Muon g-2 experiment at Fermilab

Exploring the Precision Frontier of Particle Physics

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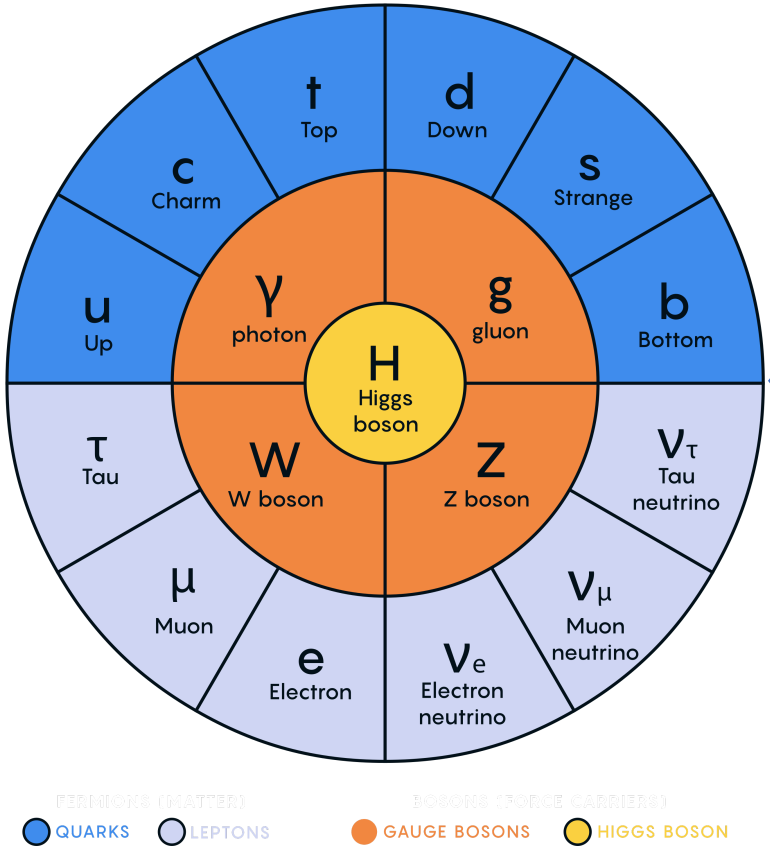
Outline

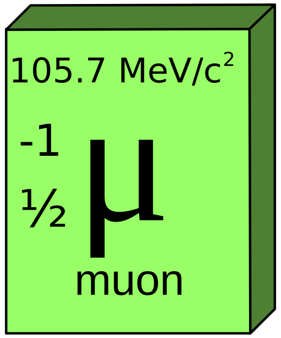
- Introduction
- Fermilab Muon g-2 experiment (Run-1)
- Improvements afterwards
- Outlook

Motivation: from the Standard Model to New Physics

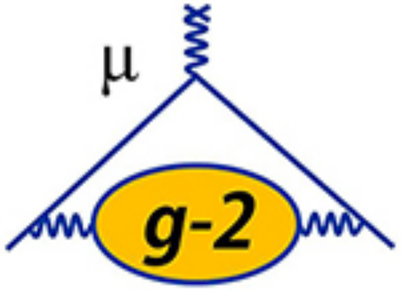
- Standard Model



Particles predicted by SM



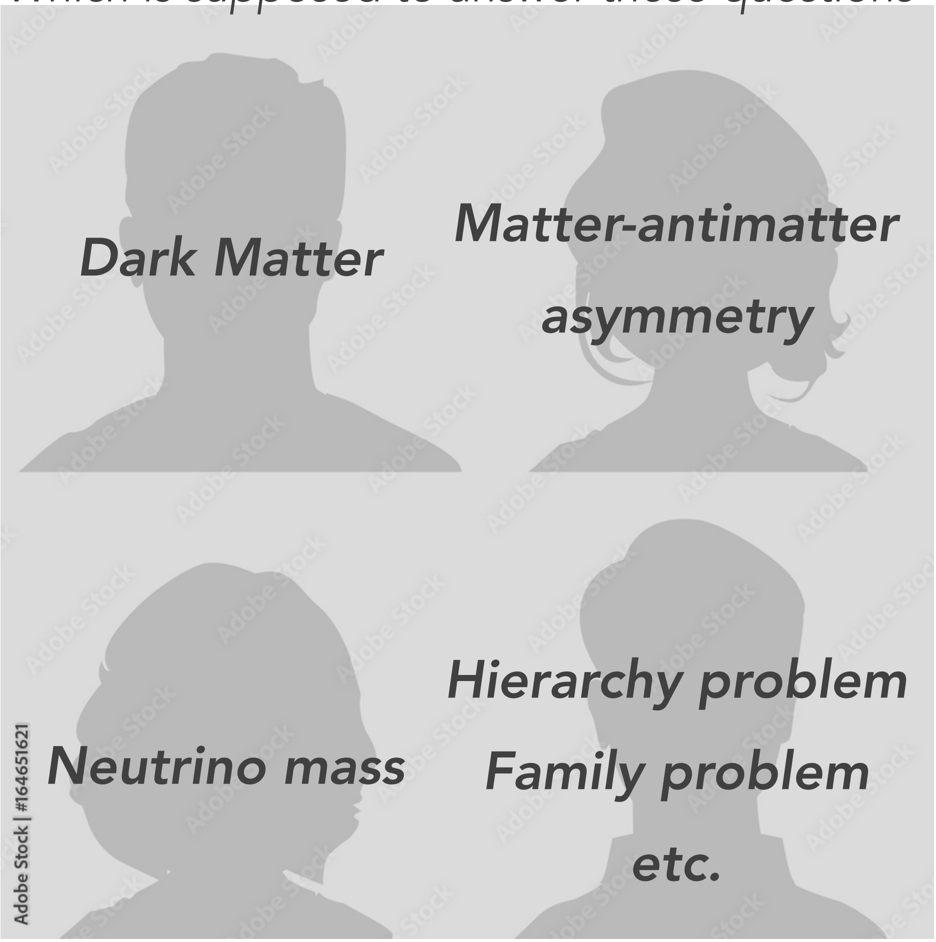
Muon as a sensitive probe



$$a_\mu = \frac{g-2}{2}$$

- New Physics

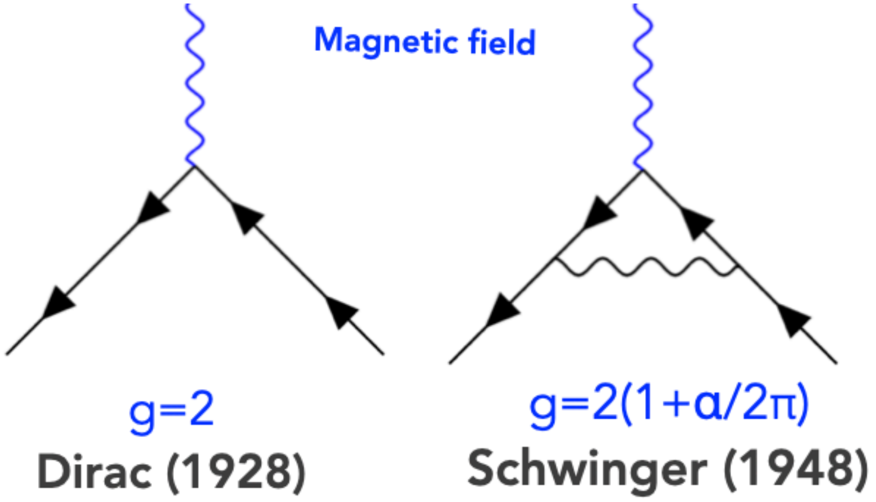
Which is supposed to answer these questions



Motivation: a story of g-2

Electron g-2 measurement

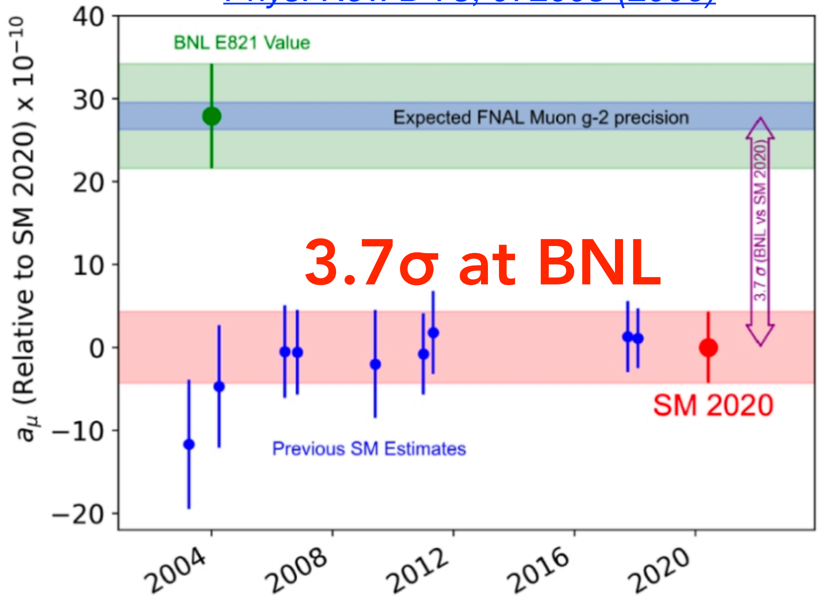
- Played an important role in the development of QFT
- Now be used to define the renormalized value of α
- Precision up to 0.13 ppt *Phys. Rev. Lett.* ***130**, 071801*



Muon g-2 measurement

- More sensitive to new types of virtual particles $(m_\mu/m_e)^2 \simeq 43,000$
- Now be used to detect new physics BSM
- Precision up to 0.53 ppm

Phys. Rev. D 73, 072003 (2006)



Standard Model Prediction of a_μ

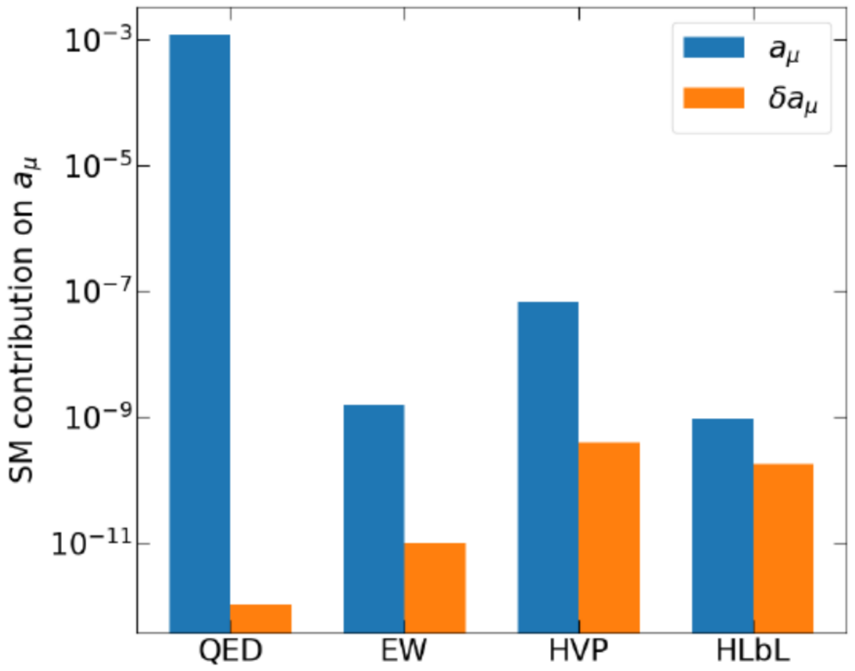
- **Perturbative terms**
  - ➡ QED: largest, evaluated up to $\mathcal{O}(\alpha^5)$
  - ➡ EW: suppressed by $(m_\mu/M_W)^2$
- **Non-perturbative terms**
  - ➡ HVP
  - ➡ HLbL

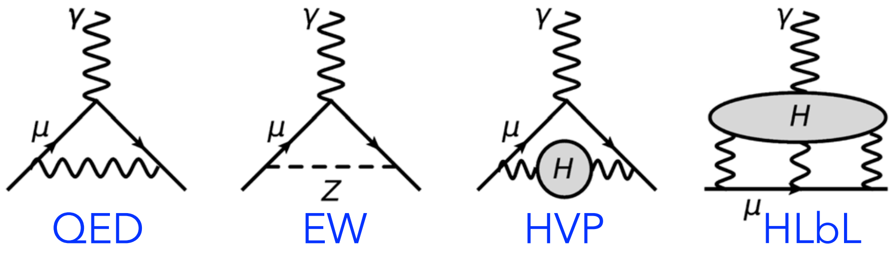
  Calculated with first principal (Lattice-QCD) or data-driven (dispersion relation) approaches
- **Data-driven calculation of $a_\mu^{\mathrm{HVP,LO}}$**

$$a_\mu^{\mathrm{HVP,LO}} = \frac{\alpha^2}{3\pi^2}\int_{M_\pi^2}^{\infty}\frac{ds}{s}\,\frac{\sigma_{e^+e^-\to \mathrm{hadrons}+\gamma}}{\sigma_{e^+e^-\to\mu^+\mu^-}}\cdot K(s)$$

- $a_\mu(\mathrm{SM}) = 116591810(43)\times 10^{-11}$ **(0.37 ppm)**
  Phys. Rep. 887, 1 (2020)

https://doi.org/10.1016/j.physrep.2020.07.006



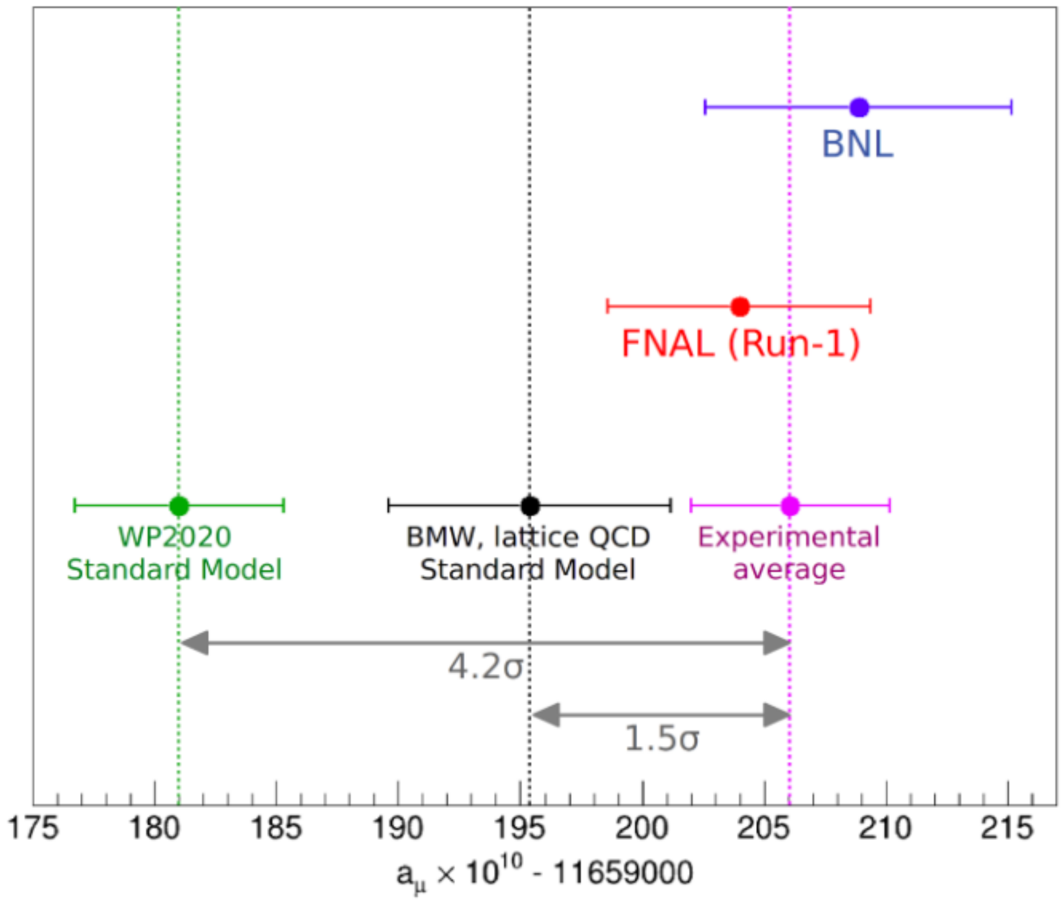


Current Status

- FNAL Run-1 measurement published on 7 April 2021 (0.46 ppm)
  - ✓ Consistent with previous BNL result
  - ✓ 4.2 σ from theoretical prediction (2020) (used dispersion relation for $a_\mu^{\mathrm{HVP,LO}}$)
- New Lattice-QCD calculation of the $a_\mu^{\mathrm{HVP,LO}}$ is in tension with the data-driven prediction
  - ✓ 4.2 σ (dispersion) → 1.5 σ (lattice-QCD)

a_μ (FNAL) = 116 592 040(54) × 10^{-11} (0.46 ppm)

a_μ (Exp) = 116 592 061(41) × 10^{-11} (0.35 ppm)



Principal of g-2 Measurement in a Storage Ring

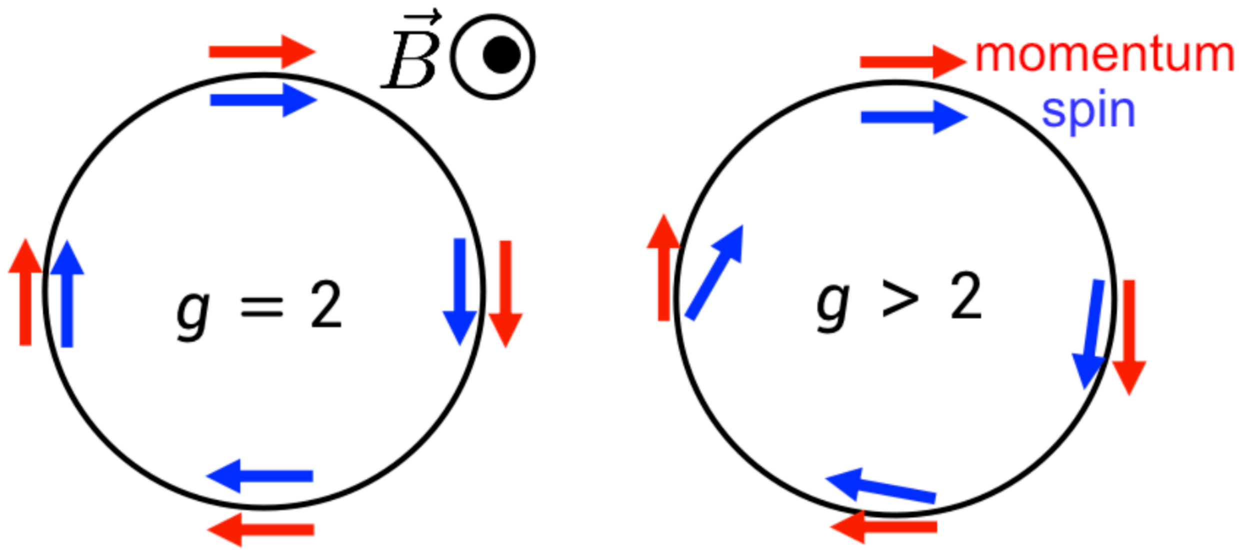
Spin precession

$$\vec{\omega}_s = -g_\mu \frac{q\vec{B}}{2m} - (1-\gamma)\frac{q\vec{B}}{m\gamma}$$

Cyclotron motion

$$\vec{\omega}_c = -\frac{q\vec{B}}{m\gamma}$$

$$\boxed{\vec{\omega}_a} = \vec{\omega}_s - \vec{\omega}_c = -\left(\frac{g_\mu - 2}{2}\right)\frac{q\vec{B}}{m} = -a_\mu \frac{q\vec{B}}{m}$$



Three main components to measure $a_\mu = \dfrac{g-2}{2}$:

- **ωa:** anomalous precession frequency
- Magnetic field **B** in terms of ω_p
- **M(r)**: the spatial distribution of μ^+

