Studying strong field QED in eA collisions

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UPC physics 2023, 复旦大学，2023. 05. 26-18

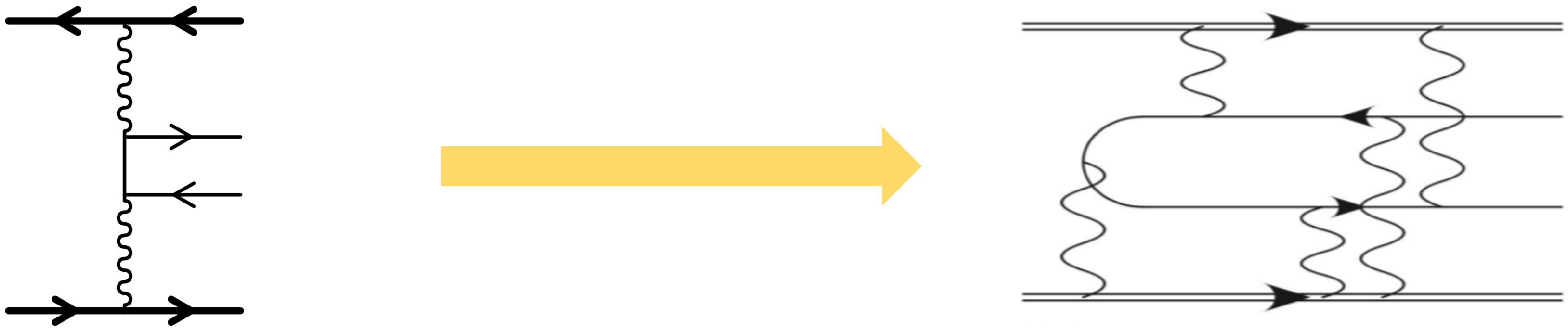
Outline:

- Why not UPC?
- Coulomb correction: formulation in TMD factorization
- Numerical results for the Bethe-Heitler process
- Summary

EM interaction with a large nuclei

For a Pb target, the effective coupling is:

$$82/137 \approx 0.6$$



Brief history

◆ Pioneered by:

Theory of Bremsstrahlung and Pair Production. I. Differential Cross Section

H. A. Bethe and L. C. Maximon*
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York
(Received October 29, 1953)

- **Solve the Dirac equation:**

- R. Jackiw, D. N. Kabat and M. Ortiz, 1992
- D. Ivanov and K. Melnikov, 1998
- U. Eichmann, J. Reinhardt and W. Greiner, 1999

….

- **Multiple photon scattering**

B. Segev and J. C. Wells, 1998
A.   J. Baltz and L. D. McLerran,1998
A. J. Baltz, F. Gelis, L. D. McLerran and A. Peshier, 1998
A.   J. Baltz, 2003
K. Tuchin,2009
….

The equivalence of two approaches, R. N. Lee and A. I. Milstein, 2000

Effective photon propagator

- t channel photon propagator:

$$F(k) = \frac{4\pi\alpha}{k^2} \quad \longrightarrow \quad F(\mathbf{k}) = \frac{4\pi\alpha Z}{k^{2-2i\alpha Z}}$$

Power suppression in di-lepton production

- According to power counting: qt^2/Q^2
- Coulomb correction in UPCs needs to be better understood

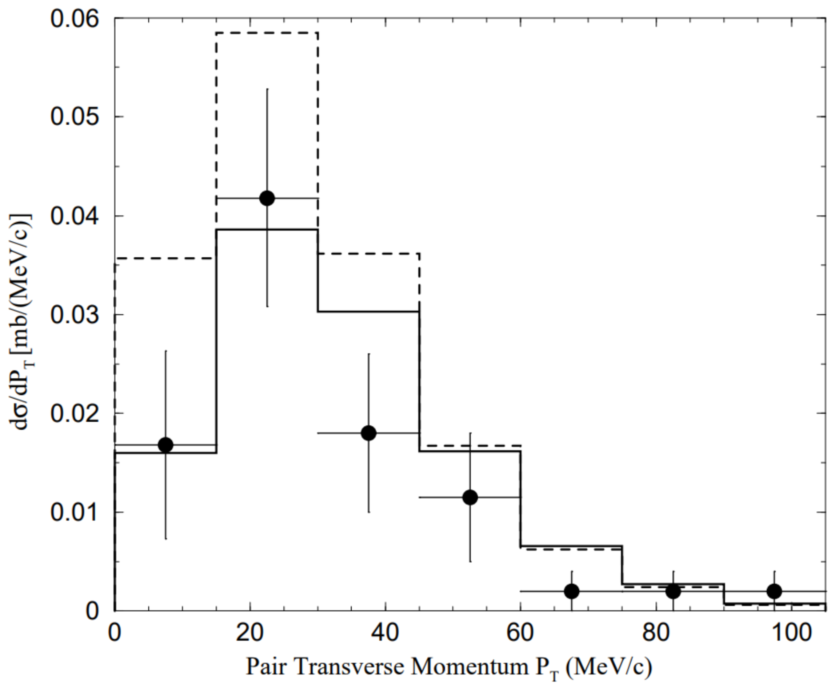
Physics Reports
Volume 453, Issue 1, November 2007, Pages 1-27

