The 29th International Workshop on Weak Interactions and Neutrinos

Complete One-loop Renormalization-group Equations in the Seesaw Effective Field Theories

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2023.07.08

Based on Y. Wang, D. Zhang and S. Zhou, JHEP 05 (2023) 044.

Outline

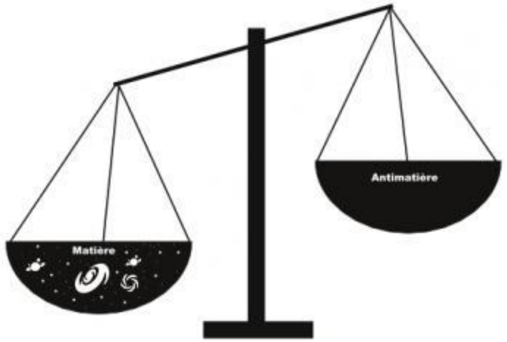
I. Background and Motivation

◆ Neutrino mass

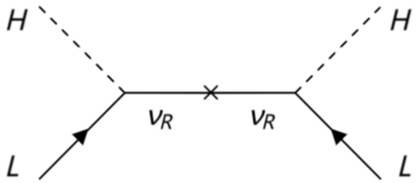
P. A. Zyla *et al*, 2020.

◆ Matter-antimatter asymmetry

M. Fukugita and T. Yanagida, 1986.



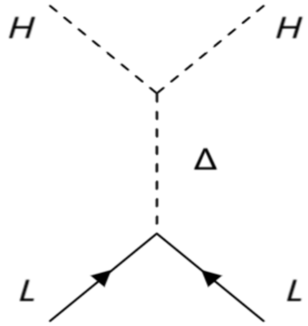
Seesaw model:



H. Fritzsch *et al.*, 75;
P. Minkowski *et al.*, 77;
T. Yanagida *et al.*, 79;
M. Gell-Mann *et al.*, 79;
S.L. Glashow *et al.*, 80;
R.N. Mohapatra *et al.*, 80.

Type-I

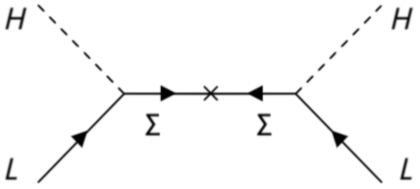
SM + **three heavy right-handed neutrinos**



W. Konetschny *et al.*, 77;
M. Magg *et al.*, 80;
J. Schechter *et al.*, 80;
T.P. Cheng *et al.*, 80;
G. Lazarides *et al.*, 81;
R.N. Mohapatra *et al.*, 81

Type-II

SM + **one heavy Higgs triplet**

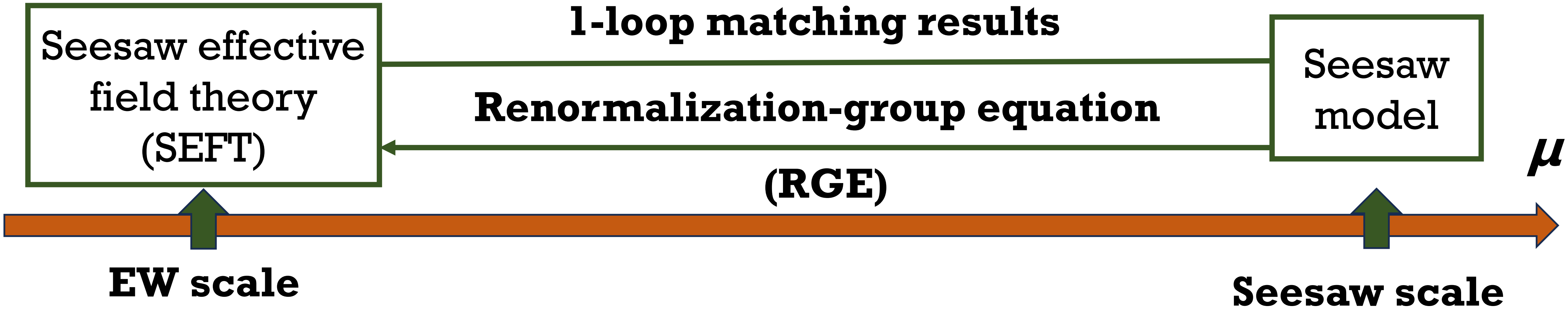


Type-III

R. Foot *et al.*, 89; E. Ma, 98.

SM + **three heavy triplet fermions**

I. How to test seesaw model?



- The information of heavy particles are contained in the Wilson coefficients.
- EFT can avoid the large logarithms in the multi-scale theory.

Why RGEs?

1-loop matching + 1-loop RGEs give complete one-loop calculations in the type-I SEFT at the electroweak scale and extract useful information about the type-I seesaw model from low-energy measurements.

