Complete One-loop Renormalization-group Equations in the Seesaw Effective Field Theories

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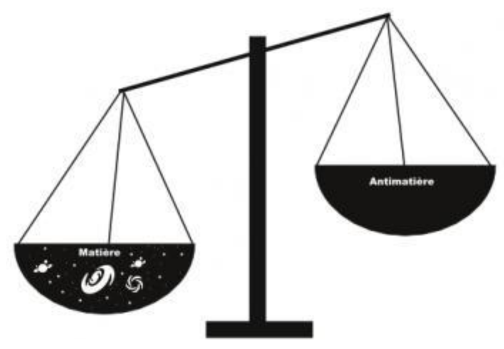
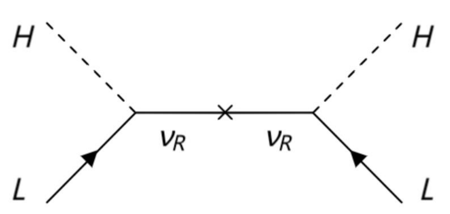
I. Background and Motivation

◆ **Neutrino mass**

◆ **Matter-antimatter asymmetry**

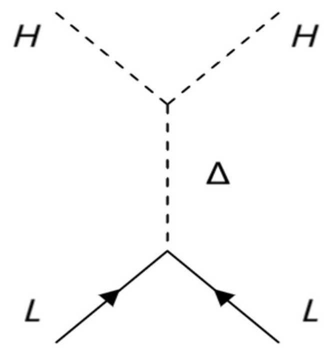


Seesaw Model :



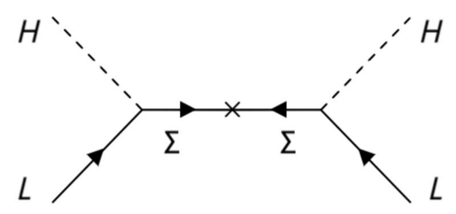
Type-I

SM + three heavy right-handed neutrinos



Type-II

SM + one heavy Higgs triplet

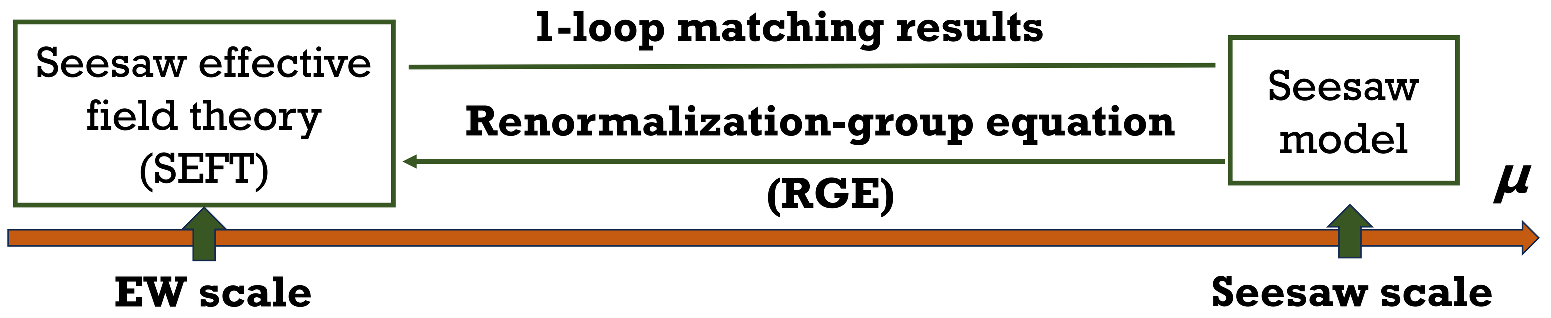


Type-III

SM + three heavy triplet fermions

◆ Seesaw model can explain tiny neutrino mass [1] naturally and matter-antimatter asymmetry [2] in Universe at the same time.

II. How to inspect seesaw model?



Why RGEs?