Electron–positron pair production in ultrarelativistic heavy ion collisions

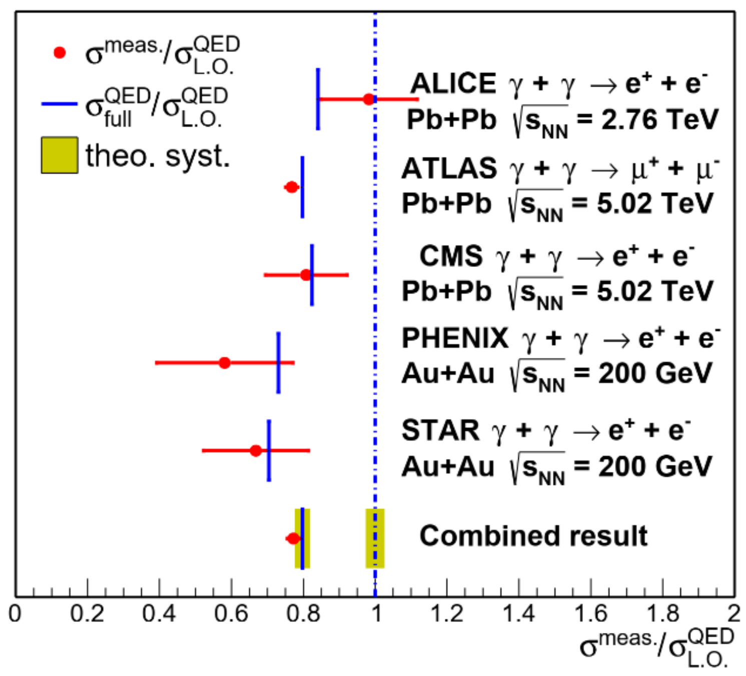
Gerhard Baur [a], Kai Hencken [b c], Dirk Trautmann [b]

In April 1990 a workshop took place in Brookhaven with the title 'Can RHIC be used to test QED?' [98]. We think that after about 17 years the answer to this question is 'no'. However, many theorists were motivated to deal with this

Yet…



A. J. Baltz, 2007

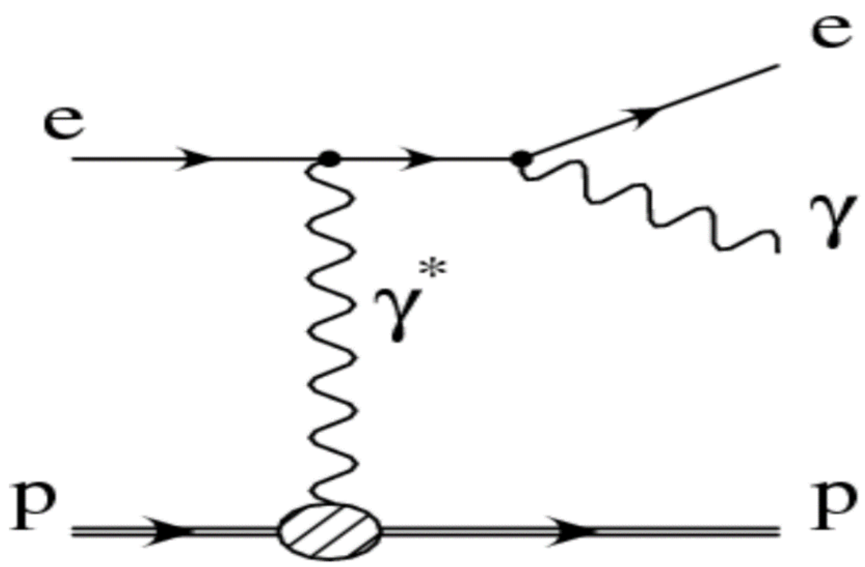


W. Zha and Z. Tang, 2021

- Cross section is reduced by the Coulomb correction!

