A quark and lepton model with flavor specific DM and muon g-2 in modular A_4 and hidden U(1) symmetry

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1. Introduction
2. Model setup
3. Phenomenology
4. Summary and discussion

Some issues in the SM suggesting new physics

- Existence of dark matter in our Universe
- Non-Zero Neutrino mass
  - **We need a mechanism to generate neutrino mass**
  - **Smallness of the mass should be explained**
- Flavor structure
  - **There is no principle to determine flavor structure in the SM**
  - **A symmetry would explain it**
- Some experimental anomalies
  - **Muon g-2**
  - **W boson mass anomaly**
  - **Etc.**

Neutrino mass and mixing

Mixing

$$\nu_l = \left(U_{PMNS}\right)_{li} \nu_i$$

Flavor eigenstate (l=e,μ,τ)

Mass eigenstate (i=1,2,3)

http://www.nu-fit.org/?q=node/228

NuFIT 5.0 (2020)

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.7$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.44^{+0.78}_{-0.75}$	$31.27 \to 35.86$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.570^{+0.018}_{-0.024}$	$0.407 \to 0.618$	$0.575^{+0.017}_{-0.021}$	$0.411 \to 0.621$
$\theta_{23}/^\circ$	$49.0^{+1.1}_{-1.4}$	$39.6 \to 51.8$	$49.3^{+1.0}_{-1.2}$	$39.9 \to 52.0$
$\sin^2\theta_{13}$	$0.02221^{+0.00068}_{-0.00062}$	$0.02034 \to 0.02430$	$0.02240^{+0.00062}_{-0.00062}$	$0.02053 \to 0.02436$
$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \to 8.97$	$8.61^{+0.12}_{-0.12}$	$8.24 \to 8.98$
$\delta_{CP}/^\circ$	195^{+51}_{-25}	$107 \to 403$	286^{+27}_{-32}	$192 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.514^{+0.028}_{-0.027}$	$+2.431 \to +2.598$	$-2.497^{+0.028}_{-0.028}$	$-2.583 \to -2.412$

$$U_{\rm PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}, \qquad \text{(II.14)}$$

Mass

$$\Delta m^2_{\rm sol} = m_2^2 - m_1^2,$$

$$({\rm NH}): \ \Delta m^2_{\rm atm} = m_3^2 - m_1^2, \quad ({\rm IH}): \ \Delta m^2_{\rm atm} = m_2^2 - m_3^2,$$

Mixing and mass differences are measured by neutrino oscillation experiments

Neutrino mass and mixing is a mystery in particle physics

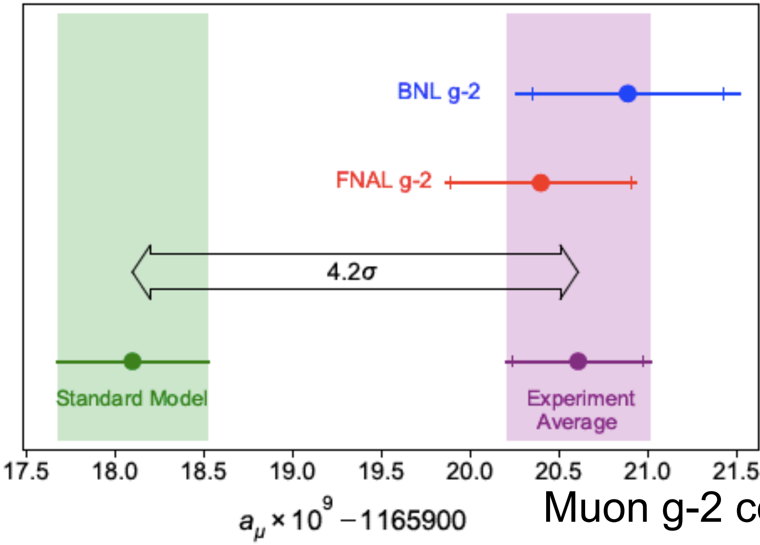
- ❖ How can we generate neutrino masses?
- ❖ How can we explain its smallness

  ⇨ Mass scale is less than O(eV)

- ❖ Ordering of neutrino mass : normal or inverted?
- ❖ Symmetry behind mixing pattern?

1. Introduction

Muon anomalous dipole magnetic moment (muon g-2)



$$a_\mu^{BNL} = (11659208.9 \pm 5.4 \pm 3.3) \times 10^{-10}$$

$$a_\mu^{FNAL} = (11659204.0 \pm 5.1 \pm 1.9) \times 10^{-10}$$

$$\Delta a_\mu = a_\mu^{\exp} - a_\mu^{SM} = (25.1 \pm 5.9) \times 10^{-10}$$

Combined result (4.2σ deviation)

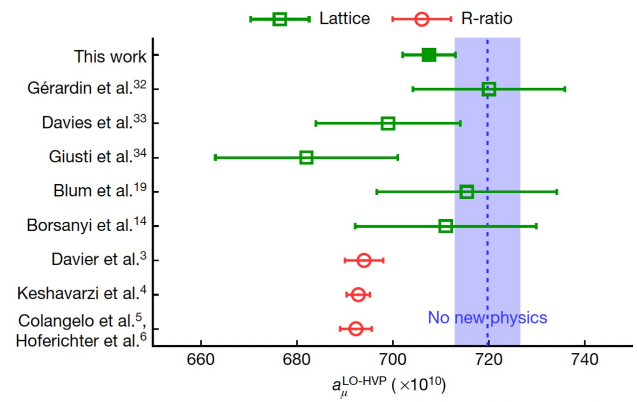
Muon g-2 collaboration, PRL126 (2021)

In fact, it is controversial

Estimation of HVP by Lattice groups: (e.g. BMWc)

Recent e+ e- → π+π- measurement :

(CMD3- Collaboration 2302.08834)



Borsanyi et al (BMWc), Nature 2021

⇨ They make SM prediction closer to experimental values (~1.5σ)

In any case the muon g-2 is good hint/test of new physics

A model with flavor symmetry is interesting in considering these issues

✓ Neutrino masses (+ charged lepton and quark)

⇨ Mass matrix is controlled by a symmetry giving predictions

✓ DM physics

⇨ Absence of DM detection would be hint of flavor specific interaction.
i.e. If DM interaction is flavor specific it is difficult to detect.

✓ Muon g-2

⇨ Realization of muon specific interaction would explain muon g-2 without other LFV constraint

Modular flavor symmetry framework is interesting possibility

Modular flavor symmetry

F. Feruglio, doi:10.1142/9789813238053_0012 [arXiv:1706.08749 [hep-ph]].
R. de Adelhart Toorop, F. Feruglio and C. Hagedorn, Nucl. Phys. B **858**, 437-467 (2012) doi:10.1016/j.nuclphysb.2012.01.017 [arXiv:1112.1340 [hep-ph]].

