Photon polarization from chiral kinetic theory in strong magnetic field

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Based on LXY, PRD 105, 074039, 2022

Outline

- Motivation: chiral transports in magnetized plasma
- Chiral kinetic theory from Landau level basis
- Response functions: photon polarization
- Summary and outlook

Anomalous chiral transports in QGP

Kharzeev, Son, Landsteiner, Yee, Neiman, Yamamoto, Stephanov, Yin, Huang, Liao ...

CME/CSE

$$\mathbf{J} = C\mu_5 e\mathbf{B} \qquad \mathbf{J}_5 = C\mu e\mathbf{B} \qquad J_5^\mu = -\epsilon^{\mu\nu} J_\nu \Rightarrow$$

CMW: expected

Vector/Axial CVE

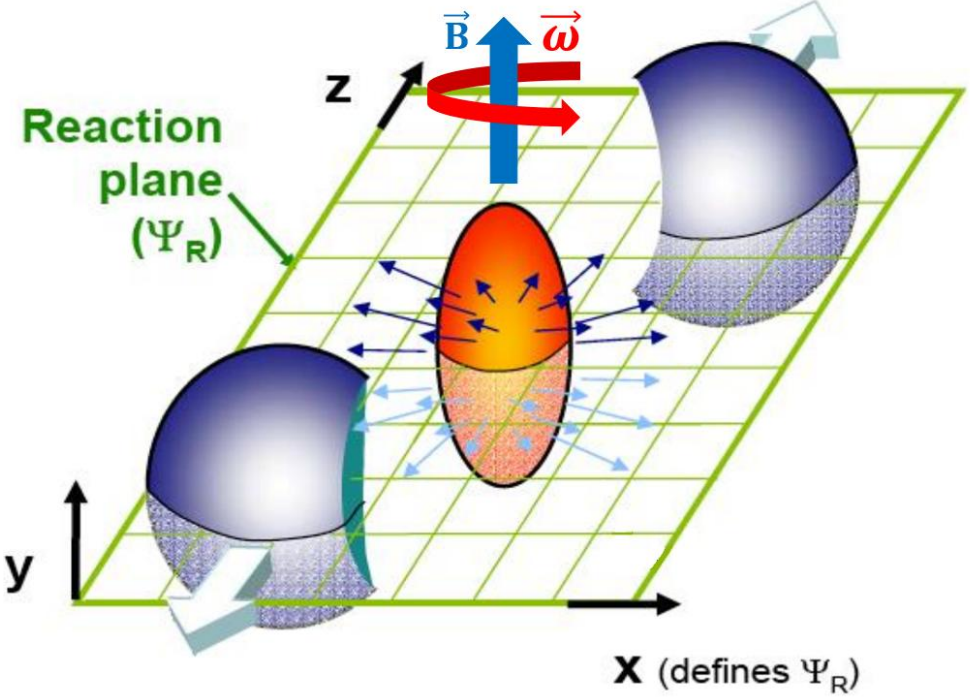
$$\mathbf{J} = C\mu\mu_5\boldsymbol{\omega} \qquad \mathbf{J}_5 = C\left(\mu^2 + \mu_5^2 + \frac{\pi^2 T^2}{3}\right)\boldsymbol{\omega}$$

CESE

$$\mathbf{J}_5 \propto \frac{\mu\mu_5}{T^2}\sigma e\mathbf{E}$$

Ohm/Hall currents(non-anomalous)

$$\mathbf{J} = \sigma e\mathbf{E} + \sigma_H e^2 \mathbf{E}\times\mathbf{B}$$



CKT with Landau level basis

Shu Lin, LXY, PRD 2020

Gao, Lin, Mo, PRD 2020

Shu Lin, LXY, JHEP 2021

CKE for chiral fermion

$$p_\mu j^\mu = 0$$

chirality
right-handed

$$\Delta_\mu j^\mu = 0$$

$$p^\mu j^\nu - p^\nu j^\mu = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\Delta_\rho j_\sigma$$

$$A_\mu \to A_\mu + a_\mu$$

$$F_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}u^\rho B^\sigma$$

$$f_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}u^\rho \mathcal{B}^\sigma + E_\mu u_\nu - E_\nu u_\mu$$

$$\Delta_\mu = \partial_\mu - \frac{\partial}{\partial p_\nu}(F_{\mu\nu} + f_{\mu\nu})$$

