\rho = \frac{\sqrt{\Delta m_{23}^2} - \sqrt{\Delta m_{23}^2 - \Delta m_{21}^2}}{\sqrt{\Delta m_{23}^2} + \sqrt{\Delta m_{23}^2 - \Delta m_{21}^2}} \quad \text{for IO,}$$

No flavor violation observed invovle charged leptons, nor CP violation.

3. The need of going beyond SM

The SM is a beautiful and successful model to describe strong and electroweak interactions. But how good is it and is there indications that is may not be the complete theory addressing all problems facing particle physics?

Yes, there are many hints. Some of the prominent phenomenological ones are:

The neutrino mass problem. Neutrino oscillations observed requires some of the neutrinos (at least two of them) to have non-zero masses. To give a mass to a fermion in the SM, one needs to pair up a left and right handed partners, example up, down quarks and charged leptons

$$- \quad \bar{Q}_L Y_u \tilde{H} u_R + \bar{Q}_L Y_d H d_R + \bar{L}_L Y_e H e_R + H.C.$$

In the minimal SM, there is not right handed neutrinos in model available, therefore need to introduce them.

Need to introduces ν_R in the model. Then one has $-\bar{L}_L Y_\nu \tilde{H} \nu_R \to \bar{\nu}_L (Y_\nu v/\sqrt{2}) \nu_R$

Then $m_\nu = Y^\nu\, v/\mathrm{sqrt}[2]$! Problem: $m_\nu/m_e = Y_\nu/Y_e < 10^{-6}$

Why such a small number?

Seesaw models

Type I seesaw model: ν_R (1, 1)(0) neutrinos, $-\bar{L}_L Y_\nu \tilde{H} \nu_R - (1/2) m_R \bar{\nu}_R^c \nu_R$, $\quad m_\nu = (Y_\nu v)^2 / 2 m_R$

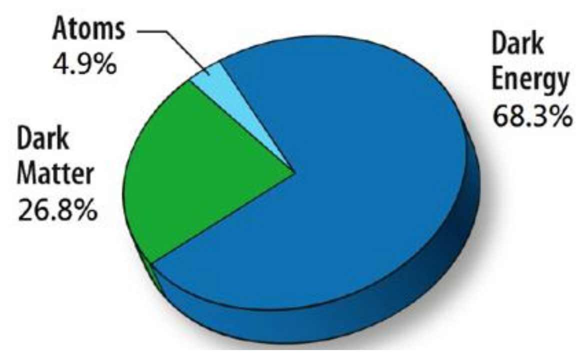
Type II seesaw model: $\chi(1,3)(-1)$ small vev v_χ, $-L_L Y_\nu \chi L_L^c \to -\nu_L (Y^\nu v_\chi / \sqrt{2}) \nu_L^c$

Type III seesaw model: N_R (1,3)(0),- $\bar{L}_L Y_\nu \tilde{H} N_R - (1/2) m_R \bar{N}_R^c N_R$, $\quad m_\nu = (Y_\nu v)^2 / 2 m_R$

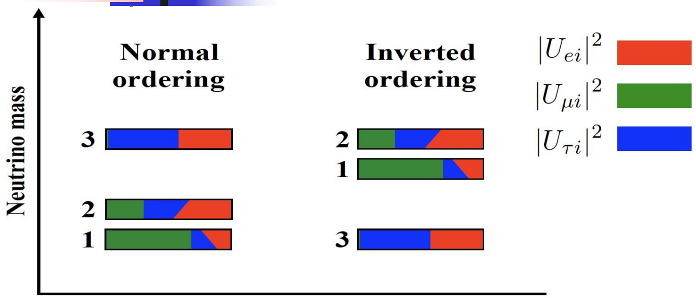
And models of generating neutrino masses at loop levels.

If only confined to leptons, flavor physics and CP violation will be affected in the lepton sector.

