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Properties of the coupling constants of J/ψ→VP decays *

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研究内容

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta_{\mathrm{P}} & -\sin\theta_{\mathrm{P}} \\ \sin\theta_{\mathrm{P}} & \cos\theta_{\mathrm{P}} \end{pmatrix} \begin{pmatrix} \eta^8 \\ \eta^0 \end{pmatrix}, \tag{1}$$

$$\begin{pmatrix} \phi \\ \omega \end{pmatrix} = \begin{pmatrix} \cos\theta_{\mathrm{V}} & -\sin\theta_{\mathrm{V}} \\ \sin\theta_{\mathrm{V}} & \cos\theta_{\mathrm{V}} \end{pmatrix} \begin{pmatrix} \omega^8 \\ \omega^0 \end{pmatrix}, \tag{2}$$

$$\omega = \sqrt{\frac{1}{2}}|u\bar{u}+d\bar{d}\rangle,\ \phi = |s\bar{s}\rangle. \tag{3}$$

J/ψ→VP的有效拉格朗日方程

$$L_{\rm int}=\psi(g_8O_1^{\rm a}O_2^{\rm a}+g_1S_1S_2), \tag{4}$$

$$m_{\rm d}(\bar{u}u+\bar{d}d)+m_{\rm s}\bar{s}s=m_0\bar{q}q+\frac{1}{\sqrt{3}}(m_{\rm d}-m_{\rm s})\bar{q}\lambda_8 q, \tag{5}$$

$$\frac{2}{3}\bar{u}\gamma_\mu u-\frac{1}{3}\bar{d}\gamma_\mu d-\frac{1}{3}\bar{s}\gamma_\mu s=\frac{1}{2}\bar{q}\gamma_\mu\left(\lambda_3+\frac{1}{\sqrt{3}}\lambda_8\right)q, \tag{6}$$

$$\begin{aligned}\mathcal{L}_{\rm eff}\ =\ \psi\Big\{&g_8O_1^{\rm a}O_2^{\rm a}+g_1S_1S_2+g_{\rm S}d_{\rm abc}O_1^{\rm a}O_2^{\rm b}O_3^{\rm c}\\&+g_{\rm A}f_{\rm abc}O_1^{\rm a}O_2^{\rm b}O_3^{\rm c}+\sqrt{\frac{2}{3}}[C_{123}O_1^{\rm a}O_2^{\rm a}S_3\\&+C_{132}O_1^{\rm a}O_3^{\rm a}S_2+C_{231}O_2^{\rm a}O_3^{\rm a}S_1+f\,S_1S_2S_3]\Big\}.\end{aligned} \tag{7}$$

J/ψ→VP的有效拉格朗日方程及 参数

$$\mathcal{L}_{\rm eff} = \psi\Big\{ g_8 P_1^{\rm a} V_2^{\rm a} + g_1 P_0 V_0 + g_{\rm M}^{88} d_{\rm abc} O_1^{\rm a} O_2^{\rm b} M_3^{\rm c} + \sqrt{\frac{2}{3}}\left[g_{\rm M}^{81} O_1^{\rm a} M_3^{\rm a} S_2 + g_{\rm M}^{18} O_2^{\rm a} M_3^{\rm a} S_1 \right] + g_{\rm E}^{88} d_{\rm abc} O_1^{\rm a} \times O_2^{\rm b} E_3^{\rm c} + \sqrt{\frac{2}{3}}\left[g_{\rm E}^{81} O_1^{\rm a} E_3^{\rm a} S_2 + g_{\rm E}^{18} O_2^{\rm a} E_3^{\rm a} S_1 \right] \Big\}, \quad (8)$$