Wigner functions in LL basis $\to j^\mu$

Valid to all order of background B & first order $O(a)$ $O(\partial a)$

$B \sim p^2 \sim O(1)$ $\quad a_\mu \sim O(a)$ $\quad \mathcal{B}$ & $E \sim O(\partial a)$

Background LLL $\quad j^\mu_{(0)} = (u + b)^\mu \delta(\bar{p}_0 - \bar{p}_3) f(\bar{p}_0) e^{\frac{p_T^2}{B}}$

1+1D, fluid velocity: u^μ
direction of B^μ: b^μ

Current & stress energy tensor

$$J^\mu = \int d^4p\, j^\mu \qquad T^{\mu\nu} = \int d^4p \frac{1}{2}(p^\mu j^\nu + p^\nu j^\mu)$$

Perturbation: vector/axial gauge field

CKE in collisionless limit

chirality
s=$\pm$1

$$p_\mu j_s^\mu = 0$$
$$\Delta_\mu j_s^\mu = 0$$
$$p^\mu j_s^\nu - p^\nu j_s^\mu = -\frac{s}{2}\epsilon^{\mu\nu\rho\sigma}\Delta_\rho^s j_\sigma^s$$

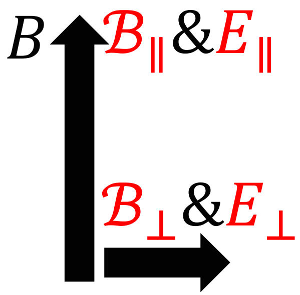
$$A_\mu \to A_\mu + a_\mu^s$$
$$F_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}u^\rho B^\sigma$$
$$f_{\mu\nu}^s = \epsilon_{\mu\nu\rho\sigma}u^\rho \mathcal{B}_s^\sigma + E_\mu^s u_\nu - E_\nu^s u_\mu$$
$$\Delta_\mu^s = \partial_\mu - \frac{\partial}{\partial p_\nu}\left(F_{\mu\nu} + f_{\mu\nu}^s\right)$$

Background LLL $\quad j_{(0)s}^\mu = (u + sb)^\mu \delta(\bar{p}_0 - s\bar{p}_3) f_s(\bar{p}_0) e^{\frac{p_T^2}{B}}$

Perturbation
expansion

$$j_s^\mu = j_{(0)s}^\mu + \underbrace{j_{as}^\mu}_{O(a)} + \underbrace{\sum_{\mathcal{A}s} j_{\mathcal{A}s}^\mu}_{O(\partial a)}$$

$$\mathcal{A}s = \mathcal{B}_\parallel^s, \mathcal{B}_\perp^s, E_\parallel^s, E_\perp^s$$



Perturbative solution: CMW

At $O(a)$ give CMW

$$j^{\mu}_{as} = (u+sb)^{\mu}\Delta\mu_s\delta(\bar{p}_0 - s\bar{p}_3)f'_s(\bar{p}_0)e^{\frac{p_T^2}{B}}$$

CMW propagating with the speed c in the limit $\boldsymbol{B} \to \infty$

Kharzeev, Yee, PRD 2011

$$\Delta\mu_s \equiv \frac{\bar{q}_3\bar{a}_0^s - \bar{q}_0\bar{a}_3^s}{s\bar{q}_0 - \bar{q}_3}$$