⇨ New framework for organizing flavor

Originated from extra dimensional symmetry (e.g. transformation on torus)

Non-Abelian discrete symmetry appears : A_4, S_4, etc.

We can get prediction for measurements in neutrino sector:
mixings, CP-phases, sum of neutrino masses, etc.

It is also interesting to apply modular symmetry
in new physics models

- We would get predictions for new physics sector
- Some nature of modular symmetry can be used
  in realizing specific neutrino mass generation mechanism

Let us focus on Modular A_4 symmetry

Yukawa couplings can be given by modular forms transformed under A_4

$$f_i(\gamma\tau) = (c\tau + d)^k \rho(\gamma)_{ij} f_j(\tau) \ ,$$

k: modular weight

ρ: A_4 transformation matrix

$$\left(\tau \longrightarrow \gamma\tau = \frac{a\tau + b}{c\tau + d} \ , \quad \text{where} \ \ a, b, c, d \in \mathbb{Z} \ \ \text{and} \ \ ad - bc = 1, \ \ \mathrm{Im}[\tau] > 0 \ , \right)$$

It can be originated from transformation of lattice vector characterizing torus; compactified extra dimension, coming from string theory

Field is also transformed by

$$\phi^{(I)} \to (c\tau + d)^{k_I} \rho^{(I)}(\gamma) \phi^{(I)}$$

Yukawa term can be invariant under modular transformation

⇨ Sum of modular weight = 0, invariant under A_4

Ex) $f \Phi_i \Phi_j \Phi_k \quad (k_f + k_{\Phi_i} + k_{\Phi_j} + k_{\Phi_k} = 0)$

1. Introduction

Yukawa couplings will be given by modular forms

$$\mathbf{Y}_{\mathbf{3}}^{(2)}(\tau)=\begin{pmatrix}Y_1(\tau)\\Y_2(\tau)\\Y_3(\tau)\end{pmatrix}=\begin{pmatrix}1+12q+36q^2+12q^3+\dots\\-6q^{1/3}(1+7q+8q^2+\dots)\\-18q^{2/3}(1+2q+5q^2+\dots)\end{pmatrix},\quad q=e^{2\pi i\tau},$$

Basis

⇩

$$Y_{1}^{(4)}=Y_1^2+2Y_2Y_3,$$

$$Y_{1'}^{(4)}=Y_3^2+2Y_1Y_2,$$

They are function of modulus τ

All modular form has even modular weight

$$\mathbf{Y}_{\mathbf{3}}^{(4)}\equiv\begin{pmatrix}Y_{3,1}^{(4)}\\Y_{3,2}^{(4)}\\Y_{3,3}^{(4)}\end{pmatrix}=\begin{pmatrix}Y_1^2-Y_2Y_3\\Y_3^2-Y_1Y_2\\Y_2^2-Y_1Y_3\end{pmatrix}$$

$$Y_{\mathbf{1}}^{(6)}=y_1^2+y_2^2+y_3^2-3y_1y_2y_3.$$

$$Y_{\mathbf{3}}^{(6)}\equiv\begin{bmatrix}f_1\\f_2\\f_3\end{bmatrix}=\begin{bmatrix}y_1^3+2y_1y_2y_3\\y_1^2y_2+2y_2^2y_3\\y_1^2y_3+2y_3^2y_2\end{bmatrix},\quad Y_{\mathbf{3'}}^{(6)}\equiv\begin{bmatrix}f_1'\\f_2'\\f_3'\end{bmatrix}=\begin{bmatrix}y_3^3+2y_1y_2y_3\\y_3^2y_1+2y_1^2y_2\\y_3^2y_2+2y_2^2y_1\end{bmatrix}$$

Applying modular symmetry to new physics model

- Predictions for neutrino sector
- Predictions for new physics sector

  ex) mass spectrum and/or BRs of extra particles
- Control flavor dependence on interactions

  ex)DM coupling, interaction realizing muon g-2
- Nature of modular symmetry can be used

  to realize structure of specific neutrino mass generation

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2.Model setup

We consider a model based on $G_{SM} \times U(1)_H \times A_4^{Modular}$

Particle contents and symmetry assignments

SM sector

	Leptons						Quarks							Higgs	
	L_e	L_μ	L_τ	$\bar{e}$	$\bar{\mu}$	$\bar{\tau}$	Q	$\bar{u}$	$\bar{c}$	$\bar{t}$	$\bar{d}$	$\bar{s}$	$\bar{b}$	H_u	H_d
$SU(2)_L$	**2**	**2**	**2**	**1**	**1**	**1**	**2**	**1**	**1**	**1**	**1**	**1**	**1**	**2**	**2**
$U(1)_Y$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	1	1	1	$\frac{1}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{2}$
A_4	1	$1'$	$1''$	$1''$	$1''$	$1'$	3	1	$1''$	$1'$	1	$1''$	$1'$	1	1
$-k$	0	-4	-4	0	-2	-2	-2	-4	-4	-4	0	0	0	0	0

Dark sector

	$\bar{U},\ U$	$\bar{D},\ D$	$\bar{E},\ E$	$\bar{N},\ N$	χ_u	φ_u	χ_d	φ_d	φ'_u	φ'_d
$SU(2)_L$	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
$U(1)_Y$	$-\frac{2}{3},\ \frac{2}{3}$	$\frac{1}{3},\ -\frac{1}{3}$	$1,\ -1$	0	0	0	0	0	0	0
$U(1)_H$	$0,\ 1$	$0,\ -1$	$0,\ -1$	$0,\ 1$	1	-1	-1	$+1$	-2	2
A_4	singlets	singlets	1	3	1	1	1	1	1	1
$-k$	$(0,\text{odd})$	$(0,\text{odd})$	$(-1,0)$	-2	-2	-3	-2	-3	0	0

DM candidate in the model (χ_u)

- **SM sector is neutral under $U(1)_H$**
- **Modular weights are chosen to control interactions**

2.Model setup

Lagrangian of the model

Superpotential (Quark sector, only for SM mass term)

$$\mathcal{W}_q = \alpha_u Y_3^{(6)} \bar{u} H_u Q + \alpha'_u Y_{3'}^{(6)} \bar{u} H_u Q + \beta_u Y_3^{(6)} \bar{c} H_u Q + \beta'_u Y_{3'}^{(6)} \bar{c} H_u Q + \gamma_u Y_3^{(6)} \bar{t} H_u Q + \gamma'_u Y_{3'}^{(6)} \bar{t} H_u Q + \alpha_d Y_3^{(2)} \bar{d} H_d Q + \beta_d Y_3^{(2)} \bar{s} H_d Q + \gamma_d Y_3^{(2)} \bar{b} H_d Q,$$