Cosmological evidences: Dark matter, Dark energy and matter-antimatter asymmetry



Neutrino mass hierarchy and CPV

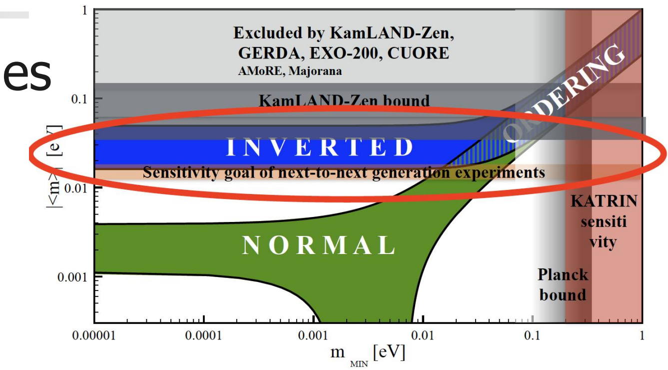


Fractional flavour content of massive neutrinos

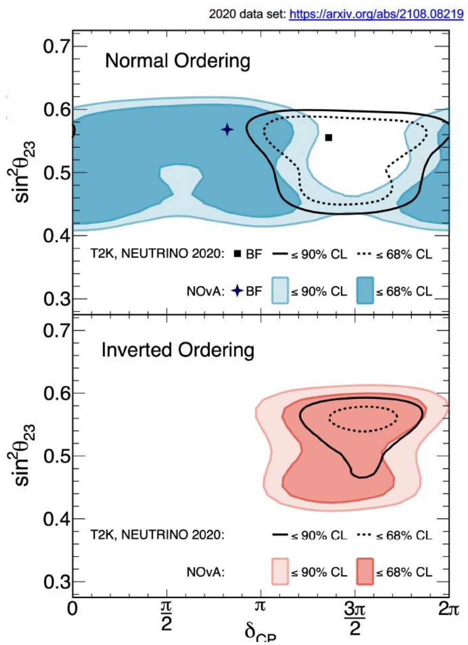
Do not know the absolute values of neutrino masses

Do not even know the mass hierarchy, Normal or Inverted

JUNO, DUNE, HyperK...

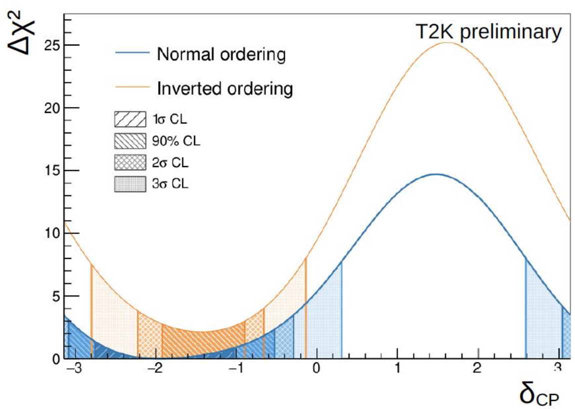


Whether neutrinos are Dirac or Majorana particles



NOvA

T2K



CP violation

Normal hiearchy seems to incline to have a phase δ = π/2

Inverted hierarchy to have a phase δ = -π/2 (NOvA) δ = π/2(T2K)

Model building for $\theta_{23} = \pi/4$ and $\delta=+(-)\pi/2$

X-G He, Chin. J. Phys. 53(2015) 100101 E Ma, PRD92(2015)051301; G-N Li, X-G He, PLB750(2015)620

In the charged lepton mass eigenstate basis,

$$m_\nu = V_{PMNS}\hat{m}_\nu V^T_{PMNS}$$

$$m_\nu = \begin{pmatrix} A & C & C^* \\ C & D^* & B \\ C^* & B & D \end{pmatrix}$$

$$\delta = -\pi/2 \text{ and } \theta_{23} = \pi/4,$$

$\mu - \tau$ conjugate symmetry

For δ=+π/2, C <-> C* and D< -> D*

W. Grimus and L. Lavoura, PLB579(2004)113
Z.-z Xing and Y. L. Zhou, PLB693(2010)584.

A A4 model to achieve this:

under A4

$\nu_R = (\nu_R^1,\ \nu_R^2,\ \nu_R^3)$ $l_L = (l_L^1,\ l_L^2,\ l_L^3)$, (l_R^1, l_R^2, l_R^3), 3 , $(1, 1'', 1')$ and 3

$\Phi = (\Phi_1,\ \Phi_2,\ \Phi_3)$ (SM doublet), ϕ (SM doublet) $\chi = (\chi_1,\ \chi_2,\ \chi_3)$ (SM singlet)

Φ and χ both transform as 3, and ϕ as 1

$$< \Phi_{1,2,3} >= v_\Phi,\ < \chi_{1,3} >= 0,\ < \chi_2 >= v_\chi, \text{ and } < \phi >= v_\phi,$$

Neutrino Chiral Oscillation

$$(i\partial\!\!\!/ - m)\psi = 0\,, \quad i\partial\!\!\!/\psi_L - m\psi_R = 0\,, \quad i\partial\!\!\!/\psi_R - m\psi_L = 0\,, \qquad \psi(t,\mathbf{x}) = U(t)\psi(0)e^{i\mathbf{p}\cdot\mathbf{x}}$$

$$\psi_L = \tfrac{1-\gamma_5}{2}\psi\,, \quad \psi_R = \tfrac{1+\gamma_5}{2}\psi\,, \quad \psi = \psi_L + \psi_R\,. \qquad U = e^{-iHt}\,, \quad H = \gamma^0\boldsymbol{\gamma}\cdot\mathbf{p} + m\gamma^0 = \boldsymbol{\alpha}\cdot\mathbf{p} + m\beta$$

How lept-handed and right-handed are entangeled in in free space?