$$\begin{aligned}
P_1 &= \frac{1}{\sqrt{2}}(\pi^+ + \pi^-), \\
P_2 &= \frac{\rm i}{\sqrt{2}}(\pi^+ - \pi^-), \\
P_3 &= \pi^0, \\
P_4 &= \frac{1}{\sqrt{2}}(K^+ + K^-), \\
P_5 &= \frac{\rm i}{\sqrt{2}}(K^+ - K^-), \\
P_6 &= \frac{1}{\sqrt{2}}(K^0 + \overline{K}^0) = K_{\rm S}^0, \\
P_7 &= \frac{\rm i}{\sqrt{2}}(K^0 - \overline{K}^0) = iK_{\rm L}^0, \\
P_8 &= \eta_8^0 = \eta\cos\theta_{\rm P} + \eta'\sin\theta_{\rm P}, \\
P_0 &= \eta_1^0 = -\eta\sin\theta_{\rm P} + \eta'\cos\theta_{\rm P}, \qquad (9)
\end{aligned}$$

$$\begin{aligned}
V_1 &= \frac{1}{\sqrt{2}}(\rho^+ + \rho^-), \\
V_2 &= \frac{\rm i}{\sqrt{2}}(\rho^+ - \rho^-), \\
V_3 &= \rho^0, \\
V_4 &= \frac{1}{\sqrt{2}}(K^{*+} + K^{*-}), \\
V_5 &= \frac{\rm i}{\sqrt{2}}(K^{*+} - K^{*-}), \\
V_6 &= \frac{1}{\sqrt{2}}(K^{*0} + \overline{K}^{*0}), \\
V_7 &= \frac{\rm i}{\sqrt{2}}(K^{*0} - \overline{K}^{*0}), \\
V_8 &= \phi\cos\theta_{\rm V} + \omega\sin\theta_{\rm V}, \\
V_0 &= -\phi\sin\theta_{\rm V} + \omega\cos\theta_{\rm V}, \qquad (10)
\end{aligned}$$

$$
\begin{aligned}
P_1^{\mathrm{a}}V_2^{\mathrm{a}} &= P^1V^1+P^2V^2+P^3V^3+P^4V^4+P^5V^5+P^6V^6 \\
&\quad +P^7V^7+P^8V^8 \\
&= \pi^+\rho^-+\pi^-\rho^+ +\pi^0\rho^0+K^+K^{*-} \\
&\quad +K^-K^{*+}+K^0\overline{K}^{*0}+\overline{K}^0K^{*0} \\
&\quad +\eta\omega\cos\theta_{\mathrm{P}}\sin\theta_{\mathrm{V}}+\eta\phi\cos\theta_{\mathrm{P}}\cos\theta_{\mathrm{V}}+\eta'\omega\sin\theta_{\mathrm{P}}\sin\theta_{\mathrm{V}} \\
&\quad +\eta'\phi\sin\theta_{\mathrm{P}}\cos\theta_{\mathrm{V}}, \qquad (11)
\end{aligned}
$$

$$
\begin{aligned}
S_1S_2 &= P^0V^0=-\eta\omega\sin\theta_{\mathrm{P}}\cos\theta_{\mathrm{V}}+\eta\phi\sin\theta_{\mathrm{P}}\sin\theta_{\mathrm{V}} \\
&\quad +\eta'\omega\cos\theta_{\mathrm{P}}\cos\theta_{\mathrm{V}}-\eta'\phi\cos\theta_{\mathrm{P}}\sin\theta_{\mathrm{V}}, \qquad (12)
\end{aligned}
$$

