光前夸克模型中重子电磁衰变$\Xi_c^{\prime+(0)} \to \Xi_c^{+(0)}\gamma$计算

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2022年11月12日

研究动机

- $M_{\Xi_c'^+} - M_{\Xi_c^+} = 2578.4 - 2467.94 = 110.46 MeV < 139.57(M_\pi)$，因此只能发生电磁衰变。
- 采用光前夸克模型(LF QM)计算非微扰物理量，概念简单，唯象可行且应用方便。
- 受郑海洋老师文章(arxiv:2109.01216)启发，我们采用LF QM计算其电磁衰变。

TABLE XXIII: Electromagnetic decay rates (in units of keV) of S-wave charmed baryons. Among the four different results listed in [260] and [266], we quote those denoted by $\Gamma_\gamma^{(0)}$ and "Present (ecqm)", respectively.

Decay	HHChPT [256, 259]	HBChPT [258]	Dey [260]	Ivanov [261]	Simonis [262]	Aliev [263]	Wang [264]	Bernotas [265]	Majethiya [266]	Hazra [267]
$\Sigma_c^+ \to \Lambda_c^+ \gamma$	91.5	65.6 ± 2	120	60.7 ± 1.5	74.1	50 ± 17		46.1	60.55	93.5 ± 0.7
$\Sigma_c^{*+} \to \Lambda_c^+ \gamma$	150.3	161.6 ± 5	310	151 ± 4	190	130 ± 45		126	154.48	231 ± 7
$\Sigma_c^{*++} \to \Sigma_c^{++} \gamma$	1.3	1.20 ± 0.6	1.6		1.96	2.65 ± 1.20	$6.36^{+6.79}_{-3.31}$	0.826	1.15	1.48 ± 0.02
$\Sigma_c^{*+} \to \Sigma_c^+ \gamma$	0.002	0.04 ± 0.03	0.001	0.14 ± 0.004	0.011	0.40 ± 0.16	$0.40^{+0.43}_{-0.21}$	0.004	$< 10^{-4}$	$(7 \pm 1)10^{-4}$
$\Sigma_c^{*0} \to \Sigma_c^0 \gamma$	1.2	0.49 ± 0.1	1.2		1.41	0.08 ± 0.03	$1.58^{+1.68}_{-0.82}$	1.08	1.12	1.38 ± 0.02
$\Xi_c'^+ \to \Xi_c^+ \gamma$	19.7	5.43 ± 0.33	14	12.7 ± 1.5	17.3	8.5 ± 2.5		10.2		21.4 ± 0.3
$\Xi_c^{*+} \to \Xi_c^+ \gamma$	63.5	21.6 ± 1	71	54 ± 3	72.7	52 ± 25		44.3	63.32	81.9 ± 0.5
$\Xi_c^{*+} \to \Xi_c'^+ \gamma$	0.06	0.07 ± 0.03	0.10		0.063	0.274	$0.96^{+1.47}_{-0.67}$	0.011		0.03 ± 0.00
$\Xi_c'^0 \to \Xi_c^0 \gamma$	0.4	0.46	0.33	0.17 ± 0.02	0.185	0.27 ± 0.6		0.0015		0.34 ± 0.01
$\Xi_c^{*0} \to \Xi_c^0 \gamma$	1.1	1.84	1.7	0.68 ± 0.04	0.745	0.66 ± 0.32		0.908	0.30	1.32 ± 0.01
$\Xi_c^{*0} \to \Xi_c'^0 \gamma$	1.0	0.42 ± 0.16	1.6		1.33	2.14	$1.26^{+0.80}_{-0.46}$	1.03		1.26 ± 0.03
$\Omega_c^{*0} \to \Omega_c^0 \gamma$	0.9	0.32 ± 0.20	0.71		1.13	0.932	$1.16^{+1.12}_{-0.54}$	1.07	2.02	1.14 ± 0.13

LF QM 计算$\mathcal{B}(1/2) \to \mathcal{B}(1/2)$ 电磁衰变形状因子

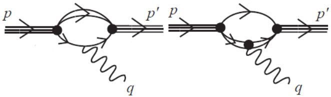


Figure 1: The electromagnetic couplings of the baryons. The left and right panels show the contribution from the quark and diquarks.