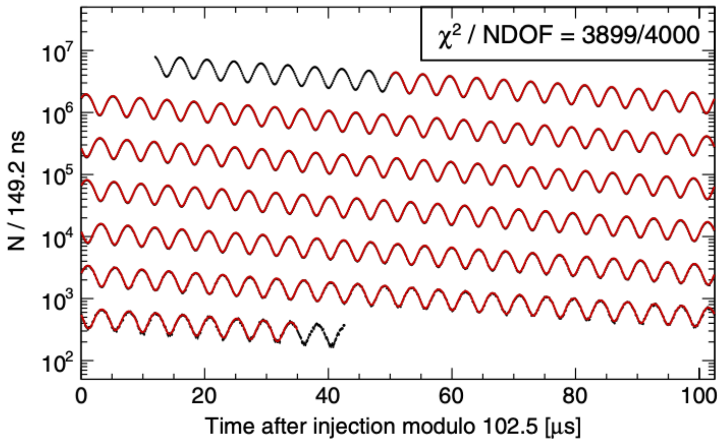
Main analyses

- **The a_μ determined from 3 main analyses**
  - ✓ Anomalous precession frequency: $\omega_a = \omega_s - \omega_c$
  - ✓ Magnetic field: ω_p
  - ✓ Beam dynamics: M(x,y,ϕ)

  } → $\tilde{\omega}'_p(T_r)$

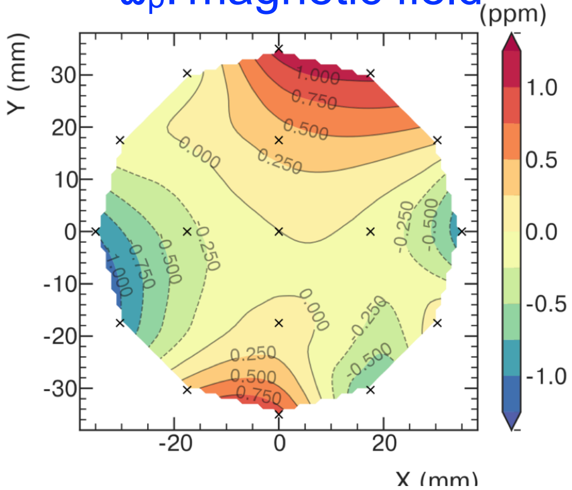
$< \omega_p(x,y,\phi,T_r) \otimes M(x,y,\phi) > \longrightarrow \tilde{\omega}'_p$

ω_a: anomalous precession frequency



Phys. Rev. D 103, 072002 (2021)

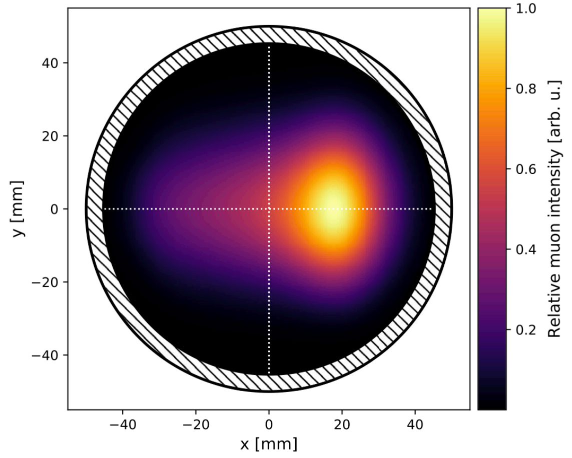
ω_p: magnetic field



Phys. Rev. A **103**, 042208

⊗

M(r): μ^+ spatial distribution



Phys. Rev. Accel. Beams 24 044002 (2021)

How the a_μ finally determined

What we measure

$$a_\mu = \frac{\omega_a}{\tilde{\omega}'_p(T_r)} \frac{\mu'_p(T_r)}{\mu_e(H)} \frac{\mu_e(H)}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2}$$

Known from other experiments (25 ppb)

$$\mathcal{R}'_\mu = \frac{\omega_a}{\tilde{\omega}'_p(T_r)} = \frac{f_{clock}\, \omega_a^m \,(1 + C_E + C_p + C_{lm} + C_{pa})}{f_{cali} \langle \omega_p(x, y, \phi, T_r) \times M(x, y, \phi) \rangle (1 + B_q + B_k)}$$

- Blinding factor: f_{clock}
- Anomalous precession frequency: ω_a^m
- Corrections from beam dynamics: $C_E + C_p + C_{lm} + C_{pa}$
- Field Calibration: f_{cali}
- Magnetic field distribution: $\omega_p(x, y, \phi, T_r)$
- Muon distribution: $M(x, y, \phi)$
- Corrections from the transient magnetic field: $B_q + B_k$

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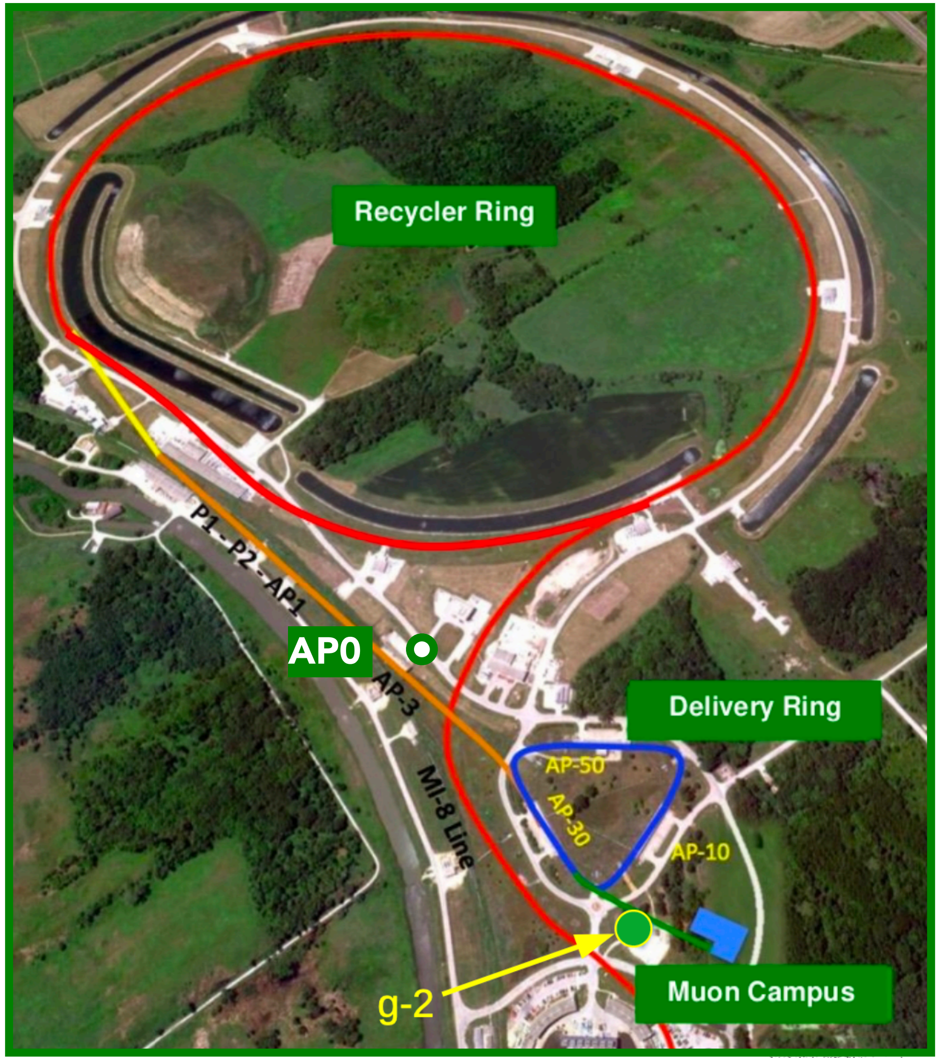
The Fermilab Muon Source

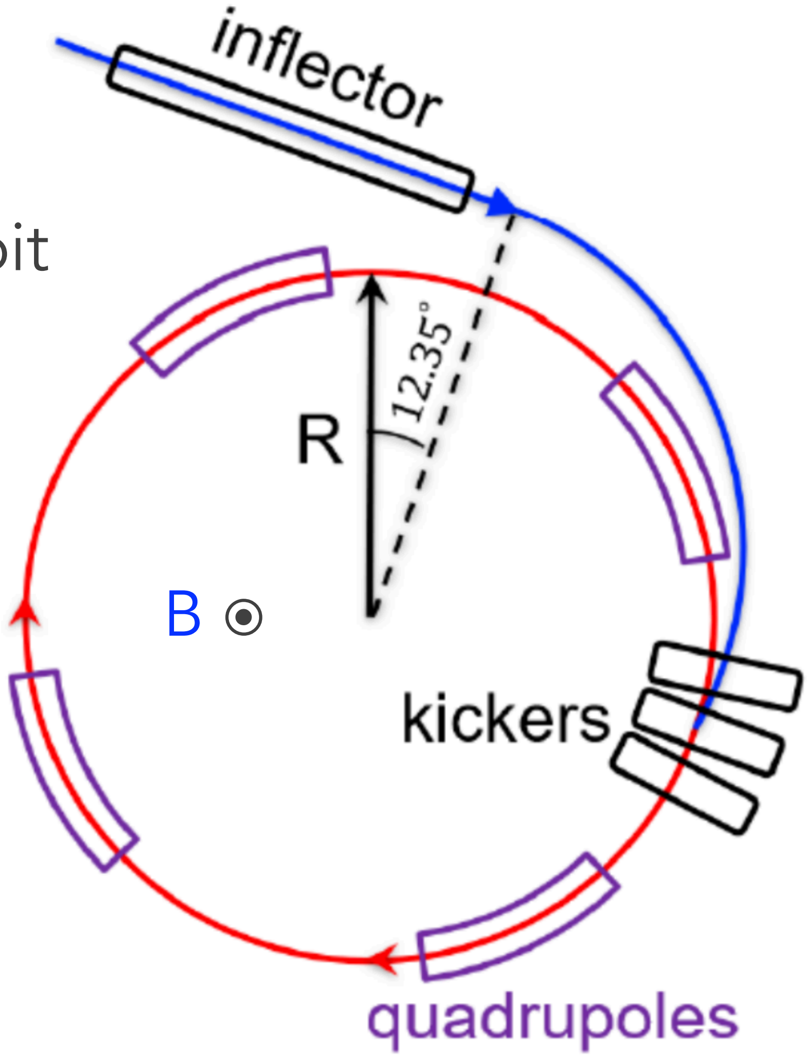
$$P + N \to \pi^+ \to \mu^+$$

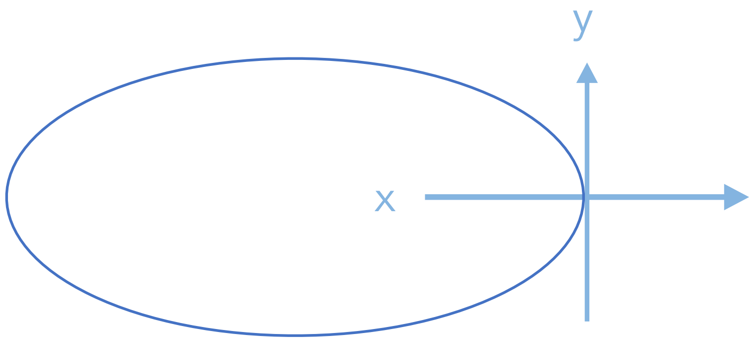
- Accelerator **protons** delivered by **Recycler Ring**
  - ➡Boosted to 8 GeV
  - ➡16 bunches x 10^{12} protons
- Hit a fixed Inconel target at **AP0** to produce π^+
  - ➡Per 1.4s
  - ➡Long beam line to collect $\pi^+ \to \mu^+$
- $p/\pi/\mu$ enter the **Delivery Ring**
  - ➡π^+ decay away, p aborted
  - ➡μ^+ extracted
- μ^+ delivered by the **Muon Campus**
  - ➡Highly polarized
  - ➡p=3.094 GeV/c
  - ➡$\tau=\gamma\tau_0\approx 64$ μs

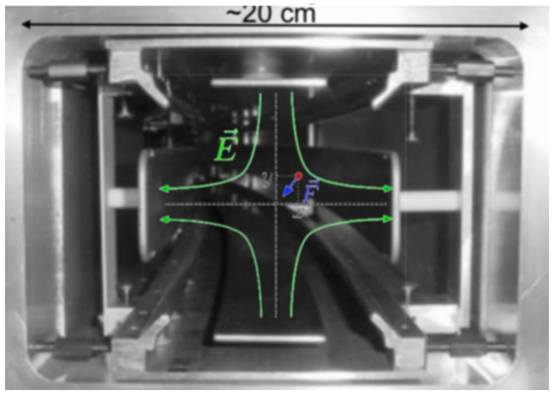


G-2 Storage Ring: Muon Injection

- Inflector to cancel the magnetic field, create field-free region for muon injection
- Fast kicker system knock μ^+ into their expected orbit
- X-direction constrained by 1.45 T magnetic field
- Y-direction constrained by $\vec{E}$ from 4 ESQs



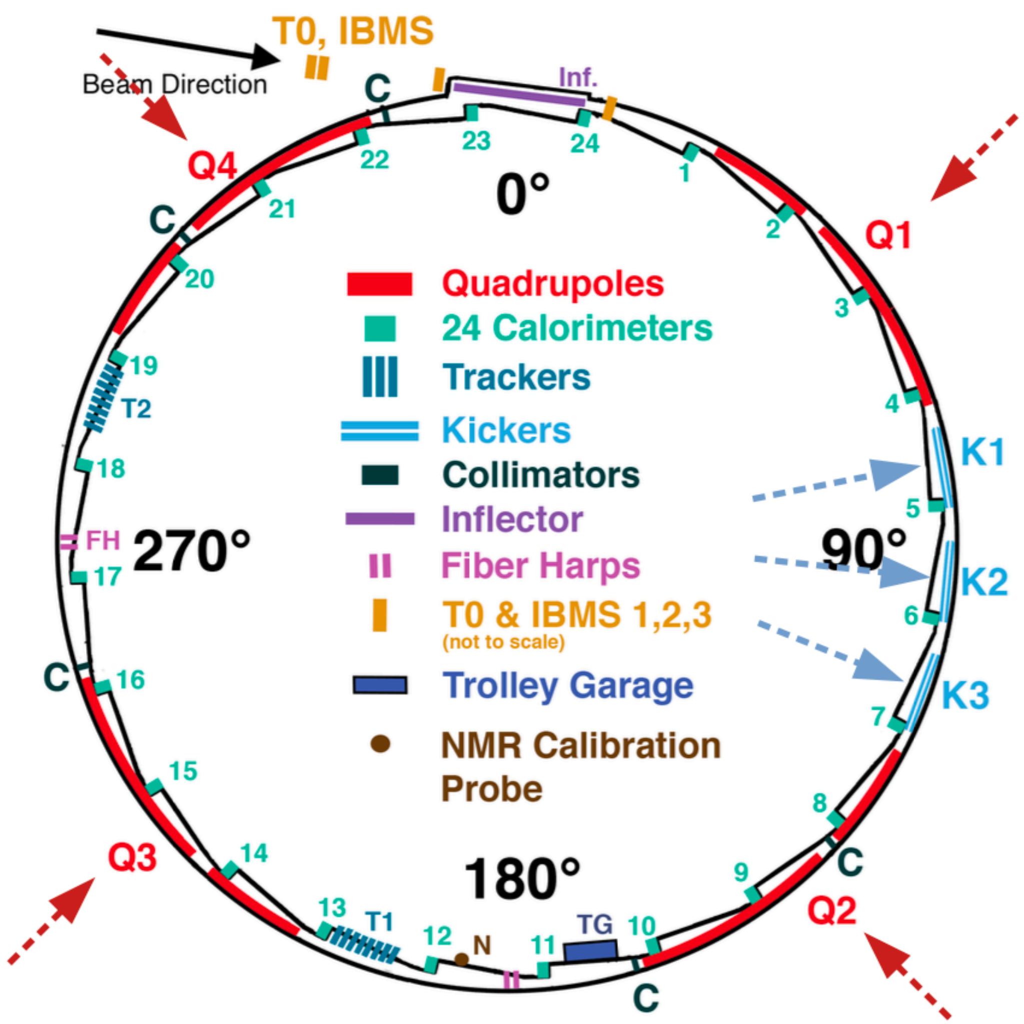




G-2 Storage Ring: Detector System

- Beam profile monitors: IBMS (x,y), T0 (time)
- 24 calorimeters to detect the energy and time of e^+ for ω_a analysis
  - ✓ Constructed with 6x9 PbF_2 crystals
  - ✓ Coupled with 144 mm^2 SiPM
- 2 tracker station to detect the decay vertex of μ^+ in beam dynamic analysis
  - ✓ 8 modules with 64+64 straw tubes

$$\mu^+ \rightarrow \bar{\nu}_\mu + \nu_e + e^+$$

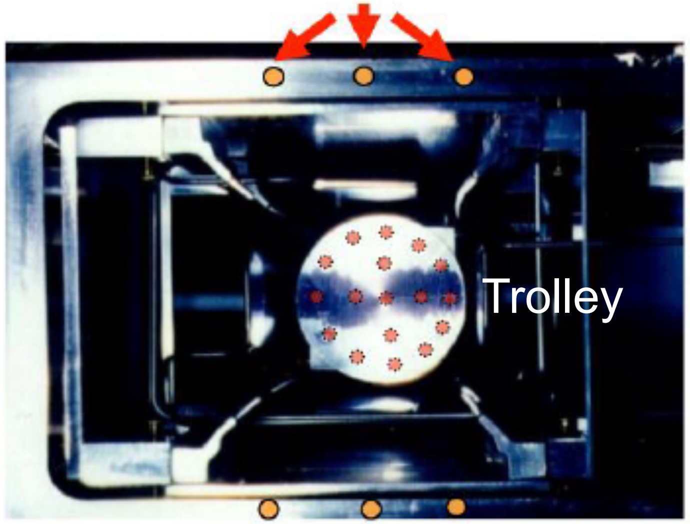


Measurement of the Magnetic Field

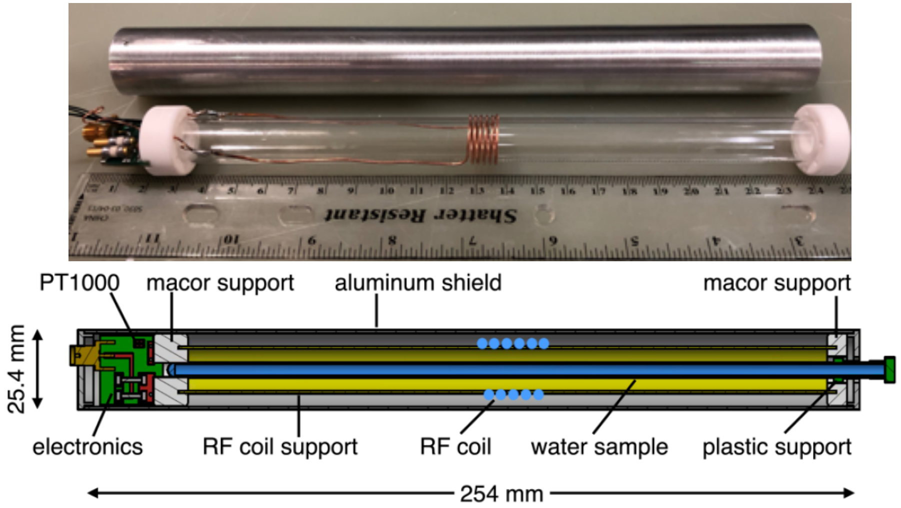
- Measure magnetic filed in terms of ω_p with NMR probes
- 378 fixed probes and 3D trolley mapping (per 3 days)
- Trolley cross-calibrated to absolute probes (water sample)