In chiral representation:

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix},\ \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix},\ \gamma^5 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}$$

$$\psi^h(t,\mathbf{x}) = U(t)\psi^h(0)e^{i\mathbf{p}\cdot\mathbf{x}} = \psi^h(0)e^{-i(Et-\mathbf{p}\cdot\mathbf{x})}\,, \quad \psi^h(0) = \frac{1}{\sqrt{2E}}\begin{pmatrix} \sqrt{E - h\cdot p}\; u^h \\ \sqrt{E + h\cdot p}\; u^h \end{pmatrix}$$

$\psi(0)^\dagger\psi(0) = 1.$ $h = \pm 1$ - helicity, $\mathbf{p}\cdot\boldsymbol{\sigma}u^h = (h\cdot p)u^h$, $\mathbf{p} = (p_x, p_y, p_z) = p(\cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta)$.

$$u^{h=+1} = \begin{pmatrix} \cos(\theta/2) \\ \sin(\theta/2)e^{i\phi} \end{pmatrix}, \quad u^{h=-1} = \begin{pmatrix} -\sin(\theta/2)e^{-i\phi} \\ \cos(\theta/2) \end{pmatrix},$$

Oscillation probability from i to k for dirac neutrinos:

$$P(\psi_i \to \psi_k) = |<\psi(0)_k|\psi(t)_i>|^2 = |\sum V_{ij}V^*_{kj}e^{-i(E_jt-\mathbf{p}\cdot\mathbf{x})}|^2$$

In the SM neutrinos are produced by W and/or Z interactions.

At production $t=0$ point, they are left-handed and normalized, $\psi_L^h(0) = \sqrt{\frac{2E}{E-h\cdot p}} \frac{1-\gamma_5}{2}\psi^h(0)$.

$$\psi_L^h(t) = \sqrt{\frac{2E}{E-h\cdot p}} e^{-iHt}\frac{1-\gamma^5}{2}\psi^h(0) = \psi_L^h(t) = \sqrt{\frac{2E}{E-h\cdot p}}\left(e^{-iEt}\frac{1-\gamma_5}{2}\psi^h(0) - i\frac{m}{E}\sin(Et)\left[\beta, \frac{1-\gamma_5}{2}\right]\psi^h(0)\right).$$

Used
$$U(t) = e^{-iHt} = \cos(Et) - i\frac{\boldsymbol{\alpha}\cdot\mathbf{p} + m\beta}{E}\sin(Et)$$

$$\psi_L^{h\dagger}\psi_L^h(t) = \left(\cos(Et) + i\frac{h\cdot p}{E}\sin(Et)\right)e^{i\mathbf{p}\cdot\mathbf{L}}\ , \quad \psi_R^{h\dagger}\psi_L^h(t) = \left(-i\frac{m}{E}\sin(Et)\right)e^{i\mathbf{p}\cdot\mathbf{L}}$$

$$P(\nu_L^h \to \nu_L^h) = |\psi_L^{h\dagger}\psi_L^h(t)|^2 = 1 - \frac{m^2}{E^2}\sin^2(Et)\ , \quad P(\nu_L^h \to \nu_R^h) = |\psi_R^{h\dagger}\psi_L^h(t)|^2 = \frac{m^2}{E^2}\sin^2(Et)$$

Left-handed neutrino oscillated into right-handed one!

$$P(\nu_{Li}^h \to \nu_{Lk}^h) = |V_{ij}V_{kj}^*(\cos(E_jt) + i\frac{h\cdot p}{E_j}\sin(E_jt))|^2\ , \quad P(\nu_{Li}^h \to \nu_{Rk}^h) = |-iV_{ij}V_{kj}^*\frac{m_j}{E_j}\sin(E_jt)|^2\psi_R^{h\dagger}\psi_L^h(t)|^2$$

S-F Ge & P Pasquini, PLB811(2020)135961; V Bittencourt, A. Bernardini & M. Blasone, EPJC81 (2021)411.

Seesaw Neutrino in Matter

The general seesaw neutrino Lagrangian in matter propagation

$$\mathcal{L} = \bar{\nu}_L i\partial\!\!\!/ \nu_L + \bar{N}_R i\partial\!\!\!/ N_R - \frac{1}{2}\left(\begin{pmatrix} \bar{\nu}_L^c & \bar{N}_R \end{pmatrix} \begin{pmatrix} M_L & M_D^T \\ M_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ N_R^c \end{pmatrix} + \text{h.c.} \right) - \begin{pmatrix} \bar{\nu}_L & \bar{N}_R^c \end{pmatrix} \begin{pmatrix} j_L^\mu & j_{RL}^\mu \\ j_{RL}^{\mu\dagger} & j_R \end{pmatrix} \gamma_\mu \begin{pmatrix} \nu_L \\ N_R^c \end{pmatrix}$$

$$= \bar{\psi}_L i\partial\!\!\!/ \psi_L - \frac{1}{2}\left(\bar{\psi}_L^c \mathcal{M} \psi_L + \text{h.c.} \right) - \bar{\psi}_L J^\mu \gamma_\mu \psi_L$$

For homogeneous, isotropic, unpolarized electrical neutrality matter medium at rest, only j_L^0 is non-zero

$$j_L^0 = \begin{pmatrix} \rho_e & 0 & 0 \\ 0 & \rho_\mu & 0 \\ 0 & 0 & \rho_\tau \end{pmatrix} = \begin{pmatrix} \sqrt{2}G_F\left(N_e - \frac{1}{2}N_n\right) & 0 & 0 \\ 0 & -\frac{G_F}{\sqrt{2}}N_n & 0 \\ 0 & 0 & -\frac{G_F}{\sqrt{2}}N_n \end{pmatrix} .$$