II. Type-I SEFT

Lagrangian @tree-level :

dim-5 **dim-6 → unitarity violation**

A. Broncano, et al., 2003.

$$\mathcal{L}_{\rm SEFT} = \mathcal{L}_{\rm SM} + \frac{1}{2}\left(C_5^{\alpha\beta}\mathcal{O}^{(5)}_{\alpha\beta} + {\rm h.c.}\right) + C^{(1)\alpha\beta}_{H\ell}\mathcal{O}^{(1)\alpha\beta}_{H\ell} + C^{(3)\alpha\beta}_{H\ell}\mathcal{O}^{(3)\alpha\beta}_{H\ell}$$

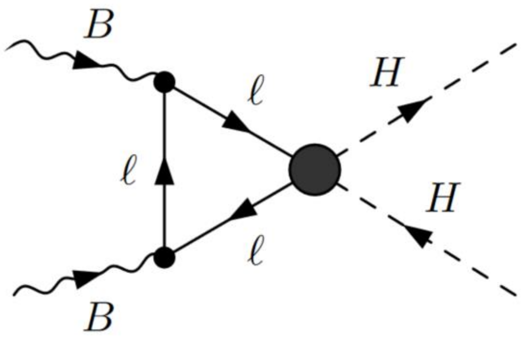
$$\mathcal{O}^{(5)}_{\alpha\beta} = \overline{\ell_{\alpha \rm L}}\widetilde{H}\widetilde{H}^{\rm T}\ell^{\rm c}_{\beta \rm L}, \quad \mathcal{O}^{(1)\alpha\beta}_{H\ell} = \left(\overline{\ell_{\alpha\rm L}}\gamma^\mu \ell_{\beta\rm L}\right)\left(H^\dagger {\rm i}\overleftrightarrow{D}_\mu H\right), \quad \mathcal{O}^{(3)\alpha\beta}_{H\ell} = \left(\overline{\ell_{\alpha\rm L}}\gamma^\mu \sigma^I \ell_{\beta\rm L}\right)\left(H^\dagger {\rm i}\overleftrightarrow{D}^I_\mu H\right)$$

Matching scale: $C_5\left(\mu_{\rm M}\right) = Y_\nu M_{\rm R}^{-1} Y_\nu^{\rm T}\,,\quad C^{(1)}_{H\ell}\left(\mu_{\rm M}\right) = -C^{(3)}_{H\ell}\left(\mu_{\rm M}\right) = \frac{1}{4} Y_\nu M_{\rm R}^{-2} Y_\nu^\dagger$

1-loop RGEs:

- ◆ single insertions of dim-6 operators

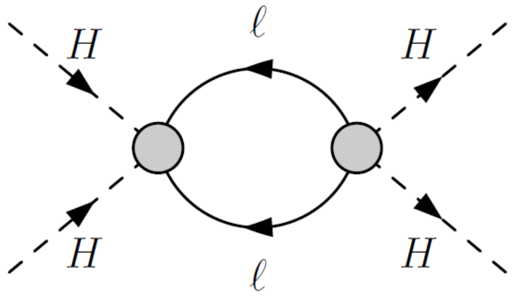
e.g.



E. E. Jenkins, et al., 2013; 2014. R. Alonso, et al., 2014.

- ◆ double insertions of dim-5 operator

e.g.

