Coevolution of AGN, BHs and their host galaxies: the observational foundations

Beijing international summer school

"The physics and evolution of AGN"

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University of Florence, Italy

Outline of lectures

- Part 1: *Supermassive black holes in galactic nuclei: detections and mass measurements* (2 lectures)
- Part 2: *Scaling relations between black holes and their host galaxies* (2 lectures)
- Part 3: *The cosmological evolution of AGN and BHs* (2 lectures)
- Part 4: *The observational signatures of coevolution* (2 lectures)

Università degli Studi di Firenze
Dipartimento di Fisica e Astronomia
Sezione di Astronomia
Università di Firenze

1. Supermassive black holes in galactic nuclei: detections and mass measurements

Brief historical introduction

- **1783 - 1795** John Michell and Pierre-Simon Laplace hypothesize existence of “dark stars” or “invisible bodies”
- **1915** Albert Einstein’s General Relativity
- **1916** Karl Schwarzschild finds the “black hole” solution for GR equations
- **1963** Maarten Schmidt, Jesse Greenstein & Thomas Matthews discover Quasars
- **1964** Edwin Salpeter and Yakov Zel’dovich independently hypothesize mass accretion onto a supermassive BH for quasars.
- **1968** John Wheeler coins the term “Black Hole”
- **1970s (beginning of)** X-ray source Cygnus X-1 is the first BH candidate with $M_{BH} \sim 12\ M_{\odot}$
- **1979** Sargent et al. showed that images and spectra of the central region of M87 indicate the presence of a BH with $M_{BH} \sim 6 \times 10^9\ M_{\odot}$

3C 273: A STAR-LIKE OBJECT WITH LARGE RED-SHIFT

By Dr. M. SCHMIDT

Mount Wilson and Palomar Observatories, Carnegie Institution of Washington, California Institute of Technology, Pasadena

THE only objects seen on a 200-in. plate near the positions of the components of the radio source 3C 273 reported by Hazard, Mackey and Shimmins in the preceding article are a star of about thirteenth magnitude and a faint wisp or jet. The jet has a width of 1″–2″ and extends away from the star in position angle 43°. It is not visible within 11″ from the star and ends abruptly at 20″ from the star. The position of the star, kindly furnished by Dr. T. A. Matthews, is R.A. 12h 26m 33·35s ±0·04s, Decl. +2° 19′ 42·0″ ±0·5″ (1950), or 1″ east of component B of the radio source. The end of the jet is 1″ east of component A. The close correla-

Table 1. Wave-lengths and Identifications

λ	$\lambda/1{\cdot}158$	λ_0	
3239	2797	2798	Mg II
4595	3968	3970	Hε
4753	4104	4102	Hδ
5032	4345	4340	Hγ
5200–5415	4490–4675		
5632	4864	4861	Hβ
5792	5002	5007	[O III]
6005–6190	5186–5345		
6400–6510	5527–5622		

Oke in a following article, and by the spectrum of another star-like object associated with the radio source 3C 48 discussed by Greenstein and Matthews in another com-

Schmidt 1963, Nature, 197, 1040

RED-SHIFT OF THE UNUSUAL RADIO SOURCE: 3C 48

By Dr. JESSE L. GREENSTEIN

Mount Wilson and Palomar Observatories, Carnegie Institution of Washington, California Institute of Technology

AND

Dr. THOMAS A. MATTHEWS

Owens Valley Radio Observatory, California Institute of Technology

THE radio source 3C 48 was announced to be a star[1] in our Galaxy on the basis of its extremely small radio diameter[2], stellar appearance on direct photographs and unusual spectrum. Detailed spectroscopic study at Palomar by Greenstein during the past year gave only partially successful identifications of its weak, broad emission lines; the possibility that they might be per-

cosmological speculation. A very interesting alternative, that the source is a nearby ultra-dense star of radius near 10 km containing neutrons, hyperons, etc., has been explored and seems to meet insuperable objections from the spectroscopic point of view. The small volume for the shell required by the observed small gradient of the gravitational potential is incompatible with the

Greenstein & Matthews 1963, Nature, 197, 1041

NOTES

ACCRETION OF INTERSTELLAR MATTER BY MASSIVE OBJECTS

Observations of quasi-stellar radio sources have indicated the existence in the Universe of extremely massive objects of relatively small size. The present note discusses the possible further growth in mass of a relatively massive object, by means of accretion of interstellar gas onto it, and the accompanying energy release. Although there is no evidence for (and possibly some evidence *against*) quasi-stellar radio sources occurring inside ordinary galaxies, for the sake of concreteness we consider the fate of an object of mass $M > 10^6$ (masses in solar units throughout) in an ordinary spiral galaxy somewhat like ours.

We first re-examine the hypothetical problem of an object of mass M moving with velocity U (in km/sec) relative to a completely uniform gas medium of density n (expressed as H-atoms per cm^3) and thermal speed U_{th}. We define (Hoyle and Lyttleton 1939) a characteristic length s_0 and express the rate of accretion in terms of a dimensionless parameter a to be determined,

$$s_0 = GM/U^2 = (M/U^2) \times 4.3 \times 10^{-3}\ \text{pc}\,,$$

$$dM/dt = 2\pi a s_0^2\, nU \equiv aM/t_0\,, \quad (1)$$

Salpeter 1964, ApJ, 140, 796

Astrophysical Black Holes

Black holes (BH) are characterized by:

mass (M_{BH}), angular momentum (also called spin) and charge.

Astrophysical black holes

- End states of stellar evolution: $M_{BH} \sim 1 - 10\ M_\odot$
- End states of Population III evolution (?): $M_{BH} \sim 10 - 1000\ M_\odot$
- Intermediate mass black holes (IMBH): $M_{BH} \sim 10^3 - 10^5\ M_\odot$
- Supermassive black holes: $M_{BH} \sim 10^6 - 10^{10}\ M_\odot$

We suppose that IMBH and SMBH grow by accreting gas, stars, other IM or SM black holes, but the origin of the seeds is still not clear.

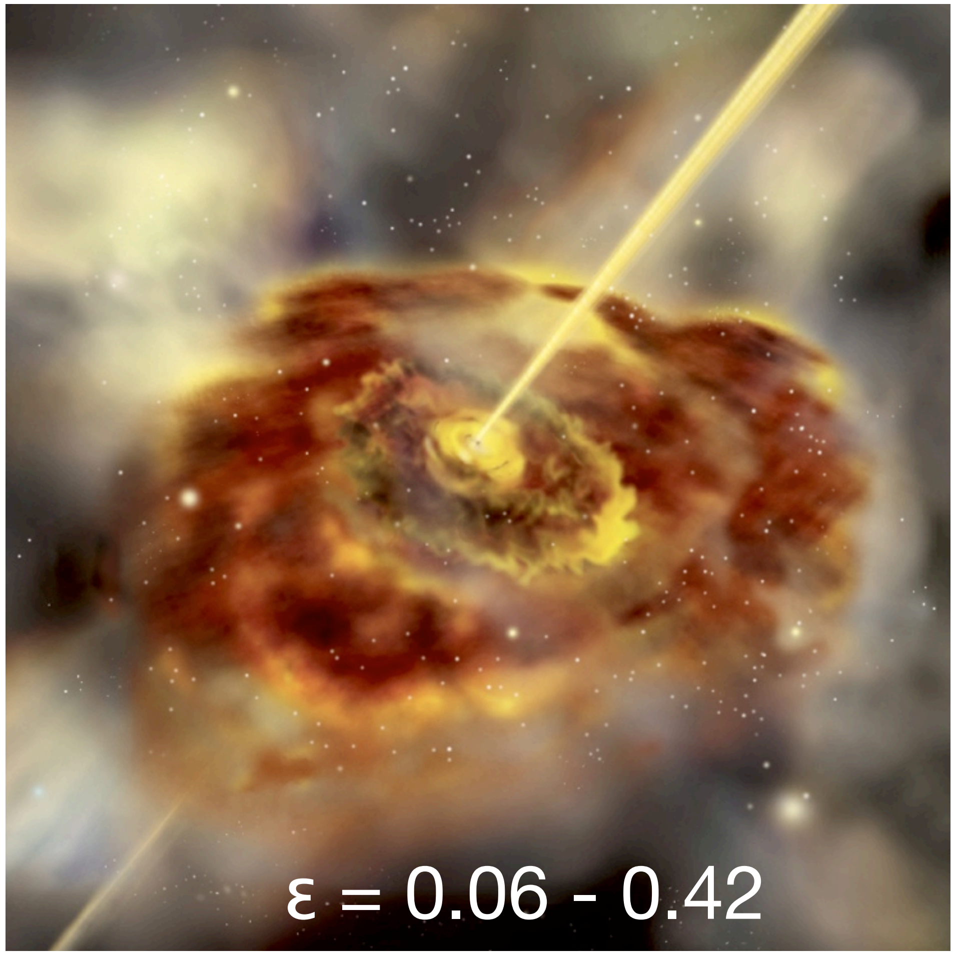
Should we expect IMBH and SMBH? Where?

Remnants of Active Galactic Nuclei

AGN are powered by accretion on supermassive BHs.
Consider an active *galactic nucleus* accreting a mass ΔM_{acc} for a time Δt_{AGN} and emitting L_{AGN},

$$L_{AGN} = \varepsilon \left(\frac{\Delta M_{acc}}{\Delta t_{AGN}} \right) c^2$$

fraction ε ΔM_{acc} is radiated away,
(1-ε) ΔM_{acc} goes into the BH



$$\Delta M_{BH} = (1 - \varepsilon)\Delta M_{acc}$$

$$\Delta M_{BH} = \frac{1 - \varepsilon}{\varepsilon\, c^2} L_{AGN} \Delta t_{AGN}$$

$$\Delta M_{BH} = 6.1 \times 10^6\,\mathrm{M}_\odot \left(\frac{L_{AGN}}{10^{12}\,\mathrm{L}_\odot} \right) \left(\frac{\Delta t_{AGN}}{10^7\,\mathrm{yr}} \right) \qquad \text{for } \varepsilon = 0.1$$

We expect IM/SMBH in the nuclei of quiescent (old) galaxies, as remnants of past AGN activity. How to find them and measure their masses?

The very first ideas ...

From the letter by John Michell read by Henry Cavendish before the Royal Society on 27 November 1783:

If there should really exist in nature any bodies whose density is not less than that of the Sun, and whose diameters are more than 500 times the diameter of the Sun, since their light could not arrive at us; or if there should exist any other bodies of a somewhat smaller size which are not naturally luminous; of the existence of bodies under either of these circumstances, we could have no information from sight;
yet, if any luminous bodies infer their existence of the central ones with some degree of probability, as this might afford a clue to some of the apparent irregularities of the revolving bodies, which would not be easily explicable on any other hypothesis;
but as the consequences of such a supposition are very obvious ...

Laplace in his book “Exposition du Systeme du Monde” (1795) called these hypothetical objects *les corps obscures*, "invisible bodies.”

The very first ideas ...

If there should really exist in nature any bodies whose density is not less than that of the Sun, and whose diameters are more than 500 times the diameter of the Sun, since their light could not arrive at us; ...

- Assume light is made of particles subjected to gravity like mass bodies [Wrong!]
- A Dark Star (mass M_{DS}, radius R_{DS}) has an escape velocity equal to or larger than the speed of light [Newtonian dynamics, wrong!]

$$v_{esc} = \left(\frac{2GM_{DS}}{R_{DS}}\right)^{1/2} = c \quad \Longrightarrow$$

correct Schwarzschild radius but wrong hypothesis!

$$R_{DS} = \frac{2GM_{DS}}{c^2} = 3.0 \times 10^{13}\,\mathrm{cm}\left(\frac{M_{\mathrm{BH}}}{10^8\,\mathrm{M}_\odot}\right) = 2.0\,\mathrm{AU}\left(\frac{M_{\mathrm{BH}}}{10^8\,\mathrm{M}_\odot}\right)$$

$$R_{DS} = \left(\frac{3c^2}{8\pi G\rho}\right)^{1/2} = 3.4 \times 10^{13}\,\mathrm{cm}\left(\frac{\rho}{\rho_\odot}\right)^{-1/2} = 487\,\mathrm{R}_\odot\left(\frac{\rho}{\rho_\odot}\right)^{-1/2}$$

... and how we do it today!

if any luminous bodies infer their existence of the central ones with some degree of probability, as this might afford a clue to some of the apparent irregularities of the revolving bodies, which would not be easily explicable on any other hypothesis;

Use the kinematics of 'test particles' (gas clouds, stars) in the nuclear region of galaxies to infer the presence of a BH.

Observables:

Gas/Stars as tracers of kinematics (V,σ) around BH

Gravitational potential of stars (Φ_{Stars}) from observed surface brightness of galaxy (assume $L \approx \Upsilon M$)

Models:

Evidence for BH?

... and how we do it today!

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Evidence for BH?

$\Phi_{\rm BH} = -G\, M_{\rm BH}\, R^{-1} \quad (R >> R_{\rm Schwarzschild})$

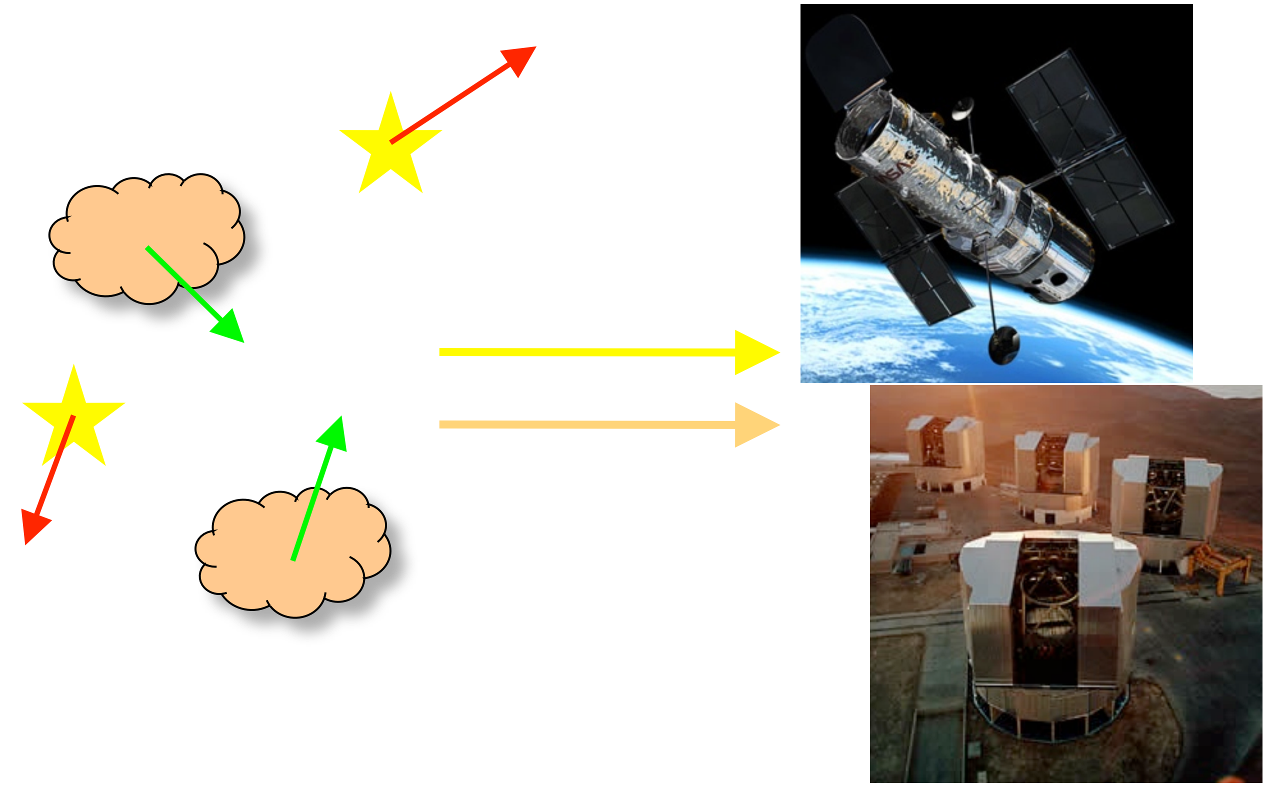
Dynamical evidences for BHs

Motions of *test particles*

- Star proper motions and radial velocities
- Radial velocities of single gas clouds (masers)

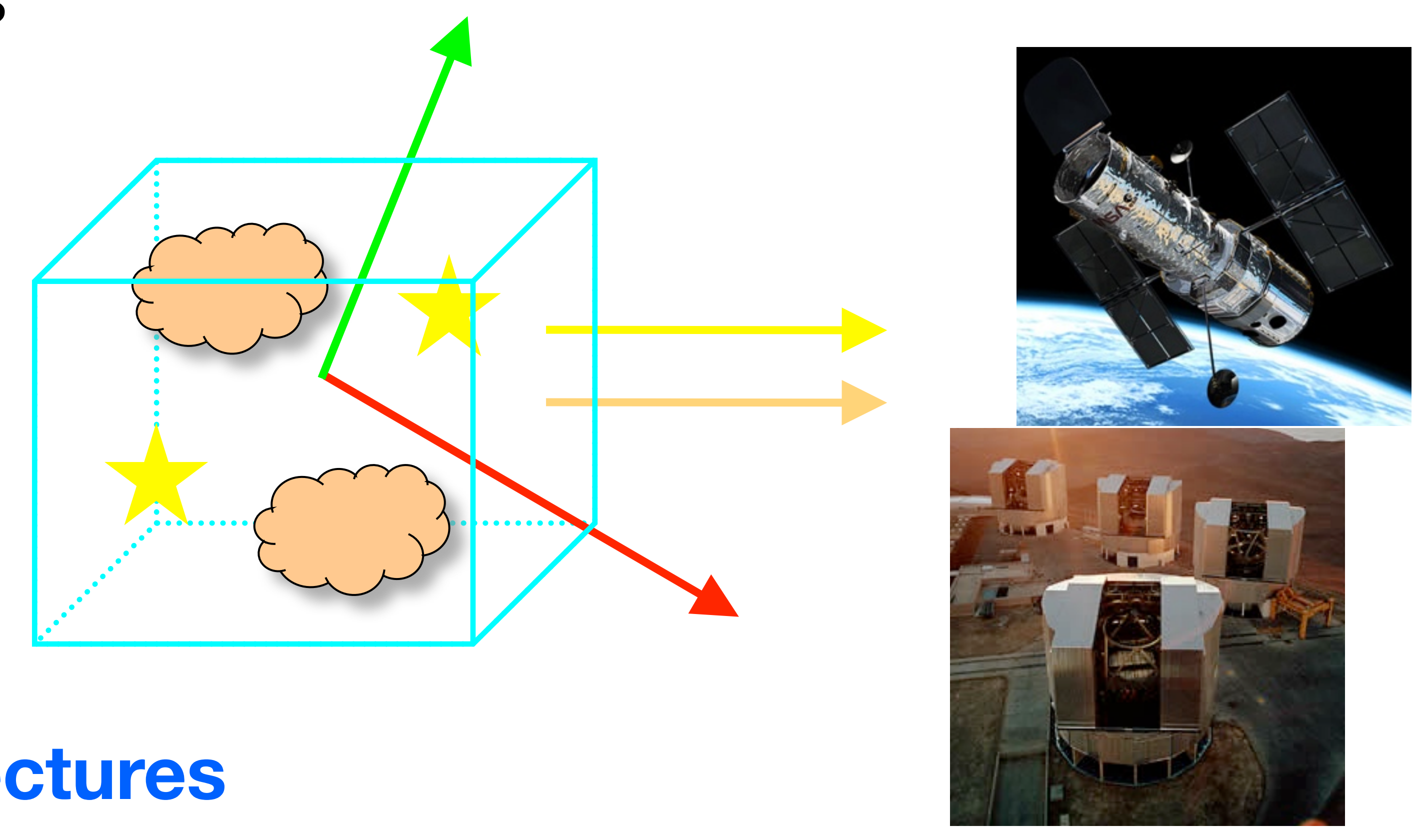
Ensemble motions
(spatially resolved)

- Stellar Dynamics
  V from Stellar Absorption Lines
- Gas Kinematics
  V from Gas Emission Lines



Ensemble motions
(time resolved)

- Reverberation Mapping
  V from line width, R from time variability → **Hagai Netzer's lectures**



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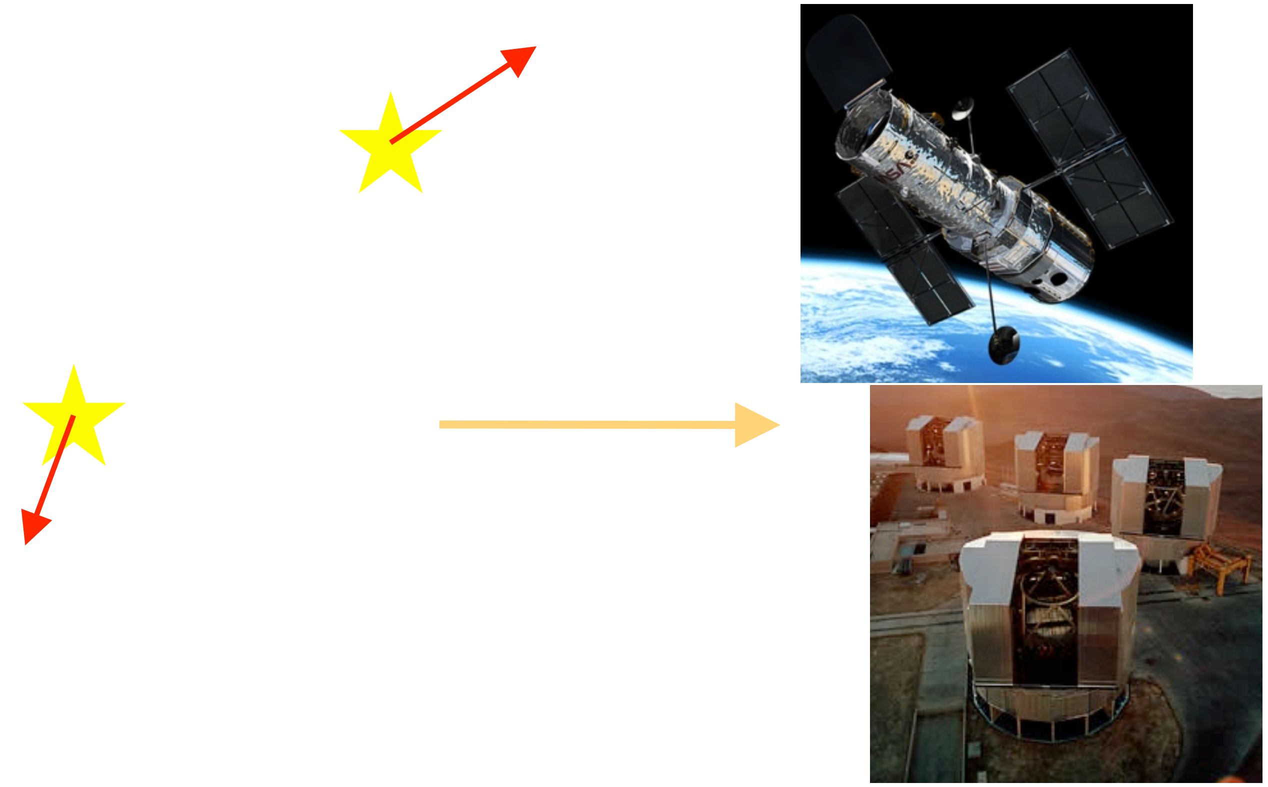
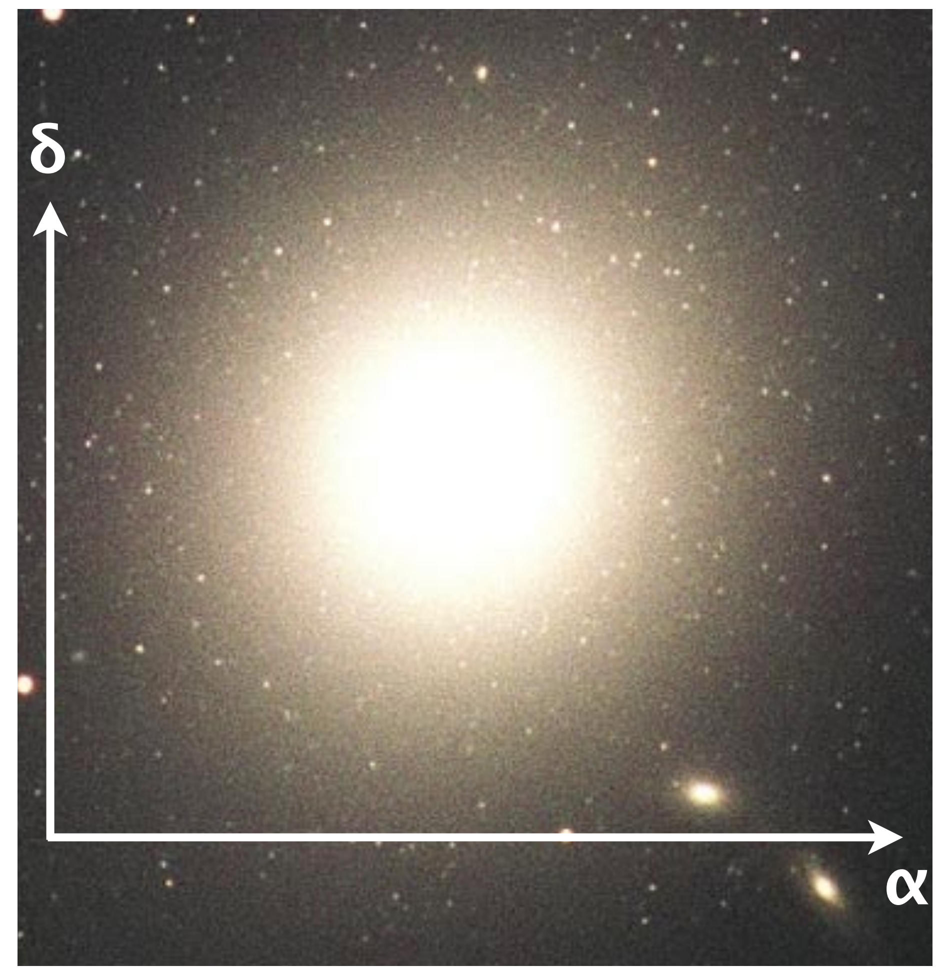


The concept of spatial resolution

A given source has "intrinsic" surface brightness $O(\alpha,\delta)$ on the plane of the sky (α,δ are angular coordinates), the observed one $I(\alpha,\delta)$ is given by

$$I(\alpha,\delta) = \int\int P(\alpha-\alpha',\delta-\delta')\,O(\alpha',\delta')\,d\alpha'\,d\delta'$$

$P(\alpha,\delta)$ is the Point Spread Function (PSF) of telescope+instrument, i.e. the response of the system to a "point-like" source like a star.



The PSF is usually characterized by its Full Width at Half Maximum (FWHM), and most of the times a Gaussian function is a good approximation for it.

The concept of spatial resolution

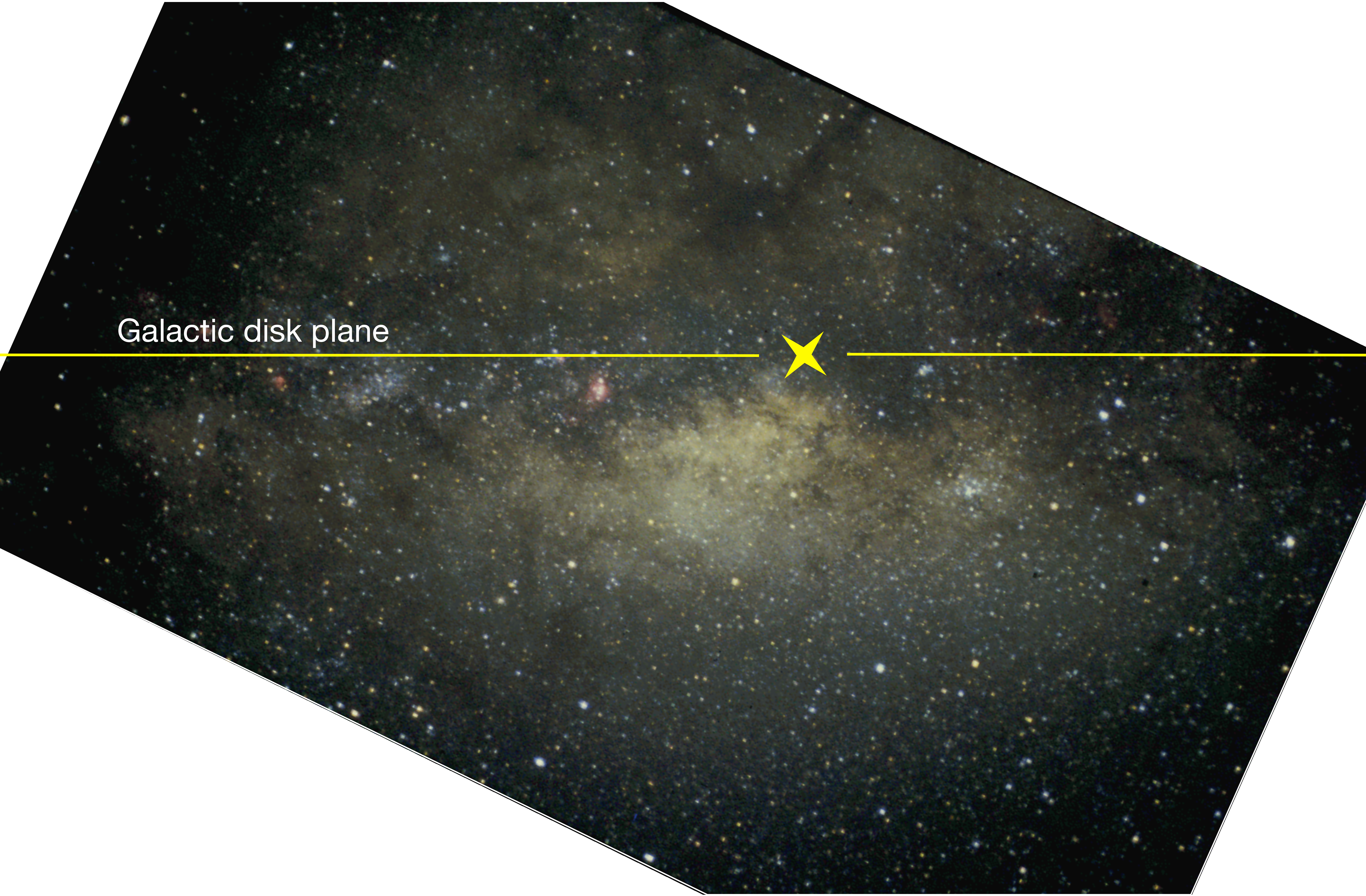
- For an observatory in space (outside Earth's atmosphere), the PSF is mostly determined by the diffraction limit of the telescope and its size depends on the telescope diameter d

$$\Delta\theta \simeq \left(\frac{\lambda}{d}\right) \text{rad}$$

- For a ground based observatory, the diffraction limited PSF is grossly degraded by refraction through the turbulent atmosphere, and is called "seeing".
  In the optical telescopes are diffraction limited only up to ~10 cm diameter.

- Adaptive Optics can correct for the atmospheric degradation by observing simultaneously with the target a reference star (natural or laser) which "maps" the degradation induced by the atmosphere and which can then be corrected in the source image.

The Galactic center in IR



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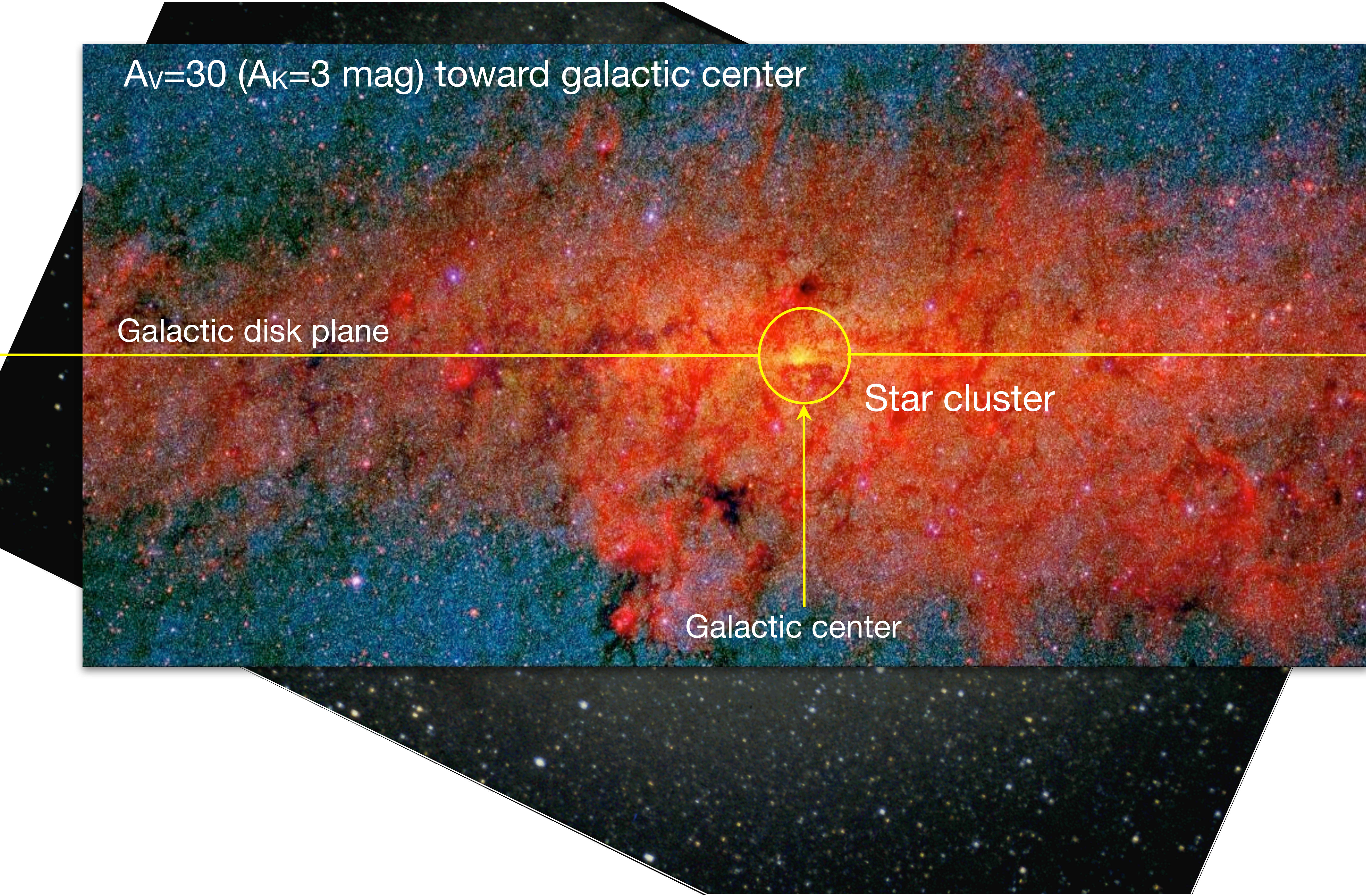
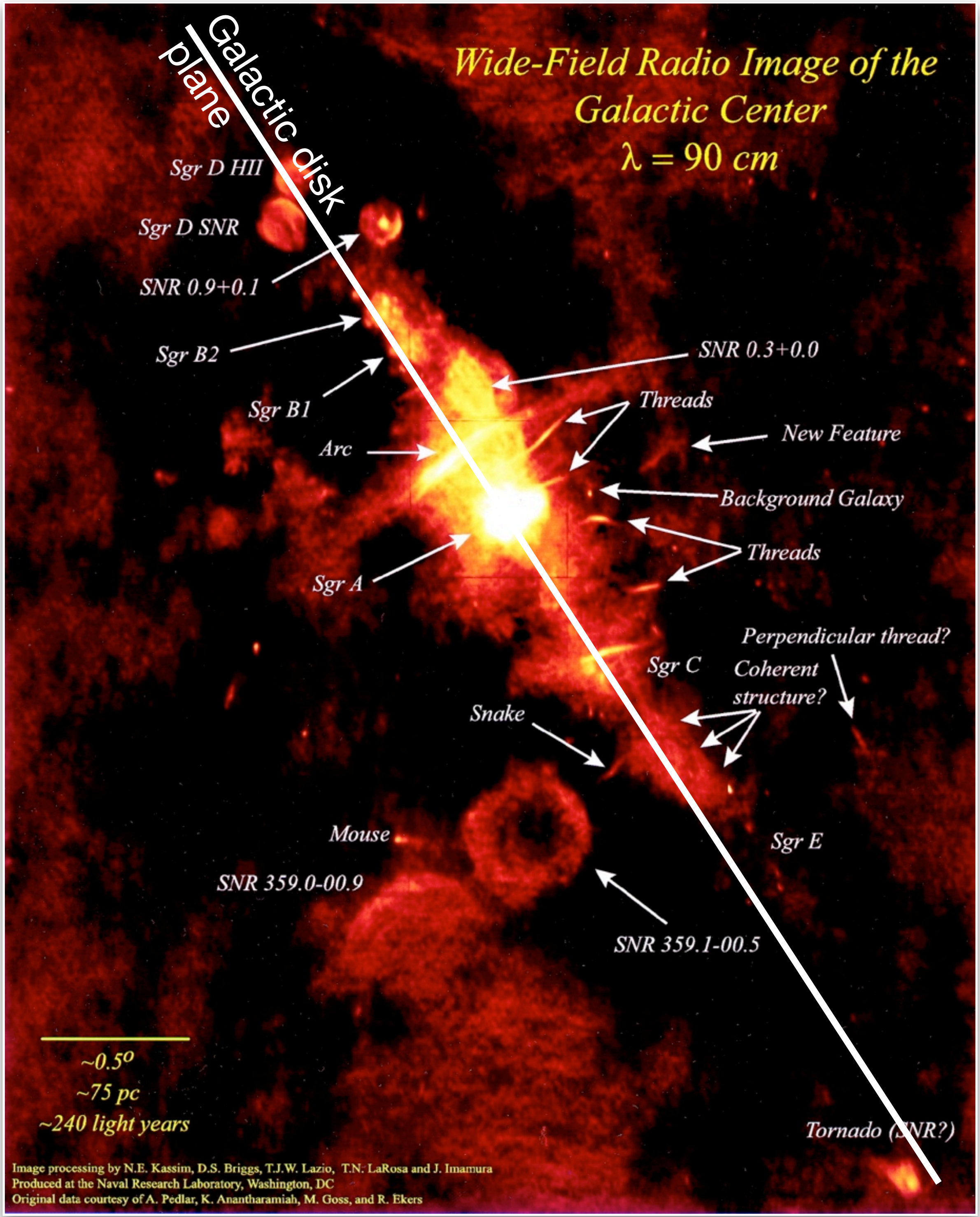


Galactic center in radio (λ = 90 cm)



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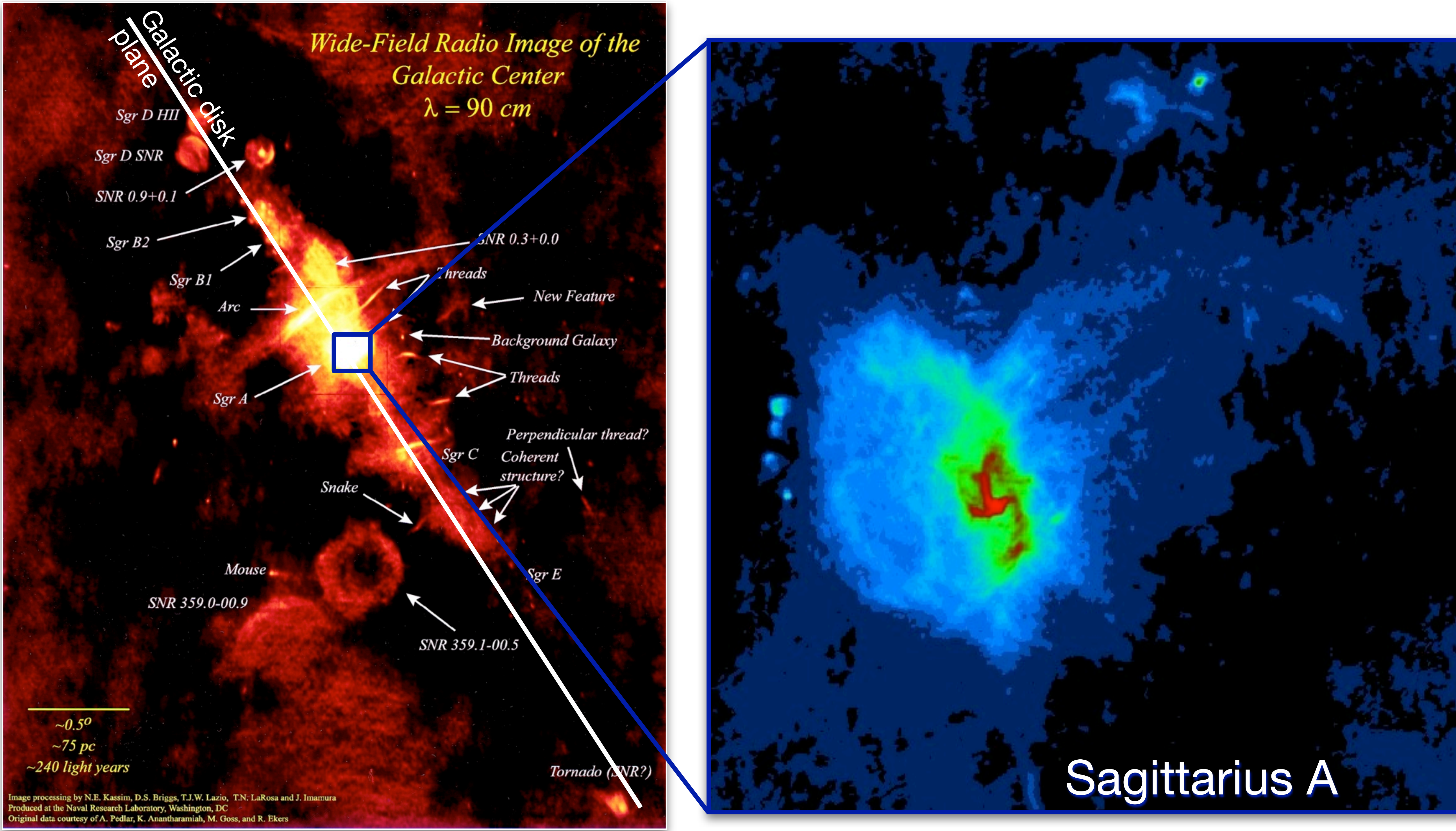

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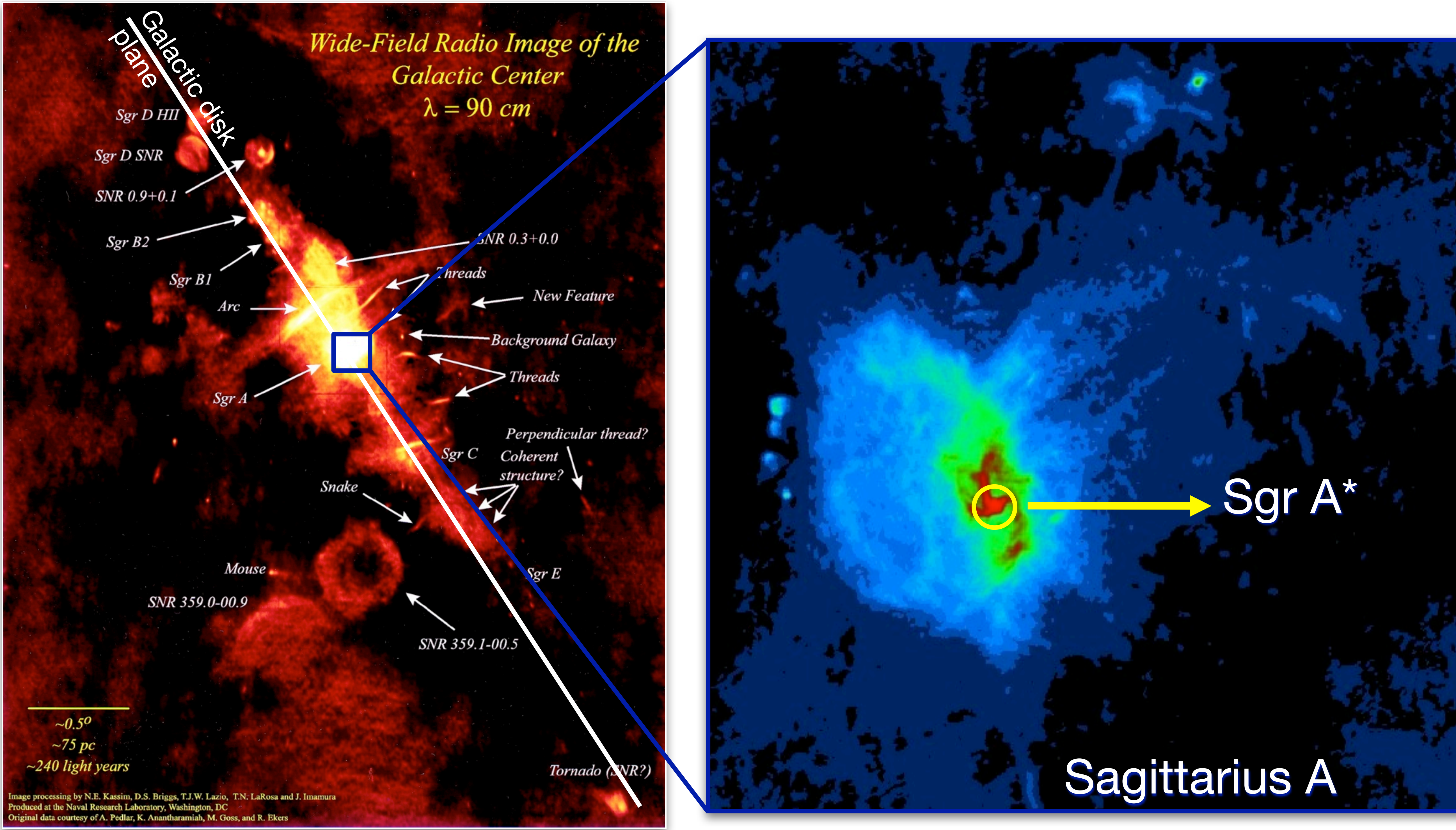
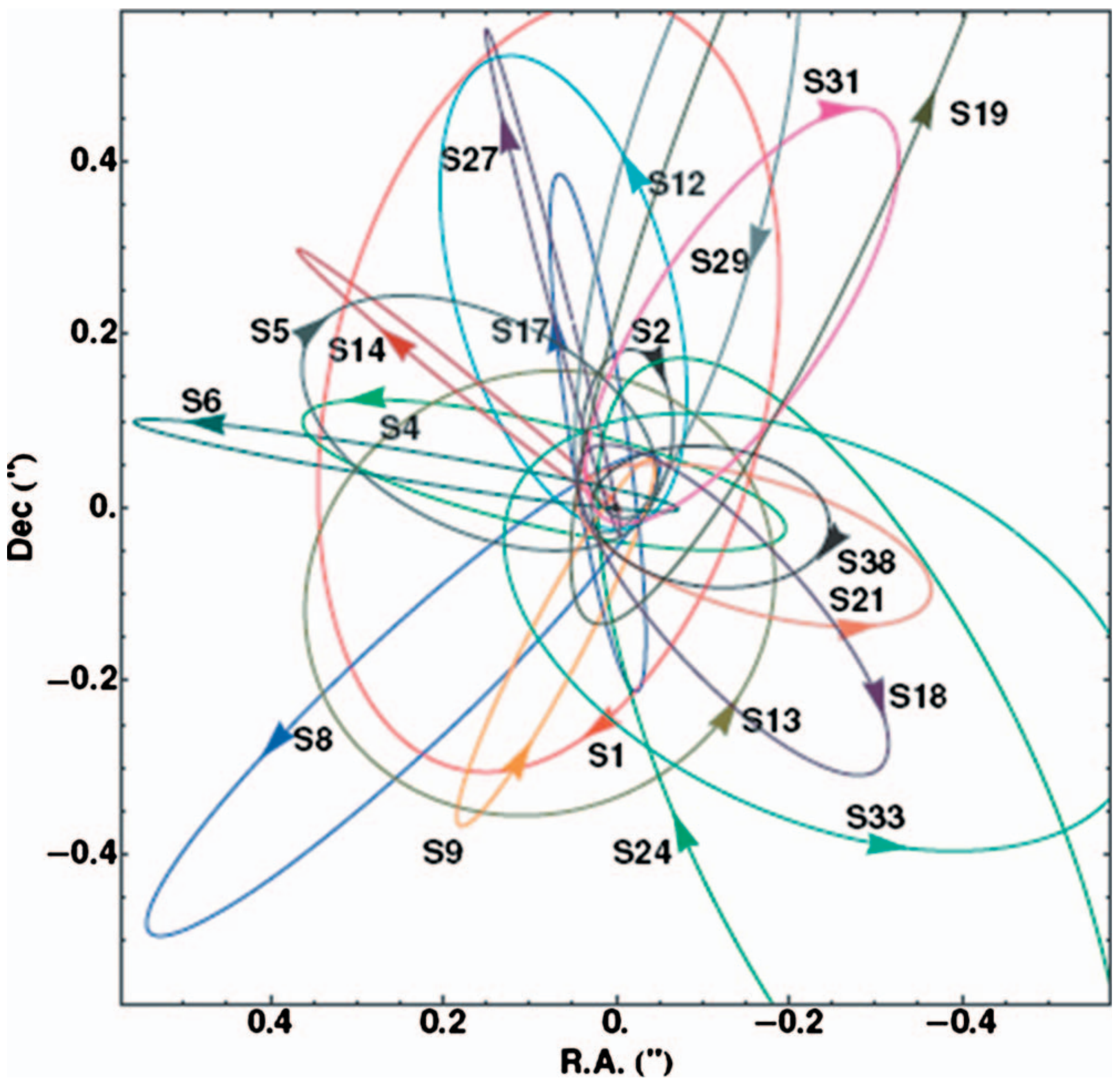


Star proper motions and orbits

Breakthrough from the detection of accelerations (i.e. curvature of proper motions), and the determination of stellar orbits.