In terms of the mass eigenstate $\psi_L = V\psi_L^m$, the Lagrangian is

$$\mathcal{L} = \frac{1}{2}\left(\bar{\psi}^m (i\partial\!\!\!/ - \widehat{M})\psi^m \right) - \bar{\psi}^m \widetilde{J}^\mu \gamma_\mu \frac{1-\gamma_5}{2}\psi^m \,, \quad \psi^m = \psi_L^m + (\psi_L^m)^c \,, \quad \widetilde{J}^\mu = V^\dagger J^\mu V.$$

$$(i\partial\!\!\!/ - \widehat{M})\psi^m - \widetilde{J}^\mu \gamma_\mu \frac{1-\gamma_5}{2}\psi^m + (\widetilde{J}^\mu)^* \gamma_\mu \frac{1+\gamma_5}{2}\psi^m = 0 \,.$$

$$H = \begin{pmatrix} \boldsymbol{\alpha}\cdot\mathbf{p} & 0 \\ 0 & \boldsymbol{\alpha}\cdot\mathbf{p} \end{pmatrix} + \begin{pmatrix} \beta\widehat{M}_l & 0 \\ 0 & \beta\widehat{M}_h \end{pmatrix} + V^\dagger \begin{pmatrix} j_L^\mu & j_{RL}^{\mu\dagger} \\ j_{RL}^\mu & -j_R^{\mu T} \end{pmatrix} V\gamma^0\gamma_\mu \frac{1-\gamma^5}{2} - \left(V^\dagger \begin{pmatrix} j_L^\mu & j_{RL}^{\mu\dagger} \\ j_{RL}^\mu & -j_R^{\mu T} \end{pmatrix} V \right)^* \gamma^0\gamma_\mu \frac{1+\gamma^5}{2}$$

Because the off-diagonal interaction, difficulty to get U(t). For just one light and one heavy neutrinos, can get a closed analytic expression.

Let $\widehat{M}_l = m$ and $\widehat{M}_h = M$, similar to Dirac case $j^\mu = (\rho, \vec{0})$ and $j_{RL}^\mu = j_R^\mu = 0$,

$$\widetilde{J}^\mu \gamma_\mu = V^\dagger J^\mu V \gamma_\mu = \begin{pmatrix} \frac{\rho}{2}(1+\cos 2\theta) & \frac{\rho}{2} e^{i\eta} \sin 2\theta \\ \frac{\rho}{2} e^{-i\eta} \sin 2\theta & \frac{\rho}{2}(1-\cos 2\theta) \end{pmatrix} \gamma_0 \,.$$

$$H = \begin{pmatrix} \frac{\rho}{2}(1+\cos 2\theta) - \mathbf{p}\cdot\boldsymbol{\sigma} & m & \frac{\rho}{2} e^{i\eta}\sin 2\theta & 0 \\ m & \mathbf{p}\cdot\boldsymbol{\sigma} - \frac{\rho}{2}(1+\cos 2\theta) & 0 & -\frac{\rho}{2} e^{-i\eta}\sin 2\theta \\ \frac{\rho}{2} e^{-i\eta}\sin 2\theta & 0 & \frac{\rho}{2}(1-\cos 2\theta) - \mathbf{p}\cdot\boldsymbol{\sigma} & M \\ 0 & -\frac{\rho}{2} e^{i\eta}\sin 2\theta & M & \mathbf{p}\cdot\boldsymbol{\sigma} - \frac{\rho}{2}(1-\cos 2\theta) \end{pmatrix}$$

For more details, see poster by Ming-Wei Li

4. More CP violating observables

CP violation with polarization measurement

a spin-1/2 -> spin-0 + spin-1/2

$$\mathcal{A} = \bar{\mathcal{F}}(A_v + iA_c\gamma_5)\mathcal{B} = \mathcal{S} + \mathcal{P}\sigma \cdot \vec{p}_c \qquad |\vec{p}_c| = \sqrt{E_{\mathcal{F}}^2 - m_{\mathcal{F}}^2}$$

$$\mathcal{S} = A_v\sqrt{\frac{(m_{\mathcal{B}} + m_{\mathcal{F}})^2 - m_{\mathcal{M}}^2}{16\pi m_{\mathcal{B}}^2}}\,, \quad \mathcal{P} = A_c\sqrt{\frac{(m_{\mathcal{B}} - m_{\mathcal{F}})^2 - m_{\mathcal{M}}^2}{16\pi m_{\mathcal{B}}^2}}\,.$$

$$\bar{\mathcal{A}} = -\bar{\mathcal{S}} + \bar{\mathcal{P}}\sigma \cdot \vec{p}_c\,.$$