EIC/EicC may offer the better opportunity to search for Coulomb correction

The Bethe-Heitler process:



Coulomb correction in TMD factorization

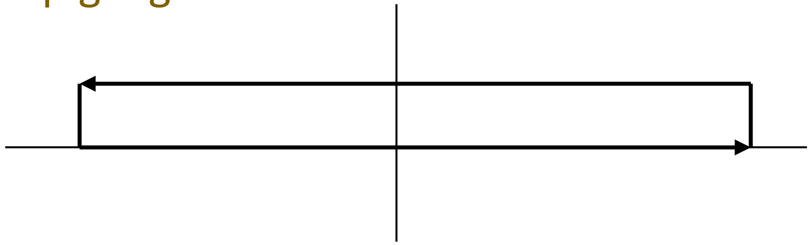
Photon TMDs

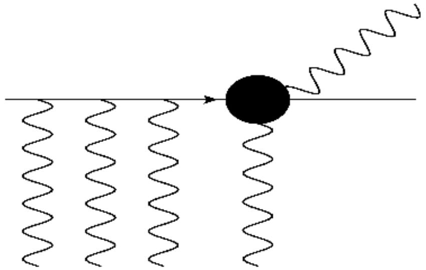
Operator definition:

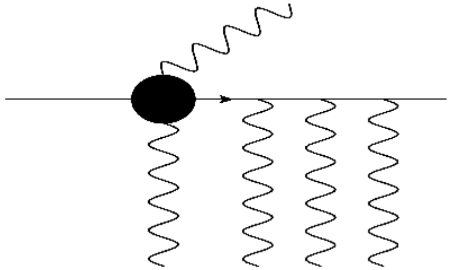
$$\int \frac{dy^- d^2 y_\perp}{P^+ (2\pi)^3} e^{ik\cdot y} \langle P| F^\mu_{+\perp}(0) U^\dagger(0_\perp) U(y_\perp) F^\nu_{+\perp}(y) |P\rangle \Big|_{y^+=0} = \frac{\delta^{\mu\nu}_\perp}{2} x f_1^\gamma(x, k_\perp^2) + \left(\frac{k^\mu_\perp k^\nu_\perp}{k^2_\perp} - \frac{\delta^{\mu\nu}_\perp}{2} \right) x h_1^{\perp\gamma}(x, k_\perp^2)$$

- Gauge link: $U(y_\perp) = \mathcal{P} e^{ie \int_{-\infty}^{+\infty} dz^- A^+(z^-, y_\perp)}$

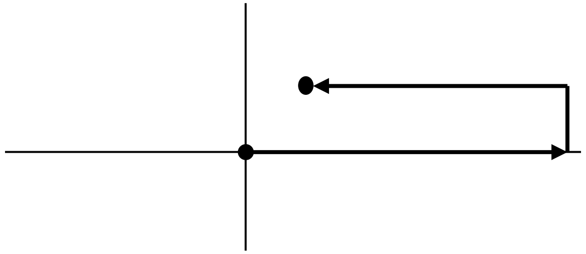
- Close loop gauge link







- Staple like gauge link in QCD



Weak field limit (single photon exchange)

$$x f_{1,0}^{\gamma}(x, k_\perp^2) = x h_{1,0}^{\perp\gamma}(x, k_\perp^2) = \frac{Z^2\alpha}{\pi^2} k_\perp^2 \left[\frac{F(k_\perp^2 + x^2 M_p^2)}{(k_\perp^2 + x^2 M_p^2)} \right]^2$$

Nuclear charge form factor

From transverse derivative

Photon propagator

Gauge link contribution: $U(y_\perp) = \mathcal{P} e^{ie\int_{-\infty}^{+\infty} dz^- A^+(z^-, y_\perp)}$

$$\mathcal{V}(y_\perp) \equiv e\int_{-\infty}^{+\infty} dz^- A^+(z^-, y_\perp) = \frac{\alpha Z}{\pi}\int d^2q_\perp e^{-iy_\perp\cdot q_\perp}\frac{F(q_\perp^2)}{q_\perp^2+\delta^2}$$

Infrared regulator

- For a point like particle: $F(q_\perp^2) = 1$

$$\mathcal{V}(y_\perp) = 2Z\alpha \lim_{\delta\to 0} K_0(|y_\perp|\delta) \approx Z\alpha\left(-2\gamma_E + \ln\frac{4}{y_\perp^2\delta^2}\right)$$

$$\mathcal{F}^\mu(x, y_\perp) = \frac{Ze}{2\pi}\frac{y_\perp^\mu}{|y_\perp|} x M_p K_1(|y_\perp| x M_p)$$

- Inserting into the operator definition of photon TMD:

Infrared regulator cancelled!