Proper motions

Gillessen+2009

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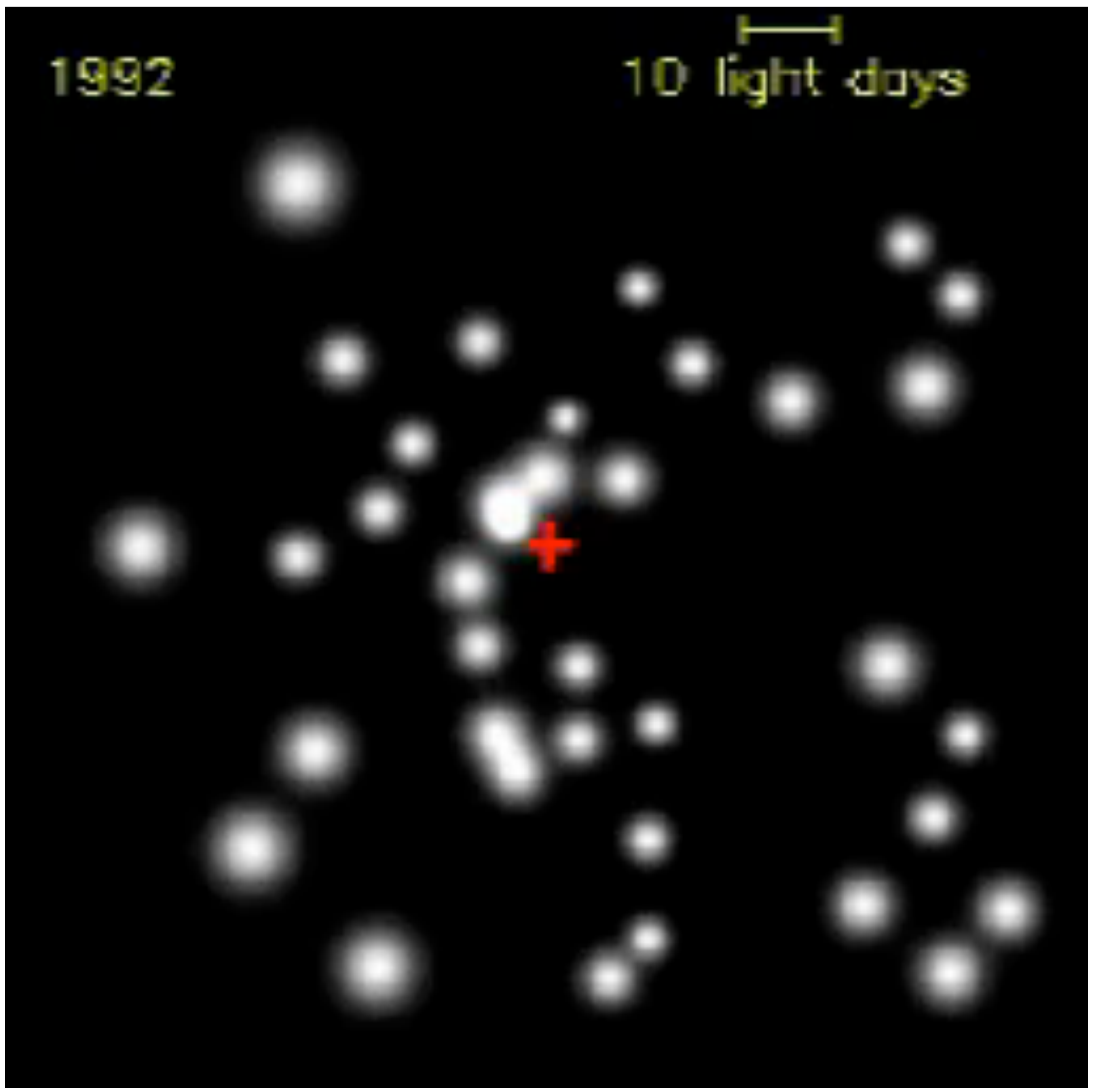

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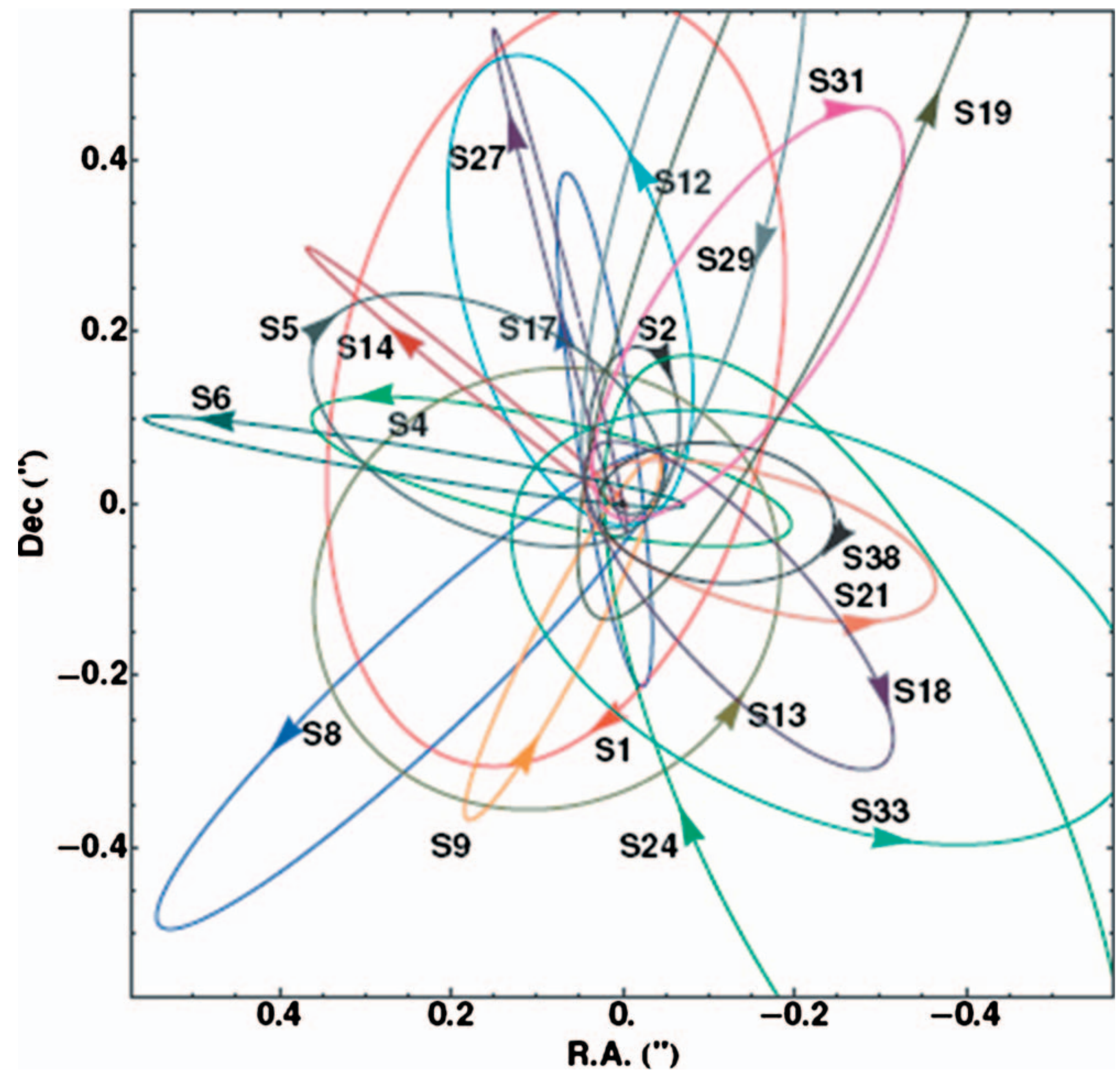


Gillessen+2009

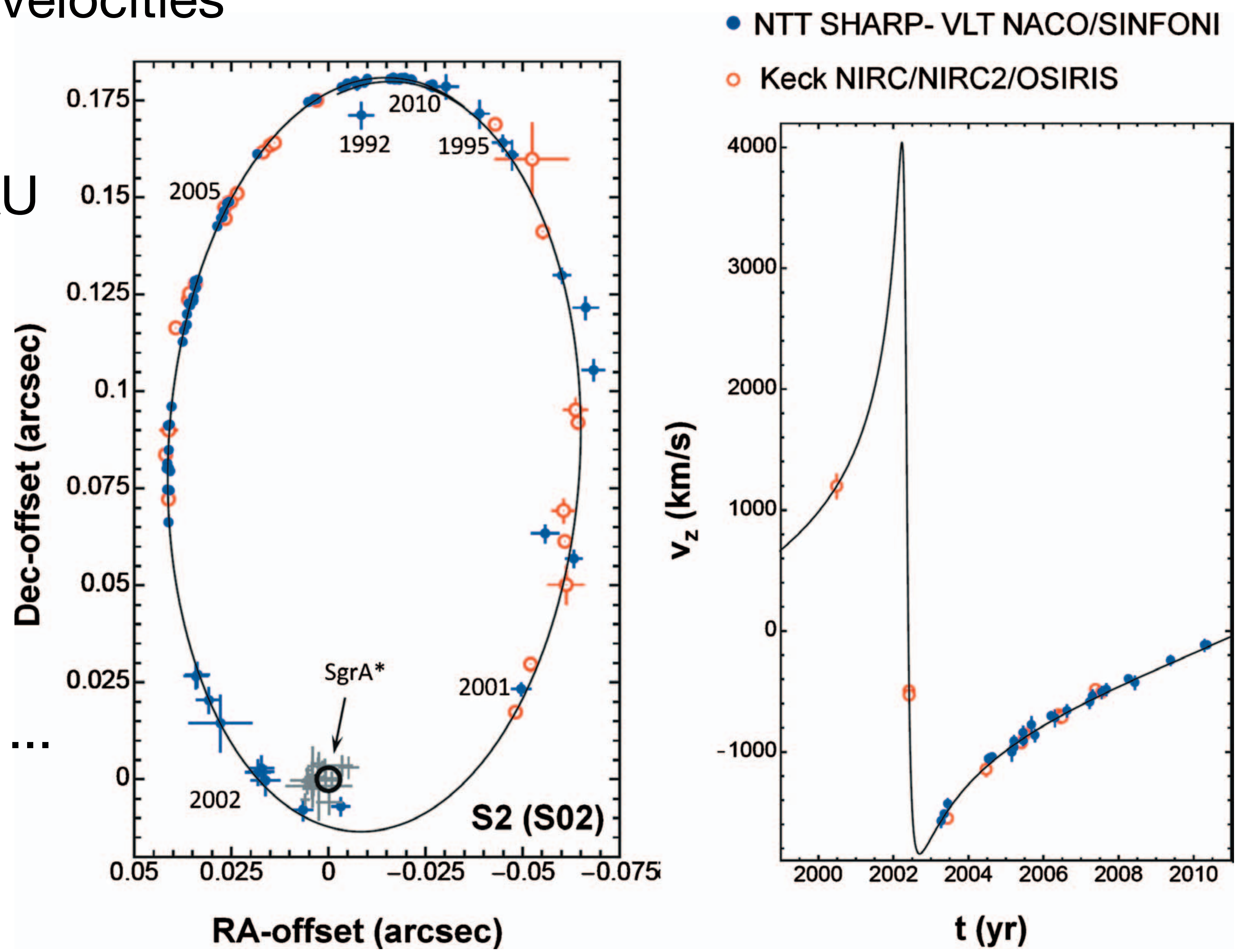
Example: the star S2

Gillessen+2009

By fitting orbit and radial velocities
Period ~15.8 y
Eccentricity ~ 0.87
Semi-major axis ~1025 AU
Pericenter ~ 125 AU

$M_{BH} \sim 4.3\times10^6\ M_\odot$

D ~ 8.3 kpc



Using Kepler's Third Law ...

$$\left(\frac{2\pi}{P}\right)^2 = \frac{GM}{a^3}$$

$$M = \mathrm{M}_\odot \left(\frac{a}{1\ \mathrm{AU}}\right)^3 \left(\frac{P}{1\ \mathrm{yr}}\right)^{-2}$$

$$M = 4.3\times10^6\ \mathrm{M}_\odot \left(\frac{a}{1025\ \mathrm{AU}}\right)^3 \left(\frac{P}{15.8\ \mathrm{yr}}\right)^{-2}$$

Orbit is not closed for possible small proper motion of BH. Gray crosses are positions of SgrA IR flares*

The case for a BH in Sgr A*

Overall from S-star orbits (Review by Genzel+2010):

- $M_{BH} = (4.3 \pm 0.2 \pm 0.3) \times 10^6\ M_\odot$
- $D = (\ 8.28 \pm 0.15 \pm 0.29\ )$ kpc
  - Mass scales roughly as D^2
    (from astrometry $\sim D^3$ and radial velocities $\sim D$)
  - One of the biggest sources of uncertainty come from possible motion of BH: line of sight velocity of BH degenerate with mass and distance
  - M_{BH} is concentrated within the pericenter of S2 i.e. $r < 125$ *AU*

Minimum density for M_{BH} of $5 \times 10^{15}\ M_\odot\ pc^{-3}$

The mass centroid lies within +/-2 mas of Sgr A*

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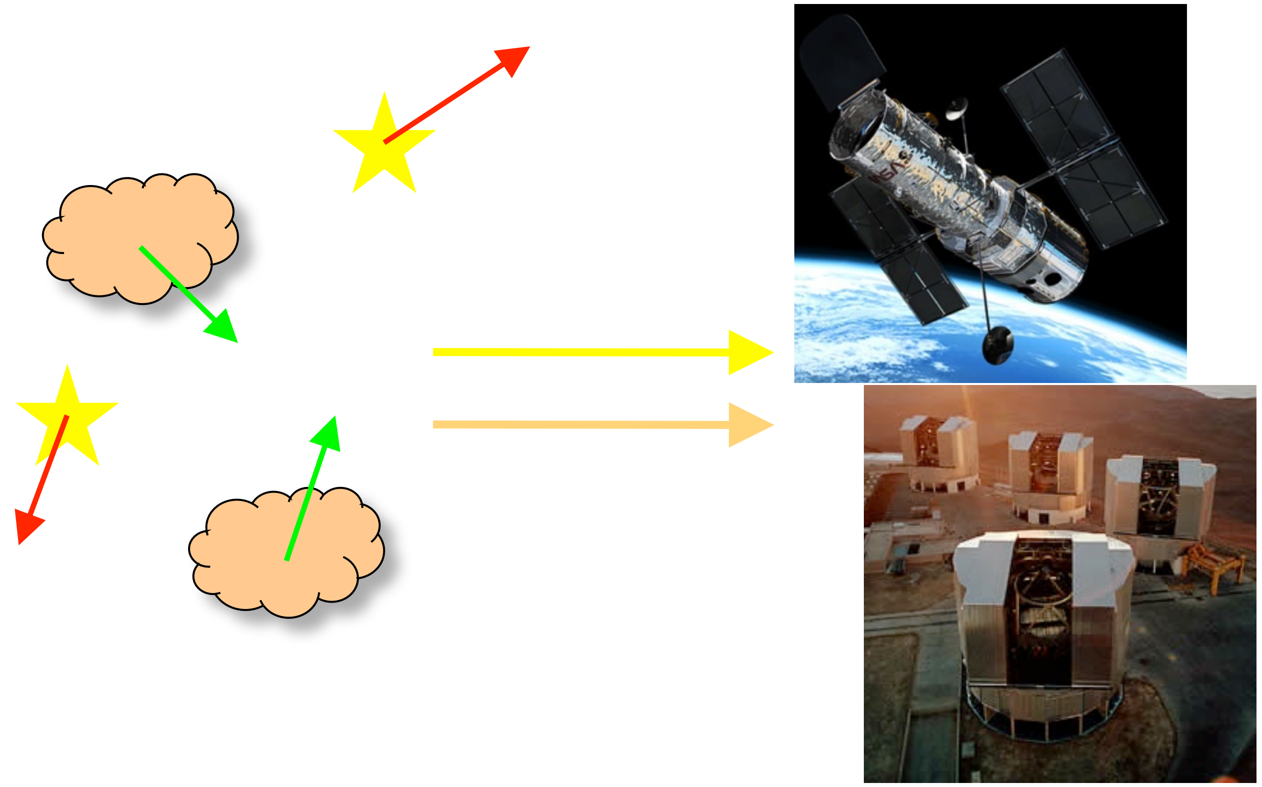
Wonderful result, but it is simple to realize that the proper motions of stars so close to the BH cannot be detected even in the Andromeda galaxy!

Dynamical evidences for BHs

Motions of *test particles*

- Star proper motions and radial velocities
- Radial velocities of single gas clouds (masers)



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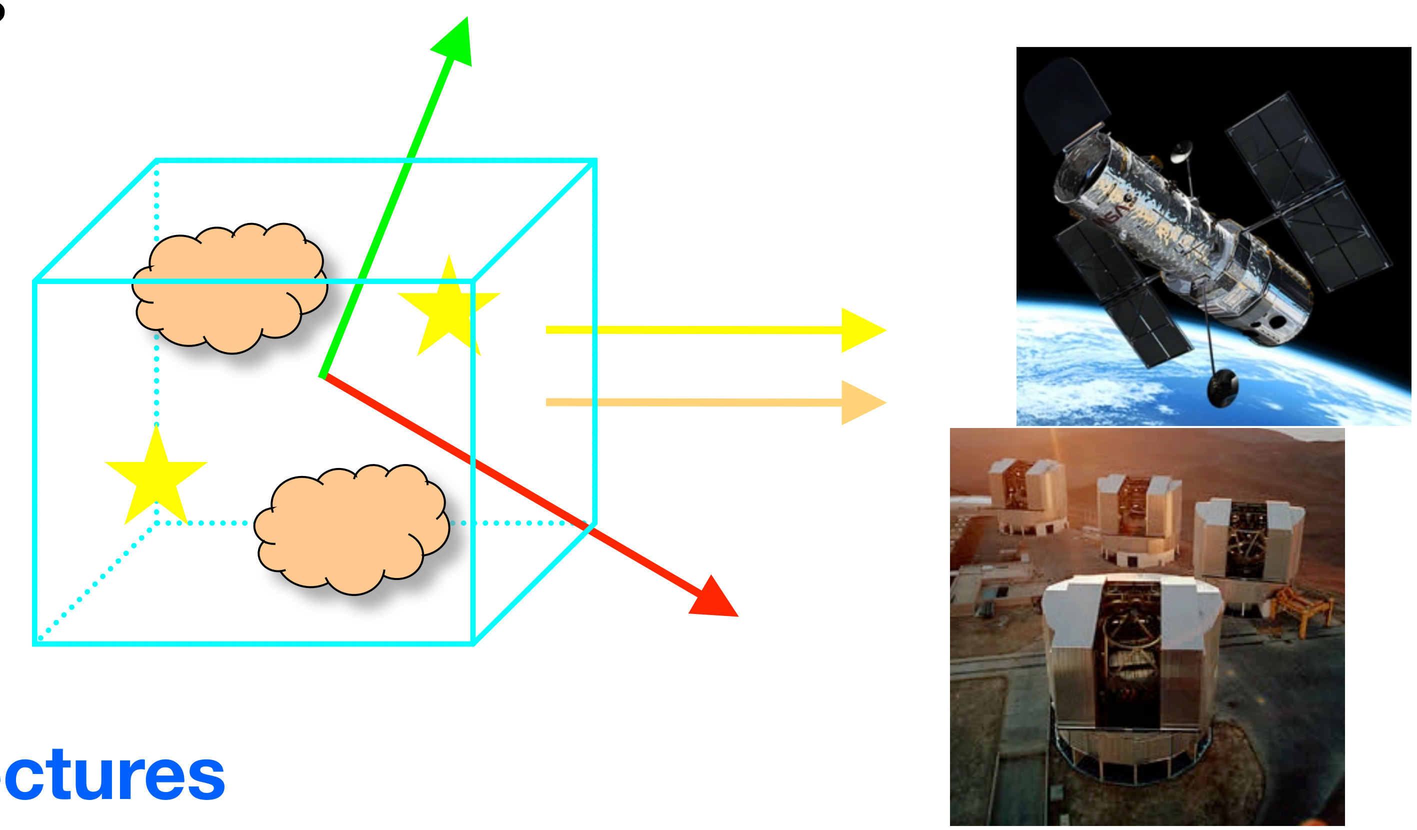

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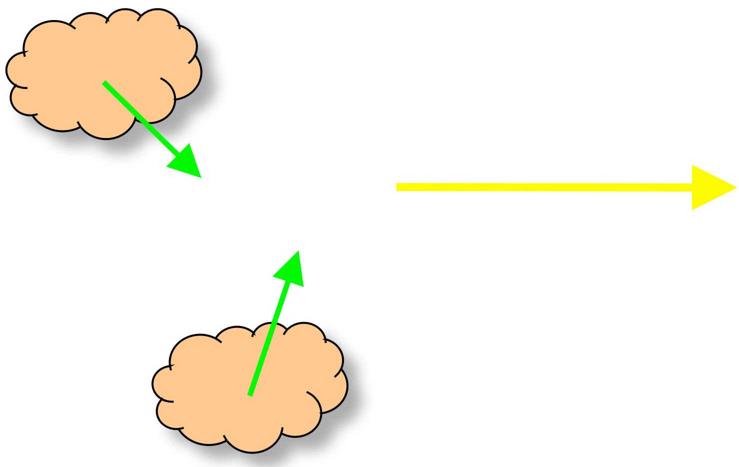
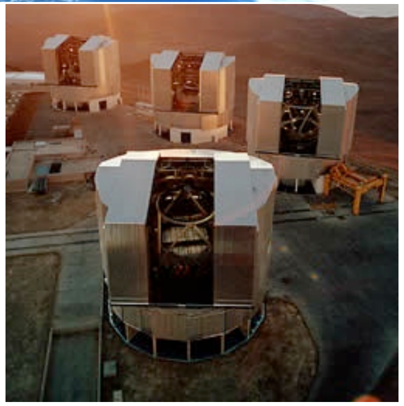




- Radial velocities of single gas clouds (masers)

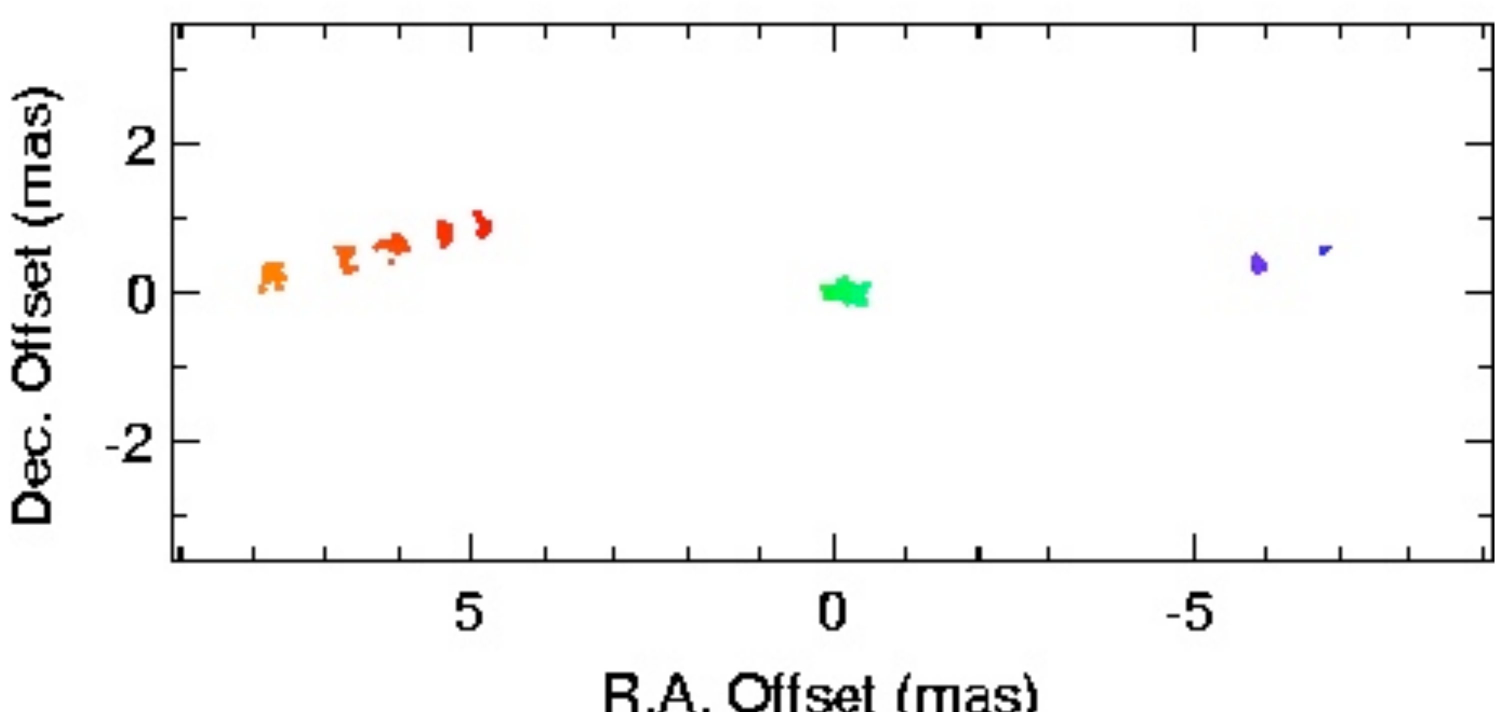
M_{BH} from H_2O megamasers: NGC 4258

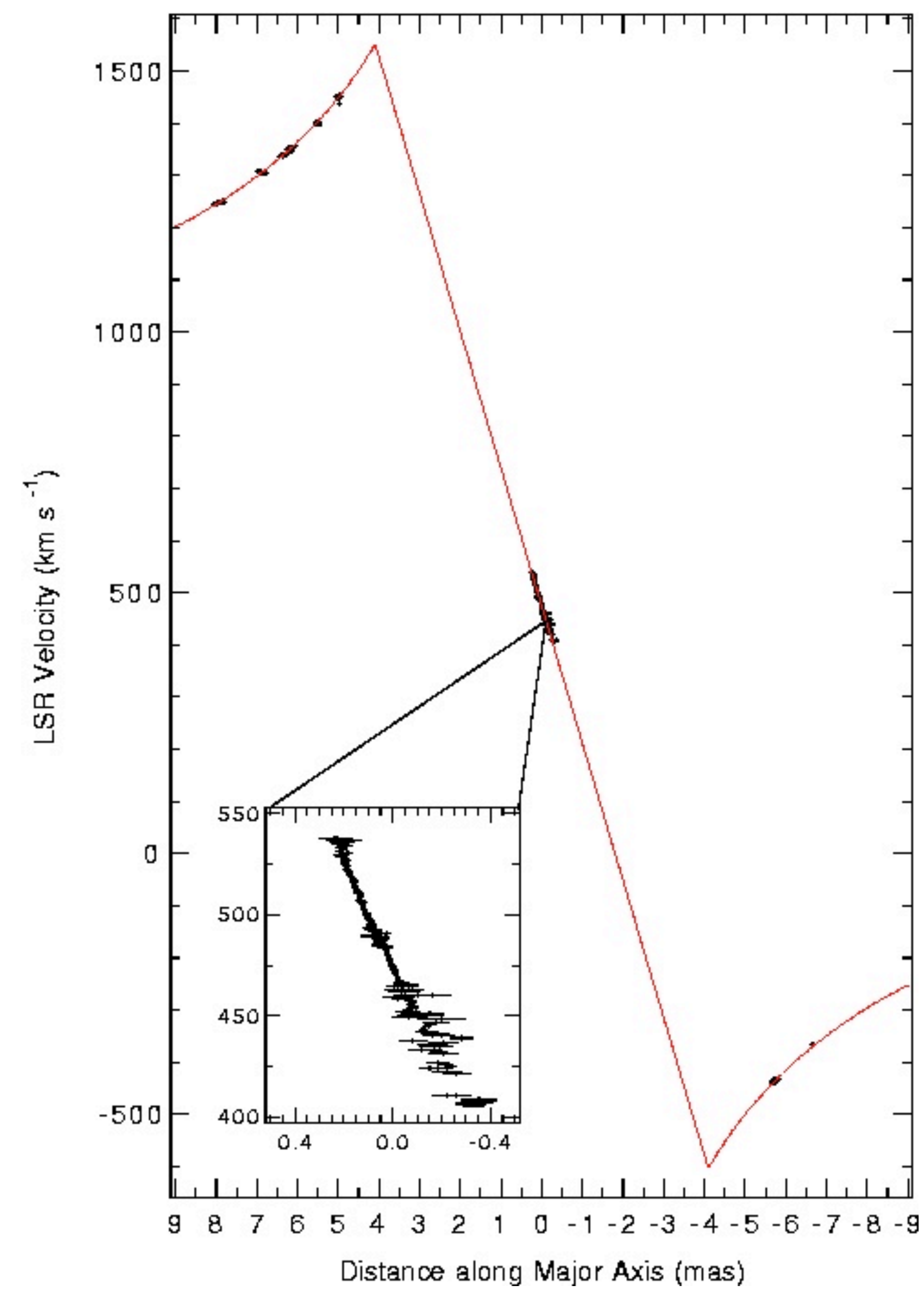
H_2O maser emission (λ=1.35 cm) can be used to trace single gas clouds orbiting massive black holes in galactic nuclei.

Radio interferometers (e.g. VLBA) can reach exceptional spatial and velocity resolution compared to optical and near-IR.

Observations of NGC4258 (D=7.2Mpc; Miyoshi et al. 1995) reached:

- $\Delta\theta$ = 0.6 mas ×0.3 mas
- ΔV = 0.2 km/s





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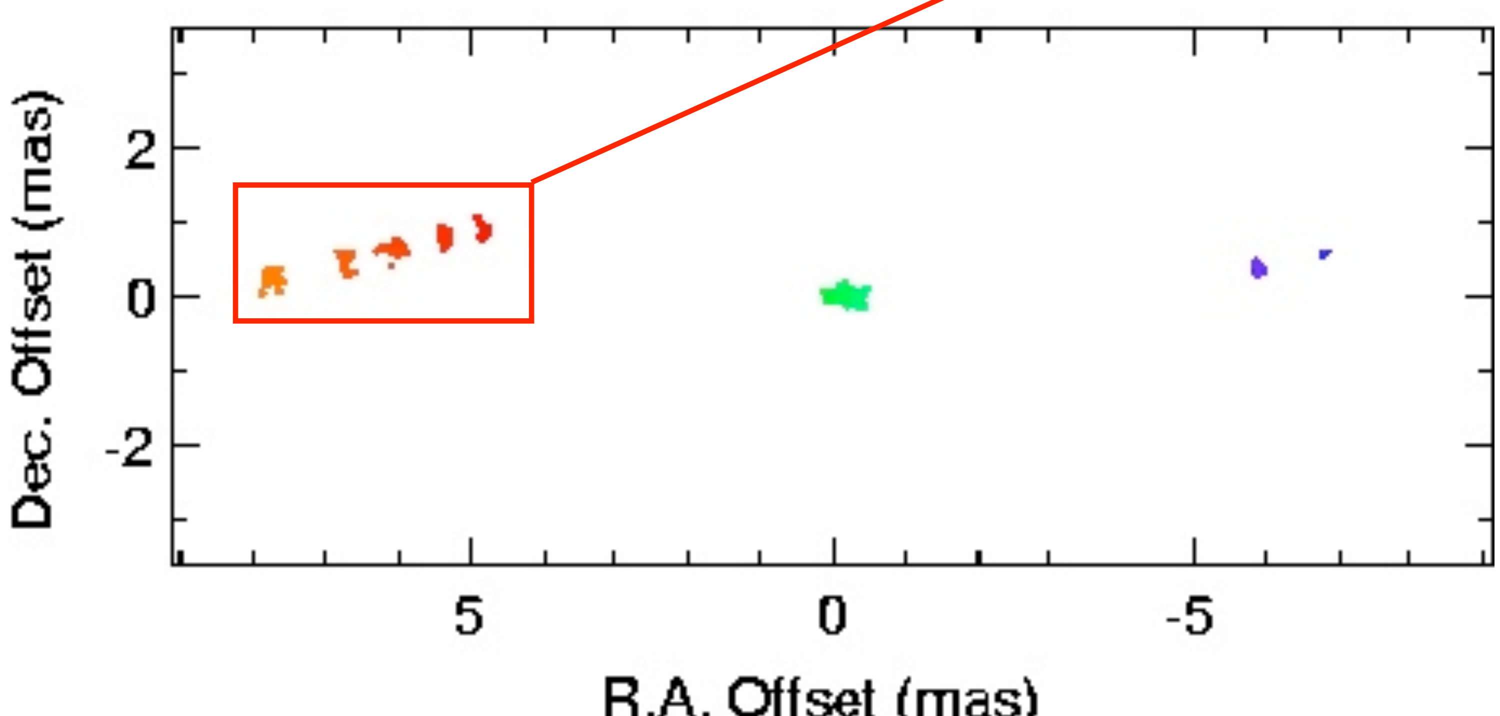

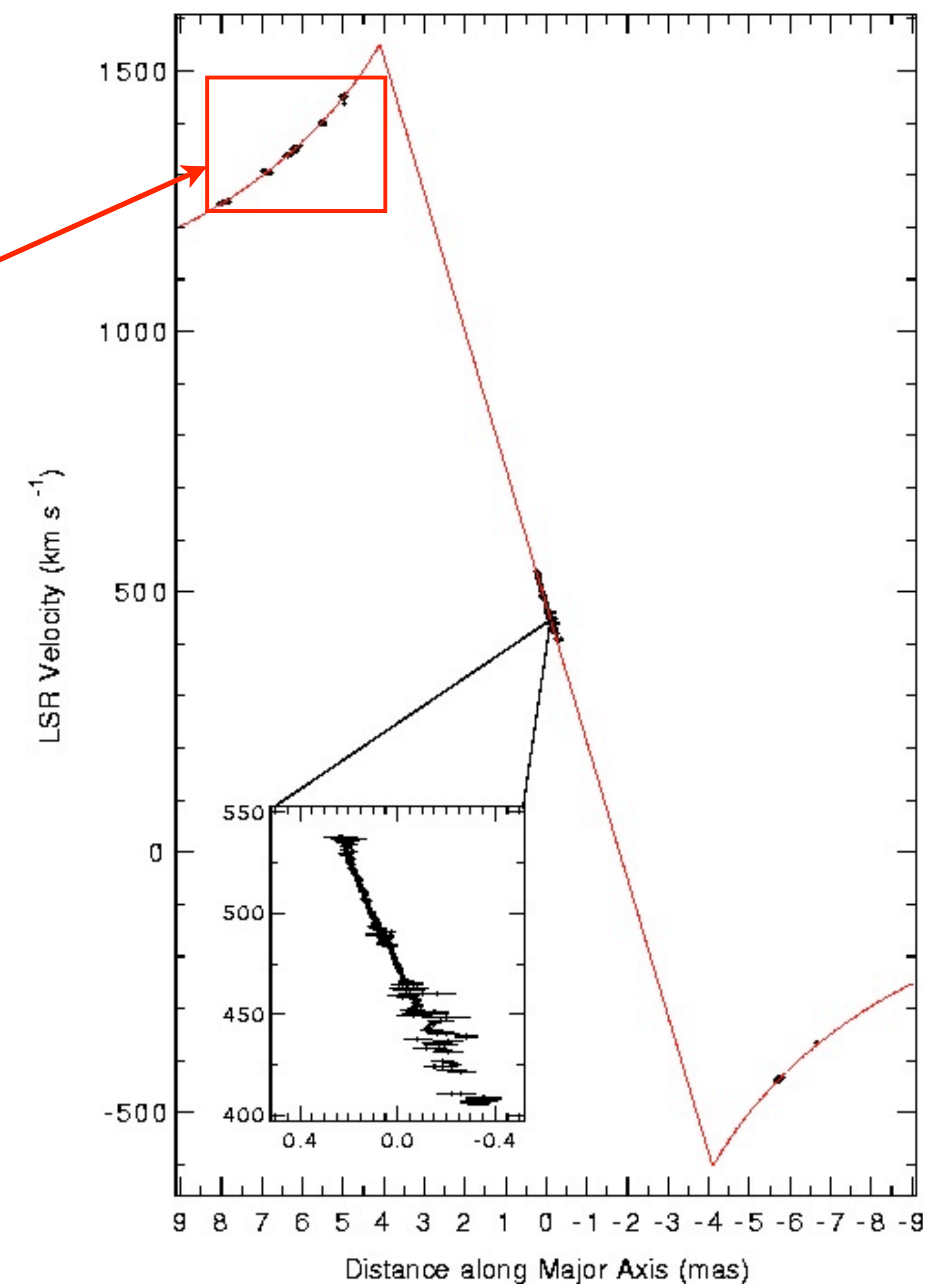

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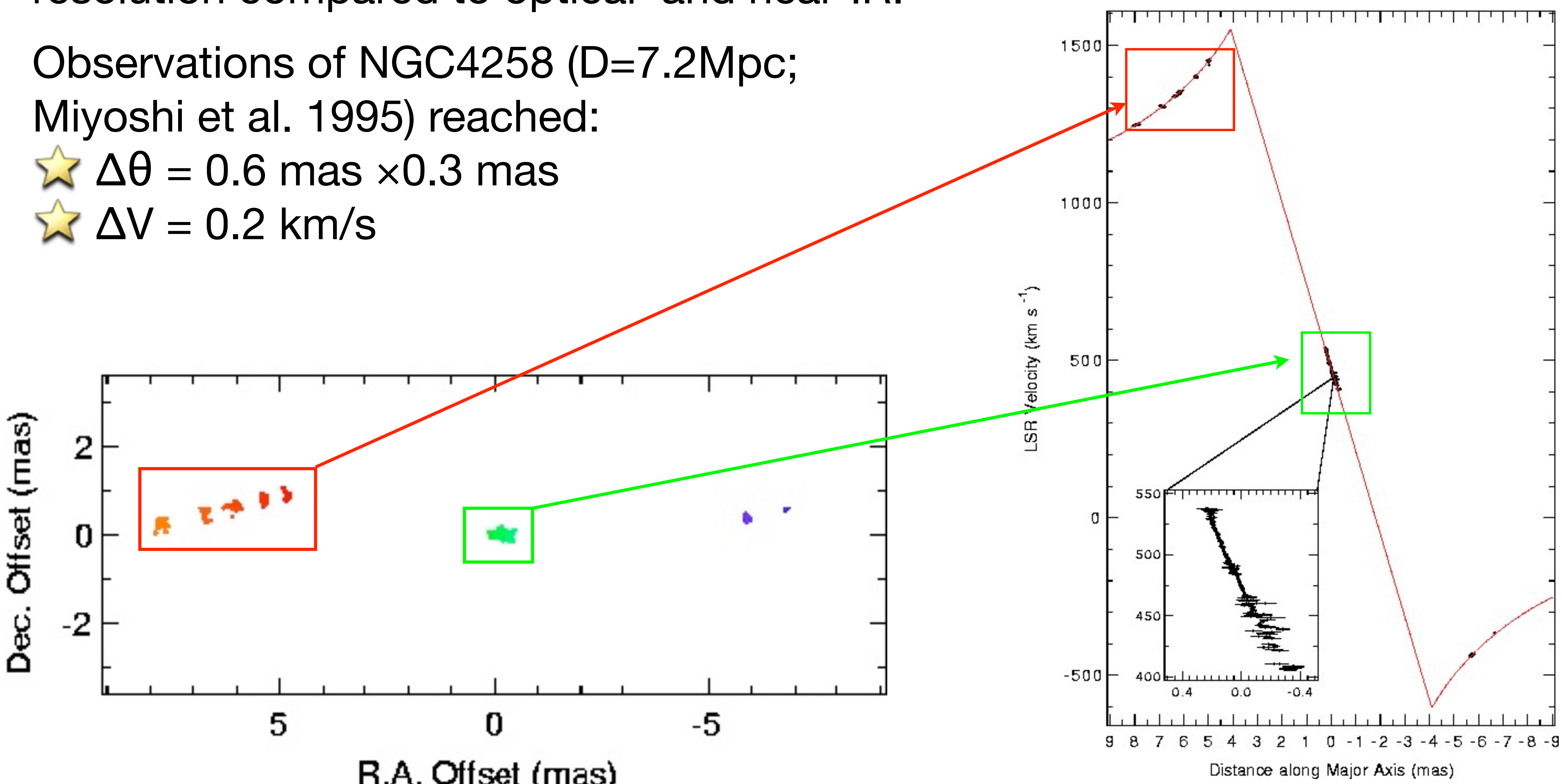

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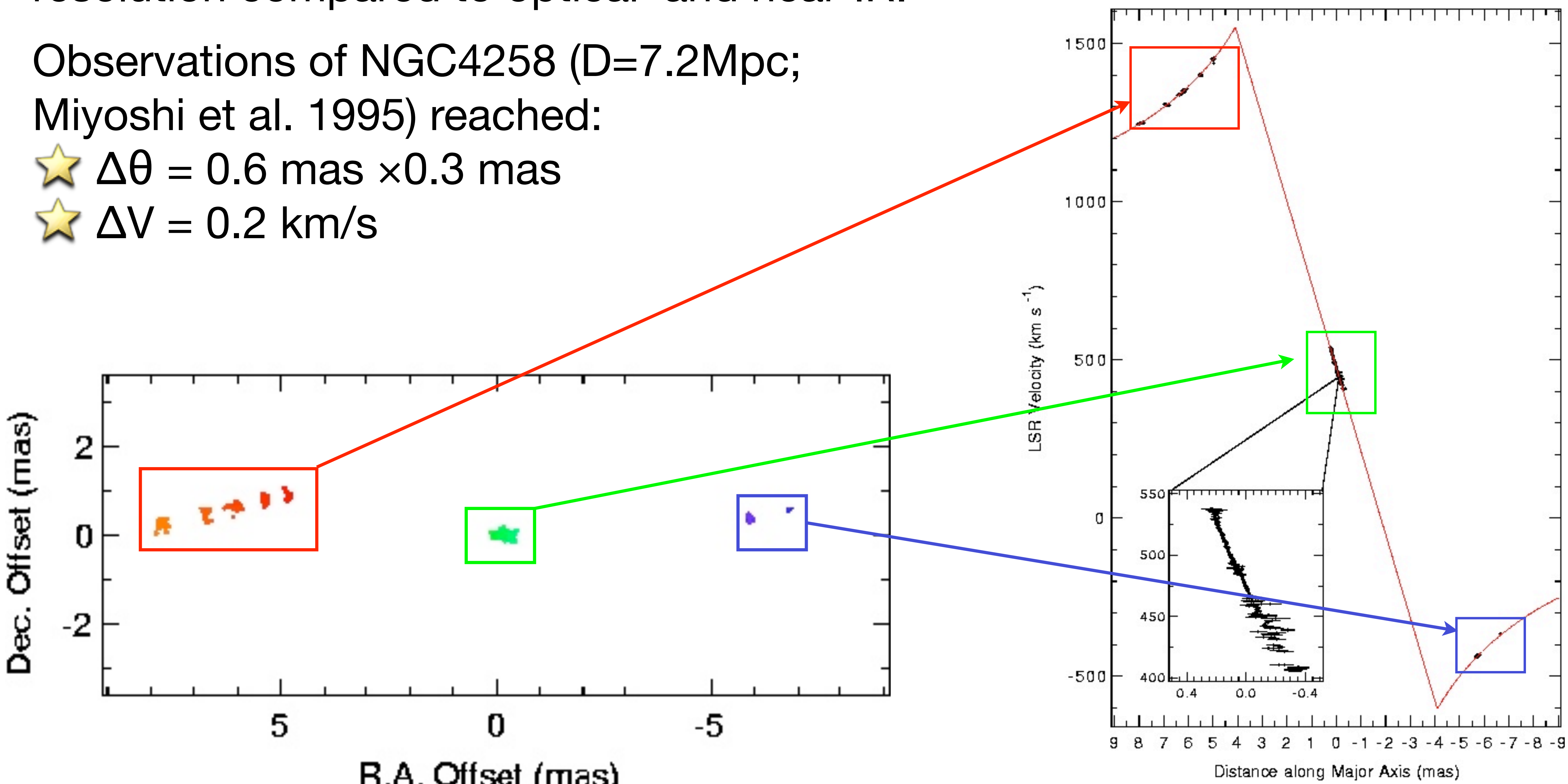
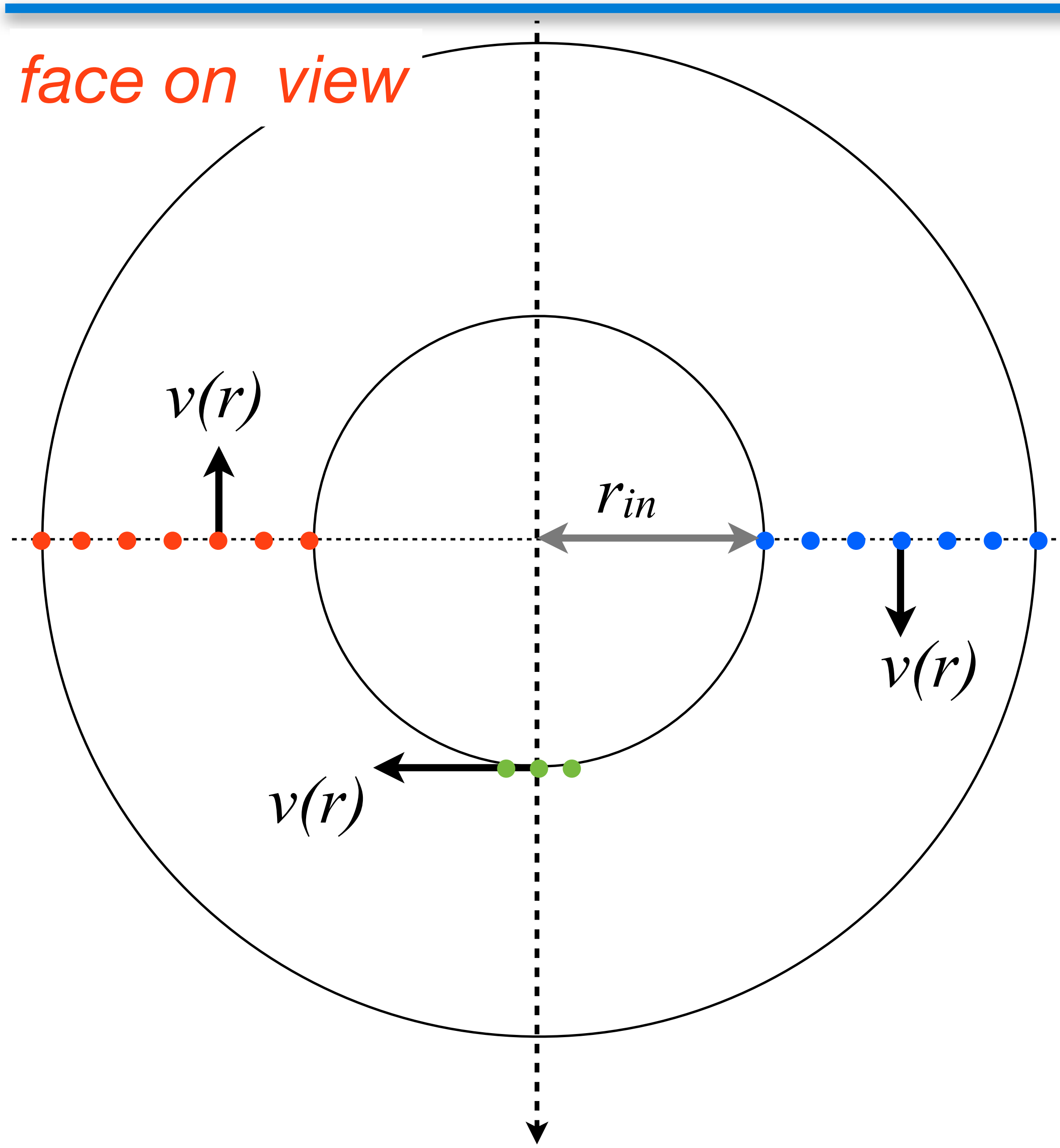


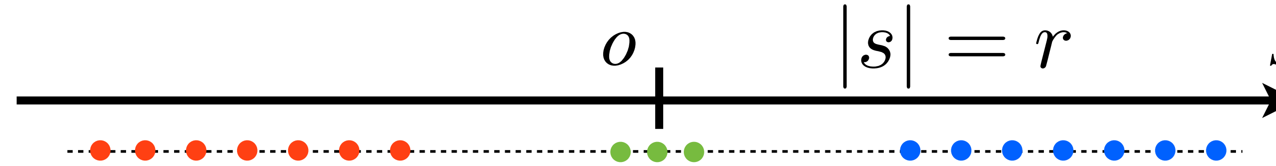
Emission from edge-on disks



edge on view (projection onto the plane of the sky, our own view)

s coordinate along line of nodes

projected vel. along line of sight:



$$v(r) = \left(\frac{GM_{\mathrm{BH}}}{r}\right)^{1/2}$$

$$v_{obs}(s) = \left(\frac{GM_{\mathrm{BH}}}{|s|}\right)^{1/2} \sin i$$

$$v_{obs}(s) = \left(\frac{GM_{\mathrm{BH}}}{r_{in}}\right)^{1/2} \sin i \, \frac{s}{r_{in}}$$

$$v_{obs}(s) = -\left(\frac{GM_{\mathrm{BH}}}{|s|}\right)^{1/2} \sin i$$

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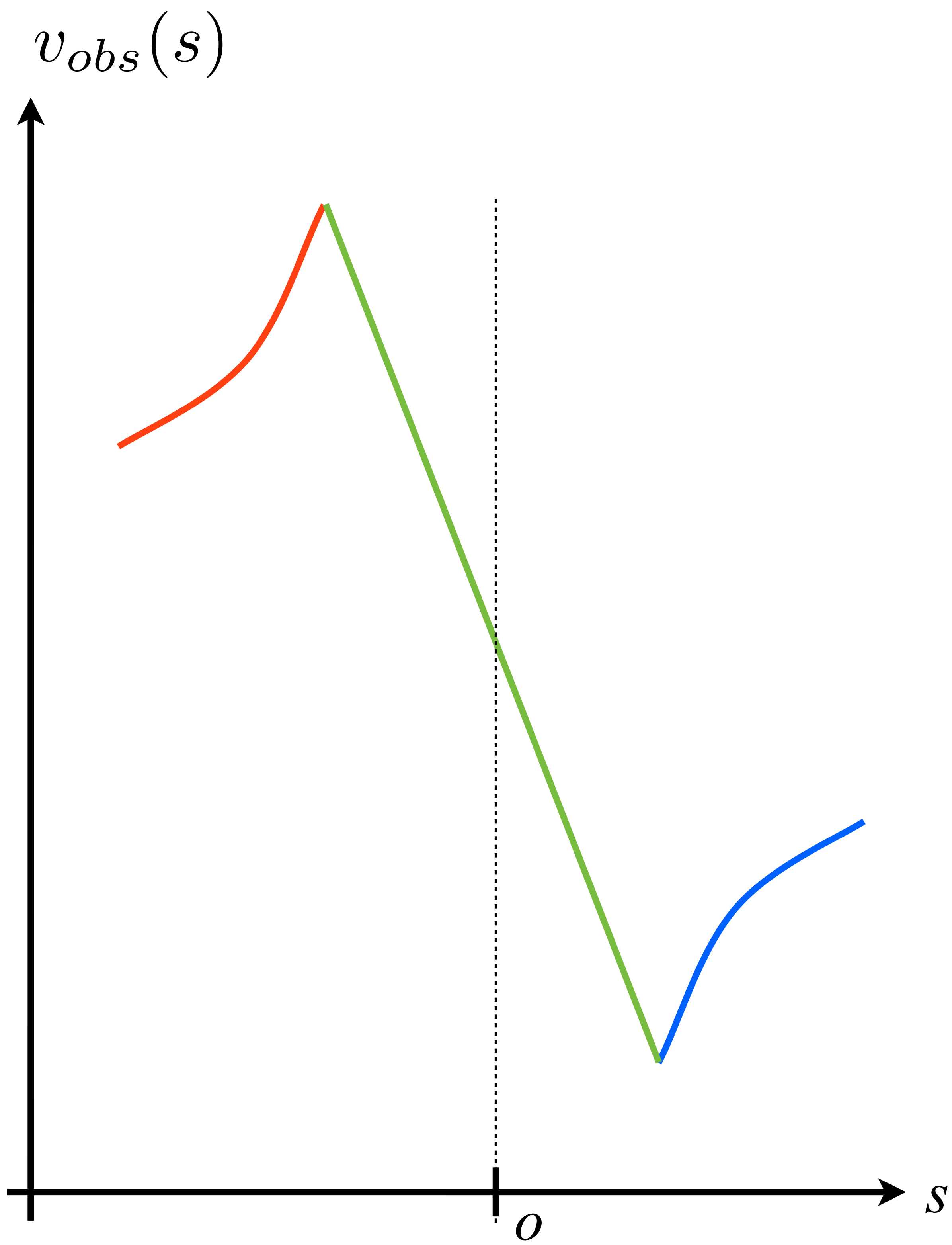

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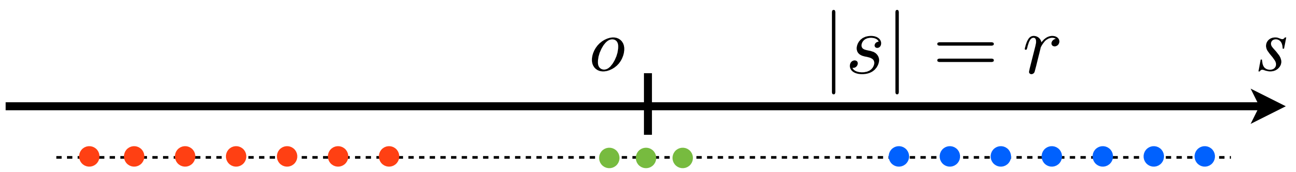
s coordinate along line of nodes

projected vel. along line of sight:



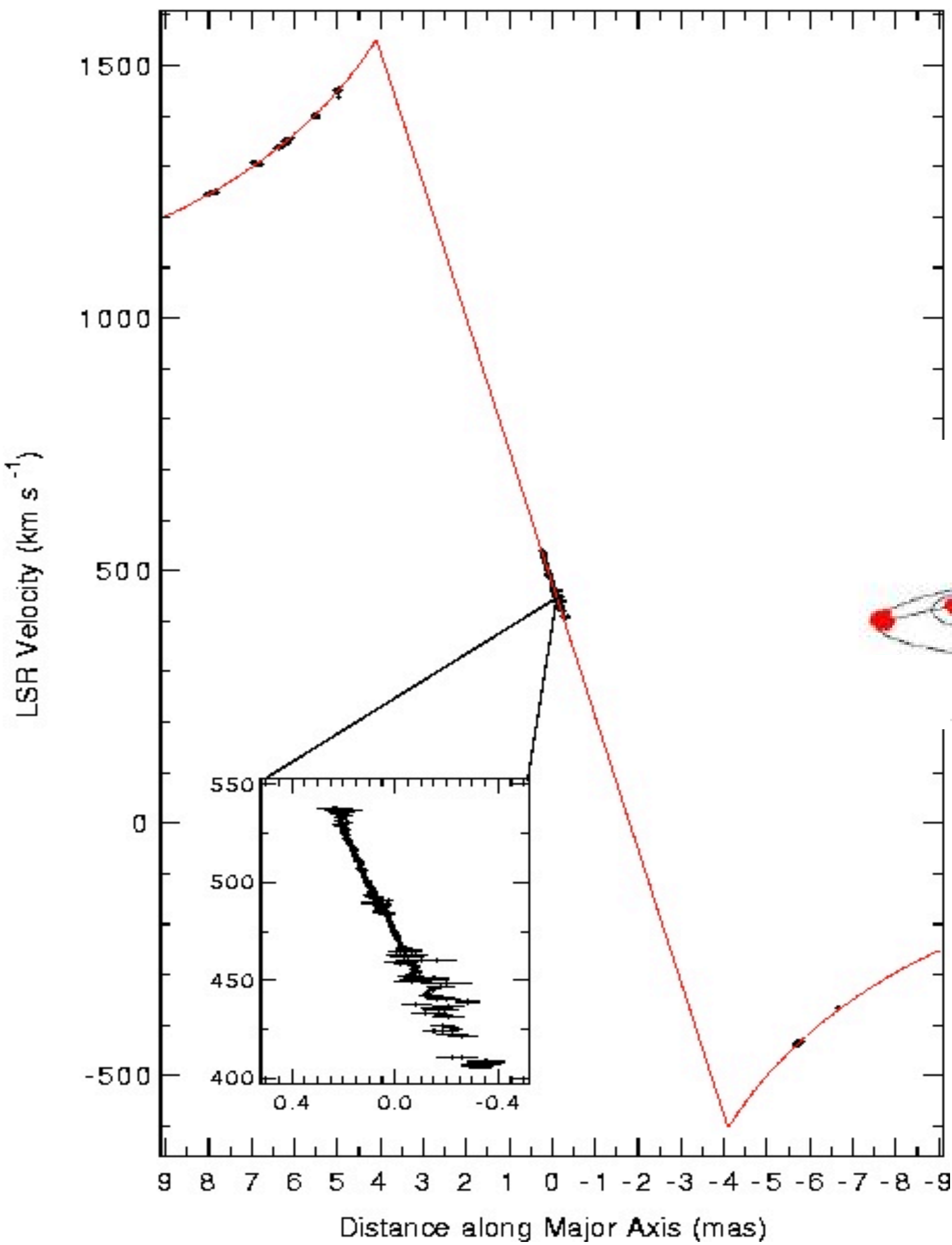
$$v(r) = \left(\frac{GM_{\mathrm{BH}}}{r}\right)^{1/2}$$

$$v_{obs}(s) = \left(\frac{GM_{\mathrm{BH}}}{|s|}\right)^{1/2} \sin i$$

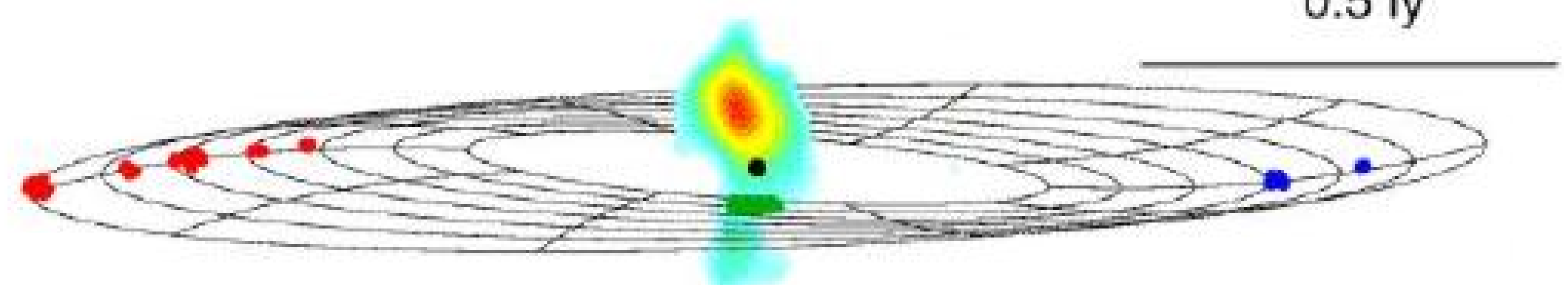
$$v_{obs}(s) = \left(\frac{GM_{\mathrm{BH}}}{r_{in}}\right)^{1/2} \sin i \, \frac{s}{r_{in}}$$

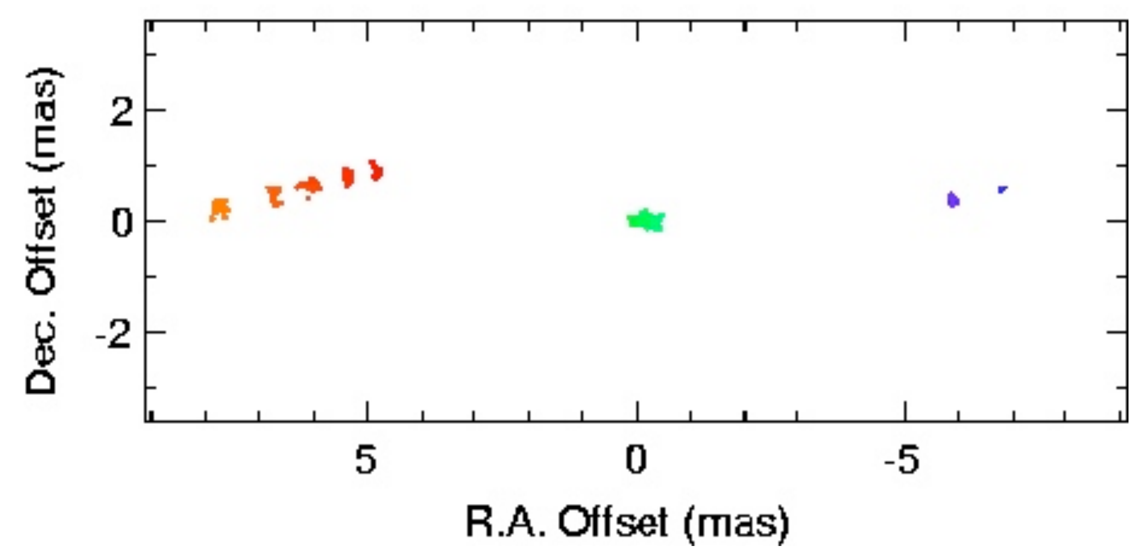
$$v_{obs}(s) = -\left(\frac{GM_{\mathrm{BH}}}{|s|}\right)^{1/2} \sin i$$

M_{BH} from H_2O megamasers: NGC 4258



- $M_{BH} = (3.9 \pm 0.1) \times 10^7\ M_\odot$ (Miyoshi+1995, Herrnstein+1999)
- Using rotation curves, it is also possible to measure centripetal acceleration of maser spots and derive galaxy distance (Herrnstein+1999)





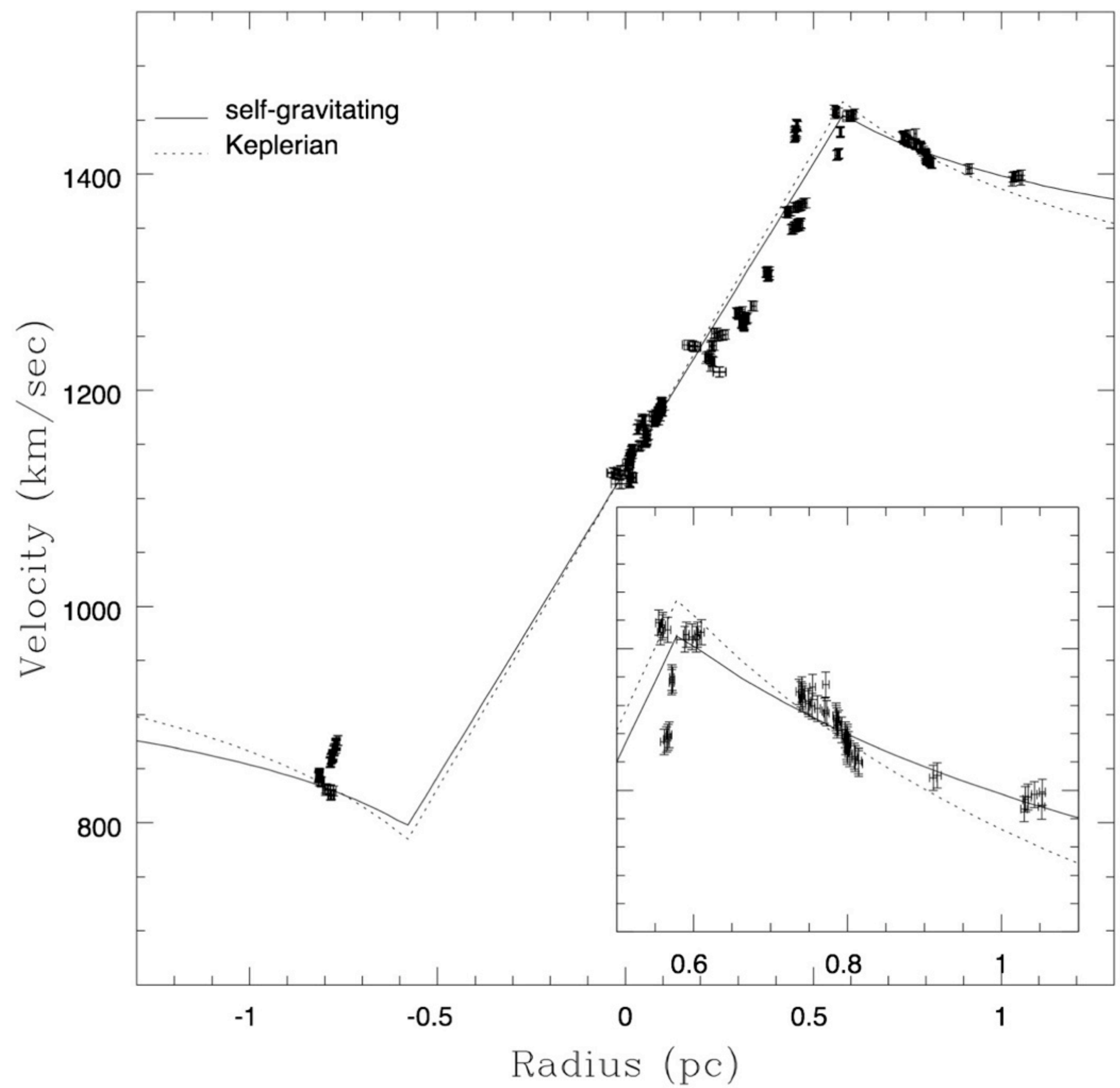
- Disk is also slightly warped

M_{BH} from H_2O megamasers: NGC 1068

M_{BH}~10^7 $M_\odot$ (Greenhill et al. 1996)

... but rotation flatter than Keplerian!

Self-gravitating disk model by Lodato & Bertin (2003) gives M_{BH}=(8.0±0.3)×10^6 $M_\odot$



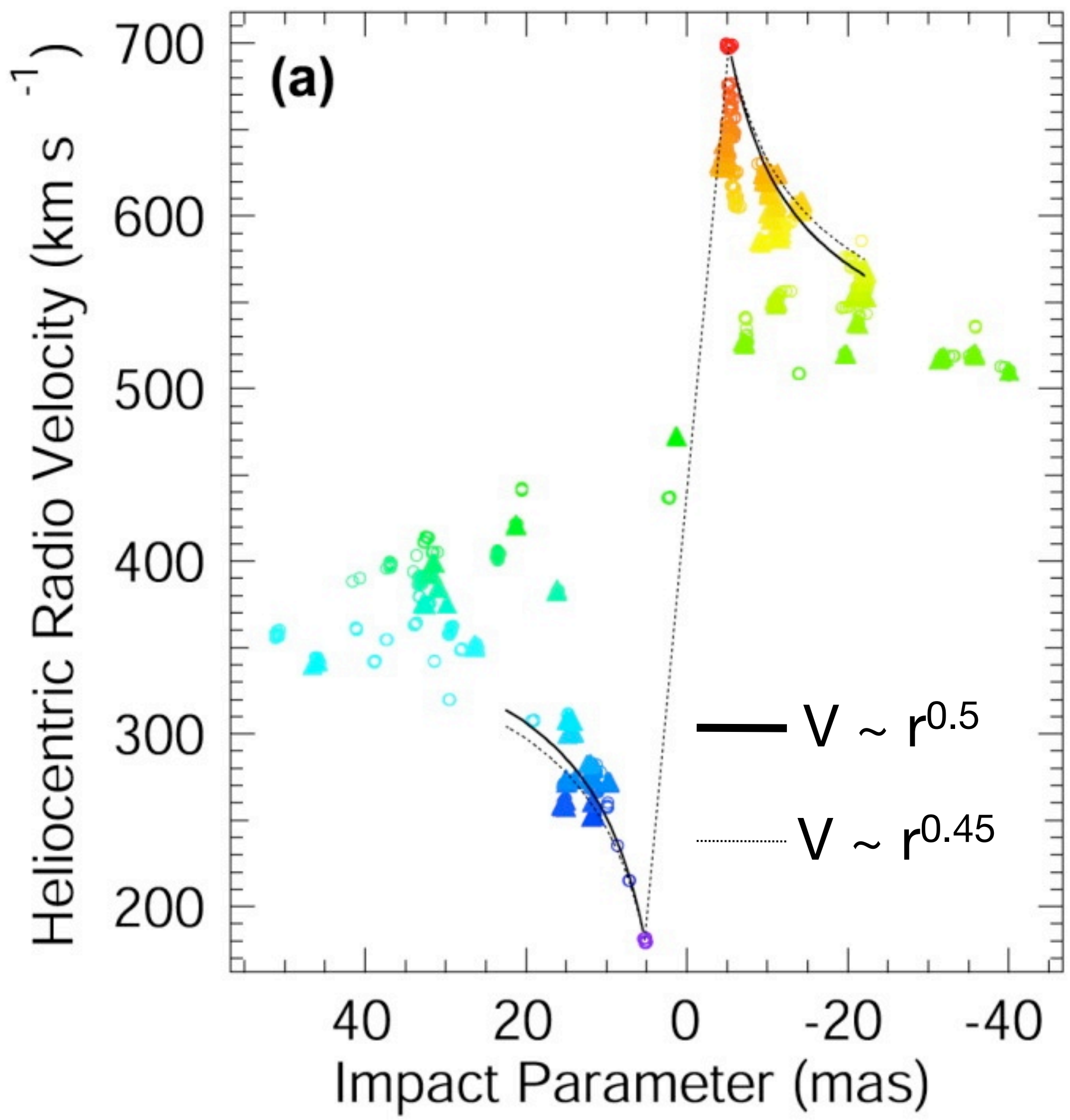
M_{BH} from H_2O megamasers: Circinus

$M_{BH}=(1.7\pm0.3)\times10^6\ M_\odot$
(Greenhill et al. 2001)

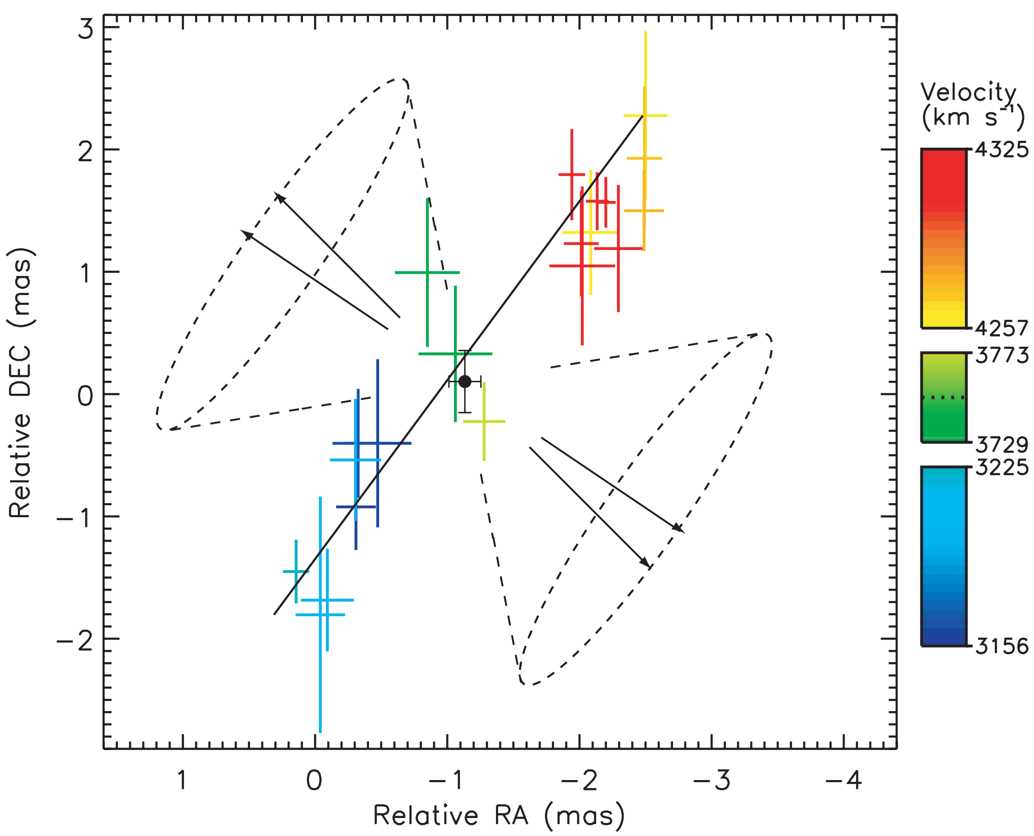
Edge-on disk extends from
0.1 to 0.4 pc.

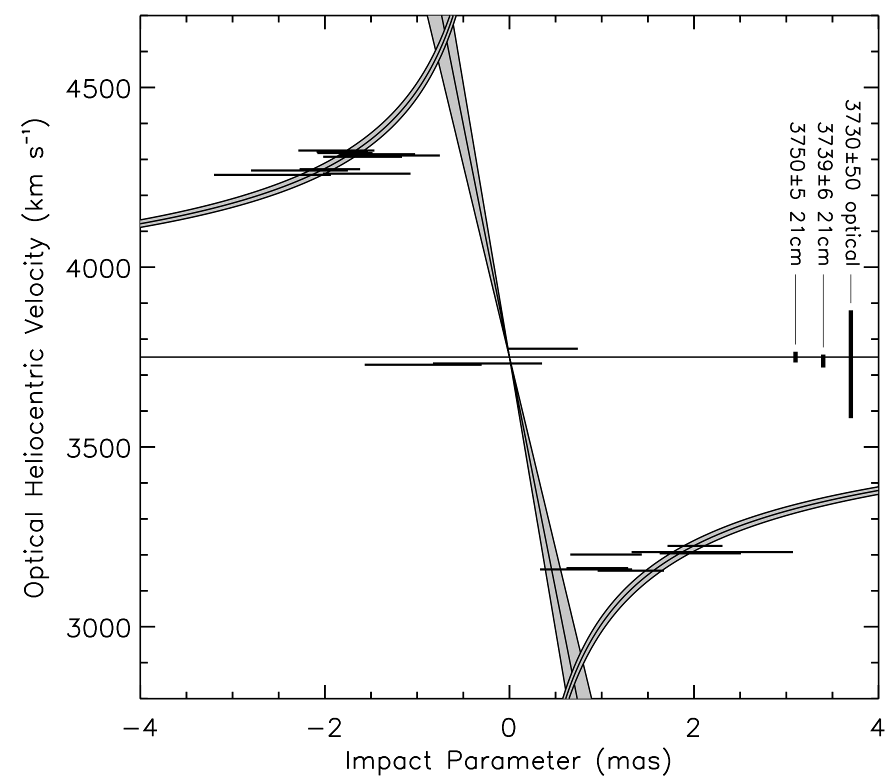
The rotation curve is nearly Keplerian (massive disk is probably massive and self-gravity is not negligible).

A second population of masers traces a wide angle outflow up to 1pc from the central engine.

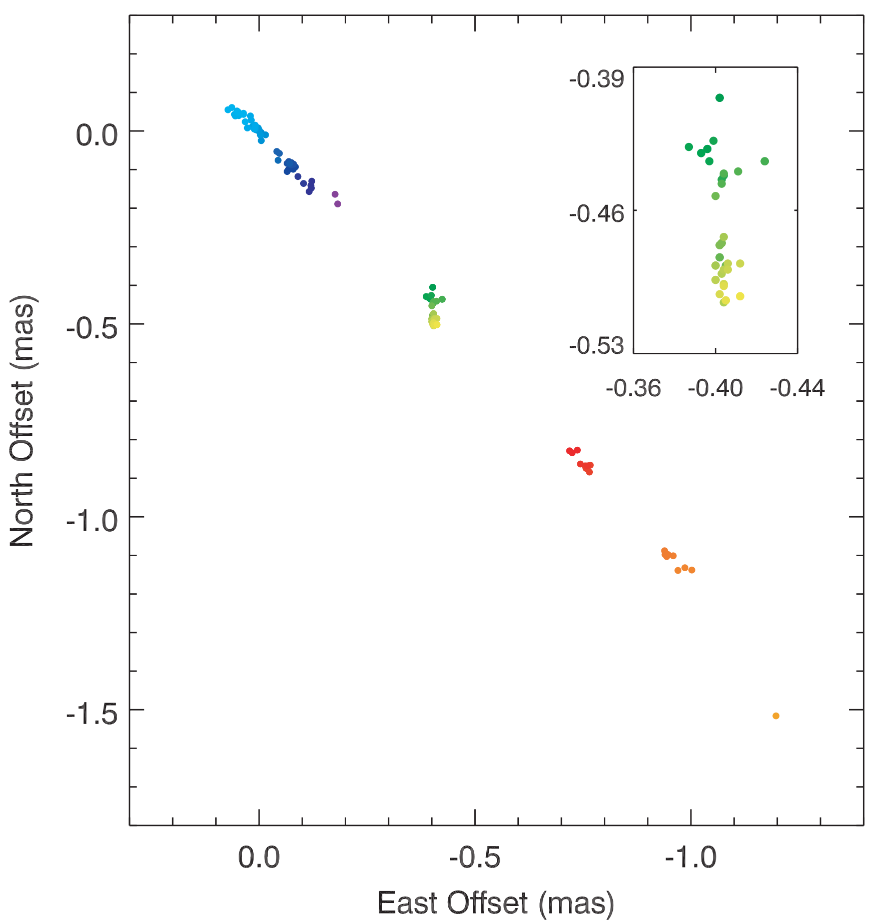


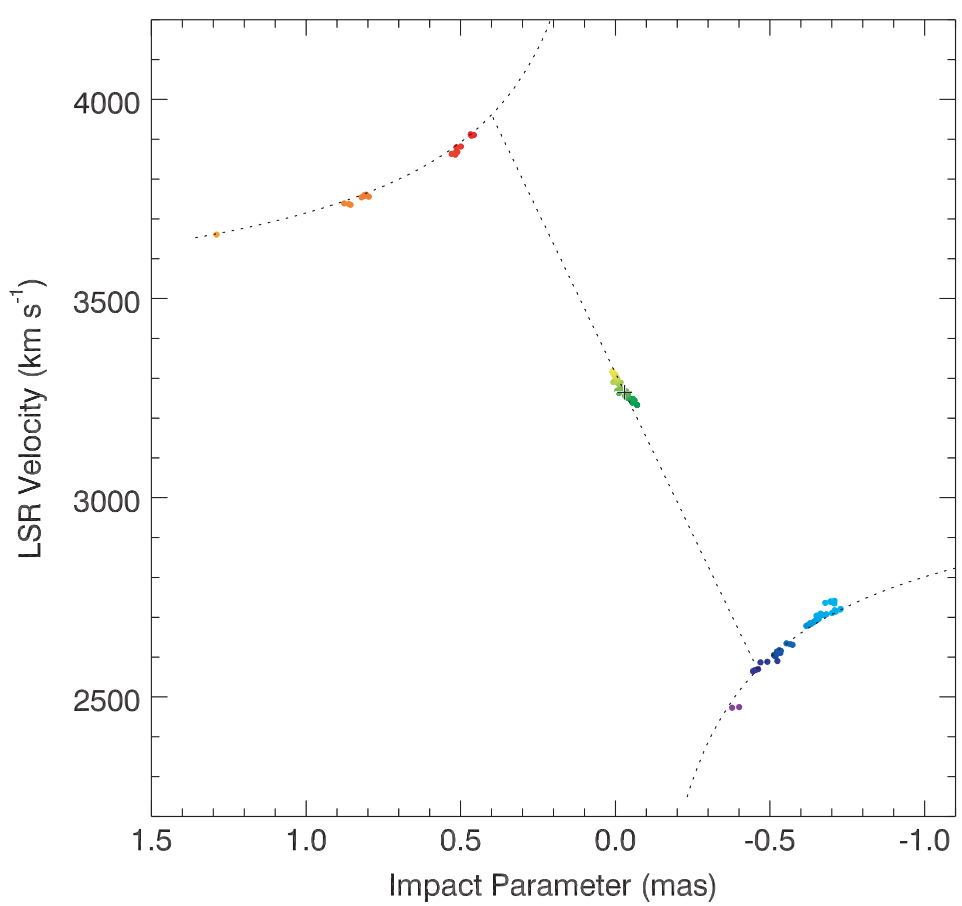
M_{BH} from H_2O megamasers: other examples





NGC 3393:
M_{BH}= 3.1 $\times 10^7$ $M_\odot$
(Kondrakto+2008)

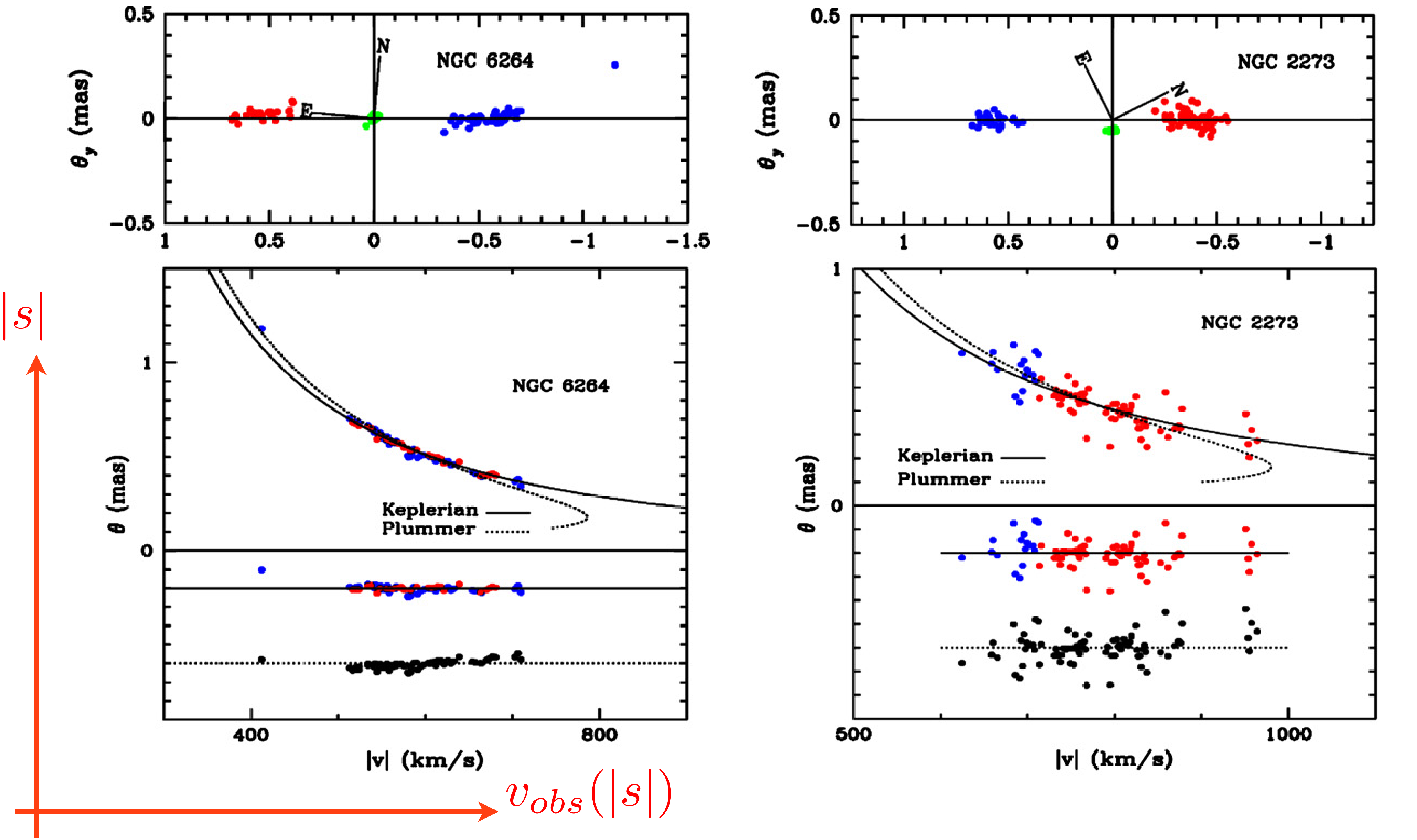




UGC 3789:
M_{BH}=1.1 $\times 10^7$ $M_\odot$
(Reid+2009)

M_{BH} from H_2O megamasers: other examples

Six estimates in Kuo et al. 2011 (NGC 1194, NGC 2273, Mrk 1419, NGC 4388, NGC 6264 and NGC 6323)



Plummer = extended mass distribution following Plummer potential

Summary on M_{BH} from megamasers

Pros [see *Kuo+2011*]

- angular (spatial) resolution with VLBI is ~2 order of magnitudes better than optical/NIR observations (see later);
- megamasers are observed in simple geometrical configurations (edge one disks) with easy and straightforward modeling;
- megamaser disks smaller than gravitational sphere of influence of BH, little influence from stellar mass;

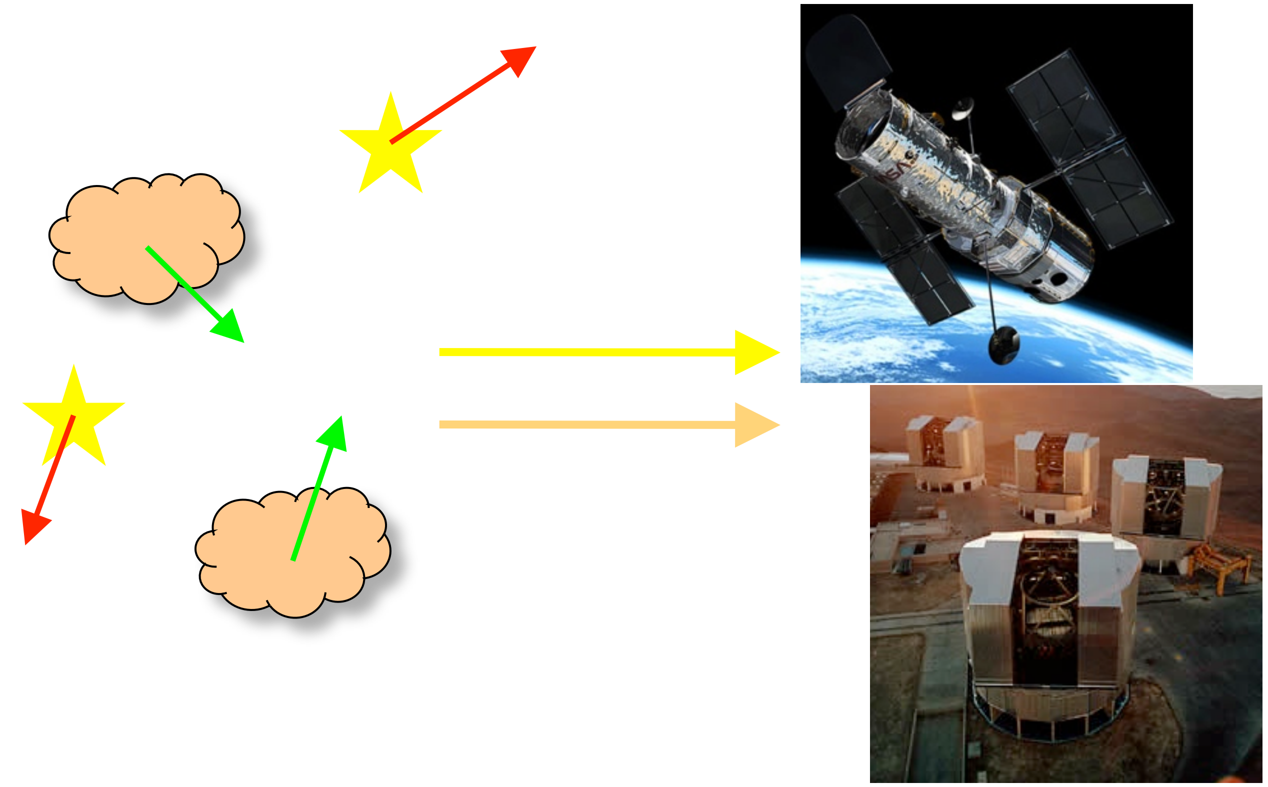
Cons

- maser emission is beamed, large column densities required for strong maser amplification;
- megamaser disks observable only if disk close to edge-on (few objects);
- not all megamasers have a clean Keplerian rotation curve ($v \sim r^{-0.5}$); in some cases there is no ordered motions, same cases are affected by clear outflows, some cases have curves flatter than Keplerian (self gravitating disks? radiation pressure?)
- ~136 megamasers detected so far, only 14 BH mass measurements.

Dynamical evidences for BHs

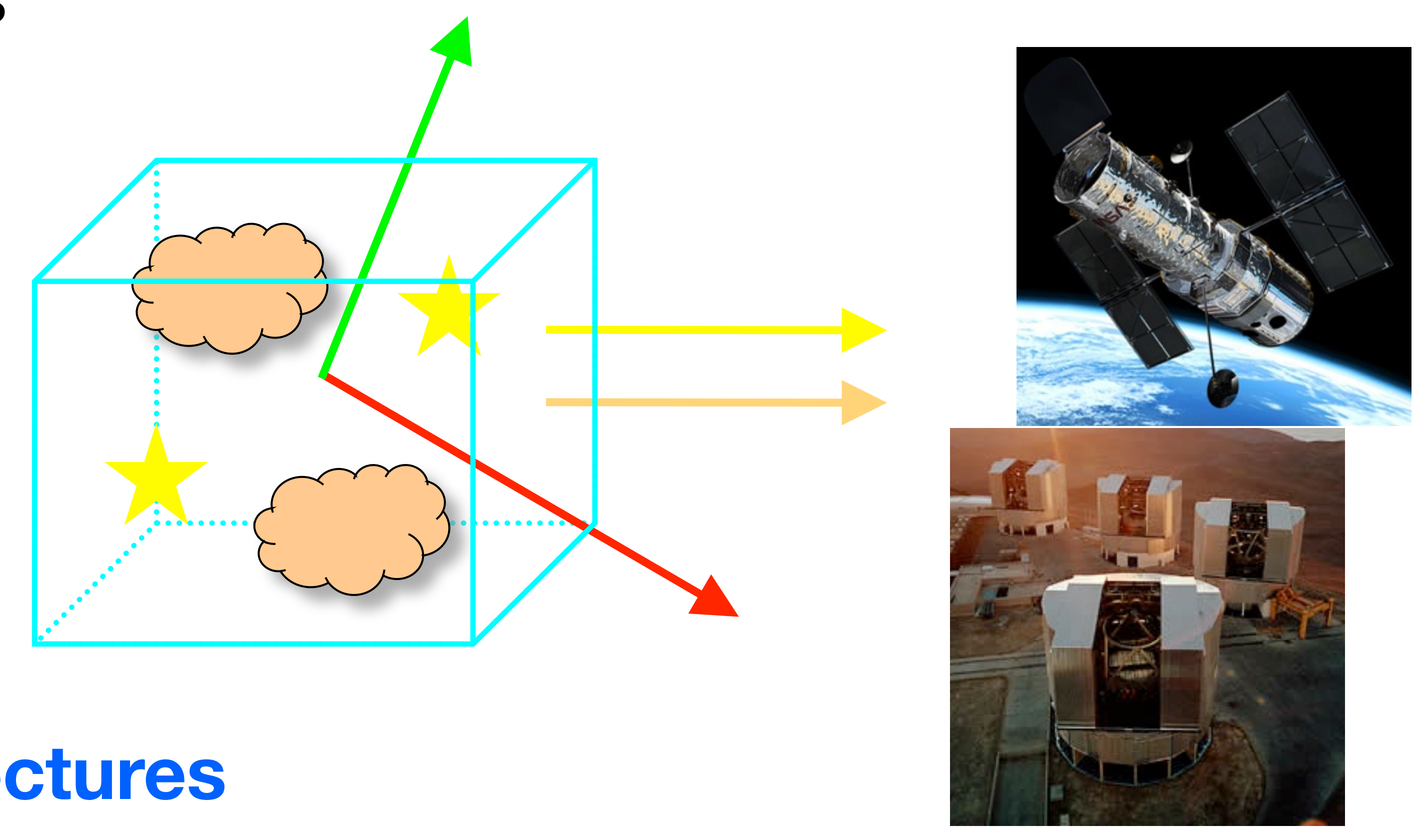
Motions of *test particles*

- Star proper motions and radial velocities
- Radial velocities of single gas clouds (masers)



Ensemble motions
(spatially resolved)

- Stellar Dynamics
V from Stellar Absorption Lines
- Gas Kinematics
V from Gas Emission Lines



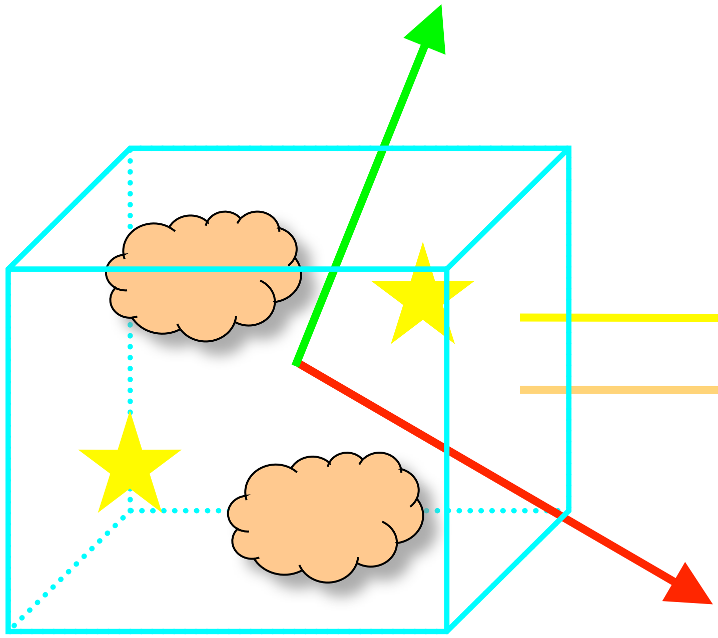
Ensemble motions
(time resolved)

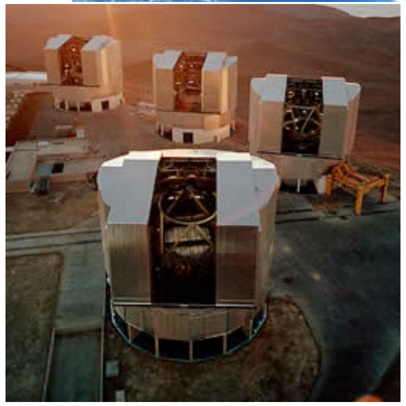
- Reverberation Mapping
V from line width, R from time variability → **Hagai Netzer's lectures**

Dynamical evidences for BHs

Ensemble motions
(spatially resolved)

- Stellar Dynamics
  V from Stellar Absorption Lines
- Gas Kinematics
  V from Gas Emission Lines





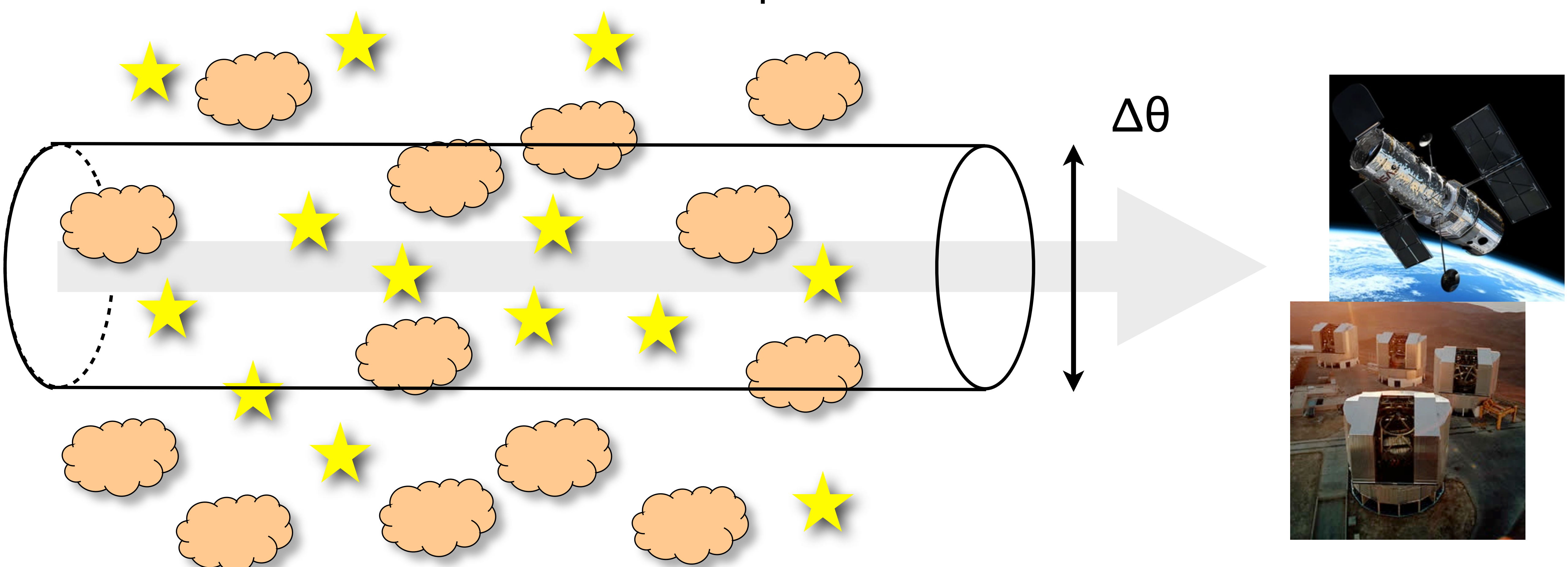
The concept of spatial resolution (2)

Let us recall that the observed surface brightness *I(α,δ)* is the convolution of the intrinsic surface brightness *O(α,δ)* with the PSF.

- This is true for any wavelength and thus affect both imaging and spectroscopy.

$$I_\lambda(\alpha,\delta) = \int\!\!\int P_\lambda(\alpha-\alpha',\delta-\delta')\,O_\lambda(\alpha',\delta')\,d\alpha'\,d\delta'$$

- The smallest volume which can be singled out in the galaxy is a column with diameter of the order of the spatial resolution



Black Hole Sphere of Influence

In general, gas clouds and stars move in the galaxy gravitational potential (mainly due to stars).

- To detect a BH it is important to explore the region where the BH gravitational potential dominates.
- Thus spatial resolution must be small enough to spatially resolve the region where the BH dominates the gravitational potential.

The size of the "BH sphere of influence" is given by the condition

BH gravitational potential ~ Star gravitational potential

Rotational velocity around BH ~ Typical star velocity in galaxy

$$\frac{G\,M_{BH}}{r_{BH}} = \sigma_\star^2$$

(see Peebles 1972)

For the typical star velocity we can use the average stellar velocity dispersion of the galaxy (measured on spatial scales where the BH DOES NOT dominate the gravitational potential).

Black Hole Sphere of Influence

$$r_{BH} = \frac{G\,M_{BH}}{\sigma_\star^2} = 10.7\,\mathrm{pc}\left(\frac{M_{BH}}{10^8\,M_\odot}\right)\left(\frac{\sigma_\star}{200\,\mathrm{km/s}}\right)^{-2}$$

Considering the projected angular dimensions on the plane of the sky:

$$\theta_{BH} = 0.11''\left(\frac{M_{BH}}{10^8\,M_\odot}\right)\left(\frac{\sigma_\star}{200\,\mathrm{km/s}}\right)^{-2}\left(\frac{D}{20\,\mathrm{Mpc}}\right)^{-1}$$

Black Hole Sphere of Influence

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- Need high spatial resolution to probe within the BH sphere of influence and detect its effects.
- Major impact of the Hubble Space Telescope (d=2m → 0.05” @ 6000 Å)
- Now 8m-class telescope with AO are being used (d=8m → 0.05” @ 2 μm)

... and how we do it today!

if any luminous bodies infer their existence of the central ones with some degree of probability, as this might afford a clue to some of the apparent irregularities of the revolving bodies, which would not be easily explicable on any other hypothesis;

Use the kinematics of ‘test particles’ (gas clouds, stars) in the nuclear region of galaxies to infer the presence of a BH.

Observables:

Gas/Stars as tracers of kinematics (V,σ) around BH

Gravitational potential of stars (Φ_{Stars}) from observed surface brightness of galaxy (assume $L \approx \Upsilon M$)

Models:

Find gravitational potential Φ to explain observed V,σ $\quad \Phi = \Phi_{Stars} + \Phi_{BH}$

Evidence for BH?

$\Phi_{BH} = -G\, M_{BH}\, R^{-1} \quad (R >> R_{Schwarzschild})$

The observables

Observables:

Gas/Stars as tracers of kinematics (V,σ) around BH

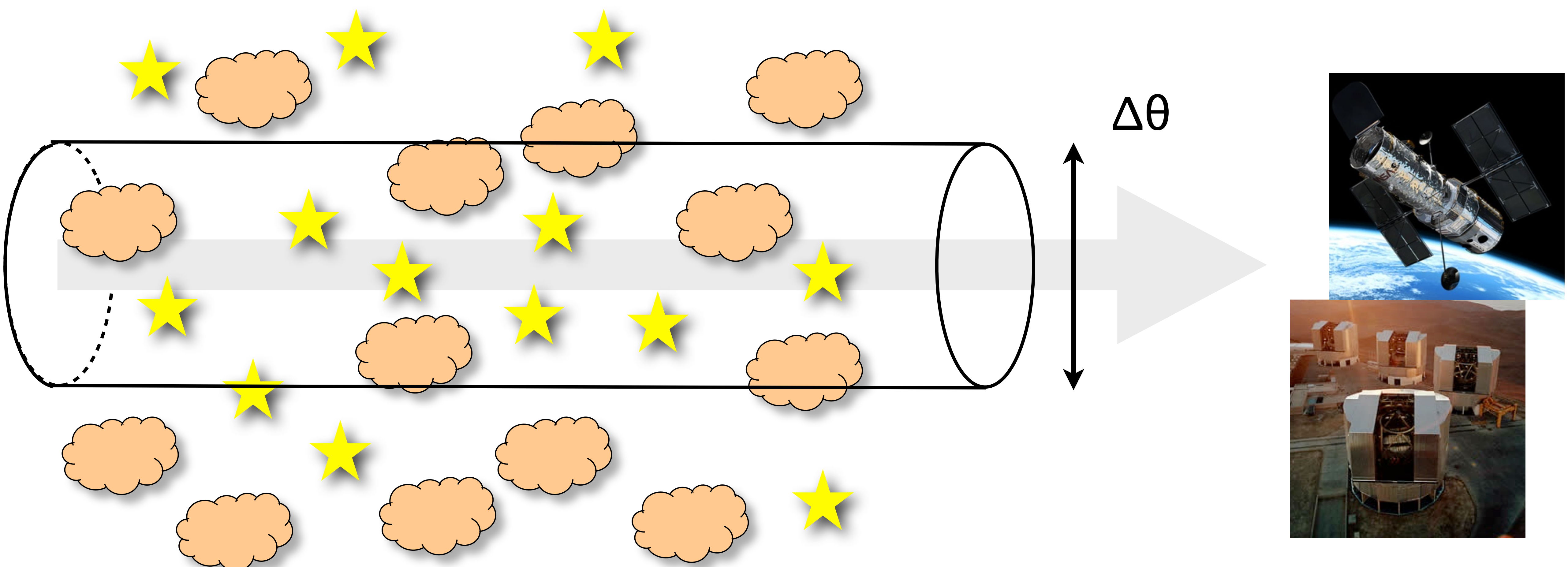
Gravitational potential of stars (Φ_{Stars}) from observed surface brightness of galaxy (assume $L \approx \Upsilon M$)

The kinematical observables

Without proper motions one can only

- measure velocities along the line of sight (Doppler)
- measure velocities over apertures with size ~Δθ (spatial resolution)

One obtains the *line-of-sight velocity distribution* [$f(v)$], i.e. the distribution of line-of-sight velocities [v] within a column sampling the whole galaxy through an aperture set by the spatial resolution of the observations



The kinematical observables

From the line-of-sight velocity distribution (within 1 resolution element!) one can trivially derive average velocity [V], and velocity dispersion [σ]

$$dn = f(v)dv$$

probability of observing a star/cloud with line-of-sight (los) velocity v

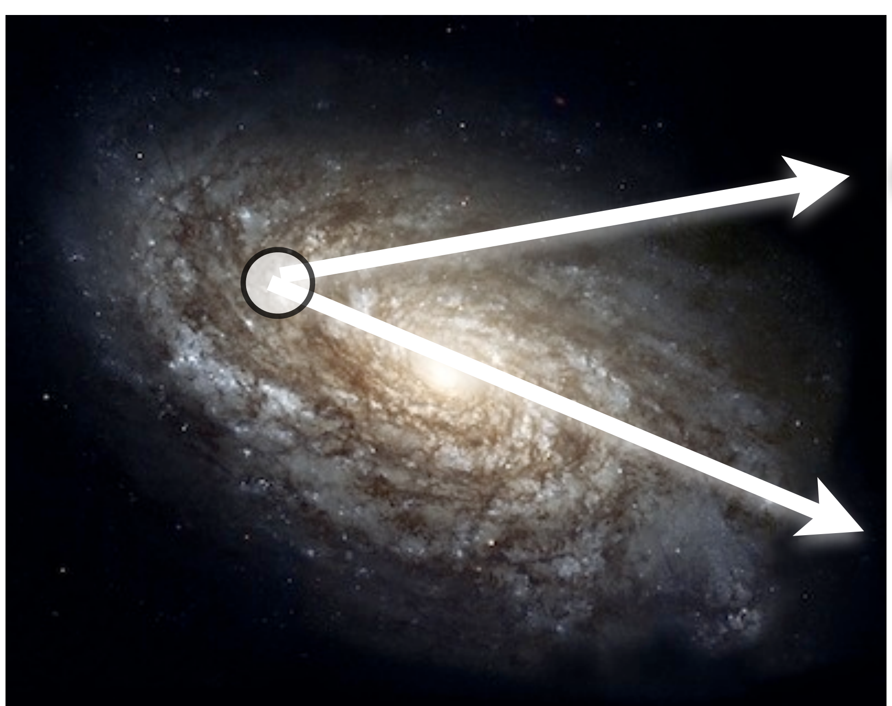
$$\int_0^{+\infty} f(v)dv = 1$$

$$V = \int_0^{+\infty} v f(v)dv$$

average velocity (“velocity”)

$$\sigma = \int_0^{+\infty} (v - V)^2 f(v)dv$$

velocity dispersion (“dispersion”)



Gas Emission lines

Stellar Continuum

V, σ

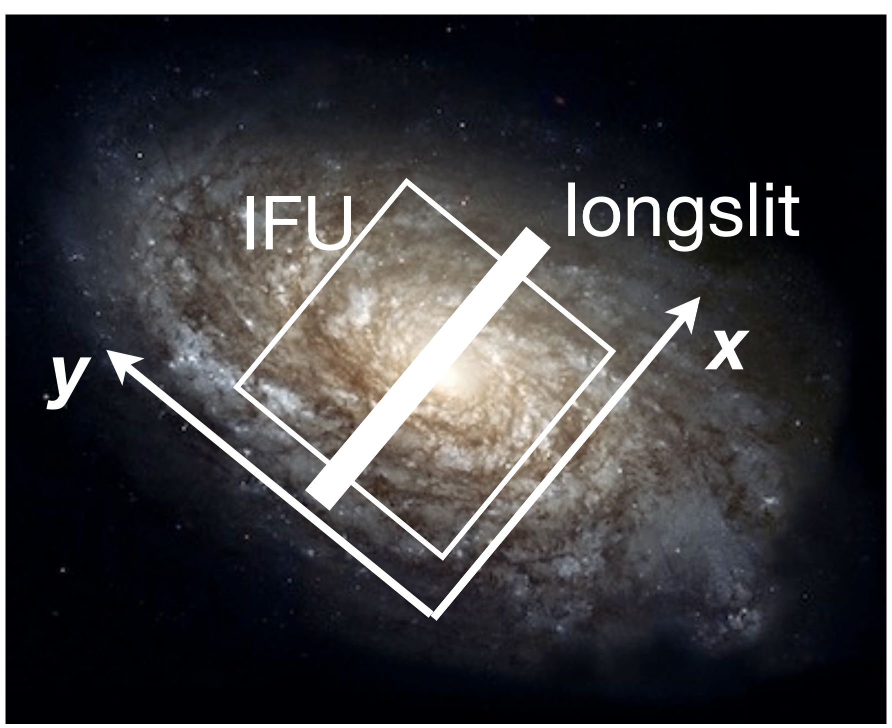
Longslit and IFU

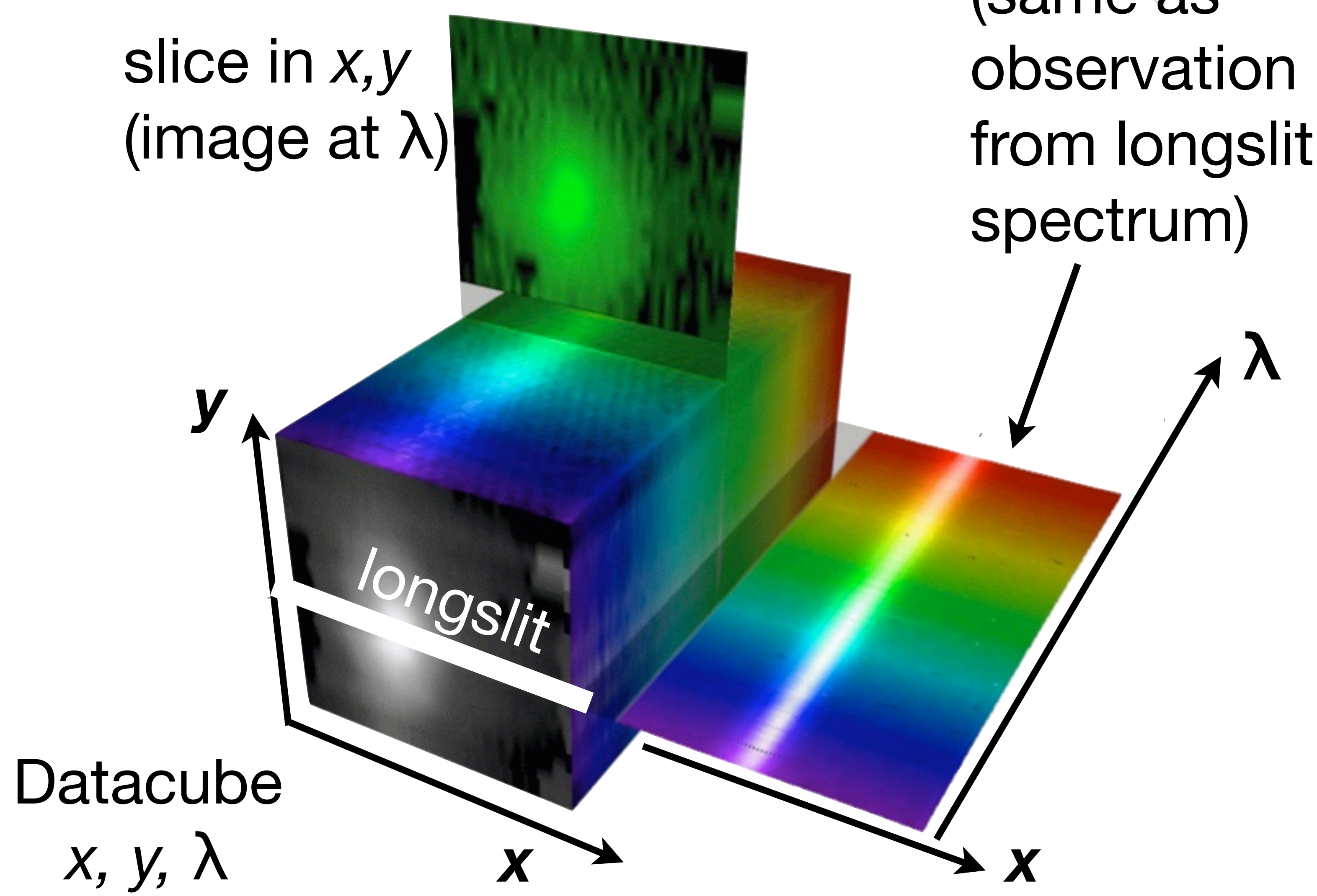
Spectrographs can provide simultaneous spectra from several regions of the source (galaxy).