Fixed probes map

Fixed Probe



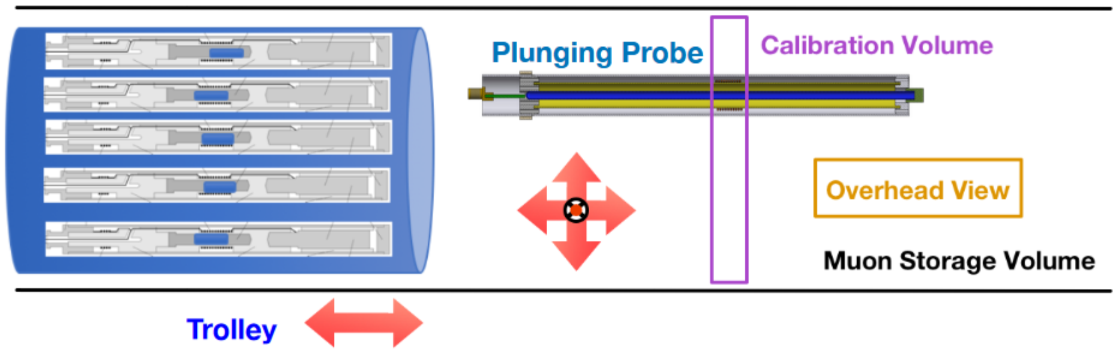
Trolley run



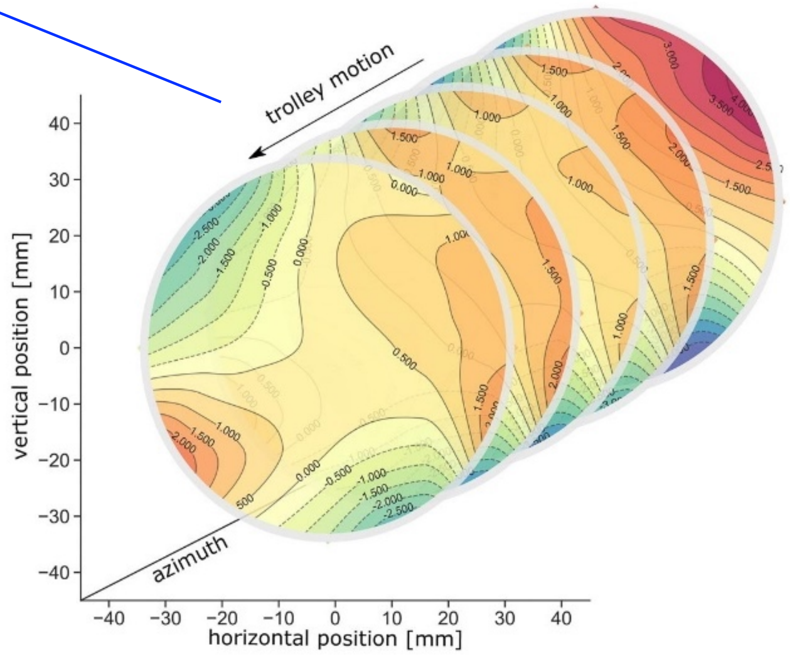
Absolute calibration

Magnetic Field Distribution and Calibration

$$\mathscr{R}'_\mu = \frac{\omega_a}{\tilde{\omega}'_p(T_r)} = \frac{f_{clock}\,\omega_a^m\,(1 + C_E + C_p + C_{lm} + C_{pa})}{f_{cali}\,\langle \omega_p(x, y, \phi, T_r) \times M(x, y, \phi)\rangle\,(1 + B_q + B_k)}$$



Trolley measurements (NMR) calibrated to a water sample in the plunging probe



3D magnetic field distribution measured with ~9,000 ϕ slices

Principal of Beam Motion

- Cyclotron motion and x-,y-oscillations:
  - $\omega_c = \frac{v}{2\pi r_0}$
  - $\omega_x \approx \omega_c\sqrt{1-n}$
  - $\omega_y \approx \omega_c\sqrt{n}$

$(e\kappa = n \cdot \omega_c^2)$

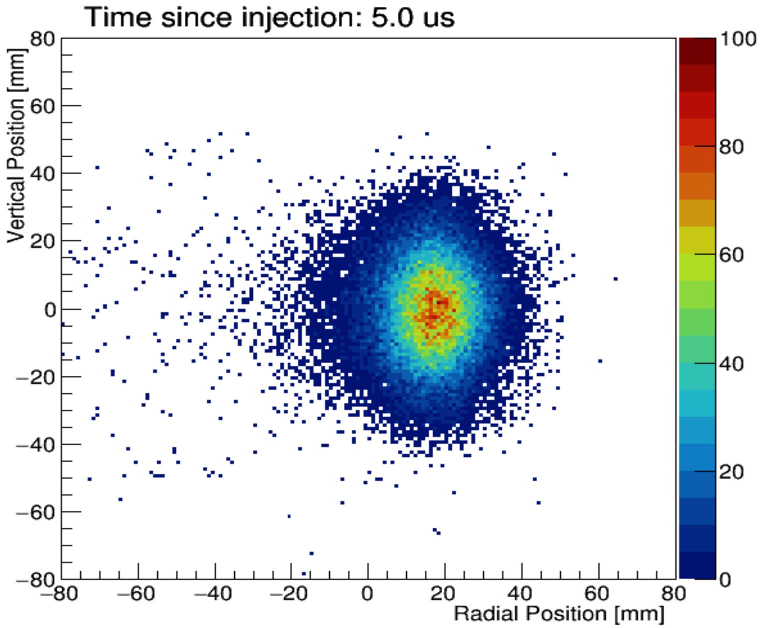
Effective field index (n)

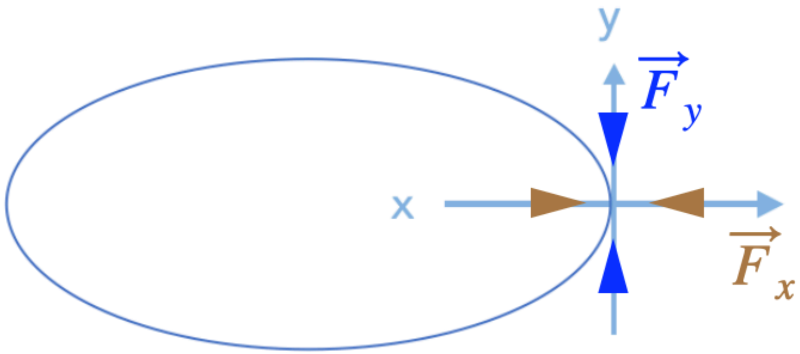
Dataset	$\delta\omega_a^m$ (stat) (ppb)	ESQ (kV)	Effective field index	Kicker (kV)
Run-1a	1206	18.3	0.108	130
Run-1b	1024	20.4	0.120	137
Run-1c	825	20.4	0.120	130
Run-1d	676[a]	18.3	0.107	125

By ESQs   By $\vec{B}$ field

$$\vec{F}_x = +e\kappa \cdot \Delta x - \omega_c^2 \cdot \Delta x$$

$$\vec{F}_y = -e\kappa \cdot \Delta y$$

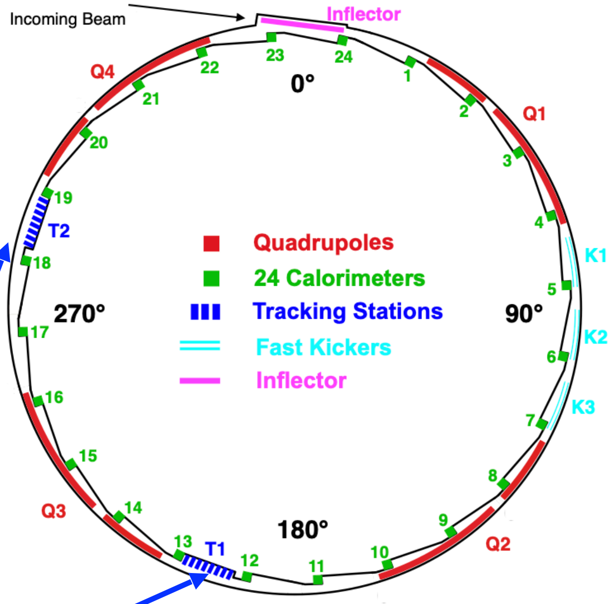


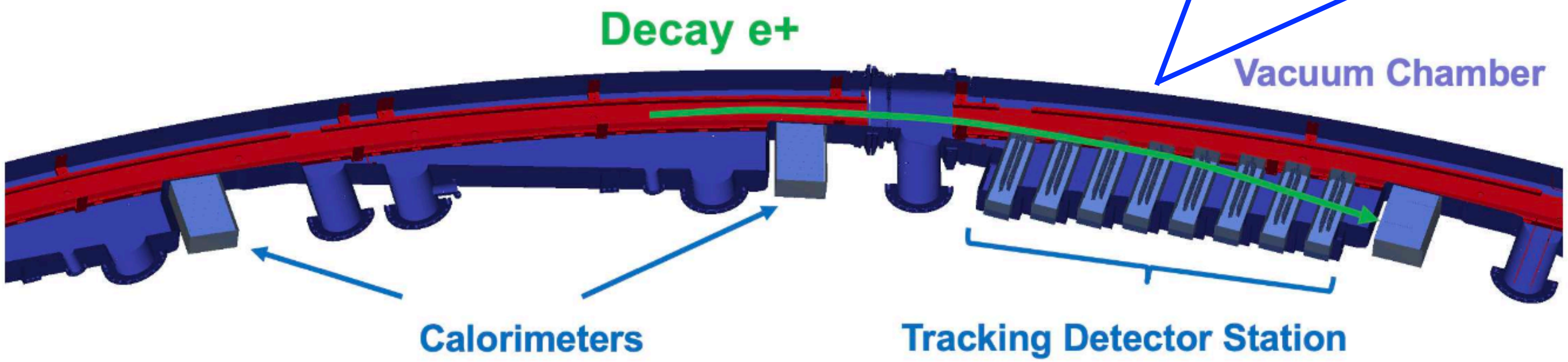


Measurement of the Beam Motion

- 2 tracker stations to reconstruct the muon decay vertex
- Match to calorimeter hit energy for particle identification
- Extrapolated to full azimuth with Geant4 based simulation tools

Storage Ring

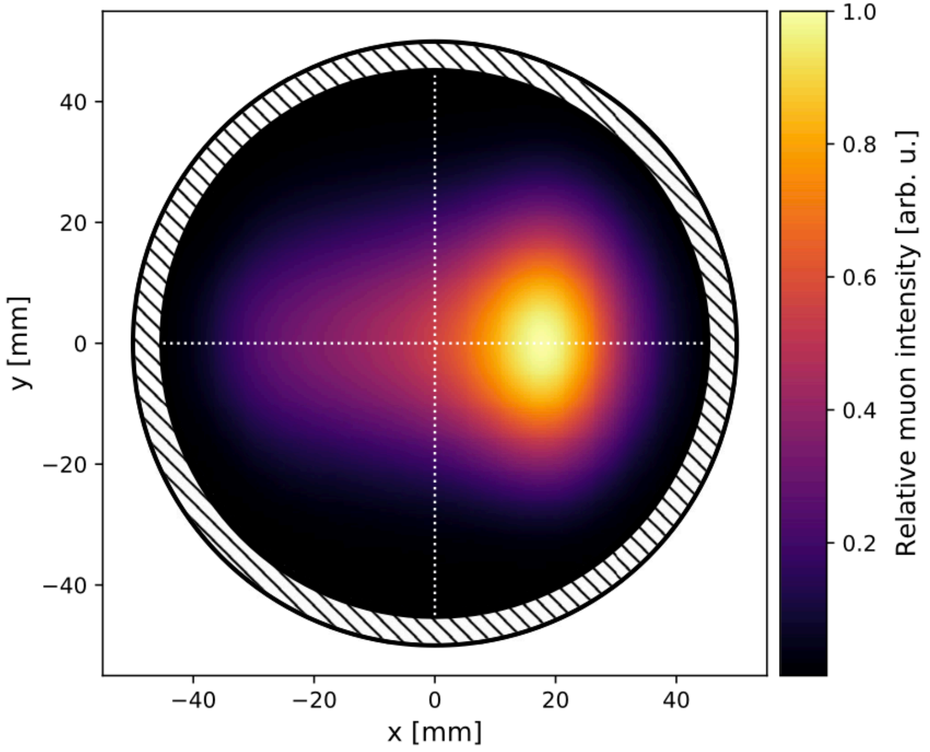




https://doi.org/10.1088/1748-0221/17/02/P02035

Muon distribution

$$\mathcal{R}'_\mu = \frac{\omega_a}{\tilde{\omega}'_p(T_r)} = \frac{f_{clock}\,\omega_a^m\,(1 + C_E + C_p + C_{lm} + C_{pa})}{f_{cali}\,\langle \omega_p(x,y,\phi,T_r) \times M(x,y,\phi)\rangle\,(1 + B_q + B_k)}$$

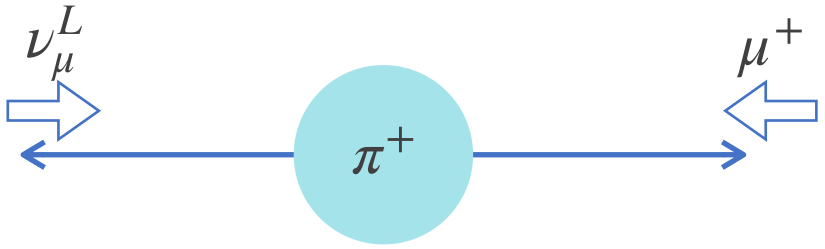


- e^+ trajectories reconstructed from tracker data
- Extrapolate the trajectories to build μ^+ decay vertex
- Extrapolate to full azimuth with Geant4 based simulation tools

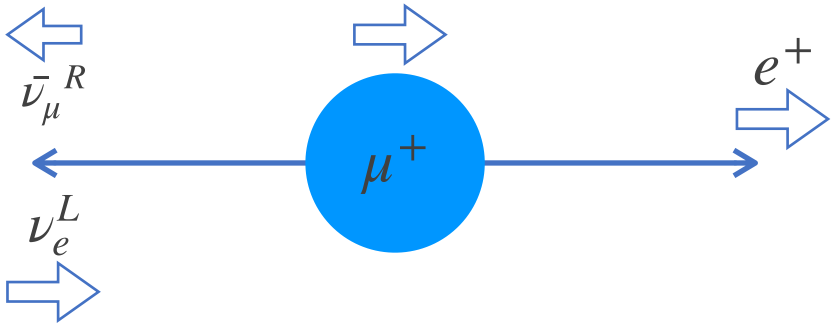
Principal of ωa measurement

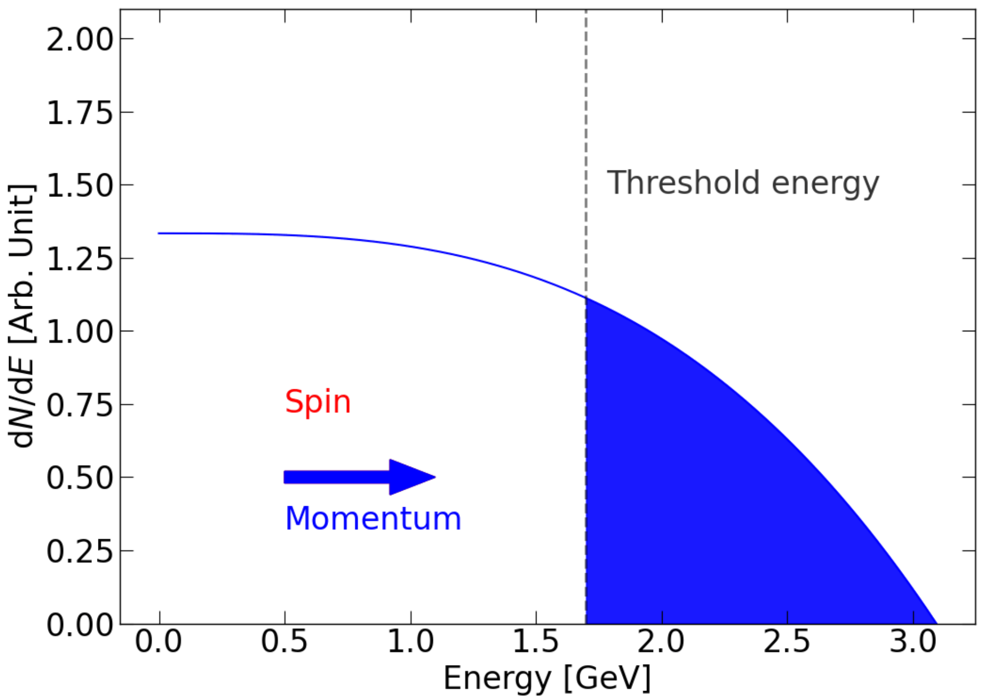
- ω_a can be measured as the time modulation of **positron energy distribution**

- Muons produced from pion decay are **highly polarized**



- **Momentum direction** of high energy **positrons** indicate the direction of muon spin



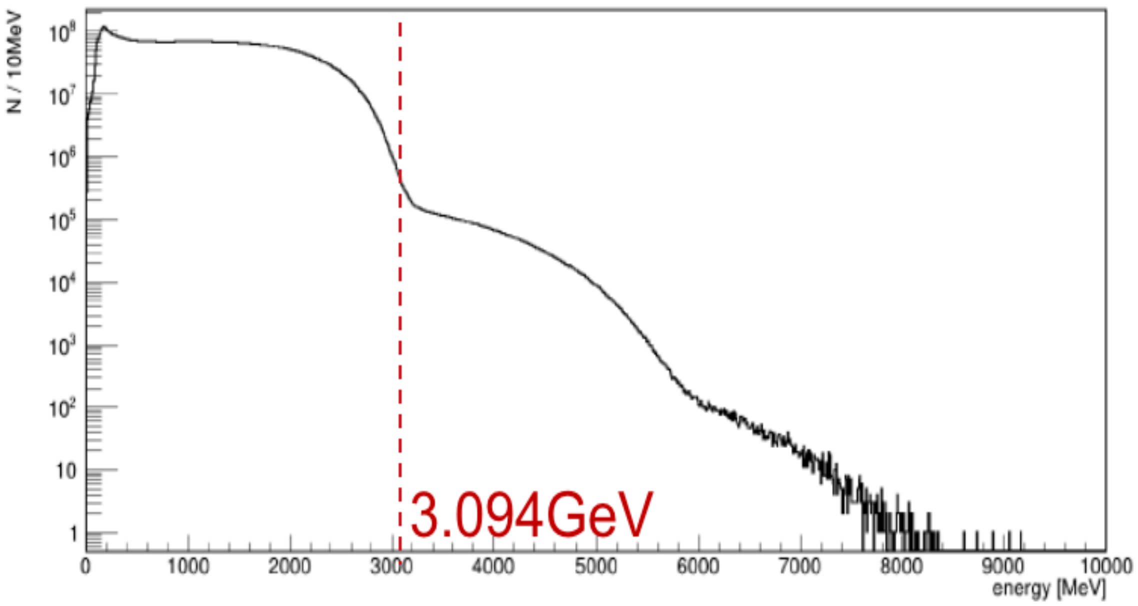


$$e^+ \text{ Signal from Muon Decay: } N_{\text{ideal}}(t) = N_0 \exp\left(-t/\gamma\tau_\mu\right)\left[1 + A\cos\left(\omega_a t + \phi\right)\right]$$

Pileup effect

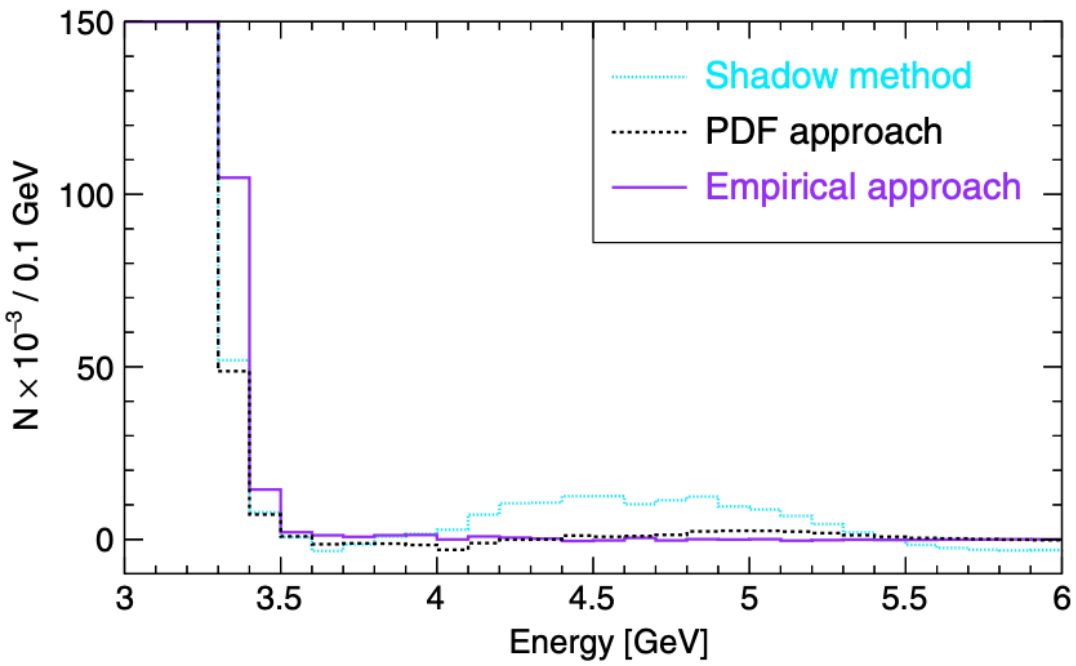
- ≧2 positrons hit at the calorimeter in a short time and reconstructed as 1 single positron

