Some Discussion on Compton Polarimeter

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Outline

- 1. FCC
  - Inverse Compton scattering
  - Layout of ICS experiments
  - Compton scattering cross section
  - Simulation parameters
  - Experiment 1
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- 2. Compare CEPC and FCC

https://arxiv.org/pdf/2208.00585.pdf

https://doi.org/10.1088/1748-0221/17/08/P08005 https://cds.cern.ch/record/1296190/files/arXiv:1009.6178.pdf

Inverse Compton scattering

- Inverse Compton Scattering (ICS) of laser light by relativistic electron beam would produce useful yields of high energy polarized photons.

Invention of lasers[1, 2]

In 1969 such a beam had been realized at SLAC for a study of γp interactions [3].

The effect of self-polarization of relativistic electrons [4] and observed at the first collider ACO [5] due to polarization effects in intra-beam scattering.

Develop a method of resonant depolarization [7]

ICS was proposed in ref. [9] as one of the processes allowing to measure the transverse spin polarization of high-energy electron beam.

Transverse polarization measurement: SPEAR [12] , PETRA [13], DORIS-II [14], LEP [15, 16], HERA [17] and VEPP-4M [18].

Longitudinal spin polarimetry: SLAC[19], AmPS ring at NIKHEF [20], HERA collider at DESY [21] and Jlab [22].

The ICS polarimeters are also planned for future collider experiments ILC [23, 24], FCC-ee [25], CEPC [26, 27] and EIC [28, 29].

FCC-ee beam polarimeter

- FCC-ee is a lepton collider with centre-of-mass energies between 90 and 350 GeV
- It is considered as a potential intermediate step towards the realization of the hadron facility.
- Beam energy calibration by resonant depolarization is the basis for the precise measurements of the Z mass and width with a precision of <100 keV, and of the W mass and width with a precision of the order of 500 keV.

Layout of ICS experiments

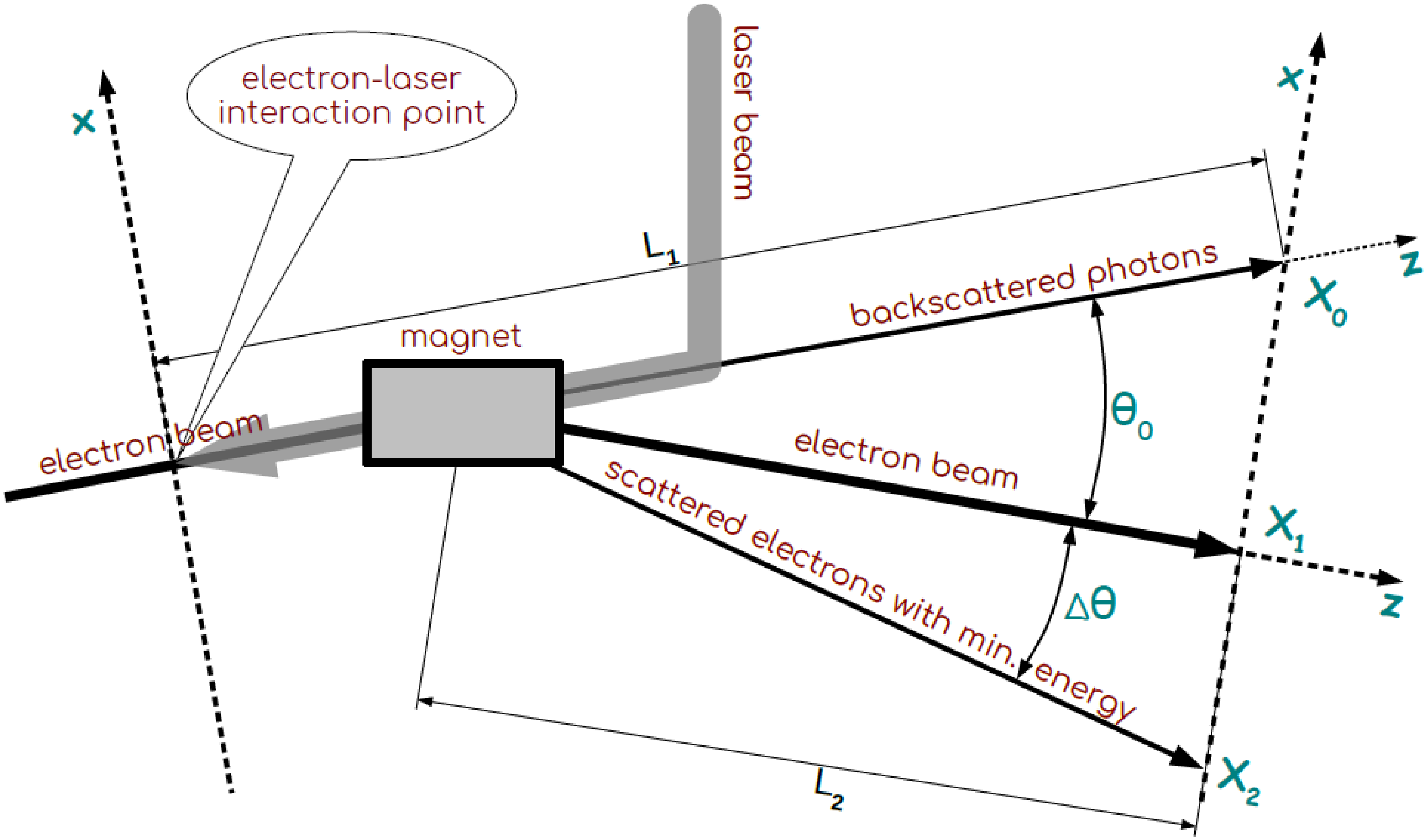


Figure 1. Layout of ICS experiment.