They are mostly of two kinds:

- Longslit spectrographs
  - provide spectra at position *x* along the slit (1 spatial dimension)
- Integral Field Units (IFU)
  - provide spectra at position *x*,*y* (2 spatial dimensions)

    → datacube (*x*, *y*, λ)



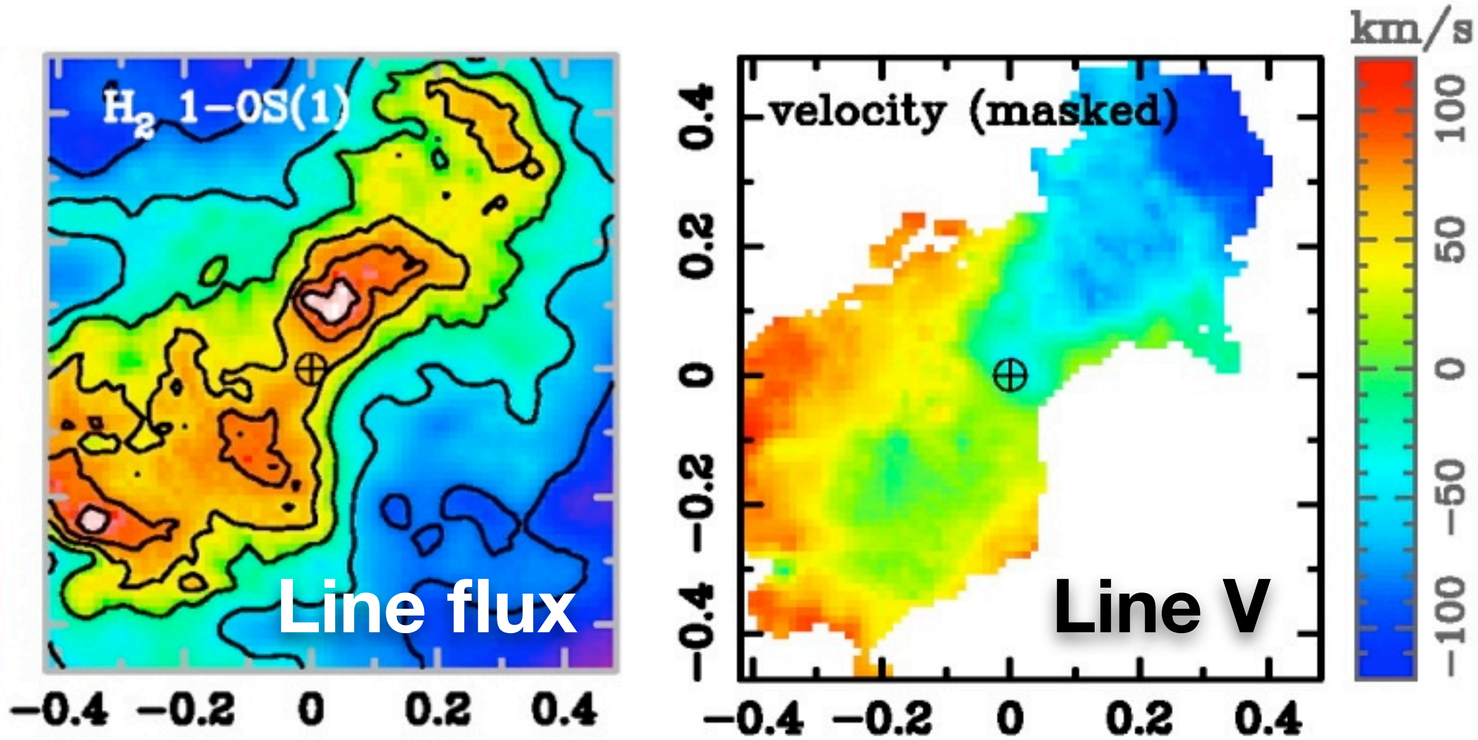


Gas kinematics

- Use emission lines, usually from ionized gas ($T_e \sim 10^4$ K);
- intrinsic line width is usually thermal broadening, negligible w.r.t. motions in galaxy nuclei

$$\sigma \simeq \left(\frac{3k_B T_e}{m_p}\right)^{1/2} = 16\,\mathrm{km\,s^{-1}} \left(\frac{T_e}{10^4\,\mathrm{K}}\right)^{1/2}$$

- observed line profile is directly line-of-sight velocity distribution;
- fit with single/multiple Gaussian functions to obtain V, σ;
- measurements at different positions → V, σ "maps"



NGC 3227, H_2 (Davies+2006)

Gas kinematics

- Use emission lines, usually from ionized gas (T_e ~10^4 K);
- intrinsic line width is usually thermal broadening, negligible w.r.t. motions in galaxy nuclei

$$\sigma \simeq \left(\frac{3k_B T_e}{m_p}\right)^{1/2} = 16\,\mathrm{km\,s^{-1}}\left(\frac{T_e}{10^4\,\mathrm{K}}\right)^{1/2}$$

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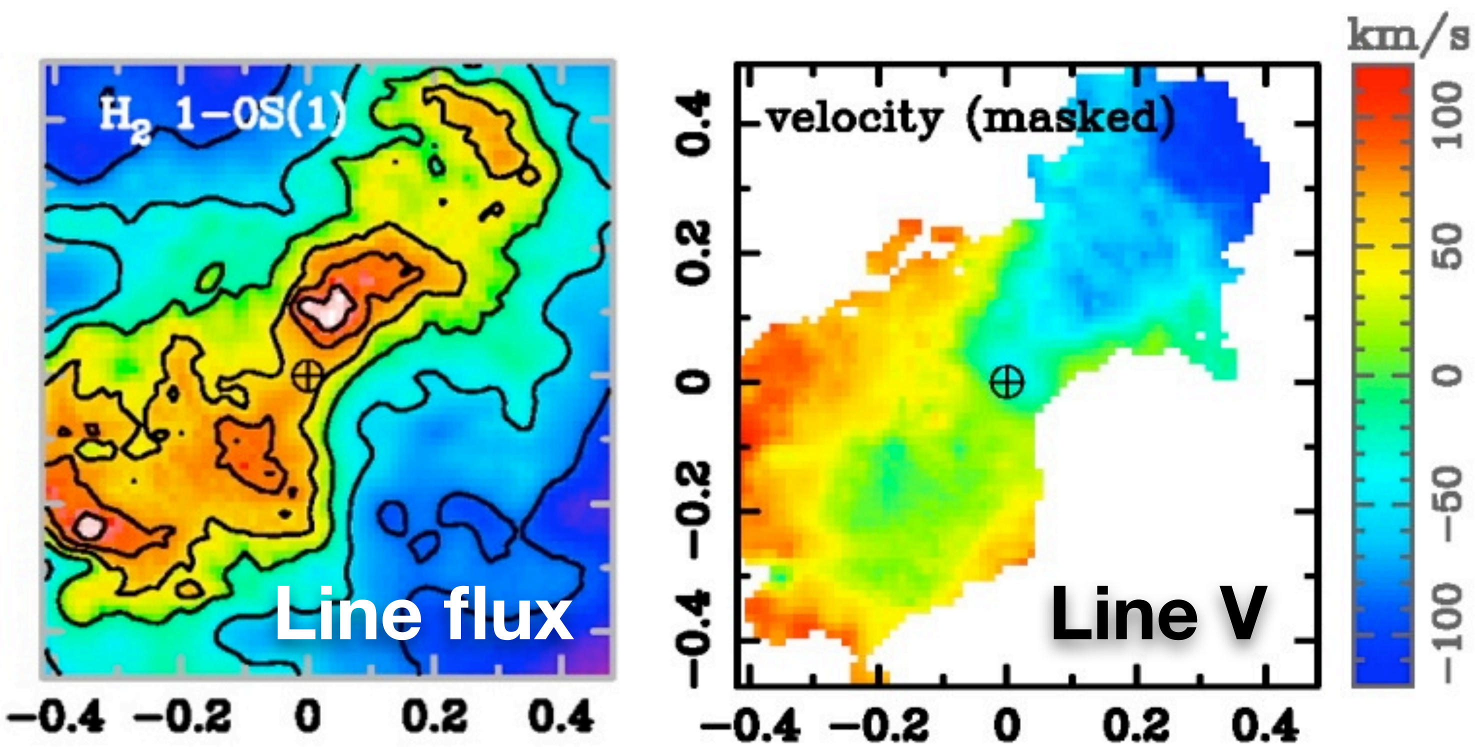
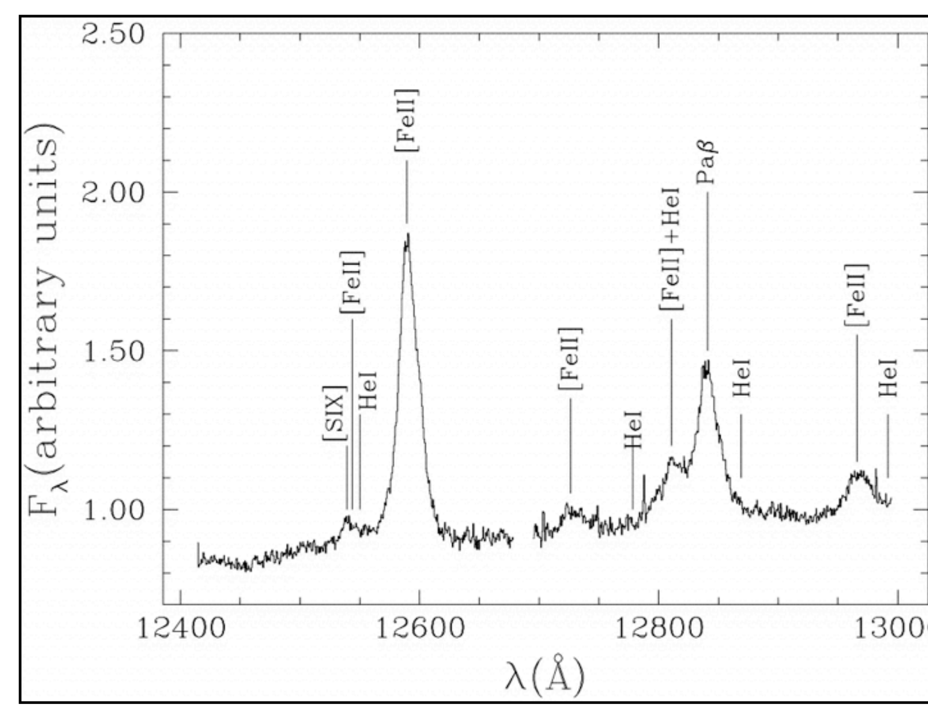


NGC 3227, H_2 (Davies+2006)



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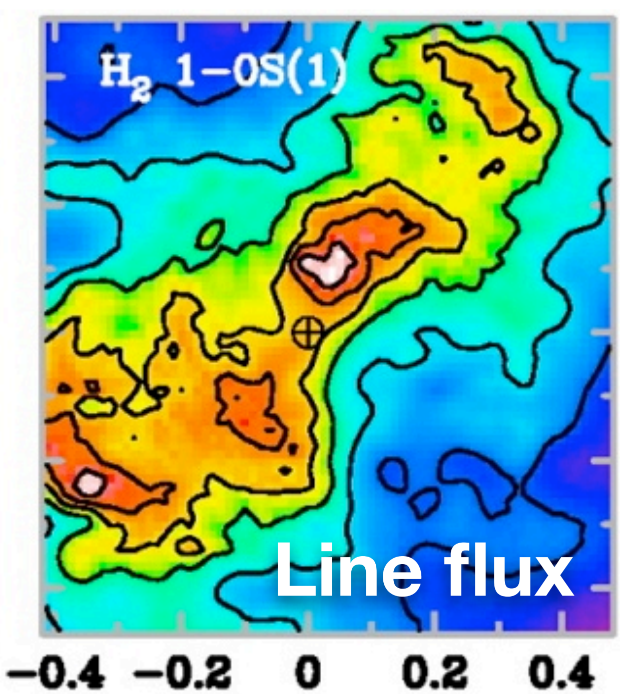
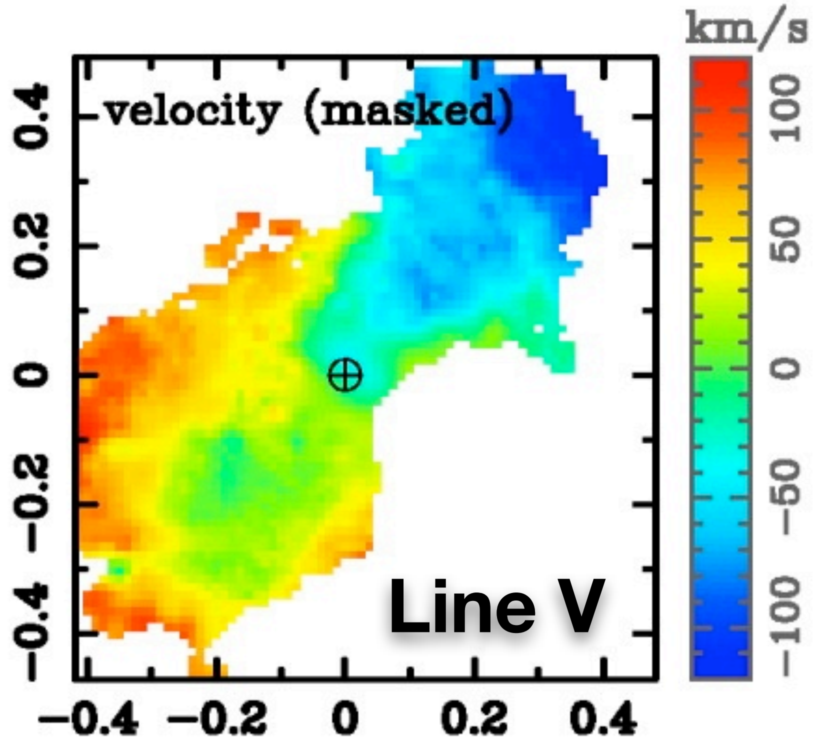
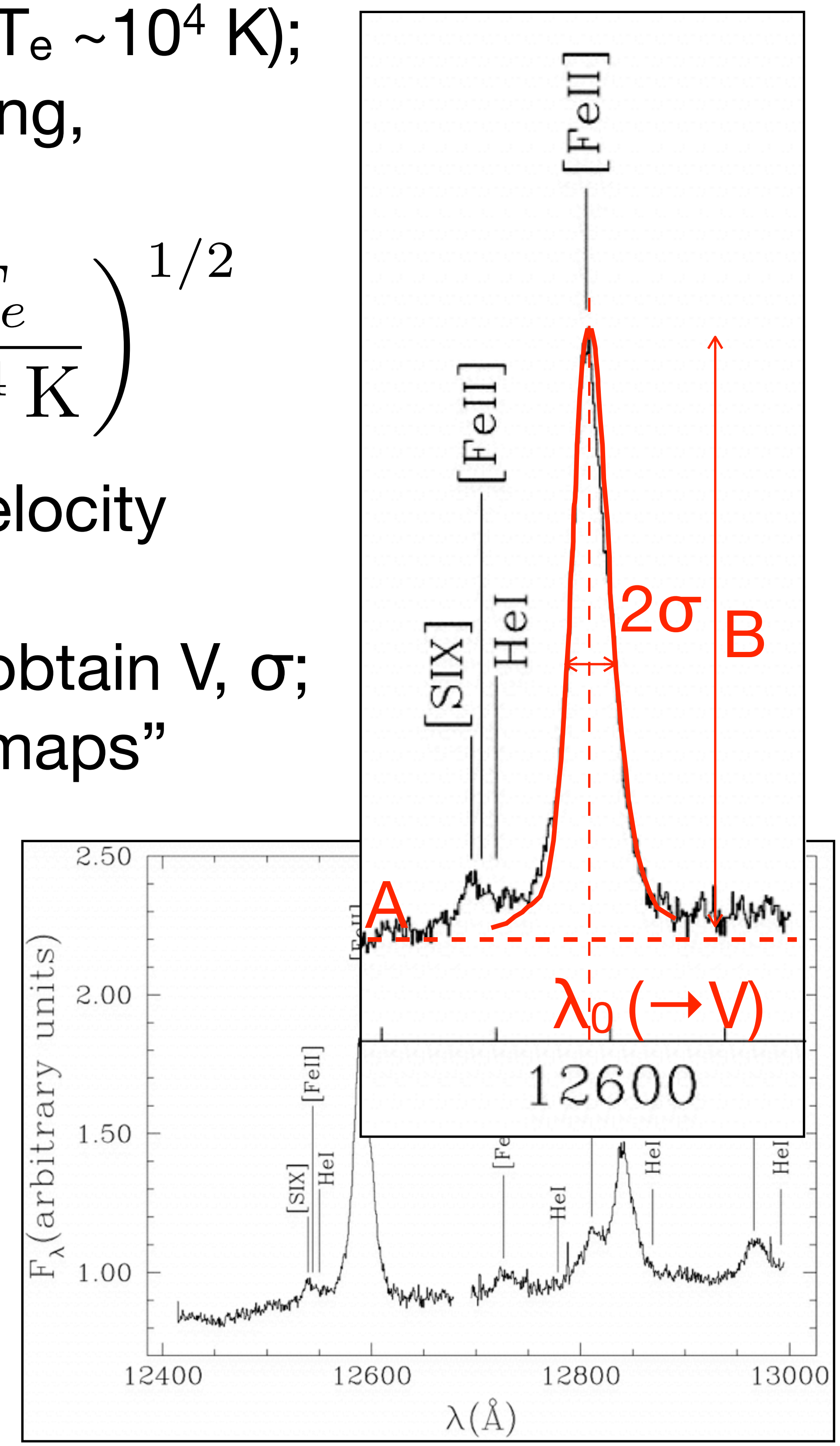






NGC 3227, H_2 (Davies+2006)

Stellar kinematics

- Stellar photospheric lines does not directly provide the line of sight velocity distribution as absorption lines are significantly broadened by electric and magnetic fields, stellar rotation etc.

- one consider a *template*, i.e. a suitable combination of stellar spectra of different spectral types (depending on the galaxy stellar population, etc.) which is then convolved with a parametric function which represents the line of sight velocity dispersion

- Recently, this is usually a Gaussian modified with Hermite polynomials (H_i, e.g. Cappellari & Emsellem 2004); h3, h4 (and superior order terms when signal-to-noise large enough) constrain the deviation of velocity distribution from Gaussian function.

$$f(v) = \frac{e^{-y^2/2}}{\sqrt{2\pi}\sigma}\left[1 + h_3\,H_3(y) + h_4\,H_4(y) + \ldots\right] \qquad y = \frac{v - V}{\sigma}$$

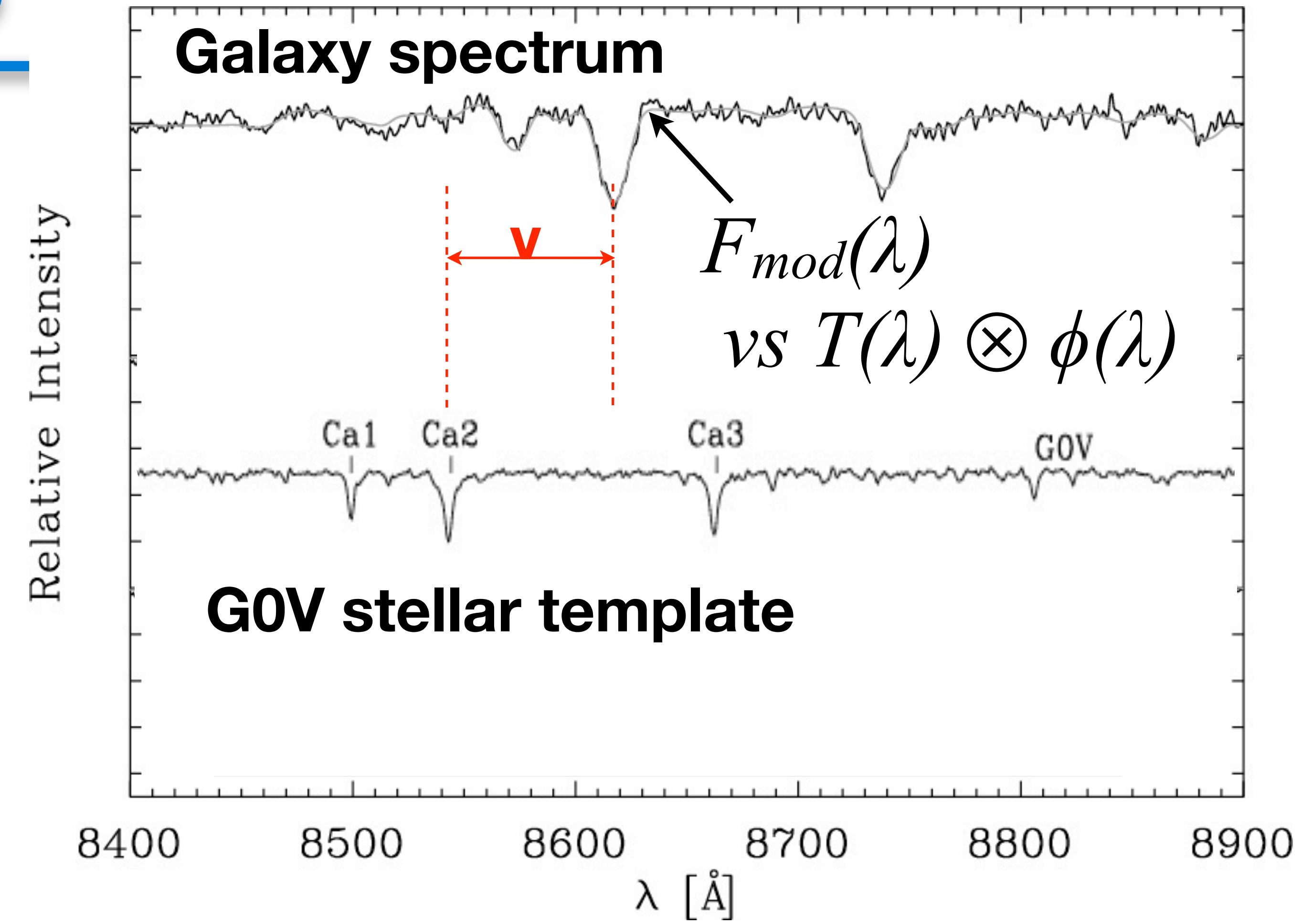
Stellar kinematics

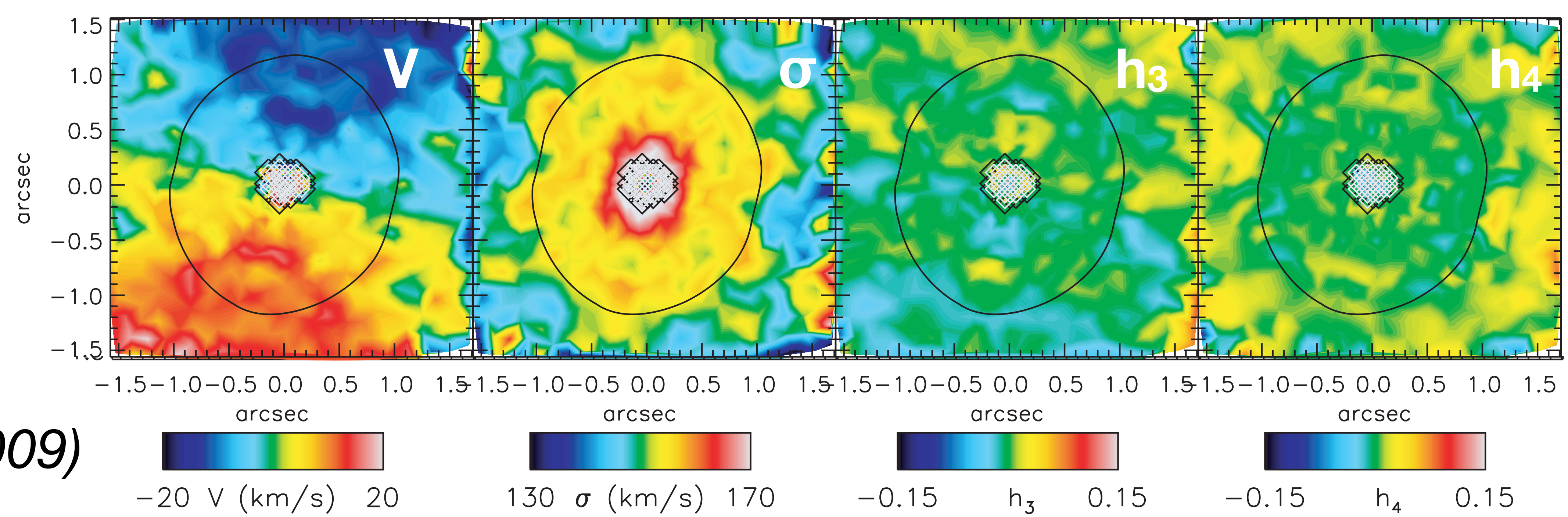
Observed spectrum is given by convolution of template with line-of-sight velocity distribution. Parameters which determine $f(v)$ are found with X^2 minimization on observed spectrum□F_{obs} vs F_{mod}

$$v = (\lambda/\lambda_{rest} - 1)c$$

$$\phi(\lambda)d\lambda = f(v)dv$$

$$F_{mod}(\lambda) = \int_0^{+\infty} T(\lambda')\phi(\lambda - \lambda')d\lambda'$$



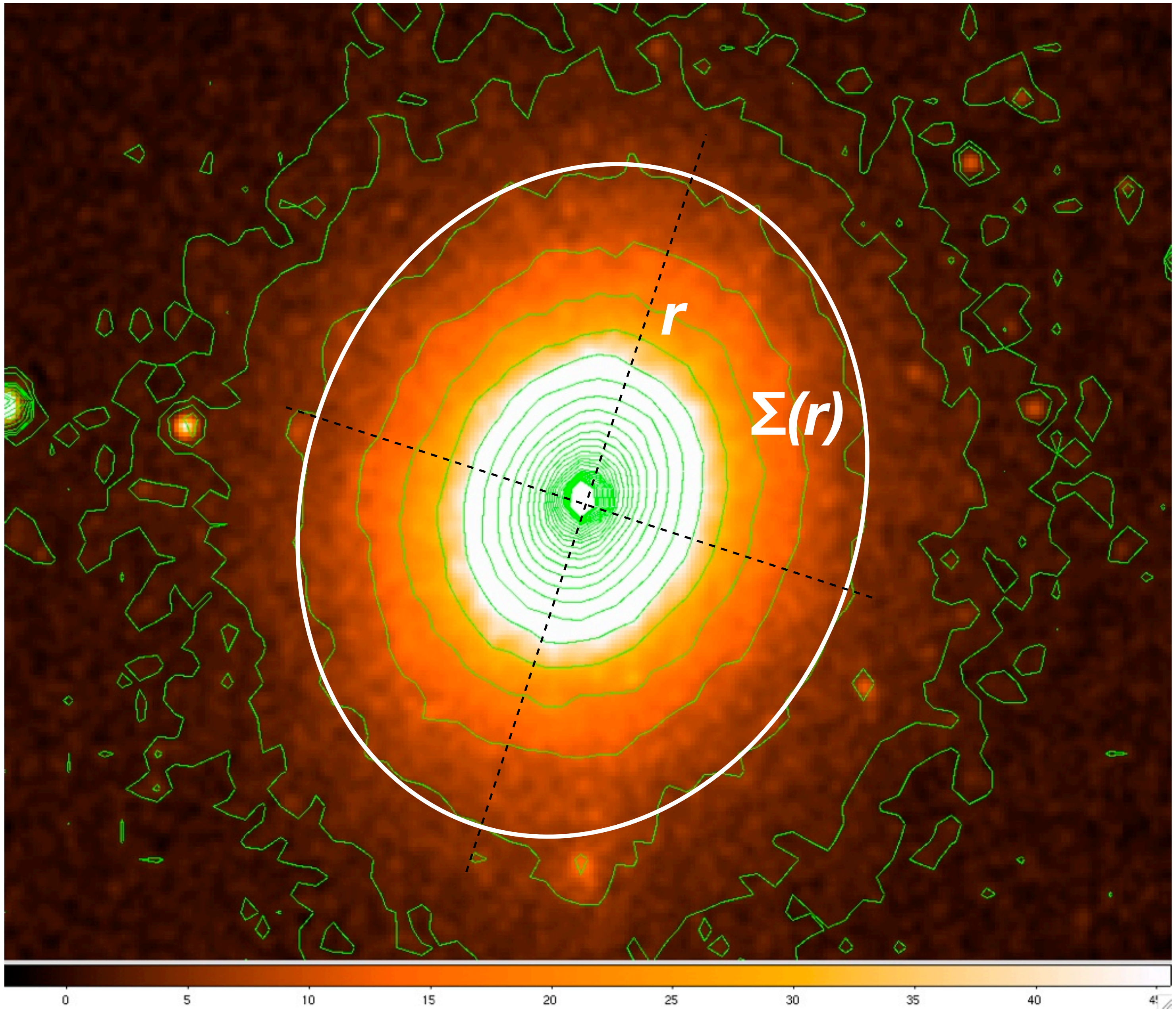


Centaurus A, CO @2.3 μm (Cappellari+2009)

Observables: galaxy surface brightness

Surface brightnesses (Σ) of galaxies (especially ellipticals) are typically regular, with a central peak which declines outwards.

It is useful to consider isophotes (curves of contant Σ) which can usually be described with ellipses.



For each isophote one can then determine

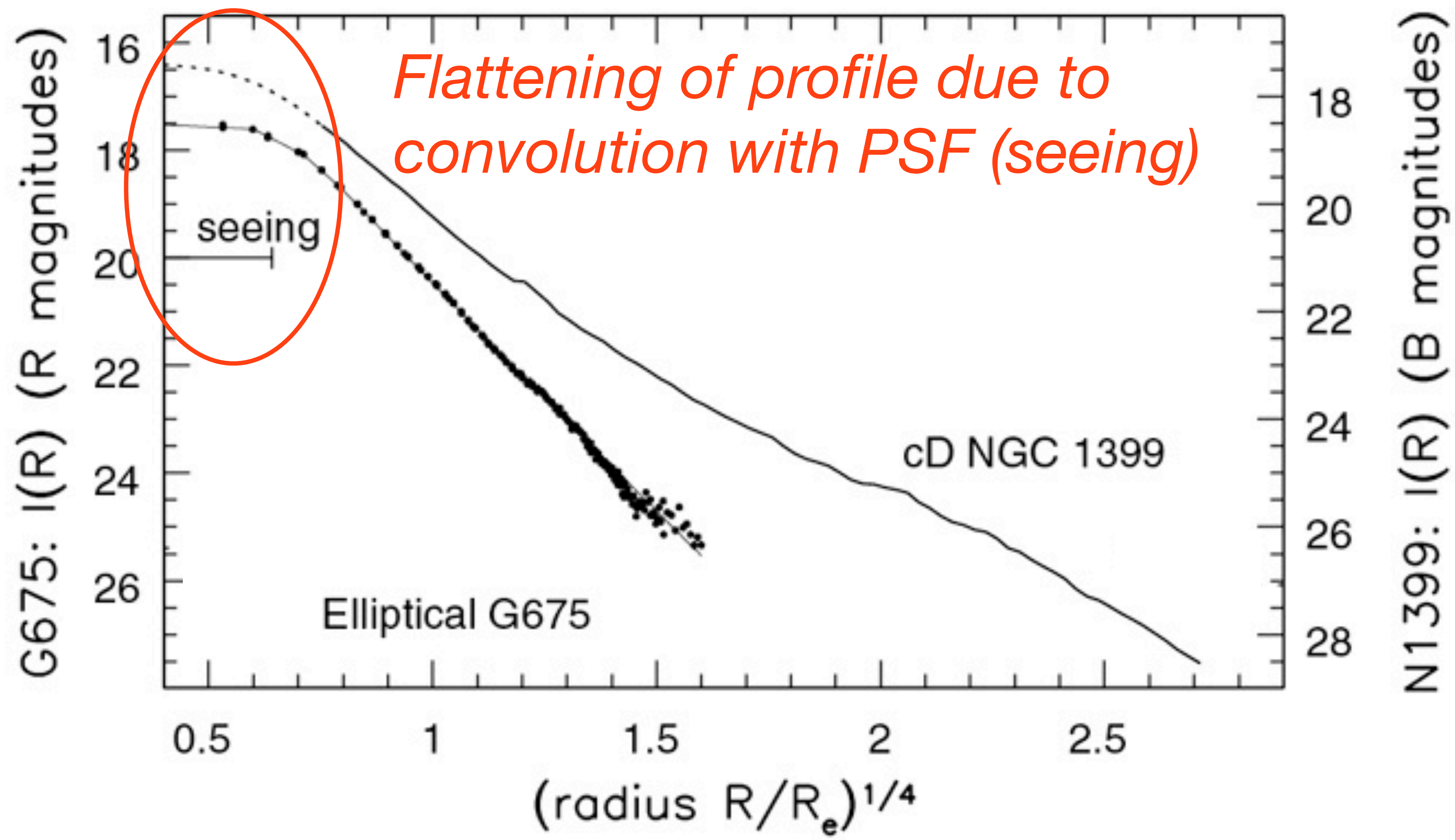
- ***r*** the semi-major axis of ellipse
- ***Σ(r)*** the average surface brightness on that ellipse
- orientation of major axis, center, ellipticity, distorsions, etc.

Elliptical isophotes are expected from spheroidal and disk-like distributions of stars.

Observables: galaxy surface brightness

The surface brightness profile well characterizes the structure of the galaxy, and the various galaxy types have well defined profiles.

Ellipticals:
- characterized by a *Sersic* profile with
  n Sersic index
  R_e half-light radius
- this is just empirical parameterization
- generalization of de Vaucouleur profile (Sersic with n=4)



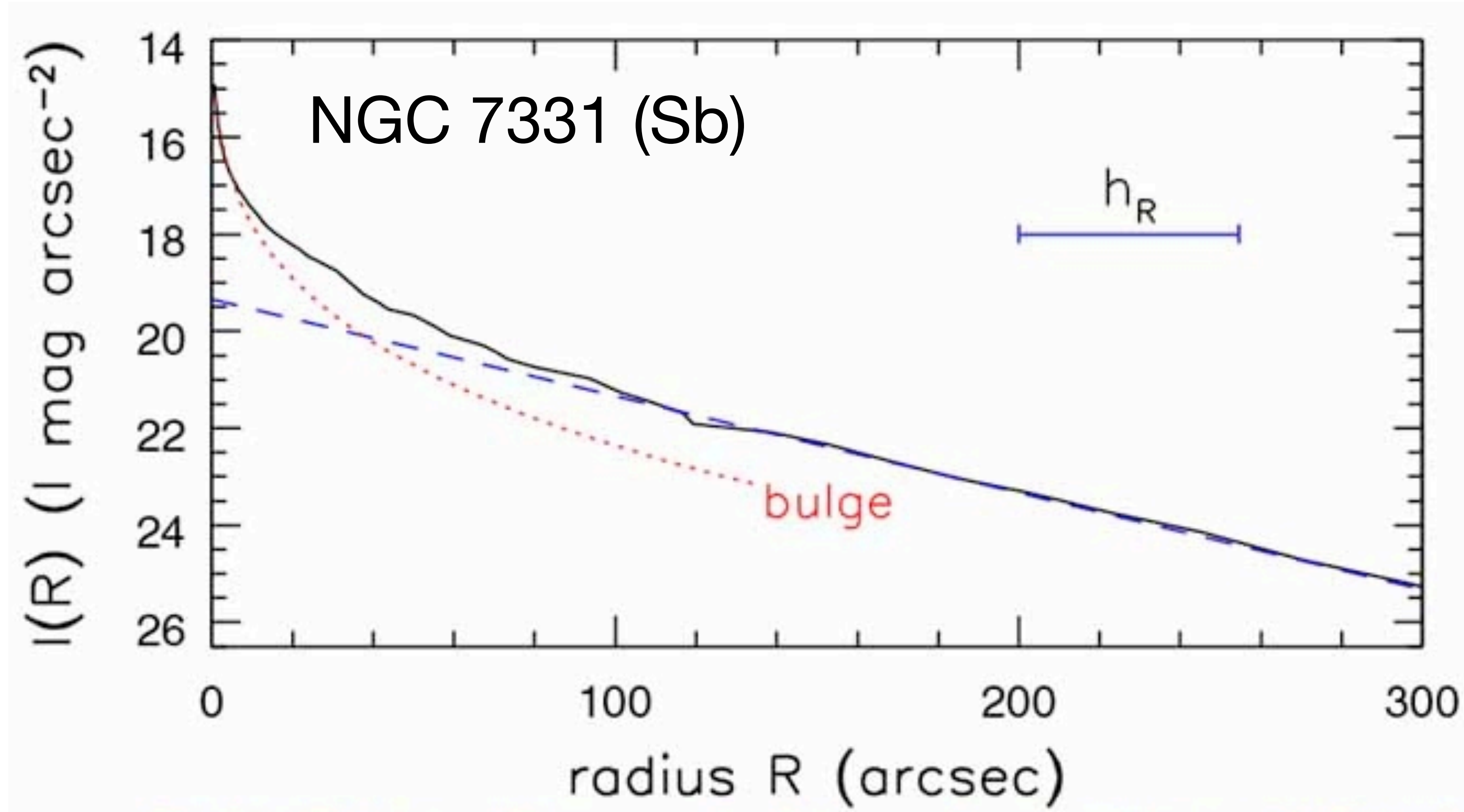
$$\Sigma(R) = \Sigma_e \exp\{-b_n[(R/R_e)^{1/n} - 1]\}$$

For n>1, $b_n \approx 1.999n-0.327$
For n=1, exponential disk
For n=0.5, Gaussian

Observables: galaxy surface brightness

Spirals:

- Bulge is characterized by a Sersic profile
- Disk is characterized by an exponential profile (Sersic with n=1)



$$\Sigma_b(R) = \Sigma_e \exp\{-b_n[(R/R_e)^{1/n} - 1]\}$$

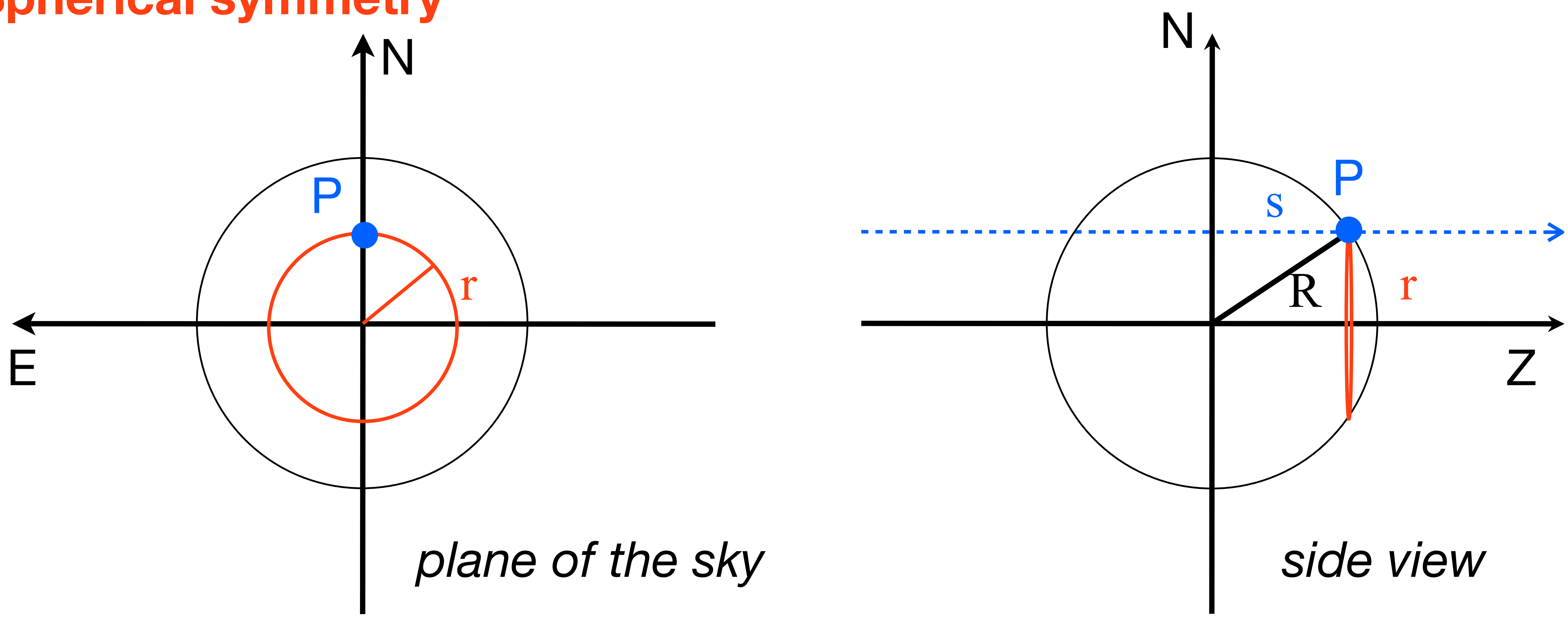
$$\Sigma_d(R) = \Sigma_0 \exp\{-(R/h_R)\}$$

- Bulge/Total:
  ~ 0.5-0.3 in S0/Sa-Sb
  ~0.1-0.02 in late type spirals
- $R_e \sim 0.1\ h_R$

Observables: galaxy grav. potential

Observed galaxy surface brightness in the plane of the sky can be converted into luminosity densities (with assumptions ...).

Spherical symmetry



Surface brightness observed at point P [$\Sigma(r)$] on the plane of the sky at projected distance *r* from center, is integrated light density of the galaxy along the direction perpendicular to the plane of the sky.

$$\Sigma(r) = \int_{-\infty}^{+\infty} J(s)ds = 2\int_{r}^{+\infty} \frac{J(R)R}{\sqrt{R^2 - r^2}}dR \qquad s = \sqrt{R^2 - r^2}$$

Observables: galaxy grav. potential

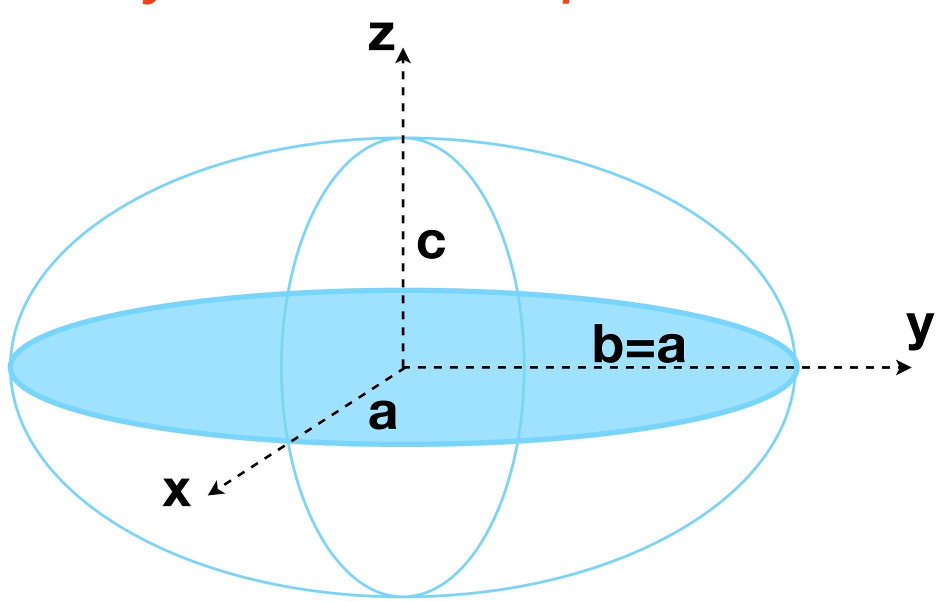
$$\Sigma(r) = 2\int_r^{+\infty} \frac{J(R)R}{\sqrt{R^2 - r^2}} dR$$

Σ(r) is observed, *J(R)* is unknown

This is Abel's Equation with solution:

$$J(r) = -\frac{1}{\pi}\int_R^{+\infty} \frac{d\Sigma(r)}{dr} \frac{dr}{\sqrt{r^2 - R^2}}$$

This approach can be generalized to oblate/prolate spheroids, i.e. *axisymmetric ellipsoids* which are a better approximation of a real galaxy (see Binney & Tremaine's book)



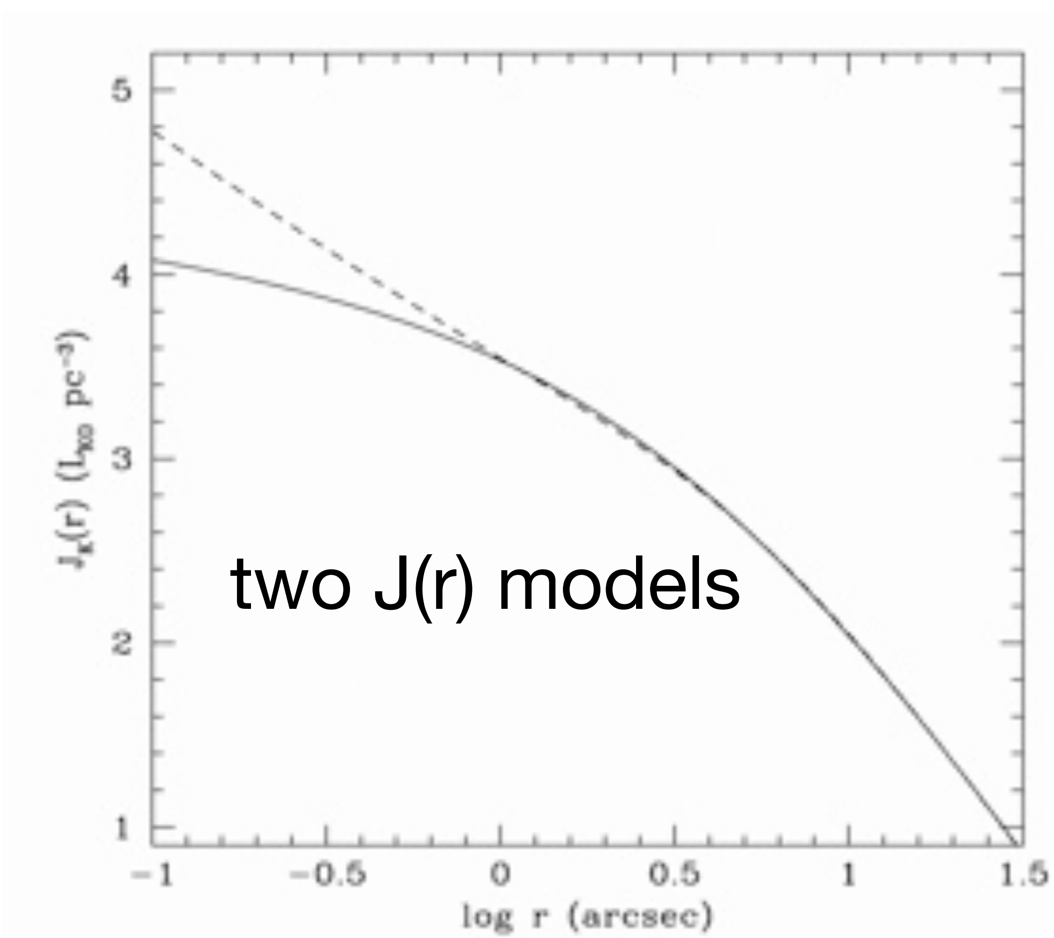
Oblate: ***a* = *b* > *c***

Prolate: ***a* = *b* < *c***

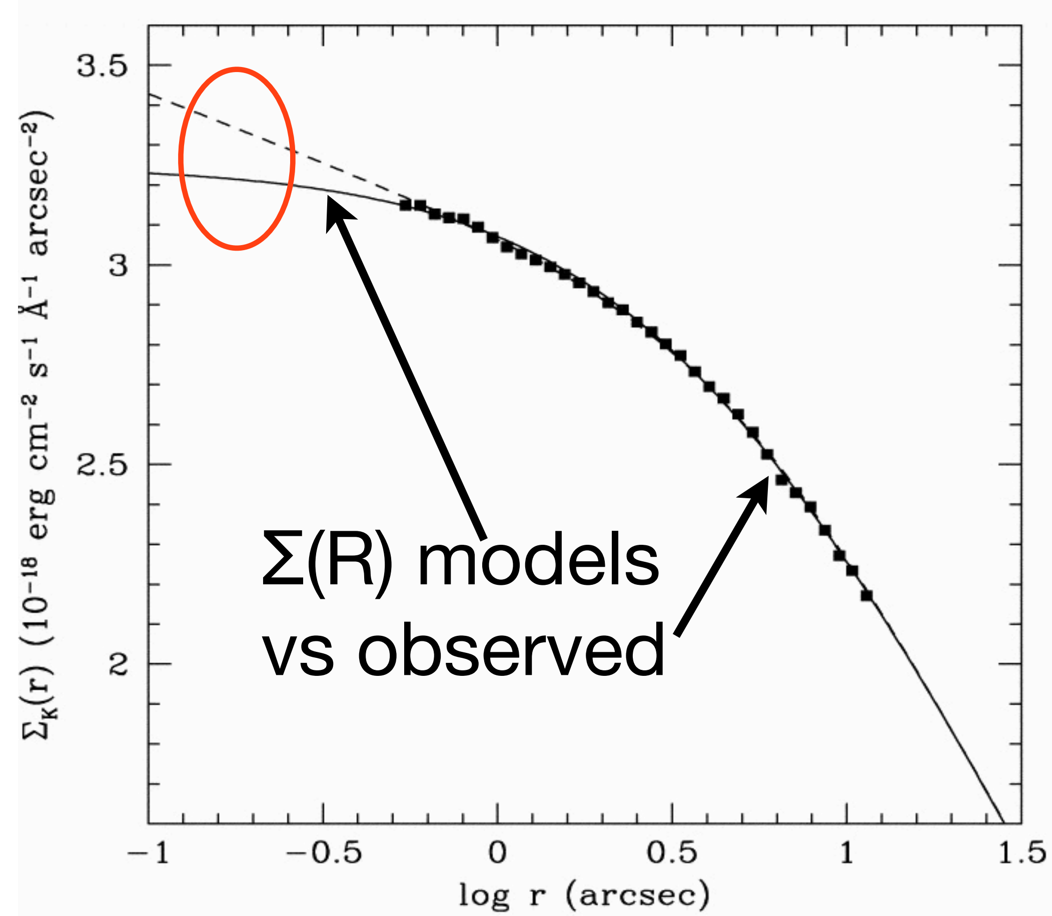
The more general ellipsoidal structure is *triaxial*: ***a* = *b* < *c***

Observables: galaxy grav. potential

Alternatively, “brute force” approach: numerically integrate and project the galaxy (of any intrinsic shape) onto the plane of the sky;
“model” surface brightness profile is then convolved with instrumental effects (see later ...) and varied to match the observed surface brightness.



uncertainties in extrapolation to small scales



Centaurus A, K band (HST/NICMOS) [Marconi+2006]

Observables: galaxy grav. potential

From observed surface brightness one infers stellar luminosity density; mass density is then obtained *assuming a constant mass-to-light ratio*

$$\rho(R) = \Upsilon\, J(R)$$

If only circular velocity is needed then

$$M(R) = \Upsilon \int_0^R J(R')4\pi R'^2 dR' = \Upsilon\, L(r)$$

$$V(R)^2 = \Upsilon \frac{GL(R)}{R}$$

or more complex formulas in non spherical cases

If gravitational potential ϕ is needed, one then solves Poisson's equation

$$\nabla^2\phi(x,y,z) = 4\pi G\, \Upsilon\, J(x,y,z)$$

We have 2D information on sky, need assumptions to get 3D structure!