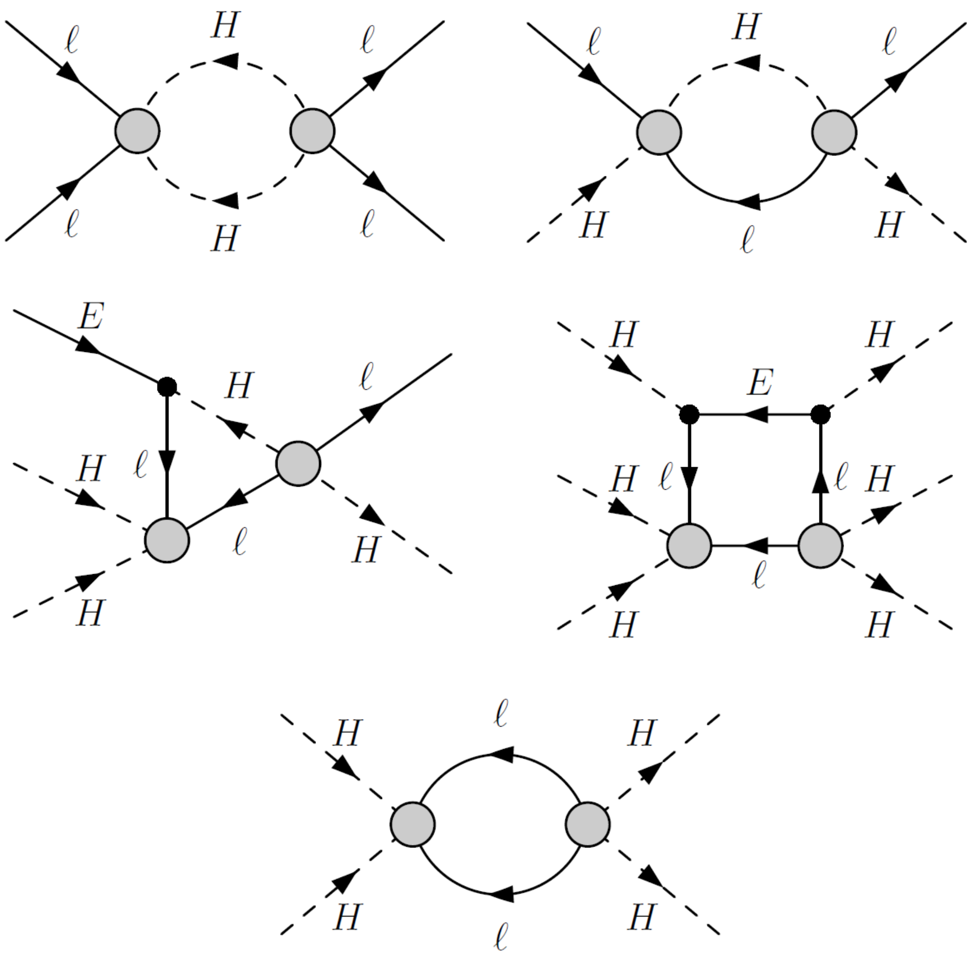


S. Davidson, et al., 2018; R. Coy and M. Frigerio, 2019.

Complete 1-loop RGEs of type-I SEFT are still lacking!

II. Strategy

- First, we choose a set of 1PI diagrams generated by the tree-level Lagrangian and covering all of dim-6 operators in the Green's basis.
- Then, one calculates these diagrams to get all the counterterms for the dim-6 operators in the Green's basis and converts them into Warsaw basis.
- Finally, we derive the RGEs from the counterterms.



II. Complete One-loop RGEs

Yukawa couplings:

$$16\pi^2\mu\frac{dY_l}{d\mu} = \left[-\frac{15}{4}g_1^2 - \frac{9}{4}g_2^2 + T + \frac{3}{2}Y_lY_l^\dagger - 2m^2\left(C_{H\ell}^{(1)} + 3C_{H\ell}^{(3)}\right)\right]Y_l ,$$

$$16\pi^2\mu\frac{dY_u}{d\mu} = \left[-\frac{17}{12}g_1^2 - \frac{9}{4}g_2^2 - 8g_s^2 + T + \frac{3}{2}\left(Y_uY_u^\dagger - Y_dY_d^\dagger\right)\right]Y_u ,$$

$$16\pi^2\mu\frac{dY_d}{d\mu} = \left[-\frac{5}{12}g_1^2 - \frac{9}{4}g_2^2 - 8g_s^2 + T - \frac{3}{2}\left(Y_uY_u^\dagger - Y_dY_d^\dagger\right)\right]Y_d ,$$

Higgs quadratic and quartic couplings:

$$T \equiv \mathrm{tr}\left(Y_lY_l^\dagger + 3Y_uY_u^\dagger + 3Y_dY_d^\dagger\right)$$

$$16\pi^2\mu\frac{dm^2}{d\mu} = \left(-\frac{3}{2}g_1^2 - \frac{9}{2}g_2^2 + 12\lambda + 2T\right)m^2 ,$$

$$16\pi^2\mu\frac{d\lambda}{d\mu} = 24\lambda^2 - 3\lambda\left(g_1^2 + 3g_2^2\right) + \frac{3}{8}\left(g_1^2 + g_2^2\right)^2 + \frac{3}{4}g_2^4 + 4\lambda T - 2\mathrm{tr}\left[\left(Y_lY_l^\dagger\right)^2 + 3\left(Y_uY_u^\dagger\right)^2 + 3\left(Y_dY_d^\dagger\right)^2\right] + m^2\mathrm{tr}\left(2C_5C_5^\dagger - \frac{8}{3}g_2^2C_{H\ell}^{(3)} + 8C_{H\ell}^{(3)}Y_lY_l^\dagger\right) .$$

Weinberg operator:

$$16\pi^2\mu\frac{dC_5}{d\mu} = \left(-3g_2^2 + 4\lambda + 2T\right)C_5 - \frac{3}{2}\left[Y_lY_l^\dagger C_5 + C_5\left(Y_lY_l^\dagger\right)^{\mathrm{T}}\right] .$$