$$
\begin{aligned}
d_{\mathrm{abc}}O_1^{\mathrm{a}}O_2^{\mathrm{b}}M_3^{\mathrm{c}} &= d_{\mathrm{ab8}}P^{\mathrm{a}}V^{\mathrm{b}}=d_{118}P^1V^1+d_{228}P^2V^2 \\
&\quad +d_{338}P^3V^3+d_{448}P^4V^4+d_{558}P^5V^5 \\
&\quad +d_{668}P^6V^6+d_{778}P^7V^7+d_{888}P^8V^8 \\
&= \frac{1}{\sqrt{3}}[\pi^+\rho^-+\pi^-\rho^+ +\pi^0\rho^0] \\
&\quad +\frac{-1}{2\sqrt{3}}[K^+K^{*-}+K^-K^{*+}+K^0\overline{K}^{*0} \\
&\quad +\overline{K}^0K^{*0}]+\frac{-1}{\sqrt{3}}[\eta\omega\cos\theta_{\mathrm{P}}\sin\theta_{\mathrm{V}} \\
&\quad +\eta\phi\cos\theta_{\mathrm{P}}\cos\theta_{\mathrm{V}}+\eta'\omega\sin\theta_{\mathrm{P}}\sin\theta_{\mathrm{V}} \\
&\quad +\eta'\phi\sin\theta_{\mathrm{P}}\cos\theta_{\mathrm{V}}], \qquad (13)
\end{aligned}
$$

$$
\begin{aligned}
O_1^{\mathrm{a}}M_3^{\mathrm{a}}S_2 &= P^8V^0=\eta\omega\cos\theta_{\mathrm{P}}\cos\theta_{\mathrm{V}}-\eta\phi\cos\theta_{\mathrm{P}}\sin\theta_{\mathrm{V}} \\
&\quad +\eta'\omega\sin\theta_{\mathrm{P}}\cos\theta_{\mathrm{V}}-\eta'\phi\sin\theta_{\mathrm{P}}\sin\theta_{\mathrm{V}}, \qquad (14)
\end{aligned}
$$

$$
\begin{aligned}
O_2^{\mathrm{a}}M_3^{\mathrm{a}}S_1 &= P^0V^8=-\eta\omega\sin\theta_{\mathrm{P}}\sin\theta_{\mathrm{V}}-\eta\phi\sin\theta_{\mathrm{P}}\cos\theta_{\mathrm{V}} \\
&\quad +\eta'\omega\cos\theta_{\mathrm{P}}\sin\theta_{\mathrm{V}}+\eta'\phi\cos\theta_{\mathrm{P}}\cos\theta_{\mathrm{V}}, \qquad (15)
\end{aligned}
$$

$$
\begin{aligned}
d_{\mathrm{ab3}}P^{\mathrm{a}}V^{\mathrm{b}} &= d_{383}P^3V^8+d_{833}P^8V^3+d_{443}P^4V^4 \\
&\quad +d_{553}P^5V^5+d_{663}P^6V^6+d_{773}P^7V^7 \\
&= \pi^0\omega\sqrt{\frac{1}{3}}\sin\theta_{\mathrm{V}}+\pi^0\phi\sqrt{\frac{1}{3}}\cos\theta_{\mathrm{V}} \\
&\quad +\eta\rho^0\sqrt{\frac{1}{3}}\cos\theta_{\mathrm{P}}+\eta'\rho^0\sqrt{\frac{1}{3}}\sin\theta_{\mathrm{P}}+\frac{1}{2}K^+K^{*-} \\
&\quad +\frac{1}{2}K^-K^{*+}-\frac{1}{2}K^0\overline{K}^{*0}-\frac{1}{2}\overline{K}^0K^{*0}, \qquad (16)
\end{aligned}
$$

$$
d_{\mathrm{abc}}O_1^{\mathrm{a}}O_2^{\mathrm{b}}E_3^{\mathrm{c}}=d_{\mathrm{ab3}}P^{\mathrm{a}}V^{\mathrm{b}}+\frac{1}{\sqrt{3}}d_{\mathrm{ab8}}P^{\mathrm{a}}V^{\mathrm{b}}, \qquad (17)
$$

$$
O_1^{\mathrm{a}}E_3^{\mathrm{a}}S_2 = P^3V^0+\frac{1}{\sqrt{3}}P^8V^0, \qquad (18)
$$

