The Two Higgs Doublets Models

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Two Higgs Doublets Models (2HDM)是SM的简单扩充，在SM的基础上额外引入一个具有相同量子数的Higgs二重态场，这样2HDM中共有两个Higgs二重态场，

$$\Phi_j = \begin{pmatrix} \varphi_j^+ \\ \varphi_j^0 \end{pmatrix},$$

在SU(2) ×U(1)下，2HDM的标量势能一般形式

$$\begin{aligned} V =& \mu_{11}^2 \Phi_1^\dagger \Phi_1 + \mu_{22}^2 \Phi_2^\dagger \Phi_2 - \left(\mu_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right) \\ &+ \frac{1}{2}\lambda_1 \left(\Phi_1^\dagger \Phi_1 \right)^2 + \frac{1}{2}\lambda_2 \left(\Phi_2^\dagger \Phi_2 \right)^2 + \lambda_3 \left(\Phi_1^\dagger \Phi_1 \right)\left(\Phi_2^\dagger \Phi_2 \right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2 \right)\left(\Phi_2^\dagger \Phi_1 \right) \\ &+ \left[\frac{1}{2}\lambda_5 \left(\Phi_1^\dagger \Phi_2 \right)^2 + \lambda_6 \left(\Phi_1^\dagger \Phi_1 \right)\left(\Phi_1^\dagger \Phi_2 \right) + \lambda_7 \left(\Phi_2^\dagger \Phi_2 \right)\left(\Phi_1^\dagger \Phi_2 \right) + \text{h.c.} \right], \end{aligned}$$

在相互作用基下，2HDM的拉氏量一般写为

$$\mathscr{L}_{\mathrm{Y}} = -\bar{Q}_L^0 \left(\Phi_1 Y_{d,1} + \Phi_2 Y_{d,2}\right) d_R^0 - \bar{Q}_L^0 \left(\tilde{\Phi}_1 Y_{u,1} + \tilde{\Phi}_2 Y_{u,2}\right) u_R^0 - \bar{L}_L^0 \left(\Phi_1 Y_{\ell,1} + \Phi_2 Y_{\ell,2}\right) \ell_R^0 - \bar{L}_L^0 \left(\tilde{\Phi}_1 Y_{\nu,1} + \tilde{\Phi}_2 Y_{\nu,2}\right) \nu_R^0 + \mathrm{h.c.}\,,$$

其中，

$$\tilde{\Phi}_j \equiv i\sigma_2 \Phi_j^* = \begin{pmatrix} \varphi_j^{0\dagger} \\ -\varphi_j^- \end{pmatrix},$$

二重态可写为

$$\Phi_j = e^{i\theta_j} \begin{pmatrix} \varphi_j^+ \\ \frac{v_j + \rho_j + i\eta_j}{\sqrt{2}} \end{pmatrix}.$$

两个二重态的电弱对称性破缺真空期望值

$$\langle 0|\Phi_1|0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 e^{i\theta_1} \end{pmatrix}, \quad \langle 0|\Phi_2|0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 e^{i\theta_2} \end{pmatrix},$$

在Higgs基下有

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \mathcal{R}_\beta \begin{pmatrix} e^{-i\theta_1}\Phi_1 \\ e^{-i\theta_2}\Phi_2 \end{pmatrix}, \quad \text{其中} \quad \mathcal{R}_\beta = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix}, \ \mathcal{R}_\beta^T = \mathcal{R}_\beta^{-1},$$

定义 $v_1^2 + v_2^2 = v^2$ ，且$tan\beta \equiv \frac{v_2}{v_1}$ ，二重态可写为

$$H_1 = \begin{pmatrix} G^+ \\ \frac{v+H^0+iG^0}{\sqrt{2}} \end{pmatrix}, \quad H_2 = \begin{pmatrix} \mathrm{H}^+ \\ \frac{R^0+iI^0}{\sqrt{2}} \end{pmatrix},$$

$$\begin{pmatrix} G^+ \\ \mathrm{H}^+ \end{pmatrix} = \mathcal{R}_\beta \begin{pmatrix} \varphi_1^+ \\ \varphi_2^+ \end{pmatrix}, \quad \begin{pmatrix} G^0 \\ I^0 \end{pmatrix} = \mathcal{R}_\beta \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}, \quad \begin{pmatrix} H^0 \\ R^0 \end{pmatrix} = \mathcal{R}_\beta \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}.$$