✓ Evidence of pileup effect in the uncorrected energy spectrum



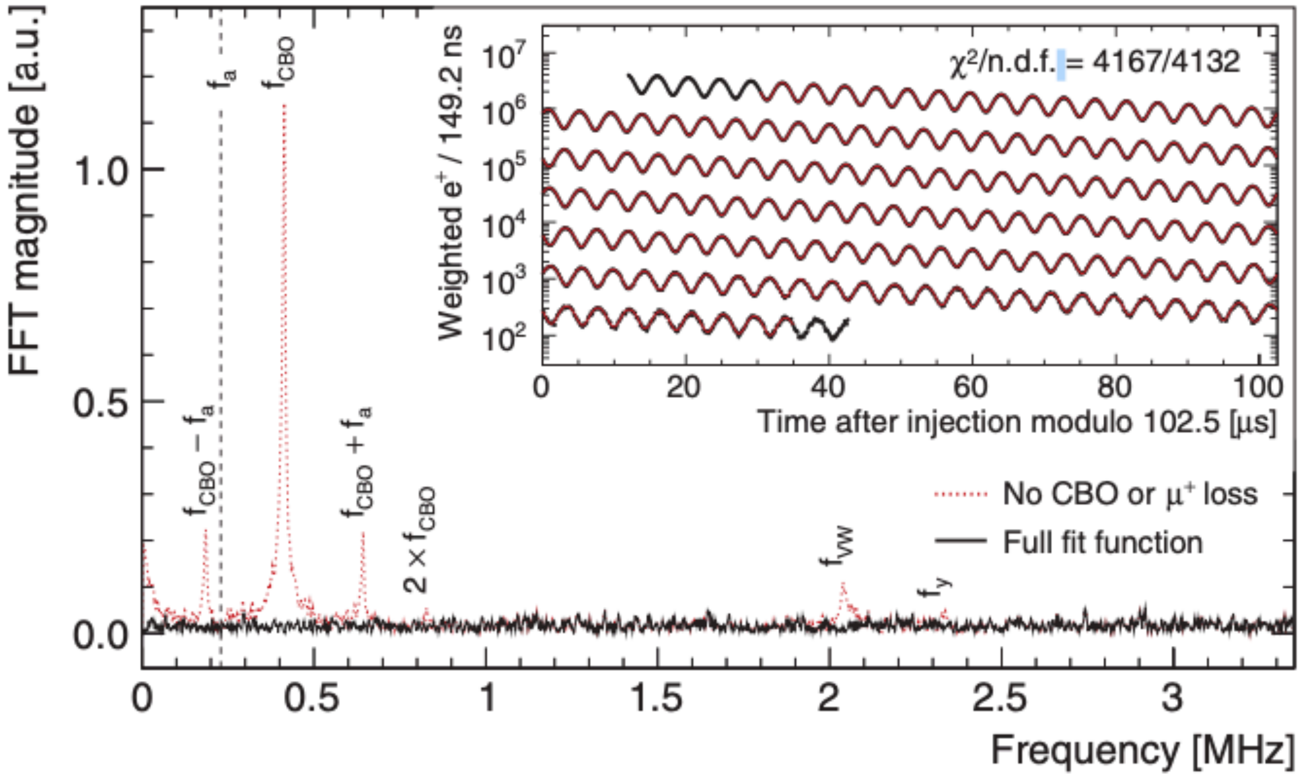
- 3 dedicated approaches to correct the pileup contamination: shadow, PDF and empirical

✓ Performance of the pileup corrections



Measurement of ω_a^m

- Extract the ω_a^m through fit to the "wiggle" histogram



Horizontal (width) oscillation

Vertical (width) oscillation

Lost muons

$$F(t) = N_0 \cdot N_x(t) \cdot N_y(t) \cdot \Lambda(t) \cdot e^{-t/\gamma\tau_\mu} \cdot [1 + A_0 \cdot A_x(t) \cdot \cos(\omega_a^m t + \phi_0 \cdot \phi_x(t))]$$

$$N_x(t) = 1 + e^{-1t/\tau_{CBO}} A_{N,x,1,1} \cos(1\omega_{CBO} t + \phi_{N,x,1,1}) + e^{-2t/\tau_{CBO}} A_{N,x,2,2} \cos(2\omega_{CBO} t + \phi_{N,x,2,2}),$$

$$N_y(t) = 1 + e^{-1t/\tau_y} A_{N,y,1,1} \cos(1\omega_y t + \phi_{N,y,1,1}) + e^{-2t/\tau_y} A_{N,y,2,2} \cos(1\omega_{VW} t + \phi_{N,y,2,2}),$$

$$A_x(t) = 1 + e^{-1t/\tau_{CBO}} A_{A,x,1,1} \cos(1\omega_{CBO} t + \phi_{A,x,1,1}),$$

$$\phi_x(t) = 1 + e^{-1t/\tau_{CBO}} A_{\phi,x,1,1} \cos(1\omega_{CBO} t + \phi_{\phi,x,1,1}).$$

How the a_μ finally determined

What we measure

$$a_\mu = \frac{\omega_a}{\tilde{\omega}'_p(T_r)} \frac{\mu'_p(T_r)}{\mu_e(H)} \frac{\mu_e(H)}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2}$$

Known from other experiments (25 ppb)

$$\mathcal{R}'_\mu = \frac{\omega_a}{\tilde{\omega}'_p(T_r)} = \frac{f_{clock}\, \omega_a^m \,(1 + C_E + C_p + C_{lm} + C_{pa})}{f_{cali} \langle \omega_p(x, y, \phi, T_r) \times M(x, y, \phi) \rangle (1 + B_q + B_k)}$$

- f_{clock}: Blinding factor
- ω_a^m: Anomalous precession frequency
- $C_E + C_p + C_{lm} + C_{pa}$: Corrections from beam dynamics
- f_{cali}: Field Calibration
- $\omega_p(x, y, \phi, T_r)$: Magnetic field distribution
- $M(x, y, \phi)$: Muon distribution
- $B_q + B_k$: Corrections from the transient magnetic field

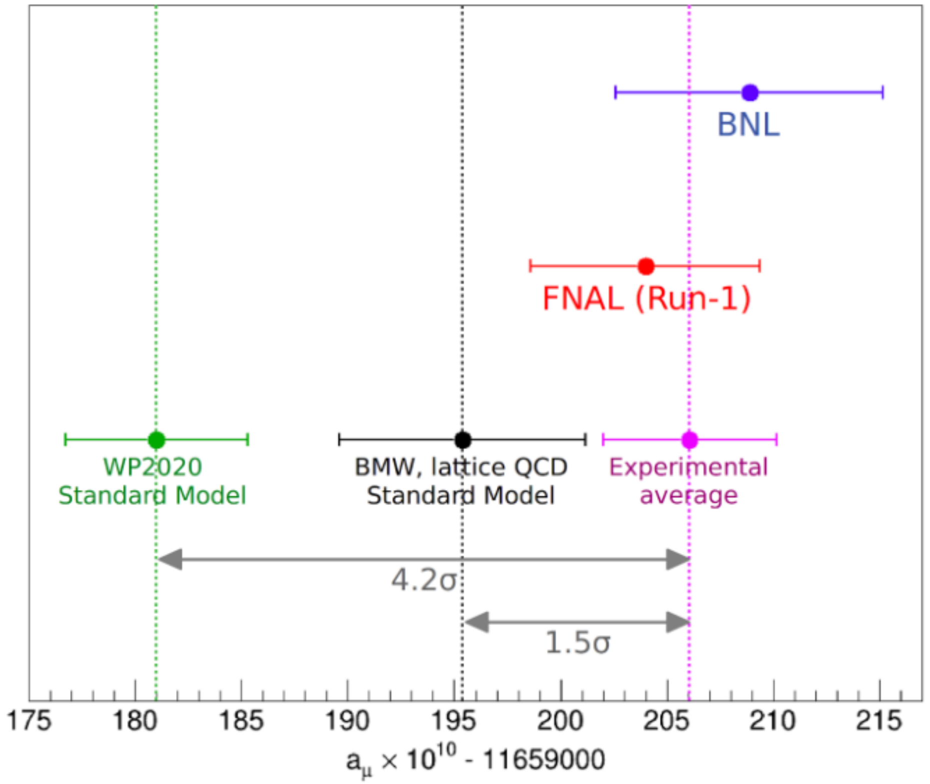
Run-1 results

- Dominated by statistical uncertainty

a_μ (FNAL) = 116 592 040(54) × 10^{-11} (0.46 ppm)

a_μ (Exp) = 116 592 061(41) × 10^{-11} (0.35 ppm)

Quantity	Correction [ppb]	Uncertainty [ppb]
ω_a (statistical)	-	434
ω_a (systematic)	-	56
C_e	489	53
C_p	180	13
C_{ml}	-11	5
C_{pa}	-158	75
$f_{calib}\langle\omega'_p(x,y,\phi)\cdot M(x,y,\phi)\rangle$	-	56
B_q	-17	92
B_k	-27	37
μ'_p/μ_e	-	10
m_μ/m_e	-	22
g_e	-	0
Total systematic	-	157
Total external factors	-	25
Total	544	462



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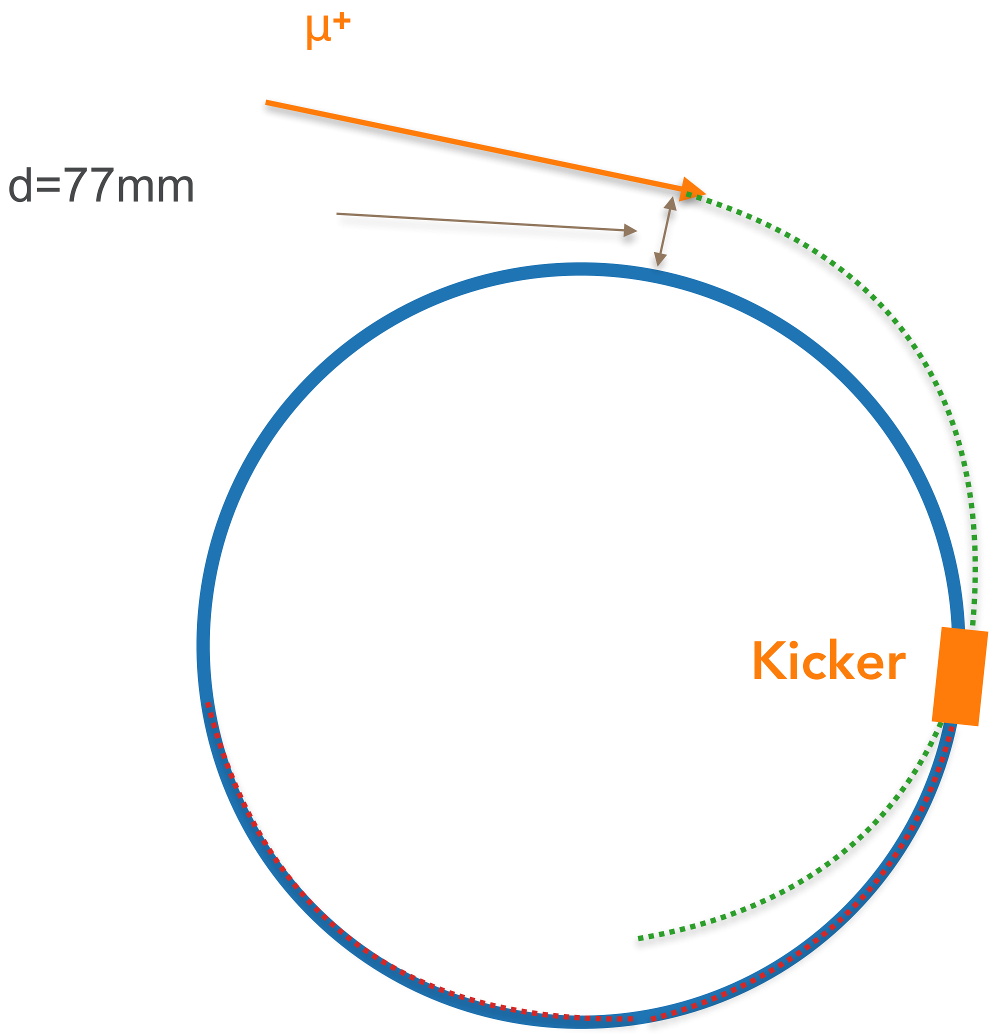
Kicker Upgrade

- Kicker "knock" the muons to their expected orbit

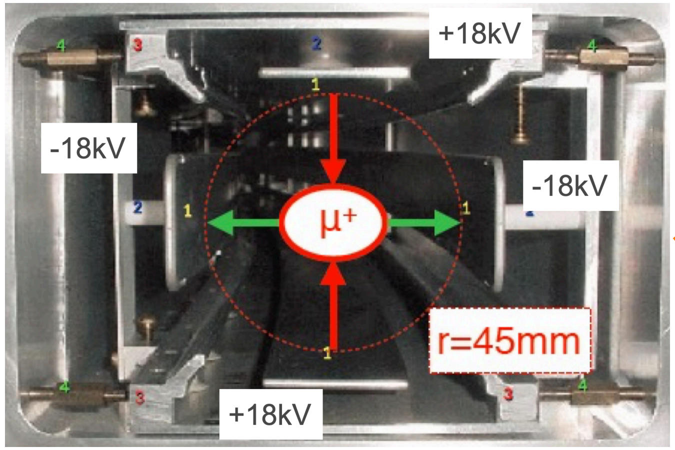


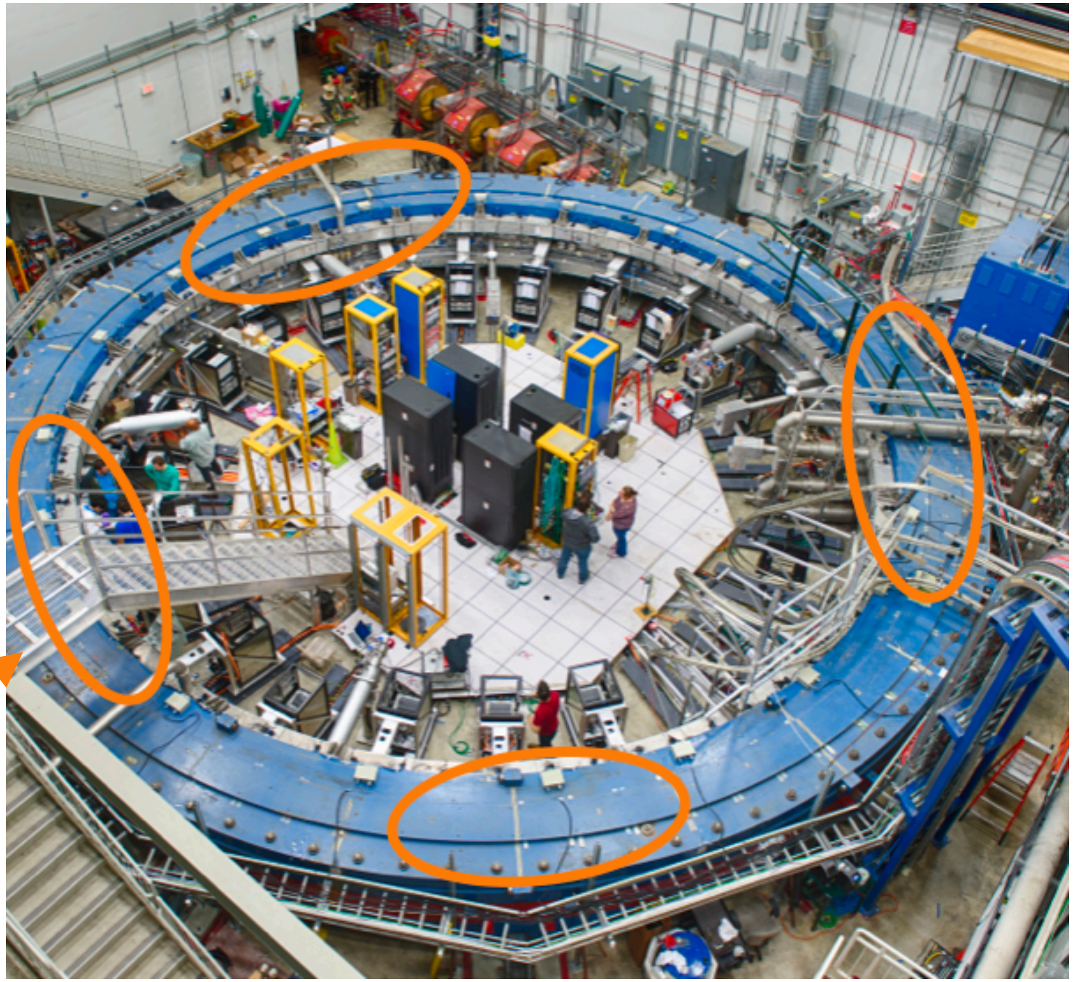
- Upgrade since run1:
  - ✓ New kicker cable allow for proper kicker
  - ✓ Reducing the equilibrium radius

ESQs Upgrade

- 4 ESQs to provide vertical focus of muons

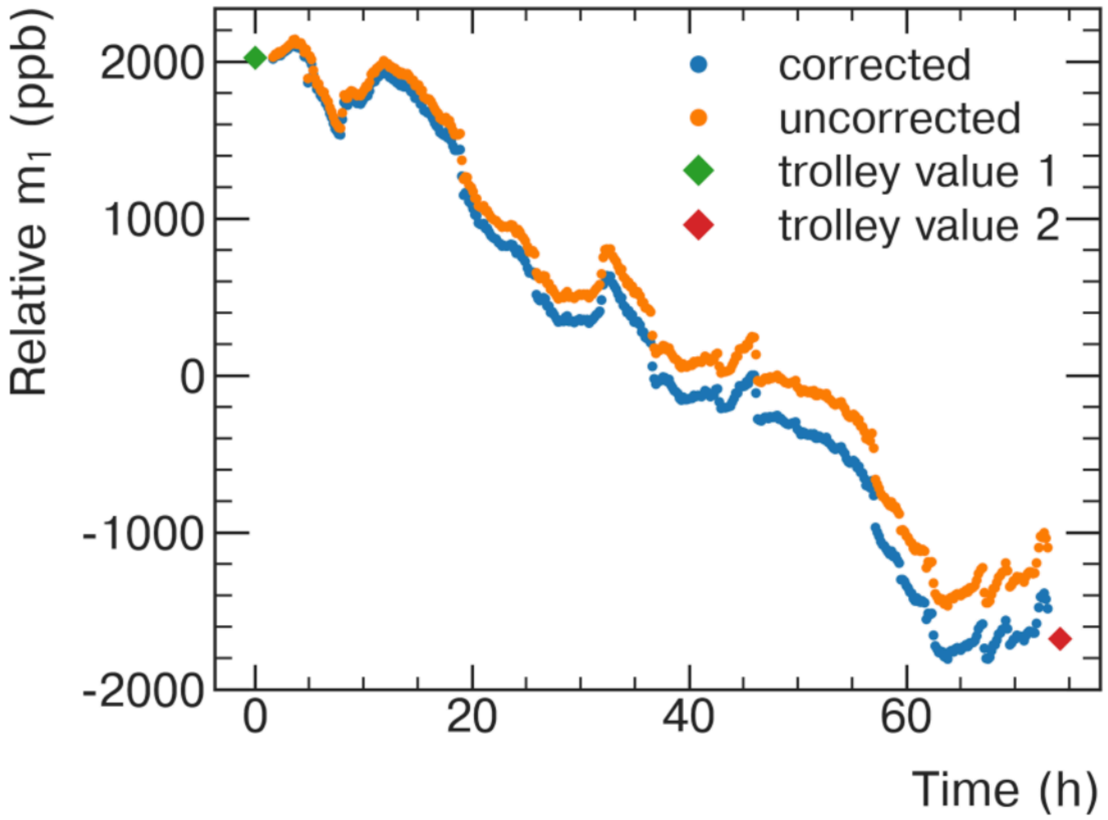
- Upgrade since run-1:
  - ✓ Fixed the broken resistors in run-1
  - ✓ Additional RF on the ESQ since the end of run-4 allows to reduce the CBO