$$\frac{4\pi}{\Gamma}\frac{d\Gamma}{d\Omega} = 1 + \alpha\vec{s}_{\mathcal{B}}\cdot\vec{n} + \vec{s}_{\mathcal{F}}\cdot[(\alpha + \vec{s}_{\mathcal{B}}\cdot\vec{n})\vec{n} + \beta\vec{s}_{\mathcal{B}}\times\vec{n} + \gamma(\vec{n}\times(\vec{s}_{\mathcal{B}}\times\vec{n}))]$$

$$\Delta = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}\,, \quad A_\alpha = \frac{\Gamma\alpha + \bar{\Gamma}\bar{\alpha}}{\Gamma\alpha - \bar{\Gamma}\bar{\alpha}}\,, \quad B_\beta = \frac{\Gamma\beta + \bar{\Gamma}\bar{\beta}}{\Gamma\beta - \bar{\Gamma}\bar{\beta}}\,.$$

$$\vec{n} = \vec{p_c}/|p_c|$$

$$\beta = (1 - \alpha^2)^{1/2} sin\phi$$

CP violation in Hyperons

	Δ	A	B
$\Lambda^0 \to p\pi^-$	-5.4×10^{-7}	-0.5×10^{-4}	3.0×10^{-3}
$\Xi^- \to \Lambda^0\pi^-$	0	-0.7×10^{-4}	8.4×10^{-4}
$\Sigma^- \to n\pi^-$	0	1.6×10^{-4}	-1.2×10^{-2}
$\Sigma^+ \to p\pi^0$	-6.2×10^{-7}	-3.2×10^{-7}	-4.2×10^{-4}
$\Sigma^+ \to n\pi^+$	6.0×10^{-7}	-1.6×10^{-4}	-8.4×10^{-7}

Signals of {CP} Nonconservation in Hyperon Decay
John F. Donoghue (Massachusetts U., Amherst), Sandip Pakvasa (Hawaii U.).
Published in **Phys.Rev.Lett. 55 (1985) 162**

Hyperon decays and *CP* nonconservation

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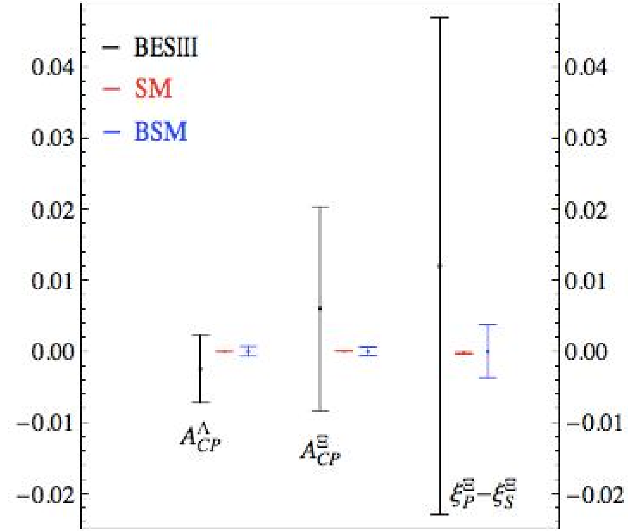
(Received 7 March 1986)

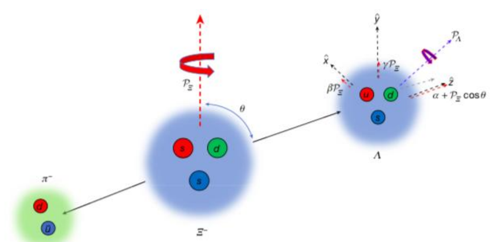
We study all modes of hyperon nonleptonic decay and consider the *CP*-odd observables which result. Explicit calculations are provided in the Kobayashi-Maskawa, Weinberg-Higgs, and left-right-symmetric models of *CP* nonconservation.



$A_{\Xi\Lambda} = A_{\Xi} + A_{\Lambda}$ HyperCP (Femilab E871): $A_{\Xi\Lambda} = [-6.0 \pm 2.1(\text{stat}) \pm 2.0(\text{syst})] \times 10^{-4}$

Recent measurement from BESIII
(Nature 606(2022)64)

$$A_{CP} = \frac{\alpha + \overline{\alpha}}{\alpha - \overline{\alpha}}, \qquad B_{CP} = \frac{\beta + \overline{\beta}}{\alpha - \overline{\alpha}}$$



$$A^{\Xi}_{CP} = (6 \pm 13 \pm 6) \times 10^{-3},$$
$$B^{\Xi}_{CP} \simeq \xi^{\Xi}_{P} - \xi^{\Xi}_{S} = (1.2 \pm 3.4 \pm 0.8) \times 10^{-2},$$
$$A^{\Lambda}_{CP} = (-4 \pm 12 \pm 9) \times 10^{-3}$$

So far not CP violation effects have been established in baryon decay!.
Similar ideas can be used for c- and b-baryon decays.