⇨ **Quark masses**

Superpotential (Lepton sector)

$$\mathcal{W}_\ell = y_e \bar{e} H_d L_e + y_\mu \bar{\mu} H_d L_\mu + y_\tau \bar{\tau} H_d L_\tau + y'_\ell \bar{e} H_d L_\tau$$

⇨ **Charged lepton masses**

$$+ \underline{f_\mu \bar{\mu} E \chi_u} + y_{D_1} Y_3^{(2)} \bar{N} H_u L_{L_e} + y_{D_2} Y_3^{(6)} \bar{N} H_u L_{L_\mu} + y'_{D_2} Y_{3'}^{(6)} \bar{N} H_u L_{L_\mu} + y_{D_3} Y_3^{(6)} \bar{N} H_u L_{L_\tau} + y'_{D_3} Y_{3'}^{(6)} \bar{N} H_u L_{L_\tau} + M_{R_1} [\bar{N}\bar{N}]_1 + M_{R_2} [\bar{N}\bar{N}]_{1''} + M_{R_3} [Y_3^{(4)} \bar{N}\bar{N}]_3 + y_{L_1} \varphi'_u [NN]_1 + y_{L_2} \varphi'_u [NN]_{1''} + y_{L_3} \varphi'_u [Y_3^{(4)} NN]_3,$$

DM int

⇨ **Neutrino mass**

We omit A_4 singlet modular forms (absorbing it in free parameter)

DM interaction is muon specific due to symmetry

Quark mass

Up-type mass matrix

$$M_u = v_u \gamma_u \begin{pmatrix} \tilde{\alpha}_u & 0 & 0 \\ 0 & \tilde{\beta}_u & 0 \\ 0 & 0 & 1 \end{pmatrix} \left[\begin{pmatrix} Y_1^{(6)} & Y_3^{(6)} & Y_2^{(6)} \\ Y_2^{(6)} & Y_1^{(6)} & Y_3^{(6)} \\ Y_3^{(6)} & Y_2^{(6)} & Y_1^{(6)} \end{pmatrix} + \begin{pmatrix} g_{u1} & 0 & 0 \\ 0 & g_{u2} & 0 \\ 0 & 0 & g_{u3} \end{pmatrix} \begin{pmatrix} Y_1^{\prime(6)} & Y_3^{\prime(6)} & Y_2^{\prime(6)} \\ Y_2^{\prime(6)} & Y_1^{\prime(6)} & Y_3^{\prime(6)} \\ Y_3^{\prime(6)} & Y_2^{\prime(6)} & Y_1^{\prime(6)} \end{pmatrix} \right]$$

$\tilde{\alpha}_u = \alpha_u/\gamma_u,\ \tilde{\beta}_u = \beta_u/\gamma_u,\ g_{u1} = \alpha'_u/\alpha_u,\ g_{u2} = \beta'_u/\beta_u$ and $g_{u3} = \gamma'_u/\gamma_u$: Complex

$\alpha_u,\ \beta_u$ and γ_u : Real

Down-type mass matrix

$$M_d = v_d \gamma_d \begin{pmatrix} \tilde{\alpha}_d & 0 & 0 \\ 0 & \tilde{\beta}_d & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} Y_1 & Y_3 & Y_2 \\ Y_2 & Y_1 & Y_3 \\ Y_3 & Y_2 & Y_1 \end{pmatrix}$$

$\tilde{\alpha}_d = \alpha_d/\gamma_d,\ \tilde{\beta}_d = \beta_d/\gamma_d$: Real

Quark mass

Some parameters, $\{\alpha_{u(d)}, \beta_{u(d)}, \gamma_{u(d)}\}$, are fixed to fit quark masses using

$$\mathrm{Tr}[M_{u,d}M_{u,d}] = |m_{u,d}|^2 + |m_{c,s}|^2 + |m_{t,b}|^2,$$

$$\mathrm{Det}[M_{u,d}^\dagger M_{u,d}] = |m_{u,d}|^2|m_{c,s}|^2|m_{t,b}|^2,$$

$$(\mathrm{Tr}[M_{u,d}^\dagger M_{u,d}])^2 - \mathrm{Tr}[(M_{u,d}^\dagger M_{u,d})^2] = 2(|m_{u,d}|^2|m_{c,s}|^2 + |m_{c,s}|^2|m_{t,b}|^2 + |m_{u,d}|^2|m_{t,b}|^2).$$

For simplicity we fixed $\tan\beta=5$

Applying observed values:

$$y_d = (4.81 \pm 1.06) \times 10^{-6}, \quad y_s = (9.52 \pm 1.03) \times 10^{-5}, \quad y_b = (6.95 \pm 0.175) \times 10^{-3},$$

$$y_u = (2.92 \pm 1.81) \times 10^{-6}, \quad y_c = (1.43 \pm 0.100) \times 10^{-3}, \quad y_t = 0.534 \pm 0.0341,$$

$$\theta_{12}^{\mathrm{CKM}} = 13.027^\circ \pm 0.0814^\circ\ , \quad \theta_{23}^{\mathrm{CKM}} = 2.054^\circ \pm 0.384^\circ\ , \quad \theta_{13}^{\mathrm{CKM}} = 0.1802^\circ \pm 0.0281^\circ\ ,$$

$$\delta_{CP}\ (\mathrm{quark}) = 69.21^\circ \pm 6.19^\circ\ ,$$

Charged lepton mass

$$m_\ell = \frac{v_d}{\sqrt{2}} \begin{bmatrix} y_e & 0 & y'_\ell \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{bmatrix},$$

Mass eigenvalues are obtained by $\mathrm{diag}(|m_e|^2, |m_\mu|^2, |m_\tau|^2) \equiv V_{e_L}^\dagger m_\ell^\dagger m_\ell V_{e_L}$

Parameters, {y_e, y_τ, y'_ℓ}, are fixed to fit quark masses using

$$y_\mu = \sqrt{2} m_\mu / v_d$$

$$\mathrm{Tr}[m_\ell {m_\ell}^\dagger] = |m_e|^2 + |m_\mu|^2 + |m_\tau|^2,$$

$$\mathrm{Det}[m_\ell {m_\ell}^\dagger] = |m_e|^2 |m_\mu|^2 |m_\tau|^2,$$

$$(\mathrm{Tr}[m_\ell {m_\ell}^\dagger])^2 - \mathrm{Tr}[(m_\ell {m_\ell}^\dagger)^2] = 2(|m_e|^2 |m_\mu|^2 + |m_\mu|^2 |m_\tau|^2 + |m_e|^2 |m_\tau|^2).$$