电磁形状因子定义：

$$\left\langle\mathcal{B}'\left(p',s'\right)\left|J_\mu(0)\right|\mathcal{B}(p,s)\right\rangle=\bar{u}\left(p',s'\right)\left[\gamma_\mu F_1\left(q^2\right)+\frac{i\sigma_{\mu\nu}q^\nu}{M+M'}F_2\left(q^2\right)\right]u(p,s). \quad (1)$$

其中$F_1\left(q^2\right)$，$F_2\left(q^2\right)$ 分别是Dirac形状因子和Pauli形状因子。电磁流$J^\mu=\bar{q}\,\Gamma^\mu\,q$，与虚光子耦合顶角$\Gamma^\mu$ [1,2]，

$$\Gamma^\mu_{Q\gamma Q}=e_Q\gamma^\mu \quad (2)$$

$$\Gamma^\mu_{S\gamma S}=-\frac{i}{3}(k_1+k_1')^\mu \quad (3)$$

$$\Gamma^\mu_{A\gamma A}=ie_A\left\{g_{\alpha\beta}(k_1+k_1')^\mu-\left[(1+\kappa)k_{1\beta}-(\kappa+\xi)k'_{1\beta}\right]g^\mu_\alpha-\left[(1+\kappa)k'_{1\alpha}-(\kappa+\xi)k_{1\alpha}\right]g^\mu_\beta\right\} \quad (4)$$

[1]J. He and Y. b. Dong, J. Phys. G 32 (2006), 189-202.

[2]T. D. Lee and C. N. Yang, Phys. Rev. 128 (1962), 885-898.

电磁形状因子

- 如果一个光子从夸克Q辐射出，(Fig. 1左图), 形状因子$F_Q(q^2)$，

$$F_{1Q}\left(q^2\right) = e_Q \int \frac{dx d^2\mathbf{k}_\perp}{2(2\pi)^3} \frac{\phi_{1s}'^{*}\left(x', k_\perp'\right)\phi_{1s}\left(x, k_\perp\right)}{16xP^+P'^+\sqrt{\left(k_1'\cdot\bar{P}' + m_1' M_0'\right)\left(k_1\cdot\bar{P} + m_1 M_0\right)}}$$
$$\times \operatorname{Tr}\left[\left(\bar{\not{P}} + M_0\right)\gamma^+\left(\bar{\not{P}}' + M_0'\right)\bar{\Gamma}'_{s00(s11)}\left(\not{k}_1' + m_1'\right)\gamma^+\left(\not{k}_1 + m_1\right)\Gamma_{s00(s11)}\right], \quad (5)$$

$$F_{2Q}\left(q^2\right) = e_Q \cdot \frac{-i(M+M')q_\perp^i}{\mathbf{q}_\perp^2}\int \frac{dx d^2\mathbf{k}_\perp}{2(2\pi)^3} \frac{\phi_{1s}'^{*}\left(x', k_\perp'\right)\phi_{1s}\left(x, k_\perp\right)}{16xP^+P'^+\sqrt{\left(k_1'\cdot\bar{P}' + m_1' M_0'\right)\left(k_1\cdot\bar{P} + m_1 M_0\right)}}$$
$$\times \operatorname{Tr}\left[\left(\bar{\not{P}} + M_0\right)\sigma^{i+}\left(\bar{\not{P}}' + M_0'\right)\bar{\Gamma}'_{s00(s11)}\left(\not{k}_1' + m_1'\right)\gamma^+\left(\not{k}_1 + m_1\right)\Gamma_{s00(s11)}\right], \quad (6)$$

其中，diquark为S时，$\Gamma_{s00}\left(k_1, k_2\right) = 1$；

diquark为A时，$\Gamma_{s11}\left(k_1, k_2, \lambda_2\right) = \frac{\gamma_5}{\sqrt{3}}\left(\not{\varepsilon}_{LF}^{*}\left(k_2, \lambda_2\right) - \frac{M_0+m_1+m_2}{\bar{P}\cdot k_2+m_2 M_0}\varepsilon_{LF}^{*}\left(k_2, \lambda_2\right)\cdot\bar{P}\right)$。

轨道波函数：$\varphi_{n=1,L=S}(\mathbf{k}_\perp, \beta) = 4(\frac{\pi}{\beta^2})^{\frac{3}{4}}\exp(-\frac{k_z^2+k_\perp^2}{2\beta^2})$。

电磁形状因子

- 如果一个光子从di-夸克$S(A)$辐射出，(Fig. 1右图), 形状因子$F_{S(A)}(q^2)$，

$$F_{1S(A)}\left(q^2\right) = \int \frac{dxd^2\mathbf{k}_\perp}{2(2\pi)^3} \frac{\phi_{1s}'^*\left(x', k_\perp'\right)\phi_{1s}\left(x, k_\perp\right)}{16\bar{x}P^+P'^+\sqrt{\left(k_2\cdot\bar{P}' + m_2M_0'\right)\left(k_2\cdot\bar{P} + m_2M_0\right)}} \times \mathrm{Tr}\left[\left(\bar{\not{P}} + M_0\right)\gamma^+\left(\bar{\not{P}}' + M_0'\right)\bar{\Gamma}'_{L'S_{[qq]}J_l'}\left(\not{k}_2 + m_2\right)\Gamma_{LS_{[qq]}J_l}\cdot\Gamma^+_{S(A)}\right], \tag{7}$$