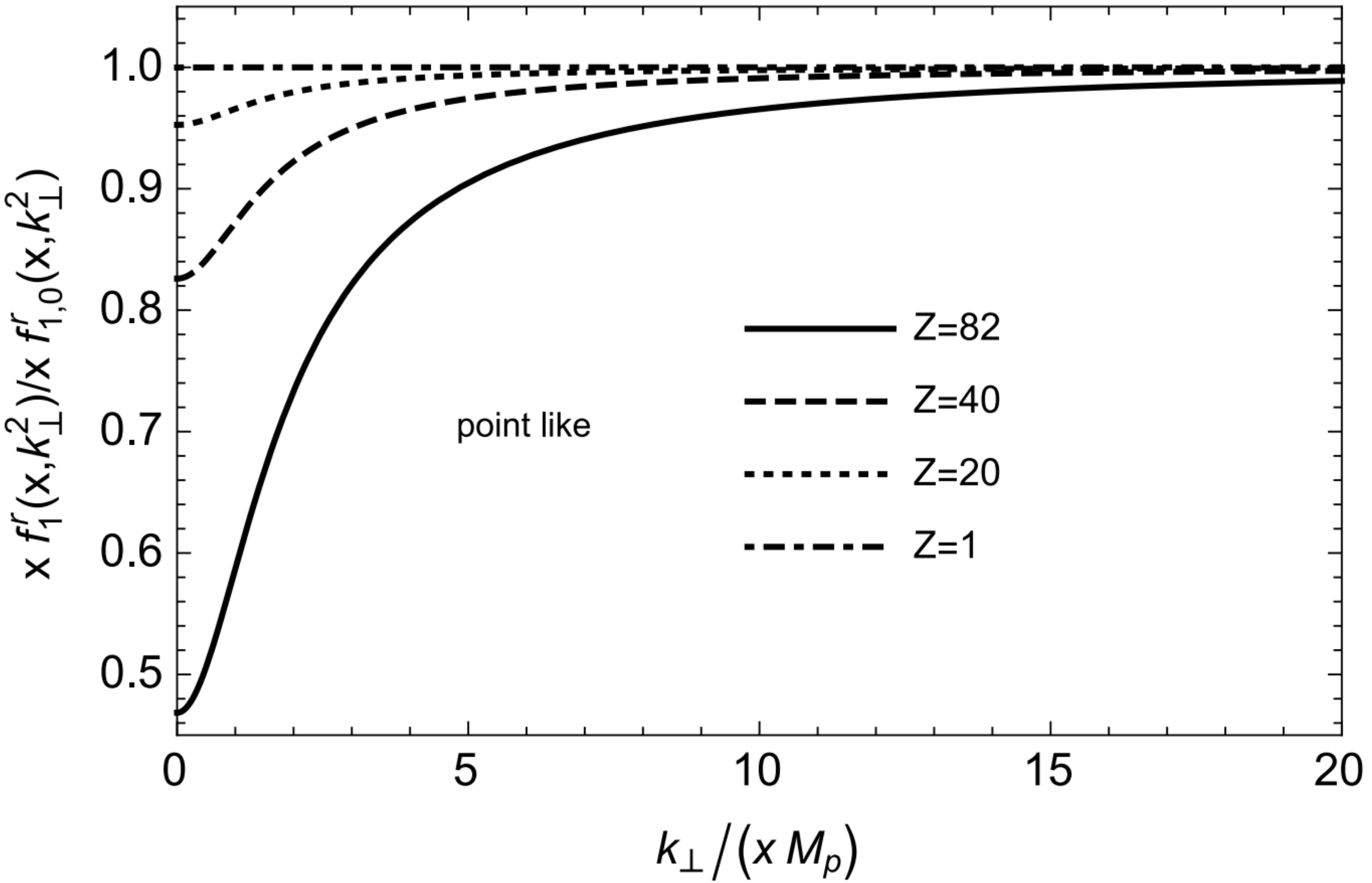
$$x f_1^\gamma(x, k_\perp^2) = x h_1^{\perp\gamma}(x, k_\perp^2)$$