Neutrino mass generation

We obtain active neutrino mass via Type-I seesaw mechanism

Dirac mass: $\bar{N} m_D \nu$

$$m_D = \frac{y_{D_1} v_d}{\sqrt{2}} \begin{bmatrix} y_1 & y_3^{(6)} + \epsilon_2 y_3'^{(6)} & y_2^{(6)} + \epsilon_3 y_2'^{(6)} \\ y_3 & y_2^{(6)} + \epsilon_2 y_2'^{(6)} & y_1^{(6)} + \epsilon_3 y_1'^{(6)} \\ y_2 & y_1^{(6)} + \epsilon_2 y_1'^{(6)} & y_3^{(6)} + \epsilon_3 y_3'^{(6)} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \tilde{y}_{D_2} & 0 \\ 0 & 0 & \tilde{y}_{D_3} \end{bmatrix} \equiv \frac{y_{D_1} v_d}{\sqrt{2}} \tilde{m}_D,$$

$\tilde{y}_{D_i} \equiv y_{D_i}/y_{D_1}$ and $\epsilon_i \equiv y_{D_i}/y'_{D_i}$ (i=2,3)

Majorana mass of N

$$M_N = M_{R_3} \left(\begin{bmatrix} 2y_1^{(4)} & -y_3^{(4)} & -y_2^{(4)} \\ -y_3^{(4)} & 2y_2^{(4)} & -y_1^{(4)} \\ -y_2^{(4)} & -y_1^{(4)} & 2y_3^{(4)} \end{bmatrix} + \epsilon_{M_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} + \epsilon_{M_2} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \right) = M_{R_3} \tilde{M},$$

$\epsilon_{M_i} \equiv M_{R_i}/M_{R_3}$ (i=1,2)

$$\Rightarrow \quad m_\nu = \frac{y_{D_1}^2 v_d^2}{2M_{R_3}} \tilde{m}_D^T \tilde{M}^{-1} \tilde{m}_D \equiv \kappa \tilde{m}_\nu$$

2.Model setup

$$m_\nu = \frac{y_{D_1}^2 v_d^2}{2M_{R_3}} \tilde{m}_D^T \tilde{M}^{-1} \tilde{m}_D \equiv \kappa \tilde{m}_\nu$$

We take κ as free parameter and use to fit neutrino mass scale

⇩

$$(\text{NH}): \ |\kappa|^2 = \frac{|\Delta m_{\text{atm}}^2|}{\tilde{D}_{\nu_3}^2 - \tilde{D}_{\nu_1}^2}, \quad (\text{IH}): \ |\kappa|^2 = \frac{|\Delta m_{\text{atm}}^2|}{\tilde{D}_{\nu_2}^2 - \tilde{D}_{\nu_3}^2},$$

$$D_\nu = |\kappa| \tilde{D}_\nu = V_\nu^T m_\nu V_\nu = |\kappa| V_\nu^T \tilde{m}_\nu V_\nu$$ Diagonalization

$$\Delta m_{\text{sol}}^2 = |\kappa|^2 (\tilde{D}_{\nu_2}^2 - \tilde{D}_{\nu_1}^2)$$

PMNS matrix : $U = V_L^\dagger V_\nu$

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Numerical analysis

- We globally scan our free parameters in the following region

$|g_{u1,u2,u3}| \in [10^{-2}, 10^{2}]$. **Quark sector**

$\{\tilde{y}_{D_{2,3}}, |\epsilon_{2,3}|, |\epsilon_{M_{2,3}}|\} \in [10^{-3}, 10^{3}]$ **Neutrino sector**

Perturbativity of original parameter is taken into account

Carrying out chi-square fitting using NuFit 5.2 data

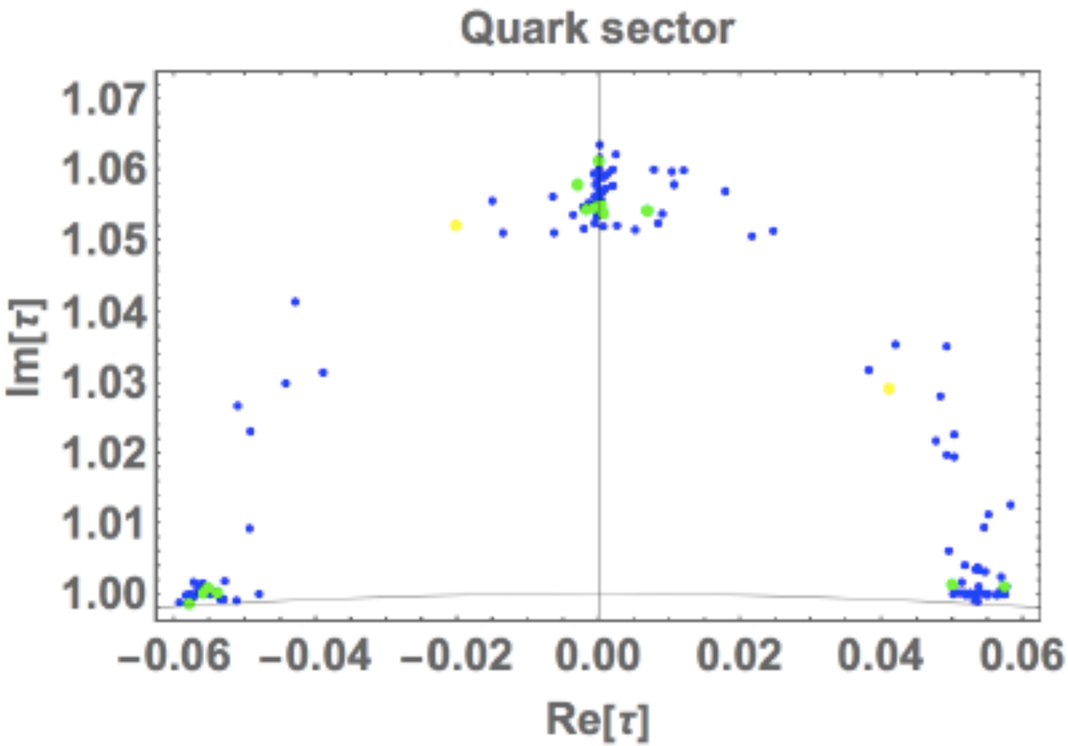
http://www.nu-fit.org/?q=node/228

$$\chi^2 = \sum_i \left(\frac{O_i^{\exp} - O_i^{theory}}{\delta_i^{\exp}} \right)^2$$