◆ **1-loop matching + 1-loop RGEs** give complete one-loop calculations in the type-I SEFT at the electroweak scale and extract useful information about the type-I seesaw model from low-energy measurements.

Lagrangian :

dim-5   dim-6 → unitarity violation

$$\mathcal{L}_{\rm SEFT} = \mathcal{L}_{\rm SM} + \frac{1}{2}\left(\boxed{C_5^{\alpha\beta}\mathcal{O}^{(5)}_{\alpha\beta}} + {\rm h.c.}\right) + \boxed{C^{(1)\alpha\beta}_{H\ell}\mathcal{O}^{(1)\alpha\beta}_{H\ell} + C^{(3)\alpha\beta}_{H\ell}\mathcal{O}^{(3)\alpha\beta}_{H\ell}}$$

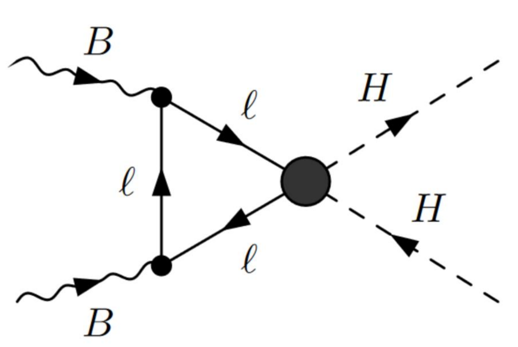
matching scale:

$$C_5\left(\mu_{\rm M}\right) = Y_\nu M_{\rm R}^{-1} Y_\nu^{\rm T} \; , \quad C^{(1)}_{H\ell}\left(\mu_{\rm M}\right) = -C^{(3)}_{H\ell}\left(\mu_{\rm M}\right) = \frac{1}{4} Y_\nu M_{\rm R}^{-2} Y_\nu^\dagger$$

1-loop RGEs:

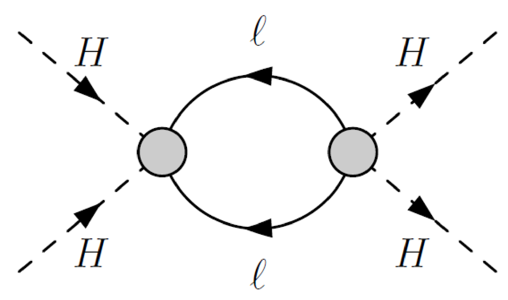
◆ single insertions of dim-6 operators:

e.g.



◆ double insertions of dim-5 operator:

e.g.



Complete 1-loop RGEs of type-I SEFT are still lacking!

III. Complete 1-loop RGEs

Strategy:

◆ First, we choose a set of 1PI diagrams generated by the tree-level Lagrangian and covering all of dim-6 operators in the Green's basis.

◆ Then, one calculates these diagrams to get all the counterterms for the dim-6 operators in the Green's basis and converts them into Warsaw basis.

◆ Finally, we bring all the conterterms into beta functions to get RGEs.

III. Complete 1-loop RGEs

Yukawa couplings:

$$16\pi^2\mu\frac{{\rm d}Y_l}{{\rm d}\mu} = \left[-\frac{15}{4}g_1^2 - \frac{9}{4}g_2^2 + T + \frac{3}{2}Y_l Y_l^\dagger - \underline{2m^2\left(C^{(1)}_{H\ell} + 3C^{(3)}_{H\ell}\right)}\right] Y_l \; ,$$

$$16\pi^2\mu\frac{{\rm d}Y_{\rm u}}{{\rm d}\mu} = \left[-\frac{17}{12}g_1^2 - \frac{9}{4}g_2^2 - 8g_s^2 + T + \frac{3}{2}\left(Y_{\rm u}Y_{\rm u}^\dagger - Y_{\rm d}Y_{\rm d}^\dagger\right)\right] Y_{\rm u} \; ,$$

$$16\pi^2\mu\frac{{\rm d}Y_{\rm d}}{{\rm d}\mu} = \left[-\frac{5}{12}g_1^2 - \frac{9}{4}g_2^2 - 8g_s^2 + T - \frac{3}{2}\left(Y_{\rm u}Y_{\rm u}^\dagger - Y_{\rm d}Y_{\rm d}^\dagger\right)\right] Y_{\rm d} \; ,$$