II. Complete One-loop RGEs

- $\boldsymbol{H^6}$ **and** $\boldsymbol{H^4D^2}$

$$16\pi^2\mu\frac{\mathrm{d}C_{H\Box}}{\mathrm{d}\mu} = -2\,\mathrm{tr}\left[\boxed{\frac{1}{2}C_5C_5^\dagger} + \frac{1}{3}g_1^2C_{H\ell}^{(1)} - g_2^2C_{H\ell}^{(3)} + \left(C_{H\ell}^{(1)} + 3C_{H\ell}^{(3)}\right)Y_lY_l^\dagger\right] ,$$

$$16\pi^2\mu\frac{\mathrm{d}C_{HD}}{\mathrm{d}\mu} = -2\,\mathrm{tr}\left(\boxed{C_5C_5^\dagger} + \frac{4}{3}g_1^2C_{H\ell}^{(1)} + 4C_{H\ell}^{(1)}Y_lY_l^\dagger\right) ,$$

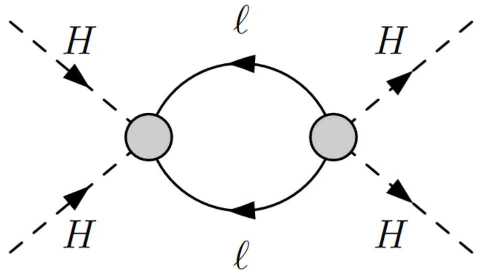
$$16\pi^2\mu\frac{\mathrm{d}C_{H}}{\mathrm{d}\mu} = 4\,\mathrm{tr}\left(\boxed{-\lambda C_5C_5^\dagger} + \frac{4}{3}\lambda g_2^2C_{H\ell}^{(3)} - 4\lambda C_{H\ell}^{(3)}Y_lY_l^\dagger\right) .$$

- $\boldsymbol{\psi^2H^3}$

$$16\pi^2\mu\frac{\mathrm{d}C_{eH}}{\mathrm{d}\mu} = 2\left[\frac{3}{4}C_5C_5^\dagger Y_l + \mathrm{tr}\left(\boxed{-\frac{1}{2}C_5C_5^\dagger} + \frac{2}{3}g_2^2C_{H\ell}^{(3)} - 2C_{H\ell}^{(3)}Y_lY_l^\dagger\right)Y_l + C_{H\ell}^{(1)}Y_lY_l^\dagger Y_l \right.$$
$$\left. + \left(2\lambda - 3g_1^2\right)C_{H\ell}^{(1)}Y_l + 3\left(2\lambda - g_1^2\right)C_{H\ell}^{(3)}Y_l\right] ,$$

$$16\pi^2\mu\frac{\mathrm{d}C_{uH}}{\mathrm{d}\mu} = \mathrm{tr}\left(\boxed{-C_5C_5^\dagger} + \frac{4}{3}g_2^2C_{H\ell}^{(3)} - 4C_{H\ell}^{(3)}Y_lY_l^\dagger\right)Y_\mathrm{u} ,$$

$$16\pi^2\mu\frac{\mathrm{d}C_{dH}}{\mathrm{d}\mu} = \mathrm{tr}\left(\boxed{-C_5C_5^\dagger} + \frac{4}{3}g_2^2C_{H\ell}^{(3)} - 4C_{H\ell}^{(3)}Y_lY_l^\dagger\right)Y_\mathrm{d} .$$



II. Complete One-loop RGEs

In the type-I SEFT, 19 dim-6 operators can be generated via the RGEs at the one-loop level.

