# GRIFFIN: A C++ library for EW radiative correction in fermion scattering and decay processes

- Introduction and motivation
- Framework set-up
- Structure of the code
- Comparison and future projections

Lisong Chen, Ayres Freitas
arxiv:2211.16272

a fighting griffin from the church in Czchów. now in muzeum narodowe w krakowie.

# Z-pole Observables

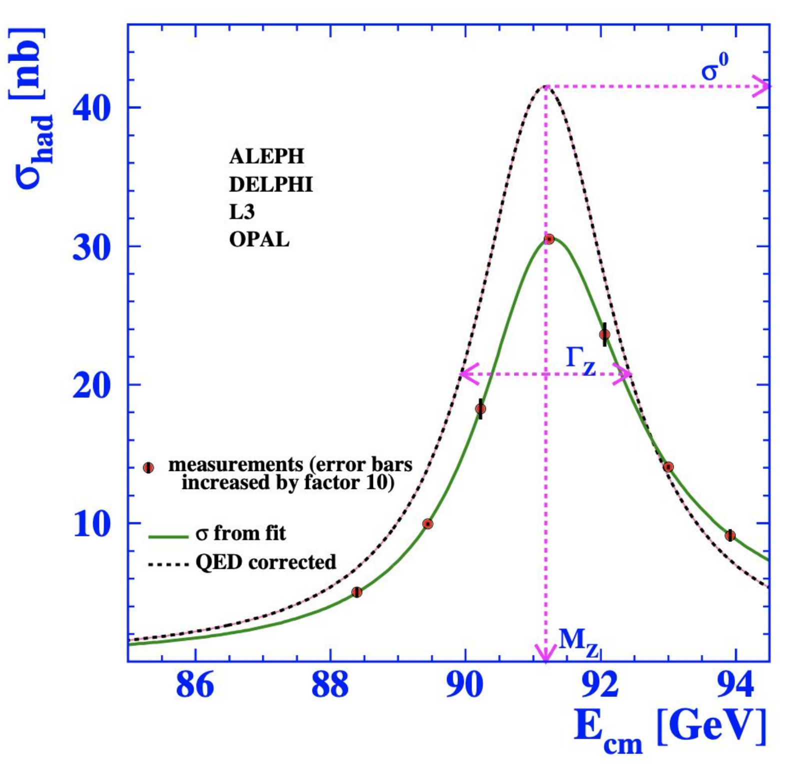
- cross section $\sigma(s = M_Z^2) \equiv \sigma_f^0$
- widths of Z boson.
- branching ratios.

$$\sigma^0_{had} = \sigma[e^+e^- \to hadrons]_{s=M_Z^2};$$

$$\Gamma_Z = \sum_f \Gamma[Z \to f\bar{f}],$$

$$R_l = \Gamma[Z \to hadrons]/\Gamma[Z \to l^+l^-], \qquad (l = e, \mu, \tau);$$

$$R_q = \Gamma[Z \to q\bar{q}]/\Gamma[Z \to hadrons], \qquad (q = b, c, s, d, u);$$



## Asymmetries and effective weak-mixing angle

$$\bar{\psi}(v_f - a_f\gamma^5)\gamma^\mu Z_\mu \psi$$

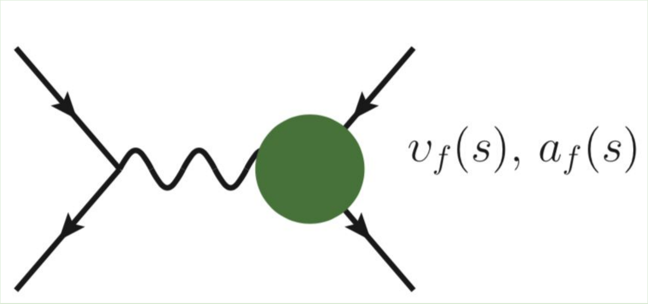
$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = \frac{2\Re v_e/a_e}{1+|v_e/a_e|^2} \equiv \mathcal{A}_e$$

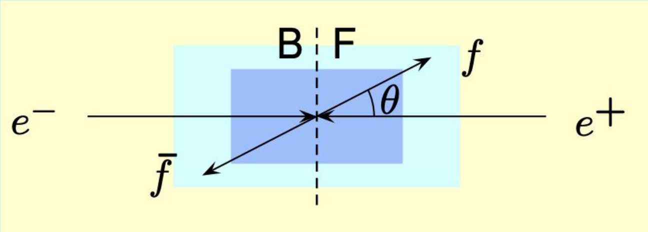
$$A_{FB} \equiv \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3\Re v_e/a_e}{1+(v_e/a_e)^2}\frac{\Re v_f/a_f}{1+(v_f/a_f)^2}$$

$$= \frac{3(1-4|Q_e|\sin^2\theta^e_{eff})}{1+(1-4|Q_e|\sin^2\theta^e_{eff})^2}\frac{(1-4|Q_f|\sin^2\theta^f_{eff})}{1+(1-4|Q_f|\sin^2\theta^f_{eff})^2}$$

$$\sin^2\theta^f_{eff} = \frac{1}{4|Q_f|}\left(1-\Re\frac{v_f}{a_f}\right) = \sin^2\theta_W(1+\Delta\kappa).$$

radiative corrections.