A new way of testing P and CP violation at BESIII

X-G He, J-P Ma, B. Mckellar, PRD 47(1993) 1744; X-G He and J-P Ma, PLB839(2023)137834

Testing of P and CP symmetries with $e^+e^- \to J/\psi \to \Lambda\bar{\Lambda}$

$$\mathcal{A}^\mu = \bar{u}(k_1)\left[\gamma^\mu F_V + \frac{i}{2m_\Lambda}\sigma^{\mu\nu}q_\nu H_\sigma + \gamma^\mu\gamma_5 F_A + \sigma^{\mu\nu}q_\nu\gamma_5 H_T\right]v(k_2),$$

$$A_{d_J} = < \frac{2}{3}\hat{l}_p \cdot \hat{p} > = 0.60 d_J = 1.52\times 10^{-4}\frac{d_J}{2.53\times 10^{-4}},$$

$$A_{F_A} = < \frac{3}{2}(\hat{l}_p - \hat{l}_{\bar{p}})\cdot\hat{k} > = 428 F_A = -0.46\times 10^{-3}\frac{F_A}{-1.07\times 10^{-6}},$$

$$A_{H_T} = \frac{3}{2} < (\hat{l}_p \times \hat{l}_{\bar{p}})\cdot\hat{k} > = 32.7 H_T = -0.73\times 10^{-2}\frac{d_\Lambda}{1.5\times 10^{-16}},$$

$\hat{l}_p$, $l_{\bar{p}}$ and $\hat{k}$ momentum directions of p, $\bar{p}$ and Λ.

$$d_J = \frac{3 - 8\sin^2\theta_W}{32\cos^2\theta_W \sin^2\theta_W}\frac{M^2_{J/\psi}}{M^2_Z} \qquad H_T = \frac{2e}{3m^2_{J/\psi}} g_V d_\Lambda$$

BESIII, accumulated 10×10^9 J/ψ, $Br(J/\psi \to \Lambda\bar{\Lambda}) = 1.89\times 10^{-3}$. Sensitivity $\delta A_i \sim 4.5\times 10^{-4}$!

STCF, $3.4\times 10^{12} J/\psi/year$, one year running, $\delta A_i \sim 1.2\times 10^{-5}$!

CP violation in Higgs h decays into $\tau^+\tau^-$

(Hayreter, He, Valencia, arXiv:1603.06326, arXiv:1606.00951)

(He, Ma, McKellar, Mod. Phys Lett. A9, 205(1994); Berge, Bereuther, Kirchner, PRD92,096012(2015))

General Higgs to fermion coupling: $L = -\bar{f}(r_f + i\tilde{r}_f\gamma_5)fh$

Define the density matrix R with polarization $\vec{n}_f(\vec{n}_{\bar{f}})$ for $f(\bar{f})$

$$R = N_f\beta_f[Im(r_f\tilde{r}_f^*)\hat{p}_f \cdot (\vec{n}_{\bar{f}} - \vec{n}_f) - Re(r_f\tilde{r}_f)\hat{p}_f \cdot (\vec{n}_f \times \vec{n}_{\bar{f}})]$$

N_f - normalization constant, $\hat{p}$ - three moment of f, $\beta_f = \sqrt{1 - 4m_f^2/m_h^2}$

Application to $h \to \tau^+\tau^-$

Using $\tau \to \pi^-\nu_\tau$ to measure $\vec{n}_f$, $\frac{4\pi}{\Gamma}\frac{d\Gamma}{d\Omega} = (1 + \alpha_\tau\vec{n}_\tau \cdot \hat{p}_\tau)$, $\alpha_\tau = 1$.

$$\hat{p}_\tau \cdot (\vec{n}_f \times \vec{n}_{\bar{f}}) \to \hat{p}_\tau \cdot (\hat{p}_{\pi^-} \times \hat{p}_{\pi^+})$$

One construct CP violating observable

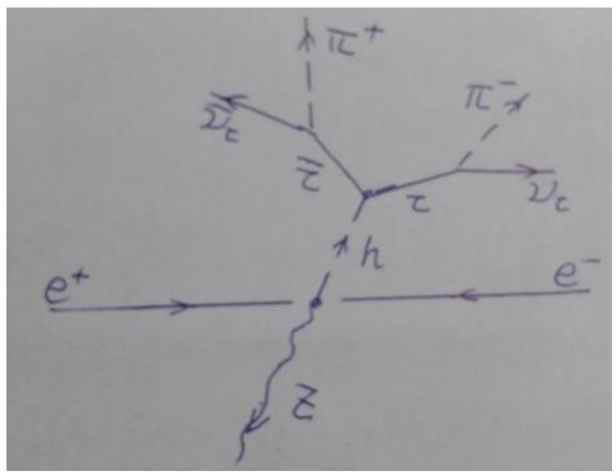
$$A_\tau = \frac{N(O_\pi > 0) - N(O_\pi < 0)}{N(O_\pi > 0) + N(O_\pi < 0)}\,, \quad O_\pi = \hat{p}_\tau \cdot (\hat{p}_{\pi^+} \times \hat{p}_{\pi^-})\,.$$

Theoretically

$$A_\tau = \frac{N(O_\pi > 0) - N(O_\pi < 0)}{N(O_\pi > 0) + N(O_\pi < 0)} = \frac{\pi}{4}\beta_\tau \alpha_\tau \alpha_{\bar{\tau}} \frac{r_\tau \tilde{r}_\tau}{\beta_\tau^2 r_\tau^2 + \tilde{r}_\tau^2}\,.$$

Data still allow A to be as large as π/8. Experiments should look such CPV.