Layout of ICS experiments

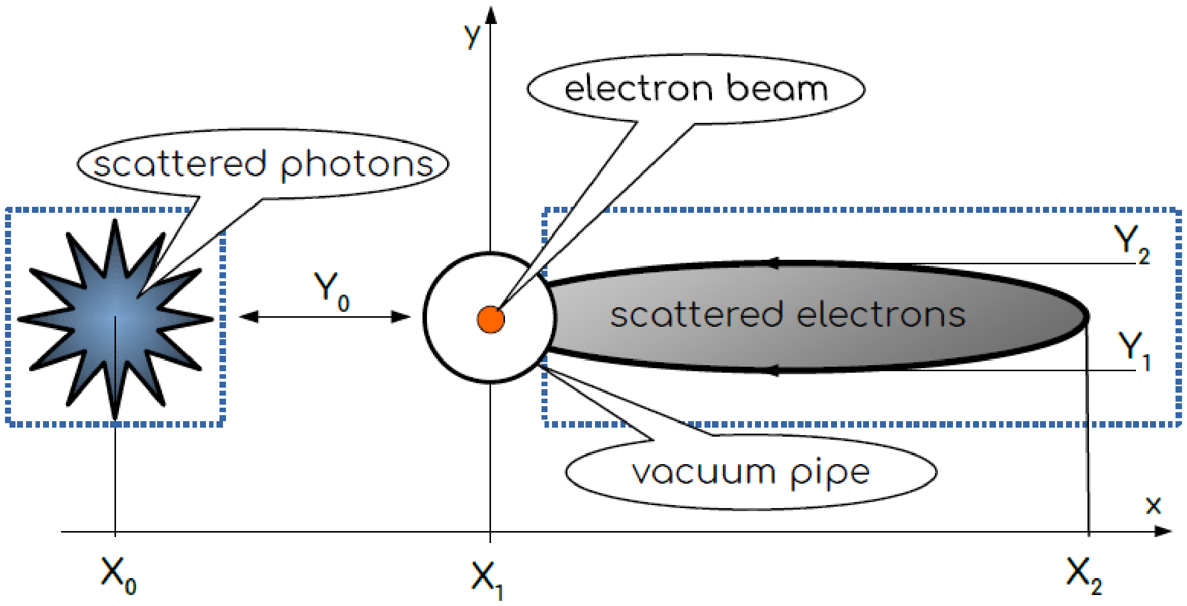


Figure 2. Detection plane. 2D pixel detectors for photons and electrons are represented by dotted rectangles.

原理：通过对 散射光子的2D分布/散射电子的二维分布进行拟合，得到 极化参数

Compton scattering cross section

https://arxiv.org/pdf/2208.00585.pdf

A Stokes parameters

The Stokes parameters describe the polarization state of electromagnetic radiation. Their definition is slightly different in different sources, so below are the definition that we use here.

- $\xi_0 = E_x^2 + E_y^2$ is the intensity of light.
  With normalization $E_x^2 + E_y^2 = 1$ for 100% polarized laser radiation $\xi_0 = \sqrt{\xi_1^2 + \xi_2^2 + \xi_3^2} = 1$.
- $\xi_1 = E_x^2 - E_y^2$.

  $E_x = 1, E_y = 0, \xi_1 = +1$: 100% linear polarization along x-axis.

  $E_x = 0, E_y = 1, \xi_1 = -1$: 100% linear polarization along y-axis.
- $\xi_2 = 2E_xE_y \cos(\delta)$.

  $E_x = E_y, \delta = 0, \xi_2 = +1$: 100% linear polarization along $\varphi = +\pi/4$.

  $E_x = E_y, \delta = \pi, \xi_2 = -1$: 100% linear polarization along $\varphi = -\pi/4$.
- $\xi_3 = 2E_xE_y \sin(\delta)$.

  $E_x = E_y, \delta = +\pi/2, \xi_3 = +1$: 100% right circular polarization.

  $E_x = E_y, \delta = -\pi/2, \xi_3 = -1$: 100% left circular polarization.

Compton scattering cross section

1.1 Cross section

ICS cross section depends on polarization states of all initial and final particles. In case of averaging over the polarizations of the final states the cross section depends solely from the initial photon and electron polarizations. Stokes parameters ξ_1, ξ_2, ξ_3 describe the polarization of laser light as it is explained in Appendix A. The electron beam polarization has three components $\zeta_x, \zeta_y, \zeta_z$, the total degree of polarization $\sqrt{\zeta_x^2 + \zeta_y^2 + \zeta_z^2} \in [0 : 1]$. We take differential cross section from ref. [34] and after Lorentz transformations it is represented in u and φ variables by the sum of the six terms:

Unpolarized polarized

Laser linearly polarized

Laser circularly polarized
&
electron three polarization

$$
\begin{aligned}
\frac{1}{r_e^2}\frac{d\sigma_0}{du\,d\varphi} &= \frac{1}{\kappa(1+u)^3}\left[1+(1+u)^2-4\frac{u}{\kappa}\left(1-\frac{u}{\kappa}\right)(1+u)\right], \\
\frac{1}{r_e^2}\frac{d\sigma_{\xi_1}}{du\,d\varphi} &= \frac{4\,\xi_1}{\kappa(1+u)^2}\frac{u}{\kappa}\left(1-\frac{u}{\kappa}\right)\cos(2\varphi), \\
\frac{1}{r_e^2}\frac{d\sigma_{\xi_2}}{du\,d\varphi} &= \frac{4\,\xi_2}{\kappa(1+u)^2}\frac{u}{\kappa}\left(1-\frac{u}{\kappa}\right)\sin(2\varphi), \\
\frac{1}{r_e^2}\frac{d\sigma_{x}}{du\,d\varphi} &= \frac{-2\,\xi_3\,\zeta_x}{(1+u)^3}\frac{u}{\kappa}\sqrt{\frac{u}{\kappa}\left(1-\frac{u}{\kappa}\right)}\cos(\varphi), \\
\frac{1}{r_e^2}\frac{d\sigma_{y}}{du\,d\varphi} &= \frac{-2\,\xi_3\,\zeta_y}{(1+u)^3}\frac{u}{\kappa}\sqrt{\frac{u}{\kappa}\left(1-\frac{u}{\kappa}\right)}\sin(\varphi), \\
\frac{1}{r_e^2}\frac{d\sigma_{z}}{du\,d\varphi} &= \frac{\xi_3\,\zeta_z}{(1+u)^3}\frac{u}{\kappa}(u+2)\left(1-2\frac{u}{\kappa}\right).
\end{aligned}
\tag{1.7}
$$