# SM Loop corrections

- 1-loop and leading 2-loop EW corrections
  Veltman, Passarino, Sirlin, Marciano, Bardin, Hollik, Riemann, Degrassi, Kniehl, …

- Full 2-loop corrections EW and mixed QCD-EW to $\Delta r$ and Z-pole observables

  Djouai, Verzegnassi '87, Djouadi '88, Kniehl, Kühn, Stuart '99, Kniehl, Sirlin '93, Djouadi, Gambino '94, Halzen Kniehl '91, Chetyrkin, Kühn '96, Fleischer et al. '92 Freitas, Hollik, Walter, Weiglein '00, Awramik, Czakon '02, Onishchenko, Vertin '02,
  Awramik, Czakon, Freitas, Weiglein '04, Awramik, Czakon, Freitas '06, Hollik, Meier, Uccirati '05 '07, Awramik, Czakon, Freitas, Kniehl '08, Freitas, Huang '12, Freitas '13'14, Dubovyk, Freitas, Gluza, Riemann, Usovitsch '18

- Approximate 3- and 4-loop corrections to universal parameters ($\rho$ parameter)

  Chetyrkin, Kühn, Steinhauser '95, Schröder, Steinhauser '05, Faisst, Kühn, Seidensticker, Veretin '03, Chetyrkin et al. '06, Boughezal, Tausk, v.d. Bij'05, Boughezal, Czakon '06

- Leading fermionic 3-loop EW&EW-QCD corrections to EWPOs. Chen, Freitas `20,

## Experimental uncertainties given by future electron-positron colliders

| | Exp | Current theo. error | CEPC | FCC-ee | ILC/GigaZ |
|---|---|---|---|---|---|
| $M_W$[MeV] | 12 | $4(\alpha^3, \alpha^2\alpha_s)$ | 1 | $0.5 \sim 1$ | 2.5 |
| $\Gamma_Z$[MeV] | 2.3 | $0.4(\alpha^3, \alpha^2\alpha_s, \alpha\alpha_s^2)$ | 0.5 | 0.1 | 1.0 |
| $\sin^2\theta_{\rm eff}^f$ $[10^{-5}]$ | 16 | $4.5(\alpha^3, \alpha^2\alpha_s)$ | $< 1$ | 0.6 | 1 |

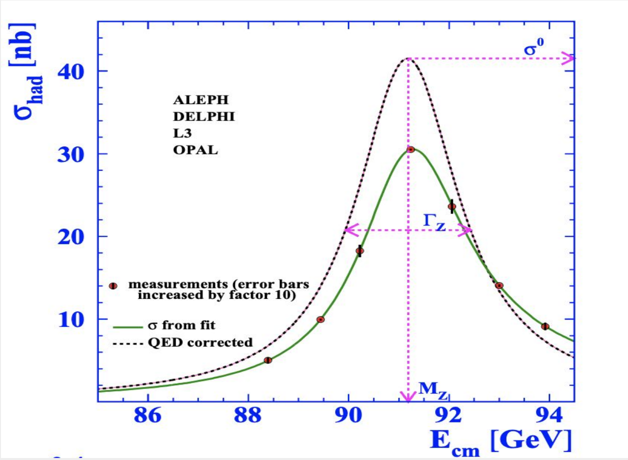
***The calculation of the next relevant order for the EWPOs will be indispensible!***

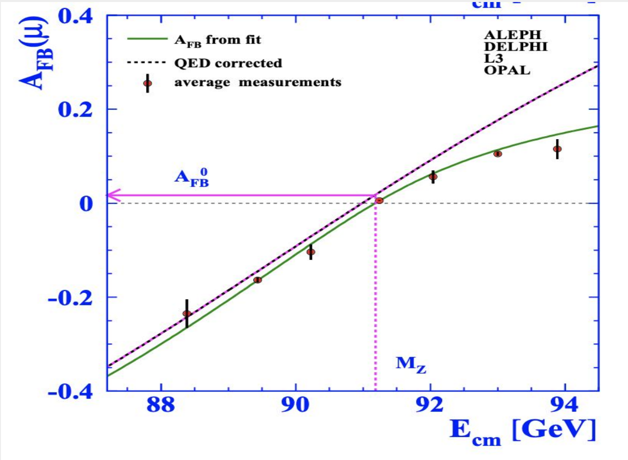
# How to connect precision observables with measurements?

- EWPOs are "pseudo-observables".
- Most of them connect to the Z boson lineshape and asymmetries. ---need theory input to extract. (Fixed-order+resummations)

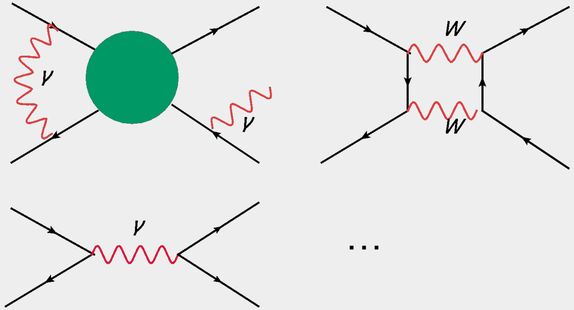
Implementation of QED effect:

1. Analytical
2. Monte Carlo tools.




LEP EWWG '05




$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{ini}(s, s') \otimes \sigma_{hard}(s')$$

$$\sigma_{\text{hard}} = \sigma_Z + \sigma_\gamma + \sigma_{\gamma Z} + \sigma_{\text{box}}$$

*shall be removed in determining EWPOs*

…..

## CERN-2019-003 (C4. by T.Riemman et al.)

In any case, we need to build a suitable theory framework. ZFITTER/DIZET will not be a useful basis for the FCC-ee, since it is structured to achieve consistent (1+1/2)-loop precision, but not beyond. No Laurent-series approach is foreseen in the kernel ZFITTER; but see Subsection C.4.5 on the SMATASY project and its applications to data. Further, later versions of the code lost modularity, owing to too-lazy additions concerning this item. We will have to begin developing a new program framework – probably object-oriented, e.g.,C++ – that is general enough to be extended to any loop order and to different assumptions about QED and inputs. All the future calculations, covering up to weak three loops and QCD four loops should be performed to fit into this new framework.

- In LEP/SLD era

  ZFITTER/DIZET(D. Bardin et al) ,TOPAZ0(G.Passarino et al), and BHM/WOH(W.Hollik et al, not public)...

- In future electron-positron colliders' era

  Formally gauge invariant setup .
  Extendability that accommodates higher precision
  and new physics.