H^6 and H^4D^2		$\psi^2 H^3$		$(\overline{\rm L}{\rm L})(\overline{\rm L}{\rm L})$	
$\mathcal{O}_H$	$(H^\dagger H)^3$	$\mathcal{O}_{eH}^{\alpha\beta}$	$(\overline{\ell_{\alpha\rm L}} H E_{\beta\rm R})(H^\dagger H)$	$\mathcal{O}_{\ell\ell}^{\alpha\beta\gamma\lambda}$	$(\overline{\ell_{\alpha\rm L}}\gamma^\mu \ell_{\beta\rm L})(\overline{\ell_{\gamma\rm L}}\gamma_\mu \ell_{\lambda\rm L})$
$\mathcal{O}_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$	$\mathcal{O}_{uH}^{\alpha\beta}$	$\left(\overline{Q_{\alpha\rm L}}\widetilde{H} U_{\beta\rm R}\right)(H^\dagger H)$	$\mathcal{O}_{\ell q}^{(1)\alpha\beta\gamma\lambda}$	$(\overline{\ell_{\alpha\rm L}}\gamma^\mu \ell_{\beta\rm L})(\overline{Q_{\gamma\rm L}}\gamma_\mu Q_{\lambda\rm L})$
$\mathcal{O}_{HD}$	$(H^\dagger D^\mu H)^*(H^\dagger D_\mu H)$	$\mathcal{O}_{dH}^{\alpha\beta}$	$(\overline{Q_{\alpha\rm L}} H D_{\beta\rm R})(H^\dagger H)$	$\mathcal{O}_{\ell q}^{(3)\alpha\beta\gamma\lambda}$	$(\overline{\ell_{\alpha\rm L}}\gamma^\mu \sigma^I \ell_{\beta\rm L})(\overline{Q_{\gamma\rm L}}\gamma_\mu \sigma^I Q_{\lambda\rm L})$
$\psi^2 H^2 D$				$(\overline{\rm L}{\rm L})(\overline{\rm R}{\rm R})$	
$\mathcal{O}_{H\ell}^{(1)\alpha\beta}$	$(\overline{\ell_{\alpha\rm L}}\gamma^\mu \ell_{\beta\rm L})\left(H^\dagger {\rm i}\overleftrightarrow{D}_\mu H\right)$	$\mathcal{O}_{Hq}^{(3)\alpha\beta}$	$(\overline{Q_{\alpha\rm L}}\gamma^\mu \sigma^I Q_{\beta\rm L})\left(H^\dagger {\rm i}\overleftrightarrow{D}^I_\mu H\right)$	$\mathcal{O}_{\ell e}^{\alpha\beta\gamma\lambda}$	$(\overline{\ell_{\alpha\rm L}}\gamma^\mu \ell_{\beta\rm L})(\overline{E_{\gamma\rm R}}\gamma_\mu E_{\lambda\rm R})$
$\mathcal{O}_{H\ell}^{(3)\alpha\beta}$	$(\overline{\ell_{\alpha\rm L}}\gamma^\mu \sigma^I \ell_{\beta\rm L})\left(H^\dagger {\rm i}\overleftrightarrow{D}^I_\mu H\right)$	$\mathcal{O}_{Hu}^{\alpha\beta}$	$(\overline{U_{\alpha\rm R}}\gamma^\mu U_{\beta\rm R})\left(H^\dagger {\rm i}\overleftrightarrow{D}_\mu H\right)$	$\mathcal{O}_{\ell u}^{\alpha\beta\gamma\lambda}$	$(\overline{\ell_{\alpha\rm L}}\gamma^\mu \ell_{\beta\rm L})(\overline{U_{\gamma\rm R}}\gamma_\mu U_{\lambda\rm R})$
$\mathcal{O}_{He}^{\alpha\beta}$	$(\overline{E_{\alpha\rm R}}\gamma^\mu E_{\beta\rm R})\left(H^\dagger {\rm i}\overleftrightarrow{D}_\mu H\right)$	$\mathcal{O}_{Hd}^{\alpha\beta}$	$(\overline{D_{\alpha\rm R}}\gamma^\mu D_{\beta\rm R})\left(H^\dagger {\rm i}\overleftrightarrow{D}_\mu H\right)$	$\mathcal{O}_{\ell d}^{\alpha\beta\gamma\lambda}$	$(\overline{\ell_{\alpha\rm L}}\gamma^\mu \ell_{\beta\rm L})(\overline{D_{\gamma\rm R}}\gamma_\mu D_{\lambda\rm R})$
$\mathcal{O}_{Hq}^{(1)\alpha\beta}$	$(\overline{Q_{\alpha\rm L}}\gamma^\mu Q_{\beta\rm L})\left(H^\dagger {\rm i}\overleftrightarrow{D}_\mu H\right)$				

Y. Wang, D. Zhang and S. Zhou, JHEP 05 (2023) 044.

III. One-loop RGEs of Physical Parameters

◆ **After spontaneous symmetry breaking (SSB) :**

$$C_{Hl}^{(1)\alpha\beta}\mathcal{O}_{Hl}^{(1)\alpha\beta} \to -\frac{g_2}{2c_{\rm W}}C_{Hl}^{(1)\alpha\beta}v^2\left(\overline{\nu_{\alpha\rm L}}\gamma^\mu\nu_{\beta\rm L}+\overline{l_{\alpha\rm L}}\gamma^\mu l_{\beta\rm L}\right)Z_\mu\ ,$$

$$C_{Hl}^{(3)\alpha\beta}\mathcal{O}_{Hl}^{(3)\alpha\beta} \to +\frac{g_2}{\sqrt{2}}C_{Hl}^{(3)\alpha\beta}v^2\left(\overline{\nu_{\alpha\rm L}}\gamma^\mu l_{\beta\rm L}W_\mu^+ +\overline{l_{\alpha\rm L}}\gamma^\mu \nu_{\beta\rm L}W_\mu^-\right) +\frac{g_2}{2c_{\rm W}}C_{Hl}^{(3)\alpha\beta}v^2\left(\overline{\nu_{\alpha\rm L}}\gamma^\mu\nu_{\beta\rm L}-\overline{l_{\alpha\rm L}}\gamma^\mu l_{\beta\rm L}\right)Z_\mu\ ,$$