$$= \frac{Z^4\alpha^3(1+Z^2\alpha^2)k_\perp^2}{M_p^4 x^4}\, {}_2F_1\left[1-iZ\alpha, 2-iZ\alpha, 2, \frac{-k_\perp^2}{M_p^2 x^2}\right] {}_2F_1\left[1+iZ\alpha, 2+iZ\alpha, 2, \frac{-k_\perp^2}{M_p^2 x^2}\right]\left(\frac{2}{e^{Z\alpha\pi} - e^{-Z\alpha\pi}}\right)^2$$

Numerical results

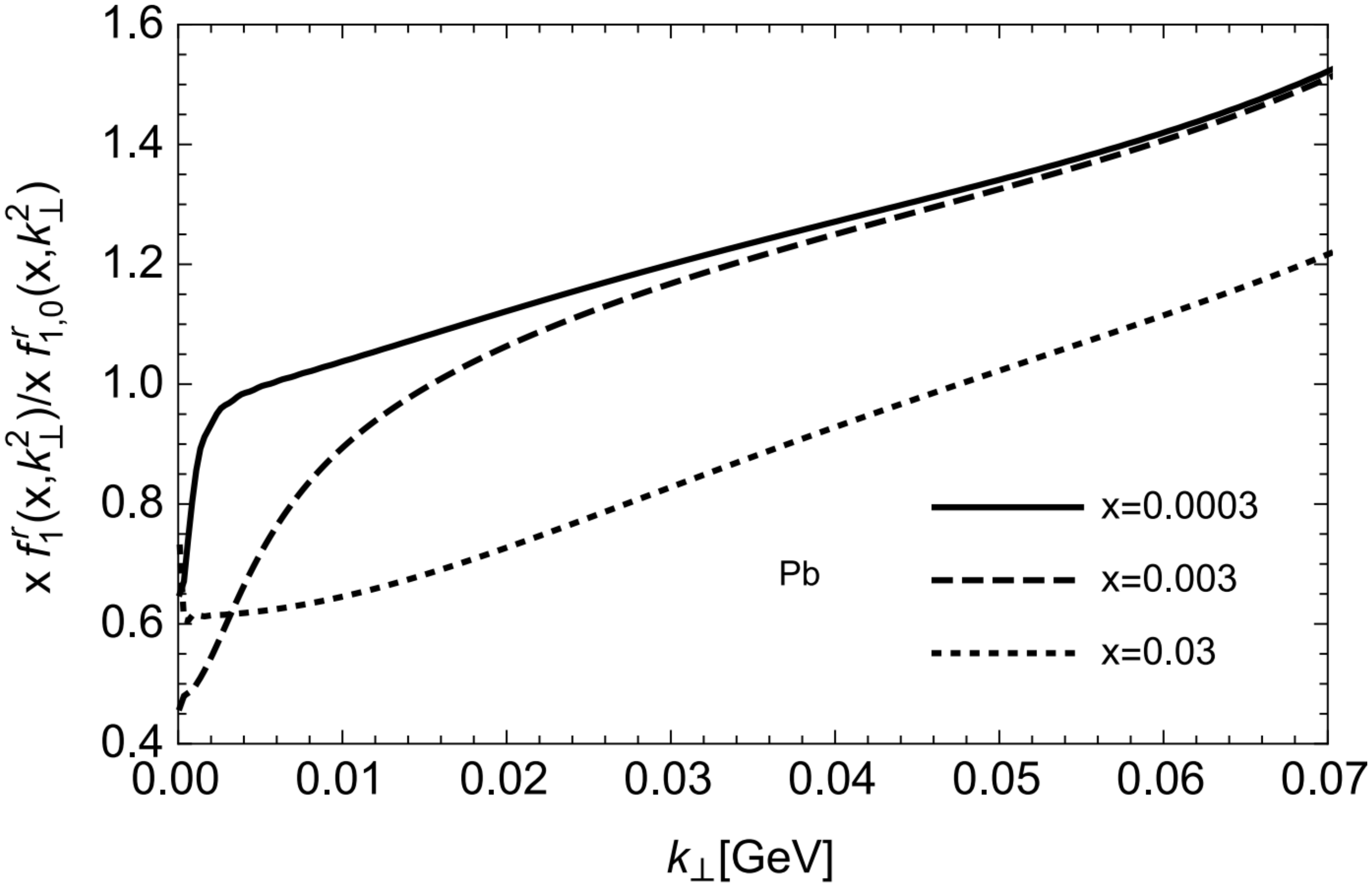
◆ Single gluon exchange for a point like charge

$$xf_1^{\gamma}(x,k_\perp^2) = xh_1^{\perp\gamma}(x,k_\perp^2) \approx \frac{Z^2\alpha}{\pi^2}\frac{k_\perp^2}{(k_\perp^2+M_p^2x^2)^2}$$