Simulation parameters

Table 2. Simulation parameters. All designations have been defined in the text above.

$\varepsilon_0 =$ 45.6 GeV	$\gamma = 89237$	$\epsilon_x = 270$ pm rad	$\beta_x = 100$ m	$L_1 = 117$ m
$\lambda_0 =$ 532 nm	$\kappa = 1.6279$	$\epsilon_y = 1$ pm rad	$\beta_y =$ 20 m	$L_2 = 100$ m
$\omega_0 = 2.331$ eV	$\vartheta_0 = 190.44$	$\theta_0 = 2.1341$ mrad	$D_x =$ 25 mm	$\sigma_\gamma/\gamma = 0.001$

- Bunch revolution frequency at FCC-ee is $3 \cdot 10^3 s^{-1}$
- The rate of Compton scattering events is estimated as $2 \cdot 10^6 s^{-1}$

Table 3. Detectors: geometry, number of pixels, size of pixels.

Detector	Size ($X \times Y$)	Npix ($X \times Y$)	Pixel size ($X \times Y$)
Photons	10×10 mm	100×100	100×100 μm
Electrons	400×4 mm	1600×80	250×50 μm

Experiment I

- The goal of the first numerical experiment is to verify the above formulae for the cross sections and check the fitting procedure.
- The subject o this study is the trivial case when the Stokes vector of laser polarization is $[\xi_1, \xi_2, \xi_3] = [0, 0, 1]$ and the electron beam is unpolarized $[\zeta_x, \zeta_y, \zeta_z] = [0, 0, 0]$

☐ The rate of Compton scattering events is estimated as $2 \cdot 10^6 s^{-1}$

☐ 5 s measurement time to obtain the 1e7 Monte-Carlo

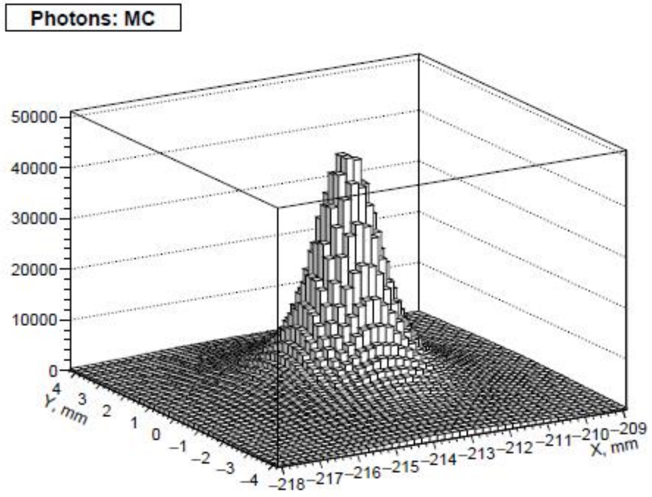


Figure 3. Photon spot at the detector. Number of bins is reduced for better visualization.

Table 4. Photon spot fit results.

$X_0 = -213.538 \pm 0.001$ mm	$Y_0 = -0.002 \pm 0.001$ mm
$\sigma_x = 255 \pm 3\ \mu$m	$\sigma_y = 30 \pm 18\ \mu$m
$\xi_1 = 0.000 \pm 0.002$	$\xi_2 = -0.001 \pm 0.001$
$\xi_3\zeta_x = 0.004 \pm 0.006$	$\xi_3\zeta_y = -0.008 \pm 0.006$
$\xi_3\zeta_z = 0.000 \pm 0.002$	$\chi^2/\mathrm{NDF} = 9796.9/9990$

◆ all polarization parameters correspond to their zero set values with absolute accuracy of better than one percent.

Experiment I

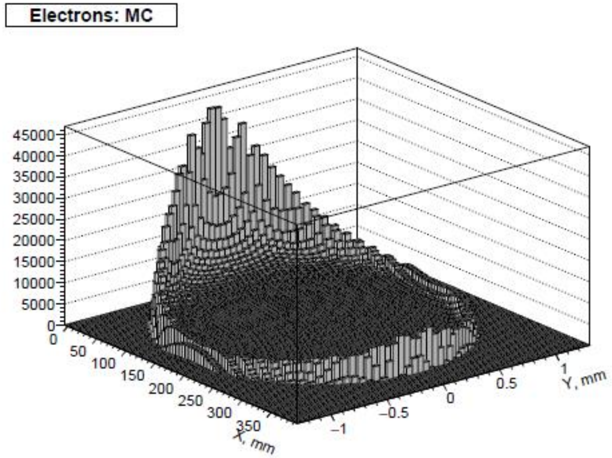


Figure 4. Recoil electrons at the detector. Number of x-bins is reduced to 100 for better visualization. Electrons detector covers the beam at the coordinates $X = 0, Y = 0$, which is impossible to do in practice.

Table 5. Scattered electrons ellipse fit results.