$$
\begin{aligned}
P^3V^0 &= \pi^0(\omega\cos\theta_{\mathrm{V}}-\phi\sin\theta_{\mathrm{V}}) \\
&= \sqrt{\frac{2}{3}}\pi^0\omega-\sqrt{\frac{1}{3}}\pi^0\phi, \qquad (19)
\end{aligned}
$$

$$
O_2^{\mathrm{a}}E_3^{\mathrm{a}}S_1 = V^3P^0+\frac{1}{\sqrt{3}}V^8P^0, \qquad (20)
$$

$$
\begin{aligned}
V^3P^0 &= \rho^0(-\eta\sin\theta_{\mathrm{P}}+\eta'\cos\theta_{\mathrm{P}}) \\
&= -\eta\rho^0\sin\theta_{\mathrm{P}}+\eta'\rho^0\cos\theta_{\mathrm{P}}, \qquad (21)
\end{aligned}
$$

decay modes	coupling constants and their corresponding strengths							
J/ψ→PV	g_8	g_1	$g_{\rm M}^{88}$	$g_{\rm M}^{81}$	$g_{\rm M}^{18}$	$g_{\rm E}^{88}$	$g_{\rm E}^{81}$	$g_{\rm E}^{18}$
$\pi^+\rho^-$	1	0	$\frac{1}{\sqrt{3}}$	0	0	$\frac{1}{3}$	0	0
$\pi^-\rho^+$	1	0	$\frac{1}{\sqrt{3}}$	0	0	$\frac{1}{3}$	0	0
$\pi^0\rho^0$	1	0	$\frac{1}{\sqrt{3}}$	0	0	$\frac{1}{3}$	0	0
K^+K^{*-}	1	0	$\frac{-1}{2\sqrt{3}}$	0	0	$\frac{1}{3}$	0	0
K^-K^{*+}	1	0	$\frac{-1}{2\sqrt{3}}$	0	0	$\frac{1}{3}$	0	0
$K^0\overline{K}^{*0}$	1	0	$\frac{-1}{2\sqrt{3}}$	0	0	$\frac{-2}{3}$	0	0
$\overline{K}^0K^{*0}$	1	0	$\frac{-1}{2\sqrt{3}}$	0	0	$\frac{-2}{3}$	0	0
$\pi^0\omega$	0	0	0	0	0	$\sqrt{\frac{1}{3}}\sin\theta_{\rm V}$	$\sqrt{\frac{2}{3}}\cos\theta_{\rm V}$	0
$\pi^0\phi$	0	0	0	0	0	$-\sqrt{\frac{1}{3}}\cos\theta_{\rm V}$	$\sqrt{\frac{2}{3}}\sin\theta_{\rm V}$	0
$\eta\rho^0$	0	0	0	0	0	$\sqrt{\frac{1}{3}}\cos\theta_{\rm P}$	0	$-\sqrt{\frac{2}{3}}\sin\theta_{\rm P}$
$\eta'\rho^0$	0	0	0	0	0	$\sqrt{\frac{1}{3}}\sin\theta_{\rm P}$	0	$\sqrt{\frac{2}{3}}\cos\theta_{\rm P}$
$\eta\omega$	$\cos\theta_{\rm P}\sin\theta_{\rm V}$	$-\sin\theta_{\rm P}\cos\theta_{\rm V}$	$\frac{-1}{\sqrt{3}}\cos\theta_{\rm P}\sin\theta_{\rm V}$	$\sqrt{\frac{2}{3}}\cos\theta_{\rm P}\cos\theta_{\rm V}$	$-\sqrt{\frac{2}{3}}\sin\theta_{\rm P}\cos\theta_{\rm V}$	$\frac{1}{\sqrt{3}}$4th colu.	$\frac{1}{\sqrt{3}}$5th colu.	$\frac{1}{\sqrt{3}}$6th colu.
$\eta\phi$	$\cos\theta_{\rm P}\cos\theta_{\rm V}$	$\sin\theta_{\rm P}\sin\theta_{\rm V}$	$-\frac{1}{\sqrt{3}}\cos\theta_{\rm P}\cos\theta_{\rm V}$	$-\sqrt{\frac{2}{3}}\cos\theta_{\rm P}\sin\theta_{\rm V}$	$-\sqrt{\frac{2}{3}}\sin\theta_{\rm P}\sin\theta_{\rm V}$	$\frac{1}{\sqrt{3}}$4th colu.	$\frac{1}{\sqrt{3}}$5th colu.	$\frac{1}{\sqrt{3}}$6th colu.
$\eta'\omega$	$\sin\theta_{\rm P}\sin\theta_{\rm V}$	$\cos\theta_{\rm P}\cos\theta_{\rm V}$	$\frac{-1}{\sqrt{3}}\sin\theta_{\rm P}\sin\theta_{\rm V}$	$\sqrt{\frac{2}{3}}\sin\theta_{\rm P}\cos\theta_{\rm V}$	$\sqrt{\frac{2}{3}}\cos\theta_{\rm P}\cos\theta_{\rm V}$	$\frac{1}{\sqrt{3}}$4th colu.	$\frac{1}{\sqrt{3}}$5th colu.	$\frac{1}{\sqrt{3}}$6th colu.
$\eta'\phi$	$\sin\theta_{\rm P}\cos\theta_{\rm V}$	$-\cos\theta_{\rm P}\sin\theta_{\rm V}$	$-\frac{1}{\sqrt{3}}\sin\theta_{\rm P}\cos\theta_{\rm V}$	$-\sqrt{\frac{2}{3}}\sin\theta_{\rm P}\sin\theta_{\rm V}$	$\sqrt{\frac{2}{3}}\cos\theta_{\rm P}\sin\theta_{\rm V}$	$\frac{1}{\sqrt{3}}$4th colu.	$\frac{1}{\sqrt{3}}$5th colu.	$\frac{1}{\sqrt{3}}$6th colu.