$$F_{2S(A)}\left(q^2\right) = \cdot\frac{-i(M+M')q_\perp^i}{\mathbf{q}_\perp^2}\int\frac{dxd^2\mathbf{k}_\perp}{2(2\pi)^3}\frac{\phi_{1s}'^*\left(x', k_\perp'\right)\phi_{1s}\left(x, k_\perp\right)}{16\bar{x}P^+P'^+\sqrt{\left(k_2\cdot\bar{P}' + m_2M_0'\right)\left(k_2\cdot\bar{P} + m_1M_0\right)}} \times \mathrm{Tr}\left[\left(\bar{\not{P}} + M_0\right)\sigma^{i+}\left(\bar{\not{P}}' + M_0'\right)\bar{\Gamma}'_{L'S_{[qq]}J_l'}\left(\not{k}_2 + m_2\right)\Gamma_{LS_{[qq]}J_l}\cdot\Gamma^+_{S(A)}\right], \tag{8}$$

最后，可以得到，

$$F_1(q^2) = F_{1Q}(q^2) + F_{1S(A)}(q^2), \tag{9}$$

$$F_2(q^2) = F_{2Q}(q^2) + F_{2S(A)}(q^2). \tag{10}$$

味道自旋波函数

物理形状因子为标量di-夸克和轴矢di-夸克跃迁形状因子的线性组合[3]，

$$[\text{ form factor }]^{\text{physical}}\left(q^2\right)=c_S\times[\text{ form factor }]_S+c_A\times[\text{ form factor }]_A$$

其中c_S 和c_A分别是重子的标量和轴矢量di- 夸克重叠因子，从重子初态和末态的味道自旋波函数推导得到。

强子矩阵元可以改写为，

$$\begin{aligned}\left\langle\Xi_c\left|j_\mu\right|\Xi_c{'}^{+}\right\rangle= & c_S\left\langle q_1\left[q_2 q_3\right]_S\left|j_\mu\right| q_1\left[q_2 q_3\right]_S\right\rangle \\ & +c_A\left\langle q_1\left[q_2 q_3\right]_A\left|j_\mu\right| q_1\left[q_2 q_3\right]_A\right\rangle .\end{aligned}$$

- $q_1=c, \quad c_S=0\,, c_A=0\,.$
- $q_1=s, \quad c_S=-\frac{\sqrt{3}}{4}\,, c_A=\frac{\sqrt{3}}{4}\,.$
- $q_1=u, \quad c_S=\frac{\sqrt{3}}{4}\,, c_A=-\frac{\sqrt{3}}{4}\,.$

[3]X. H. Hu, R. H. Li and Z. P. Xing, Eur. Phys. J. C **80** (2020) no.4, 320.

输入参数

- $\Xi_c^{+(0)}$ 和$\Xi_c^{\prime+(0)}$的质量(GeV)

$$\Xi_c^{+} = 2.468\,, \quad \Xi_c^{0} = 2.471\,, \quad \Xi_c^{\prime+} = 2.578\,, \quad \Xi_c^{\prime0} = 2.579\,.$$

- 夸克质量(GeV)

$$m_{u(d)} = 0.23\,, \quad m_s = 0.43\,, \quad m_c = 1.6\,, \quad m_b = 4.9\,.$$

diquark $[cu(cd)]$ 和$[cs]$ 的质量近似取$m_c + m_{u(d)}$ 和$m_c + m_s$。

- $\beta_{q_1[q_2q_3]}$
由于重-轻diquark与重重夸克表现出的颜色三重态一样，所以单重味道重子波函数中β 参数近似等于相应介子波函数中β参数 [4]。

$$\beta_{u[cs]} = 0.473\,, \quad \beta_{s[cq]} = 0.543\,.$$

- diquark 电荷: $e_{[q_1q_2]} = e_{q_1} + e_{q_2}$
轴矢矢量diquark的磁矩反常贡献: $\kappa = 1.6$ [5].