$X_1 = 0.0035 \pm 0.0016$ mm	$X_2 = 347.635 \pm 0.003$ mm
$Y_1 = -1.0682 \pm 0.0001$ mm	$Y_2 = 1.0682 \pm 0.0001$ mm
$\sigma_x = 320.7 \pm 1.5\ \mu$m	$\sigma_y = 27.06 \pm 0.03\ \mu$m
$\xi_1 = 0.001 \pm 0.001$	$\xi_2 = 0.432 \pm 0.198$
$\xi_3\zeta_x = 1.000 \pm 0.195$	$\xi_3\zeta_y = -0.001 \pm 0.002$
$\xi_3\zeta_z = 0.000 \pm 0.001$	χ^2/NDF = 51568.9/52270
$\varepsilon_0 = 45.5997 \pm 0.0007$ GeV	

(compared to e. g. ref. [38]). As for polarization parameters, ξ_2 and $\xi_3\zeta_x$ determination is wrong. This happens because the sum of u_+ and u_- solutions in corresponding terms of the cross section (1.21) have opposite signs and almost cancel each other when $\vartheta_0 \gg 1$ (in our case $\vartheta_0 = \gamma\theta_0 \simeq 190$). The other three polarization parameters ξ_1, $\xi_3\zeta_y$ and $\xi_3\zeta_z$ however are determined even more precise than from the photons distribution. The value of the beam energy ε_0 in the last row of table 5 is

➢ **真实物理解的问题**

Experiment 2

- In this experiment we study more realistic case when the recoil electron detector is shifted 15 mm away from the electron beam. It is assumed that this gap will provide a sufficient physical aperture for the electron beam.

- Stokes vector of laser polarization is $[\xi_1, \xi_2, \xi_3] = [0.1, 0.1, 0.99]$ and the electron beam is $[\zeta_x, \zeta_y, \zeta_z] = [0.1, 0.25, 0.1]$

- The expected polarization of the pilot electron bunches at FCC-ee (averaged over thousands beam revolutions) has ζy component only.
- The ζx and ζz components are added to the simulations in order to investigate the possibility of measuring the electron beam polarization in general case

- From the table we can conclude that all polarization parameters are determined correctly with absolute accuracies from 0.1% to 0.7%.

Experiment 2

- 真值

$[\xi_1, \xi_2, \xi_3] = [0.1, 0.1, 0.99]$ & $[\zeta_x, \zeta_y, \zeta_z] = [0.1, 0.25, 0.1]$

$\xi_1 = 0.1$ $\xi_2 = 0.1$ $\xi_3\zeta_x = 0.099$ $\xi_3\zeta_y = 0.2475$ $\xi_3\zeta_z = 0.099$

- 散射光子拟合结果
- 散射电子拟合结果

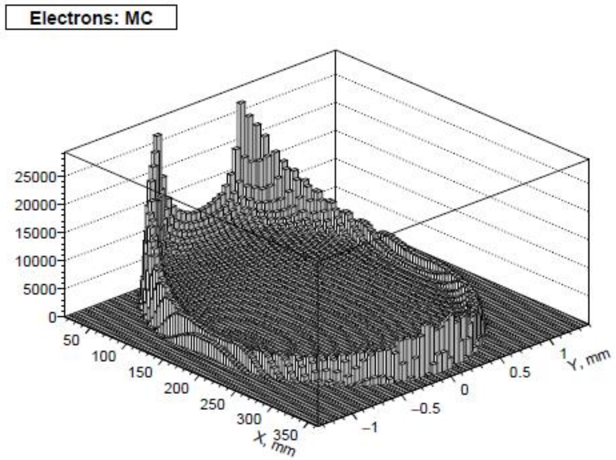


Figure 5. Recoil electrons at the detector. Number of x-bins is reduced to 100 for better visualization. Electrons detector is shifted in X by 15 mm away from the beam coordinates $X = 0, Y = 0$.

Table 6. Photon spot fit results.

$X_0 = -213.539 \pm 0.002$ mm	$Y_0 = 0.000 \pm 0.001$ mm
$\sigma_x = 246 \pm 4\ \mu m$	$\sigma_y = 13 \pm 70\ \mu m$
$\xi_1 = 0.102 \pm 0.002$	$\xi_2 = 0.100 \pm 0.001$
$\xi_3\zeta_x = 0.095 \pm 0.007$	$\xi_3\zeta_y = 0.247 \pm 0.006$
$\xi_3\zeta_z = 0.105 \pm 0.002$	$\chi^2/\mathrm{NDF} = 9935.8/9990$

Table 7. Scattered electrons ellipse fit results.

$X_1 = 0.013 \pm 0.007$ mm	$X_2 = 347.632 \pm 0.004$ mm
$Y_1 = -1.0682 \pm 0.0001$ mm	$Y_2 = 1.0684 \pm 0.0001$ mm
$\sigma_x = 319.6 \pm 4.3\ \mu m$	$\sigma_y = 27.15 \pm 0.03\ \mu m$
$\xi_1 = 0.100 \pm 0.001$	$\xi_2 = 0.100$
$\xi_3\zeta_x = 0.099$	$\xi_3\zeta_y = 0.246 \pm 0.002$
$\xi_3\zeta_z = 0.099 \pm 0.001$	$\chi^2/\mathrm{NDF} = 50152.7/51245$
$\varepsilon_0 = 45.5959 \pm 0.0025$ GeV	

The polarization parameters ξ_1, $\xi_3\zeta_y$ and $\xi_3\zeta_z$ are determined with absolute accuracy of 0.1%, 0.2% and 0.1%.