$$g_{\rm E}^{\rm i} = |g_{\rm E}^{\rm i}|e^{\delta_{\rm E}}$$

假设：

$$g_{\rm M}^{88} = g_{\rm M}^{81} = g_{\rm M}^{18} = g_{\rm M}$$

$$g_{\rm E}^{88} = g_{\rm E}^{81} = g_{\rm E}^{18} = g_{\rm E}$$

$$\theta_{\rm V} \approx 35.3^\circ$$

Decay mode	Coupling constant			
$J/\psi\to X$	g_8	g_1	g_M	g_E
$\pi^+\rho^-$	1	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{3}$
$\pi^-\rho^+$	1	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{3}$
$\pi^0\rho^0$	1	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{3}$
K^+K^{*-}	1	0	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$
K^-K^{*+}	1	0	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$
$K^0\bar{K}^{*0}$	1	0	$-\frac{1}{2\sqrt{3}}$	$-\frac{2}{3}$
$\bar{K}^0K^{*0}$	1	0	$-\frac{1}{2\sqrt{3}}$	$-\frac{2}{3}$
$\eta\phi$	$(\frac{2}{3})^{1/2}\cos\theta_P$	$(\frac{1}{3})^{1/2}\sin\theta_P$	$-\frac{2}{\sqrt{3}}[(\frac{2}{3})^{1/2}\cos\theta_P+(\frac{1}{3})^{1/2}\sin\theta_P]$	$-\frac{2}{3}[(\frac{2}{3})^{1/2}\cos\theta_P+(\frac{1}{3})^{1/2}\sin\theta_P]$
$\eta\omega$	$(\frac{1}{3})^{1/2}\cos\theta_P$	$-(\frac{2}{3})^{1/2}\sin\theta_P$	$\frac{1}{\sqrt{3}}[(\frac{1}{3})^{1/2}\cos\theta_P-(\frac{2}{3})^{1/2}\sin\theta_P]$	$\frac{1}{3}[(\frac{1}{3})^{1/2}\cos\theta_P-(\frac{2}{3})^{1/2}\sin\theta_P]$
$\eta\rho^0$	0	0	0	$[(\frac{1}{3})^{1/2}\cos\theta_P-(\frac{2}{3})^{1/2}\sin\theta_P]$
$\eta'\phi$	$(\frac{2}{3})^{1/2}\sin\theta_P$	$-(\frac{1}{3})^{1/2}\cos\theta_P$	$-\frac{2}{\sqrt{3}}[(\frac{2}{3})^{1/2}\sin\theta_P-(\frac{1}{3})^{1/2}\cos\theta_P]$	$-\frac{2}{3}[(\frac{2}{3})^{1/2}\sin\theta_P-(\frac{1}{3})^{1/2}\cos\theta_P]$
$\eta'\omega$	$(\frac{1}{3})^{1/2}\sin\theta_P$	$(\frac{2}{3})^{1/2}\cos\theta_P$	$\frac{1}{\sqrt{3}}[(\frac{1}{3})^{1/2}\sin\theta_P+(\frac{2}{3})^{1/2}\cos\theta_P)]$	$\frac{1}{3}[(\frac{1}{3})^{1/2}\sin\theta_P+(\frac{2}{3})^{1/2}\cos\theta_P]$
$\eta'\rho^0$	0	0	0	$[(\frac{1}{3})^{1/2}\sin\theta_P+(\frac{2}{3})^{1/2}\cos\theta_P]$
$\pi^0\phi$	0	0	0	0
$\pi^0\omega$	0	0	0	1