➔ Motivates this project! (GRIFFIN: **G***auge*-**R***esonance*-**I***n*-**F***our*-**F***ermion*-**IN***teraction*)

# Combining on- and off-resonance

$$f\, f \to f' \bar{f}' \qquad (i,j) = (V,A)$$

- Laurent expansion is suitable for describing the physics in the vicinity of the resonance. (R.Stuart 91')

$$\mathcal{A}_{ij}\Big|_{s_0} = \frac{R_{ij}}{s - s_0} + S_{ij} + S'_{ij}(s - s_0)$$

- Away from the resonance, non-expanded matrix elements (and non-Dyson-resummed ,real mass only) gives a better description.

- Full description of the Z-lineshape ?

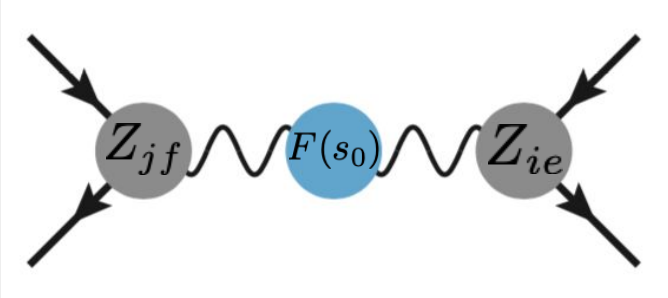
@NNLO @NLO @NLO

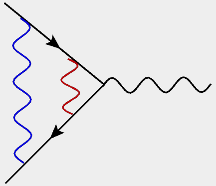
$$\mathcal{A}_{ij} = \mathcal{A}_{ij}\Big|_{s_0} + \mathcal{A}_{ij}^{noexp} - \mathcal{A}_{ij}\Big|_{M_Z^2}$$

Implemented in GRIFFIN !

# Leading Pole Term $R$

$$\mathcal{A}_{ij} = \frac{R_{ij}}{s - s_0} + S_{ij} + (s - s_0) S'_{ij}$$



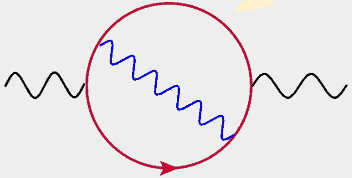



**For NNLO**

$$R_{ij} = \frac{Z_{ie} Z_{jf}}{1 + \Sigma'_Z}(1 + b^R_{\gamma Z})$$

Freitas, Hollik, Walter, Weiglein'00;Amramik, Czakon '02;Onishchenko, Vertin '02; Dubovyk, Freitas, Gluza, Riemann Usovitsch '18; Freitas '14; 13Awramik, Czakon, Freitas, Weiglein '04;Hollik, Meier, Uccirati'05; Awramik,Czakon, Feritas '06...

already exists!



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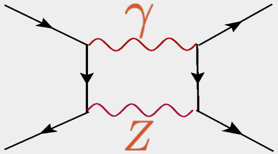
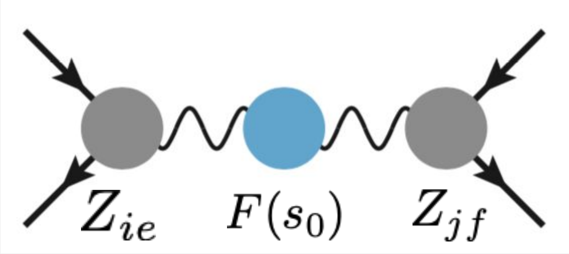
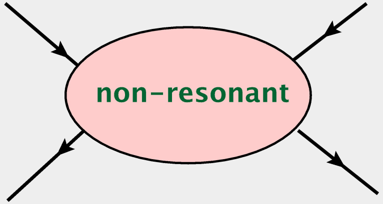


# Non-resonant terms $S$, $S'$

$$\mathcal{A}_{ij} = \frac{R_{ij}}{s - s_0} + S_{ij} + (s - s_0) S'_{ij}$$

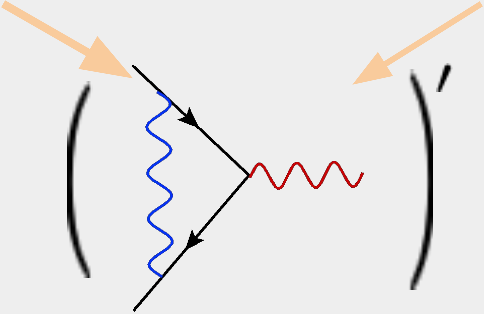



$+$




$$S_{ij} = Z'_{ie} Z_{jf} F(s_0) + Z_{ie} Z'_{jf} F(s_0) + Z_{ie} Z_{jf} F'(s_0) + B_{ij} + B_{ij}^{\gamma Z, S} \ldots$$

**For NNLO :**



$+ \quad \ldots$

the derivative of triangle-loop needs to be carried out.

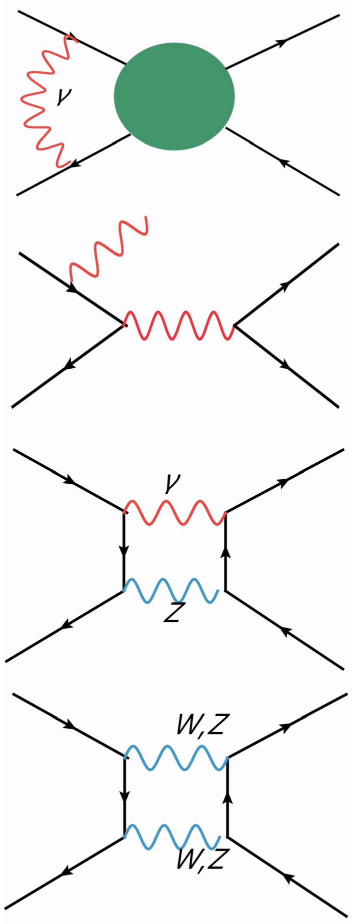
- Current state-of-art : *R*@NNLO+leading N3(4)LO,
  *S*@NNLO,
  *S'*@LO.
  *Off-resonance matrix elements* @NLO

- Future projection, FCC, e.g., requires at least one order higher for each!)