Numerical results for Pb target

- Nuclear charge form factor: $$F(|\vec{k}|) = \frac{4\pi\rho^0}{|\vec{k}|^3 A}\left[\sin(|\vec{k}|R_A) - |\vec{k}|R_A\cos(|\vec{k}|R_A)\right]\frac{1}{a^2\vec{k}^2+1}$$



A puzzle

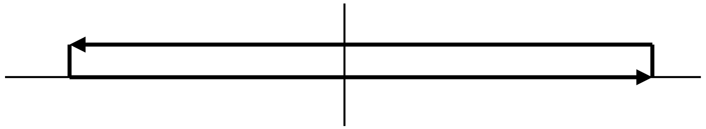
TMDs are process dependent, but PDFs are supposed to be universal!

- Two different photon TMDs: no gauge link, close loop gauge link

$$\int \frac{dy^- d^2 y_\perp}{P^+ (2\pi)^3} e^{ik\cdot y} \langle P | F^\mu_{+\perp}(0) U^\dagger(0_\perp) U(y_\perp) F^\nu_{+\perp}(y) | P \rangle \Big|_{y^+=0}$$

- Carrying out kt integration, produce a delta function

- Close loop gauge link become unity



=1, when $y_\perp$=0

Are they really identical?

Coulomb correction

No gauge link: $$xf_{1,0}^{\gamma}(x,k_\perp^2) = \frac{Z^2\alpha}{\pi^2}\frac{k_\perp^2}{(k_\perp^2+M_p^2x^2)^2}$$

Close loop gauge link: $$xf_1^{\gamma}(x,k_\perp^2) = \frac{Z^4\alpha^3(1+Z^2\alpha^2)k_\perp^2}{M_p^4x^4}\,{}_2F_1\left[1-iZ\alpha,2-iZ\alpha,2,\frac{-k_\perp^2}{M_p^2x^2}\right]{}_2F_1\left[1+iZ\alpha,2+iZ\alpha,2,\frac{-k_\perp^2}{M_p^2x^2}\right]\left(\frac{2}{e^{Z\alpha\pi}-e^{-Z\alpha\pi}}\right)^2$$

$$\int d^2k_\perp\left[xf_1^{\gamma}(x,k_\perp^2) - xf_{1,0}^{\gamma}(x,k_\perp^2)\right] = -\frac{2Z^2\alpha}{\pi}f(Z\alpha)$$

Coulomb correction term

$$f(Z\alpha)\equiv Re\psi(1+iZ\alpha)+\gamma_E \text{ with } \psi(x)=d\ln\Gamma(x)/dx$$

Are PDFs universal?

Observables

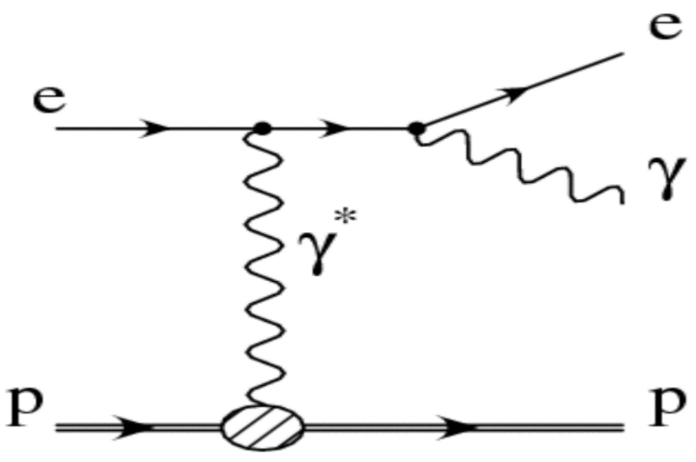
The Bethe-Heitler process

$$\frac{d\sigma}{dP.S} = H_{\rm Born} \int \frac{d^2 r_\perp}{(2\pi)^2} e^{i r_\perp \cdot q_\perp} e^{-\frac{\alpha_e}{2\pi}\ln^2 \frac{P_\perp^2}{\mu_r^2}} \int d^2 k_\perp e^{i r_\perp \cdot k_\perp} x f_1^\gamma(x, k_\perp^2)$$