其中，G和$G^\pm$为Goldstone玻色子， $H^\pm, H^0, R^0, I^0$分别是物理的带电、标量和赝标Higgs玻色子。

在Higgs基下，拉氏量写作

$$\begin{aligned} \mathscr{L}_\mathrm{Y} = &- \frac{\sqrt{2}}{v}\bar{Q}_L^0 \left(H_1 \mathrm{M}_d^0 + H_2 \mathrm{N}_d^0\right) d_R^0 - \frac{\sqrt{2}}{v}\bar{Q}_L^0 \left(\tilde{H}_1 \mathrm{M}_u^0 + \tilde{H}_2 \mathrm{N}_u^0\right) u_R^0 \\ &- \frac{\sqrt{2}}{v}\bar{L}_L^0 \left(H_1 \mathrm{M}_\ell^0 + H_2 \mathrm{N}_\ell^0\right) \ell_R^0 - \frac{\sqrt{2}}{v}\bar{L}_L^0 \left(\tilde{H}_1 \mathrm{M}_\nu^0 + \tilde{H}_2 \mathrm{N}_\nu^0\right) \nu_R^0 + \mathrm{h.c.}\ . \end{aligned}$$

Fernando Cornet-Gomez, Taming Flavor in Two Higgs Doublet Models, Valencia U., IFIC 2021.

其中， $M_f^0(f=u,d,l,v)$是非对角化的费米子质量矩阵，矩阵 $N_f^0(f=u,d,l,v)$包含了标量二重态的Yukawa 耦合。

$$\mathrm{M}_d^0 = \frac{v\,e^{i\theta_1}}{\sqrt{2}}\left[c_\beta Y_{d,1} + e^{i\theta} s_\beta Y_{d,2}\right], \qquad \mathrm{N}_d^0 = \frac{v\,e^{i\theta_1}}{\sqrt{2}}\left[-s_\beta Y_{d,1} + e^{i\theta} c_\beta Y_{d,2}\right],$$

$$\mathrm{M}_u^0 = \frac{v\,e^{-i\theta_1}}{\sqrt{2}}\left[c_\beta Y_{u,1} + e^{-i\theta} s_\beta Y_{u,2}\right], \quad \mathrm{N}_u^0 = \frac{v\,e^{-i\theta_1}}{\sqrt{2}}\left[-s_\beta Y_{u,1} + e^{-i\theta} c_\beta Y_{u,2}\right],$$

$$\mathrm{M}_\ell^0 = \frac{v\,e^{i\theta_1}}{\sqrt{2}}\left[c_\beta Y_{\ell,1} + e^{i\theta} s_\beta Y_{\ell,2}\right], \qquad \mathrm{N}_\ell^0 = \frac{v\,e^{i\theta_1}}{\sqrt{2}}\left[-s_\beta Y_{\ell,1} + e^{i\theta} c_\beta Y_{\ell,2}\right],$$

$$\mathrm{M}_\nu^0 = \frac{v\,e^{-i\theta_1}}{\sqrt{2}}\left[c_\beta Y_{\nu,1} + e^{-i\theta} s_\beta Y_{\nu,2}\right], \quad \mathrm{N}_\nu^0 = \frac{v\,e^{-i\theta_1}}{\sqrt{2}}\left[-s_\beta Y_{\nu,1} + e^{-i\theta} c_\beta Y_{\nu,2}\right],$$

$$\theta \equiv \theta_2 - \theta_1 .$$

$$\mathrm{N}_d^0 = t_\beta^{-1}\mathrm{M}_d^0 - (t_\beta + t_\beta^{-1})\frac{v_1\,e^{i\theta_1}}{\sqrt{2}} Y_{d,1}, \quad \mathrm{N}_u^0 = t_\beta^{-1}\mathrm{M}_u^0 - (t_\beta + t_\beta^{-1})\frac{v_1\,e^{-i\theta_1}}{\sqrt{2}} Y_{u,1},$$

$$\mathrm{N}_\ell^0 = t_\beta^{-1}\mathrm{M}_\ell^0 - (t_\beta + t_\beta^{-1})\frac{v_1\,e^{i\theta_1}}{\sqrt{2}} Y_{\ell,1}, \quad \mathrm{N}_\nu^0 = t_\beta^{-1}\mathrm{M}_\nu^0 - (t_\beta + t_\beta^{-1})\frac{v_1\,e^{-i\theta_1}}{\sqrt{2}} Y_{\nu,1},$$

$$\mathrm{N}_d^0 = -t_\beta\mathrm{M}_d^0 + (t_\beta + t_\beta^{-1})\frac{v_2\,e^{i\theta_2}}{\sqrt{2}} Y_{d,2}, \quad \mathrm{N}_u^0 = -t_\beta\mathrm{M}_u^0 + (t_\beta + t_\beta^{-1})\frac{v_2\,e^{-i\theta_2}}{\sqrt{2}} Y_{u,2},$$

$$\mathrm{N}_\ell^0 = -t_\beta\mathrm{M}_\ell^0 + (t_\beta + t_\beta^{-1})\frac{v_2\,e^{i\theta_2}}{\sqrt{2}} Y_{\ell,2}, \quad \mathrm{N}_\nu^0 = -t_\beta\mathrm{M}_\nu^0 + (t_\beta + t_\beta^{-1})\frac{v_2\,e^{-i\theta_2}}{\sqrt{2}} Y_{\nu,2}.$$