- QED vertex contributions can be fully taken care by MC tools (e.g. KKMC S. Jadach, B.F.L.Ward, Z.Wąs).

- photon-Z boxes needs special care since they also contribute to resonant part.

- Other finite EW boxes, as pure background, are computed.



# The Structure of the Library

| `class inval` | | `class psobs` | |
|---|---|---|---|
| input parameters (in the SM) | | output observables | |
| Boson masses and widths | $M_{\mathrm{W,Z,H}}$<br>$\Gamma_{\mathrm{W,Z}}$ | pesudo-observables defined at Z-peak | $F_{V,A}$, $\sin^2\theta^f_{eff}$<br>$\Gamma_{Z\to f\bar{f}}$ , $\Delta r$,<br>etc. |
| Fermion masses | $m^{\mathrm{OS}}_{\mathrm{e},\mu,\tau}$<br>$m^{\overline{\mathrm{MS}}}_{\mathrm{d,u,s,c}}(M_{\mathrm{Z}})$<br>$m^{\mathrm{OS}}_{\mathrm{t}}$ | amplitude coefficients under pole scheme | $R$ , $S$ , and $S'$ |
| Couplings | $\alpha(0)$<br>$\Delta\alpha \equiv 1-\alpha(0)/\alpha(M^2_{\mathrm{Z}})$<br>$\alpha_s^{\overline{\mathrm{MS}}}(M^2_{\mathrm{Z}})$, $G_\mu$ | (polarized) matrix element near or away Z-peak | $M_{ij}$ |

$$\sin^2\theta^f_{\mathrm{eff}} = \frac{1}{4|Q_f|}\left[1-\mathrm{Re}\,\frac{Z_{Vf}}{Z_{Af}}\right]_{s=M^2_{\mathrm{Z}}},$$

$$F^f_A = \left[\frac{|Z_{Af}|^2}{1+\mathrm{Re}\,\Sigma'_Z} - \frac{1}{2}M_{\mathrm{Z}}\Gamma_{\mathrm{Z}}|a^{\mathrm{Z}}_{f(0)}|^2\,\mathrm{Im}\,\Sigma''_Z\right]_{s=M^2_{\mathrm{Z}}} + \mathcal{O}(\alpha^3),$$

$$F^f_V = \left[\frac{|Z_{Vf}|^2}{1+\mathrm{Re}\,\Sigma'_Z} - \frac{1}{2}M_{\mathrm{Z}}\Gamma_{\mathrm{Z}}|v^{\mathrm{Z}}_{f(0)}|^2\,\mathrm{Im}\,\Sigma''_Z\right]_{s=M^2_{\mathrm{Z}}} + \mathcal{O}(\alpha^3)$$

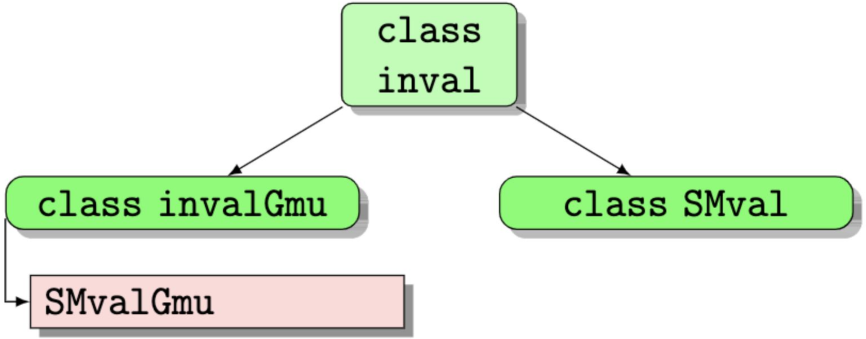
$$= F^f_A\left[(1-4|Q_f|\sin^2\theta^f_{\mathrm{eff}})^2 + \left(\mathrm{Im}\,\frac{Z_{Vf}}{Z_{Af}}\right)^2\right]$$

$$\Gamma_f[Z\to f\bar{f}] \equiv \frac{N^f_c M_Z}{12\pi}(\mathcal{R}_V F^f_V + \mathcal{R}_A F^f_A)$$

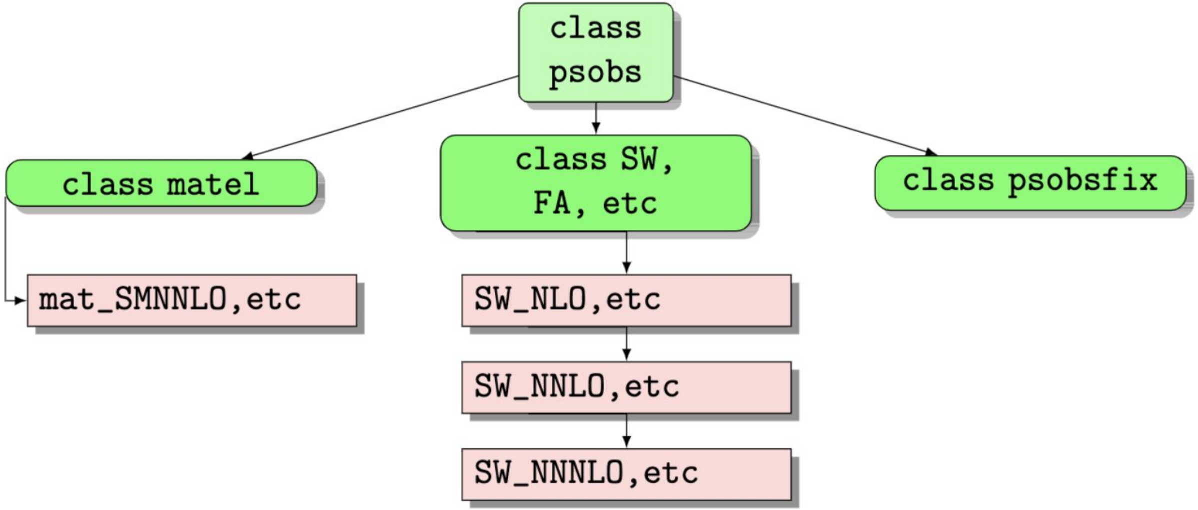
$$R = \mathcal{R}(F_A, \sin^2\theta_{eff}, b^R_{\gamma Z}, \ldots)$$

$$S = \mathcal{S}(Z_{ie}, Z_{jf}, Z'_{ie}, Z'_{jf}, \Sigma_Z, B_{ij}, \ldots)$$

- Class structure of the input:




- Class structure of the output:



## Example of the Code

- setting the input.

$$\overline{M} = M^{\exp}/\sqrt{1 + (\Gamma^{\exp}/M^{\exp})^2}\ , \overline{\Gamma} = M^{\exp}/\sqrt{1 + (\Gamma^{\exp}/M^{\exp})^2}.$$

```
#include "SMval.h"
int main()
{
  SMval myinput; //defining the input set as an object of class SMval
  myinput.set(MZ, 91.1876);
  myinput.set(GamZ, 2.4966);

  cout << myinput.get(MZc) << endl; //output the Z-boson mass in complex-
   pole mass scheme
}
```

**Example 2.2.1** Setting input values, with conversion of the gauge-boson masses and widths from PDG value to complex-pole masses and widths

- defining the virtual function that evaluates the form factors or observables,...

**Example 2.6.1** An example of defining function `result()` in the scope of the derived class `SW_SMLO`

```
Cplx SW_SMLO::result(void) const
{
  return(...); // the expression of effective weak-mixing angle defined at
   the LO SM
}
```

- the main file.

```
#include "EWOPZ2.h"
#include "SMval.h"
int main()
{
  SMval myinput; //defining the input set as an object of class SMval
  myinput.set(MZ, 91.1876);
  myinput.set(GamZ, 2.4966);
  ...             // more input parameters to be set up
  //defining objects from classes FA_SMLO and SW_SMLO
  FA_SMNNLO FA21(LEP, myinput);
  SW_SMNNLO SW21(LEP, myinput);
  //radiative correction for FA and SW due to delta_rho at O(alpha_t*alpha_s
   ^2)
  cout <<FA21.drho3aas2()<<endl;
  cout <<SW21.drho3aas2()<<endl;

  cout <<"the_leading-order_FA^lep"<< FA1.result() << endl; //output the F_A
   ^l at NNLO
  cout <<"the_leading-order_SW^lep"<< SW1.result() << endl; //output the SW^
   l at NNLO
}
```

**Figure 3: The example of outputting numerical results of $F^f_{V,A}$ and $\sin^2\theta^f_{\text{eff}}$ at NNLO in the SM.**

- An example of output from `testmatel.cc`

the form factors/EWPOs used in building matrix elements.

evaluations of helicity matrix elements at demanded order.

differential x-section of Z-lineshape (hard) at demanded order

```
======================================================

======================================================
   __________  ________________________   __
  / ____/ __ \/  _/ ____/ ____/  _/ | / /
 / / __/ /_/ // // /_  / /_   / //  |/ /
/ /_/ / _, _// // __/ / __/ _/ // /|  /
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======================================================
                 version 1.0
======================================================

Complex-pole masses: MW=80.35, MZ=91.1535

=== Matrix element for ee->dd (i=e, f=d) ===

F_A^i (NNLO+) = (0.034499,0)
F_A^f (NNLO+) = (0.0345443,0)
sineff^i (NNLO+) = (0.231172,0)
sineff^f (NNLO+) = (0.230985,0)

SM matrix element M_VV for cos(theta)=0.5:
sqrt(s)         tot. result                     off-resonance contrib.
10      (0.000316739,-5.58082e-06)      (0.000309429,-5.53734e-06)
30      (3.53793e-05,-5.99317e-07)      (2.84458e-05,-5.59139e-07)
50      (1.25851e-05,-1.90789e-07)      (6.4247e-06,-1.59184e-07)
70      (6.07798e-06,-5.97311e-08)      (1.19433e-06,-4.81728e-08)
90      (-7.31188e-07,-3.55673e-06)     (8.7104e-09,-1.80673e-09)
110     (3.14635e-06,-1.62001e-07)      (4.59289e-07,1.10821e-08)
130     (2.12596e-06,-7.90095e-08)      (1.82894e-06,1.92144e-08)
150     (1.5668e-06,-5.34561e-08)       (3.83515e-06,2.49419e-08)
170     (1.20884e-06,-3.97403e-08)      (6.35319e-06,2.97998e-08)
190     (9.60973e-07,-3.33532e-08)      (9.31833e-06,3.12732e-08)

diff. cross-section for cos(theta)=0.5:
sqrt(s) dsig/dcos [nb]
10      0.14256
30      0.0133563
50      0.00341281
70      0.00700565
90      2.36867
110     0.0325627
130     0.0106802
150     0.00583374
170     0.00383656
190     0.00276578
```

# Implementation of the higher-order contributions.

| | | | |
|---|---|---|---|
| Corrections entering through $\delta\rho$: | | | |
| | drho2aas | $\mathcal{O}(\alpha_t\alpha_s)$ | |
| | drho2a2 | $\mathcal{O}(\alpha_t^2)$ | |
| * | drho3aas2 | $\mathcal{O}(\alpha_t\alpha_s^2)$ | |
| * | drho3a2as | $\mathcal{O}(\alpha_t^2\alpha_s)$ | |
| * | drho3a3 | $\mathcal{O}(\alpha_t^3)$ | |
| * | drho3aas3 | $\mathcal{O}(\alpha_t\alpha_s^3)$ | |
| Full corrections to $F_A^f$, $\sin^2\theta_{\text{eff}}^f$: | | | |
| * | res2ff | $\mathcal{O}(\alpha_f^2)$ | |
| * | res2fb | $\mathcal{O}(\alpha_f\alpha_b)$ | |
| * | res2bb | $\mathcal{O}(\alpha_b^2)$ | |
| * | res2aas | $\mathcal{O}(\alpha\alpha_s)$ | (correction to internal gauge-boson self-energies) |
| * | res2aasnf | $\mathcal{O}(\alpha\alpha_s)$ | (non-factorizable final-state corrections for $f = q$) |
| * | res3fff | $\mathcal{O}(\alpha_f^3)$ | |
| * | res3ffa2as | $\mathcal{O}(\alpha_f^2\alpha_s)$ | |

* asterisk indicates the contribution that can be summed up as a meaningful result.