redistribution of the chiral plasma

$$\Delta\mu = \frac{\Delta\mu_R + \Delta\mu_L}{2}$$

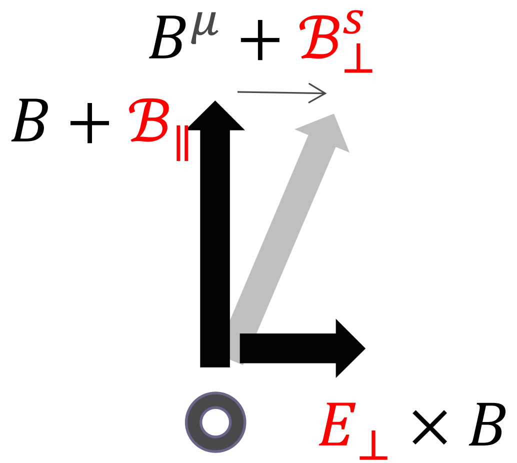
$$\Delta\mu_5 = \frac{\Delta\mu_R - \Delta\mu_L}{2}$$

At $O(\partial a)$ $\propto [\#\delta(\bar{p}_0 - s\bar{p}_3) + \#\delta'(\bar{p}_0 - s\bar{p}_3)\,]e^{\frac{p_T^2}{B}}$

non-vanishing $j^{\mu}_{\mathcal{B}\|s}, j^{\mu}_{E\perp s}, j^{\mu}_{\mathcal{B}\perp s}$ give CME, CSE, Hall current

collisionless limit $\to$ non-dissipative $\to$ $j^{\mu}_{E\|s} = 0$

Suggestive form: LLL, on-shell



At $O(a)$ shifted by $\Delta\mu_s$

$$j^\mu_{(0)s} + j^\mu_{as} = j^\mu_{(0)s}(\mu_s \to \mu_s + \Delta\mu_s)$$

At $O(\partial a)$ in static limit, shifted by $\mathcal{B}_\parallel^s, \mathcal{B}_\perp^s, U^s$

$$j^\mu_{(0)s} + j^\mu_{\mathcal{B}\parallel s} = j^\mu_{(0)s}\left(B \to B + \mathcal{B}_\parallel^s\right)$$ magnitude enhanced

$$j^\mu_{(0)s} + j^\mu_{\mathcal{B}\perp s} = j^\mu_{(0)s}\left(b^\mu \to b^\mu + \mathcal{B}_\perp^s/B\right)$$ tilted

$$j^\mu_{(0)s} + j^\mu_{E\perp s} = j^\mu_{(0)s}\left(u^\mu \to u^\mu + U_s^\mu/B\right)$$ boosted

$$U_s^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} b_\nu f_{\rho\sigma}^{E\perp s}$$

Constitutive relation of currents

At $O(1)$

$$J^{\mu}_{(0)} = \frac{\mu B}{2\pi^2} u^{\mu} + \overbrace{\frac{\mu_5 B}{2\pi^2} b^{\mu}}^{\text{CME}}$$

$$J^{\mu}_{(0)5} = \frac{\mu_5 B}{2\pi^2} u^{\mu} + \underbrace{\frac{\mu B}{2\pi^2} b^{\mu}}_{\text{CSE}}$$

At $O(a)$

$$J^{\mu}_{a} = \frac{\Delta\mu B}{2\pi^2} u^{\mu} + \frac{\Delta\mu_5 B}{2\pi^2} b^{\mu}$$

$$J^{\mu}_{a5} = \frac{\Delta\mu_5 B}{2\pi^2} u^{\mu} + \frac{\Delta\mu B}{2\pi^2} b^{\mu}$$

At $O(\partial a)$

$$J^{\mu}_{\mathcal{A}} = \frac{\mu\mathcal{B}_{\parallel} + \mu_5\mathcal{B}^5_{\parallel}}{2\pi^2} u^{\mu} \overbrace{+ \frac{\mu U^{\mu}}{2\pi^2}}^{\text{Hall flow}} + \frac{\mu_5 U^{\mu}_5}{2\pi^2} + \frac{\mu_5\left(\mathcal{B}^{\mu}_{\perp} + \mathcal{B}^{\mu}_{\parallel}\right)}{2\pi^2} + \frac{\mu\left(\mathcal{B}^{\mu}_{\perp 5} + \mathcal{B}^{\mu}_{\parallel 5}\right)}{2\pi^2}$$