We separately consider quark and lepton sector

Numerical results in quark sector

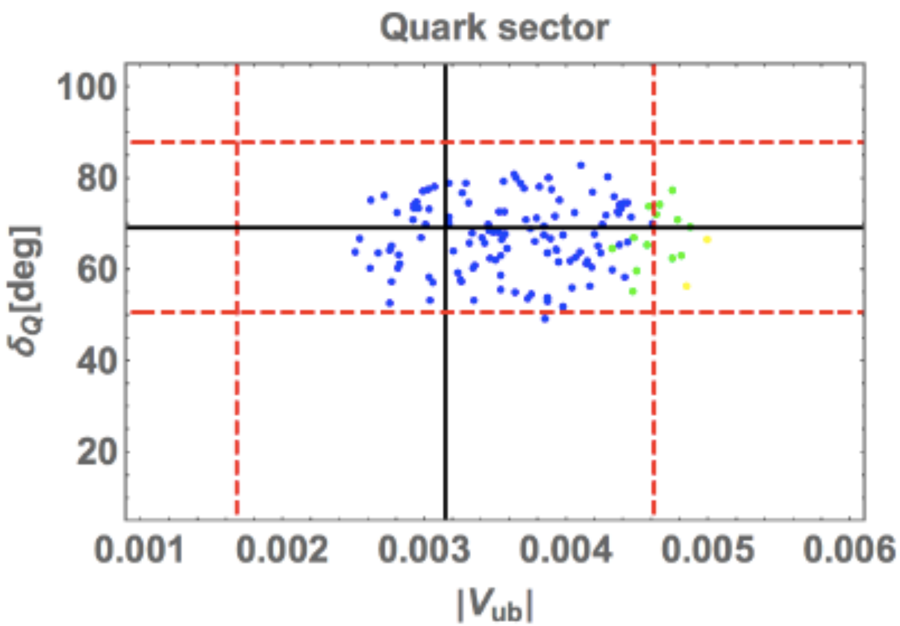
Allowed region of τ

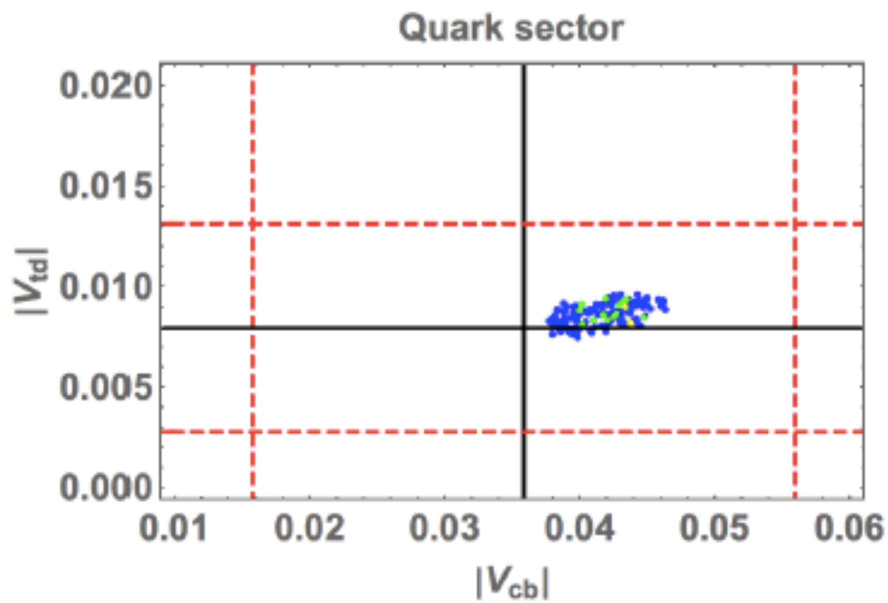


Blue: within 1σ, Green: 1σ to 2σ, Yellow: 2σ to 3σ

3.Phenomenology

Numerical results in quark sector





Range of allowed input parameters

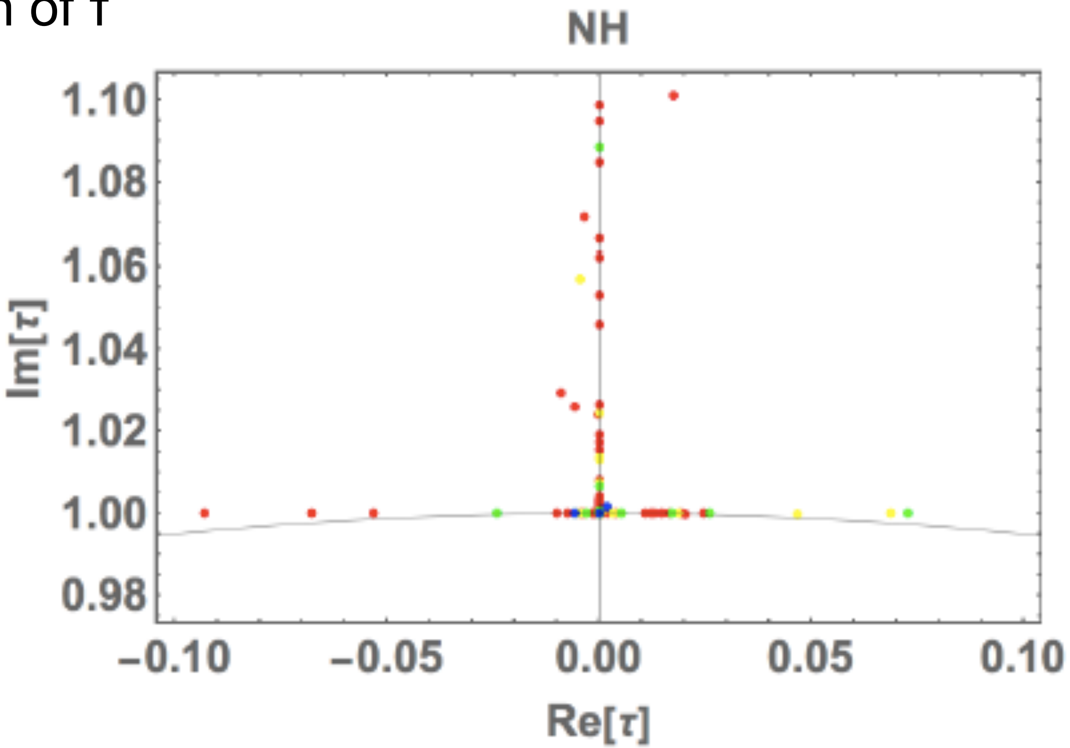
$\lvert g_{u1}\rvert$	$\lvert g_{u2}\rvert$	$\lvert g_{u3}\rvert$	$\tilde{\alpha}_u$	$\tilde{\beta}_u$	γ_u
$[0.071, 62]$	$[0.014, 50]$	$[0.012, 0.23]$	$[2.6 \times 10^{-5}, 330]$	$[9.2 \times 10^{-7}, 8.6]$	$[4.6 \times 10^{-5}, 0.27]$

$\tilde{\alpha}_d$	$\tilde{\beta}_d$	γ_d
$[0.0033, 0.015]$	$[0.0036, 0.015]$	$[0.026, 0.029]$