Field Measurement Upgrade

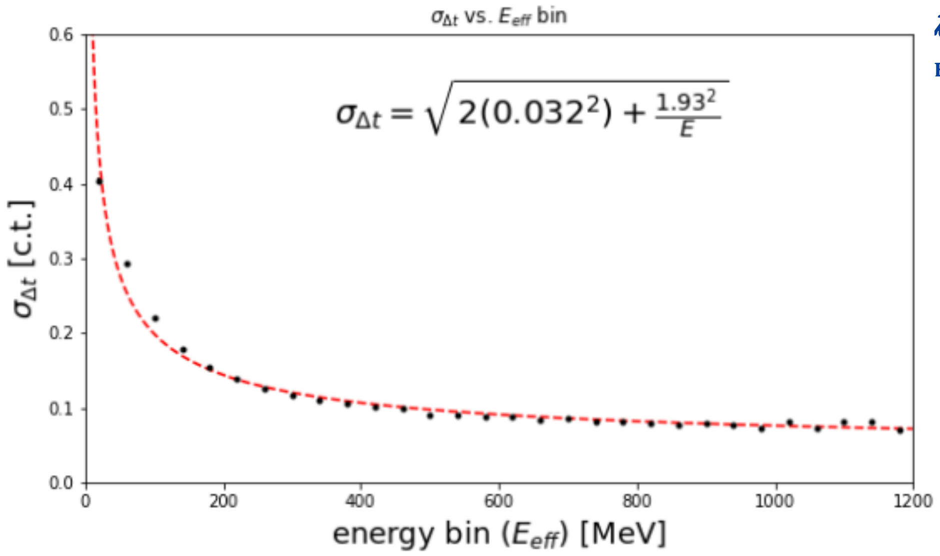
- Magnetic field provided by the super-conducting magnets

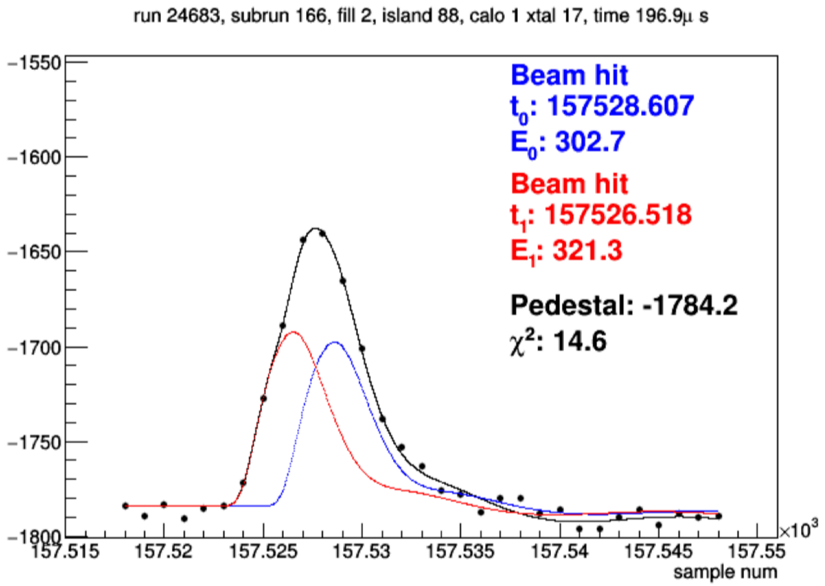


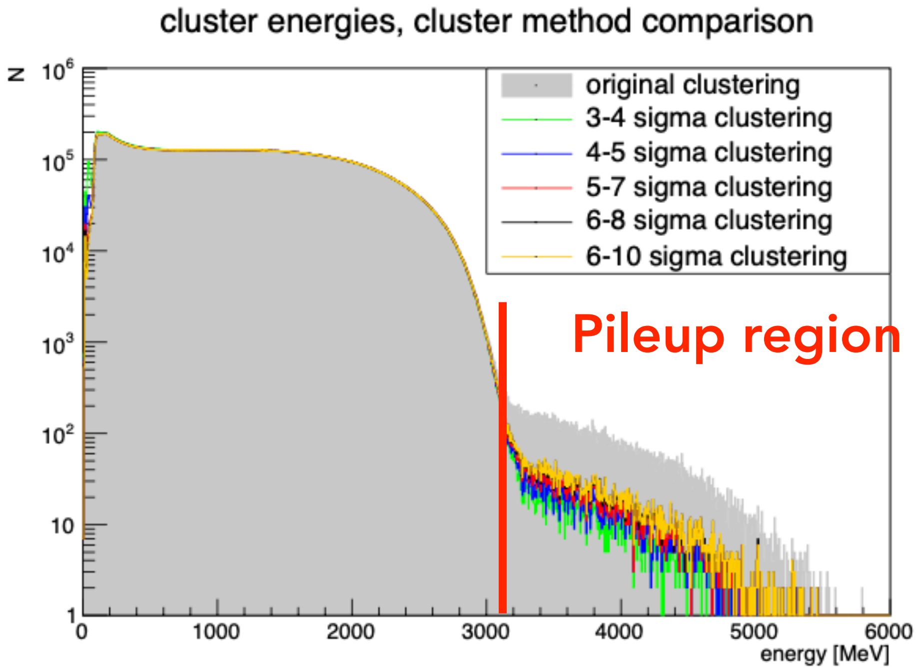
- **Field stability** improved since run-1:
  - ✓ Thermal insulation (run-2)
  - ✓ Improved AC (run-3)

Positron Reconstruction Upgrade

- Run-1 used constant time resolution to separate positrons
- Energy dependent time resolution introduced to reduce pileup, by a factor of 4

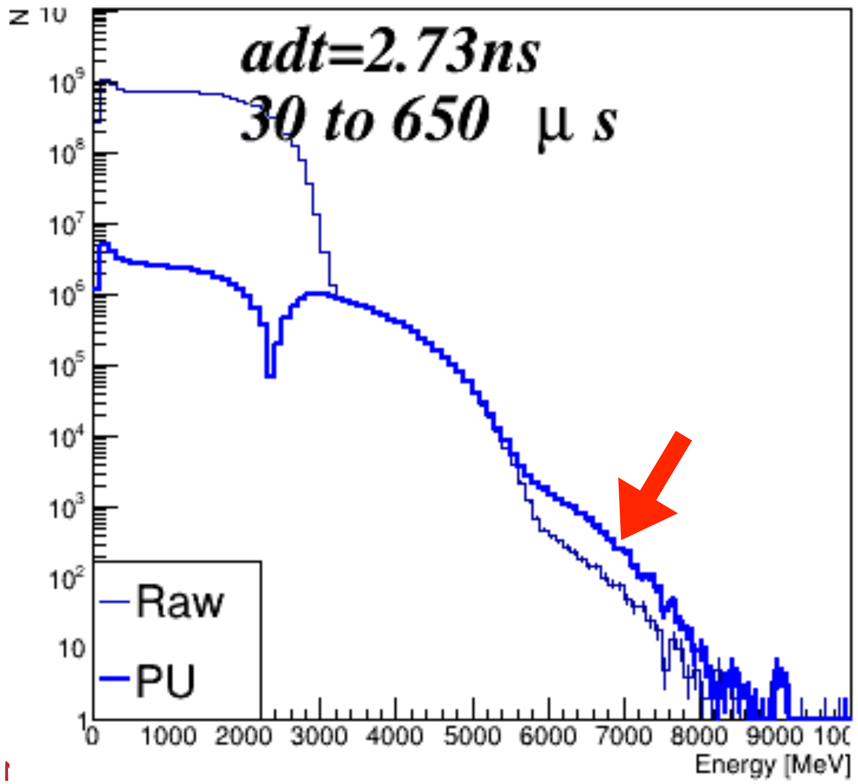


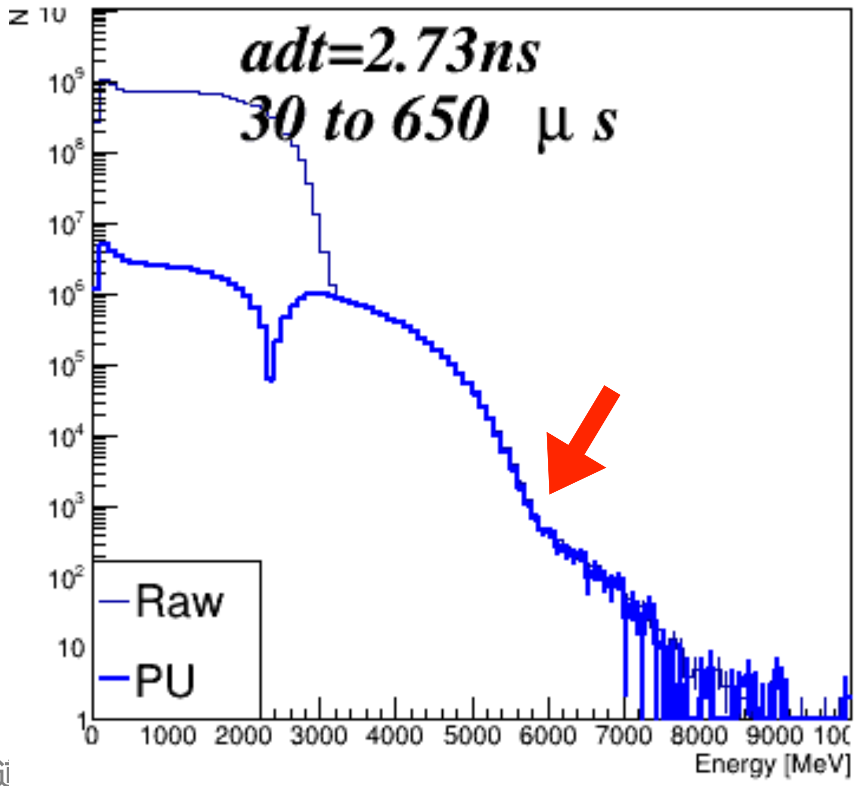




Pileup formula improvement (SJTU)

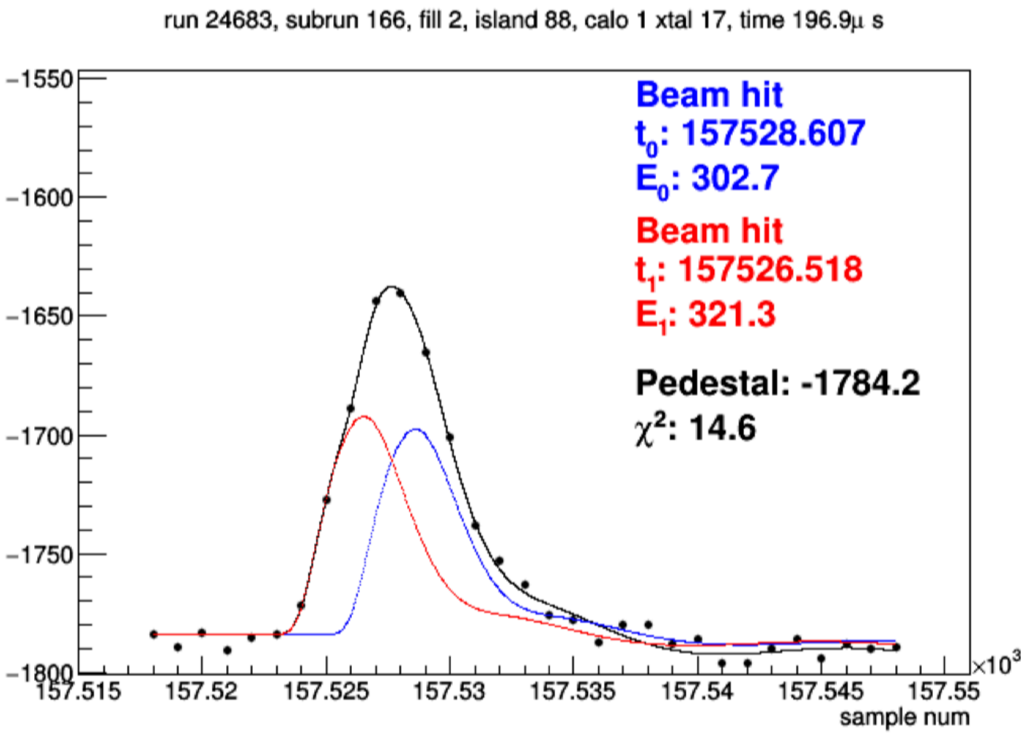
- Estimated the pileup to second order, corrected the formula
- Better agreement between raw and reconstructed pileup spectrum in pileup region

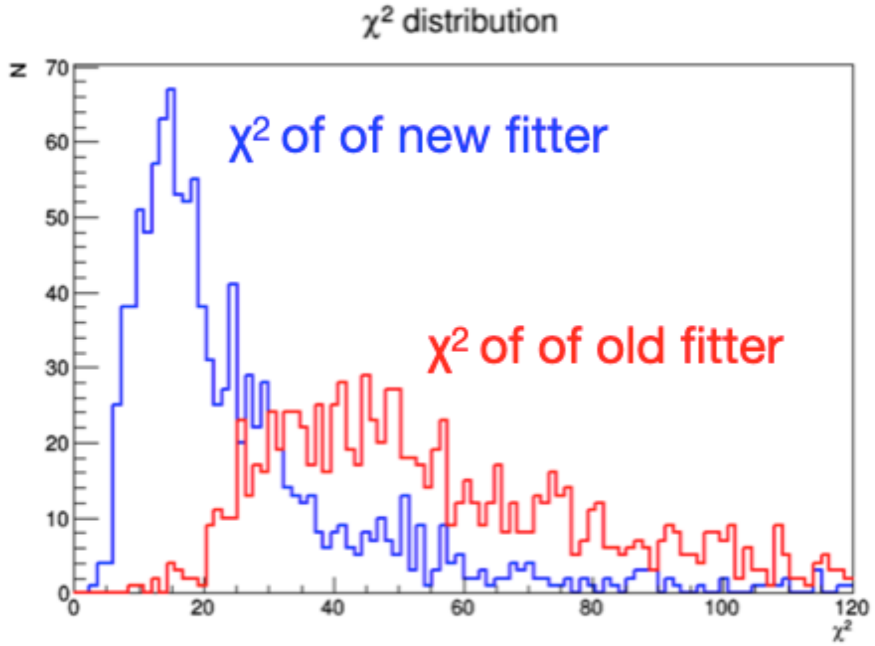


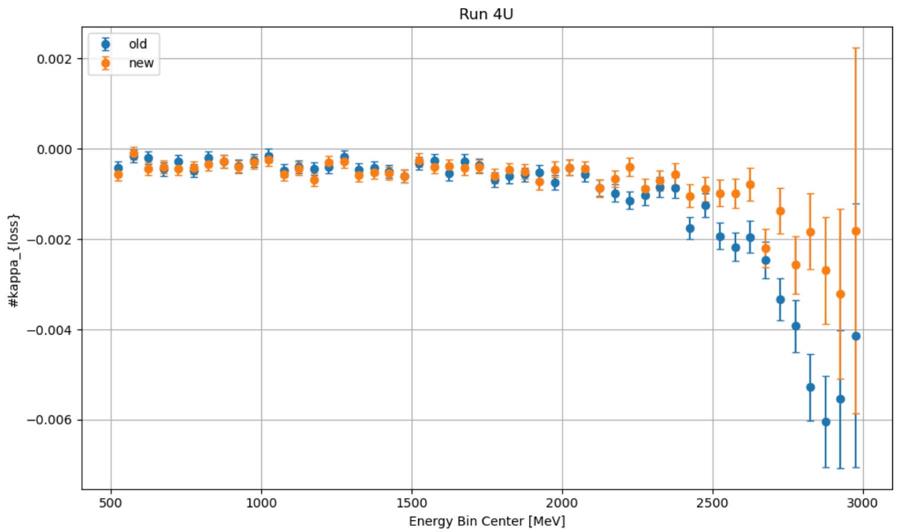


Improved Pulse-fitter (SJTU)

- Pulse-fitter reconstruct the (E,t) of digitized waveforms
- Changed the fitter behavior in fitting multiple-pulses
- Improved the energy dependence of the muon loss rate





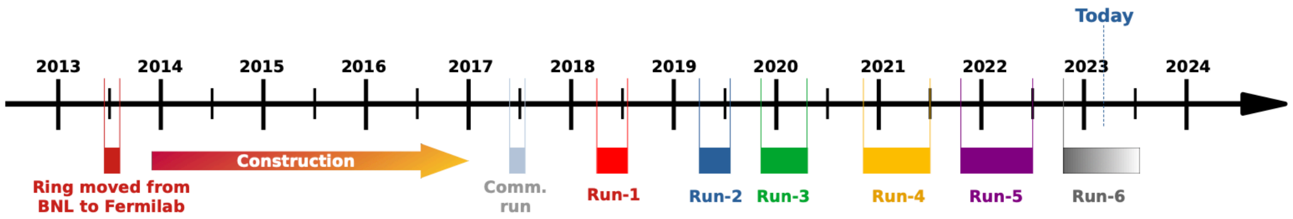


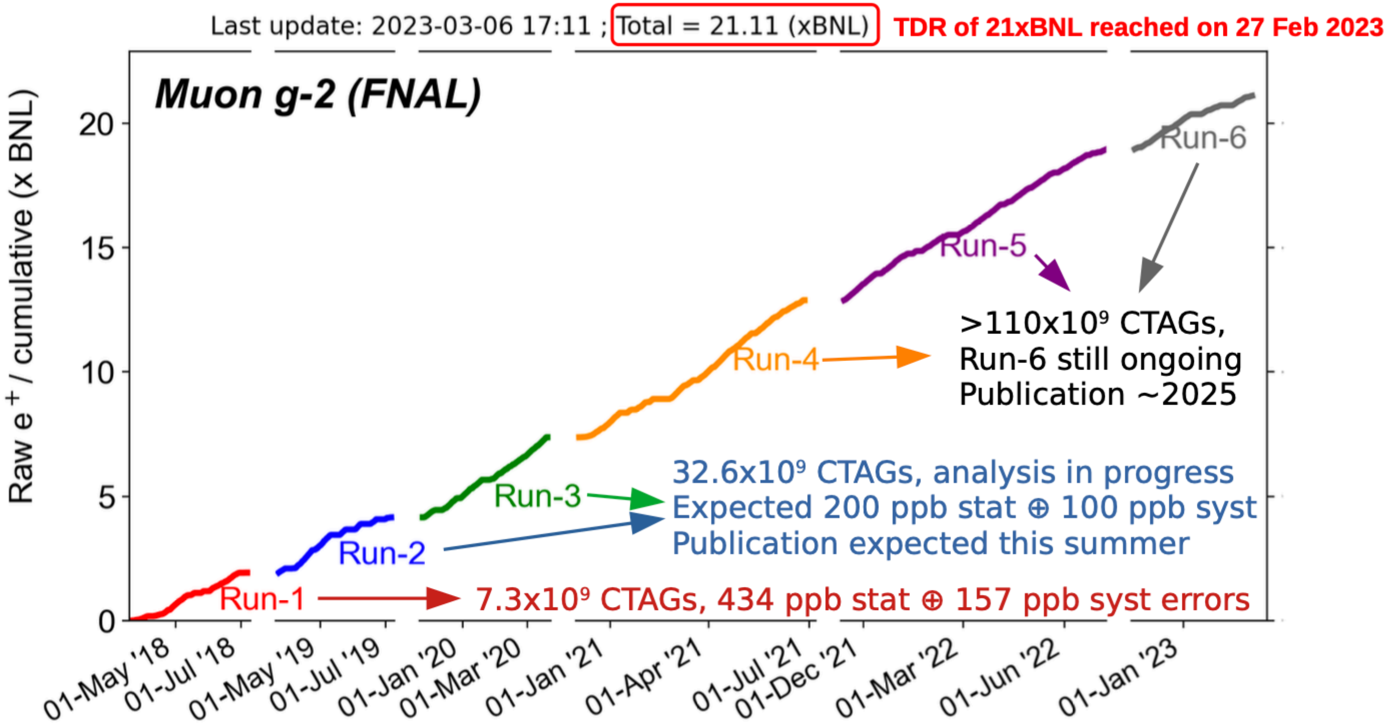
Outline

- Introduction
- Fermilab Muon g-2 experiment (Run-1)
- Improvements afterwards
- Outlook