In the SM $A_T = 0$



Br(h ->ττ) ~ 5×10^{-2},

Br(τ –> π ν) ~ 0.1

10^6 Higgs bosons, sensitivity to A_T can be 10% at CEPC.

The EDM of a fundamental particle

Classically a EDM $\vec{D} = \int d^3x \vec{x} \rho(\vec{x})$ interacts with an electric field $\vec{E}$

The interaction energy is given by $H = \vec{D} \cdot \vec{E}$, allowed by P and T symmetries.

Under P, $\vec{D} \to -\vec{D}$ and $\vec{E} \to -\vec{E}$, H conserves both P and T.

A fundamental particle, $\vec{D}$ is equal to $d\vec{S}$, $H_{edm} = d\vec{S} \cdot \vec{E}$.

Since under P, $\vec{S} \to \vec{S}$ and under T, $\vec{S} \to -\vec{S}$

H_{edm} violates both P and T, CPT is conserved, CP is also violated!

Quantum field theory, $H_{edm} = -i\frac{1}{2} d\bar{\psi}\sigma^{\mu\nu}\psi\tilde{F}_{mu\nu} = -i\frac{1}{2} d\bar{\psi}\sigma^{\mu\nu}\gamma_5\psi F_{\mu\nu}$

In non-relativestic limit H_{edm} reduce to $d\frac{\vec{\sigma}}{2} \cdot \vec{E} = d\vec{S} \cdot \vec{E}$.

One easily sees that H_{edm} violates P and T, violates CP, but conserve CPT.

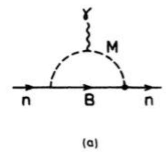
A non-zero fundamental particle EDM, violates P, T and CP!

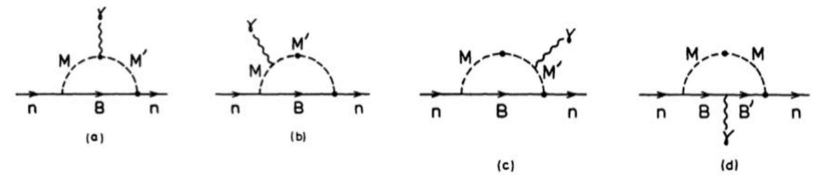
In KM model, quark EDM only generated at two electroweak and one strong loop level (3 loop effects)l, very small ~ 10^{-33} e.cm. (Shabalin, 1978, 1980)

In fact with two weak and one strong interaction vertices, EDM can also be generated!

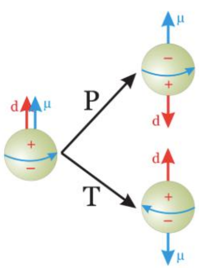
(He, McKellar and Pakvasa, PLB197, 556(1987), J. Mod. Phys. A4, 5011(1989)

$$1.6 \times 10^{-31}\ \text{e.cm} \geq |D_n| \geq 1.4 \times 10^{-33}\ \text{e.cm}$$





Electron EDM is even smaller, generated at fourth loop level, $D_e <\ 10^{-38}$ecm



Magnetic Dipole conserves P and T

$H_{mdm} = d_m \vec{S} \cdot \vec{B}$,

Under P: $\vec{B} \to \vec{B}$ and under T: $\vec{B} \to -\vec{B}$

Relativistic expression: $d_m \bar{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu}$.

$$D_n \sim -3.8 \times 10^{-16}\theta\ \text{ecm}$$

Including all SU(3) octet contributions:

$$2.5 \times 10^{-16}\theta ecm < |D_n| < 4.6 \times \theta ecm$$

Using data $|D_n| < 3 \times 10^{-27} ecm$, $|\theta| < 10^{-11}$!

Neutron $|D_n| < 1.8 \times 10^{-26}$ ecm,
electron $|D_e| < 1.1 \times 10^{-29}$ ecm

Why θ is small is the strong CP problem.

5. Theory efforts to reduce model parameters

SM has many free parameters. Possible to reduce them?

Extensions of SM usually introduce more paramters in the model!

SUSY, Multi-Higgs, New symmetries, usually, introduce more parameters

(some of them may reduce the parameter in certain sectors)...

Unification is one wayto try: Unify forces - reduce gauge couplings, Unify representation - reduce Yukawa coupling, relate masses of particles and etc...

Have more particles with higher masses scale than electroweak scale... but a progress for us looking at electroweak scale physics.

Examples: SO(10)

Gauge boson in 45 representation, Fermions in 16, Higgs fields 10 and 120, anti-126, 210...