... and how we do it today!

if any luminous bodies infer their existence of the central ones with some degree of probability, as this might afford a clue to some of the apparent irregularities of the revolving bodies, which would not be easily explicable on any other hypothesis;

Use the kinematics of ‘test particles’ (gas clouds, stars) in the nuclear region of galaxies to infer the presence of a BH.

Observables:

Gas/Stars as tracers of kinematics (V,σ) around BH

Gravitational potential of stars (Φ_{Stars}) from observed surface brightness of galaxy (assume $L \approx \Upsilon M$)

Models:

Find gravitational potential Φ to explain observed V,σ $\quad \Phi = \Phi_{Stars} + \Phi_{BH}$

Evidence for BH?

$\Phi_{BH} = -G\, M_{BH}\, R^{-1} \quad (R >> R_{Schwarzschild})$

... and how we do it today!

if any luminous bodies infer their existence of the central ones with some degree of probability, as this might afford a clue to some of the apparent irregularities of the revolving bodies, which would not be easily explicable on any other hypothesis;

Use the kinematics of 'test particles' (gas clouds, stars) in the nuclear region of galaxies to infer the presence of a BH.

Models:

Find gravitational potential Φ to explain observed V,σ $\quad \Phi = \Phi_{Stars} + \Phi_{BH}$

Gas kinematics: assumptions

Basic assumptions

- gas is in circularly rotating disks
- hydrodynamical effects are negligible
- motions are entirely determined by gravity

If these assumptions are not valid then gas kinematics cannot be used to measure BH masses.

It is then straightforward to show that the rotation velocity of the disk at radius *r* is

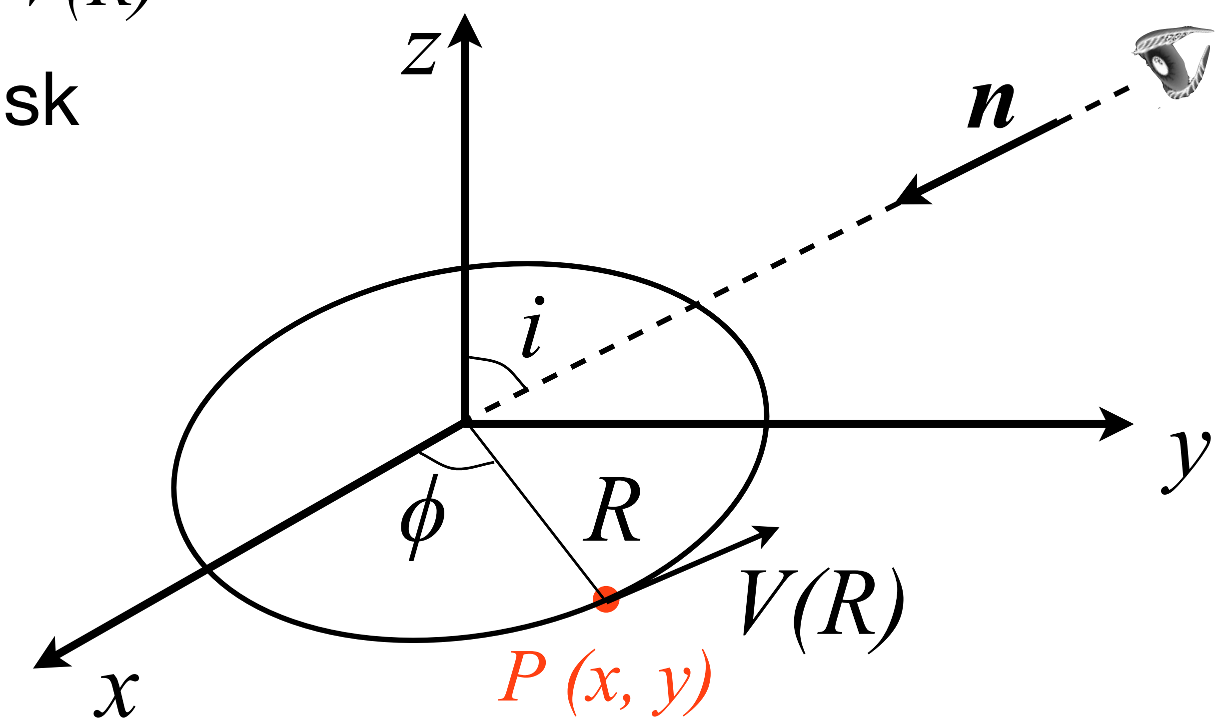
$$\begin{aligned} V_{\rm circ}(r) &= \left[G\,\frac{M_{\rm BH} + M_\star(r)}{r} \right]^{1/2} \\ &= 207\,{\rm km/s} \left[\frac{M_{\rm BH} + M_\star(r)}{10^8\,{\rm M}_\odot} \right]^{1/2} \left(\frac{r}{10\,{\rm pc}} \right)^{-1/2} \end{aligned}$$

The rest is just geometrical projection, instrumental and finite spatial resolution effects.

The contribution from stellar mass is more complex if mass is not in a spherical distribution (e.g. Binney & Tremaine).

Projected line-of-sight velocity

- Thin disk in circular rotation with $V = V(R)$
- Reference system xyz centered on disk
- $\boldsymbol{n}$ is line-of-sight (*los*) versor
- Galaxy has systemic velocity V_{sys}



$$\vec{n} = -\sin i\,\vec{j} - \cos i\,\vec{k}$$

Velocity of point P is

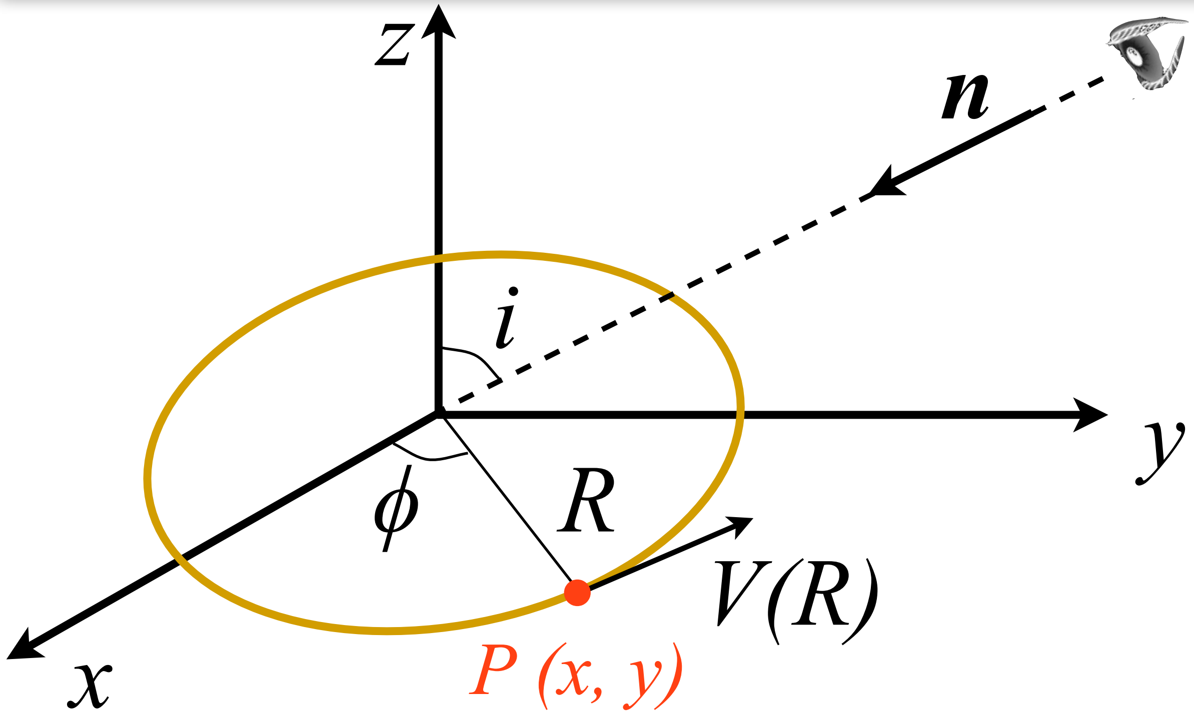
$$\vec{v} = V_{sys}\vec{n} + V(R)\left(-\sin\phi\,\vec{i} + \cos\phi\,\vec{j}\right)$$

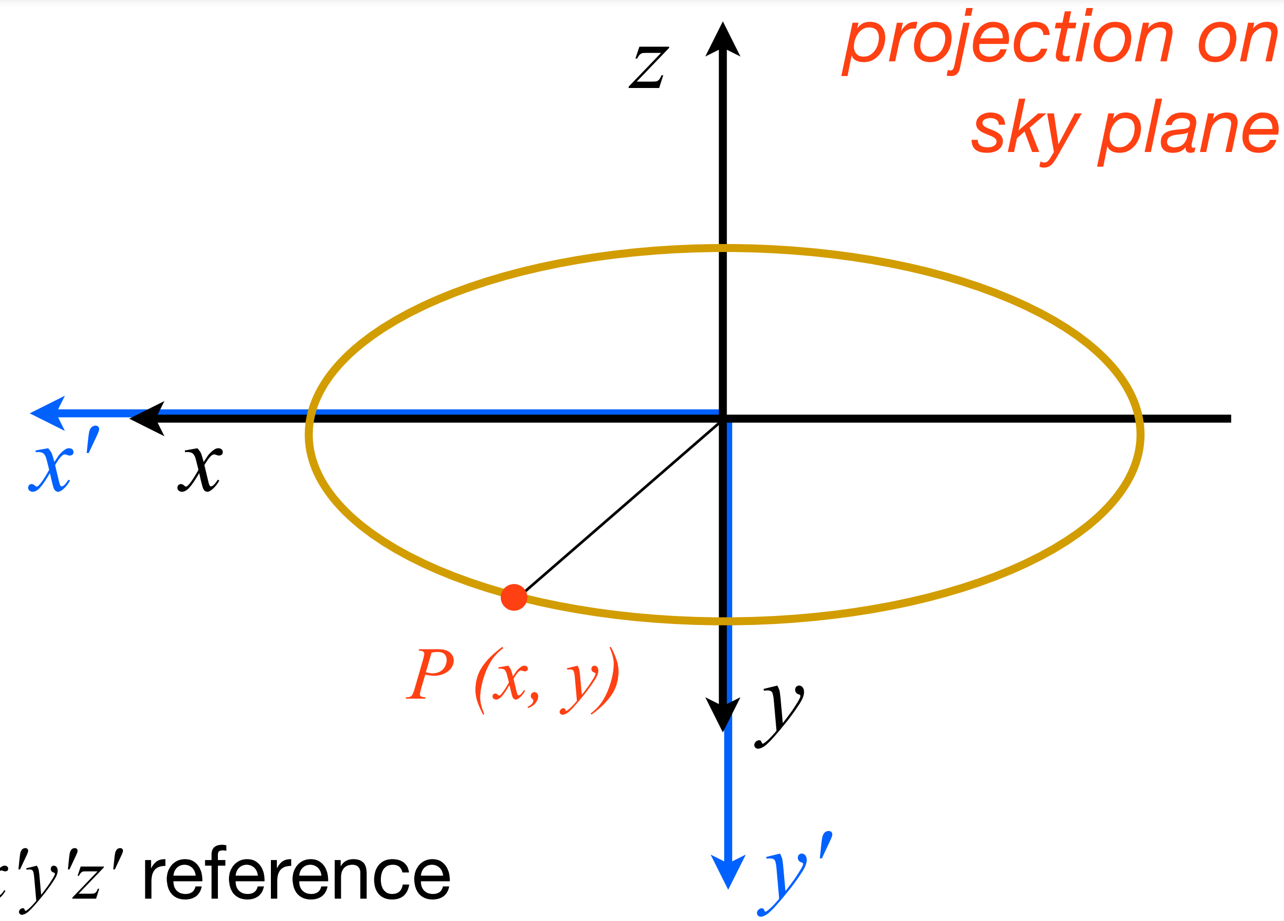
Velocity of P along line of sight is then

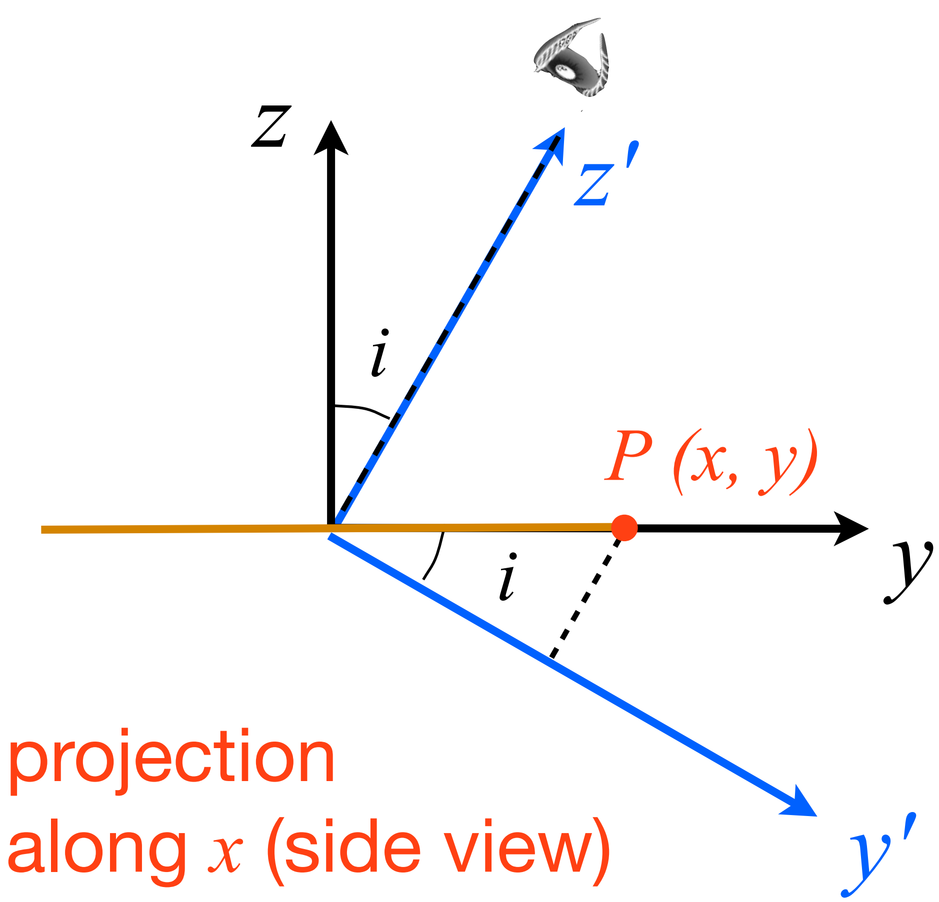
$$V_{obs} = \vec{v}\cdot\vec{n} = V_{sys} + V(R)\sin i\cos\phi$$

We now need to project disk on the plane of the sky to write R and $\cos\phi$ as a function of sky coordinates.

Projected line-of-sight velocity







$x'y'z'$ reference
($x'y'$ sky plane, z' line of sight)

$$x' = x \qquad y' = y \cos i$$

we can then obtain R and $cos\phi$ as a function of sky coordinates $x'y'$

$$R = \sqrt{x^2 + y^2} = \sqrt{x'^2 + \left(\frac{y'}{\cos i}\right)^2}$$

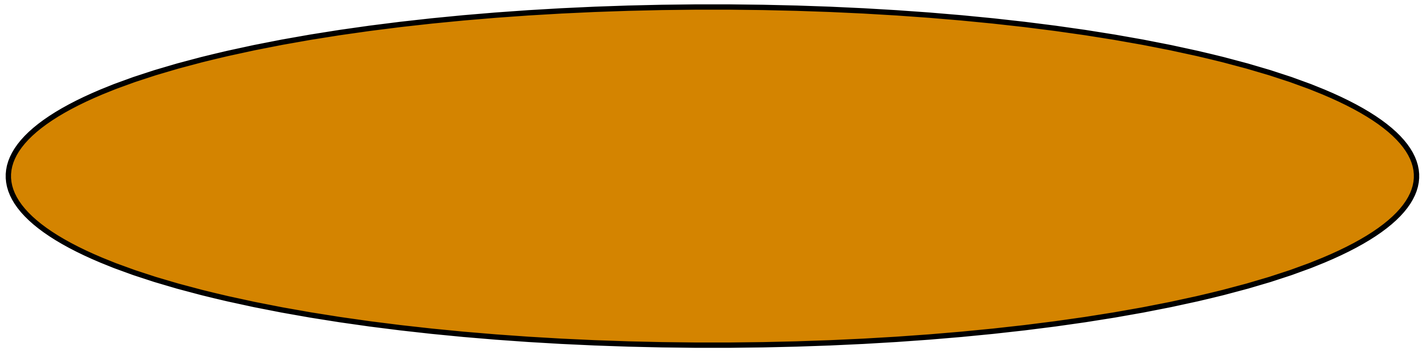
$$\cos\phi = \frac{x}{R} = \frac{x'}{\sqrt{x'^2 + (y'/\cos i)^2}}$$

Projected line-of-sight velocity

We finally obtain

$$V_{los}(x', y') = V_{sys} + \left[GM(R)\sin^2 i\right]^{1/2} \frac{x'}{\left[x'^2 + (y'/\cos i)^2\right]^{3/2}}$$

$$M(R) = M_{\rm BH} + M_\star(R)$$



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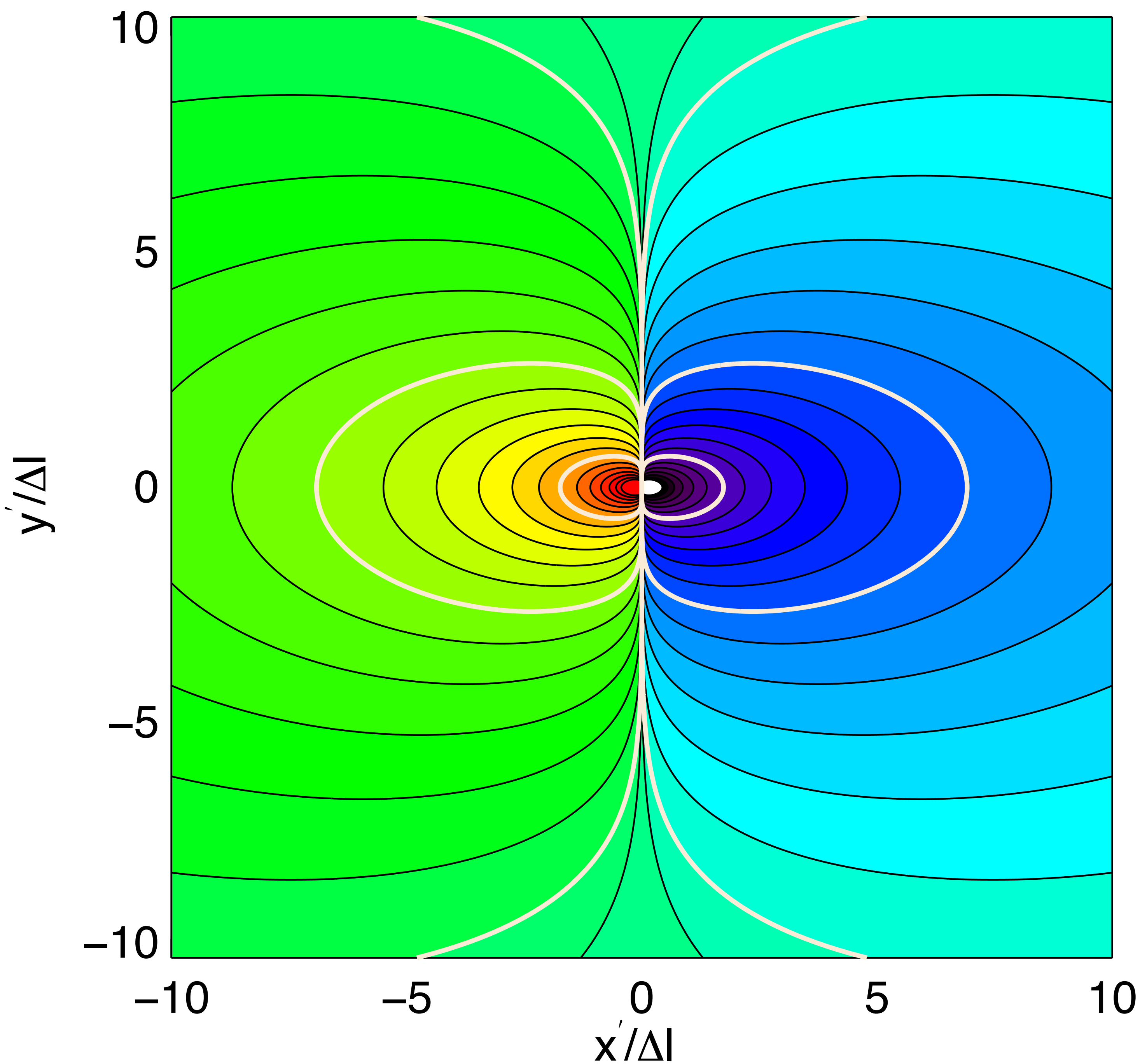

$$M(R) = M_{\rm BH} + M_\star(R)$$

Isovelocity contours on the plane of the sky: spider diagram

Contours are log spaced by 0.5 dex, V_{sys}=0

Projected line-of-sight velocity

We finally obtain

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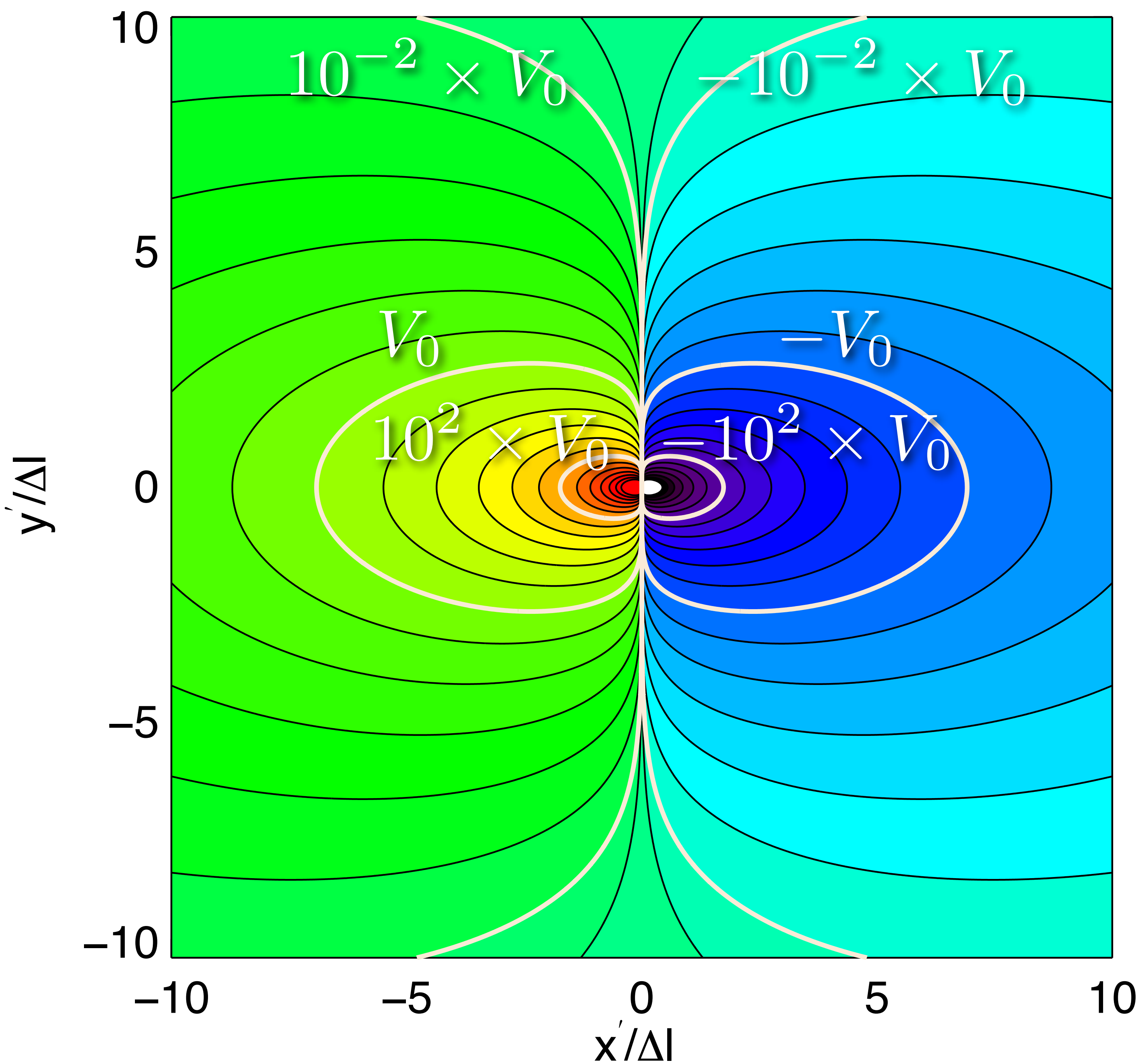

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Isovelocity contours on the plane of the sky: spider diagram

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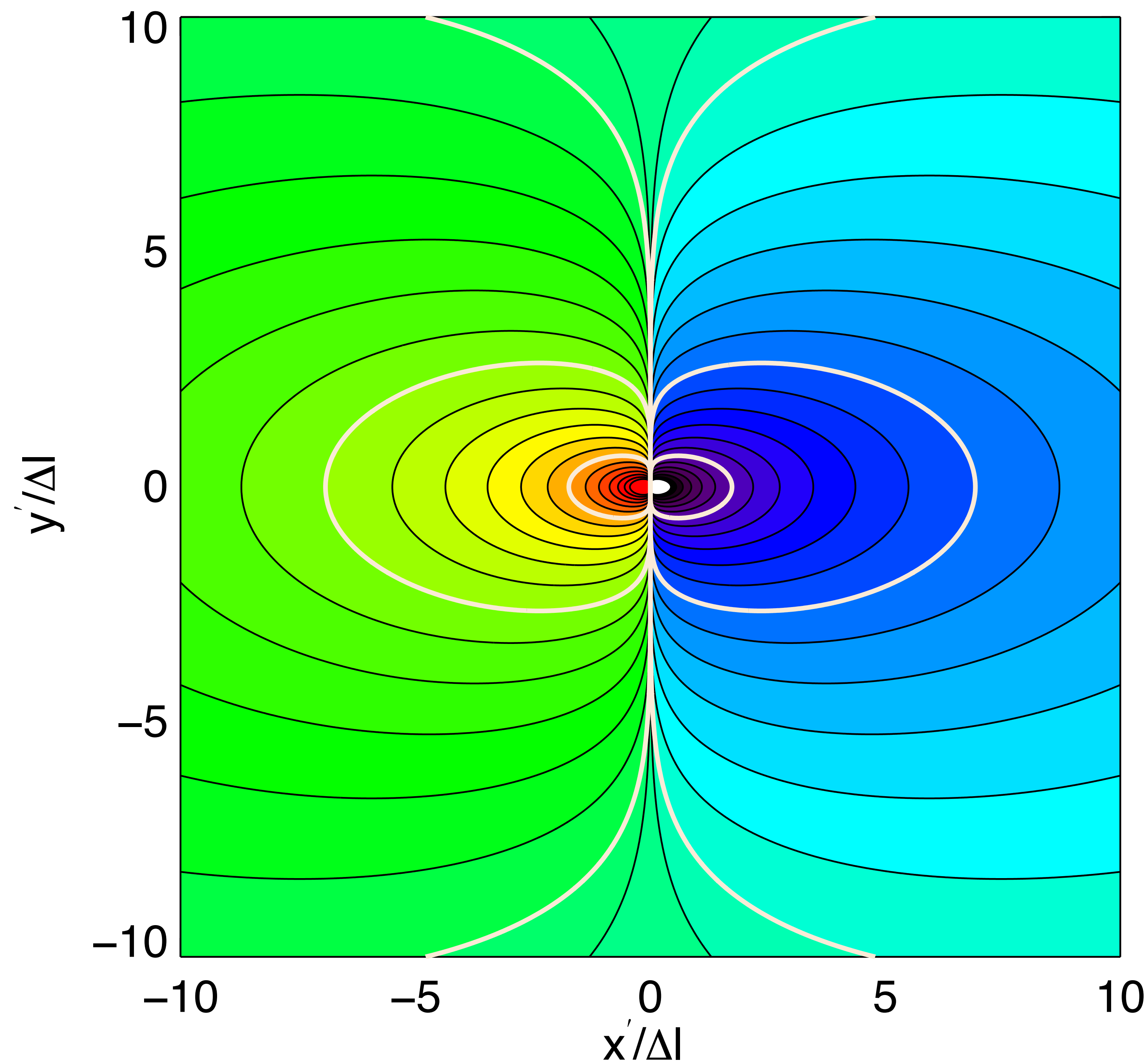
Projected line-of-sight velocity

Let us put infinitely thin spectrograph slits and observe the velocities along the slit.



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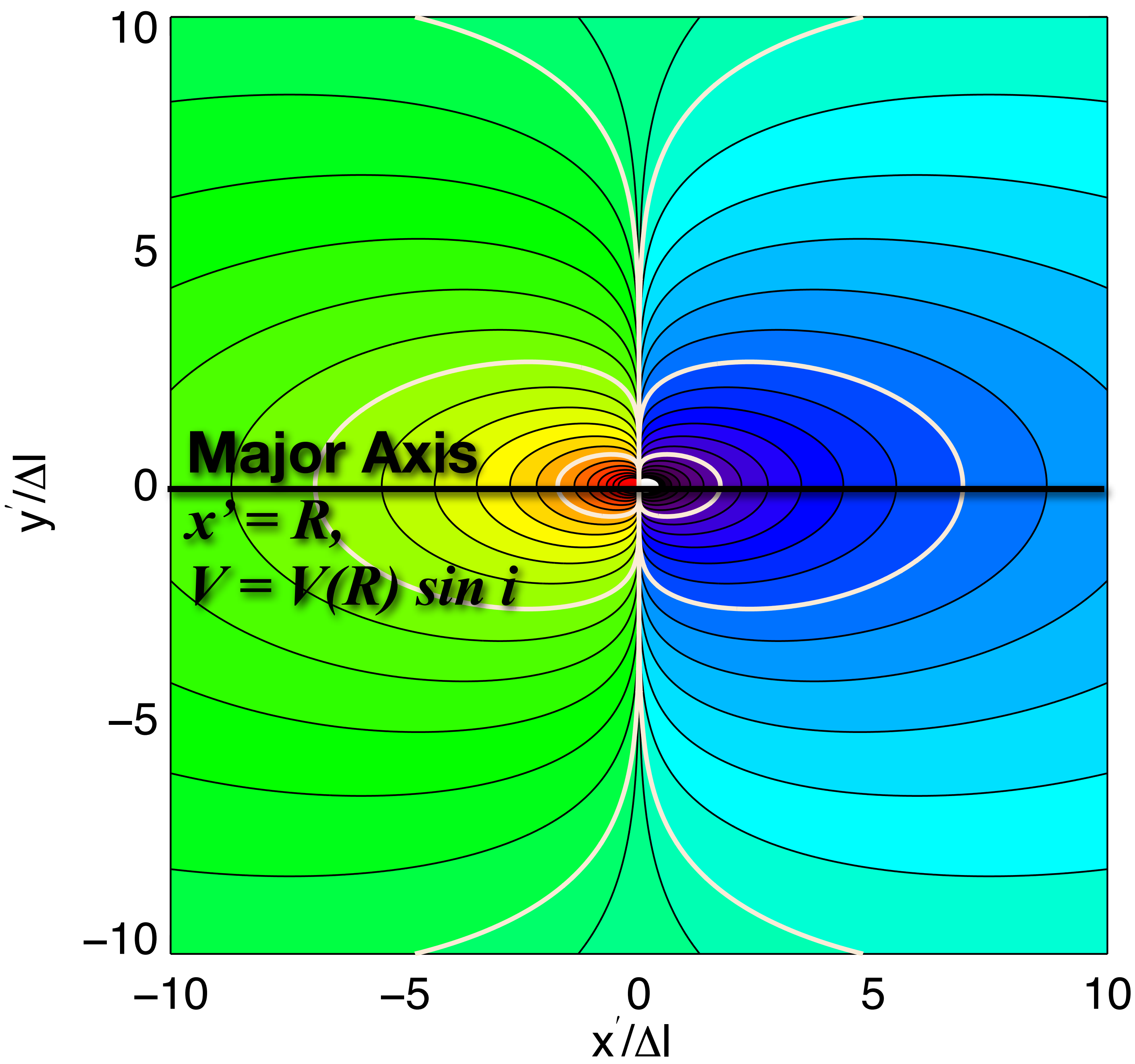

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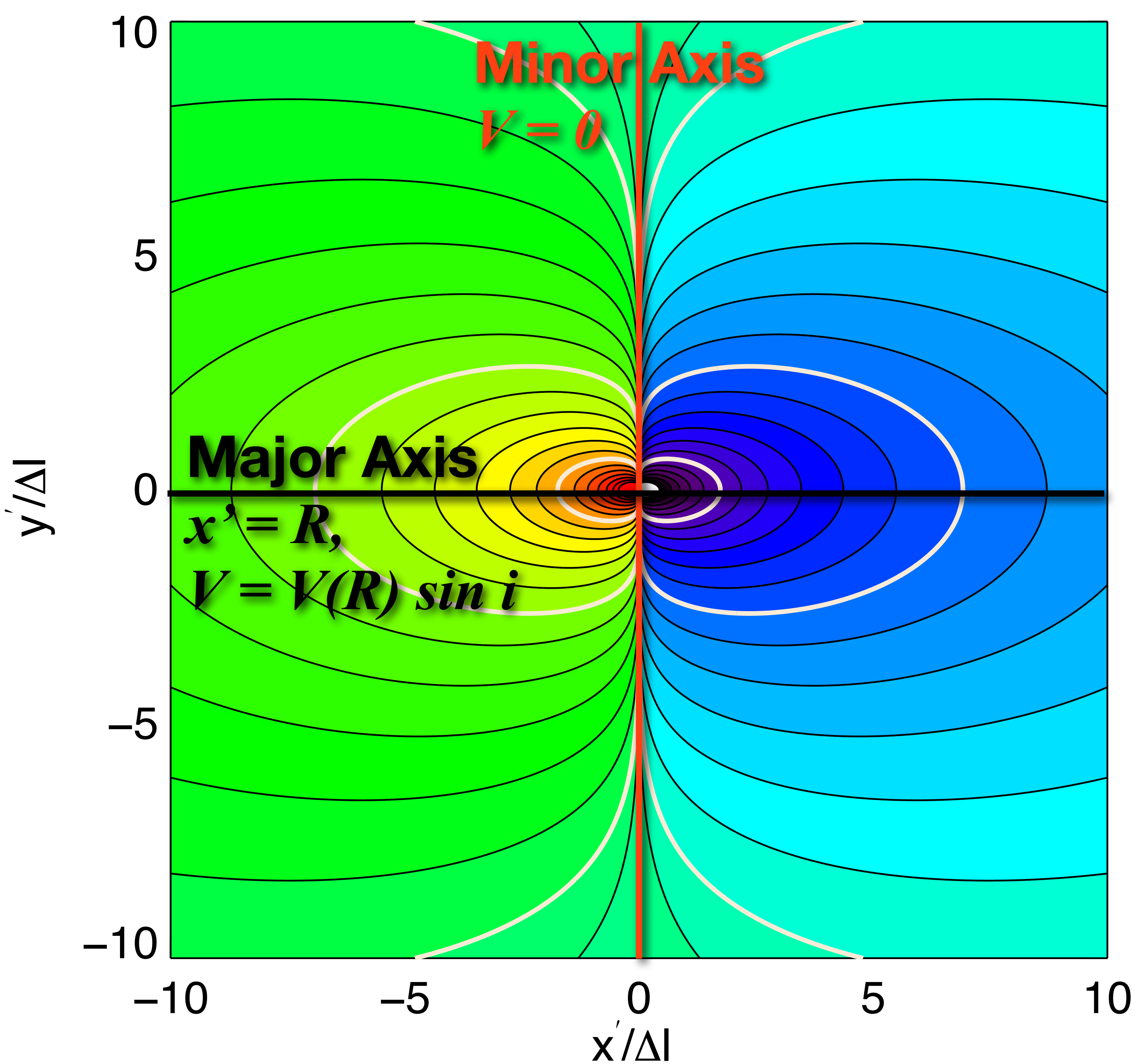

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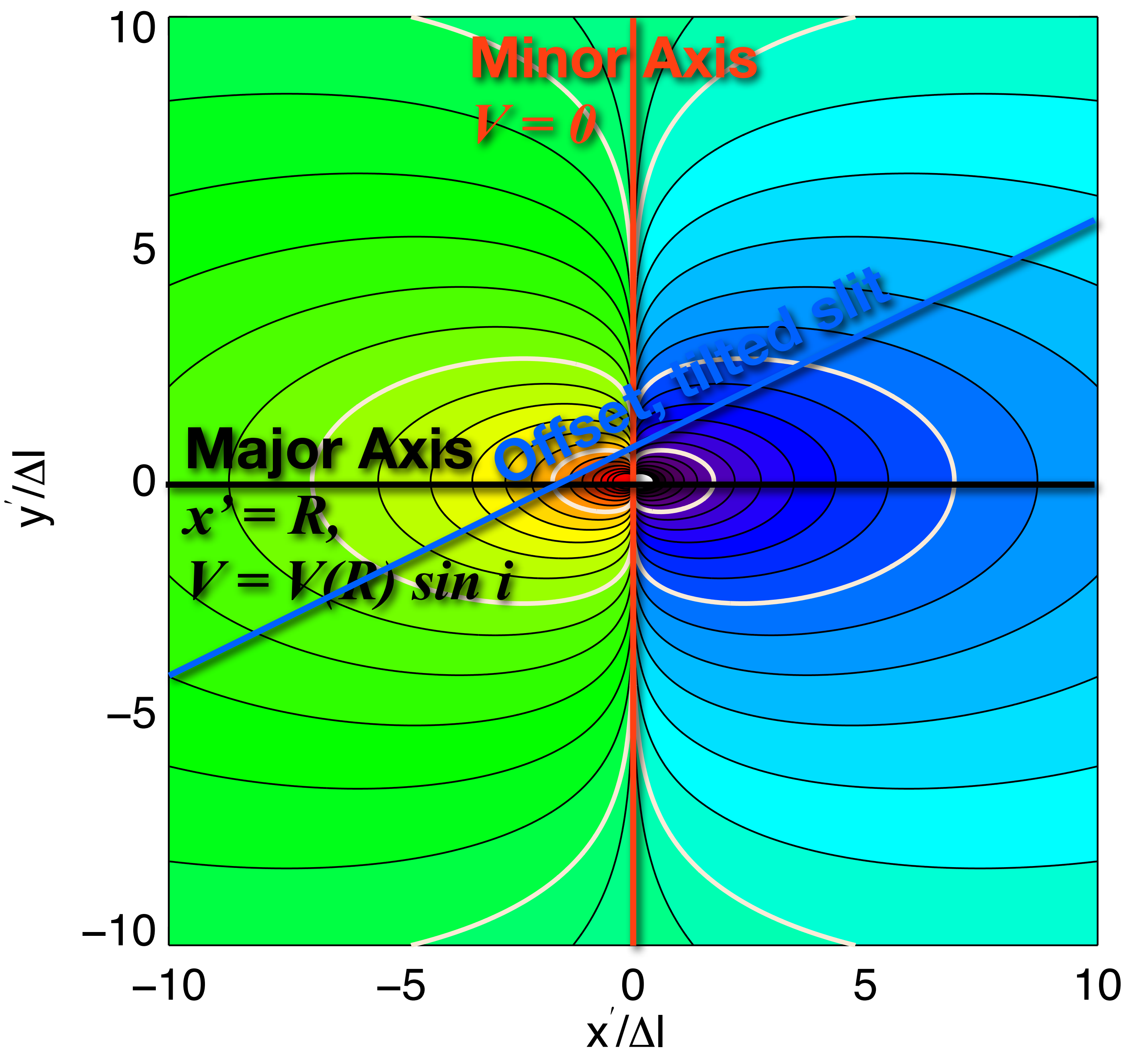

Projected line-of-

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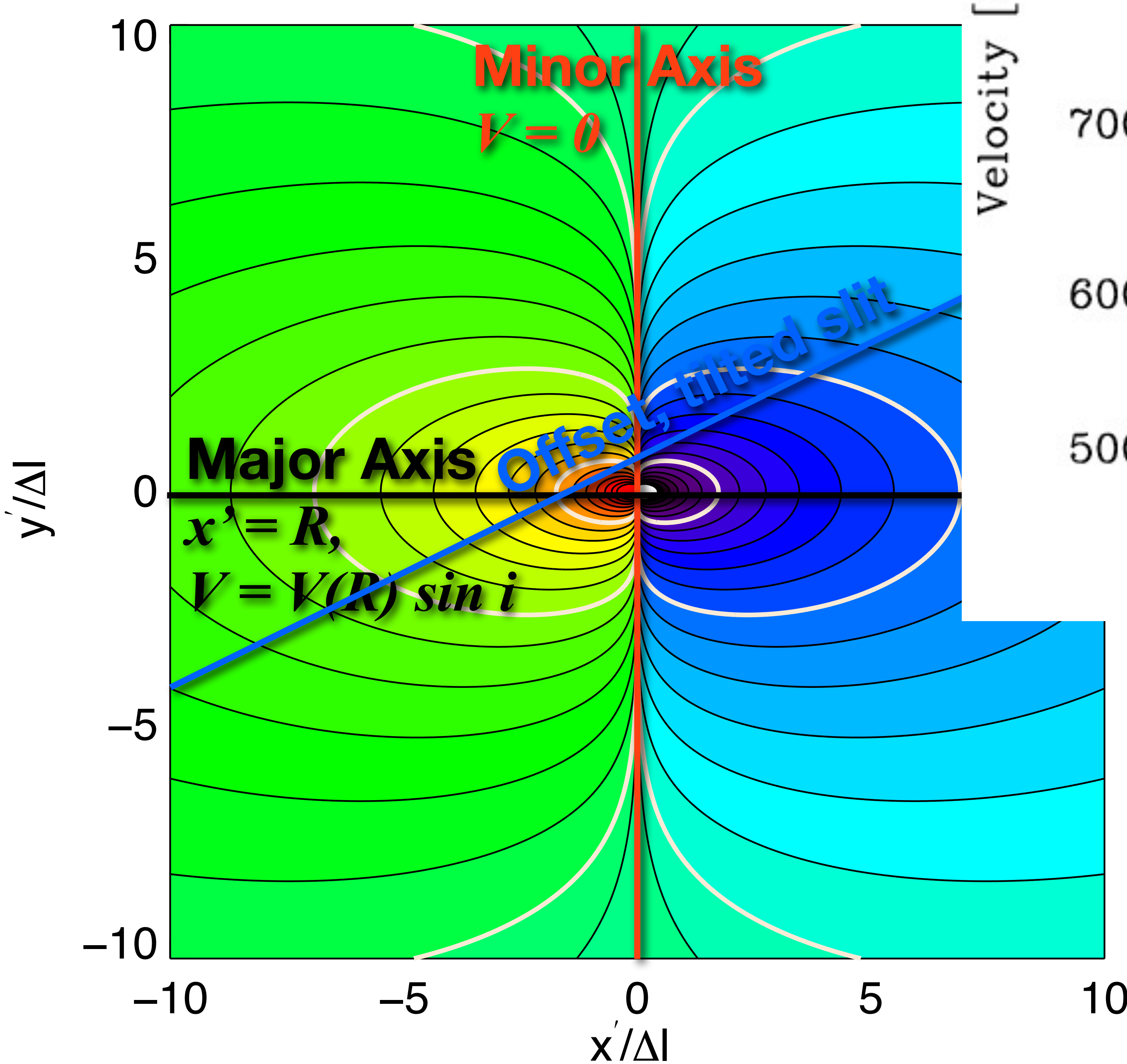
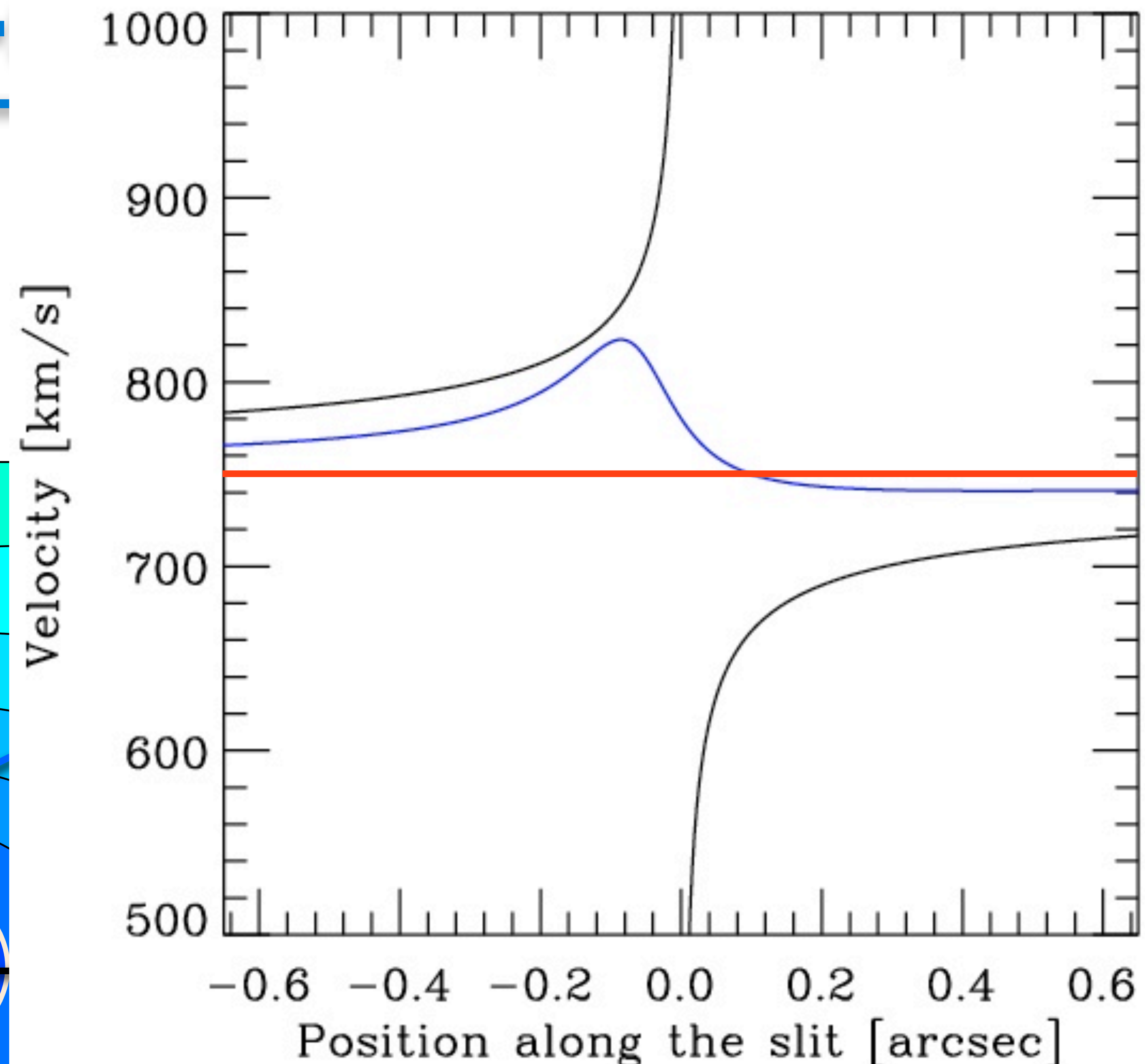




Instrumental effects

In reality there is a finite spatial resolution (either diffraction limit, or seeing, or intermediate resolution from AO assisted observations).

Given

- $\Sigma(x',y')$ instrinsic surface brightness of line emission
- $P(x',y')$ Point Spread Function (PSF) of observations (e.g. Gaussian)
- $V_{los}(x',y')$ velocity along the line of sight

we have

$$\Sigma_{conv}(x'_p, y'_p) = \int_{-\infty}^{+\infty} dx' \int_{-\infty}^{+\infty} dy' \Sigma(x', y') \times P(x'_p - x', y'_p - y')$$

intrinsic surface brightness on the plane of the sky, convolved with PSF

$$\Sigma_{obs}(x_p, y_p) = \frac{1}{\Delta x_p \Delta y_p} \int_{x_p - \Delta x_p}^{x_p + \Delta x_p} dx'_p \int_{y_p - \Delta y_p}^{y_p + \Delta y_p} dy'_p \Sigma_{conv}(x'_p, y'_p)$$

convolved surface brightness averaged over aperture
(pixel × pixel, slit × pixel, etc.)