From BNL to FNAL Run-6

- Run-1 is only ~5% of the final dataset
  - ✓434 ppb stat ⊕ 157 ppb syst
  - ✓Finalized in April 2021
- Run-2/3 analysis is about to finalize
  - ✓200 ppb stat ⊕ 100 ppb syst (expected)
  - ✓Publication this summer (expected)
- Run-6 is still ongoing
  - ✓21.11xBNL in total
  - ✓150 billion of raw e^+ in total





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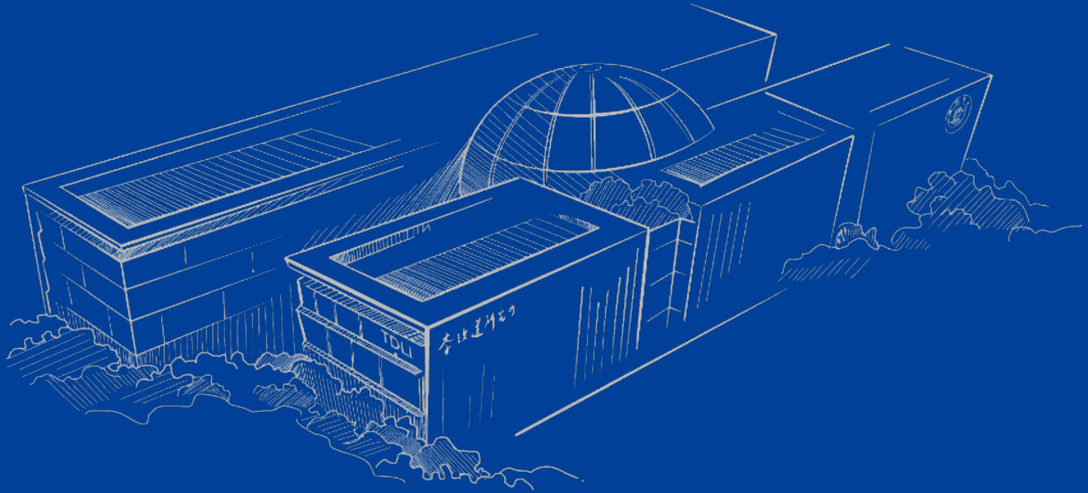
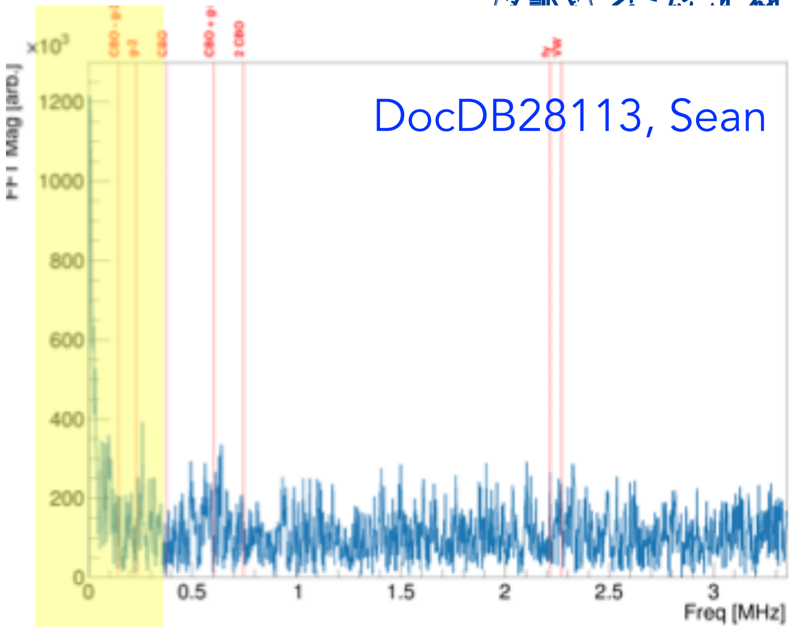
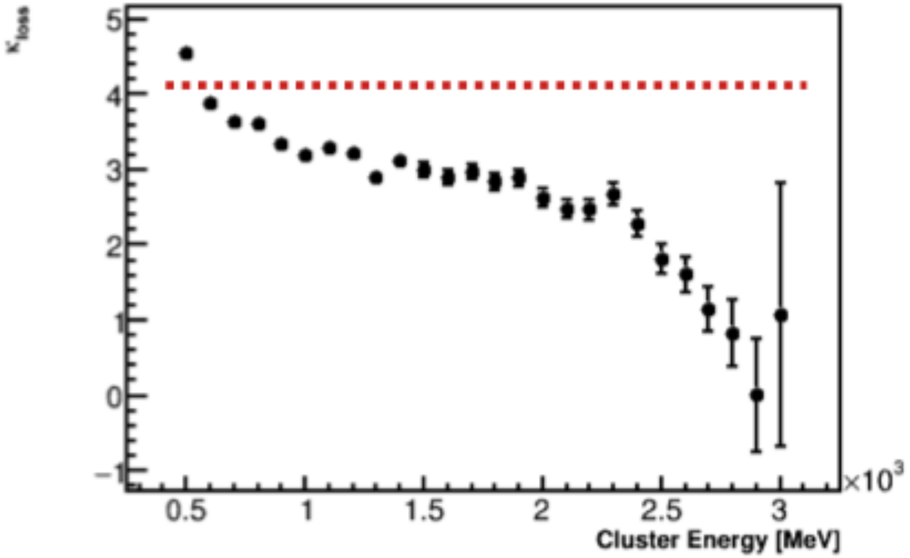


Residual Slow Effect in Run-1

- In Run-1, there are remaining early-to-late slow effect after all corrections
- Evidence observed in residual FFT and kloss vs energy
- Tested different models to remove this slow effect
  - ✓ No correction applied
  - ✓ ~20 ppb difference among models considered as systematic uncertainty



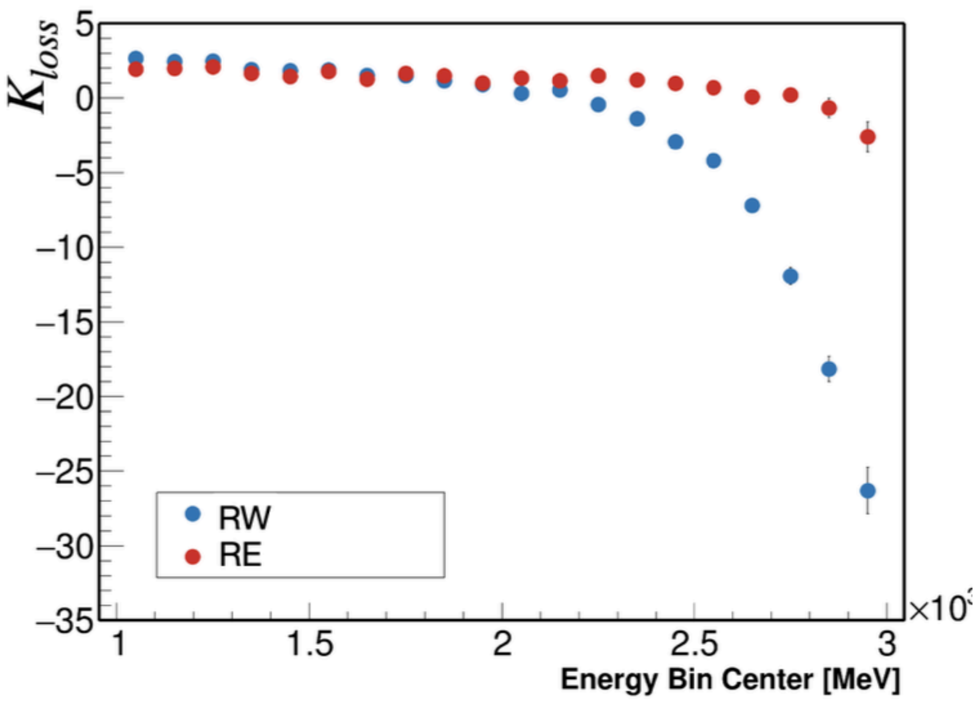
Peak in low frequencies residual FFT

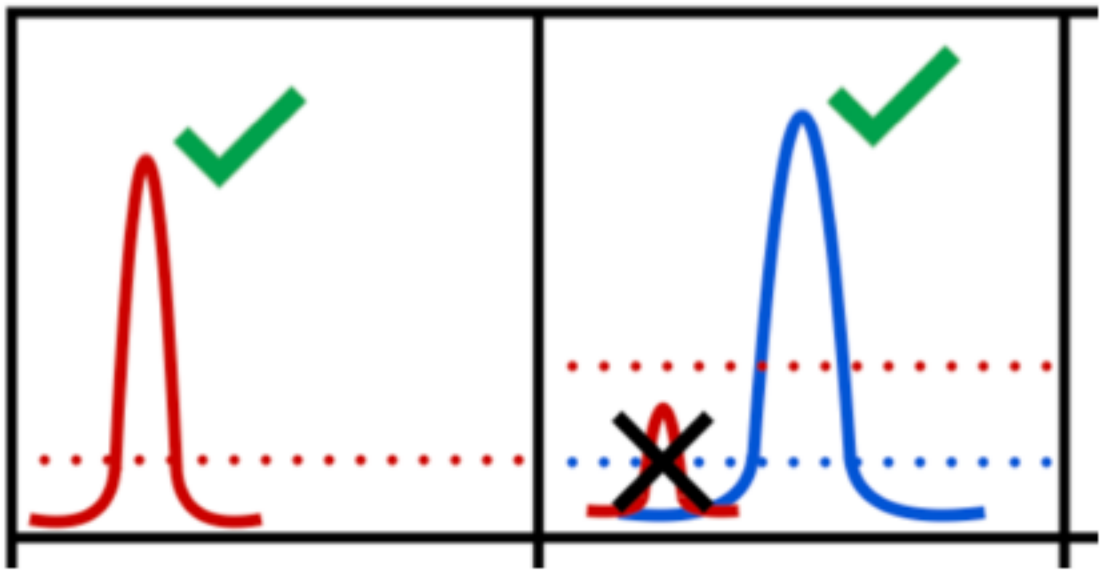


Drop of kloss in high energy region

Possible reason of the residual slow effect

- More significant slow effect in RW compared to RE
- Higher residual threshold in RW pulse-fitter can introduce early-to-late effect





Residual threshold in RW pulse-fitter

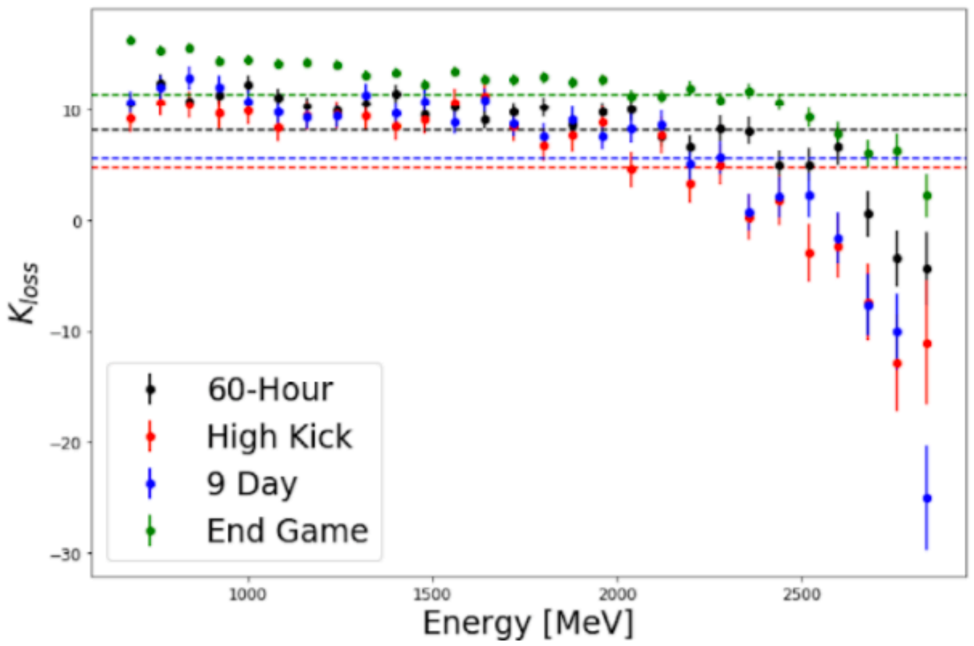
Ad-hoc gain correction in Run-1

- Gain-like effect first introduced by Aaron

$$G_{ad-hoc} = 1 + \delta_N \times 10^{-3} \times e^{-t/\tau} \cdot [1 - \delta_A \cdot \cos(\omega_a t + \phi)]$$

- Explained as "a gain perturbation" in the start time

- Improves the kloss vs energy in high energy region



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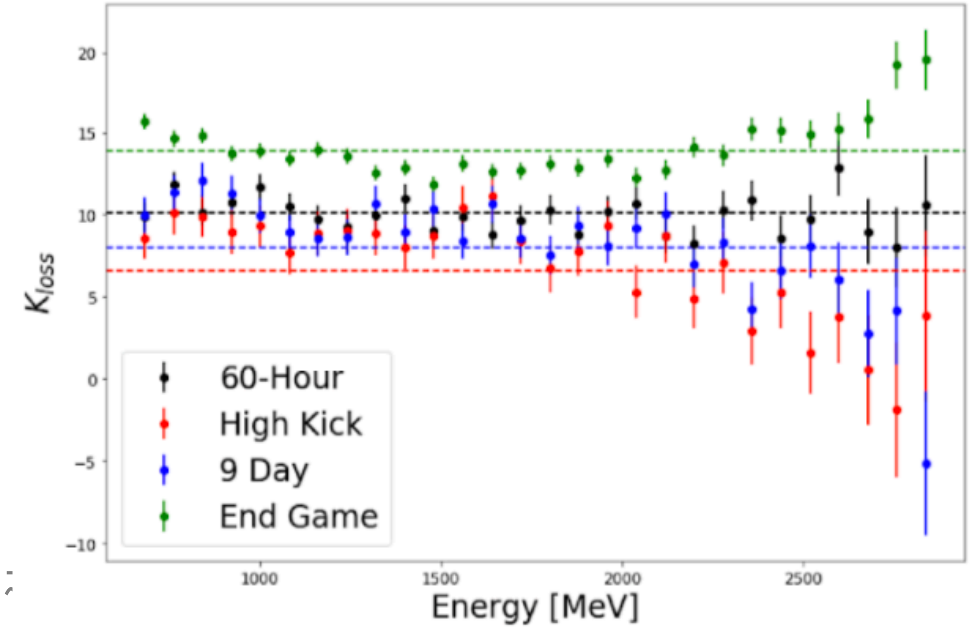


Pileup Correction with Shadow Method

- Pileup rate is proportional to the $\rho(t)^2$

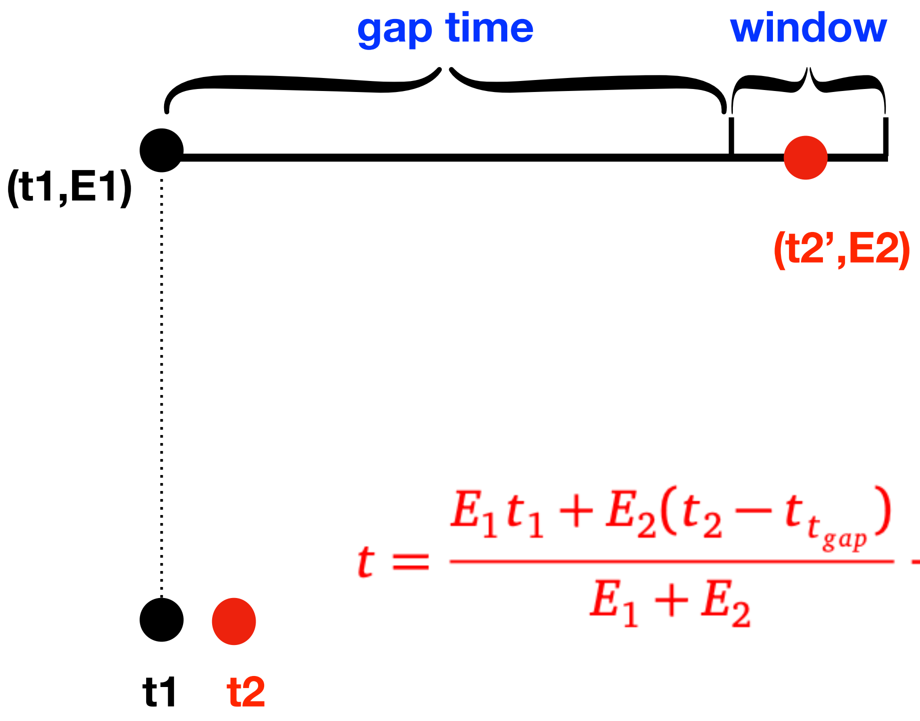
$$\rho(t)^2 \propto N(t)^2$$

- Pileup rate is proportional to the $N(t)^2$

$$\rho(t)^2 \cong \rho(t')\rho(t'+\Delta t)$$

- Reconstructed from shadow clusters

$$\rho_{PU}(E,t) = d_{12} - d_1 - d_2$$



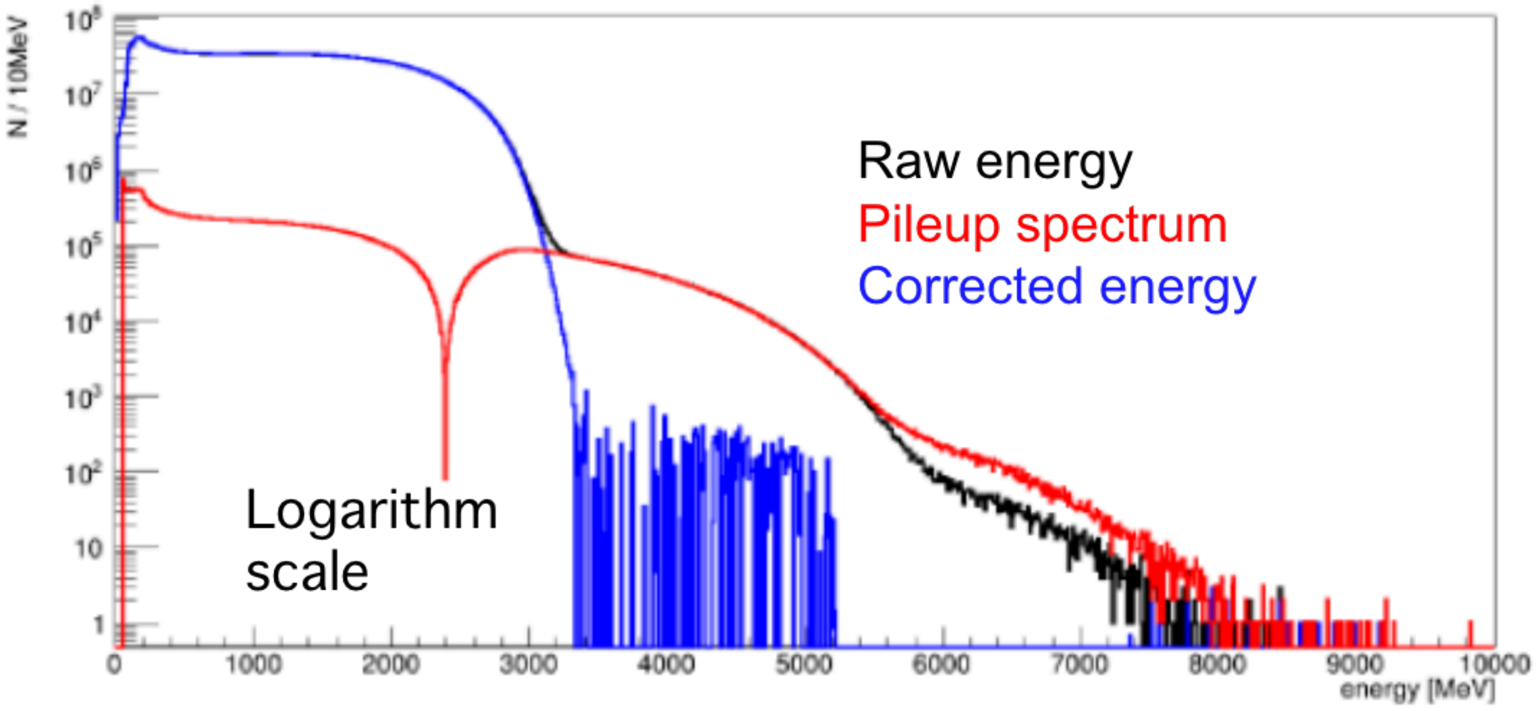
$$t = \frac{E_1 t_1 + E_2(t_2 - t_{t_{gap}})}{E_1 + E_2} + \frac{t_{gap}}{2}$$