CC $\tilde{\eta}\equiv -C_{Hl}^{(3)}v^2$

NC $\tilde{\eta}'\equiv\left(C_{Hl}^{(1)}-C_{Hl}^{(3)}\right)v^2$

◆ **Non-unitary parameters:**

RGEs of dim-6 operators → **RGEs of non-unitary parameters**

$$16\pi^2\mu\frac{{\rm d}C_{H\ell}^{(1)}}{{\rm d}\mu}=-\frac{3}{2}C_5C_5^\dagger+\frac{2}{3}g_1^2{\rm tr}\left(C_{H\ell}^{(1)}\right)\mathbb{1}+\left(\frac{1}{3}g_1^2+2T\right)C_{H\ell}^{(1)}+\frac{1}{2}\left[\left(4C_{H\ell}^{(1)}+9C_{H\ell}^{(3)}\right)Y_lY_l^\dagger+Y_lY_l^\dagger\left(4C_{H\ell}^{(1)}+9C_{H\ell}^{(3)}\right)\right]$$

$$16\pi^2\mu\frac{{\rm d}C_{H\ell}^{(3)}}{{\rm d}\mu}=C_5C_5^\dagger+\frac{2}{3}g_2^2{\rm tr}\left(C_{H\ell}^{(3)}\right)\mathbb{1}+\left(-\frac{17}{3}g_2^2+2T\right)C_{H\ell}^{(3)}+\frac{1}{2}\left[\left(3C_{H\ell}^{(1)}+2C_{H\ell}^{(3)}\right)Y_lY_l^\dagger+Y_lY_l^\dagger\left(3C_{H\ell}^{(1)}+2C_{H\ell}^{(3)}\right)\right]$$

III. One-loop RGEs of Physical Parameters

◆ **Unitary parameters：**

PMNS matrix : $V \equiv (\mathbb{1} - \eta) \cdot \boxed{U} \cdot \boxed{Q}$ → Majorana phase matrix

U → standard parametrization

Diagonalization of lepton mass matrices → RGEs of $V' \equiv U_l^\dagger U_\nu \equiv \boxed{P} \cdot U \cdot Q$

P → unphysical phase matrix

$$M_l = Y_l v/\sqrt{2} \qquad U_l^\dagger M_l U_l' = \widehat{M}_l \equiv \mathrm{diag}\{m_e, m_\mu, m_\tau\}$$

$$M_\nu = -C_5 v^2/2 \qquad U_\nu^\dagger M_\nu U_\nu^* = \widehat{M}_\nu \equiv \mathrm{diag}\{m_1, m_2, m_3\}$$

Bridge : $\mathcal{T} \equiv V'^\dagger \dot{V}'$ **&&** $\mathcal{T}' \equiv Q \cdot \mathcal{T} \cdot Q^\dagger = \dot{Q} Q^\dagger + U^\dagger \dot{U} + U^\dagger P^\dagger \dot{P} U$.

⇒ **RGEs of unitary parameters :** $\theta_{ij}, \delta, \rho, \sigma, m_i, m_\alpha$

III. One-loop RGEs of Physical Parameters

◆ **Results**：

$$\dot{\theta}_{13} = 2\,\frac{m^2}{v^2}\left[y_{e\mu} s_{23}\left(\left|\eta'_{e\mu}\right| c'_{e\mu+\delta} - 4\left|\eta_{e\mu}\right| c_{e\mu+\delta}\right) + y_{e\tau}\, c_{23}\left(\left|\eta'_{e\tau}\right| c'_{e\tau+\delta} - 4\left|\eta_{e\tau}\right| c_{e\tau+\delta}\right)\right]$$

SM+$\mathcal{O}^{(5)}$

$$\begin{aligned}
&+C_\kappa\, \zeta_{13}^{-1} c_{12}\, c_{13}\left[s_{12} s_{23} c_{23}\left(y_\mu^2 - y_\tau^2\right) c_\rho - c_{12} s_{13}\left(y_e^2 - s_{23}^2\, y_\mu^2 - c_{23}^2\, y_\tau^2\right) c_{\rho+\delta}\right] c_{\rho+\delta}\\
&+C_\kappa\, \zeta_{13}\, c_{12}\, c_{13}\left[s_{12} s_{23} c_{23}\left(y_\mu^2 - y_\tau^2\right) s_\rho - c_{12} s_{13}\left(y_e^2 - s_{23}^2\, y_\mu^2 - c_{23}^2\, y_\tau^2\right) s_{\rho+\delta}\right] s_{\rho+\delta}\\
&-C_\kappa\, \zeta_{23}^{-1} s_{12}\, c_{13}\left[c_{12} s_{23} c_{23}\left(y_\mu^2 - y_\tau^2\right) c_\sigma + s_{12} s_{13}\left(y_e^2 - s_{23}^2\, y_\mu^2 - c_{23}^2\, y_\tau^2\right) c_{\sigma+\delta}\right] c_{\sigma+\delta}\\
&-C_\kappa\, \zeta_{23}\, s_{12}\, c_{13}\left[c_{12} s_{23} c_{23}\left(y_\mu^2 - y_\tau^2\right) s_\sigma + s_{12} s_{13}\left(y_e^2 - s_{23}^2\, y_\mu^2 - c_{23}^2\, y_\tau^2\right) s_{\sigma+\delta}\right] s_{\sigma+\delta}\;,
\end{aligned}$$

$$\zeta_{ij} \equiv (\kappa_i - \kappa_j)/(\kappa_i + \kappa_j) \qquad C_\kappa = -3/2 \qquad y_{\alpha\beta} \equiv (y_\beta^2 + y_\alpha^2)/(y_\beta^2 - y_\alpha^2)$$