Compare CEPC and FCC

➢ 区别1：关于同时测量 电子束流极化的讨论

- Longitudinal polarization + vertical polarization + Radial polarization

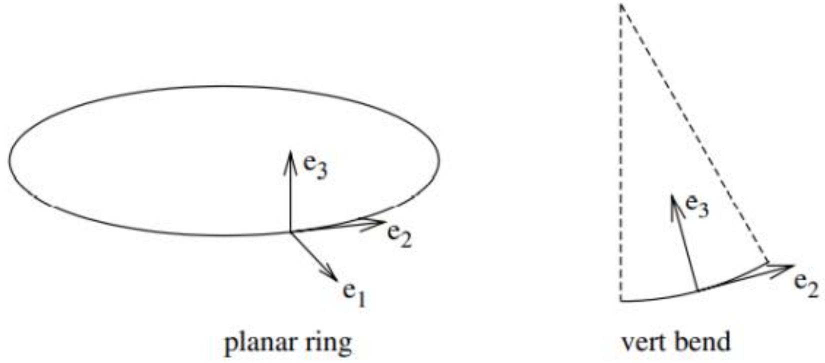


Figure 23. Sketch of the coordinate basis for a planar ring and a vertical bend.

Ref: Mane_2005_Rep._Prog._Phys._68_1997

new

$$\frac{1}{r_e^2}\frac{d\sigma_0}{du\,d\varphi} = \frac{1}{\kappa(1+u)^3}\left[1+(1+u)^2-4\frac{u}{\kappa}\left(1-\frac{u}{\kappa}\right)(1+u)\right],$$

$$\frac{1}{r_e^2}\frac{d\sigma_{\xi_1}}{du\,d\varphi} = \frac{4\,\xi_1}{\kappa(1+u)^2}\frac{u}{\kappa}\left(1-\frac{u}{\kappa}\right)\cos(2\varphi),$$

$$\frac{1}{r_e^2}\frac{d\sigma_{\xi_2}}{du\,d\varphi} = \frac{4\,\xi_2}{\kappa(1+u)^2}\frac{u}{\kappa}\left(1-\frac{u}{\kappa}\right)\sin(2\varphi),$$

$$\frac{1}{r_e^2}\frac{d\sigma_x}{du\,d\varphi} = \frac{-2\,\xi_3\,\zeta_x}{(1+u)^3}\frac{u}{\kappa}\sqrt{\frac{u}{\kappa}\left(1-\frac{u}{\kappa}\right)}\cos(\varphi),$$

$$\frac{1}{r_e^2}\frac{d\sigma_y}{du\,d\varphi} = \frac{-2\,\xi_3\,\zeta_y}{(1+u)^3}\frac{u}{\kappa}\sqrt{\frac{u}{\kappa}\left(1-\frac{u}{\kappa}\right)}\sin(\varphi),$$

$$\frac{1}{r_e^2}\frac{d\sigma_z}{du\,d\varphi} = \frac{\xi_3\,\zeta_z}{(1+u)^3}\frac{u}{\kappa}(u+2)\left(1-2\frac{u}{\kappa}\right).$$

old

$$d\sigma_0 = \frac{r_e^2}{\kappa^2(1+u)^3}\left(\kappa(1+(1+u)^2)-4\frac{u}{\kappa}(1+u)(\kappa-u)\left[1-\xi_\perp\cos(2(\varphi-\varphi_\perp))\right]\right)\,du\,d\varphi,$$

$$d\sigma_\parallel = \frac{\xi_\circlearrowleft\zeta_\circlearrowleft r_e^2}{\kappa^2(1+u)^3}\,u(u+2)(\kappa-2u)\,du\,d\varphi,$$

$$d\sigma_\perp = -\frac{\xi_\circlearrowleft\zeta_\perp r_e^2}{\kappa^2(1+u)^3}\,2u\sqrt{u(\kappa-u)}\cos(\varphi-\phi_\perp)\,du\,d\varphi.$$

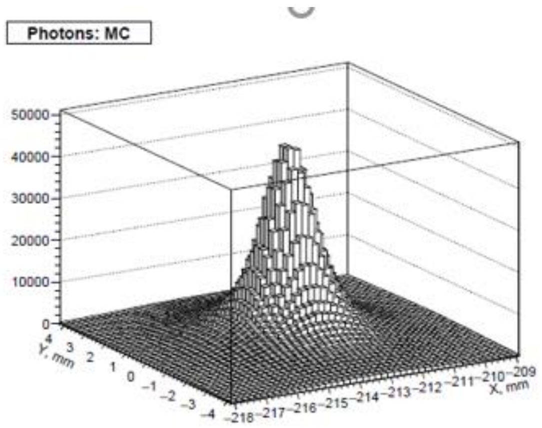
- 均讨论的是Z mode，为RDP 提供条件
- 从第一性原理的角度上，推导公式（场论+Compton backscattering）
- ? 探测器这个位置测不出来radial polarization