In the type-I SEFT, **19** dim-6 operators can be generated via the RGEs at the one-loop level.[3]

Higgs quadratic and quartic couplings:

$$16\pi^2\mu\frac{{\rm d}m^2}{{\rm d}\mu} = \left(-\frac{3}{2}g_1^2 - \frac{9}{2}g_2^2 + 12\lambda + 2T\right) m^2 \; , \qquad T \equiv {\rm tr}\left(Y_l Y_l^\dagger + 3Y_{\rm u}Y_{\rm u}^\dagger + 3Y_{\rm d}Y_{\rm d}^\dagger\right)$$

$$16\pi^2\mu\frac{{\rm d}\lambda}{{\rm d}\mu} = 24\lambda^2 - 3\lambda\left(g_1^2 + 3g_2^2\right) + \frac{3}{8}\left(g_1^2 + g_2^2\right)^2 + \frac{3}{4}g_2^4 + 4\lambda T - 2{\rm tr}\left[\left(Y_l Y_l^\dagger\right)^2 + 3\left(Y_{\rm u}Y_{\rm u}^\dagger\right)^2 + 3\left(Y_{\rm d}Y_{\rm d}^\dagger\right)^2\right] + m^2{\rm tr}\left(\boxed{2C_5 C_5^\dagger} - \underline{\frac{8}{3}g_2^2 C^{(3)}_{H\ell} + 8C^{(3)}_{H\ell}Y_l Y_l^\dagger}\right) \; .$$

IV. 1-loop RGEs of Physical Parameters

After spontaneous symmetry breaking (SSB) :

$$C^{(1)\alpha\beta}_{Hl}\mathcal{O}^{(1)\alpha\beta}_{Hl} \to -\frac{g_2}{2c_{\rm W}}C^{(1)\alpha\beta}_{Hl}v^2\left(\overline{\nu_{\alpha\rm L}}\gamma^\mu\nu_{\beta\rm L} + \overline{l_{\alpha\rm L}}\gamma^\mu l_{\beta\rm L}\right)Z_\mu \; ,$$

$$C^{(3)\alpha\beta}_{Hl}\mathcal{O}^{(3)\alpha\beta}_{Hl} \to +\frac{g_2}{\sqrt{2}}C^{(3)\alpha\beta}_{Hl}v^2\left(\overline{\nu_{\alpha\rm L}}\gamma^\mu l_{\beta\rm L}W^+_\mu + \overline{l_{\alpha\rm L}}\gamma^\mu\nu_{\beta\rm L}W^-_\mu\right) + \frac{g_2}{2c_{\rm W}}C^{(3)\alpha\beta}_{Hl}v^2\left(\overline{\nu_{\alpha\rm L}}\gamma^\mu\nu_{\beta\rm L} - \overline{l_{\alpha\rm L}}\gamma^\mu l_{\beta\rm L}\right)Z_\mu \; ,$$

CC $\quad \tilde{\eta} \equiv -C^{(3)}_{Hl}\,v^2$

NC $\quad \tilde{\eta}' \equiv \left(C^{(1)}_{Hl} - C^{(3)}_{Hl}\right)v^2$

RGEs of dim-6 operators ➡ **RGEs of non-unitary parameters**

PMNS matrix : $V \equiv (\mathbb{1} - \eta)\cdot\boxed{U}\cdot\boxed{Q}$ → Majorana phase matrix (U: standard parametrization)