Soft photon radiation can produce finite kt as well

$$H_{\rm Born} = 2\alpha_e^2 z^2 \frac{1+(1-z)^2}{P_\perp^4}$$

- We compare the BH cross section in ep and eA collisions at low kt



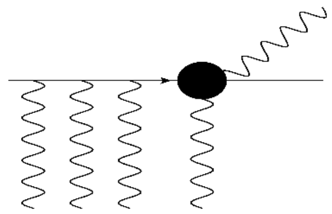
- Proton charge form factor well constrained:

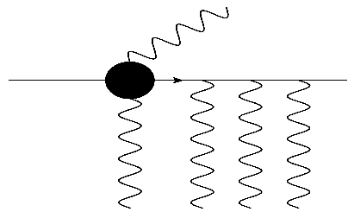
$$F(|\vec{k}|) = 1/(1+\frac{\vec{k}^2}{Q_0^2})^2 \text{ with } Q_0^2 = 0.71 \text{ GeV}^2$$

Coulomb correction at EicC

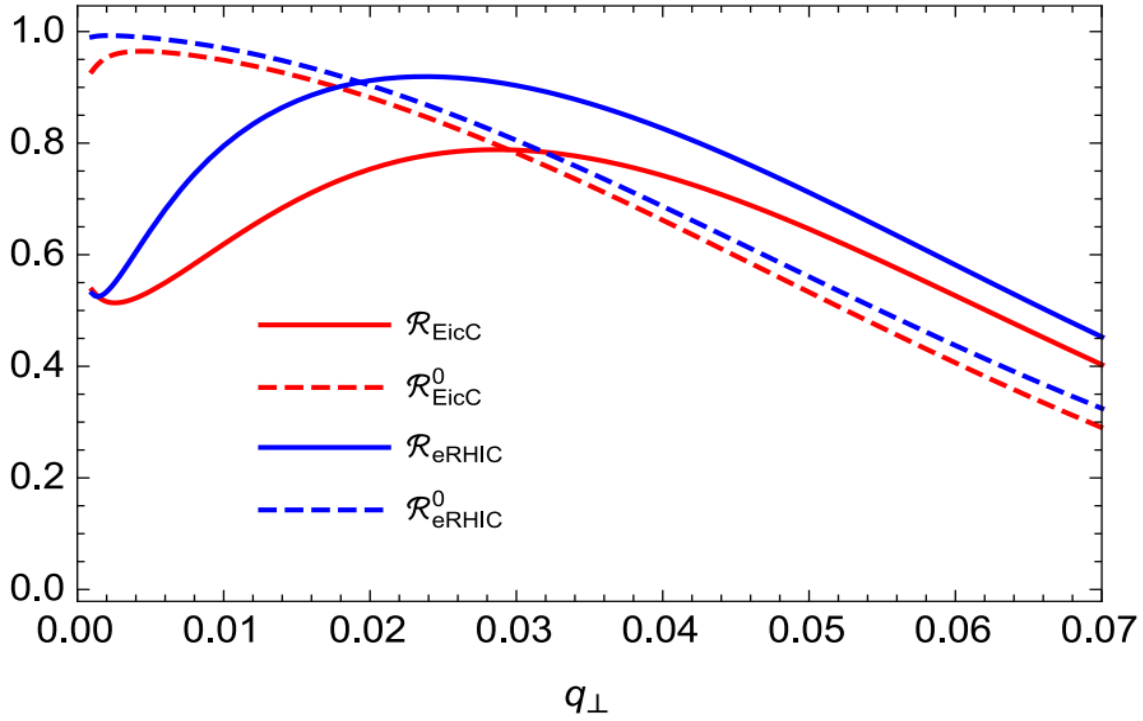
Multiple photon exchange enhance by a factor Z in eA collions

The ratios between the BH cross sections in ep and eA collisions





$$\mathcal{R}(q_\perp) = \frac{d\sigma_{eA}}{Z^2 d\sigma_{ep}} \qquad \mathcal{R}^0(q_\perp) = \frac{d\sigma^0_{eA}}{Z^2 d\sigma^0_{ep}}$$