## Preliminary results and comparison with ZFITTER/DIZET

- **Benchmark inputs:**

| GRIFFIN input parameters | |
|---|---|
| DIZET input parameters | DIZET output |
| $\alpha_s(M_Z^2) = 0.118, \quad \alpha = 1/137.035999084$ | $\Gamma_Z = 2.495890$ GeV |
| $\Delta\alpha = 0.059, \quad M_Z = 91.1876$ GeV$, \quad G_\mu = 1.166137 \times 10^{-5}$ | $M_W = 80.3599$ GeV |
| $m_t = 173.0$ GeV$, \quad M_H = 125.0$ GeV$, \quad m_{e,\mu,\tau,u,d,s,c,b} = 0$ GeV | $\Gamma_W = 2.090095$ GeV |

- **using the W-mass and W-width output from dizet to minimize the parametrical shift between two schemes.**

## Numerical Results:

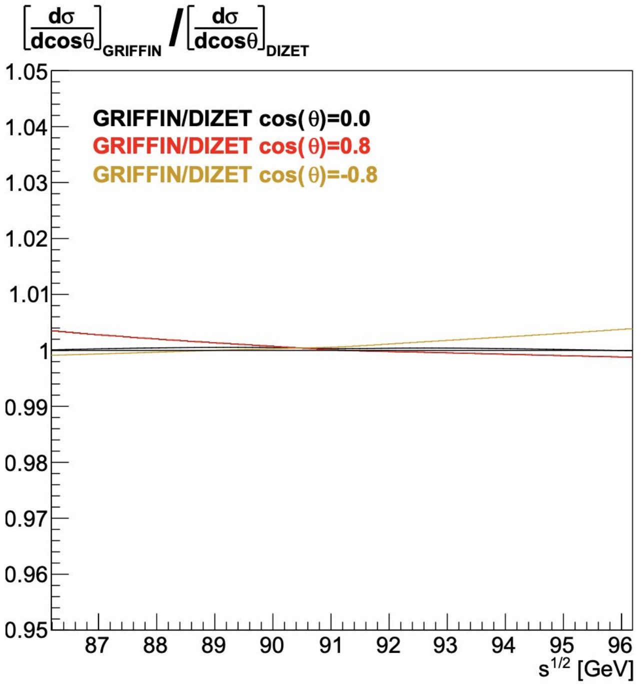
$$|\rho_Z^f| = \frac{2\sqrt{2}F_A^f}{G_\mu M_Z^2}$$

| | $\lvert\rho_Z^f\rvert$ | | $\sin^2\theta_{\text{eff}}^f$ | | $\Gamma_{Z\to f\bar{f}}$ | |
|---|---|---|---|---|---|---|
| | DIZET 6.45 | GRIFFIN | DIZET 6.45 | GRIFFIN | DIZET 6.45 | GRIFFIN |
| $\nu\bar{\nu}$ | 1.00800 | 1.00814 | 0.231119 | NAN | 0.167206 | 0.167197 |
| $\ell\bar{\ell}$ | 1.00510 | 1.00519 | 0.231500 | 0.231534 | 0.083986 | 0.083975 |
| $u\bar{u}$ | 1.00578 | 1.00573 | 0.231393 | 0.231420 | 0.299938 | 0.299958 |
| $d\bar{d}$ | 1.00675 | 1.00651 | 0.231266 | 0.231309 | 0.382877 | 0.382846 |
| $b\bar{b}$ | 0.99692 | 0.99420 | 0.232737 | 0.23292 | 0.376853 | 0.377432 |

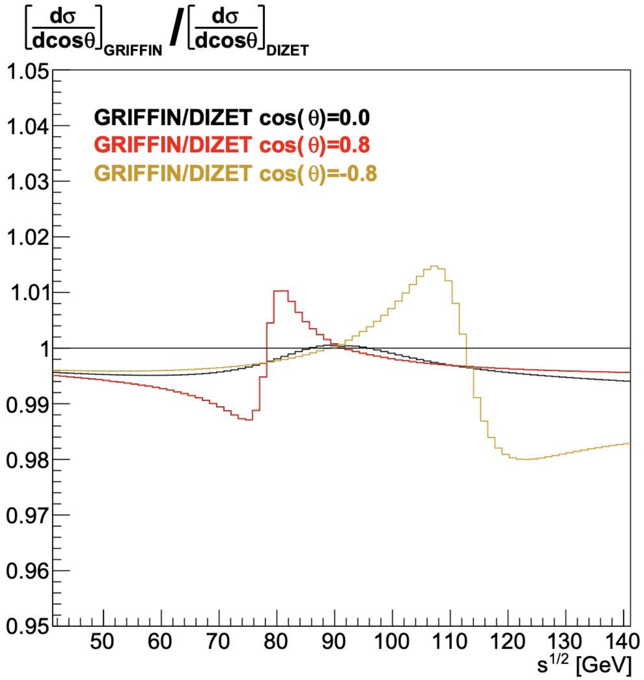
| | DIZET 6.45 | GRIFFIN all orders | GRIFFIN $\mathcal{O}(\alpha, \alpha^2, \alpha_t\alpha_s, \alpha_t\alpha_s^2)$ |
|---|---|---|---|
| $\Delta r$ | $3.63947 \times 10^{-2}$ | $3.68836 \times 10^{-2}$ | $3.63987 \times 10^{-2}$ |

- Not a **one-one-one match.** (no leading N3LO implemented in dizet v.6.45)
- most numbers are in agreement up to at least **4-digit**. The actual discrepancy is in the realm of missing N3(4)LO.
- fictitious discrepancies stem from the input scheme/definition of the form factors/EWPOs.