一般而言， Yukawa 耦合矩阵$M_f^0(f=u,d,l,v)$和$N_f^0(f=u,d,l,v)$在味空间中不能同时被对角化。从而产生树图阶味道改变中性流。大量实验数据证实，味道改变的中性流（FCNC）是树图强烈压低的。为了避免味道改变的中性流在树图阶出现，我们会引入其他一些不同的限制条件，让耦合系数矩阵可以被同时对角化。因此2HDM方案可以分为两种：

- 通过引入不同的分离对称性来直接消除树图阶的味道改变中性流，如$\boldsymbol{Z_2}$分立对称性
  - Type-I 2HDM（Φ_2与所有夸克及轻子耦合）
  - Type-II 2HDM（Φ_2与上型夸克耦合， Φ_1与下型夸克及轻子耦合）
  - Lepton-specific (Type-X) 2HDM （Φ_2与夸克耦合， Φ_1与轻子耦合）
  - Flipped (Type-Y) 2HDM （Φ_2与上型夸克及轻子耦合， Φ_1与下型夸克耦合）
- 直接引入新的耦合参数来压低树图的味道改变的中性流
  - Type-III 2HDM
  - Aligned 2HDM

前四种经典2HDM中各种场在Z_2分立对称性下的赋荷如下，其他剩余的场在Z_2变换下均为“+”。

Model	Φ_1	Φ_2	u_R	d_R	l_R
Type-I	-	+	+	+	+
Type-II	-	+	+	-	-
Lepton-specific(Type-X)	-	+	+	+	-
Flipped(Type-Y)	-	+	+	-	+

Type-I 2HDM中，Φ_2与所有夸克及轻子耦合，其耦合矩阵满足

$$\mathrm{N}_f^0 = t_\beta^{-1}\mathrm{M}_f^0\,, \quad (Y_{u,1} = Y_{d,1} = Y_{\ell,1} = 0)\,.$$

Type-II 2HDM中， Φ_2与上型夸克耦合， Φ_1与下型夸克及轻子耦合，耦合矩阵满足

$$\mathrm{N}_u^0 = t_\beta^{-1}\mathrm{M}_u^0 \quad (Y_{u,1} = 0)\,,$$

$$\mathrm{N}_d^0 = -t_\beta\mathrm{M}_d^0 \quad (Y_{d,2} = 0)\,,$$

$$\mathrm{N}_\ell^0 = -t_\beta\mathrm{M}_\ell^0 \quad (Y_{\ell,2} = 0)\,.$$

Lepton-specific 2HDM及Flipped (Type-Y) 2HDM 的耦合矩阵

$$\mathrm{N}_u^0 = t_\beta^{-1}\mathrm{M}_u^0 \quad (Y_{u,1} = 0)\,,$$

$$\mathrm{N}_d^0 = t_\beta^{-1}\mathrm{M}_d^0 \quad (Y_{d,1} = 0)\,,$$

$$\mathrm{N}_\ell^0 = -t_\beta\mathrm{M}_\ell^0 \quad (Y_{\ell,2} = 0)\,.$$

$$\mathrm{N}_u^0 = t_\beta^{-1}\mathrm{M}_u^0 \quad (Y_{u,1} = 0)\,,$$

$$\mathrm{N}_d^0 = -t_\beta\mathrm{M}_d^0 \quad (Y_{d,2} = 0)\,,$$

$$\mathrm{N}_\ell^0 = t_\beta^{-1}\mathrm{M}_\ell^0 \quad (Y_{\ell,1} = 0)\,.$$

·我们还可以将拉氏量转换到质量本征态，夸克部分和轻子部分

$$d_L = \mathcal{U}^\dagger_{d_L} d^0_L,\ d_R = \mathcal{U}^\dagger_{d_R} d^0_R,\ u_L = \mathcal{U}^\dagger_{u_L} u^0_L,\ u_R = \mathcal{U}^\dagger_{u_R} u^0_R\,,$$

$$\ell_L = \mathcal{U}^\dagger_{\ell_L} \ell^0_L\,,\quad \ell_R = \mathcal{U}^\dagger_{\ell_R} \ell^0_R\,,\quad \nu_L = \mathcal{U}^\dagger_{\nu_L} \nu^0_L\,,\quad \nu_R = \mathcal{U}^\dagger_{\nu_R} \nu^0_R\,,$$