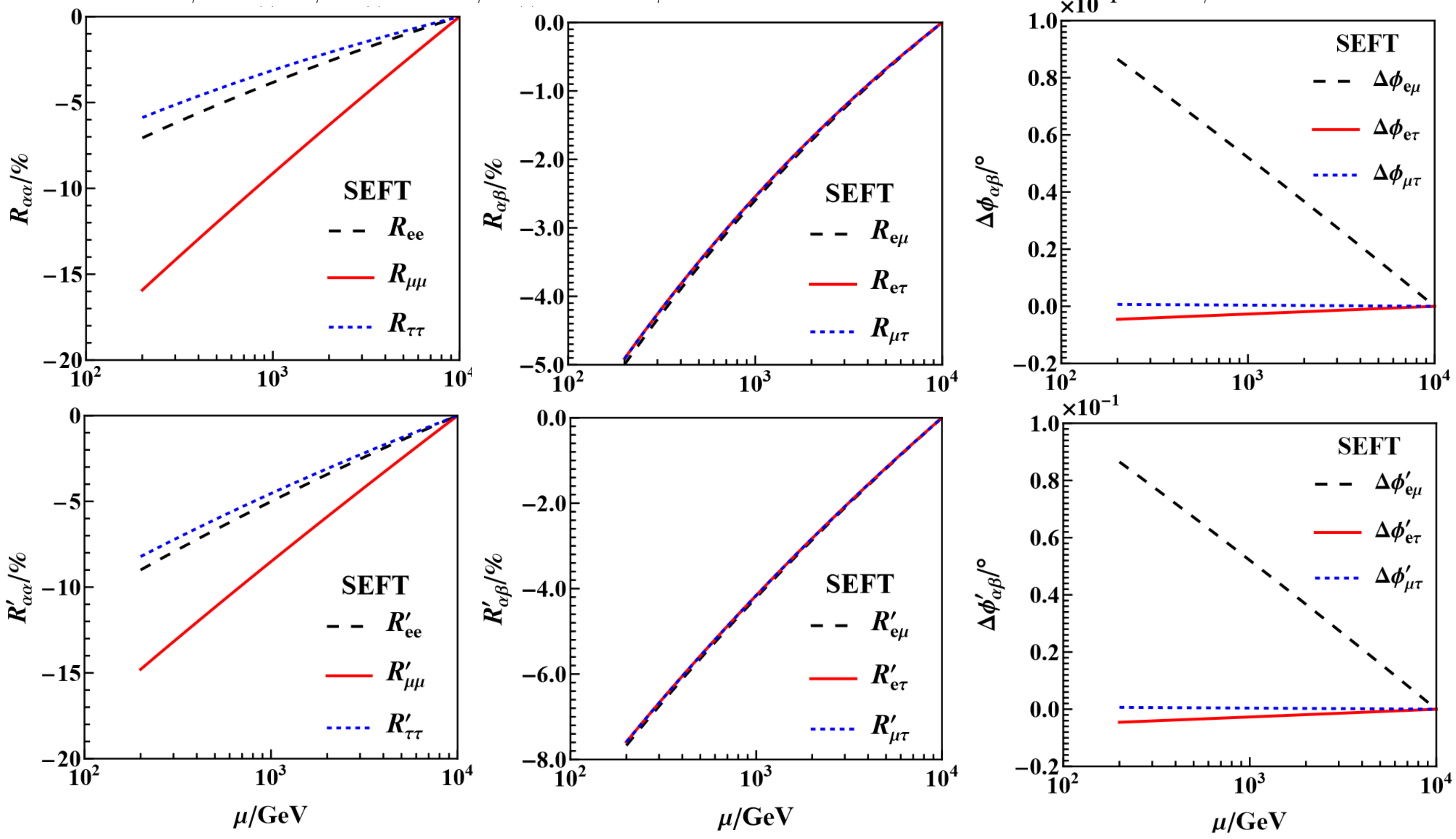
RGEs of the mixing parameters in the standard parametrization of a unitary leptonic mixing matrix are obtained for the first time.