Instrumental effects

Obtaining (with a simplified notation):

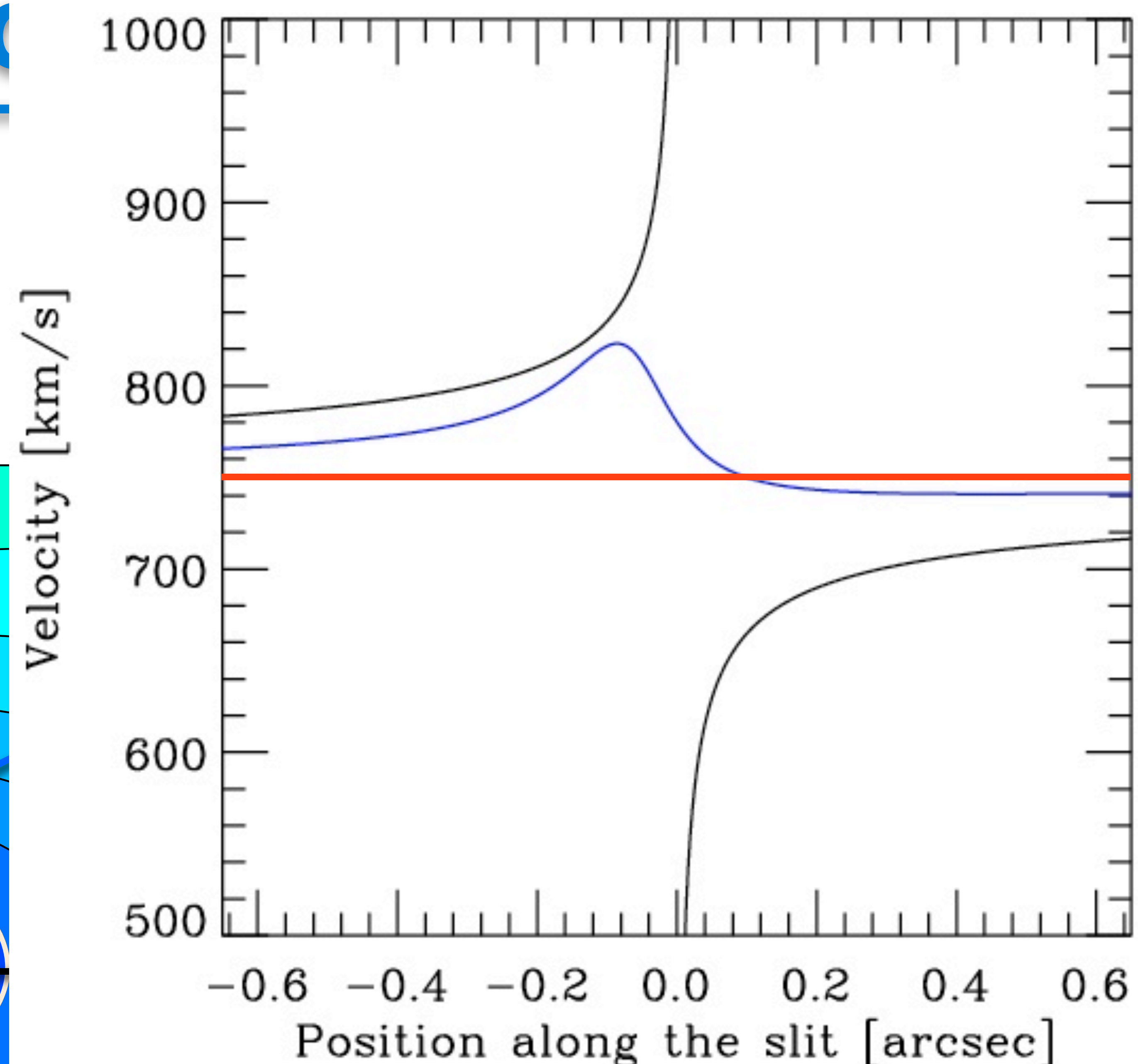
$$\Sigma_{obs}(x_p, y_p) = \frac{1}{\Delta A} \int_{\Delta A} \Sigma \otimes P(x'_p, y'_p)\, dx'_p dy'_p$$

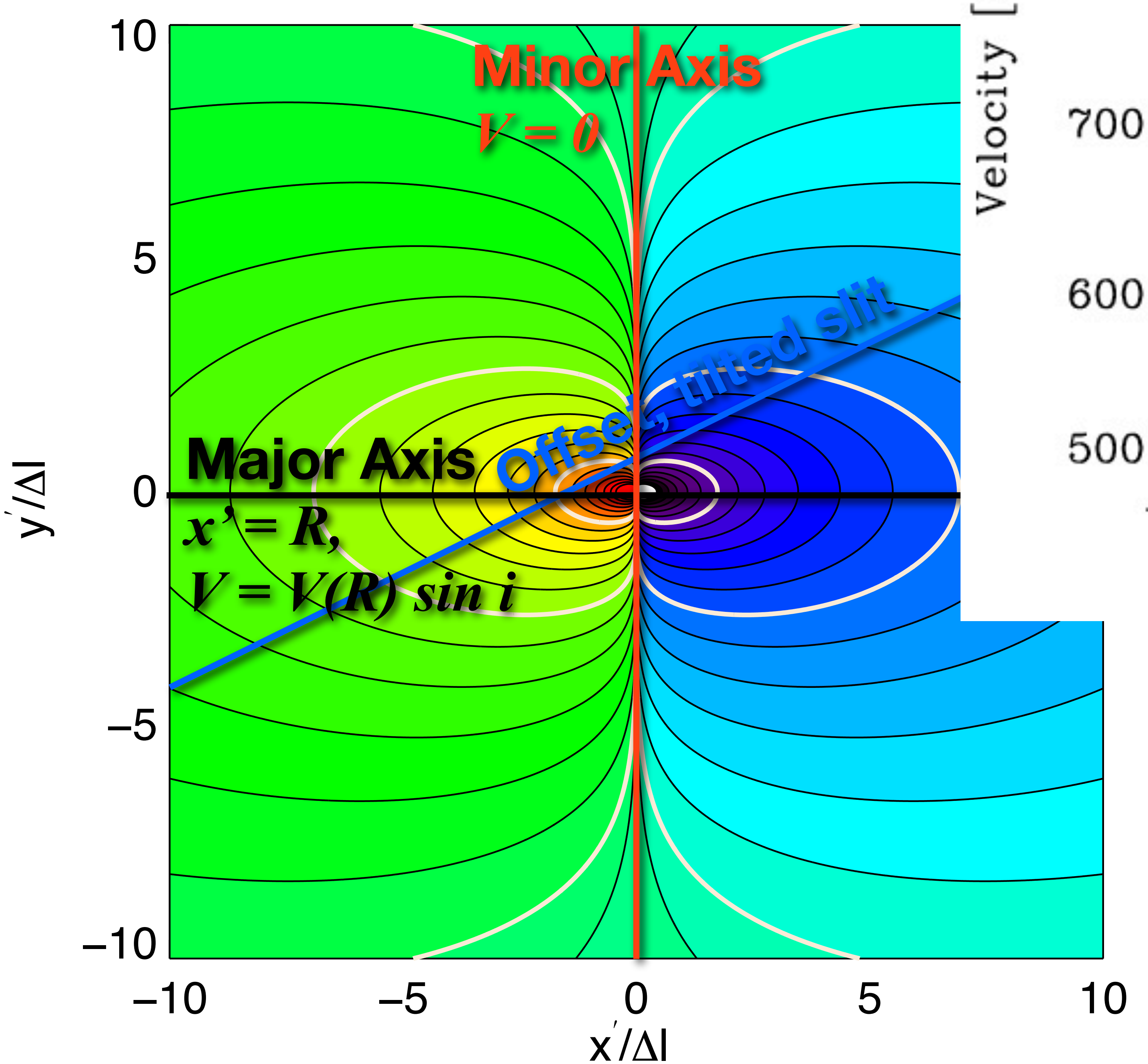
Observed velocity and velocity dispersion are weighted by the gas emissivity i.e.

$$V_{obs}(x_p, y_p) = \frac{1}{\Sigma_{obs}(x_p, y_p)} \int_{\Delta A} (V\Sigma) \otimes P(x'_p, y'_p)\, dx'_p dy'_p$$

$$\sigma^2_{obs}(x_p, y_p) = \frac{1}{\Sigma_{obs}(x_p, y_p)} \int_{\Delta A} (V^2\Sigma) \otimes P(x'_p, y'_p)\, dx'_p dy'_p - V^2_{obs}(x_p, y_p)$$

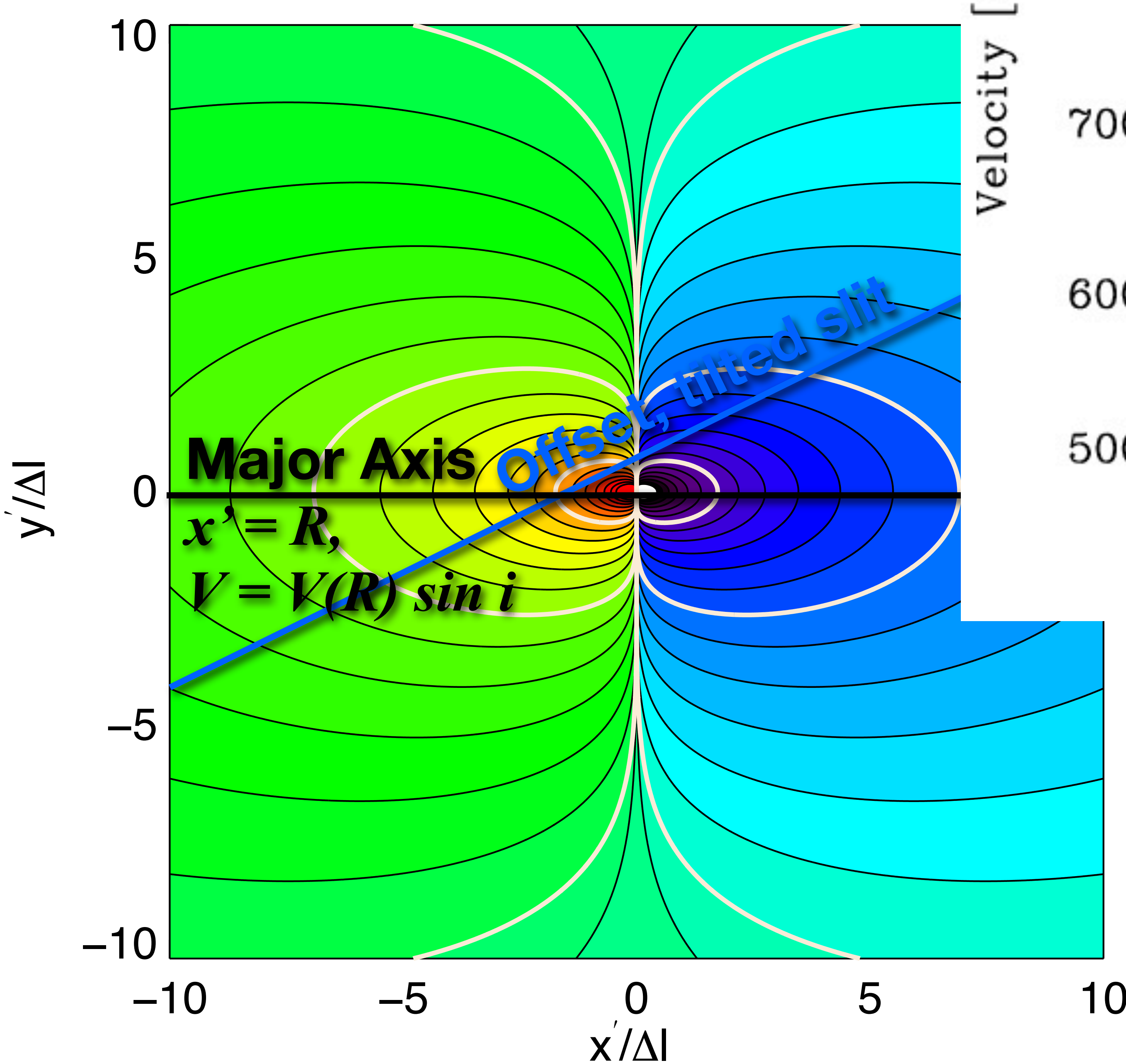
Instrumental effec

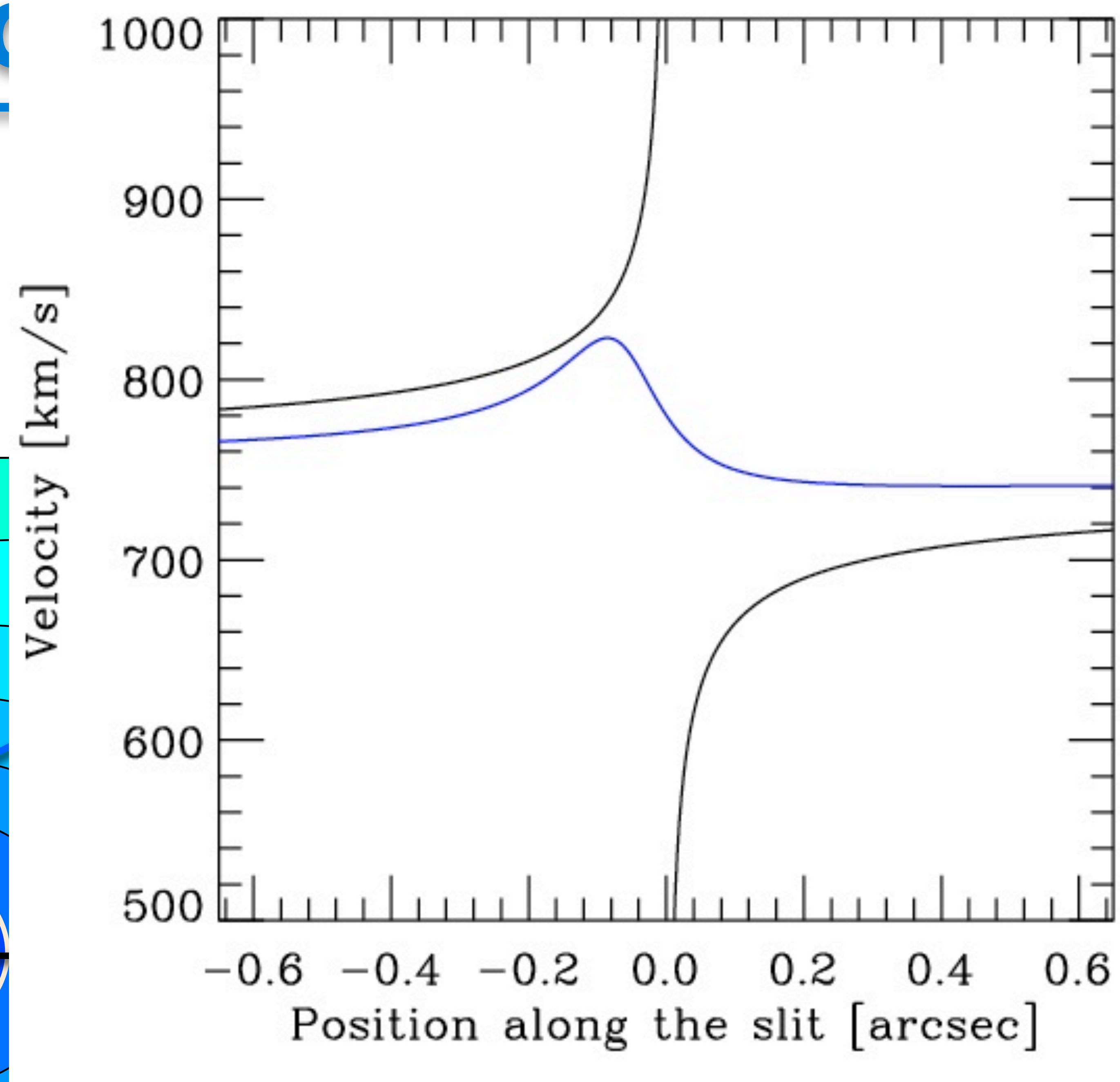




Rotation curves are "smoothed".
High velocities which are signature of BH are lost.

Instrumental effec





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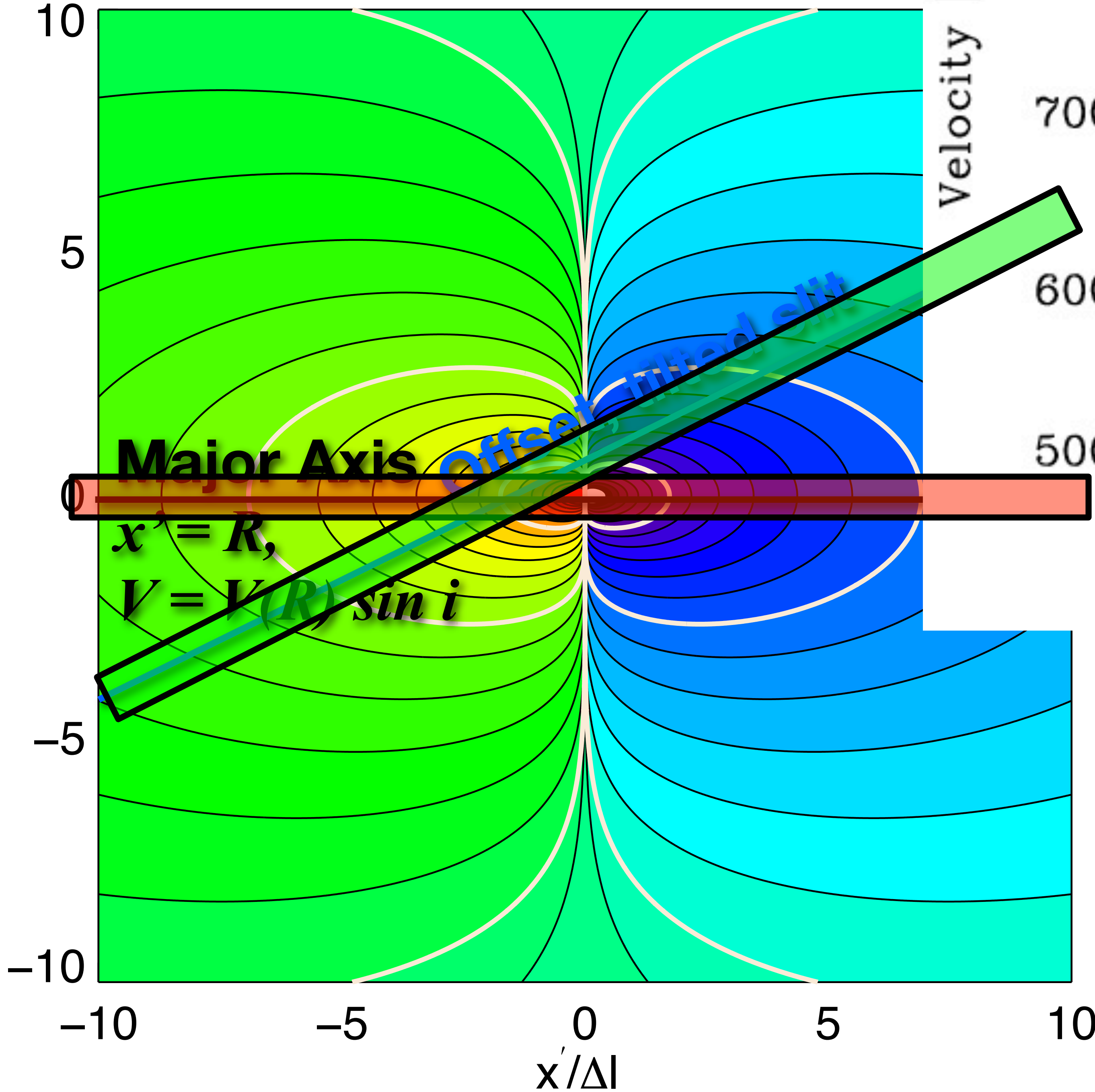

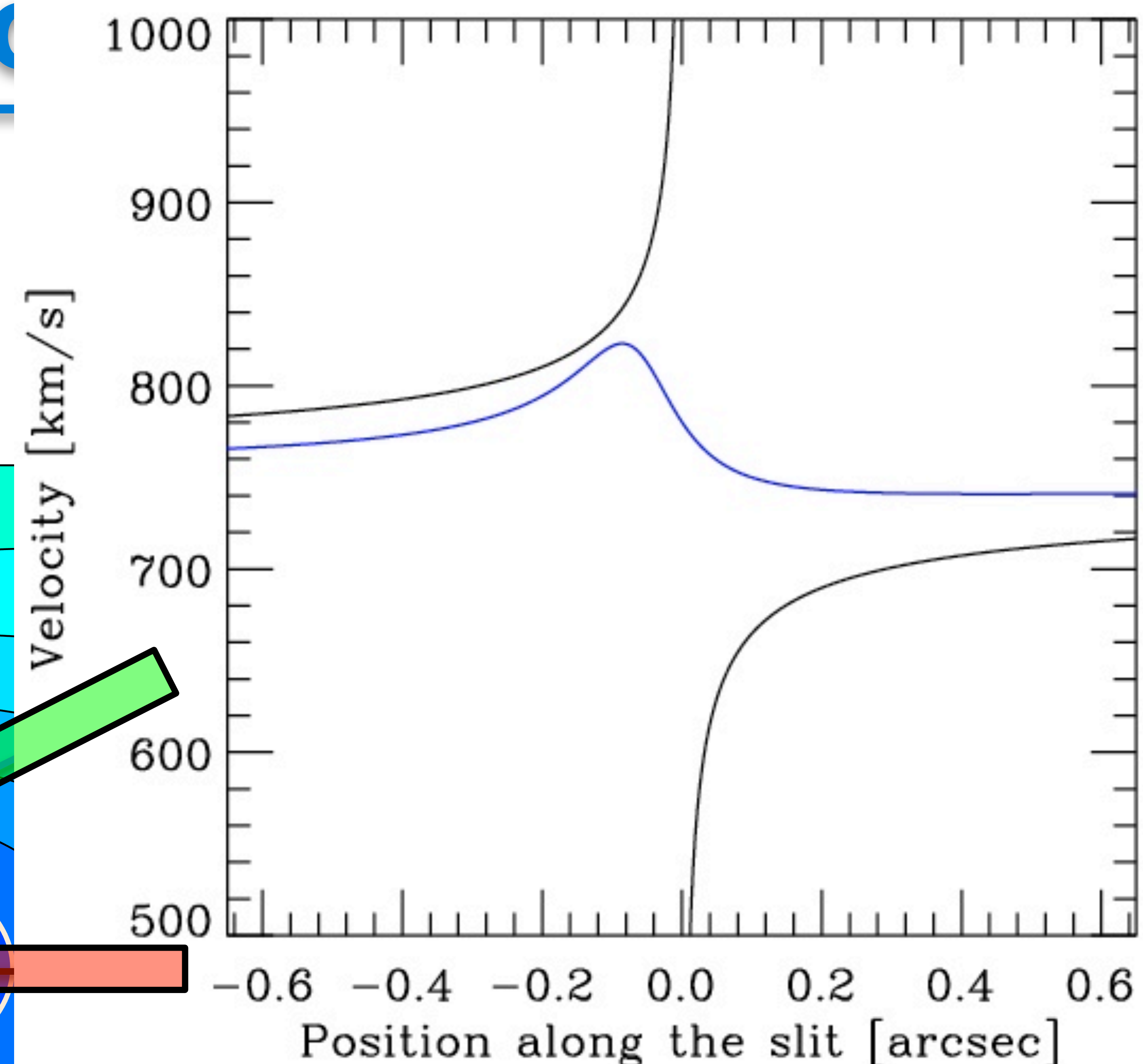

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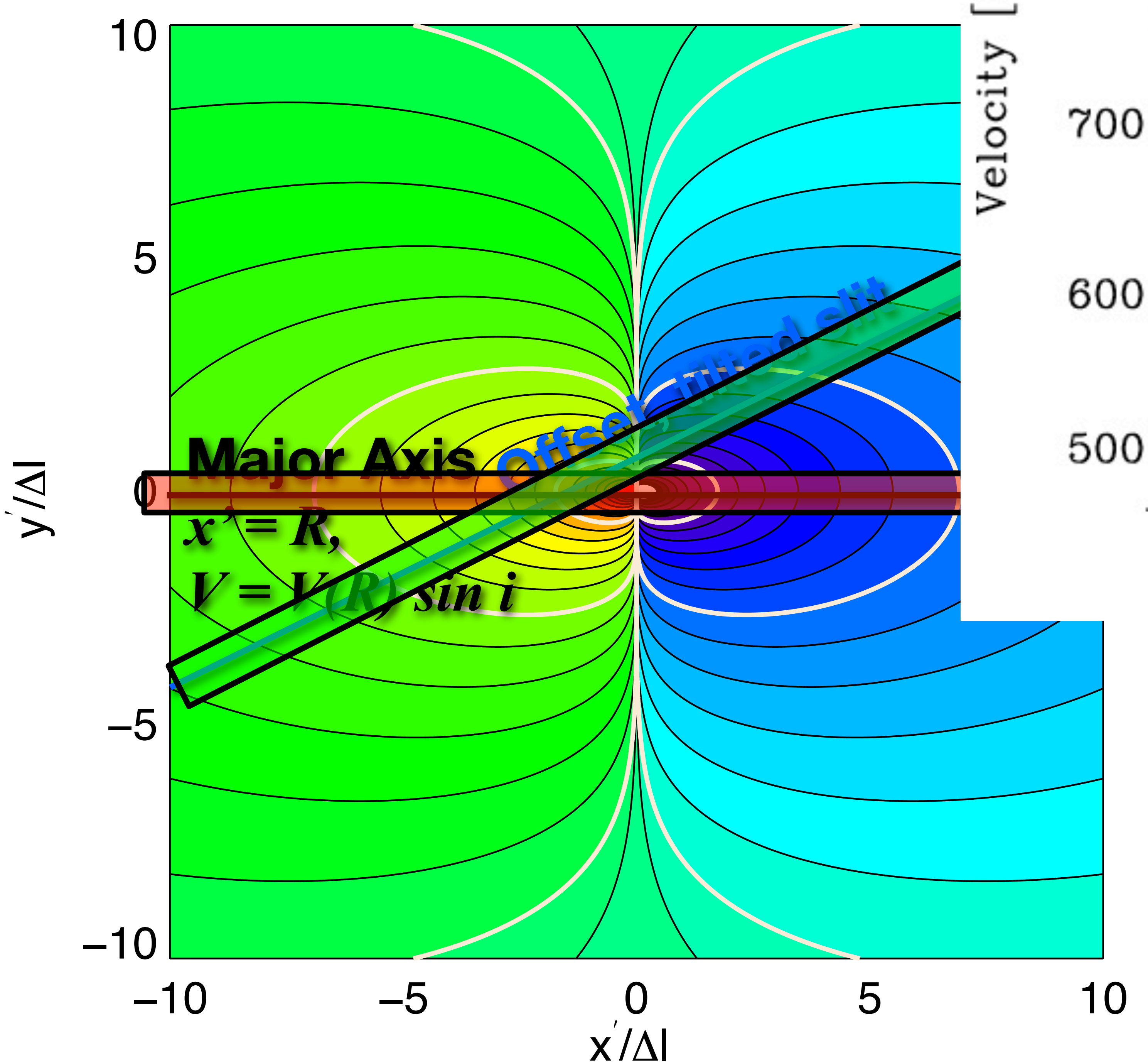

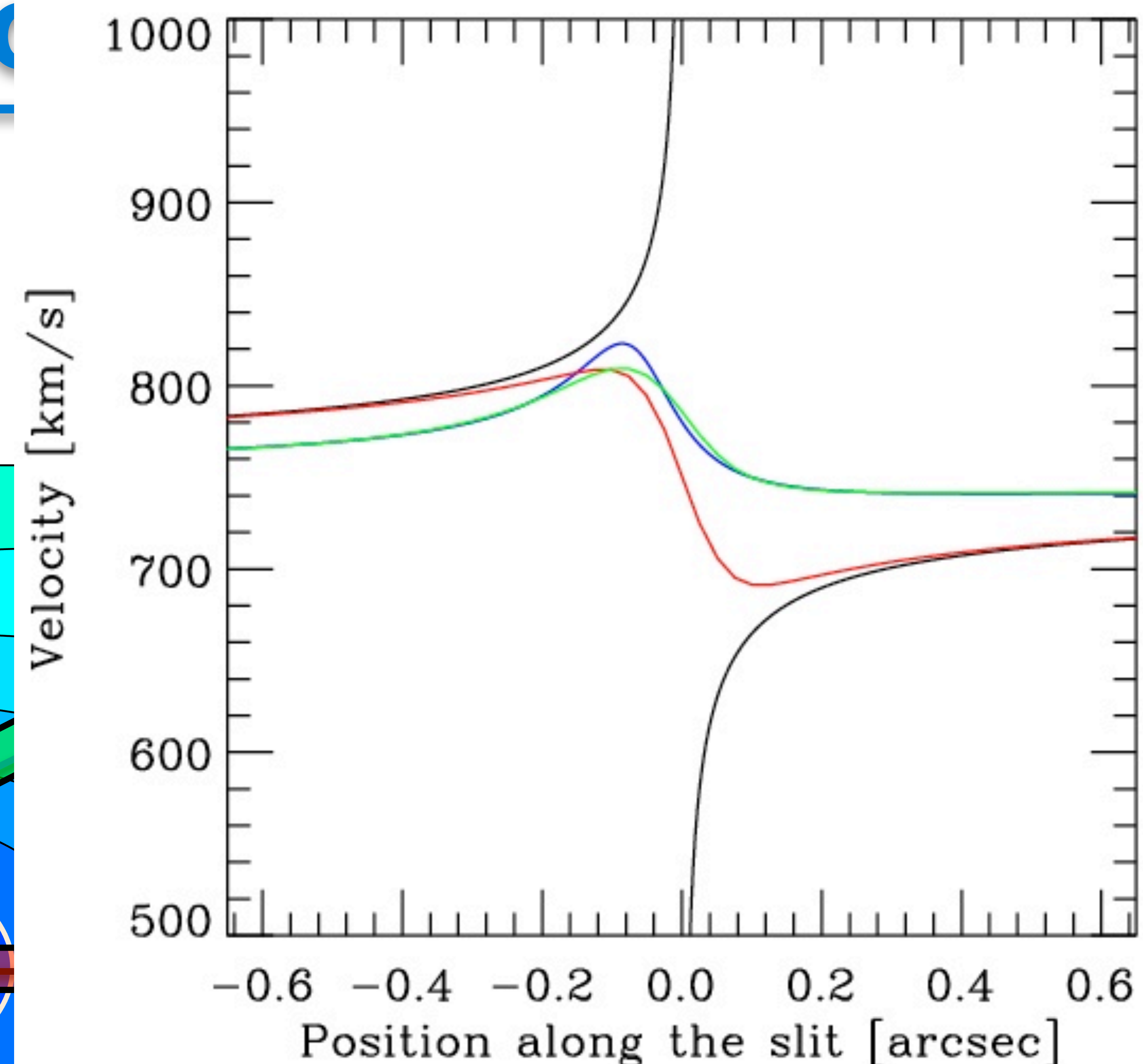

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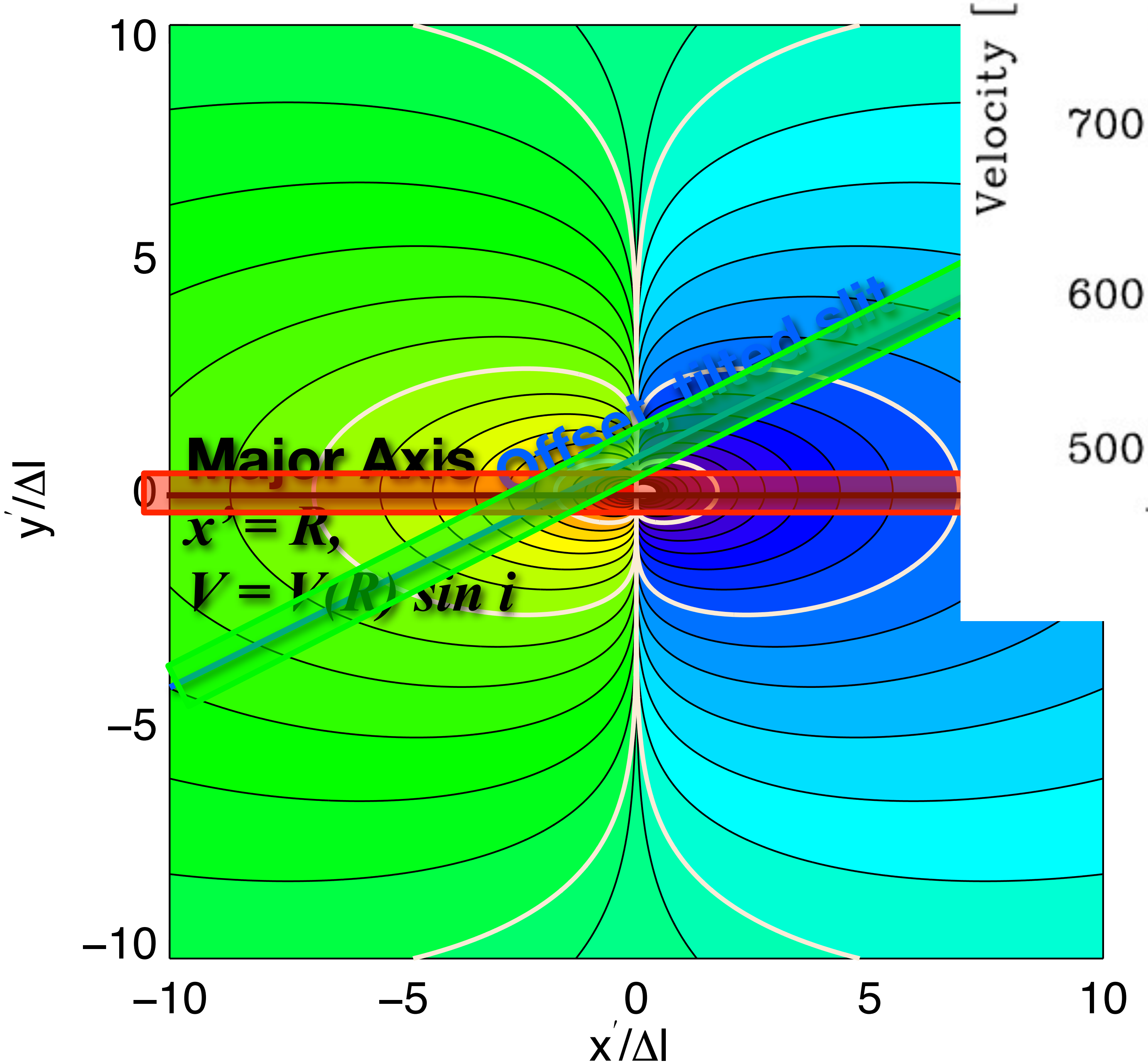
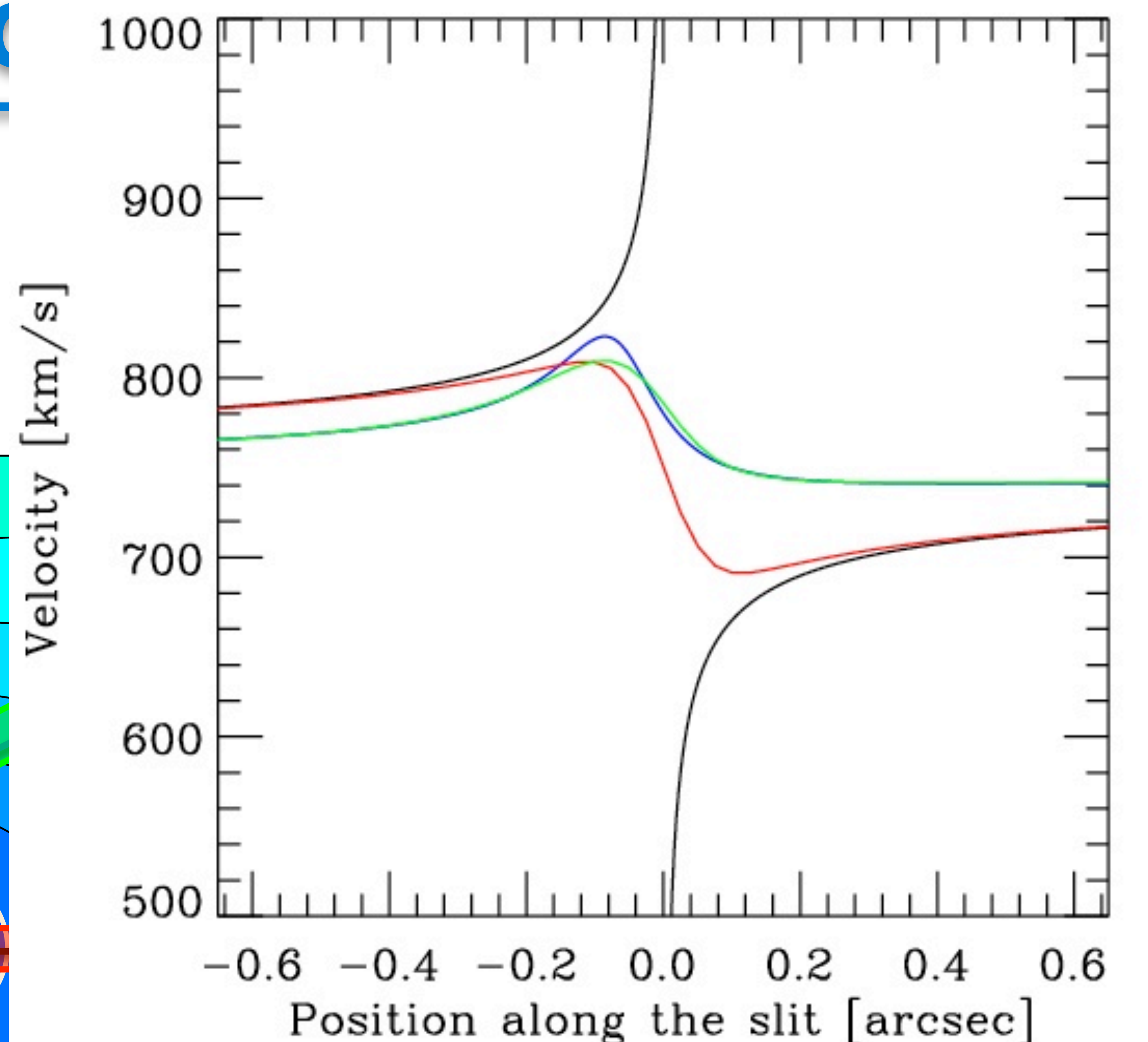
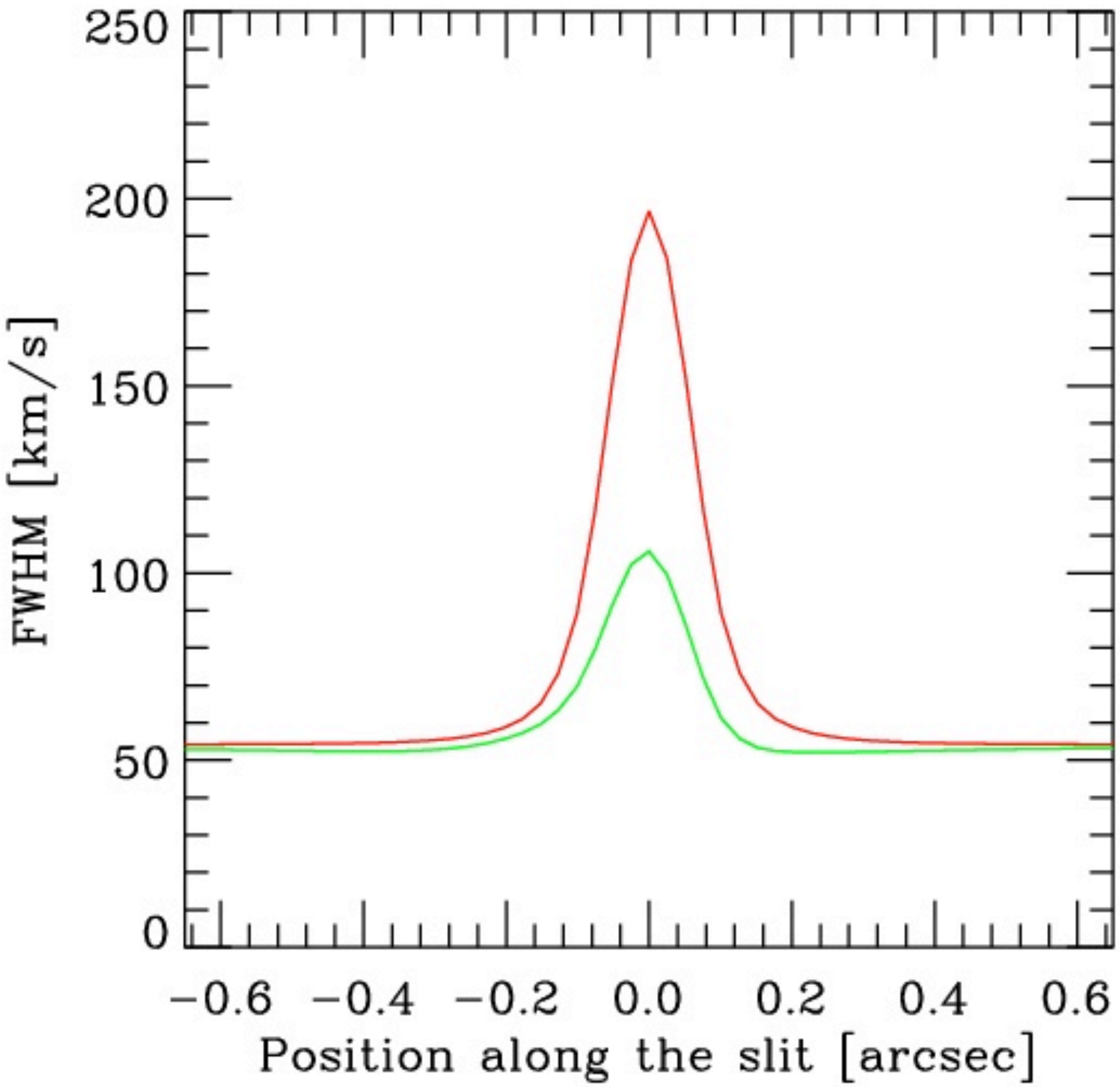
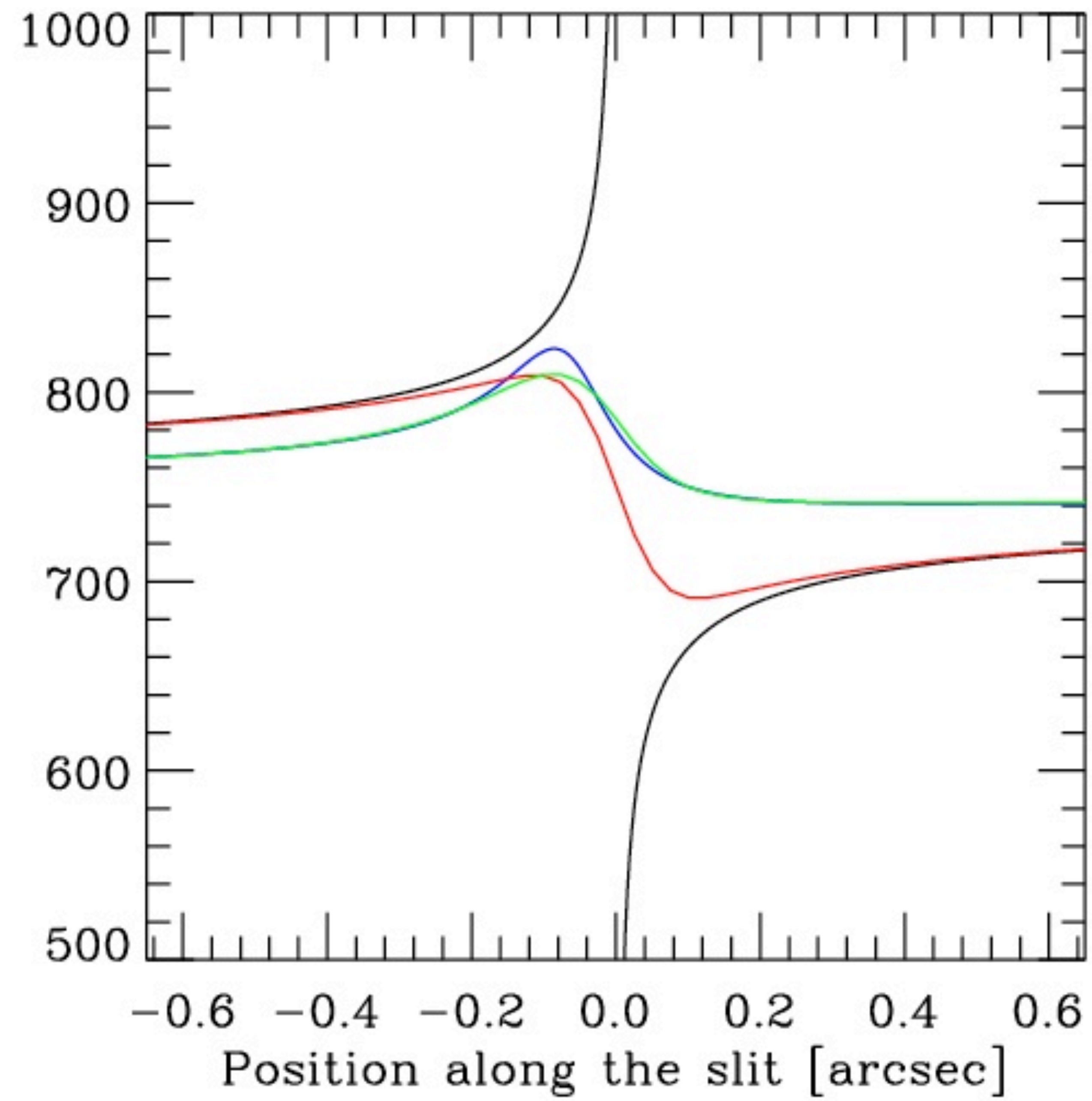
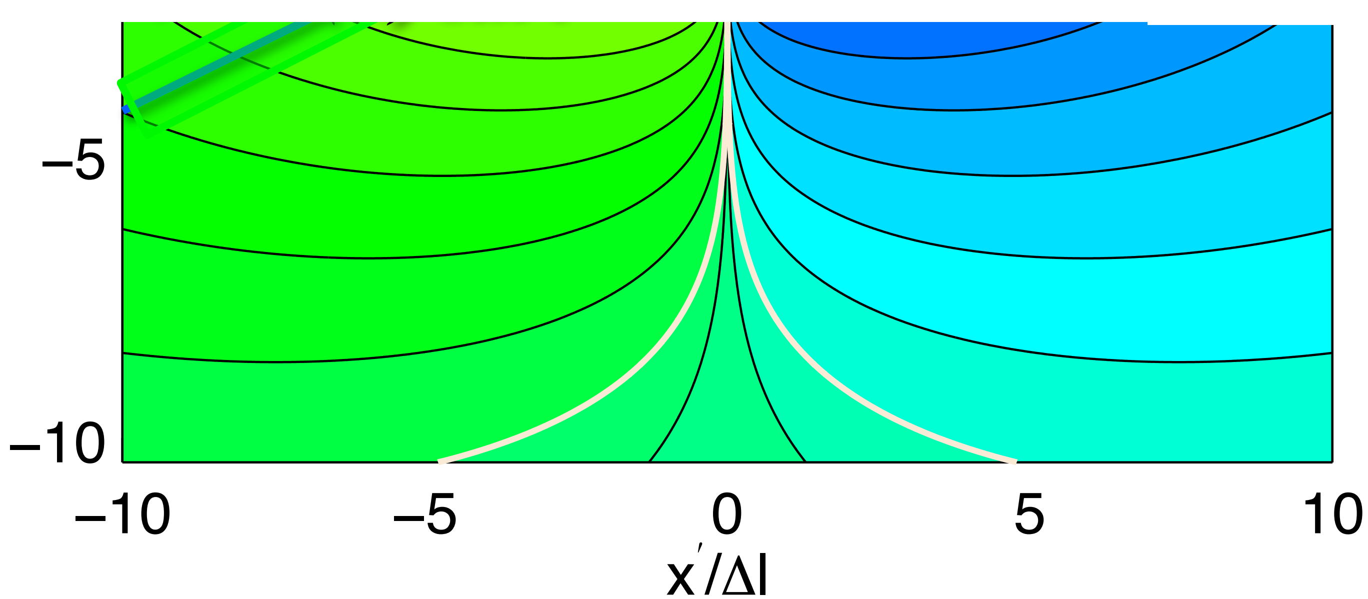
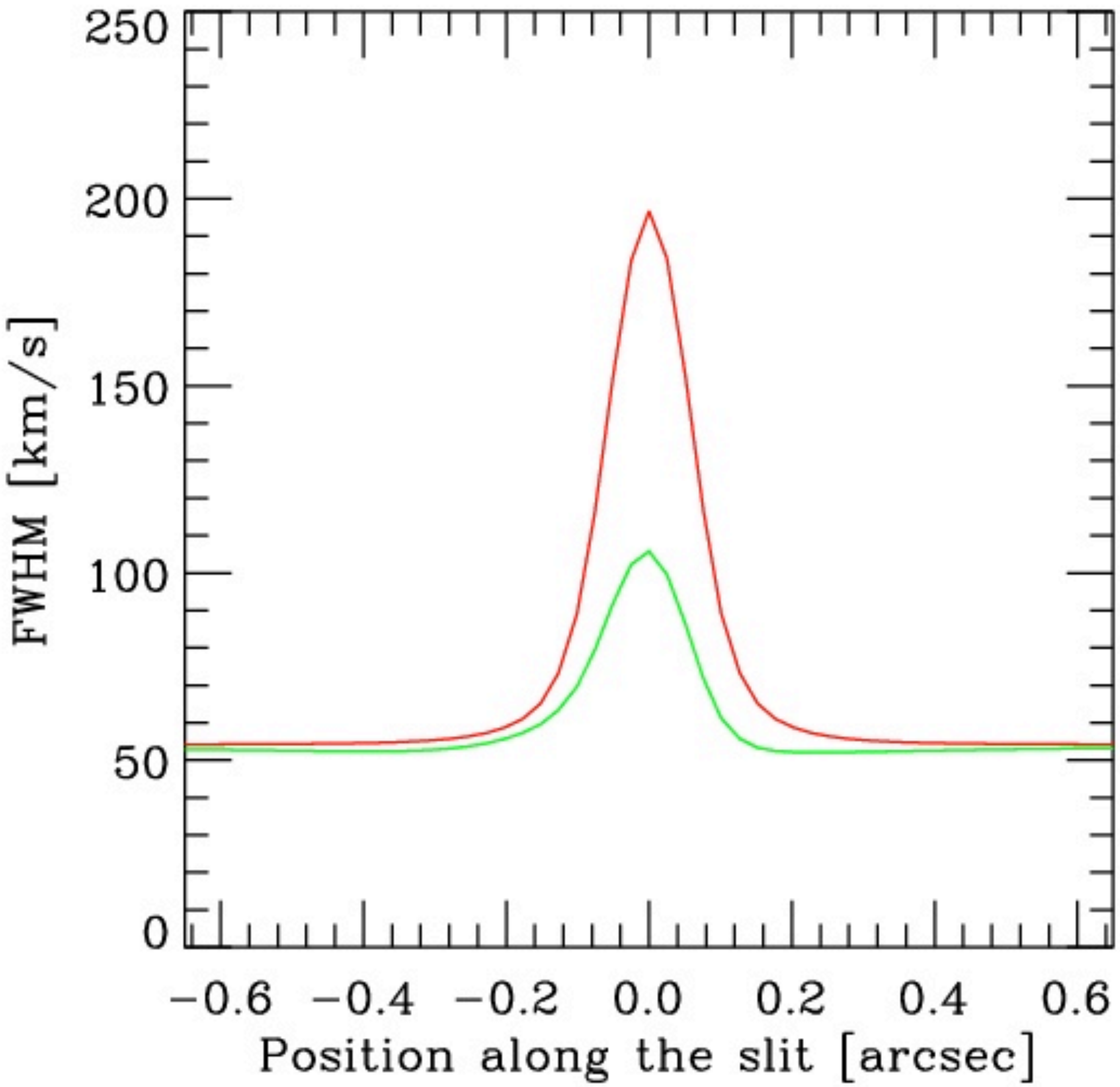