[4]W. Wang, F. S. Yu and Z. X. Zhao, Eur. Phys. J. C 77, no.11, 781 (2017)

[5]V. Keiner, Phys. Rev. C **54**, 3232-3239 (1996)

数值结果

重子的衰变宽度$\mathcal{B} \to \mathcal{B}'\gamma$只与形状因子$F_2(0)$ 有关,

$$\Gamma(\mathcal{B} \to \mathcal{B}'\gamma) = \frac{4\alpha \mid \vec{q} \mid^3}{(M+M')^2} \mid F_2(0) \mid^2 \tag{11}$$

光子动量$\mid \vec{q} \mid = \frac{M^2 - M'^2}{2M}$。

- $q_1 = s$, $[q_2 q_3] = [cu]$

$$F_1^{\Xi_c'^+ \to \Xi_c^+}(0) = -12.57/-5.07\,, \qquad F_2^{\Xi_c'^+ \to \Xi_c^+}(0) = -3.35/-10.29 \tag{12}$$

$$F_1^{\Xi_c'^0 \to \Xi_c^0}(0) = -3.78/3.72\,, \qquad F_2^{\Xi_c'^0 \to \Xi_c^0}(0) = 0.54/-6.41 \tag{13}$$

$$\Gamma_{\Xi_c'^+ \to \Xi_c^+ \gamma} = 1.60 * 10^{-5}/1.52 * 10^{-4}\,\mathrm{GeV} \tag{14}$$

$$\Gamma_{\Xi_c'^0 \to \Xi_c^0 \gamma} = 0.39 * 10^{-6}/5.56 * 10^{-5}\,\mathrm{GeV} \tag{15}$$

- $q_1 = u$, $[q_2 q_3] = [cs]$

$$F_1^{\Xi_c'^+ \to \Xi_c^+}(0) = 1.16/39.34\,, \qquad F_2^{\Xi_c'^+ \to \Xi_c^+}(0) = 6.35/-22.62 \tag{16}$$

$$F_1^{\Xi_c'^0 \to \Xi_c^0}(0) = 6.12/-12.97\,, \qquad F_2^{\Xi_c'^0 \to \Xi_c^0}(0) = -1.12/13.38 \tag{17}$$

$$\Gamma_{\Xi_c'^+ \to \Xi_c^+ \gamma} = 5.78 * 10^{-5}/7.32 * 10^{-4}\,\mathrm{GeV} \tag{18}$$

$$\Gamma_{\Xi_c'^0 \to \Xi_c^0 \gamma} = 1.68 * 10^{-6}/2.42 * 10^{-4}\,\mathrm{GeV} \tag{19}$$

Table 1: 电磁衰变$\Xi_c^{\prime +(0)} \to \Xi_c^{+(0)}\gamma$ 衰变宽度(单位： keV)

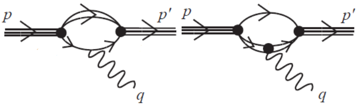
decay	$\Xi_c^{\prime +} \to \Xi_c^+\gamma$	$\Xi_c^{\prime 0} \to \Xi_c^0\gamma$	$\Sigma_c^+ \to \Lambda_c^+\gamma$
this work(diquark[cu]])	16/152	0.39/55.6	106
this work(diquark[cs]/[cd])	57.8/732	1.68/242	187
Chiral Lagrangians	19.7	0.4	91.5
Chiral Perturbation	5.43	0.46	65.6
Non-Relativistic Quark Potential Model	14.00	0.33	120
Relativistic Three-Quark Model	12.7	0.17	60.7
Bag Model	17.3	0.185	74.1
LC SR	8.5	0.27	50
LQCD	5.468	0.002	–
Effective Quark Mass Scheme	21.4	0.34	93.5

Table 2: 电磁衰变$\Xi_b^{\prime -} \to \Xi_b^- \gamma$ 衰变宽度(单位： keV)

decay	$\Xi_b^{\prime -} \to \Xi_b^- \gamma$
this work(diquark[cd]])	78.8
this work(diquark[cs])	138
Bag Model	0.118
Bag Model	0.357
Chiral Perturbation	1.00
LC SM	3.3

谢谢!

关于diquark 辐射光子



$$\varepsilon^*_{I\mu}(k,s_2)\cdot\varepsilon_{I\nu}(k,s_2) = -g_{\mu\nu} + \frac{k_\mu k_\nu}{m^2}$$

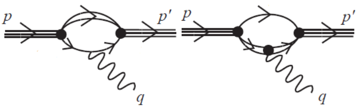
左图，强子矩阵元

$$F_{1Q}\left(q^2\right) = e_Q \int \frac{dx d^2\mathbf{k}_\perp}{2(2\pi)^3} \frac{\phi'^*_{1s}\left(x', k'_\perp\right)\phi_{1s}\left(x, k_\perp\right)}{16xP^+P'^+\sqrt{\left(k'_1\cdot\bar{P}' + m'_1 M'_0\right)\left(k_1\cdot\bar{P} + m_1 M_0\right)}}$$
$$\times \operatorname{Tr}\left[\left(\bar{\not{P}} + M_0\right)\gamma^+\left(\bar{\not{P}}' + M'_0\right)\Gamma'^{\mu}_{s11}\left(\not{k}'_1 + m'_1\right)\gamma^+\left(\not{k}_1 + m_1\right)\Gamma^{\nu}_{s11}\right]\cdot\varepsilon^*_\mu(k_2)\cdot\varepsilon_\nu(k_2) \quad (20)$$