Y. Wang, D. Zhang and S. Zhou, JHEP 05 (2023) 044.

IV. Numerical Results

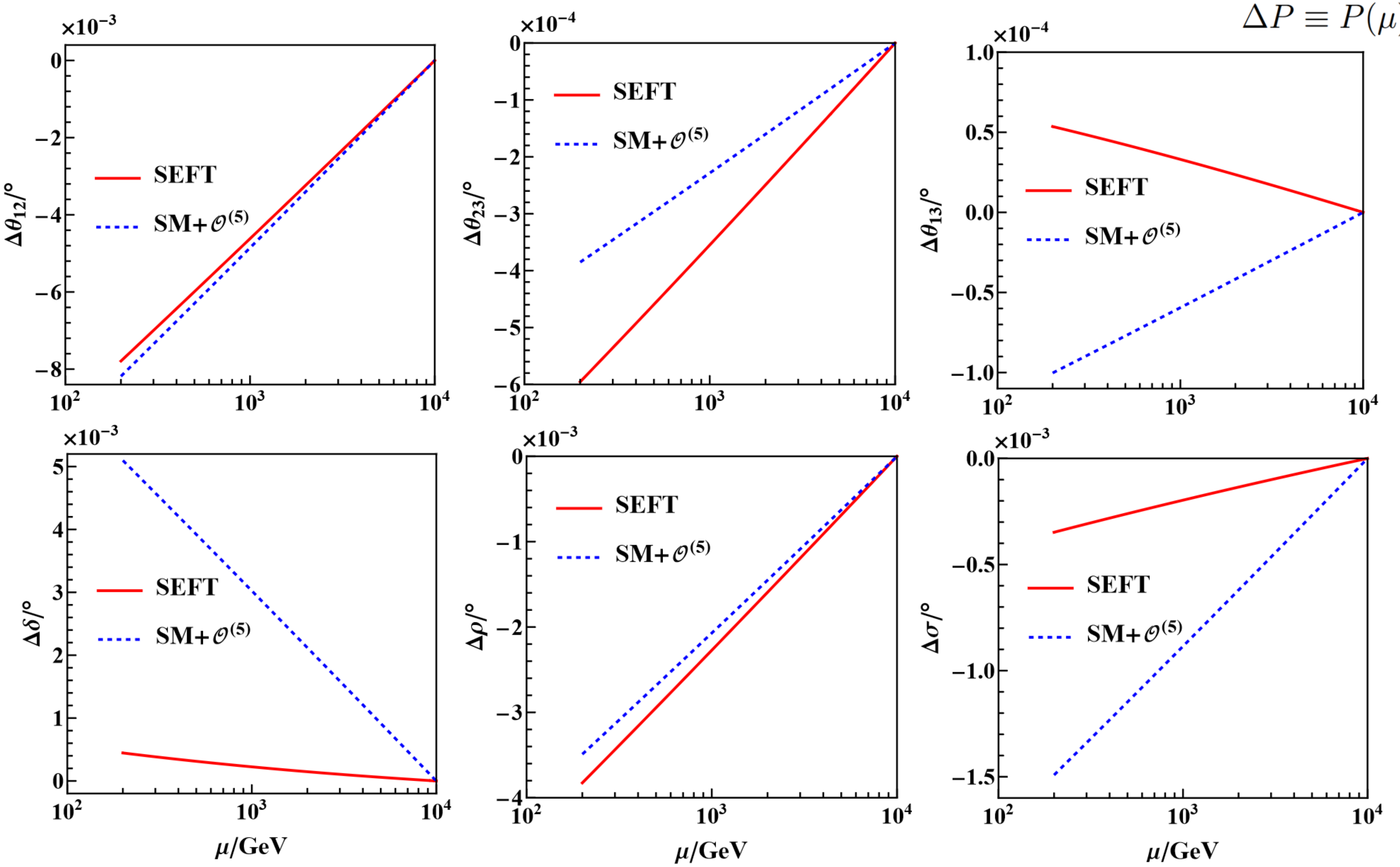
$$R^{(\prime)}_{\alpha\beta} \equiv [|\eta^{(\prime)}_{\alpha\beta}(\mu)| - |\eta^{(\prime)}_{\alpha\beta}(\mu_{\rm M})|]/|\eta^{(\prime)}_{\alpha\beta}(\mu_{\rm M})| \times 100\%$$

$$\Delta\phi^{(\prime)}_{\alpha\beta} \equiv \phi^{(\prime)}_{\alpha\beta}(\mu) - \phi^{(\prime)}_{\alpha\beta}(\mu_{\rm M})$$



IV. Numerical Results

$$\Delta P \equiv P(\mu) - P(\mu_{\rm M})$$



V. Summary

- We derive the complete set of one-loop RGEs for all SM couplings and Wilson coefficients of operators up to dim-6 in the type-I SEFT.
- There are 19 dim-6 operators in total, which can be generated by the RGEs at the one-loop level.
- After SSB, two tree-level dim-6 operators result in a non-unitary leptonic flavor mixing matrix appearing in the cc- and nc-interaction of leptons.
- As a by-product, the RGEs of the mixing parameters in the standard parametrization of a unitary leptonic mixing matrix are obtained for the first time.

Thanks for your attention!