First-order	d_1	d_2	d_{12}
Energy	E_1	E_2	E_{12}

Shadow clusters

Performance of pileup correction

- Most of the high energy entries (>3.1 GeV) are removed
- Only first-order correction in SJTU-Run1
- Included the second-order correction for Run2/3



In-Fill Gain Correction

- Gain drop during a fill monitored with laser calibration system

- Gain correction function is extracted as

$$G(t;\alpha,\tau) = 1 - \alpha \cdot \exp(-\frac{t}{\tau})$$

- Typical uncertainties (DS-2C):

$$\langle\frac{\Delta\alpha}{\alpha}\rangle \sim 5\,\%\,,\ \langle\frac{\Delta\tau}{\tau}\rangle \sim 0.7\,\%$$

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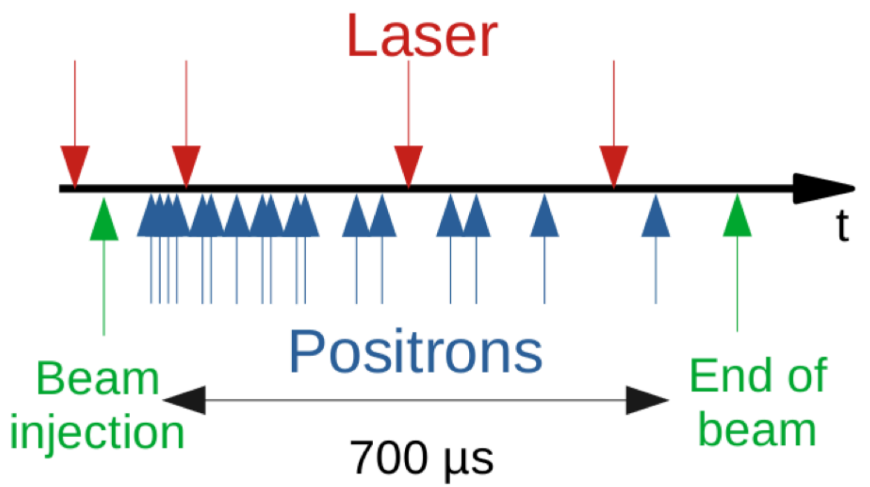
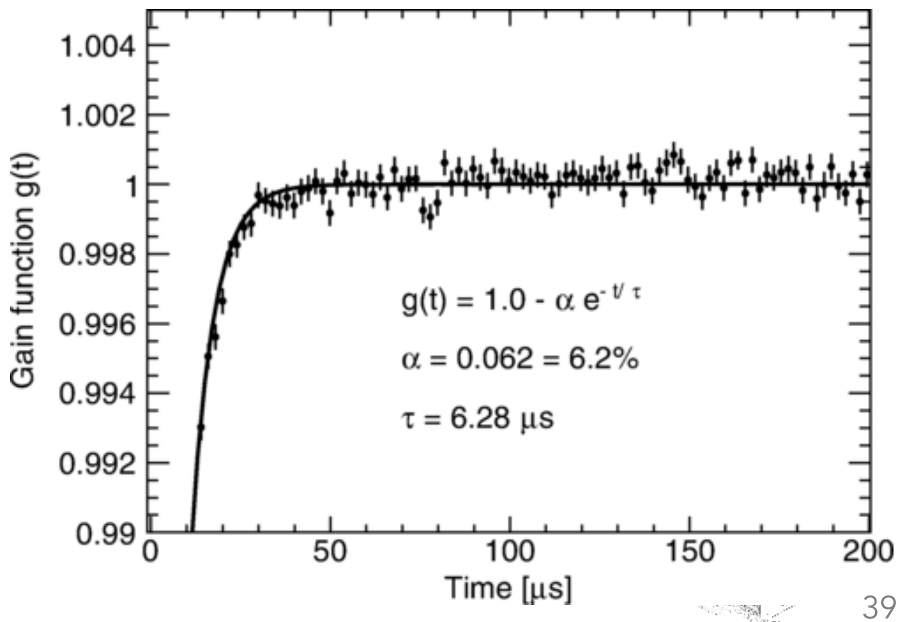
L. Cotrozzi

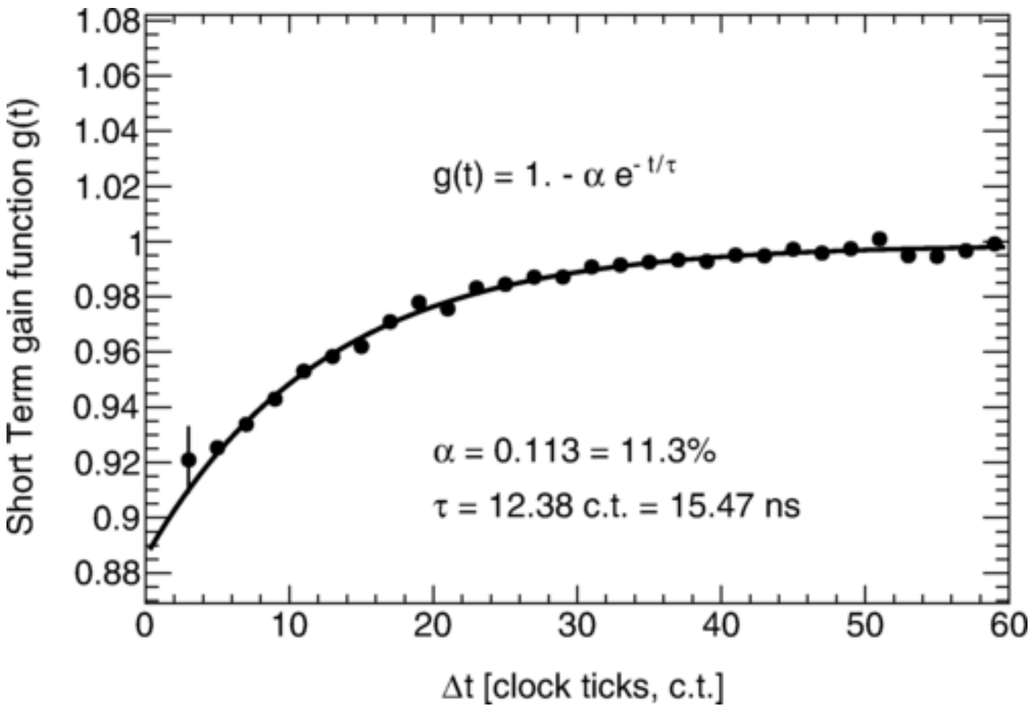




Short-Term Double-Pulse (STDP) Correction

- Caused by the finite recovery time of SiPM and amplifier
- Correction for consecutive hits within Δt~15 ns
- Typical uncertainties for Run-2/3
  - ✓ $\alpha \sim 2\,\%$ /GeV, $\langle\frac{\Delta\alpha}{\alpha}\rangle \sim 2\,\%$
  - ✓ $\tau \sim 15$ ns, $\langle\frac{\Delta\tau}{\tau}\rangle \sim 2\,\%$
  - ✓ $\beta \sim 5\,\%/^{\circ}C$ (Run-2 only)

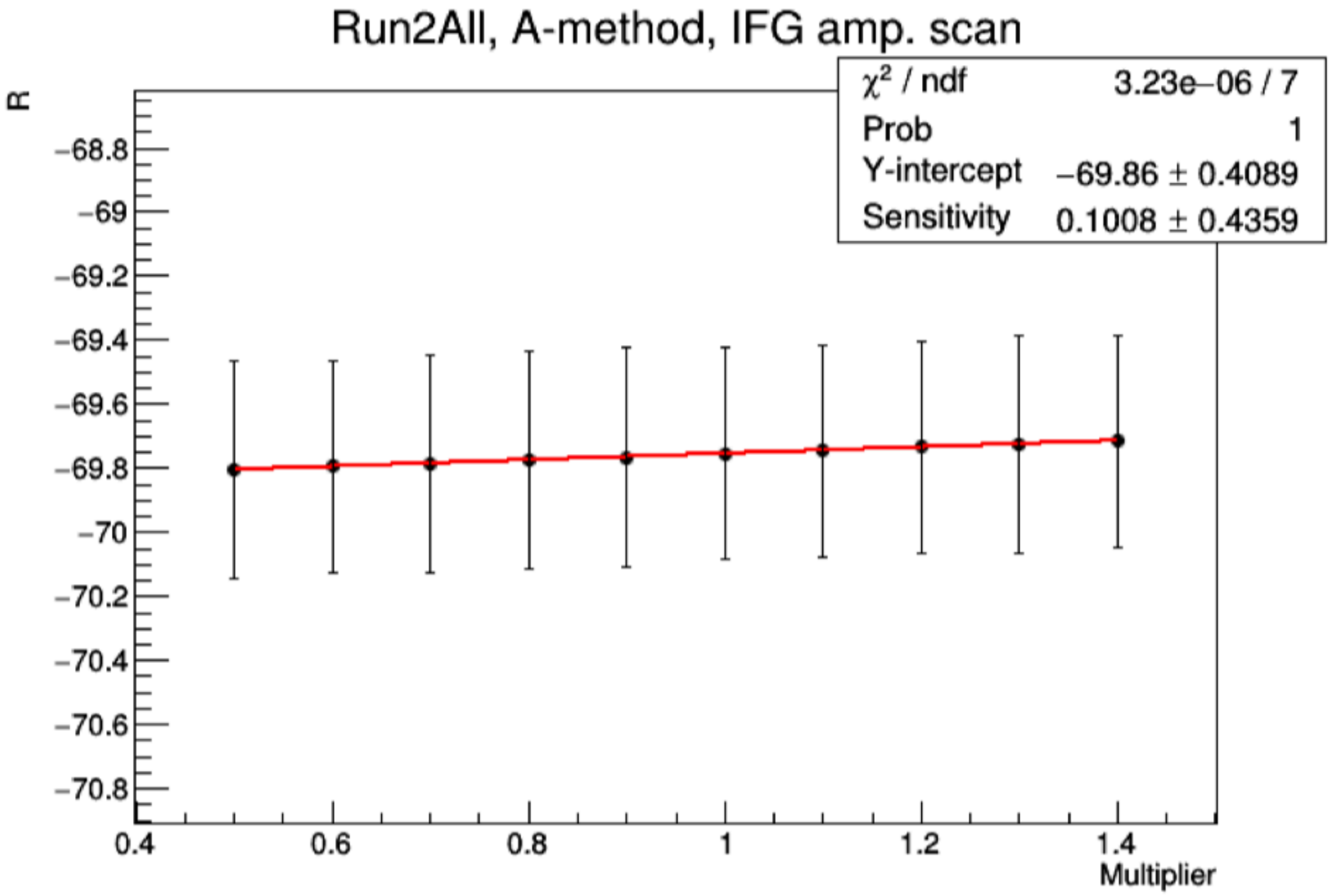
$$G(E_1, \Delta t, T; \alpha, \tau, \beta) =$$
$$1 - E_1 \cdot \alpha \cdot (1 + \beta T) \cdot \exp(-\frac{\Delta t}{\tau})$$



General Strategy of Gain Systematics

- Rescale the parameters (α,τ) with a multiplier m
- Extract the R-sensitivity as $\frac{dR}{dm}$
- Systematic uncertainty is then estimated as

$$\sigma(R) = \frac{dR}{dm} \cdot \sigma(m)$$



Electric Field Correction

李政道研究所 TSUNG-DAO LEE INSTITUTE

$$\mathcal{R}'_\mu = \frac{\omega_a}{\tilde{\omega}'_p(T_r)} = \frac{f_{clock}\,\omega_a^m\,(1 + C_E + C_p + C_{lm} + C_{pa})}{f_{cali}\,\langle \omega_p(x,y,\phi,T_r) \times M(x,y,\phi)\rangle\,(1 + B_q + B_k)}$$

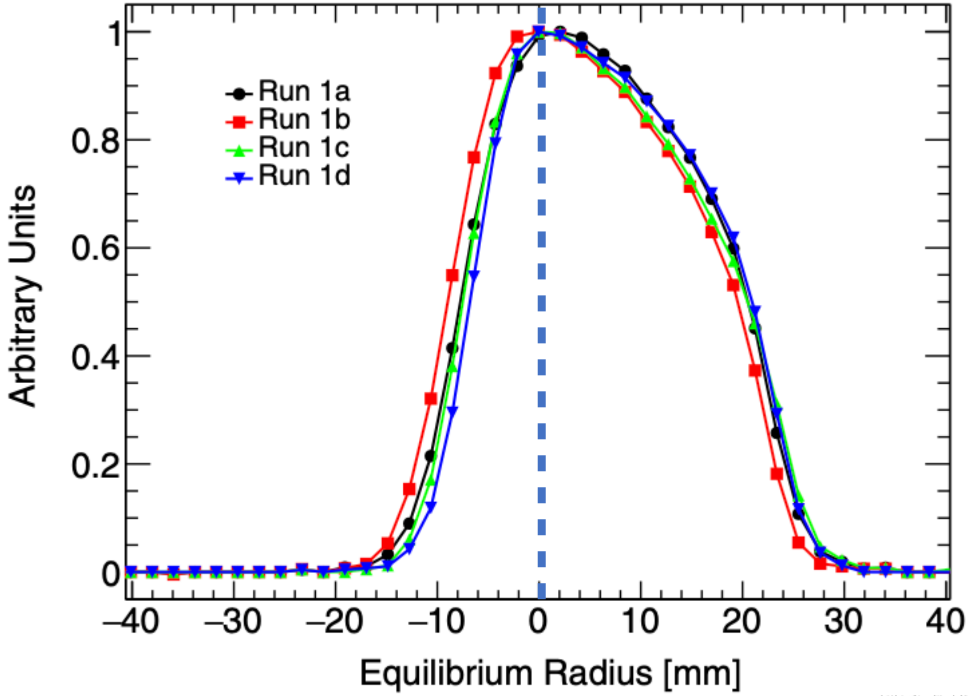
$$C_e \approx 2n(1-n)\beta_0^2 \frac{\langle x_e^2 \rangle}{R_0^2}$$

- "Magic momentum" to cancel the effect of $\mathbf{E_r}$ → p_0=3.094 GeV

- Based on muon radial distribution measurement

Cancelled with "magic momentum"

$$\frac{d(\hat{\beta}\cdot\vec{S})}{dt} = -\frac{q}{m}\vec{S}_T \cdot \left[a_\mu \hat{\beta} \times \vec{B} + \beta\left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{E}}{c}\right]$$



Pitch Correction

$$\mathcal{R}'_\mu = \frac{\omega_a}{\tilde{\omega}'_p(T_r)} = \frac{f_{clock}\, \omega_a^m \;(1 \;+ C_E + C_p + C_{lm} + C_{pa})}{f_{cali}\, \langle \omega_p(x,y,\phi,T_r) \times M(x,y,\phi) \rangle\, (1 + B_q + B_k)}$$

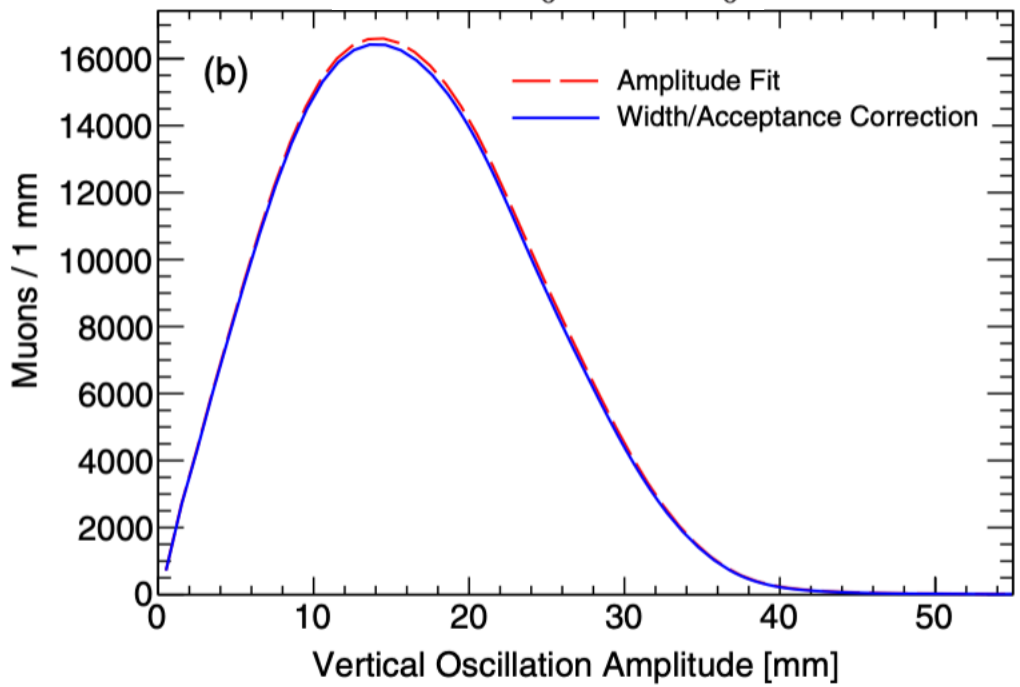
- Correction needed since $\hat{\beta}$, $\vec{B}$ are not exactly perpendicular

- Based on the vertical distribution of muons

$$\frac{d(\hat{\beta}\cdot\vec{S})}{dt} = -\frac{q}{m}\vec{S}_T \cdot \left[a_\mu \hat{\beta}\times\vec{B} + \beta\left(a_\mu - \frac{1}{\gamma^2-1}\right)\frac{\vec{E}}{c}\right]$$

Not perpendicular

$$C_p \approx \frac{n}{2}\frac{\langle y^2\rangle}{R_0^2} = \frac{n}{4}\frac{\langle A^2\rangle}{R_0^2}$$



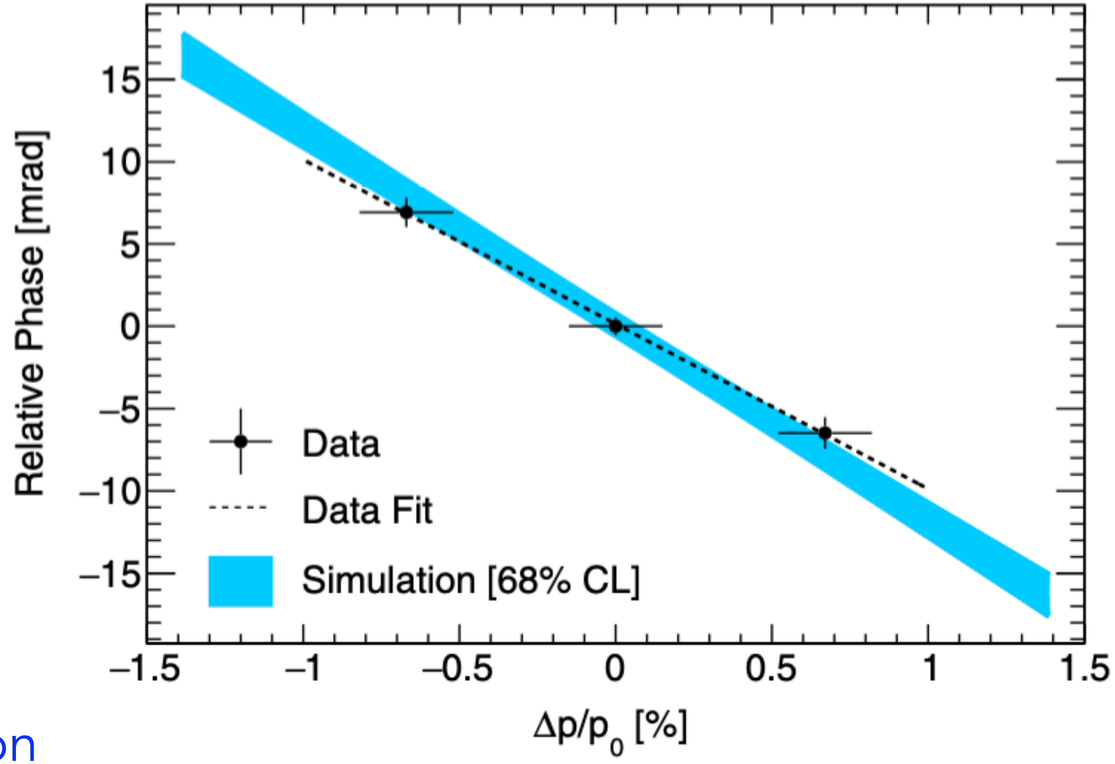
Lost muon correction

$$\mathscr{R}'_\mu = \frac{\omega_a}{\tilde{\omega}'_p(T_r)} = \frac{f_{clock}\, \omega_a^m \, (1 + C_E + C_p + \boxed{C_{lm}} + C_{pa})}{f_{cali} \left\langle \omega_p(x,y,\phi,T_r) \times M(x,y,\phi) \right\rangle (1 + B_q + B_k)}$$