右图，强子矩阵元

$$F_{1A}\left(q^2\right) = \int \frac{dx d^2\mathbf{k}_\perp}{2(2\pi)^3} \frac{\phi'^*_{1s}\left(x', k'_\perp\right)\phi_{1s}\left(x, k_\perp\right)}{16\bar{x}P^+P'^+\sqrt{\left(k_2\cdot\bar{P}' + m_2 M'_0\right)\left(k_2\cdot\bar{P} + m_2 M_0\right)}}$$
$$\times \operatorname{Tr}\left[\left(\bar{\not{P}} + M_0\right)\gamma^+\left(\bar{\not{P}}' + M'_0\right)\Gamma'^{\mu}_{s11}\left(\not{k}_2 + m_2\right)\Gamma^{\nu}_{s11}\cdot\Gamma^{+}_{\mu\nu}(A)\right] \quad (21)$$

关于diquark 辐射光子



$$\varepsilon^*_{I\mu}(k, s_2) \cdot \varepsilon_{I\nu}(k, s_2) = -g_{\mu\nu} + \frac{k_\mu k_\nu}{m^2}$$

左图，强子矩阵元

$$F_{1Q}\left(q^2\right) = e_Q \int \frac{dx d^2\mathbf{k}_\perp}{2(2\pi)^3} \frac{\phi'^*_{1s}\left(x', k'_\perp\right)\phi_{1s}\left(x, k_\perp\right)}{16xP^+P'^+\sqrt{\left(k'_1\cdot\bar{P}' + m'_1 M'_0\right)\left(k_1\cdot\bar{P} + m_1 M_0\right)}}$$
$$\times \operatorname{Tr}\left[\left(\bar{\not{P}} + M_0\right)\gamma^+\left(\bar{\not{P}}' + M'_0\right)\bar{\Gamma}'^{\mu}_{s11}\left(\not{k}'_1 + m'_1\right)\gamma^+\left(\not{k}_1 + m_1\right)\Gamma^{\nu}_{s11}\right]\cdot\left(-g_{\mu\nu} + \frac{k_{2\mu}k_{2\nu}}{m_2^2}\right) \tag{22}$$

右图，强子矩阵元

$$F_{1A}\left(q^2\right) = \int \frac{dx d^2\mathbf{k}_\perp}{2(2\pi)^3} \frac{\phi'^*_{1s}\left(x', k'_\perp\right)\phi_{1s}\left(x, k_\perp\right)}{16\bar{x}P^+P'^+\sqrt{\left(k_2\cdot\bar{P}' + m_2 M'_0\right)\left(k_2\cdot\bar{P} + m_2 M_0\right)}}$$
$$\times \operatorname{Tr}\left[\left(\bar{\not{P}} + M_0\right)\gamma^+\left(\bar{\not{P}}' + M'_0\right)\bar{\Gamma}'_{s11}\left(\not{k}_2 + m_2\right)\Gamma_{s11}\cdot\varepsilon^{*\mu}\cdot\Gamma^{+}_{\mu\nu}(A)\cdot\varepsilon^{\nu}\right] \tag{23}$$

光前夸克模型中介子衰变常数的流分量，极化矢量和参考系独立性分析

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◆研究动机

◆研究发现

◆分析与讨论

Independence of current components, polarization vectors, and reference frames in the light-front quark model analysis of meson decay constants

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研究动机

1 存在问题

在光前夸克模型（LF QM）中，采用不同的流分量计算非微扰物理量，不是总能得到相同的结果，存在自恰性问题。

2 在以前的一些LF QM自洽性研究工作中，发现：在壳条件（介子正反夸克在壳），用介子不变质量M_0替换物理介子质量M：$M \to M_0$，可解决模型自洽性问题。

研究发现： The Bakamjian-Thomas (BT) construction

自洽性条件：$M \to M_0$

$$M := M_0 + V_{Q\bar{Q}}$$

self-consistency condition, $P = p_1 + p_2$ or equivalently $M \to M_0$. This condition reflects effectively the BT construction in the computation of the one-body current matrix elements where the meson state is described in the non-interacting $Q\bar{Q}$ basis while the interaction is added to the mass operator via $M := M_0 + V_{Q\bar{Q}}$.