$10 \to 5 + \bar{5}$

$16 \to 10 + \bar{5} + 1$

$45 \to 24 + 10 + \overline{10} + 1$

$54 \to 15 + \overline{15} + 24$

$120 \to 5 + \bar{5} + 10 + \overline{10} + 45 + \overline{45}$

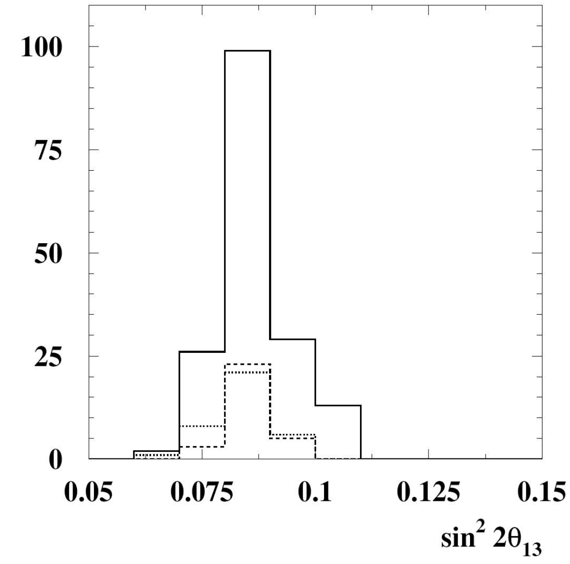
$126 \to 1 + \bar{5} + 10 + \overline{15} + 45 + \overline{50}$

$210 \to 1 + 5 + \bar{5} + 10 + \overline{10} + 24 + 40 + \overline{40} + 75$

$$16 \Rightarrow 1_F = \nu^c + \bar{\mathbf{5}}_F = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ -\nu \end{pmatrix} + \mathbf{10}_F = \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e_R \\ -d_1 & -d_2 & -d_3 & -e_R & 0 \end{pmatrix}$$

SO(10) Predictions

$$16_F(Y_{10}10_H + Y_{\overline{126}}\overline{126}_H + Y_{120}120_H)16_F$$



Minimal SO(10) Model without 120

$$\mathcal{L}_{\text{Yukawa}} = Y_{10}\,\mathbf{16}\,\mathbf{16}\,\mathbf{10}_H + Y_{126}\,\mathbf{16}\,\mathbf{16}\,\overline{\mathbf{126}}_H$$

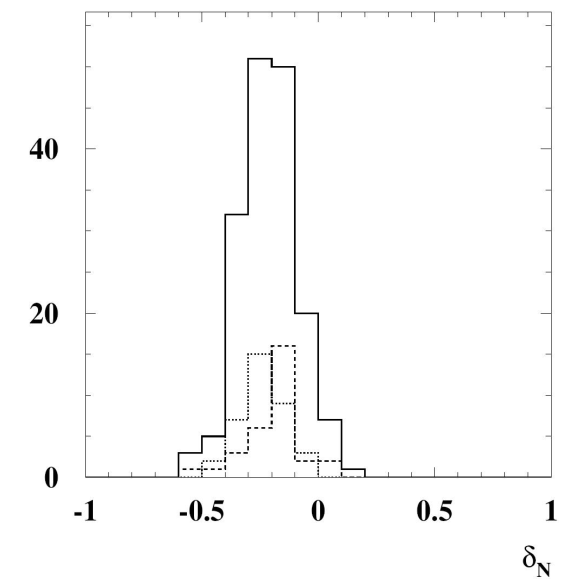
Two Yukawa matrices determine all fermion masses and mixings, including the neutrinos

$$\begin{aligned} M_u &= \kappa_u Y_{10} + \kappa'_u Y_{126} \\ M_d &= \kappa_d Y_{10} + \kappa'_d Y_{126} \\ M_\nu^D &= \kappa_u Y_{10} - 3\kappa'_u Y_{126} \\ M_l &= \kappa_d Y_{10} - 3\kappa'_d Y_{126} \end{aligned}$$

$$\begin{aligned} M_{\nu R} &= \langle \Delta_R \rangle Y_{126} \\ M_{\nu L} &= \langle \Delta_L \rangle Y_{126} \end{aligned}$$

Good prediction for θ_{13}

δ Away from $-\pi/2$!!! Tobe tested!!



Model has only 11 real parameters plus 7 phases

Babu, Mohapatra (1993)
Fukuyama, Okada (2002)
Bajc, Melfo, Senjanovic, Vissani (2004)
Fukuyama, Ilakovac, Kikuchi, Meljanac,
Okada (2004)
Aulakh et al (2004)
Bertolini, Frigerio, Malinsky (2004)
Babu, Macesanu (2005)
Bertolini, Malinsky, Schwetz (2006
Dutta, Mimura, Mohapatra (2007)
Bajc, Dorsner, Nemevsek (2009)
Jushipura, Patel (2011).

6. Conclusions

Flavor Physics is a very lively field of research with a lot of new data coming from experiments. SM is being tested to better precision, perturbative and global fitting..., Data now demands more accurate theoretical hadronic matrix element calculations.

SM is in good shape except in neutrino sector. There are some anomalies..., but the error bars are shrinking, still posing chanllenges to theoretical studies. A lot of new ideas have been proposed to to explain possible anomalies, and new experients are going on to provide data to test SM and provide hints for new physics beyond. Stay tuned!

Thank you!