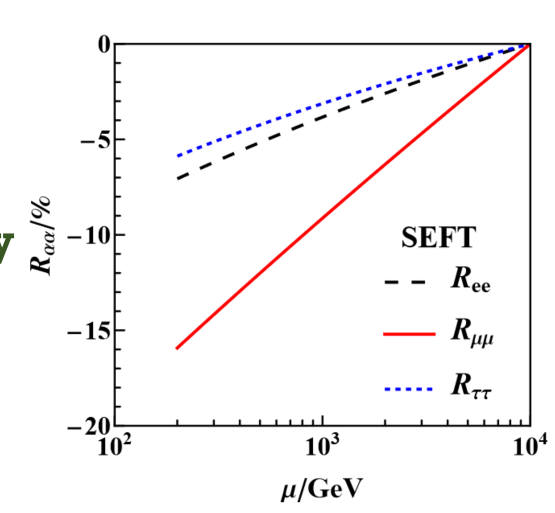
Diagonalization of lepton mass matrices ⟶ RGEs of $V' \equiv U_l^\dagger U_\nu \equiv \boxed{P}\cdot U\cdot Q$ (P: unphysical phase matrix)

Bridge : $\mathcal{T} \equiv V'^\dagger\dot{V}'$ **&&** $\mathcal{T}' \equiv Q\cdot\mathcal{T}\cdot Q^\dagger = \dot{Q}Q^\dagger + U^\dagger\dot{U} + U^\dagger P^\dagger\dot{P}U$.

➡ **RGEs of unitary parameters :** $\theta_{ij}, \delta, \rho, \sigma, m_i, m_\alpha$

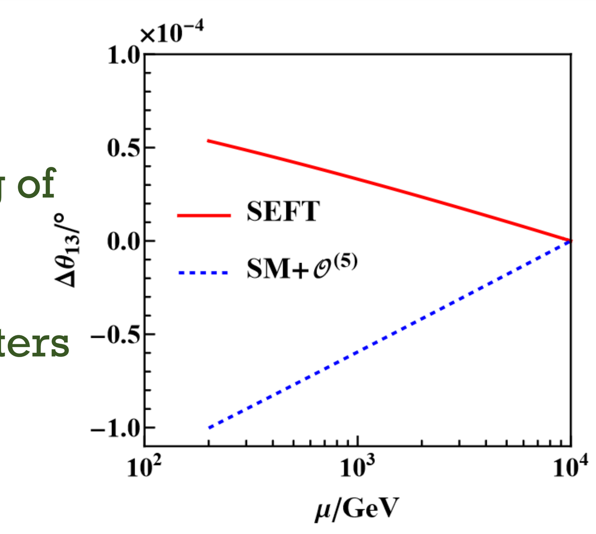
V. Numerical Results

Running of **non-unitary** parameters



$$R^{(\prime)}_{\alpha\beta} \equiv \left[|\eta^{(\prime)}_{\alpha\beta}(\mu)| - |\eta^{(\prime)}_{\alpha\beta}(\mu_{\rm M})|\right]/|\eta^{(\prime)}_{\alpha\beta}(\mu_{\rm M})| \times 100\%$$

Running of **unitary** parameters



$$\Delta P \equiv P(\mu) - P(\mu_{\rm M})$$

◆ **Non-unitary parameters may affect significantly the running of leptonic flavor mixing parameters.**

VI. Summary

➢ We derive the complete set of **one-loop RGEs** for all SM couplings and Wilson coefficients of operators up to dim-6 in the type-I SEFT.

➢ There are 19 dim-6 operators in total, which can be generated by the RGEs at the one-loop level.

➢ After SSB, two tree-level dim-6 operators result in a non-unitary leptonic flavor mixing matrix appearing in the cc- and nc-interaction of leptons.

➢ As a by-product, the RGEs of the mixing parameters in the standard parametrization of a unitary leptonic mixing matrix are obtained for the first time.

References:

[1] Z. z. Xing, Phys. Rept. 854, 1-147 (2020).
[2] M. Fukugita and T. Yanagida, Phys. Lett. B 174 (1986), 45-47.
[3] Y. Wang, D. Zhang and S. Zhou, JHEP 05 (2023) 044.