Relativistic Particle Dynamics. II*

B. Bakamjian and L. H. Thomas†
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(Received June 19, 1953)

The relativistic dynamics for a system of non-interacting particles in Hamiltonian form is separated by a contact transformation into motion of their center of mass and internal motion. Interaction at a distance between them is then introduced into the expression for the rest-mass in terms of the internal variables. This gives a dynamics for which invariance over space displacements and rotations is trivial and which is rigorously ... contact transf...

Spin variables are treated as quasi-momenta, referred, in classical mechanics, to true coordinates like Eulerian angles. Their commutation relations are sufficient, however, to justify the transformations without this reference.

The introduction of interaction by replacing m by the function M of the internal variables in Eqs. (2.21) and (2.22), with $\omega = \sum_u\{[\mathbf{s}_u \times \mathbf{S}_u] + \mathfrak{n}_u\}$, gives a dynamics which may be modified by any further contact transformation of the variables.

衰变常数:

定义

$$\begin{aligned}
\langle 0|\,\bar{q}\gamma^{\mu}\gamma_5 q\,|\mathrm{P}(P)\rangle &= i f_{\mathrm{P}} P^{\mu}, \\
\langle 0|\,\bar{q}\gamma^{\mu} q\,|\mathrm{V}(P,J_z)\rangle &= f_{\mathrm{V}} M \epsilon^{\mu}(J_z), \\
\langle 0|\,\bar{q}\sigma^{\mu\nu} q\,|\mathrm{V}(P,J_z)\rangle &= i f_{\mathrm{V}}^{T}\left[\epsilon^{\mu}(J_z)P^{\nu} - \epsilon^{\nu}(J_z)P^{\mu}\right]
\end{aligned}$$

$$\begin{aligned}
\mathcal{F} &= \sqrt{N_c}\int_0^1 \mathrm{d}x \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3}\,\phi(x,\mathbf{k}_\perp) \\
&\quad \times \frac{1}{\mathcal{P}}\sum_{\lambda_1,\lambda_2}\mathcal{R}^{JJ_z}_{\lambda_1\lambda_2}\left[\frac{\bar{v}_{\lambda_2}(p_2)}{\sqrt{x_2}}\mathcal{G}\frac{u_{\lambda_1}(p_1)}{\sqrt{x_1}}\right] \\
&= \sqrt{6}\int_0^1 \mathrm{d}x \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3}\,\frac{\phi(x,\mathbf{k}_\perp)}{\sqrt{m^2+\mathbf{k}_\perp^2}}\,\mathcal{O}(x,\mathbf{k}_\perp)
\end{aligned}$$

分析与讨论

TABLE I: Operators $\mathcal{O}$ and the helicity contributions $H_{\lambda_1\lambda_2}$ to $\mathcal{O}$ defined in Eq. (4) for all possible components of the current $\mathcal{G}$ and the polarization vectors $\epsilon(J_z)$, where $x_1 = x, x_2 = 1-x$, and $\mathcal{D}_0 = M_0 + 2m$.