分析与总结

parameter	Mark-III	DM2	PDG2012
g_8 $(\times 10^{-3})$	5.89±0.18	5.57±0.22	5.98±0.13
g_1 $(\times 10^{-3})$	2.72±0.20	3.12±0.40	2.77±0.26
g_M $(\times 10^{-4})$	8.08±3.21	5.84±4.22	9.44±2.72
$\lvert g_E \rvert$ $(\times 10^{-3})$	2.11±0.10	1.95±0.11	2.06±0.08
δ_E	71.4±11.5	74.9±17.0	75.8±6.87
θ_P	−19.5±1.52	−20.0±0.68	−19.0±1.59
χ^2/d.o.f	7.19/4	3.63/4	19.4/4

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decay modes	Mark-III	DM2	BES	PDG2012
$\rho\pi$	14.2±0.1±1.9	13.2±2.0	21.8±0.05±2.01	16.9±1.5
$\rho\eta$	0.193±0.013±0.029	0.194±0.017±0.029		0.193±0.023
$\rho\eta'$	0.114±0.014±0.016	0.083±0.030±0.012		0.105±0.018
$\phi\pi$	<0.0068		<0.0064	<0.0064
$\phi\eta$	0.661±0.045±0.078	0.64±0.04±0.11	0.898±0.024±0.089	0.75±0.08
$\phi\eta'$	0.308±0.034±0.036	0.41±0.03±0.08	0.546±0.031±0.056	0.40±0.07
$\omega\pi$	0.482±0.019±0.064	0.360±0.028±0.054	0.538±0.012±0.065	0.45±0.05
$\omega\eta$	1.71±0.08±0.20	1.43±0.10±0.21	2.352±0.273	1.74±0.20
$\omega\eta'$	0.166±0.017±0.019	$0.18^{+0.10}_{-0.08}$±0.03	0.226±0.043	0.182±0.021
$K^{*-}K^{+}$+c.c.	5.26±0.13±0.53	4.57±0.17±0.70		5.12±0.30
$K^{*0}\bar{K}^{0}$+c.c.	4.33±0.12±0.45	3.96±0.15±0.60		4.39±0.31

parameter	Mark-III	DM2	PDG2012
g_8 $(\times10^{-3})$	5.89±0.18	5.57±0.22	5.98±0.13
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θ_P	−19.5±1.52	−20.0±0.68	−19.0±1.59
χ^2/d.o.f	7.19/4	3.63/4	19.4/4