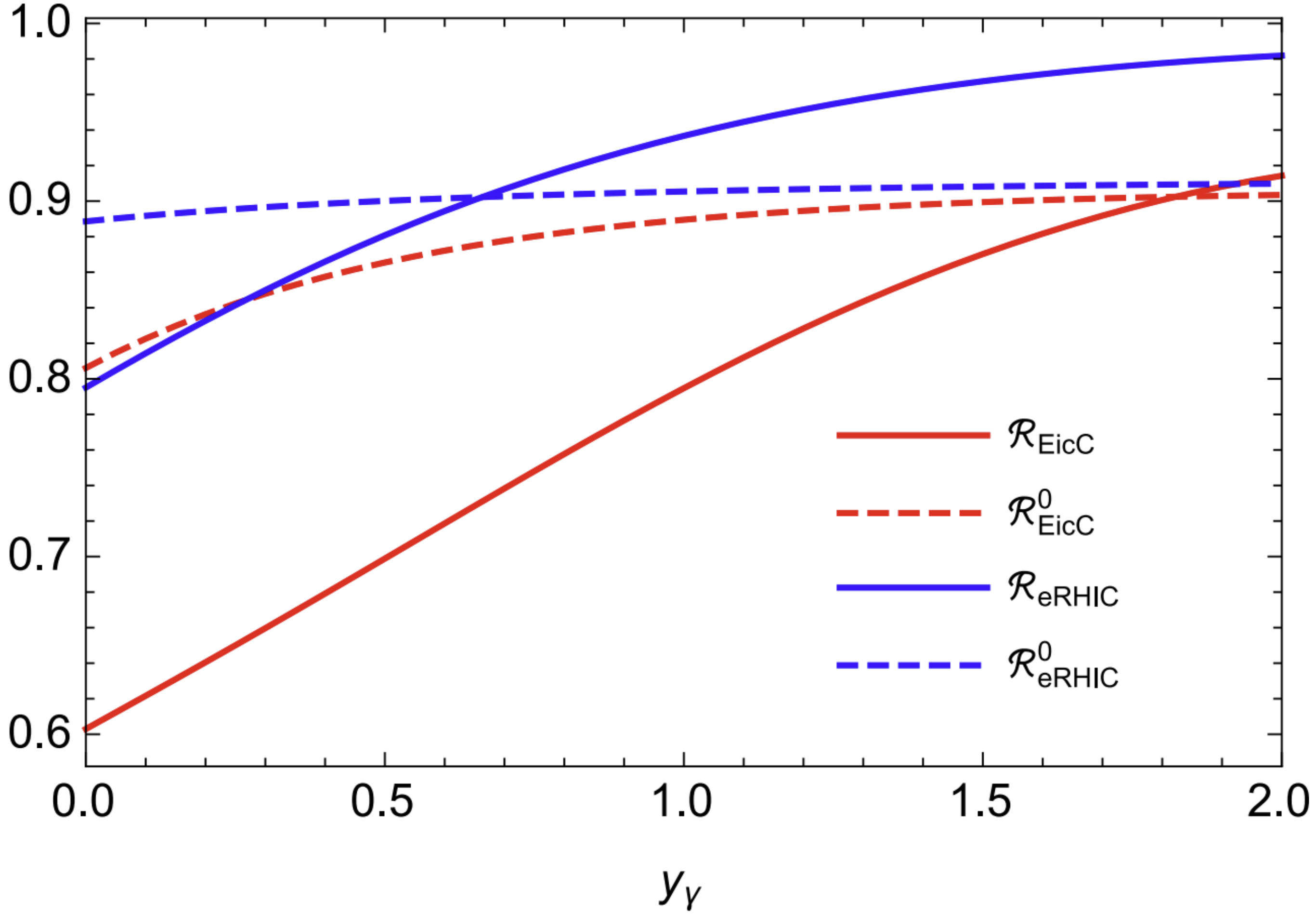


Z.h. Sun, D.x. Zheng, J. Zhou, Y.j. Zhou, 2020

- Requires high momentum resolution!
- It is more promising to observe Coulomb correction at EicC as compared to EIC.

Rapidity dependence

$$\mathcal{R}(q_\perp) = \frac{d\sigma_{eA}}{Z^2 d\sigma_{ep}} \qquad \mathcal{R}^0(q_\perp) = \frac{d\sigma^0_{eA}}{Z^2 d\sigma^0_{ep}}$$



Summary

- It is promising to study Coulomb correction at EicC!
- Search for golden observable in UPCs to address the Coulomb correction.

Thank you!

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