3.Phenomenology

Numerical results in neutrino sector

Allowed region of τ



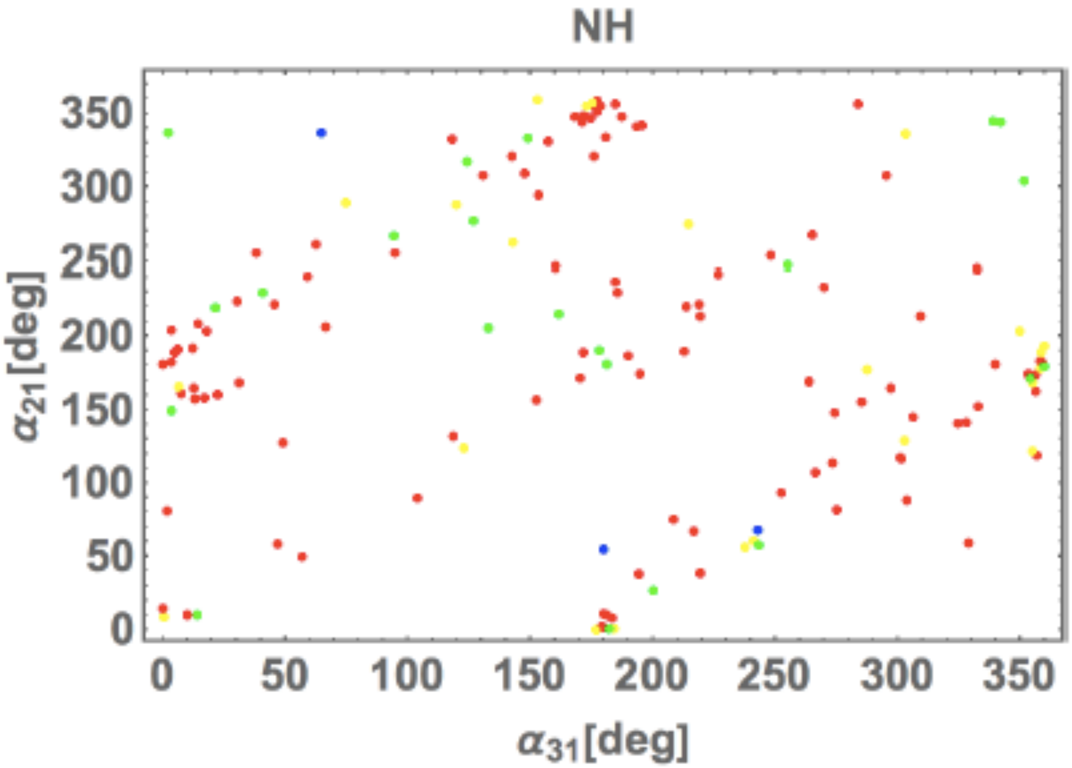
Blue: within 1σ, Green: 1σ to 2σ, Yellow: 2σ to 3σ, red: 3σ to 5σ

Ranges for other allowed parameters:

$\lvert\epsilon_{M_1}\rvert$	$\lvert\epsilon_{M_2}\rvert$	$\tilde{y}_{D_2}$	$\tilde{y}_{D_3}$	$\lvert\epsilon_2\rvert$	$\lvert\epsilon_3\rvert$
[0.00626, 867]	[0.00278, 896]	[0.00124, 1.828]	[0.0425, 0.975]	[0.0164, 947]	[0.00219, 2.67]