$$J^{\mu}_{\mathcal{A}\mathbf{5}} = \frac{\mu_5\mathcal{B}_{\parallel} + \mu\mathcal{B}^5_{\parallel}}{2\pi^2} u^{\mu} + \frac{\mu_5 U^{\mu}}{2\pi^2} + \frac{\mu U^{\mu}_5}{2\pi^2} + \frac{\mu\left(\mathcal{B}^{\mu}_{\perp} + \mathcal{B}^{\mu}_{\parallel}\right)}{2\pi^2} + \frac{\mu_5\left(\mathcal{B}^{\mu}_{\perp 5} + \mathcal{B}^{\mu}_{\parallel 5}\right)}{2\pi^2}$$

No Ohm current in collisionless limit

Response functions as functional derivatives

Response functions from generating functional Γ

$$\mathcal{G}_{VA}^{\mu\nu}(q)\delta^{(4)}(k-q) = \frac{\delta \mathcal{J}^{\mu}(k)}{\delta a_{\nu}^{5}(q)} = \frac{\delta^{2}\Gamma}{\delta a_{\nu}^{5}(q)\delta a_{\mu}(-k)} \qquad \mathcal{G}_{VV}^{\mu\nu} \quad \mathcal{G}_{AA}^{\mu\nu} \quad \mathcal{G}_{AV}^{\mu\nu}$$

$$\mathcal{G}_{TV}^{\lambda\nu,\mu}(q)\delta^{(4)}(k-q) = \frac{\delta T^{\lambda\nu}(k)}{\delta a_{\mu}(q)} = \frac{\delta^{2}\Gamma}{\delta a_{\mu}(q)\delta g_{\lambda\nu}(-k)} \qquad \mathcal{G}_{VT}^{\mu,\lambda\nu} \quad \mathcal{G}_{TA}^{\lambda\nu,\mu} \quad \mathcal{G}_{AT}^{\mu,\lambda\nu}$$

Derivative symmetry $\qquad \mathcal{G}_{VA}^{\mu\nu}(q) = \mathcal{G}_{AV}^{\nu\mu}(-q)$

General form $\qquad \mathcal{G}_{ab}(q) = \mathcal{G}_{ba}(-q) \qquad a, b = \mathcal{J}_{V}^{\mu}, \mathcal{J}_{A}^{\mu}, T^{\mu\nu}\ldots$

CKT solutions do not satisfy derivative symmetry?

CKT with consistent & covariant anomaly

Consistent $\mathcal{J}$ & covariant J anomaly

$$\partial_\mu \mathcal{J}^\mu = \partial_\mu J^\mu + \frac{1}{8\pi^2}\epsilon^{\mu\nu\rho\lambda}F_{\mu\nu}f^5_{\rho\lambda} = 0 \qquad \partial_\mu \mathcal{J}^\mu_5 = \partial_\mu J^\mu_5 = -\frac{1}{8\pi^2}\epsilon^{\mu\nu\rho\lambda}F_{\mu\nu}f_{\rho\lambda}$$

Bardeen, Zumino, Nucl. Phys. B 1984

Landsteiner, Phys. Pol. B 2016

CKT with covariant anomaly

Son, Yamamoto, PRL 2012, PRD 2013

Manuel, Torres-Rincon, PRD 2014

Gorbar, Miransky, Shovkovy, Sukhachov, PRL 2017

in contrast to consistent anomaly

Carignano, Manuel, Torres-Rincon, PRD 2018

Relation between consistent & covariant current

$$\mathcal{J}^\mu = J^\mu + \frac{1}{4\pi^2}\epsilon^{\mu\nu\rho\lambda}a^5_\nu F_{\rho\lambda} \qquad \mathcal{J}^\mu_5 = J^\mu_5$$