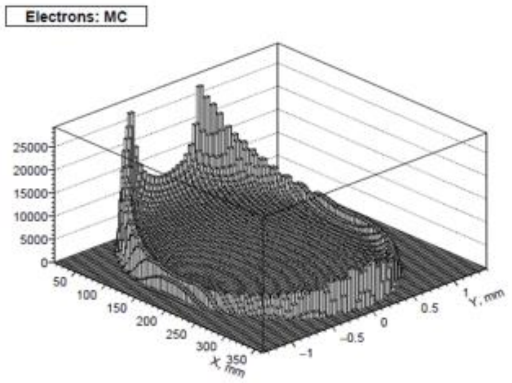
Compare CEPC and FCC

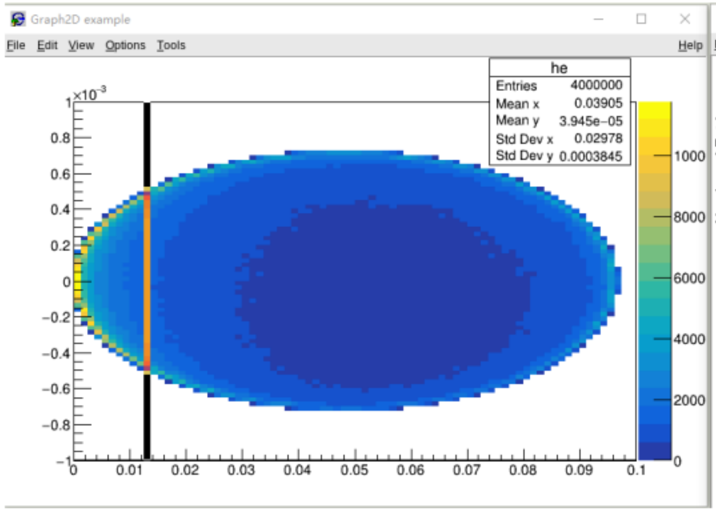
- 区别2：测量极化方法上的差异

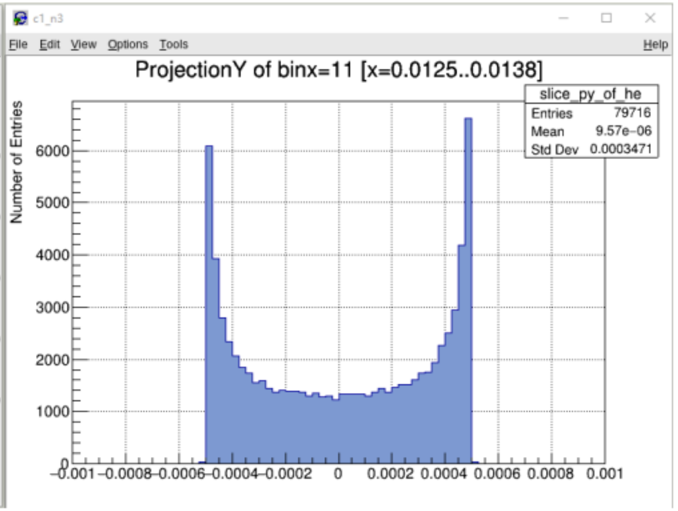
我的方案：asymmetry 信息测 $< y > |_x$

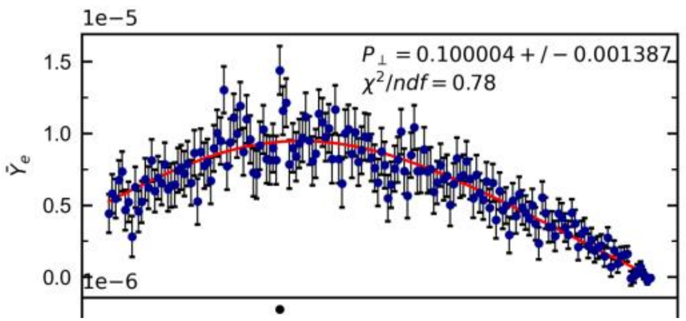
Muchnoi方案：散射粒子2D spatial distribution 拟合











Compare CEPC and FCC

➢ 区别3：关于u+ u-，即两个物理解的讨论

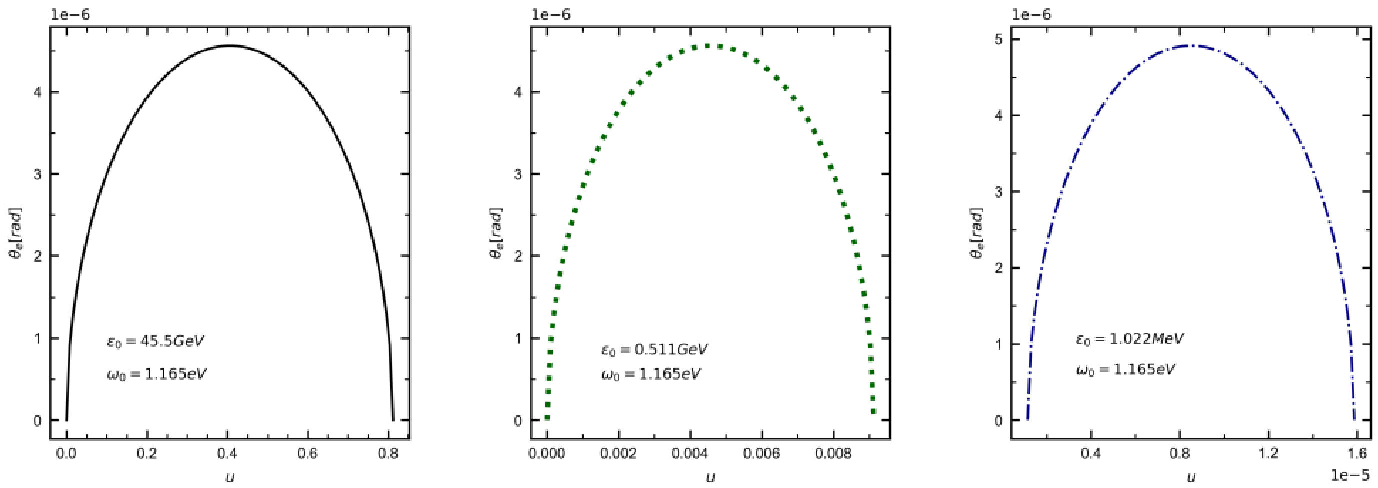


Figure 2. Kinematic of the Compton scattering. The relationships between the scattered angle of the electrons θ_e and kinetic parameter u are verified for an electron beam energy of 45.5 GeV, 0.511 GeV, 1.022 MeV and a laser photon of 1.165 eV. The symmetric axis of the distribution of the scattering angle, θ_e, is $u = \kappa/2$.