$\mathcal{F}$	$\mathcal{G}$	$\epsilon(J_z)$	$H_{\uparrow\uparrow}$	$H_{\uparrow\downarrow}$	$H_{\downarrow\uparrow}$	$H_{\downarrow\downarrow}$	$\mathcal{O}$
$f_{\rm P}$	$\gamma^{(+,\perp)}\gamma_5$		0	m	m	0	$2m$
	$\gamma^-\gamma_5$		$\frac{2m\mathbf{k}_\perp^2}{x_1x_2(M_0^2+\mathbf{P}_\perp^2)}$	$m-\frac{2m\mathbf{k}_\perp^2}{x_1x_2(M_0^2+\mathbf{P}_\perp^2)}$	$m-\frac{2m\mathbf{k}_\perp^2}{x_1x_2(M_0^2+\mathbf{P}_\perp^2)}$	$\frac{2m\mathbf{k}_\perp^2}{x_1x_2(M_0^2+\mathbf{P}_\perp^2)}$	$2m$
$f_{\rm V}$	$\gamma^{(+,\perp)}$	$\epsilon(0)$	0	$m+\frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$	$m+\frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$	0	$2m+\frac{4\mathbf{k}_\perp^2}{\mathcal{D}_0}$
	γ^-	$\epsilon(0)$	0	$m+\frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$	$m+\frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$	0	$2m+\frac{4\mathbf{k}_\perp^2}{\mathcal{D}_0}$
	$\gamma^{(\perp,-)}$	$\epsilon(+1)$	$M_0-\frac{(M_0+m)\mathbf{k}_\perp^2}{x_1x_2M_0\mathcal{D}_0}$	$\frac{x_1(x_1M_0+m)\mathbf{k}_\perp^2}{x_1x_2M_0\mathcal{D}_0}$	$\frac{x_2(x_2M_0+m)\mathbf{k}_\perp^2}{x_1x_2M_0\mathcal{D}_0}$	0	$M_0-\frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$
$f_{\rm V}^T$	$\sigma^{\perp+}$	$\epsilon(+1)$	$2m+\frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$	0	0	0	$2m+\frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$
	$\sigma^{\perp-}$	$\epsilon(+1)$	$2m-\frac{2m(m+M_0)\mathbf{k}_\perp^2}{x_1x_2M_0^2\mathcal{D}_0}$	$\frac{2m(m+x_1M_0)\mathbf{k}_\perp^2}{x_1x_2M_0^2\mathcal{D}_0}$	$\frac{2m(m+x_2M_0)\mathbf{k}_\perp^2}{x_1x_2M_0^2\mathcal{D}_0}$	$\frac{2\mathbf{k}_\perp^4}{x_1x_2M_0^2\mathcal{D}_0}$	$2m+\frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$
	σ^{+-}	$\epsilon(0)$	$\frac{\mathbf{k}_\perp^2}{2x_1x_2\mathcal{D}_0}-\frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$	$\frac{M_0}{2}-\frac{\mathbf{k}_\perp^2}{2x_1x_2\mathcal{D}_0}$	$\frac{M_0}{2}-\frac{\mathbf{k}_\perp^2}{2x_1x_2\mathcal{D}_0}$	$\frac{\mathbf{k}_\perp^2}{2x_1x_2\mathcal{D}_0}-\frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$	$M_0-\frac{4\mathbf{k}_\perp^2}{\mathcal{D}_0}$

$$f_{\rm P} + f_{\rm V}(J_z) = 2f_{\rm V}^T(J_z)$$

分析与讨论

$$\mathcal{F} = \sqrt{6}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\,\frac{\hat{\phi}(\mathbf{k})}{M_0^{3/2}}\,\mathcal{O}(\mathbf{k}),$$

定义：

$$\tilde{\mathcal{O}}_V^{(T)} = \mathcal{O}_V^{(T)}(J_z=1) - \mathcal{O}_V^{(T)}(J_z=0) = \frac{2}{\mathcal{D}_0}(k_\perp^2 - 2k_z^2)$$

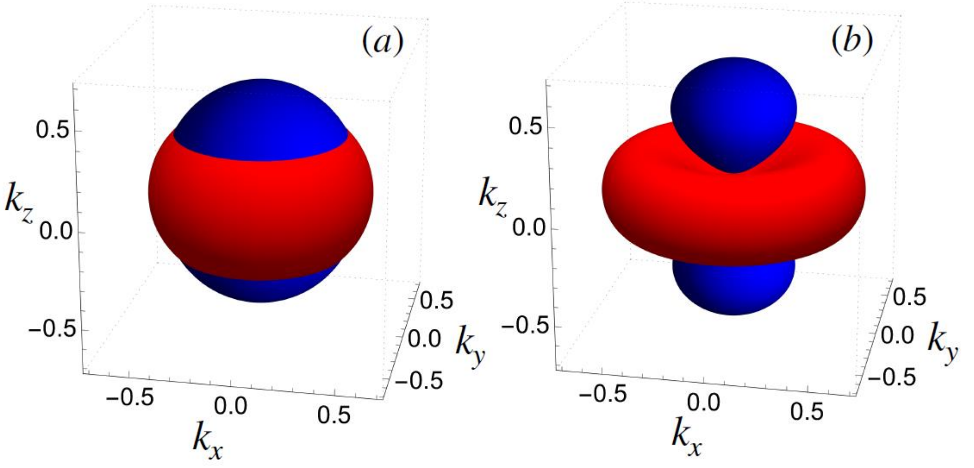

FIG. 1: The 3D plots of the wave functions (a) $\psi_\rho^{(J_z)}(\mathbf{k})$ for the ρ meson and (b) $\tilde{\psi}_\rho(\mathbf{k}) = \psi_\rho^{(0)} - \psi_\rho^{(+1)}$ defined by $f_\rho(J_z) = \int \mathrm{d}^3\mathbf{k}\,\psi_\rho^{(J_z)}(\mathbf{k})$, where $\psi_\rho^{(0)}$ (red) and $\psi_\rho^{(+1)}$ (blue).