$G^{\mu\lambda}_{VA,a}(q) \neq G^{\lambda\mu}_{AV,a}(-q)$ → Chern-Simons term → $\mathcal{G}^{\mu\lambda}_{VA,a}(q) = \mathcal{G}^{\lambda\mu}_{AV,a}(-q)$

Photon polarization

Structures of response functions

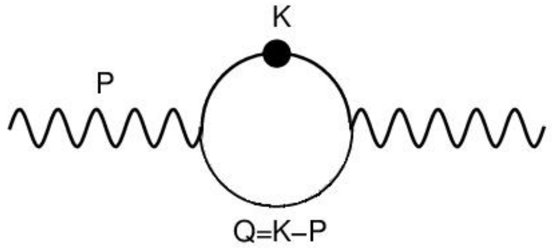
$$\mathcal{G}^{\mu\lambda}_{VV,a}(q) = \mathcal{G}^{\mu\lambda}_{AA,a}(q) = \frac{B}{2\pi^2}\frac{\bar{q}_3^2 u^\mu u^\lambda + \bar{q}_0^2 b^\mu b^\lambda + \bar{q}_0\bar{q}_3 u^{\{\mu}b^{\lambda\}}}{\bar{q}_0^2 - \bar{q}_3^2}$$

$$\mathcal{G}^{\mu\lambda}_{VV,\mathcal{A}}(q) = \mathcal{G}^{\mu\lambda}_{AA,\mathcal{A}}(q)$$

$$= \underbrace{\frac{i\mu_5}{2\pi^2}}_{\text{CME}}\left(\epsilon^{\mu\lambda\rho\sigma}\bar{q}_3 - b^{[\mu}\epsilon^{\lambda]\nu\rho\sigma}q^T_\nu\right)u_\rho b_\sigma - \underbrace{\frac{i\mu}{2\pi^2}}_{\text{CSE}}\left(\epsilon^{\mu\lambda\rho\sigma}\bar{q}_0 + u^{[\mu}\epsilon^{\lambda]\nu\rho\sigma}q^T_\nu\right)u_\rho b_\sigma$$

LLL photon polarization

Fukushima, PRD 2011



Correlators of consistent currents satisfy Onsager relations

$$\mathcal{G}_{ab}(q_0, \mathbf{q}, \widetilde{\mathbf{B}}) = \gamma_a\gamma_b\mathcal{G}_{ba}(q_0, -\mathbf{q}, -\widetilde{\mathbf{B}})$$

Phenomenology of LLL in HIC

Estimate parameters in QGP

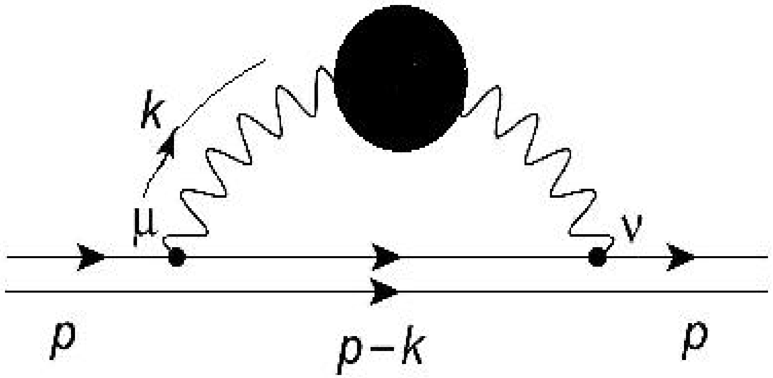
$$eB = m_\pi^2 \sim 10 m_\pi^2$$

$$T = 350 MeV$$

$$\frac{eB}{T^2} = 0.2 \sim 1.6$$

higher LLs to be included

UPC? $$\frac{eB}{T^2} \gg 1$$ lower LLs summation



Effect on photon splitting and polarization → spin polarization of probe fermion

Lihua Dong, Shu Lin, poster at Guangzhou & upcoming works

Summary

- Response functions from CKT gives photon polarization in strong B

Outlook

- Higher Landau levels contribution to photon polarization
- Effect on photon splitting and polarization of probe fermion

Thanks for your attention!