On the Z-pole. δ=griffin/dizet < 0.001




Off-resonance region. δ ~ 0.001- 0.02




*the authors thank S.Jadach and his group for providing the test program on KKMCee.

# Summary

- ❏ GRIFFIN provides a gauge-invariant, theoretically consistent description of 4-fermion scattering with a wider range of cme. It can systematically include higher-order contributions.

- ❏ In version 1.0, on shell renormalization, $G_\mu$- and $M_W$- input schemes are implemented. CEEX scheme have been implemented to separate IR physics for $\gamma\gamma$, $\gamma$Z boxes. The error estimation of EWPOs are also implemented.

- ❏ The results has been validated and checked with DIZET v6.45(A. Arbuzov, J.Gluza, et al. ‘19&‘23)

# Future projections:

- Interfacing with MC tools (KKMC, Sherpa, POWHEG-EW, etc)
- Alternative schemes regarding to the resonance, IR-subtractions/factorizations, renormalizations…
- Including orders *beyond* NNLO @ Z-pole, NNLO *away from* Z-pole, *Bhabhar* ME, etc.
- study of BSM, SMEFT.
- Other 4-fermion interaction processes. (e.g. charge-current Drell-Yan at the HL-LHC)

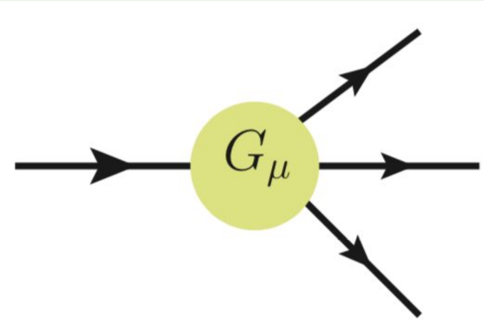
https://github.com/lisongc/GRIFFIN/releases/tag/v1.0.0

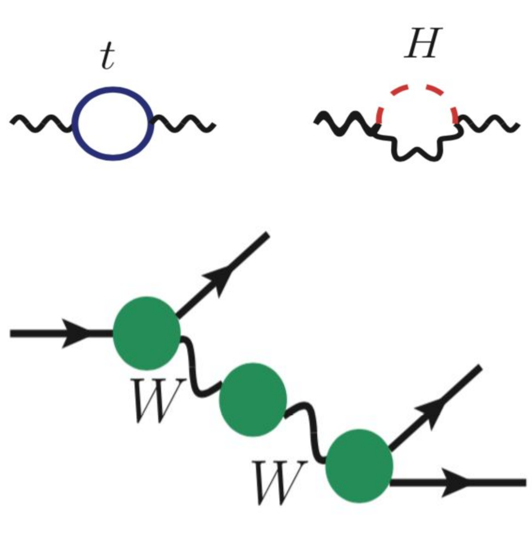
***we welcome feedbacks, suggestions, contributions/collaborations from the community!***


THANK YOU

# ***Backup Slides***

# $\Delta r$ and $M_{\mathrm{W}}$



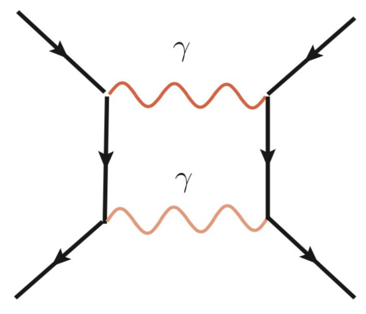




$$\Gamma_\mu = \frac{{G_F}^2 m_\mu^5}{192\pi^2} F(\frac{m_e^2}{m_\mu})(1 + \Delta q)$$

**2-loop QED**

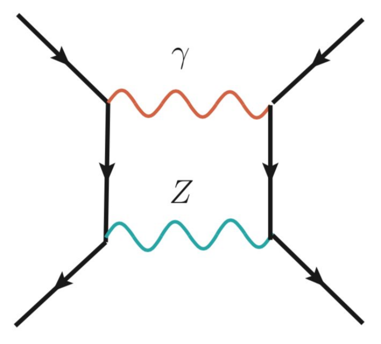
$$G_\mu = \frac{\pi\alpha}{\sqrt{2} s_W^2 M_W^2}(1 + \Delta r(M_H^2, M_t^2, \ldots))$$

$$M_W^2 = M_Z^2(\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\alpha\pi}{\sqrt{2} G_\mu M_Z^2}(1 + \Delta r)})$$




$$\gamma\gamma \text{ box:} \quad B_{\mathrm{VV}(1)} \to \overline{B}_{\mathrm{VV}(1)} = B_{\mathrm{VV}(1)} - S^{(0)}_{VV} \frac{\alpha}{\pi} Q_e Q_f \, f_{\mathrm{IR}}(m_\gamma, t, u),$$

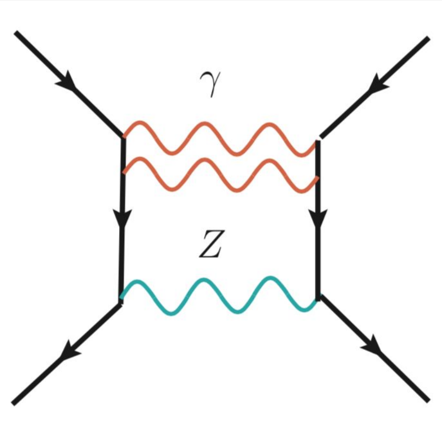
$$f_{\mathrm{IR}}(m_\gamma, t, u) = \ln\left(\frac{t}{u}\right)\left[\ln\left(\frac{m_\gamma^2}{\sqrt{tu}}\right) + \frac{1}{2}\right] = \ln\left(\frac{1-c_\theta}{1+c_\theta}\right)\left[\ln\left(\frac{2m_\gamma^2}{s\sqrt{1-c_\theta^2}}\right) + \frac{1}{2}\right]$$




$$\gamma Z \text{ box:} \quad B_{\gamma Z,ij(1)} \to \overline{B}_{\gamma Z,ij(1)} = B_{\gamma Z,ij(1)} - \frac{R^{(0)}_{ij}}{s - s_0} \frac{\alpha}{\pi} Q_e Q_f \left[f_{\mathrm{IR}}(m_\gamma, t, u) + \delta_G(s, t, u)\right],$$

$$\delta_G(s, t, u) = -2\ln\left(\frac{t}{u}\right)\,\ln\left(\frac{s_0 - s}{s_0}\right) = -2\ln\left(\frac{1-c_\theta}{1+c_\theta}\right)\,\ln\left(\frac{s_0 - s}{s_0}\right)$$