Numerical results in neutrino sector

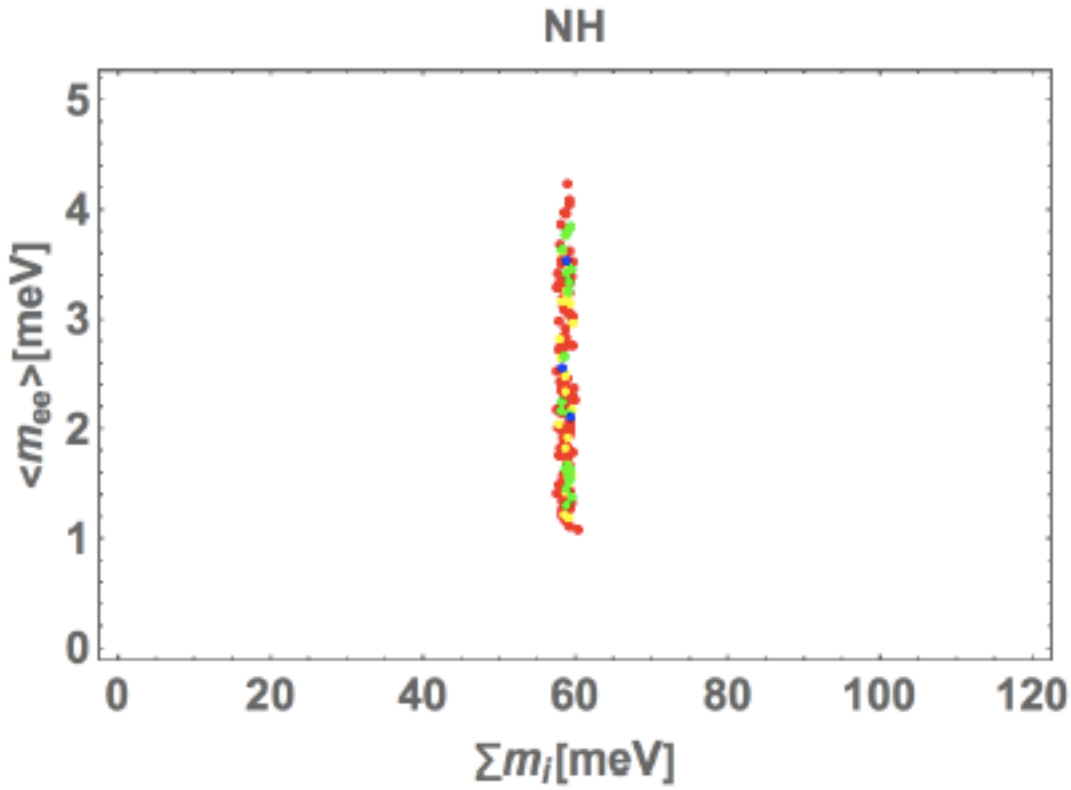
Majorana phases for allowed parameters



We find slight correlation between phases

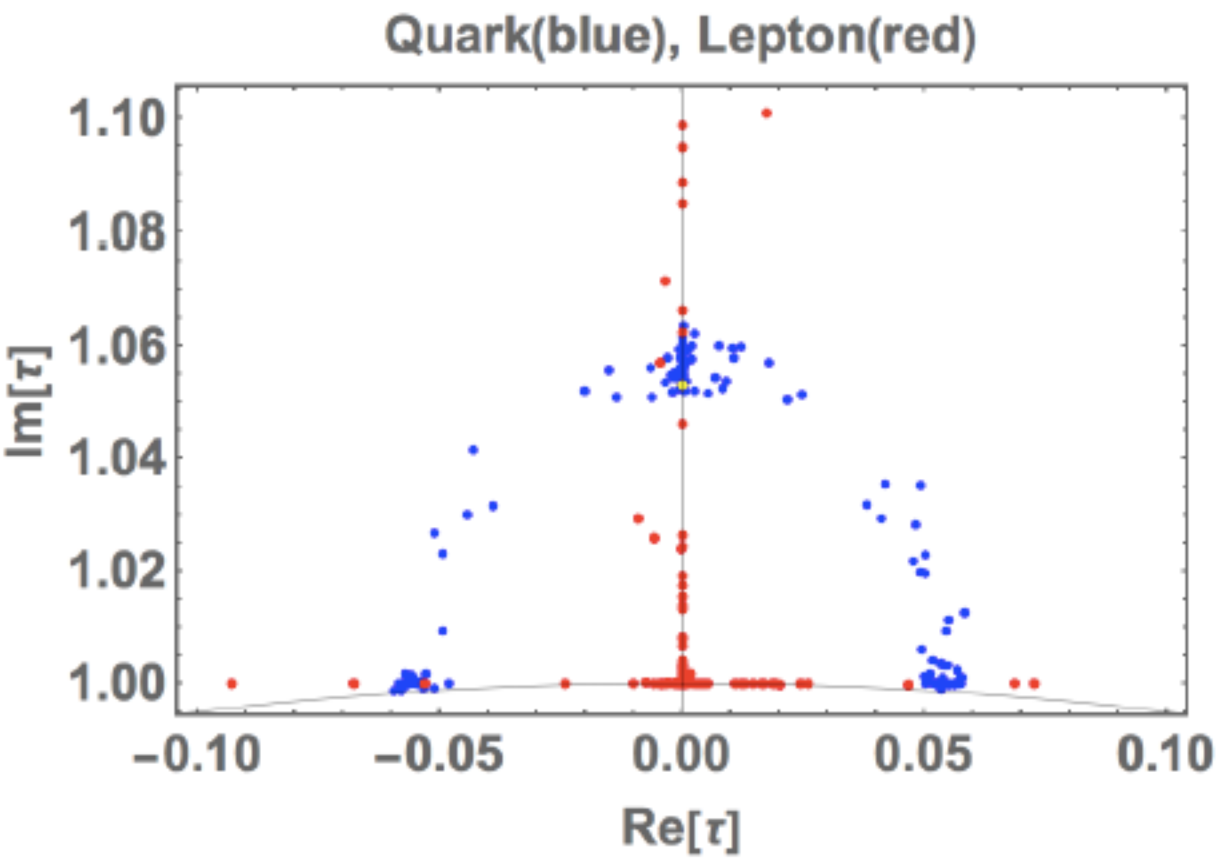
Numerical results in neutrino sector

Sum of neutrino mass and effective mass for neutrinoless double β decay



- ❖ The effective mass is small and difficult for detection
- ❖ Sum of neutrino mass is around 60 meV

Checking overlap of τ for quark and lepton sector



We have overlap at around some specific region

A benchmark point with common τ for both sector

τ	$1.053\,i$
g_{u1}	$-20.4551 - 3.89777i$
g_{u2}	$0.0422226 + 0.308798i$
g_{u3}	$0.0130486 + 0.0159581i$
$\hat{\alpha}_u$	0.713292
$\hat{\beta}_u$	0.000426951
γ_u	0.023187
$\hat{\alpha}_d$	0.00392545
$\hat{\beta}_d$	0.0152835
γ_d	0.028698
$\|V_{us}\|$	0.224981
$\|V_{cb}\|$	0.0428095
$\|V_{ub}\|$	0.00413679
δ_Q	61.3284°
χ^2	10.6623

τ	$1.053\,i$
ϵ_{M_1}	$-1.34226 + 0.00413708i$
ϵ_{M_2}	$0.00945163 - 2.28048i$
$\tilde{y}_{D_2}$	-0.0919822
$\tilde{y}_{D_3}$	-0.324861
ϵ_2	$-7.77596 - 0.0157862i$
ϵ_3	$0.0415398 - 0.047376i$
$\sin^2\theta_{12}$	0.320588
$\sin^2\theta_{23}$	0.457686
$\sin^2\theta_{13}$	0.0213432
δ^{ℓ}_{CP}	56.1248°
$\alpha_{21},\ \alpha_{31}$	$241.826^\circ,\ 205.144^\circ$
$\sum m_i$	$57.4937\,\mathrm{meV}$
$\langle m_{ee}\rangle$	$2.53293\,\mathrm{meV}$
χ^2	5.62011

Dark matter physics and muon g-2

Relevant Lagrangian (non supersymmetric writing)

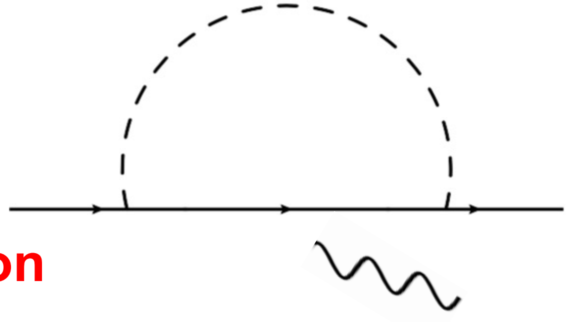
$$-\mathcal{L}_{DM} = f_\mu \bar{\mu} E \chi_u - \mu_\chi^2 \chi_u^2 + y_E \bar{E} E \varphi_d + |m_\chi|^2 |\chi_u|^2 + \text{h.c.},$$

In our scenario, real component of χ_u is scalar DM (χ)

Dominant DM interaction: $-\mathcal{L}_{DM}^{\text{int.}} = \frac{f_\mu}{\sqrt{2}} \bar{\mu} E \chi + \text{h.c.}.$

Muon specific interaction → contributing to muon g-2 at one-loop

$$\Delta a_\mu = \frac{|f_\mu|^2}{16\pi^2} \int_0^1 dx \frac{m_\mu^2 x^2 (1-x)}{x(x-1)m_\mu^2 + x m_E^2 + (1-x) m_\chi^2} \,.$$



✓ **We do not have other LFV from the interaction**

Dark matter physics and muon g-2

Dominant DM annihilation cross section

$$(\sigma v_{\rm rel}) \approx \frac{|f_\mu|^4}{240\pi}\frac{m_\chi^6}{(m_\chi^2+m_E^2)^4}v_{\rm rel}^4 \equiv d_{\rm eff}v_{\rm rel}^4, \qquad m_\mu << m_\chi \lesssim m_E$$

Relic density:

$$\Omega h^2 \approx \frac{5.35\times 10^7 x_F^3 {\rm GeV}^{-1}}{\sqrt{g_*(x_F)}M_{\rm PL}d_{\rm eff}}$$