Consistent & covariant anomaly

Consistent anomaly & covariant anomaly

$$\partial_\mu \mathcal{J}_s^\mu = \pm \frac{1}{96\pi^2} \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu}^s F_{\rho\lambda}^s \qquad \partial_\mu J_s^\mu = \pm \frac{1}{32\pi^2} \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu}^s F_{\rho\lambda}^s$$

Bardeen, Zumino, Nucl. Phys. B 1984

Landsteiner, Phys. Pol. B 2016

Bardeen counterterms

Consistent axial anomaly & covariant anomaly

$$\partial_\mu J^\mu + \frac{1}{8\pi^2} \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu} f_{\rho\lambda}^5 = \partial_\mu \mathcal{J}^\mu = 0 \qquad \partial_\mu J_5^\mu = \partial_\mu \mathcal{J}_5^\mu = -\frac{1}{8\pi^2} \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu} f_{\rho\lambda}$$

CKT with free particle basis

$O(1)$: spinless particle $\partial_t f + \mathbf{v}\cdot\nabla_x f + (\boldsymbol{E} + \mathbf{v}\times\mathbf{B})\cdot\nabla_p f = 0$

$\delta(p^2)$

$O(\hbar)$: particle with Berry curvature $\boldsymbol{\Omega} = \dfrac{\mathbf{p}}{2|\mathbf{p}|^3}$

$$(1 + \hbar\boldsymbol{\Omega}\cdot\mathbf{B})\partial_t f$$
$$+ [\mathbf{v} + \hbar(\mathbf{E}\times\boldsymbol{\Omega}) + \hbar(\mathbf{v}\cdot\boldsymbol{\Omega})\mathbf{B}]\cdot\nabla_x f$$
$$+ [\mathbf{E} + \mathbf{v}\times\mathbf{B} + \hbar(\mathbf{E}\cdot\mathbf{B})\boldsymbol{\Omega}]\cdot\nabla_p f = 0$$

magnetic moment $\dfrac{\mathbf{p}}{2|\mathbf{p}|^2}$ $\delta(p^2) \to \delta(\tilde{p}^2)$

$$\tilde{p}^2 \equiv p^2 + \hbar\frac{\mathbf{B}\cdot\mathbf{p}}{p_0}$$

valid when $\sqrt{\hbar E}, \sqrt{\hbar B}, \hbar\partial_X \ll p$

$O(\hbar^2)$: particle no longer on-shell, simple picture lost

$$p_\mu G^\mu_{(0)}[f\delta(\tilde{p}^2)] + \frac{\hbar s}{2}\mathbf{G}^{(0)}\cdot\left\{\frac{1}{p_0}\mathbf{G}^{(0)}\times[\mathbf{p}f\delta(\tilde{p}^2)]\right\} + \hbar^2 C(f) = 0$$

$\delta(p^2) \nrightarrow \delta(\tilde{p}^2)$

$C(f)$: off-shell effect

Gao, Liang, Q. Wang, X.N. Wang, PRD 2018

Noncommutable limits

Derivative symmetry in static limit

Shu Lin, LXY, JHEP 2021

$$\mathcal{G}_{TV}^{\lambda\nu,\mu}(q)\delta^{(4)}(k-q)=\frac{\delta T^{\lambda\nu}(k)}{\delta a_\mu(q)} \qquad \mathcal{G}_{VT}^{\mu,\lambda\nu}(q)\delta^{(4)}(k-q)=\frac{\delta J_\nu^\mu(k)}{\delta g_{\lambda\nu}(q)}$$

With limits $\bar{q}_3 \to 0$ before $\bar{q}_0 \to 0$ Son, Yamamoto, PRD 2013

$$\mathcal{G}_{TV}^{\lambda\nu,\mu}(q)=\mathcal{G}_{VT}^{\mu,\lambda\nu}(-q)$$

Two limits noncommutable, finite interactions needed Satow, Yee, PRD 2014