# Do we need?




- power counting $\alpha \sim \frac{\Gamma_Z}{M_Z}$
- For σ at $N^nLO$, we need n-loop at ***R,*** n-1-loop at ***S,*** n-2 at ***S'***
- Since

$$\delta_{ifi} = \frac{\alpha}{\pi}\frac{k_{0,min}}{E_{\text{beam}}} \ll \frac{\alpha}{\pi}\frac{\Gamma_Z}{M_Z} \sim \mathcal{O}(\alpha^2)$$

*(S. Jadach et al. '00)*

near the resonance. Can hence be safely neglected.

**In ZFITTER/DIZET:**

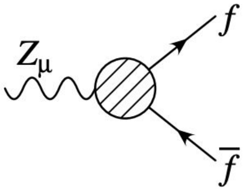
$$\Gamma_{Zf\bar{f}} = \Gamma_0 c_f \left|\rho_Z^f\right| \left( \left|g_Z^f\right|^2 R_V^f + R_A^f \right) + \delta_{\alpha\alpha_s}$$

$$\sin^2\theta_{eff}^f = \left(1 - \frac{M_W^2}{M_Z^2}\right)(1 + \Delta\kappa_f)$$

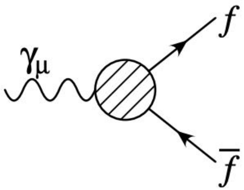
**Conversion:**

$$\left|\rho_Z^f\right| = \frac{2\sqrt{2}F_A^f}{G_\mu M_Z^2}$$

## An example of the numerical impact given by non-consistenly using pole scheme (M. Awramik, M. Czakon, A. Freitas ‘06)




$$\equiv \Gamma[Z_\mu f\bar{f}] \equiv z_{\mathrm{f},\mu} = i\gamma_\mu(v_\mathrm{f} + a_\mathrm{f}\gamma_5)$$




$$\equiv \Gamma[\gamma_\mu f\bar{f}] \equiv g_{\mathrm{f},\mu} = i\gamma_\mu(q_\mathrm{f} + p_\mathrm{f}\gamma_5),$$

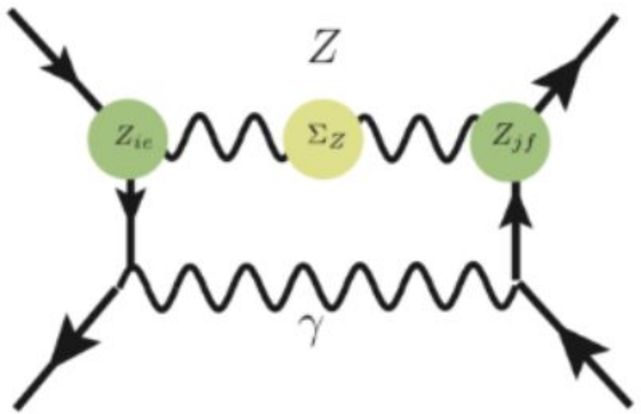
$$\delta s_W^2 = \sin^2\theta^f_{eff,ZFITTER} - \sin^2\theta^f_{eff,pole\,scheme}$$

$$= -\frac{\Gamma_Z}{M_Z}\frac{q_f^{(0)}}{a_e^{(0)}(a_f^{(0)} - v_f^{(0)})}(\Im p_e^{(1)} + \Im B_{ij}^{(1)}) \sim \mathcal{O}(10^{-6})$$

$$\delta^{exp}\sin^2\theta^f_{eff,\,FCC,\,CEPC} \sim \mathcal{O}(10^{-6})$$

## ❑ Pole scheme for gamma-Z box diagram.

$$B_{\gamma Z} \sim \int \frac{d^4q}{(2\pi)^4} \frac{\cdots}{q^2(\not{q} - \not{p}_2)(\not{q} - \not{k}_2)} \underbrace{\frac{Z_i(s', s_i) Z_f(s', s_f)}{s' - m_{Z0}^2 + \Sigma_Z(s')}}_{W(s', s_i, s_f)}$$




$$s' = (q + p_2 + p_1)^2, \quad s_i = (q + p_2)^2, \quad s_f = (q + k_2)^2$$

$$\begin{aligned} W(s', s_i, s_f) &= \frac{Z_i(s', s_i) Z_f(s', s_f)}{s' - m_Z^2 + \Sigma_Z(s')} \\ &= \frac{Z_i(s_0, 0) Z_f(s_0, 0) + Z_i(s', s_i) Z_f(s', s_f) - Z_i(s_0, 0) Z_f(s_0, 0)}{s' - s_0 + \Sigma_Z(s') - \Sigma_Z(s_0)} \\ &= \frac{Z_i(s_0, 0) Z_f(s_0, 0)}{(s' - s_0)(1 + \Sigma'_Z(s_0))} + \frac{Z_i(s', s_i) Z_f(s', s_f) - Z_i(s_0, 0) Z_f(s_0, 0)}{s' - s_0 + \Sigma_Z(s') - \Sigma_Z(s_0)} \\ &\equiv \frac{P(s_0)}{s' - s_0} + N(s', s_i, s_f) \end{aligned}$$