⇒

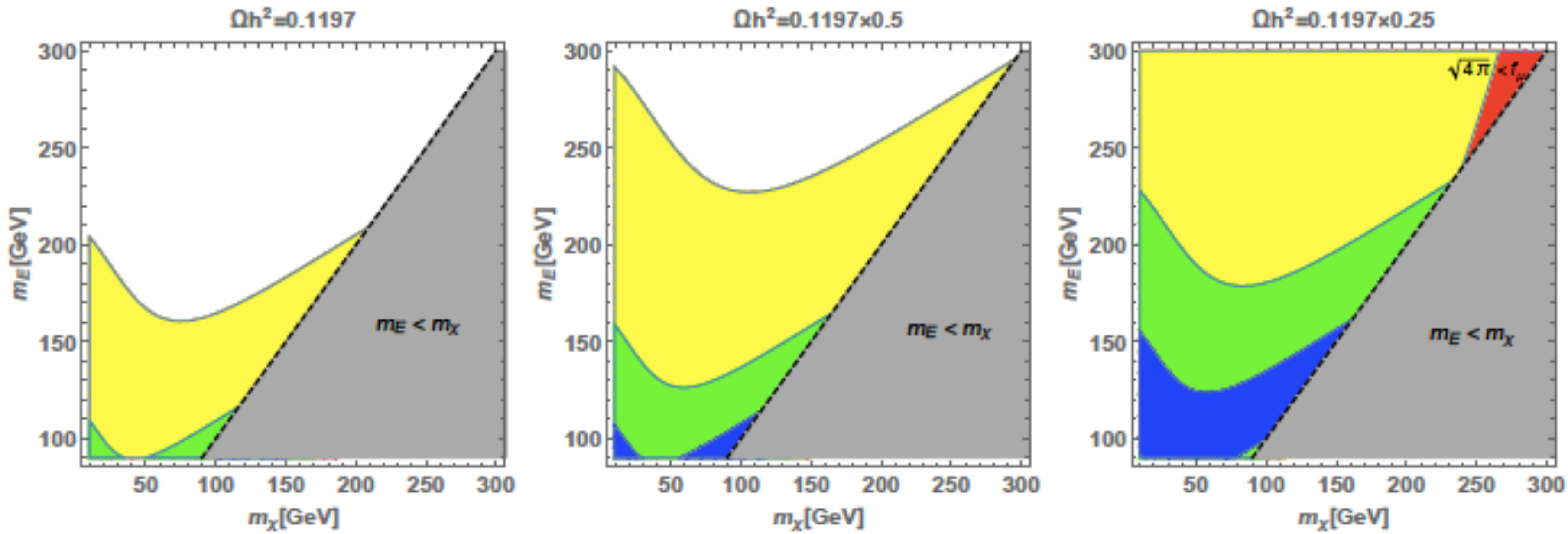
$$|f_\mu|^4 = \frac{5.35\times 10^7 x_F^3}{\sqrt{g_*(x_F)}M_{\rm PL}}\frac{240\pi(m_\chi^2+m_E^2)^4}{m_\chi^6(\Omega h^2)\ {\rm GeV}} \approx (4.32\times 10^{-5})\times\left(\frac{0.1197}{\Omega h^2}\right)\times\frac{(m_\chi^2+m_E^2)^4}{m_\chi^6\ {\rm GeV}^2},$$

⇓ Combining with muon g-2

$$\Delta a_\mu = \frac{(6.57\times 10^{-3})}{16\pi^2}\times\left(\frac{0.1197}{\Omega h^2}\right)^{\frac{1}{2}}\times\frac{(m_\chi^2+m_E^2)^2}{m_\chi^3\ {\rm GeV}}\int_0^1 dx\frac{m_\mu^2 x^2(1-x)}{x(x-1)m_\mu^2+xm_E^2+(1-x)m_\chi^2}$$

3.Phenomenology

Dark matter physics and muon g-2



Blue: within 1σ, Green: 1σ to 2σ, Yellow: 2σ to 3σ of $\Delta a_\mu = (25.1 \pm 5.9) \times 10^{-10}$

- ❖ We can obtain muon g-2 within 2σ when relic density is satisfied
- ❖ 1σ region can be obtained when relic density is smaller than observed

Summary and discussion

Applying modular flavor A_4 symmetry to dark sector model

- A model construction
  - Neutrino mass model with dark sector
  - DM interaction is muon specific due to $U(1)_H$ and A4 symmetry
  - Mass matrices are ristricted from symmetry
- Phenomenology
  - We obtain parameter region realizing both quark and lepton masses
  - Muon specific DM interaction realizing muon g-2

Appendix

Kinetic term and quadratic term of scalar fields

$$\sum_I \frac{|\partial_\mu \phi^{(I)}|^2}{(-i\tau + i\bar{\tau})^{k_I}}, \quad \sum_I \frac{|\phi^{(I)}|^2}{(-i\tau + i\bar{\tau})^{k_I}}$$

Invariant under

$$\tau \longrightarrow \gamma\tau = \frac{a\tau + b}{c\tau + d} \qquad \phi^{(I)} \to (c\tau + d)^{-k_I} \rho^{(I)}(\gamma)\phi^{(I)},$$

Factor of denominator will be absorbed by field redefinition after modular symmetry breaking (fixing modulus)

Mixing obtained from PMNS matrix

$$\sin^2\theta_{13} = |(U_{\rm PMNS})_{13}|^2, \quad \sin^2\theta_{23} = \frac{|(U_{\rm PMNS})_{23}|^2}{1-|(U_{\rm PMNS})_{13}|^2}, \quad \sin^2\theta_{12} = \frac{|(U_{\rm PMNS})_{12}|^2}{1-|(U_{\rm PMNS})_{13}|^2}. \tag{II.15}$$

$$J_{CP} = {\rm Im}[U_{e1}U_{\mu 2}U^*_{e2}U^*_{\mu 1}] = s_{23}c_{23}s_{12}c_{12}s_{13}c^2_{13}\sin\delta_{CP},$$

$$I_1 = {\rm Im}[U^*_{e1}U_{e2}] = c_{12}s_{12}c^2_{13}\sin\left(\frac{\alpha_{21}}{2}\right), \; I_2 = {\rm Im}[U^*_{e1}U_{e3}] = c_{12}s_{13}c_{13}\sin\left(\frac{\alpha_{31}}{2}-\delta_{CP}\right). \tag{II.17}$$

$$\langle m_{ee}\rangle = |m_1\cos^2\theta_{12}\cos^2\theta_{13} + m_2\sin^2\theta_{12}\cos^2\theta_{13}e^{i\alpha_{21}} + m_3\sin^2\theta_{13}e^{i(\alpha_{31}-2\delta_{CP})}|,$$