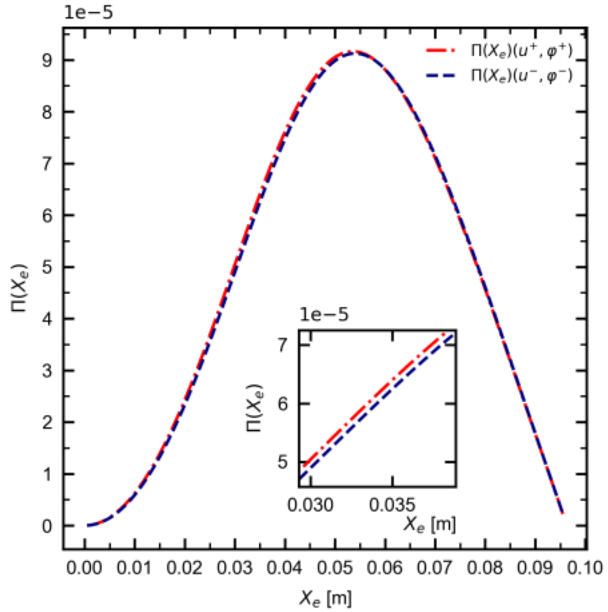
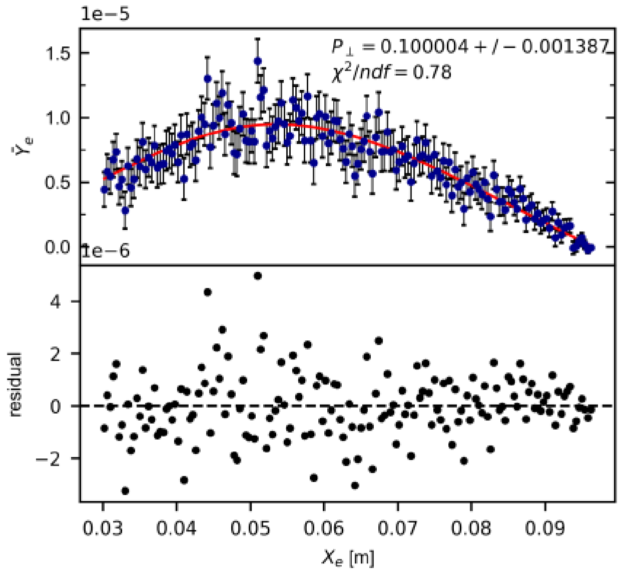


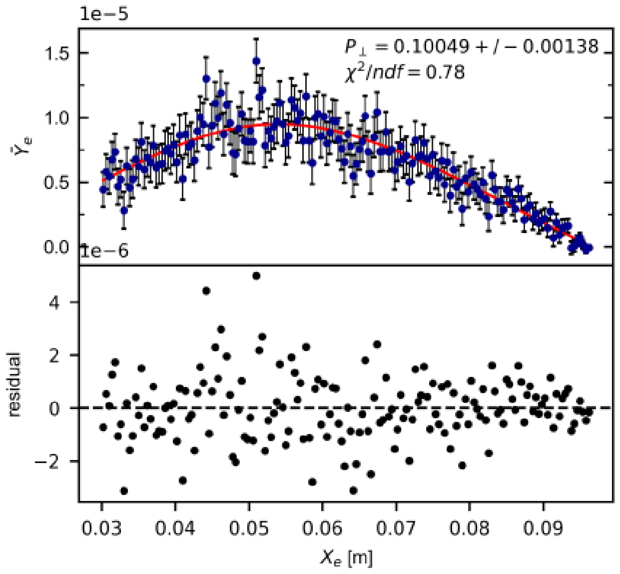
Figure 5. Graph of the analyzing power for CEPC Compton polarimeter. It is seen that in terms of the value of analyzing power, two sets of data denoted by blue and red curves are quite close and the maximum difference is merely 10−6.

Compare CEPC and FCC

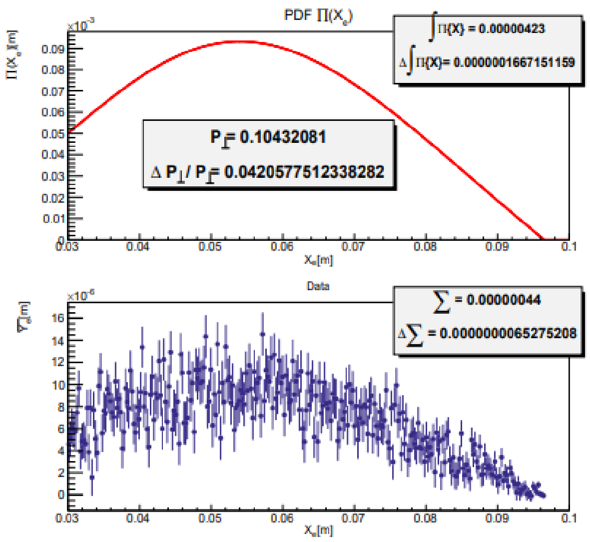
➢ 区别4：拟合结果

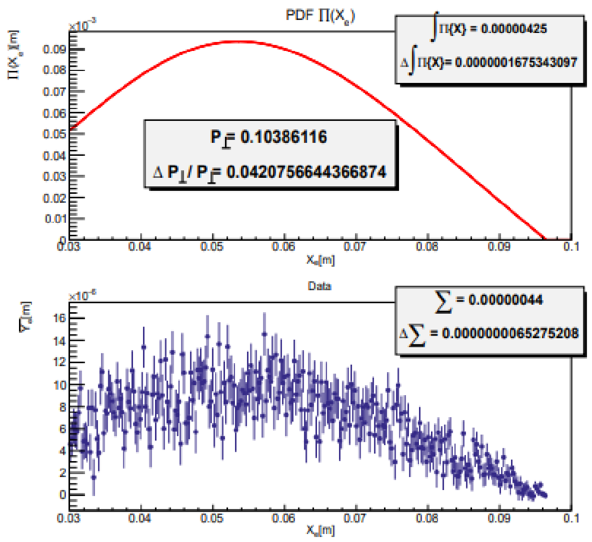


(a) $\Pi(X_e)$ as the function of (u^+, φ^+).