$$\mathrm{M}_d = \mathcal{U}^\dagger_{d_L} \mathrm{M}^0_d \mathcal{U}_{d_R} = \mathrm{diag}(m_d, m_s, m_b)\,,\qquad \mathrm{M}_\ell = \mathcal{U}^\dagger_{\ell_L} \mathrm{M}^0_\ell \mathcal{U}_{\ell_R} = \mathrm{diag}(m_e, m_\mu, m_\tau),$$

$$\mathrm{M}_u = \mathcal{U}^\dagger_{u_L} \mathrm{M}^0_u \mathcal{U}_{u_R} = \mathrm{diag}(m_u, m_c, m_t)\,,\qquad \mathrm{M}_\nu = \mathcal{U}^\dagger_{\nu_L} \mathrm{M}^0_\nu \mathcal{U}_{\nu_R} = \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}),$$

$$\mathrm{N}_d = \mathcal{U}^\dagger_{d_L} \mathrm{N}^0_d \mathcal{U}_{d_R}\,,\ \mathrm{N}_u = \mathcal{U}^\dagger_{u_L} \mathrm{N}^0_u \mathcal{U}_{u_R}\,,\qquad \mathrm{N}_\ell = \mathcal{U}^\dagger_{\ell_L} \mathrm{N}^0_\ell \mathcal{U}_{\ell_R},\quad \mathrm{N}_\nu = \mathcal{U}^\dagger_{\nu_L} \mathrm{N}^0_\nu \mathcal{U}_{\nu_R}\,.$$

$$\Rightarrow\quad \mathscr{L}^{[q]}_{\mathrm{Y}} = -\frac{\sqrt{2}}{v}\bar{Q}_L[\mathrm{M}_d H_1 + \mathrm{N}_d H_2]d_R - \frac{\sqrt{2}}{v}\bar{Q}_L[\mathrm{M}_u \tilde{H}_1 + \mathrm{N}_u \tilde{H}_2]u_R + \mathrm{h.c.}\,.$$

$$\Rightarrow\quad \mathscr{L}^{[\ell]}_{\mathrm{Y}} = -\frac{\sqrt{2}}{v}\bar{L}_L[\mathrm{M}_\ell H_1 + \mathrm{N}_\ell H_2]\ell_R - \frac{\sqrt{2}}{v}\bar{L}_L[\mathrm{M}_\nu \tilde{H}_1 + \mathrm{N}_\nu \tilde{H}_2]\nu_R + \mathrm{h.c.}\,.$$

上述变换中 $\mathcal{U}^\dagger_{u_L}\mathcal{U}_{d_L}$ 即为CKM夸克混合矩阵。N矩阵一般是不对角的，导致树图阶产生味道改变中性流。

在质量本征态下，Aligned 2HDM(A2HDM)引入了新的对齐参数ς_f，使Yukawa耦合矩阵和质量矩阵之间只相差一个常数，两个矩阵得以同时对角化，消除树图阶的FCNC，即

$$\mathrm{N}_u = \varsigma_u^\dagger\, \mathrm{M}_u\,, \quad \mathrm{N}_d = \varsigma_d\, \mathrm{M}_d\,, \quad \mathrm{N}_\ell = \varsigma_\ell\, \mathrm{M}_\ell\,,$$

当对齐参数取特定值时，A2HDM可以回到经典的四种2HDM，对应关系如下，

Model	ς_u	ς_d	ς_ℓ
Type I	t_β^{-1}	t_β^{-1}	t_β^{-1}
Type II	t_β^{-1}	$-t_\beta$	$-t_\beta$
Lepton-Specific (X)	t_β^{-1}	t_β^{-1}	$-t_\beta$
Flipped (Y)	t_β^{-1}	$-t_\beta$	t_β^{-1}
Inert	0	0	0

Type-III 2HDM允许费米子与两个二重态都耦合，存在FCNC，且依据Cheng-Sher假定，Yukawa耦合矩阵的形式与参与耦合的不同代费米子质量有关，写为

$$\xi_{ij} = \lambda_{ij}\sqrt{m_i m_j}\,\frac{\sqrt{2}}{v},$$

其中λ_{ij}为一阶。即假设Yukawa耦合矩阵为

$$\lambda_i' = \frac{\sqrt{2}}{v_i}\begin{pmatrix} 0 & A_i & 0 \\ A_i & 0 & B_i \\ 0 & B_i & C_i \end{pmatrix},$$

其中

$$A_i = a_i\sqrt{m_1 m_2},\ B_i = b_i\sqrt{m_2 m_3},\ C_i = c_i m_3,$$

$$\sum_i a_i = \sum_i b_i = \sum_i c_i = 1$$

T. P. Cheng and M. Sher, Mass Matrix Ansatz and Flavor Nonconservation in Models with Multiple Higgs Doublets, Phys. Rev. D 35, (1987) 3484.

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