- Lost muon have different <p> leads to time dependent $\varphi(t)$
- Corrections estimated as bias between constant φ_0 and time-dependent $\varphi(t)$

$$\frac{d\varphi_0}{dt} = \frac{d\varphi_0}{d\langle p\rangle} \boxed{\frac{d\langle p\rangle}{dt}} \neq 0$$

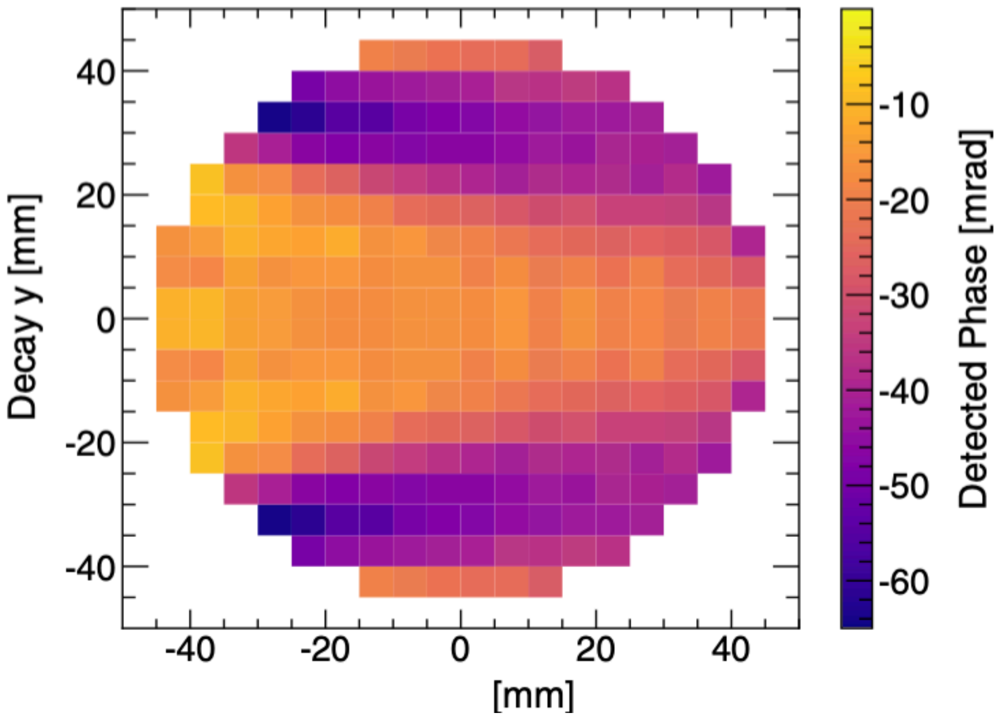
from different <p> of lost muon



Phase acceptance correction

$$\mathcal{R}'_\mu = \frac{\omega_a}{\tilde{\omega}'_p(T_r)} = \frac{f_{clock}\,\omega_a^m\,(1 + C_E + C_p + C_{lm} + C_{pa})}{f_{cali}\,\langle \omega_p(x,y,\phi,T_r) \times M(x,y,\phi)\rangle\,(1 + B_q + B_k)}$$

- Muons have coordinate dependent phase which not reflected in the nominal fit function

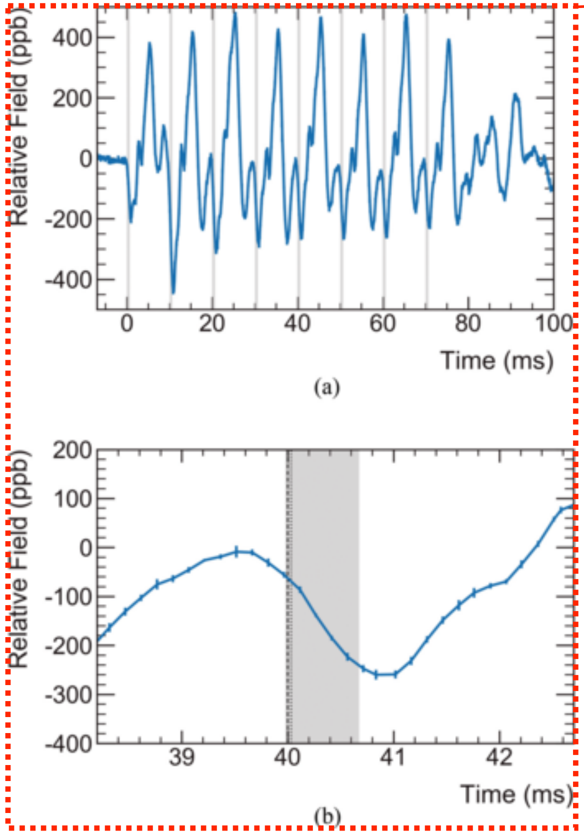


- How to estimate the correction:
  1. Measure the phase map
  2. Create pseudo data
  3. Fit pseudo data and get the bias

$$N(t,E) = N_0(E)e^{-t/\gamma\tau_\mu}\{1 + A(E)\cos[\omega_a t + \varphi_0(E)]\}$$

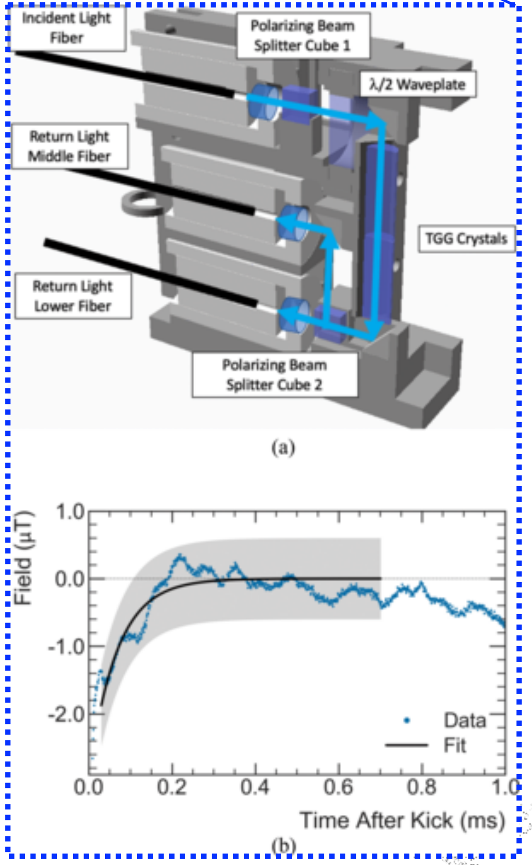
Magnetic Field Corrections

$$\mathcal{R}'_\mu = \frac{\omega_a}{\tilde{\omega}'_p(T_r)} = \frac{f_{clock}\,\omega_a^m\,(1 + C_E + C_p + C_{lm} + C_{pa})}{f_{cali}\,\langle \omega_p(x,y,\phi,T_r) \times M(x,y,\phi)\rangle\,(1 + B_q + B_k)}$$



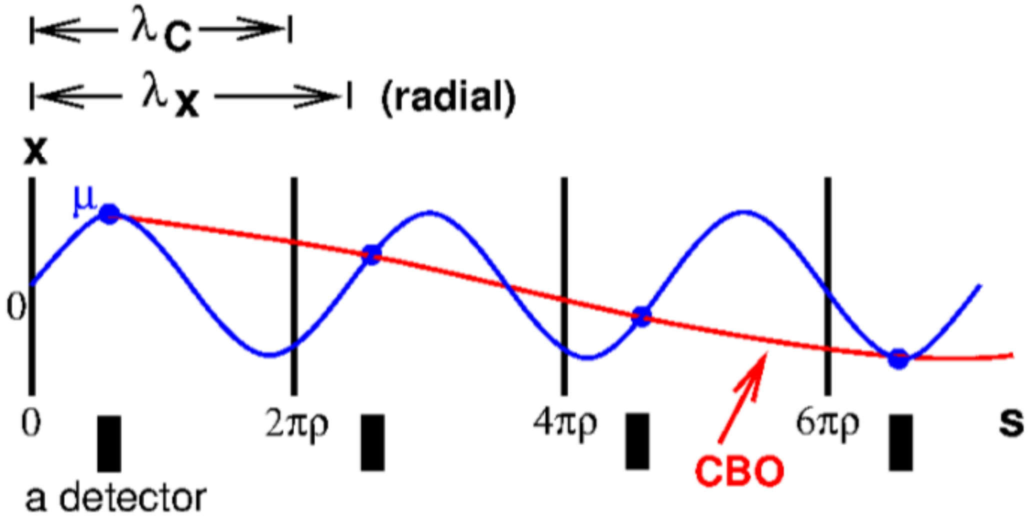
- ESQ transient field (μs scale) measured with dedicated fixed probe
- Zoomed in time structure of the field oscillation

- Kicker transient fields (ms scale) correction measured with fiber magnetometer
- Measured data (blue) after Kick



What's the Calorimeter Observes

- 24 Calorimeters sampling the beam by cyclotron frequency ω_c
- Radial oscillation observed as (aliased) $\omega_{CBO} = \omega_c - \omega_x$
- Beam width oscillation observed as $2\omega_{CBO}$
- Vertical oscillation observed as ω_y
- Vertical width oscillation observed as (aliased) $\omega_{VW} = \omega_c - 2\omega_y$

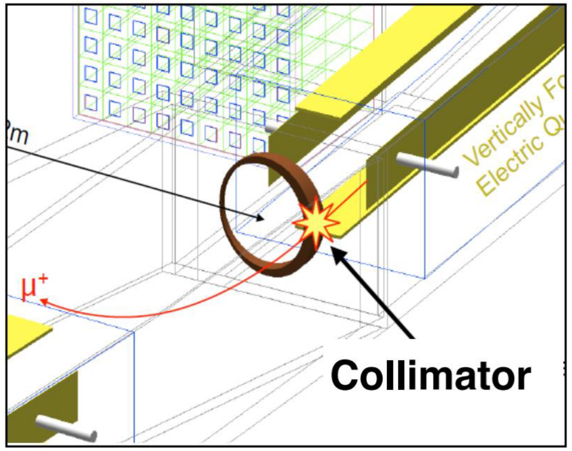


Physical frequency	Calculated expression	Frequency (rad/μs) $n = 0.108$	Frequency (rad/μs) $n = 0.120$
ω_c	v/R_0	42.15	42.15
ω_x	$\sqrt{1-n}\omega_c$	39.81	39.54
ω_y	$\sqrt{n}\omega_c$	13.85	14.60
ω_{CBO}	$\omega_c - \omega_x$	2.34	2.61
ω_{VW}	$\omega_c - 2\omega_y$	14.45	12.95
ω_a	$ea_\mu B/m$	1.44	1.44

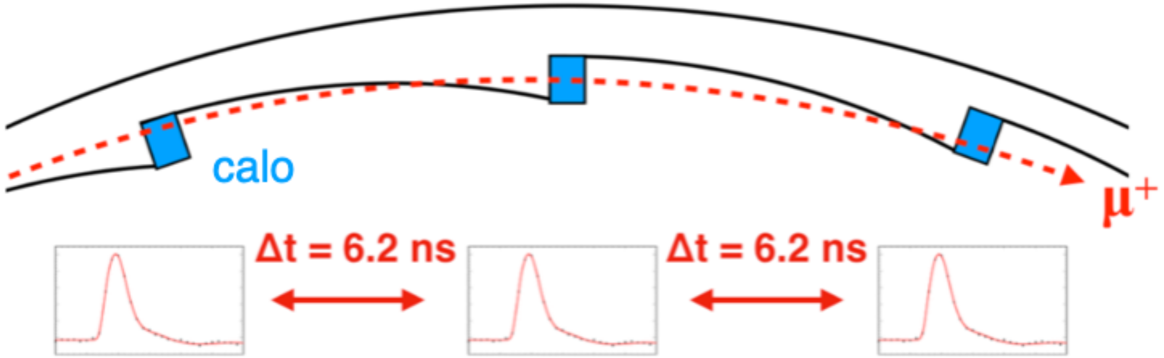
Muon Loss in the Storage Ring

- Muon losses due to interact with collimator or other effects

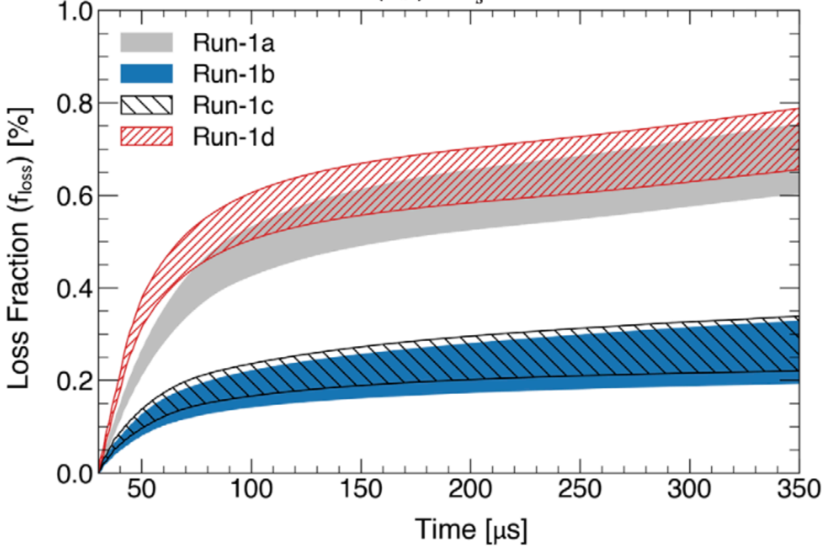
$$N_0 \to N_0\Lambda(t) = N_0\left(1 - K_{\text{loss}}\int_0^t e^{t'/\gamma\tau_\mu}L(t')dt'\right)$$



- Identification: μ⁺ sequentially hit ≧3 calorimeters with MIP energy deposits (≈170 MeV)



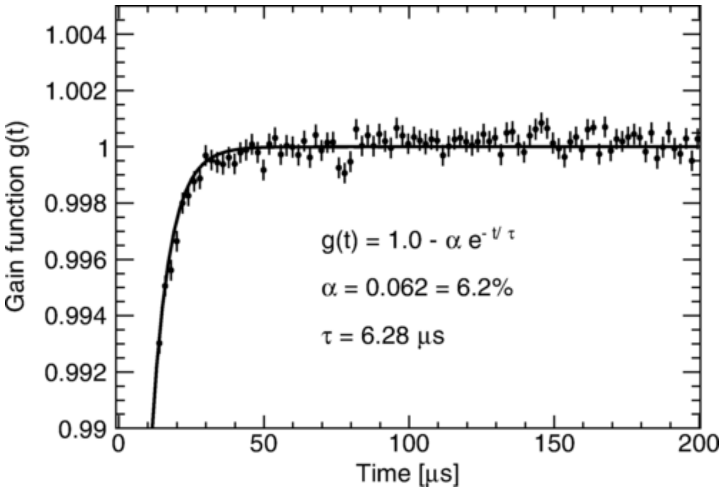
$$f_{\text{loss}}(t) = \frac{K_{\text{loss}}}{\Lambda(t_s)}\int_{t_s}^{t} e^{t'/\gamma\tau_\mu}L(t')dt'$$



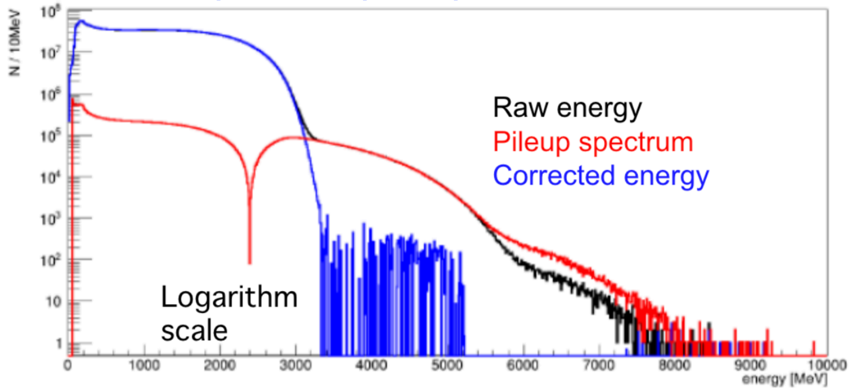
Source of systematic uncertainties

- Calorimeter gain changes
- Imperfect pileup correction
- Beam dynamics modeling: CBO, muon loss
- Others: randomization

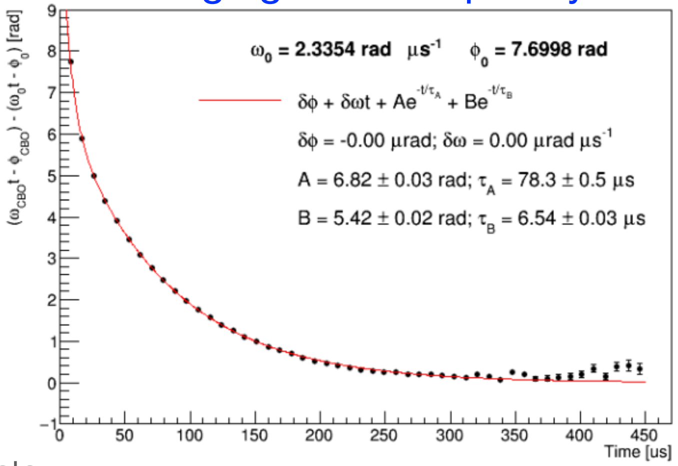
Gain change during a muon fill



Imperfect pileup correction

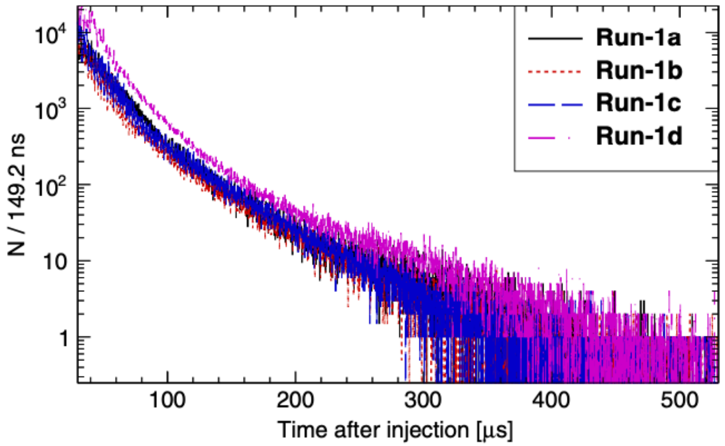


Time-changing CBO frequency model



Changing CBO frequency data from tracker data

Measured muon loss spectrum



Systematic uncertainties in ω_a^m analysis

- Gain effects
- Pileup correction
- Beam dynamics
- Time randomization

Systematic	Run1a	Run1b	Run1c	Run1d
in fill gain amplitude	2	7	4	5
in fill gain time constant	2	1	1	4
STDP gain amplitude	<1	<1	<1	<1
residual gain	77	14	4	39
pileup amplitude	14	13	9	7
pileup time model	47	53	44	41
pileup energy model	11	8	12	7
unseen pileup	1	2	2	4
triple pileup	4	5	4	4
CBO frequency	7	13	13	13
CBO envelope	20	3	8	3
CBO time constant	2	9	6	1
lost muon	<1	<1	<1	<1
time randomization	27	17	15	11
total systematic	116	87	77	77