(b) $\Pi(X_e)$ as the function of (u^-, φ^-).





Compare CEPC and FCC

➢ 区别5：拟合结果

- Muchnoi results
  - From the table we can conclude that all polarization parameters are determined correctly with absolute accuracies from 0.1% to 0.7%.

- My results
  - Accordingly, the transverse polarization in the same measurement period, yielded $P\perp$ = 0.1000 $\pm$ 0.0014(u + , φ+) and $P\perp$ = 0.1005 $\pm$ 0.0014(u − , φ−).

Table 2. Comparison of fitting results of different formula forms of analyzing power for 10% transverse polarization.

Function	$P_\perp = 10\%$	$\Delta P_\perp / P_\perp \%$
Fit results	$P_\perp = 0.1002$	0.98
Integration results	$P_\perp = 0.1041$	3.1

Compare CEPC and FCC

➢ 区别6：系统参数的差异

- Muchnoi results

Table 2. Simulation parameters. All designations have been defined in the text above.

$\varepsilon_0 =$ 45.6 GeV	$\gamma = 89237$	$\epsilon_x = 270$ pm rad	$\beta_x = 100$ m	$L_1 = 117$ m
$\lambda_0 =$ 532 nm	$\kappa = 1.6279$	$\epsilon_y = 1$ pm rad	$\beta_y =$ 20 m	$L_2 = 100$ m
$\omega_0 = 2.331$ eV	$\vartheta_0 = 190.44$	$\theta_0 = 2.1341$ mrad	$D_x =$ 25 mm	$\sigma_\gamma/\gamma = 0.001$

- The rate of Compton scattering events is estimated as $2 \cdot 10^6 s^{-1}$

- My results

Table 1. Electron beam [29] and laser properties used in simulation.

E_b, GeV	N_e	σ'_x/σ'_y, μrad	energy, mJ	pulse length, ps	λ, nm	σ_γ, μm
45.5	8×10^{10}	1.22/0.4	2.8	28	1064	160

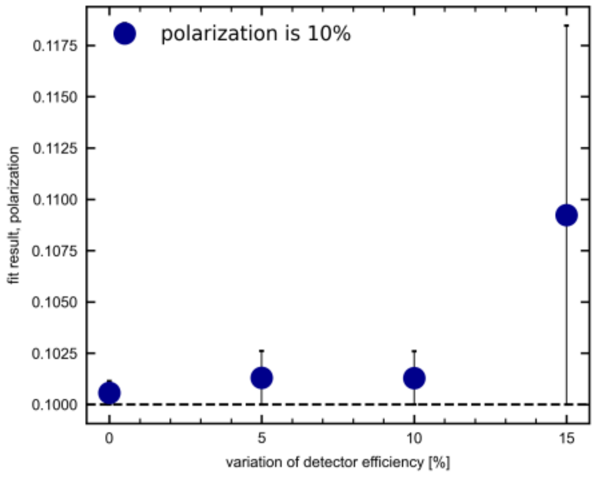
- The corresponding luminosity for the different collision angles is shown in figure 7. The luminosity can reach 7.0405×1033 m−2 s −1 with α = 2.35 mrad. The total cross section of Compton back-scattering is 402 mb. Therefore, the maximum scattering rate reaches 6.3506 × 1e5 pulse−1 for one electron bunch. The frequency of laser is 1 Hz, the Compton scattering rate is $2.83 \cdot 10^5 s^{-1}$

Compare CEPC and FCC

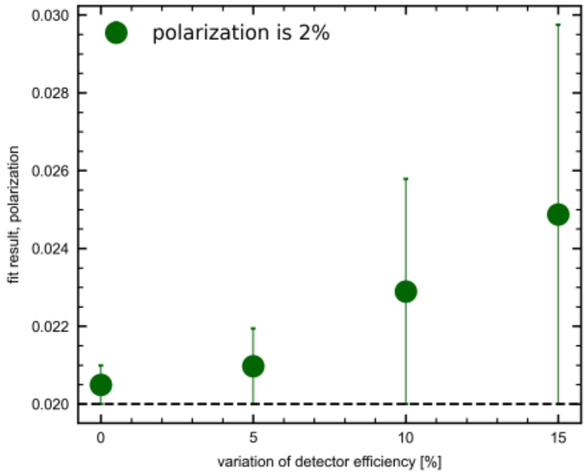
➢ 区别7：系统误差的讨论

Table 3. A list of the polarimeter setup parameters that are expected to contribute to the systematic errors of the transverse polarization measurement for 10% polarization. And the total of them is about 0.6%.

Source	Uncertainty	$\Delta P_\perp / P_\perp$%
Dipole strength	3.3×10^{-7} T	0.062%
L1 (IP-to-detector)	1 cm	0.007%
L2 (Dipole-to-detector)	1 cm	0.051%
Beam energy	100 keV	0.0001%
Detector resolution	115 µ × 7 µm	0.278%
$\Delta\alpha$ deviation of the detector	0.1°	negligible
$\Delta\beta$ deviation of the detector	0.1°	negligible
Detector placement		ignored
laser polarization	0.2%	0.2%
Total		0.6%



(a) $P_\perp$ = 10% as the pixel cell size is 400 µm×25 µm.



(b) $P_\perp$ = 2% as the pixel cell size is 200 µm×25 µm.

Discussion and Plan

- 第一性原理，公式推导 cross-section 去理解 三个极化测量的可能性

Thanks !