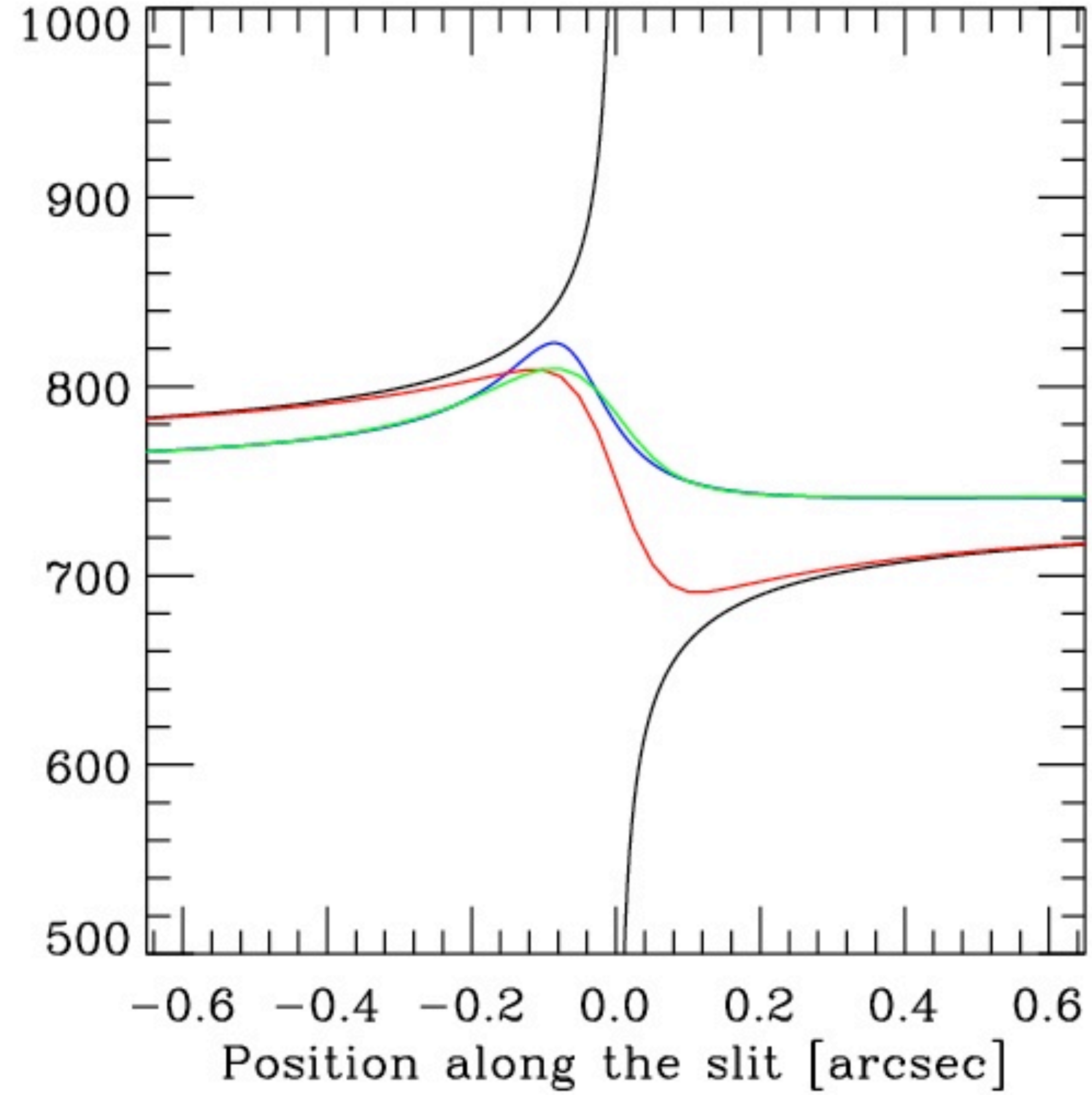


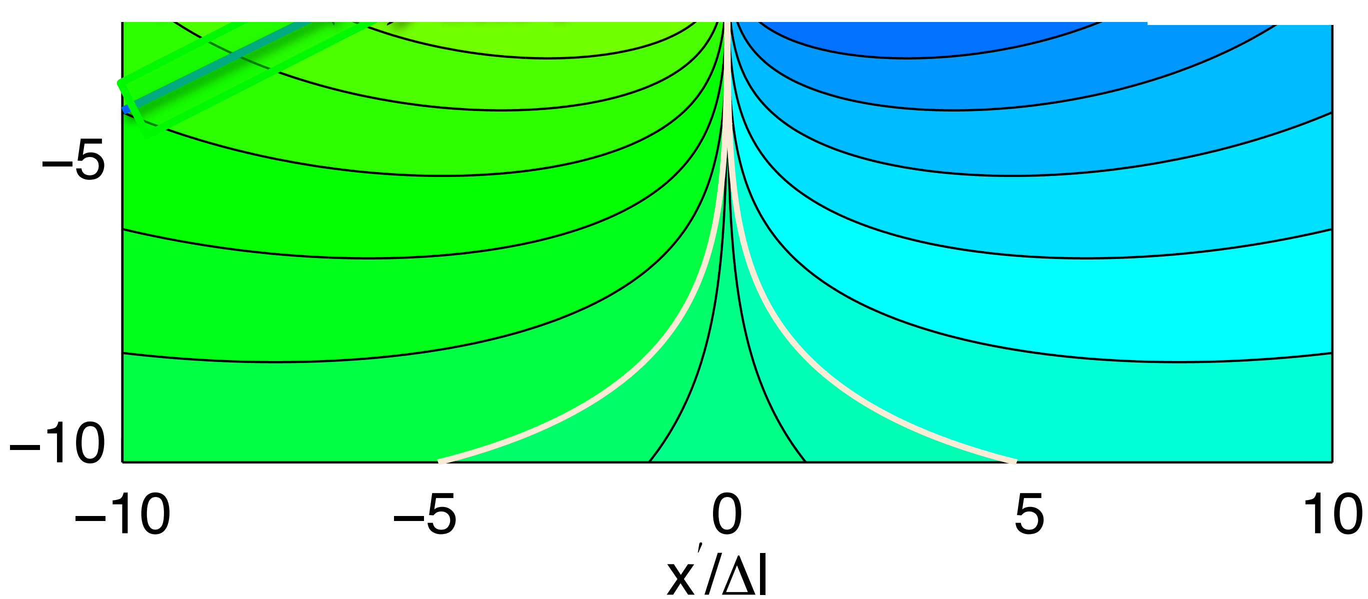

FWHM [km/s]
250
200
150
100
50
0
−0.6 −0.4 −0.2 0.0 0.2 0.4 0.6
Position along the slit [arcsec]


1000
900
800
700
600
500
−0.6 −0.4 −0.2 0.0 0.2 0.4 0.6
Position along the slit [arcsec]

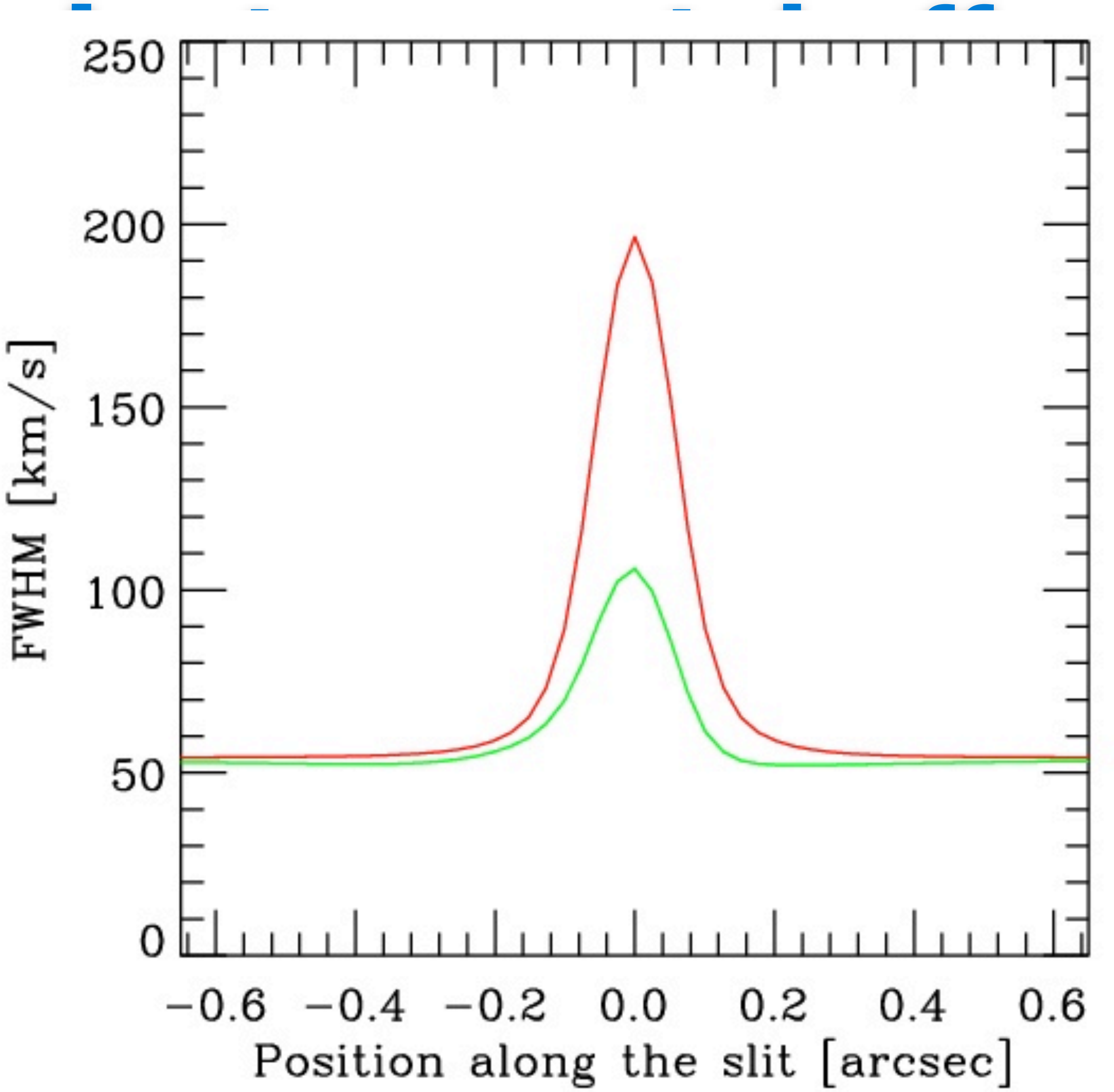

−5
−10
−10 −5 0 5 10
x′/Δl

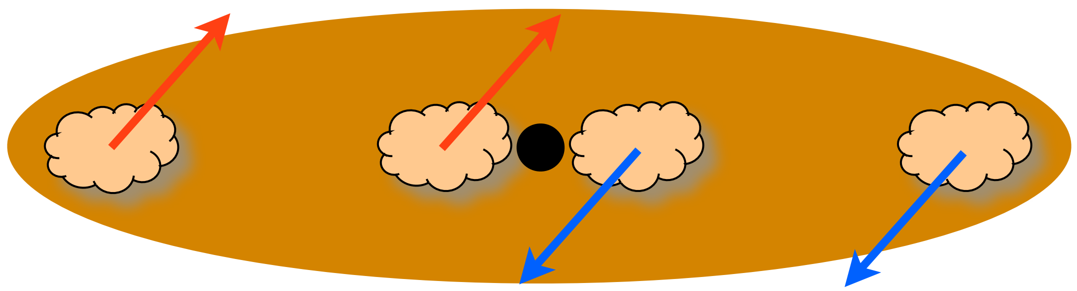


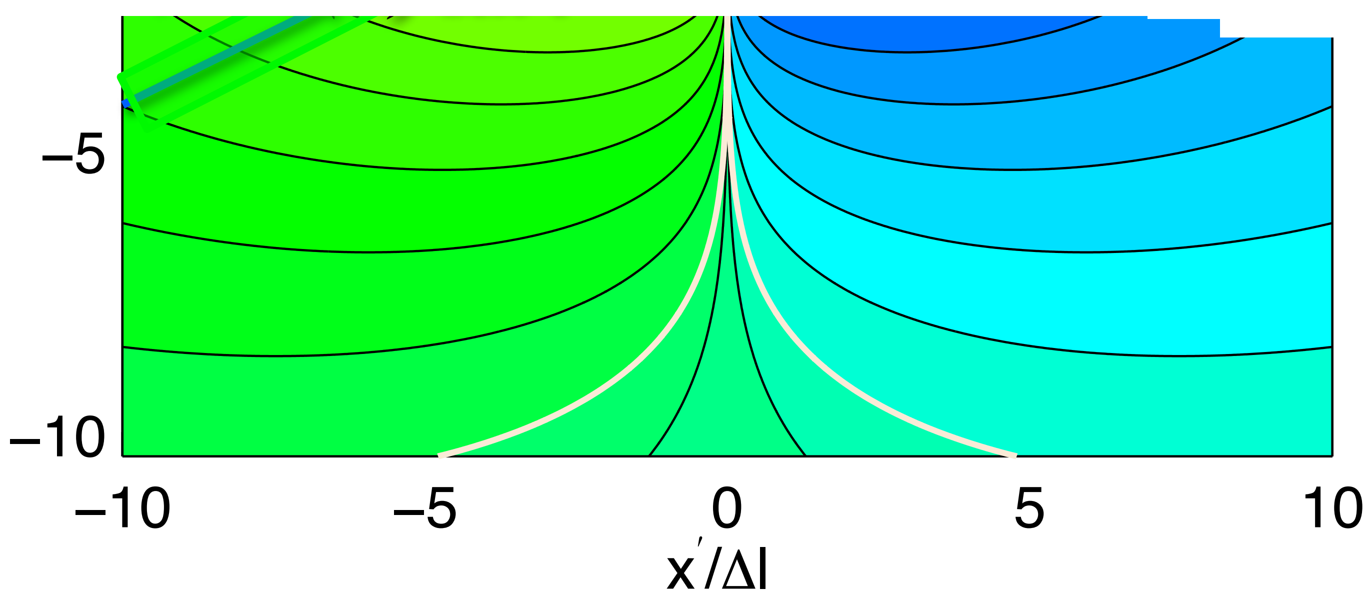




Line width increase around BH on region with size ~spatial resolution: this is due to *unresolved rotation*







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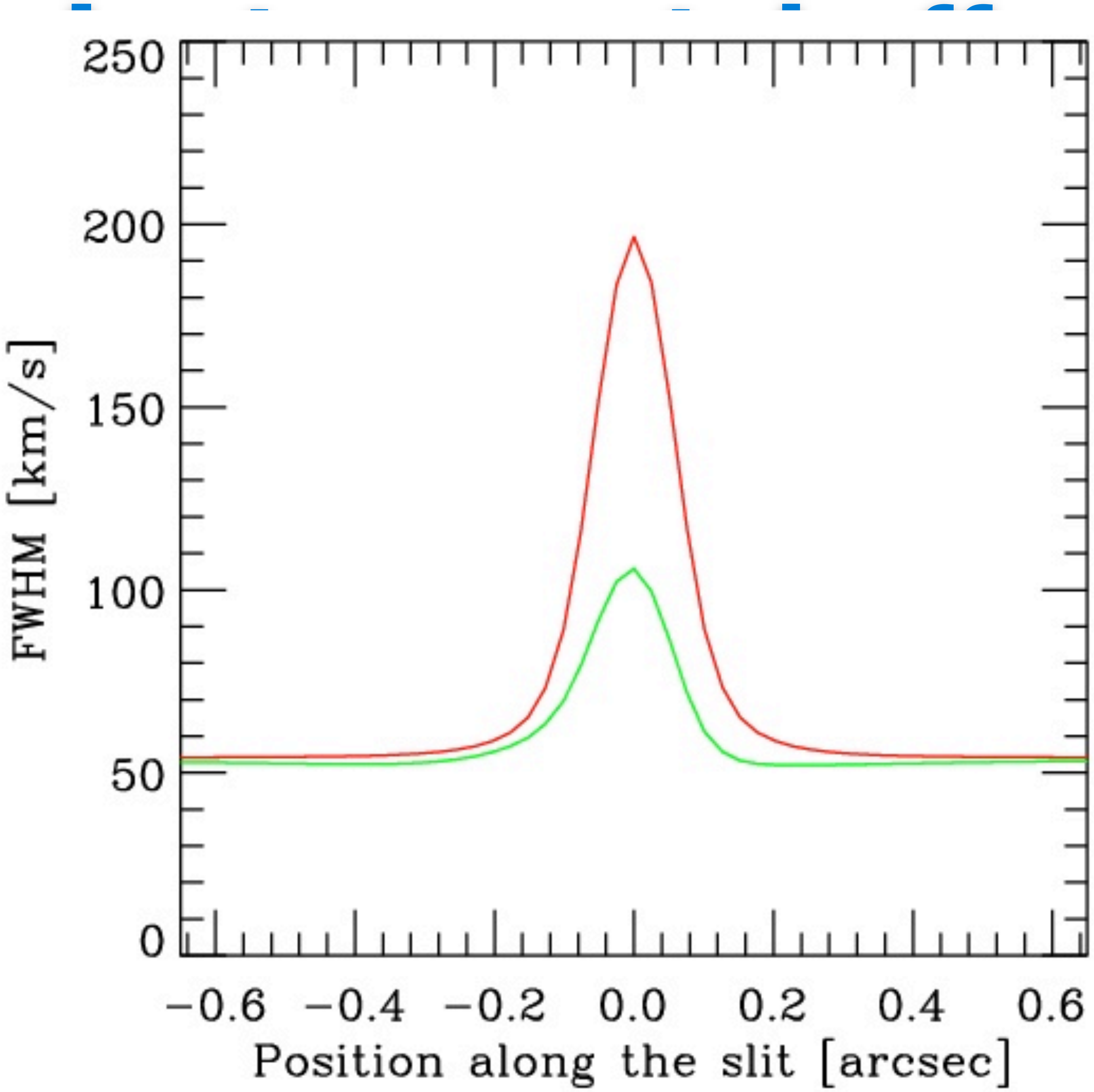

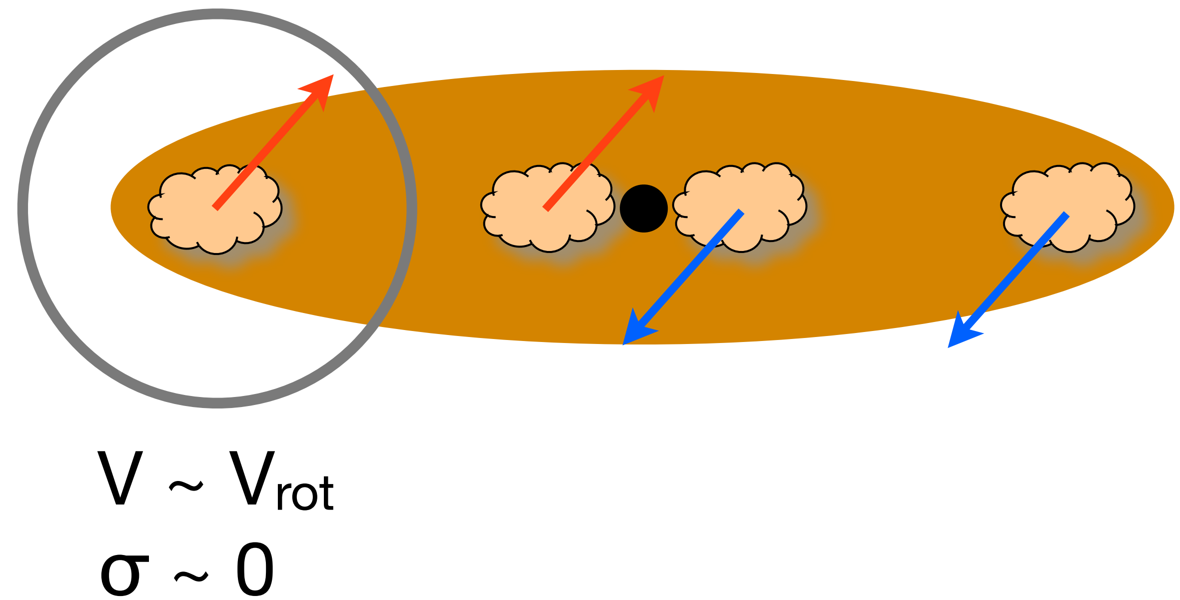

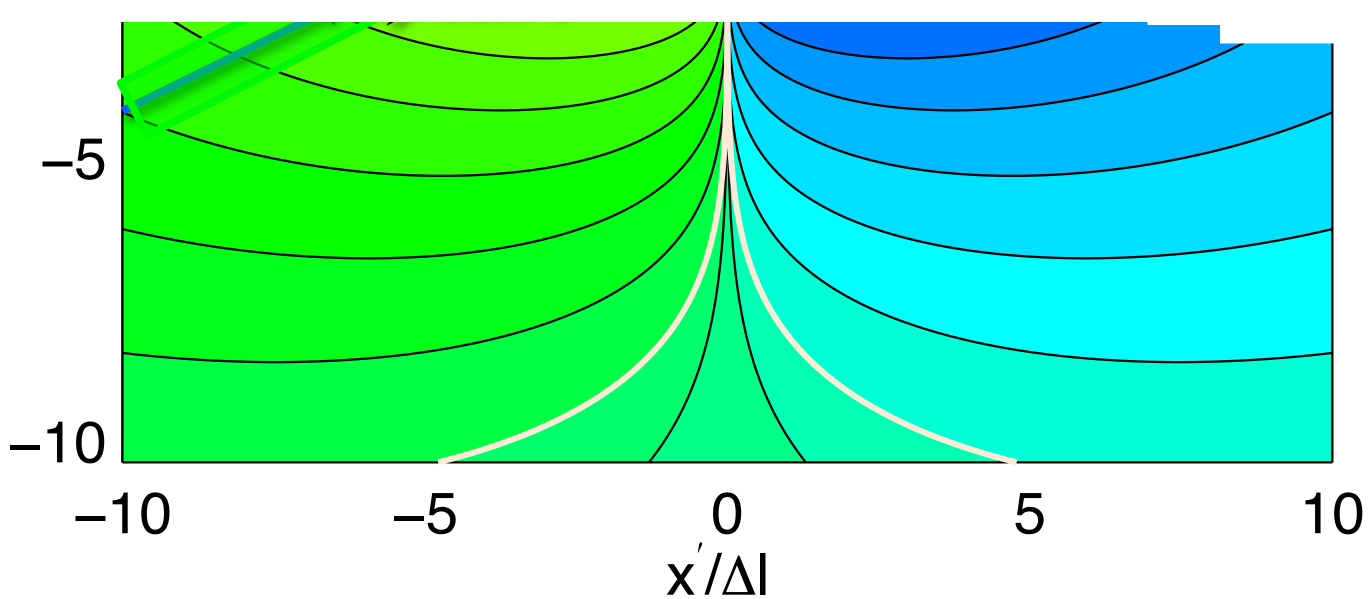

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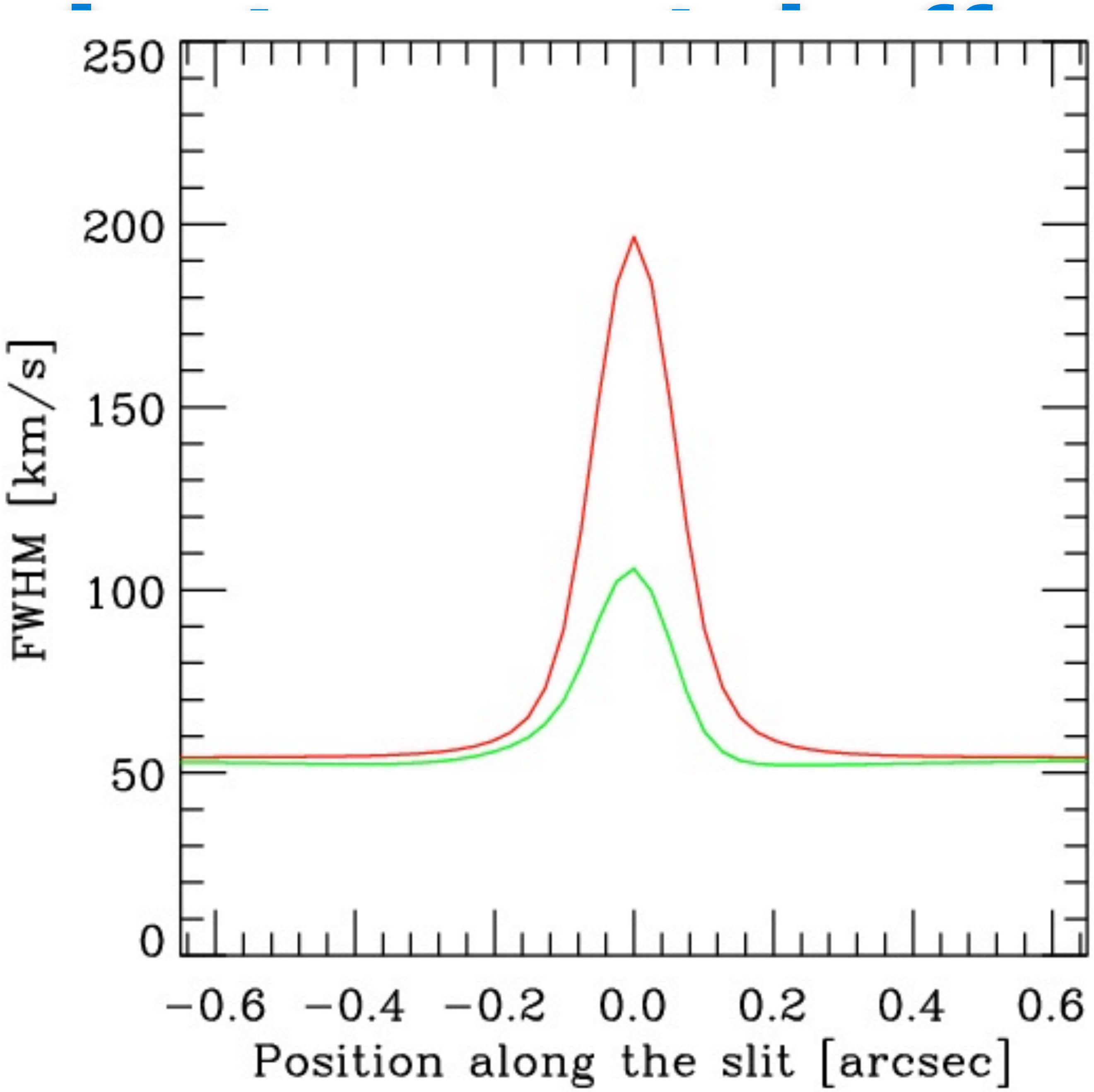

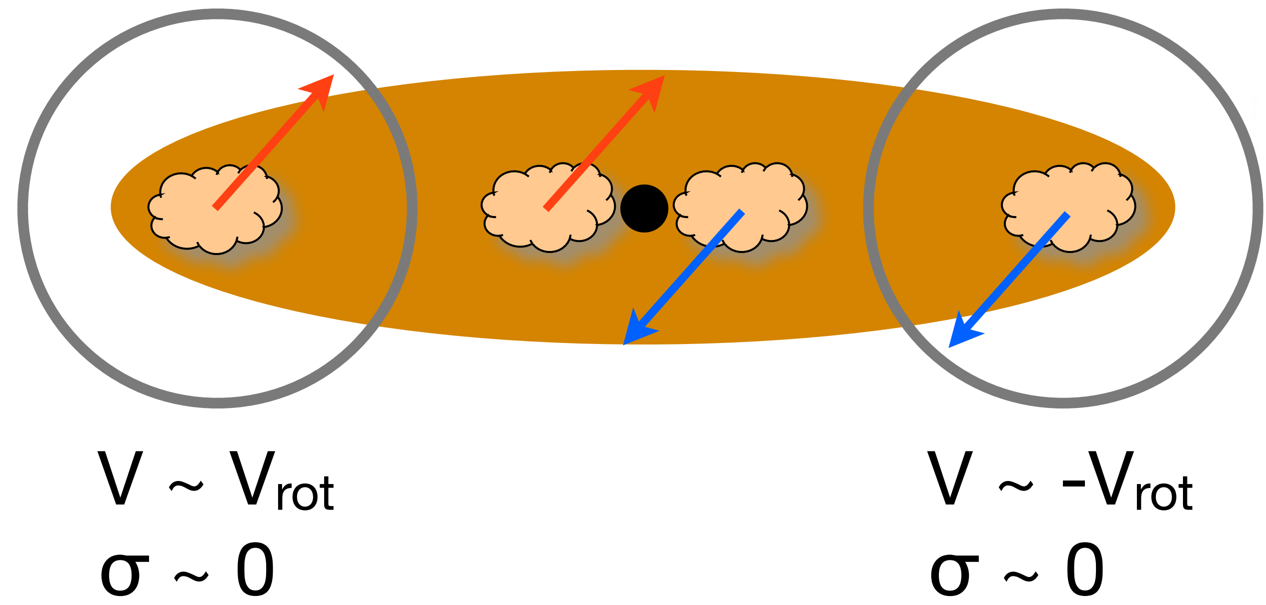

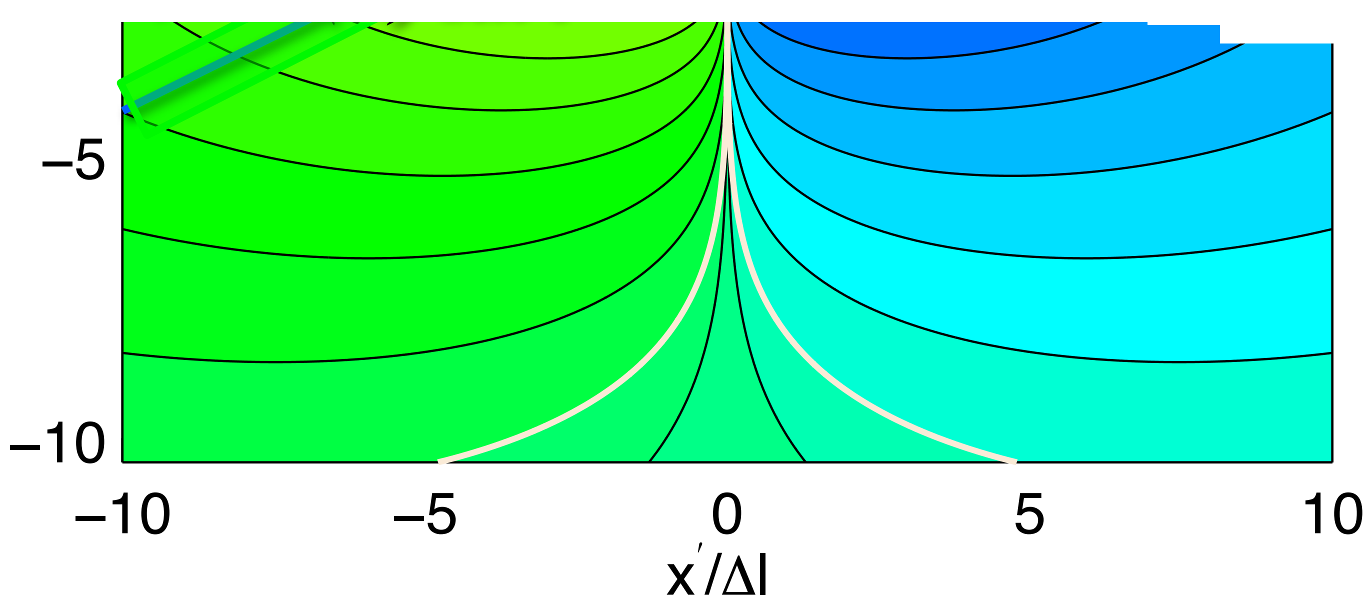

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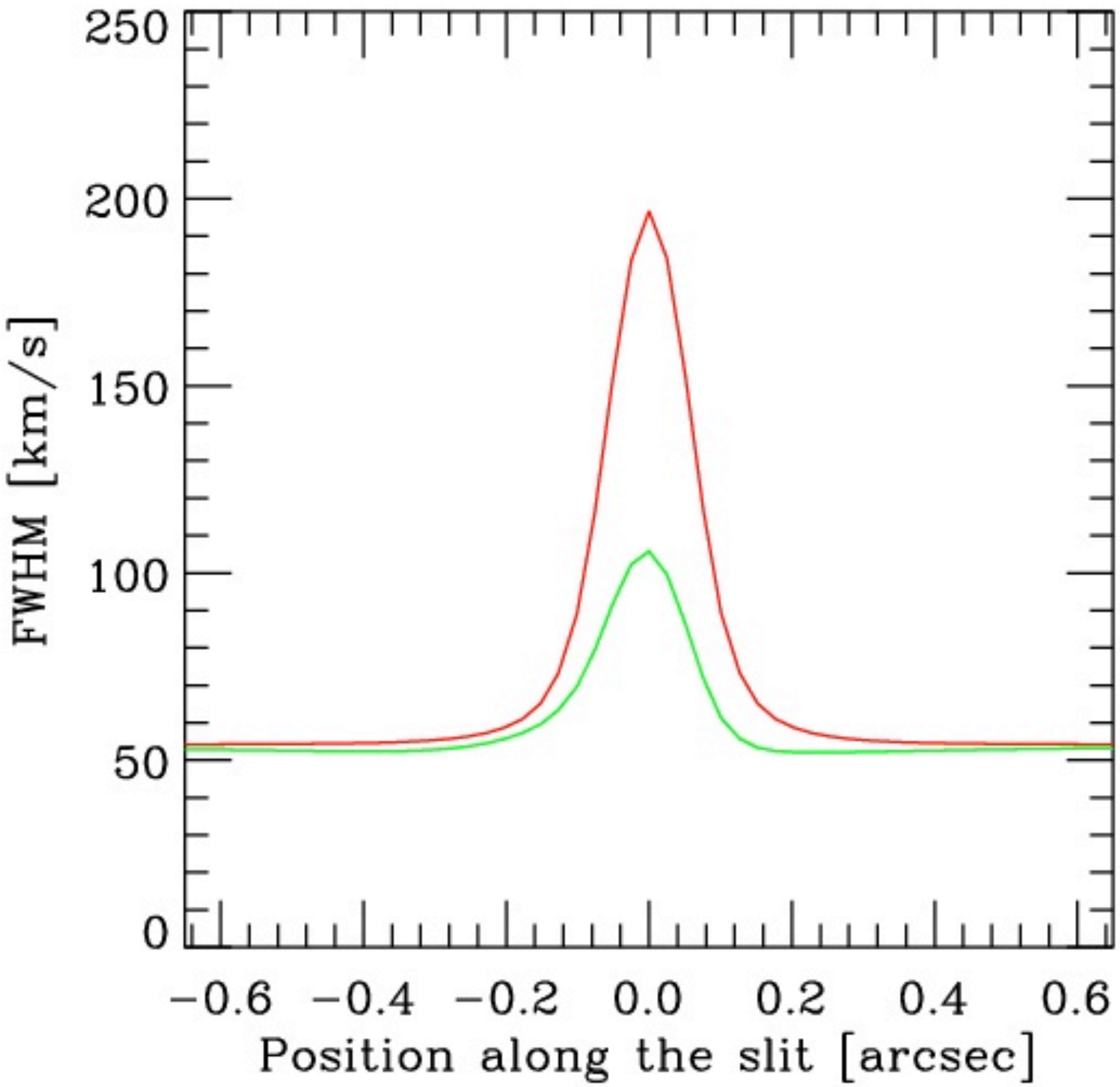
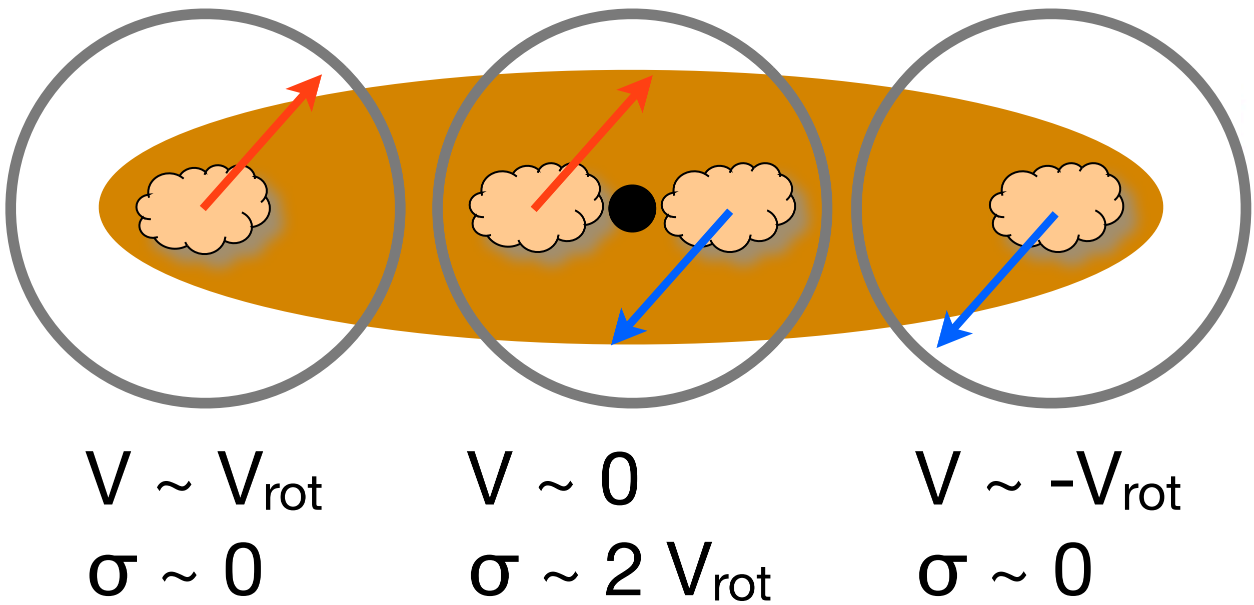
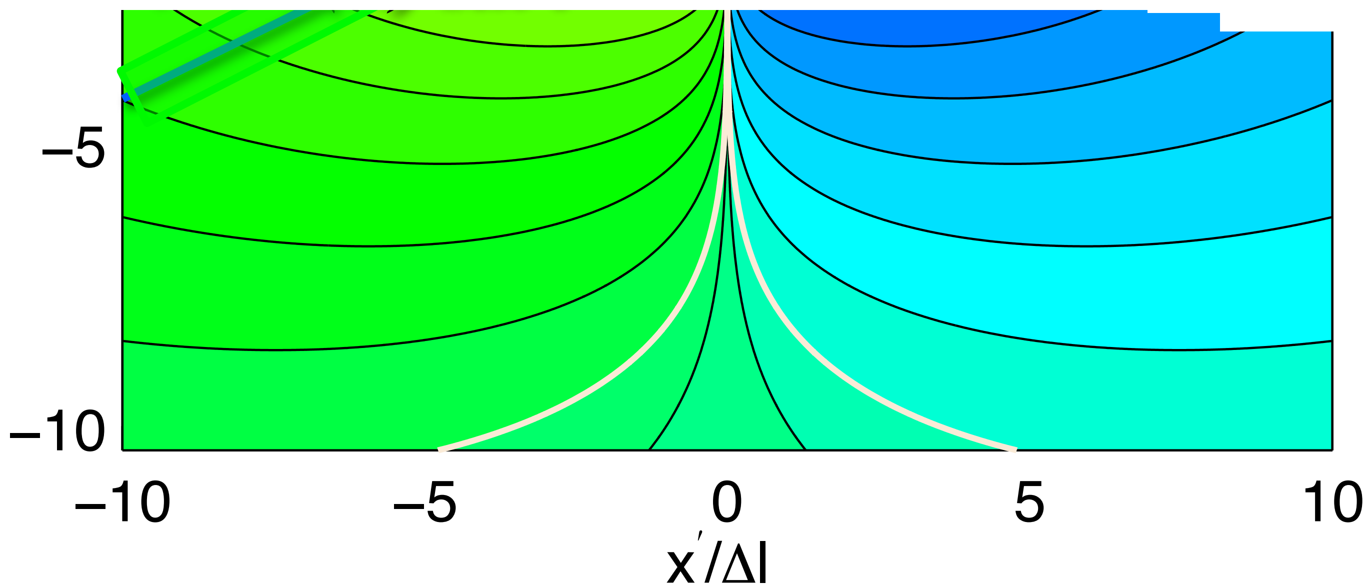
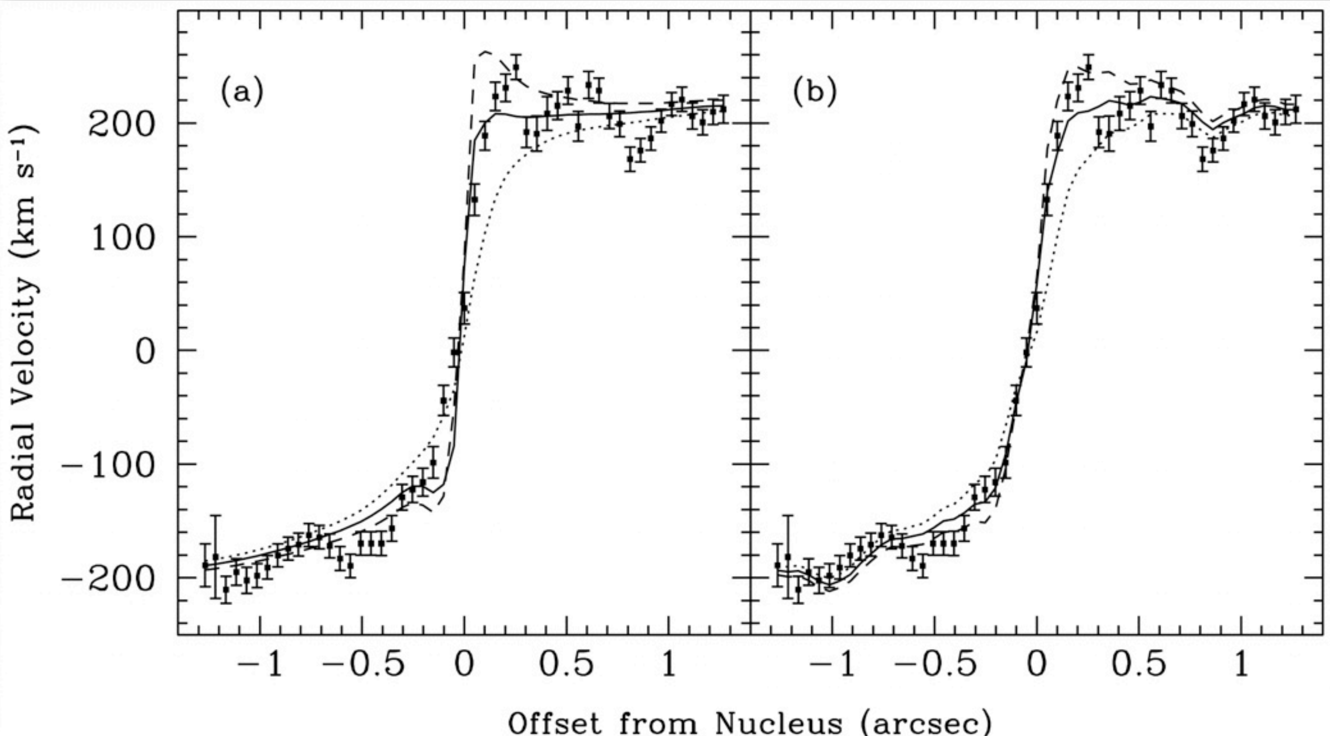




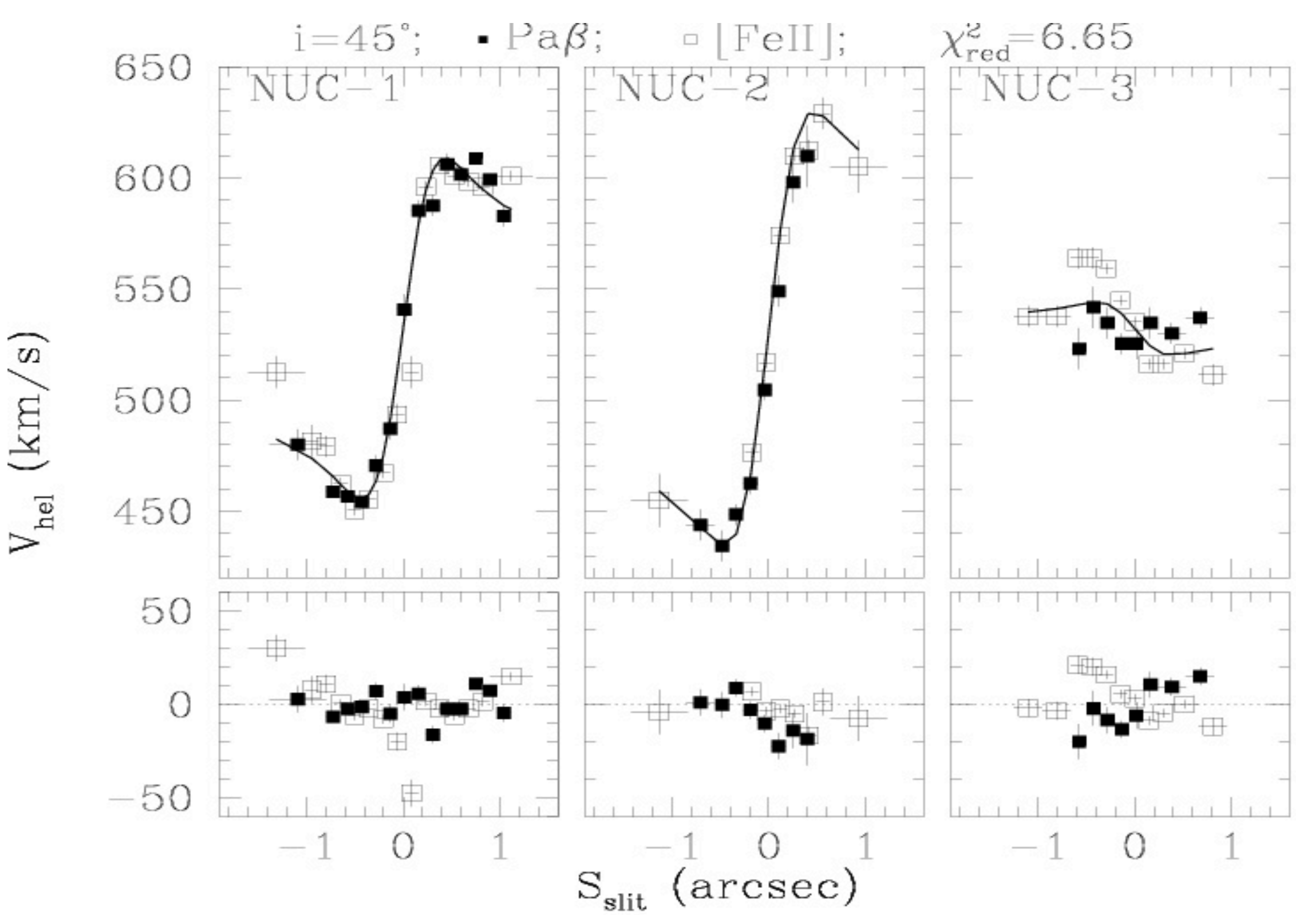


Line width increase around BH on region with size ~spatial resolution: this is due to *unresolved rotation*

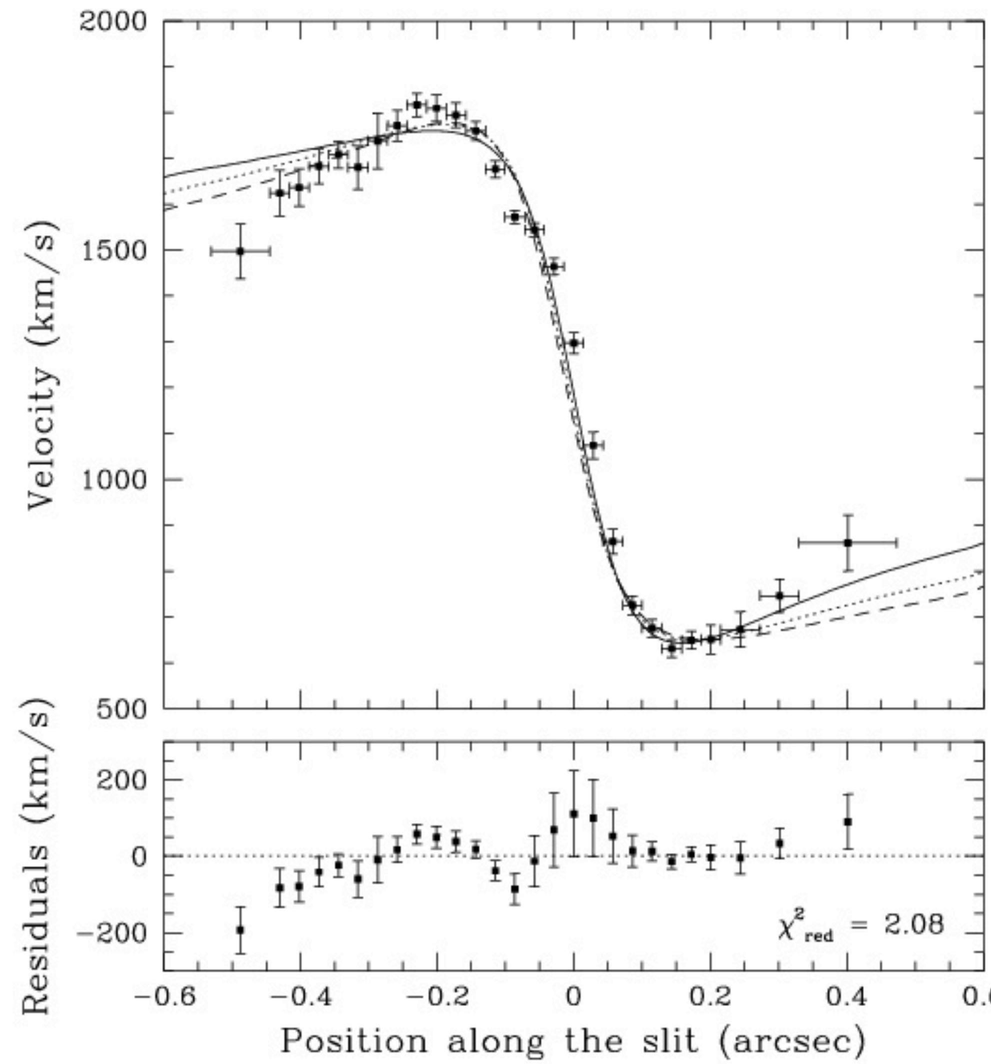
First attempts ...



NGC 3250: Barth et al. 2001

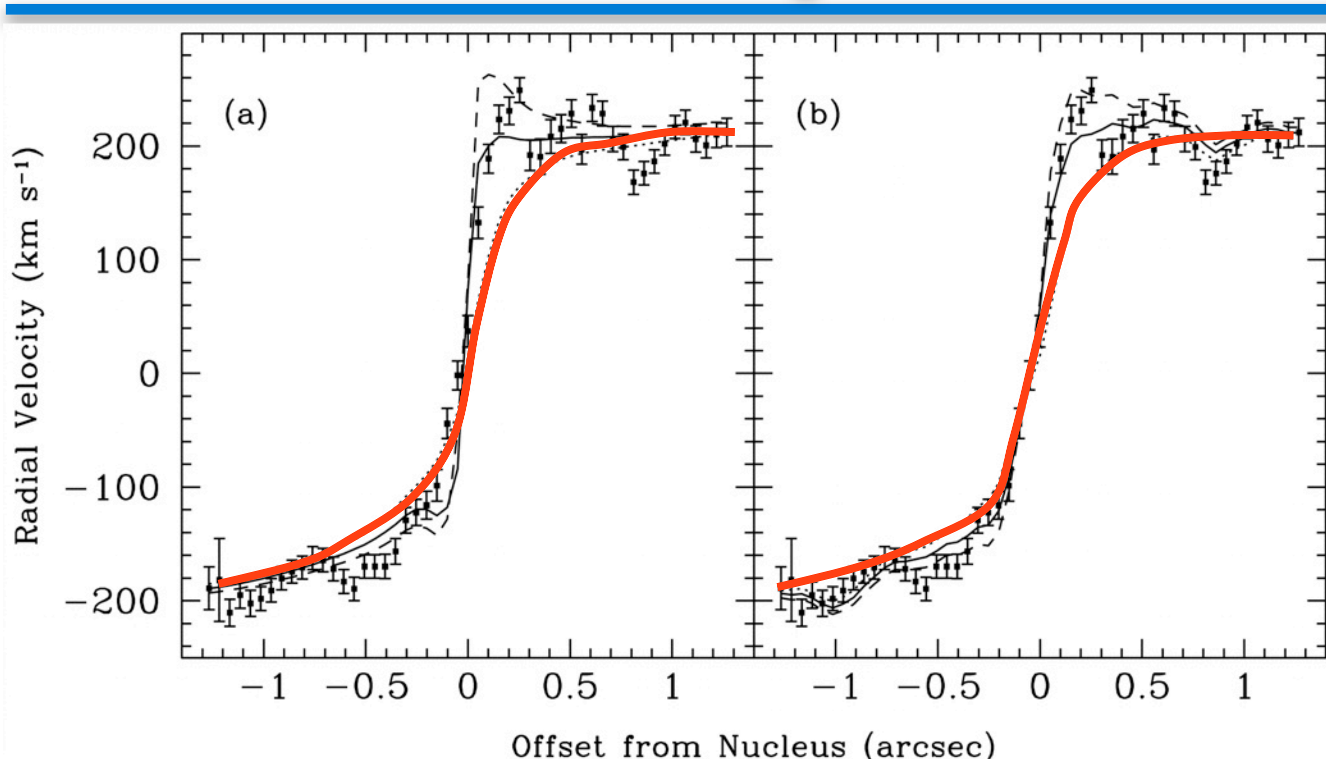


Centaurus A: Marconi et al. 2001



M87: Macchetto, AM, et al. 1997

First attempts ...