$$\tilde{\psi}_\rho(\mathbf{k}) \propto (2k_z^2 - \mathbf{k}_\perp^2)$$

分析与讨论

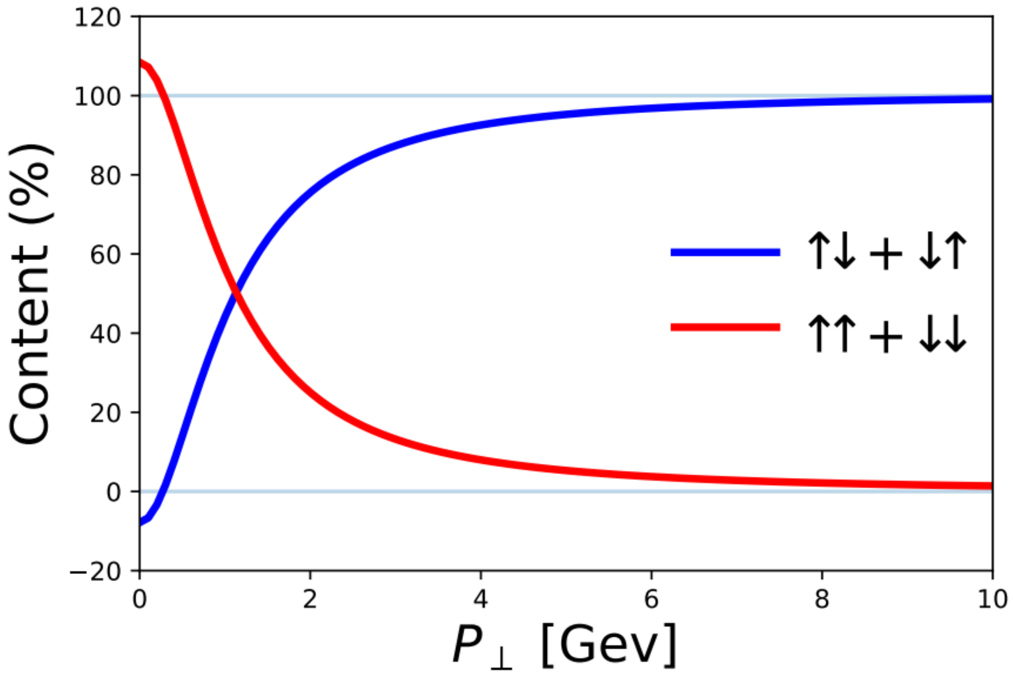

FIG. 2: The relative helicity contributions to f_π as a function of $\mathbf{P}_\perp$ calculated with the minus current. The blue and red lines represent the ordinary helicity (↑↓, ↓↑) and the higher helicity (↑↑, ↓↓) contributions, respectively. The sum is always the same regardless of $\mathbf{P}_\perp$.

分析与讨论

定义： $\tilde{\mathcal{O}}_{\mathrm{P}} \equiv \mathcal{O}_{\mathrm{P}}^{-} - \mathcal{O}_{\mathrm{P}}^{+}$

$$\tilde{\mathcal{O}}_{\mathrm{P}} = \frac{4(m_1 - m_2)M_0}{(\mathbf{P}_\perp^2 + M_0^2)} \boxed{k_z} \longleftarrow \quad p^- - p^+ = -2p^3$$

反对称： $k_z \leftrightarrow -k_z \Rightarrow \mathcal{F}^-(0) = \mathcal{F}^+(0)$

谢谢！

研究方法

Dirac spinors:
$$u(k,\lambda)=\frac{1}{\sqrt{k^+}}(\not{k}+m)u(\lambda),\quad v(k,\lambda)=\frac{1}{\sqrt{k^+}}(\not{k}-m)v(\lambda),$$

$$u\left(\frac{1}{2}\right)=\begin{pmatrix}1\\0\\0\\0\end{pmatrix},\quad u\left(-\frac{1}{2}\right)=\begin{pmatrix}0\\0\\0\\1\end{pmatrix},\quad v(\lambda)=u(-\lambda)$$

The spin wave functions:

P: $$\mathcal{R}^{00}_{\lambda\bar{\lambda}}(x,\mathbf{k}_\perp)=\frac{-1}{\sqrt{2}[M_0^2-(m_1-m_2)^2]^{1/2}}\bar{u}(p_1,\lambda)\gamma_5 v(p_2,\bar{\lambda}),$$

V: $$\mathcal{R}^{1J_3}_{\lambda\bar{\lambda}}(x,\mathbf{k}_\perp)=\frac{-1}{\sqrt{2}[M_0^2-(m_1-m_2)^2]^{1/2}}\times\bar{u}(p_1,\lambda)\left[\not{\varepsilon}(J_3)-\frac{\varepsilon\cdot(p_1-p_2)}{M_0+m_1+m_2}\right]v(p_2,\bar{\lambda}).$$

文献：arXiv：2210.12780