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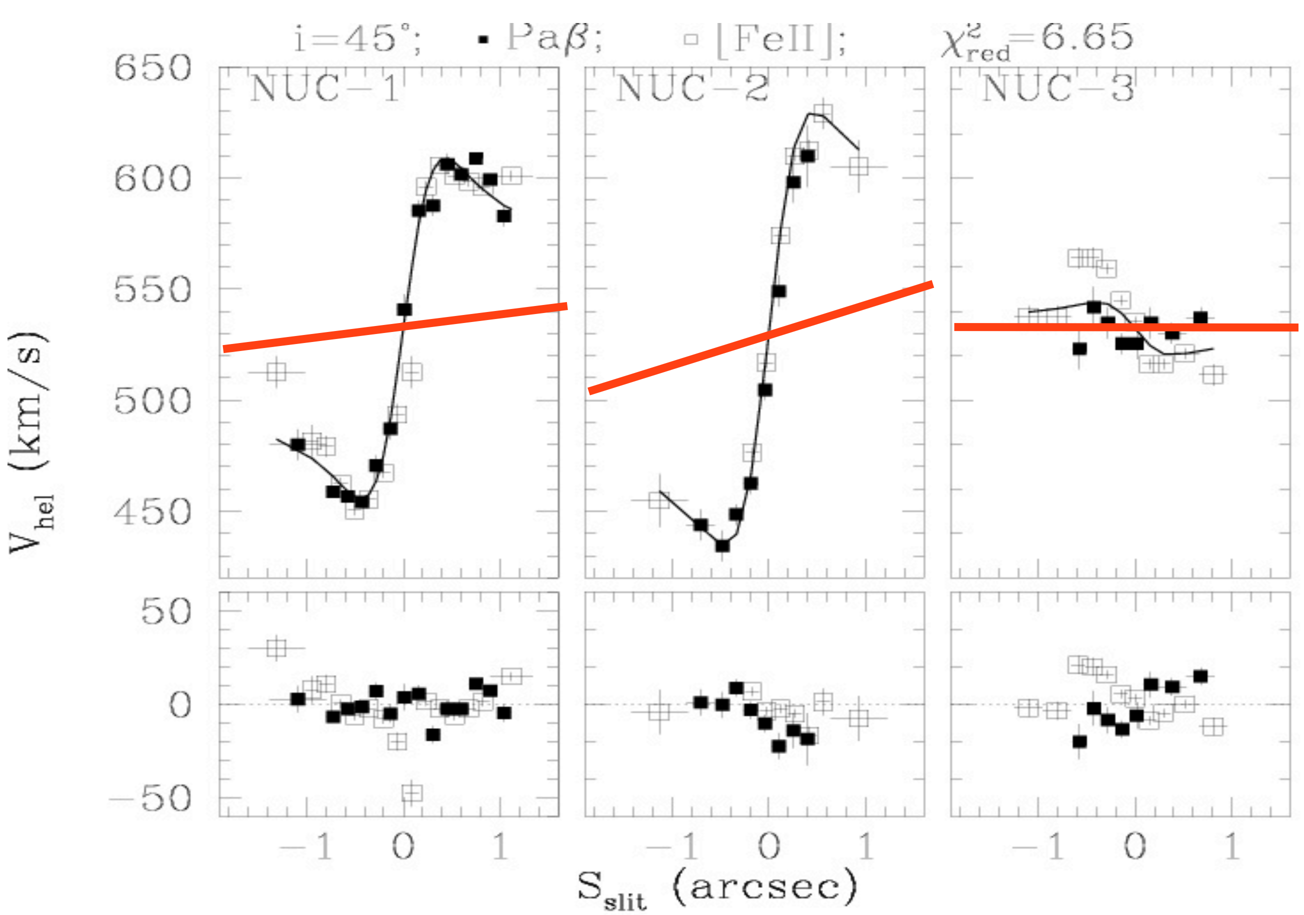

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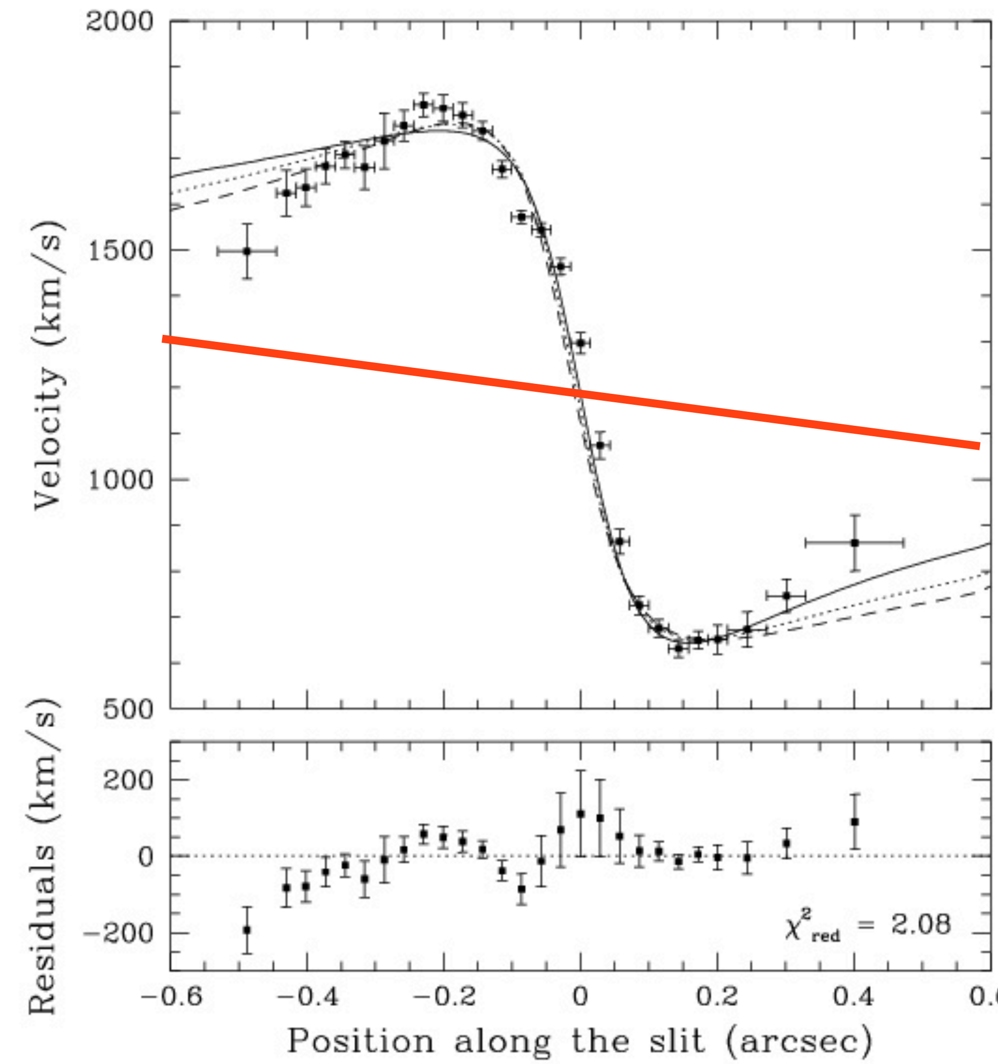


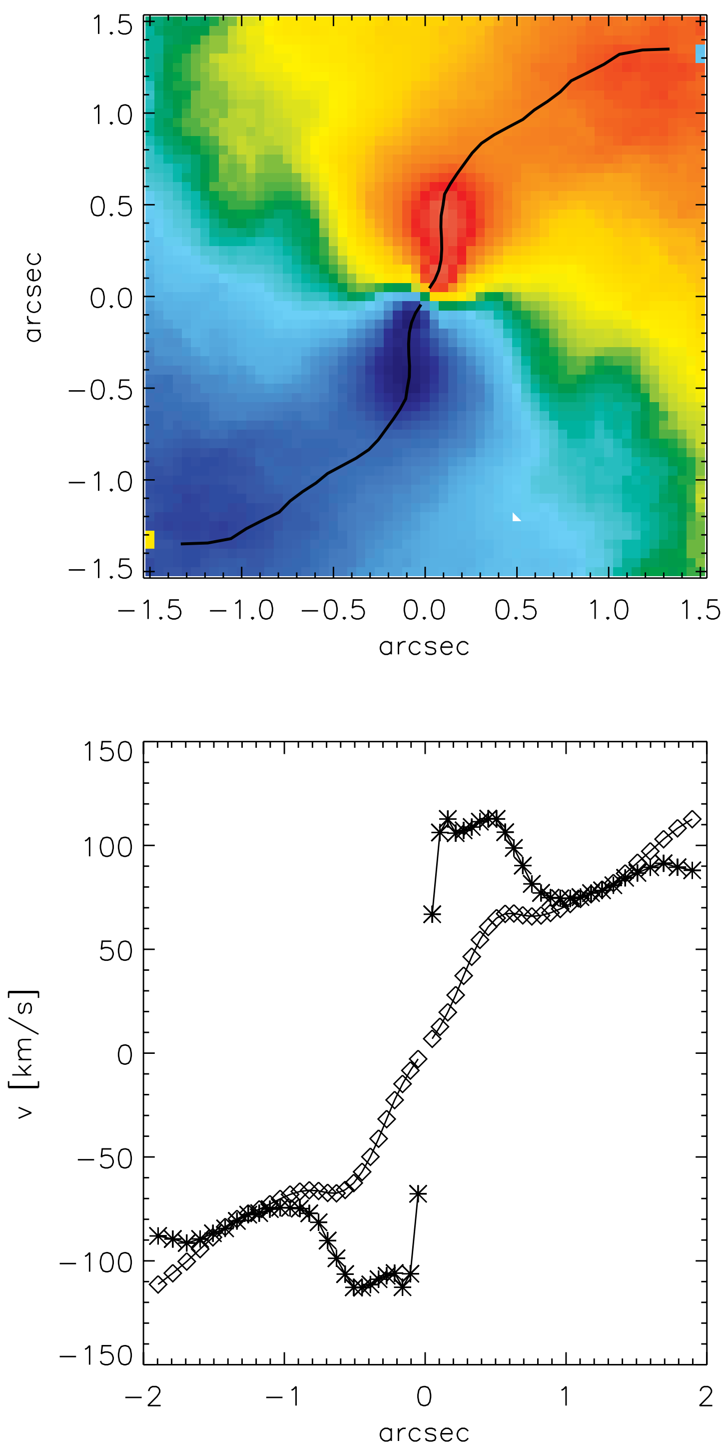
M87: Macchetto, AM, et al. 1997

Rotation curves
with stars only
and no BH!

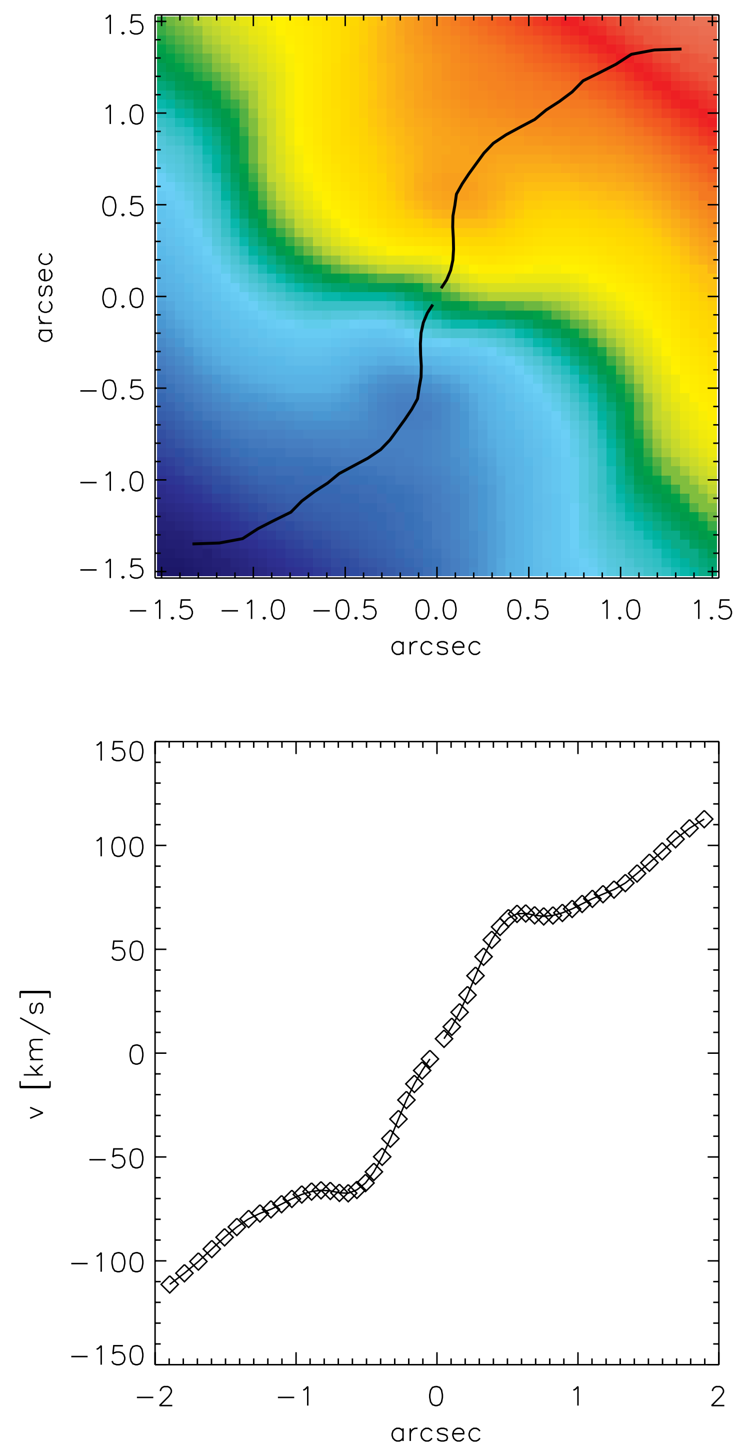
AO/IFU observations of Centaurus A

Velocity field from H_2 (2.12)

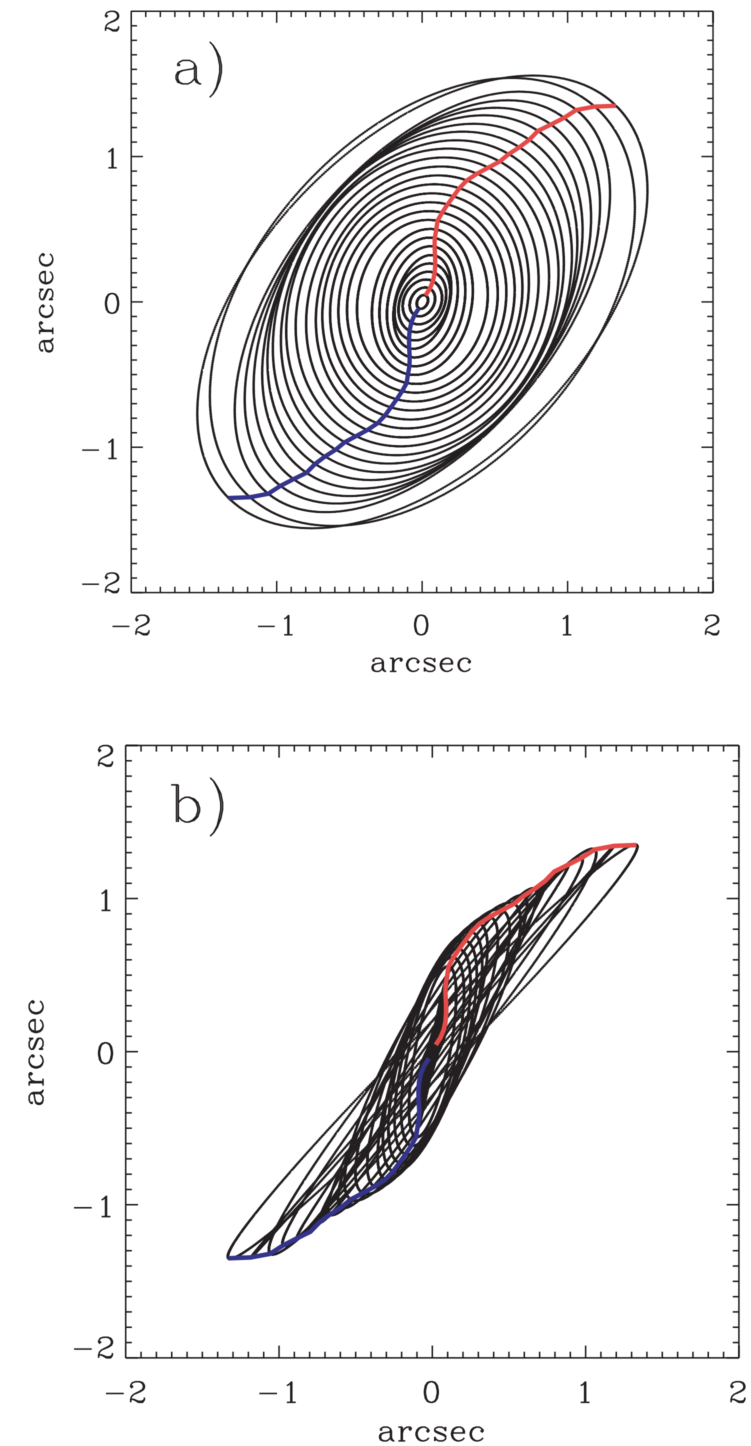
$M_{BH} = 4.5 \times 10^7\ M_\odot$



$M_{BH} = 0$

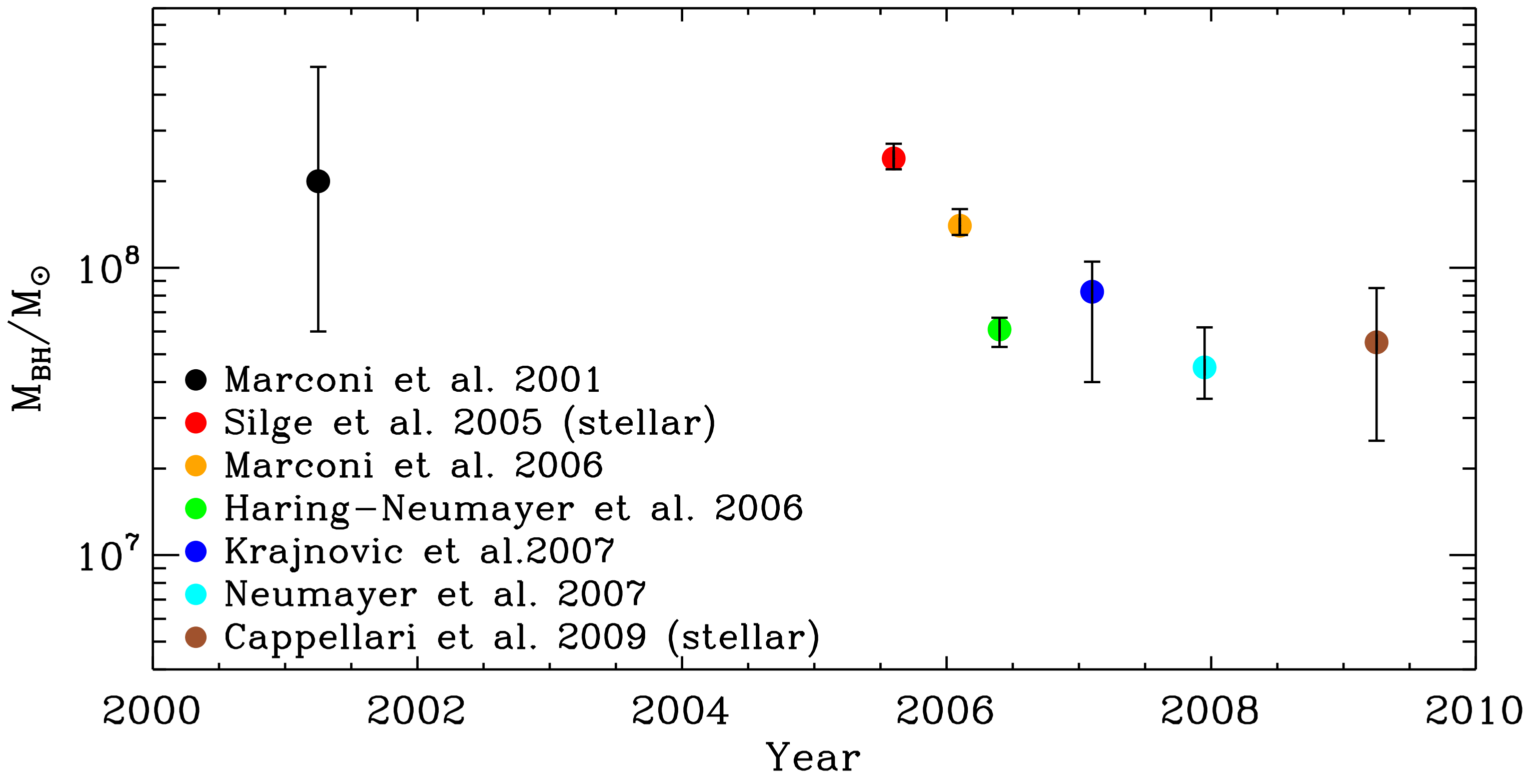


Warped disk model

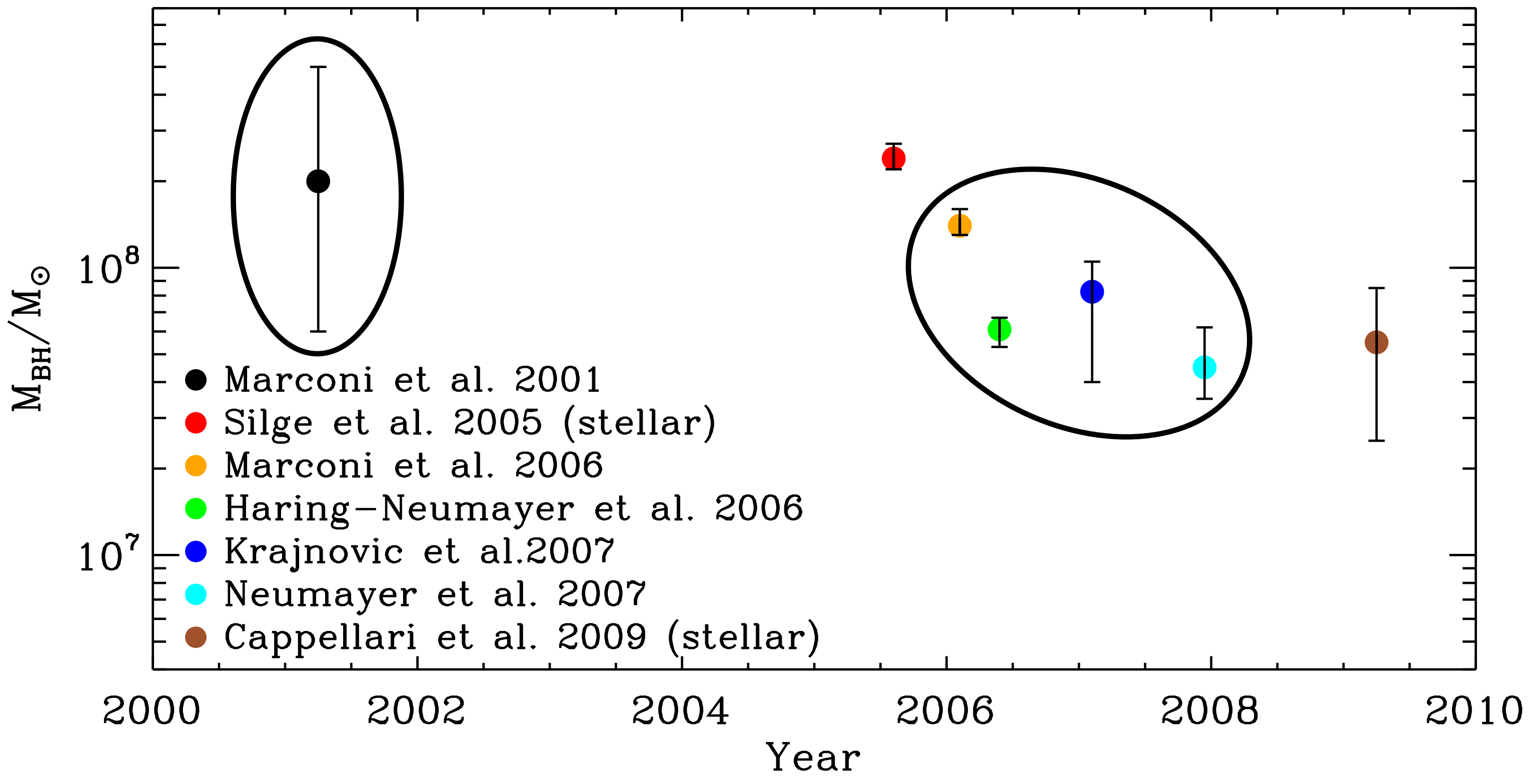


Neumayer et al. 2007

Centaurus A: a case study

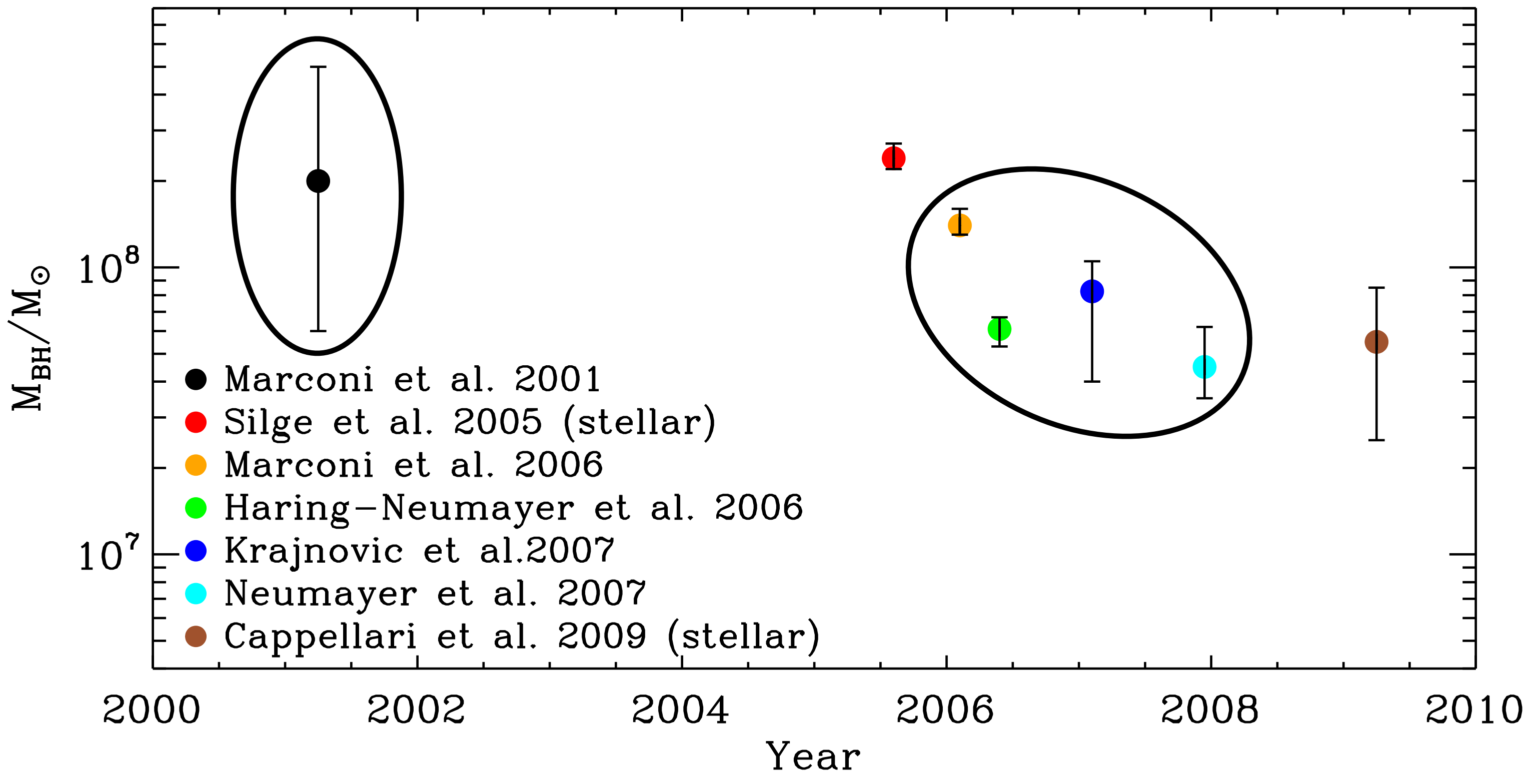


Centaurus A: a case study



Large error bars, discrepant measurements?

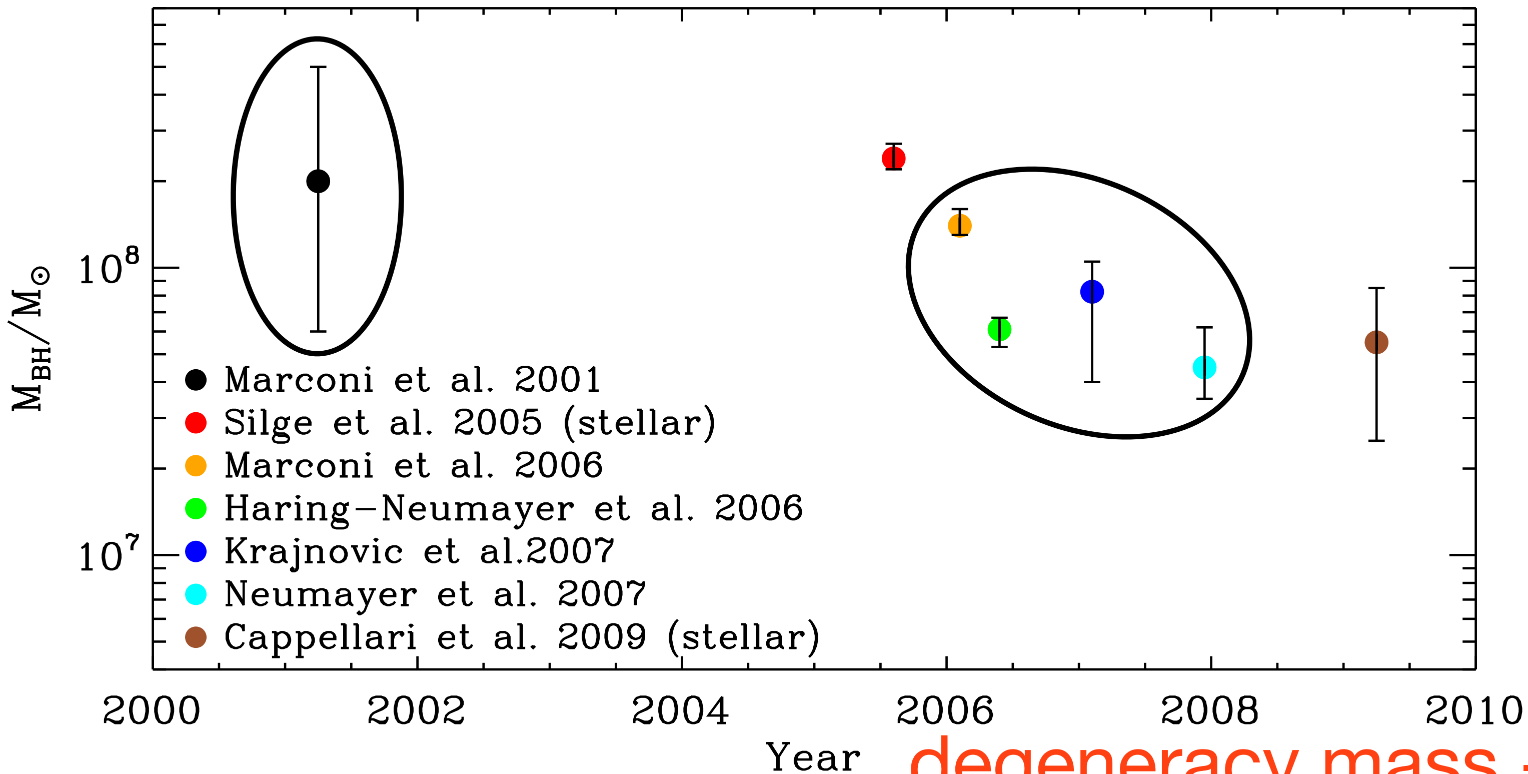
Centaurus A: a case study



Large error bars, discrepant measurements?
No, badly constrained inclination ...

$$V_{los}(x', y') = V_{sys} + \left[GM(R)\sin^2 i\right]^{1/2} \frac{x'}{\left[x'^2 + (y'/\cos i)^2\right]^{3/2}}$$

Centaurus A: a case study



Large error bars, discrepant measurements?
No, badly constrained inclination ...

degeneracy mass - inclination

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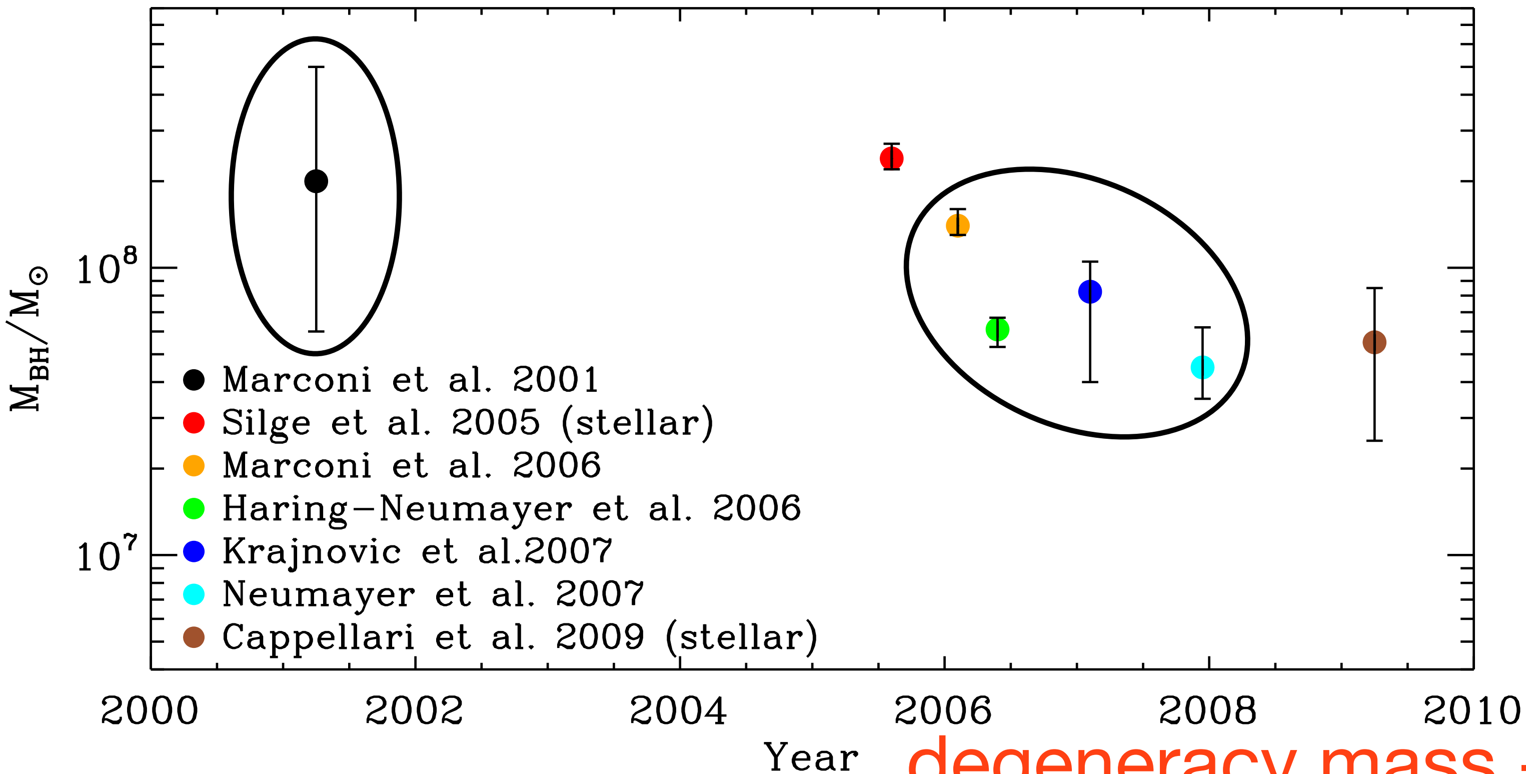

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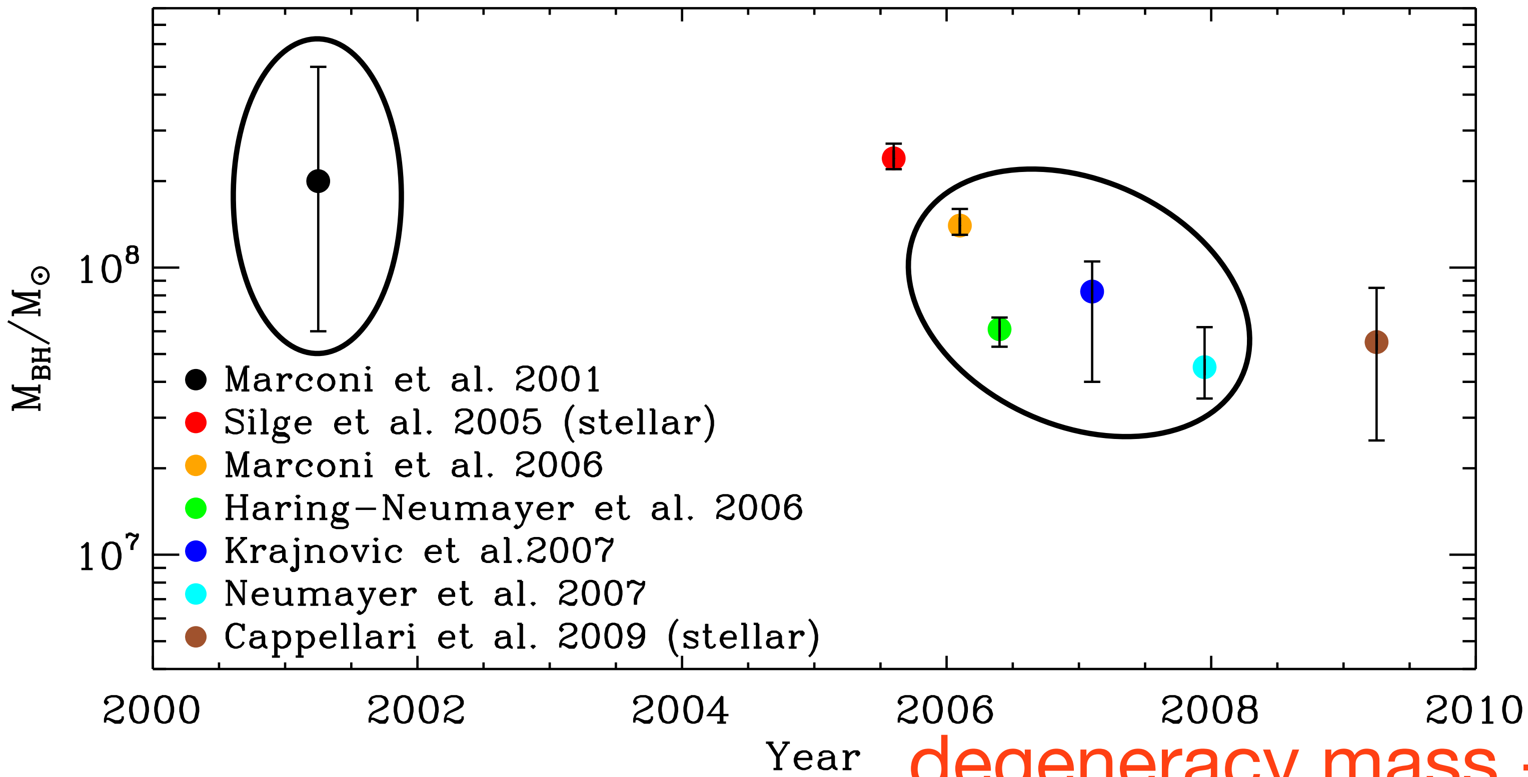

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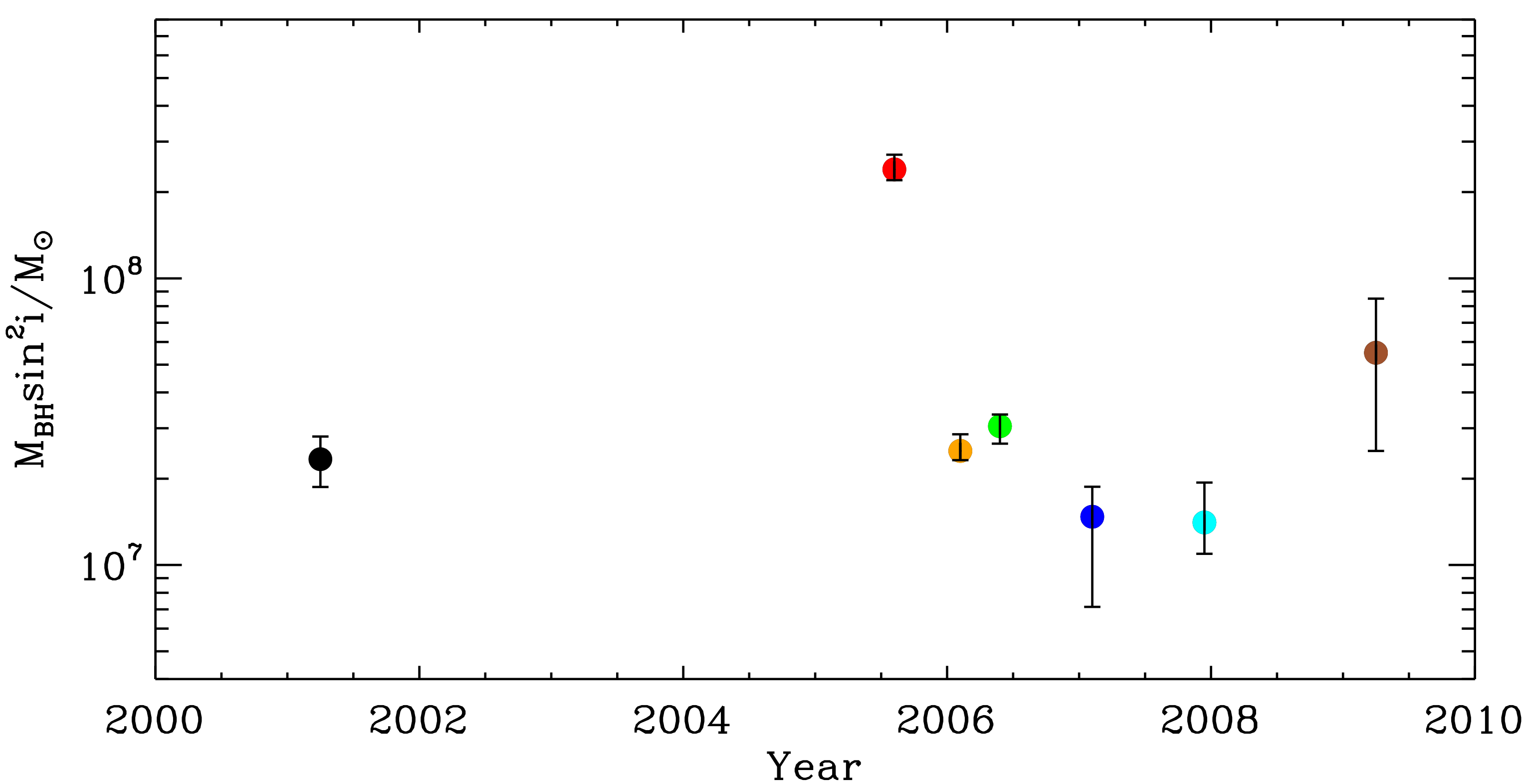

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$$M_{\rm BH}(\sin i)^2$$

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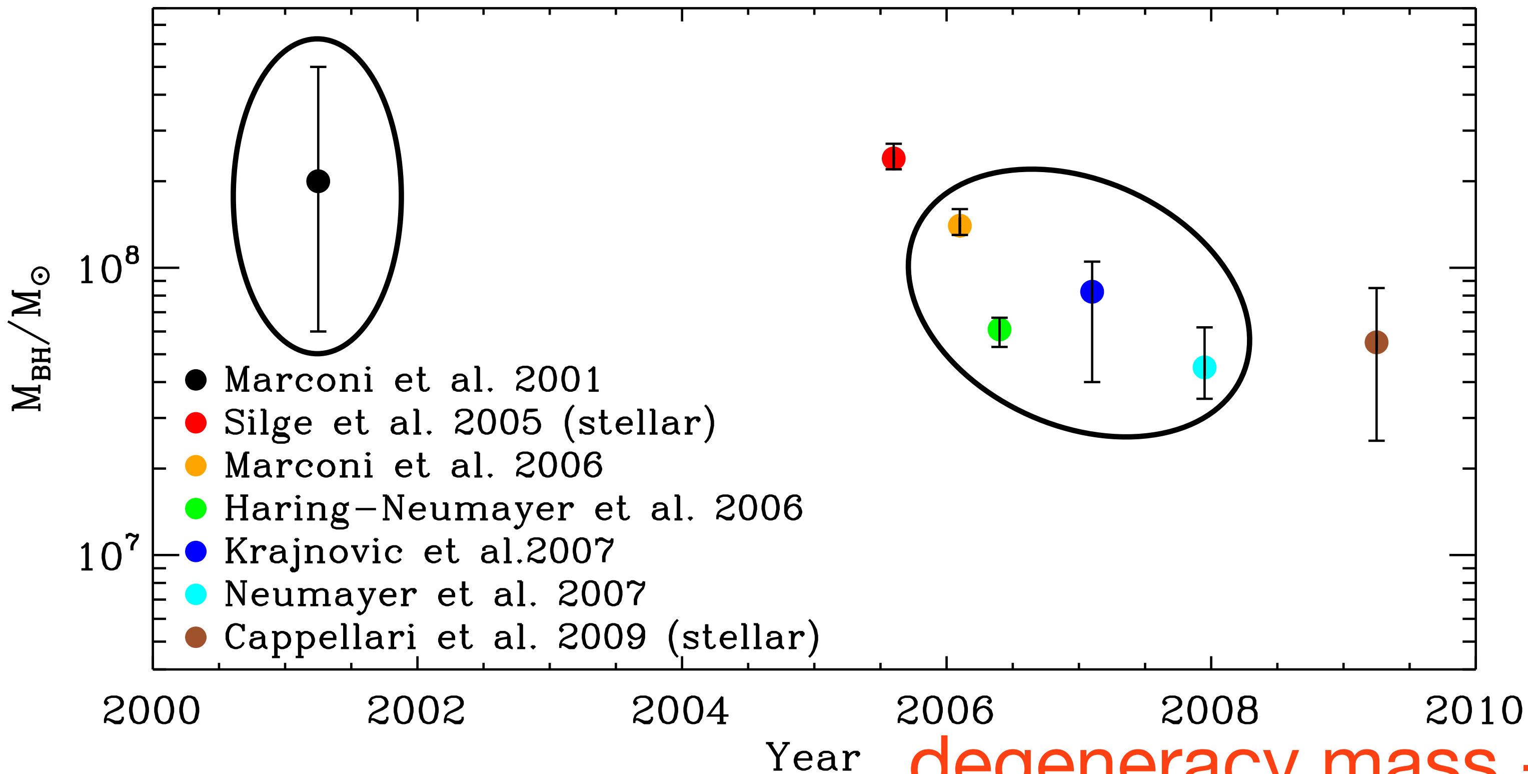

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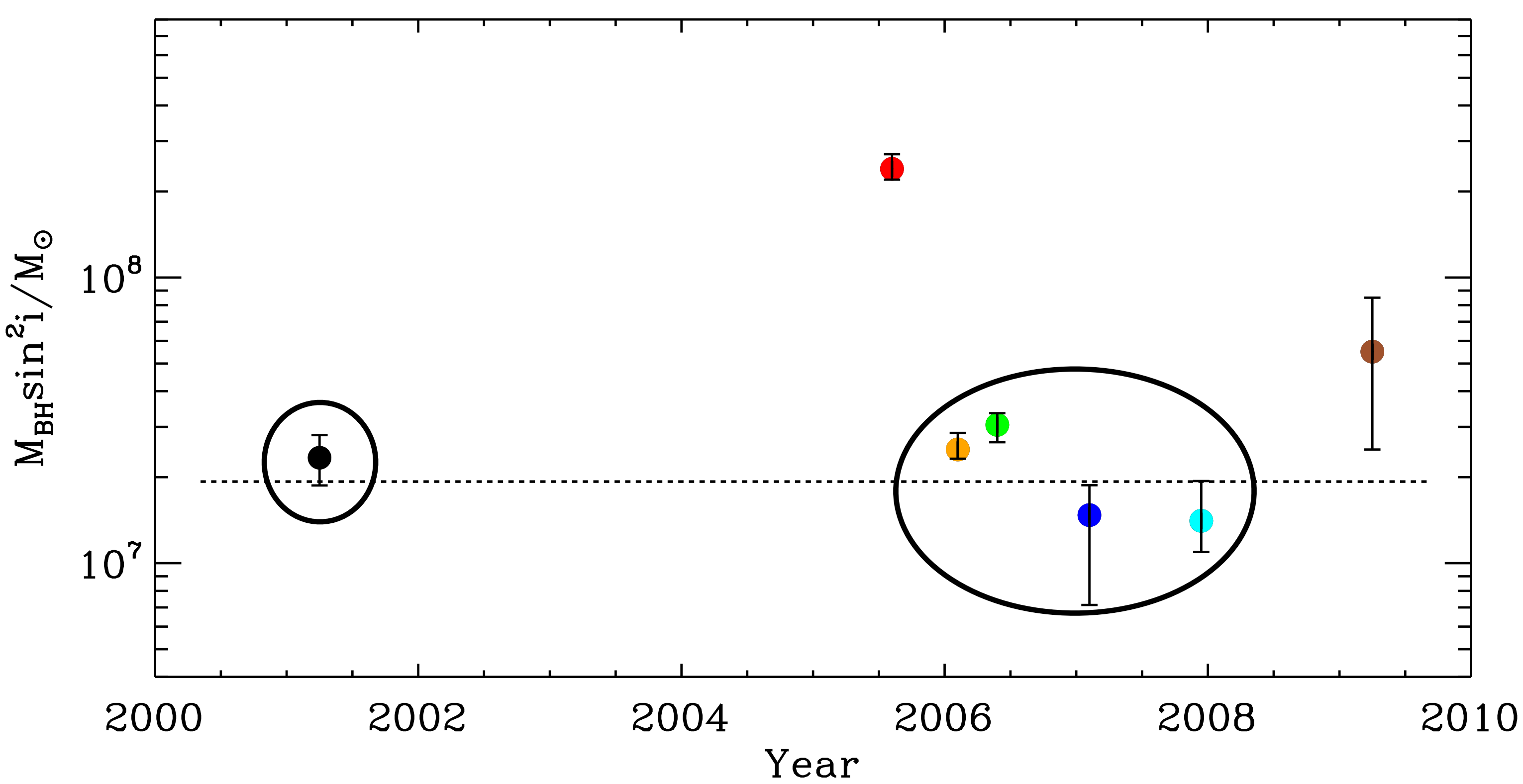

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Stellar dynamics

A galaxy is made of N stars (typically N~10^{11}) thus one has to use a statistical approach to find its gravitational potential and kinematics.

In the case of a galaxy,

- galaxy lifetime << relaxation time
  (time over which 2 body interactions affect star orbits)
- a stellar system can be considered as a collisionless gas
- to compute star orbits, gravitational field of a galaxy can be considered "smooth" and not concentrated in nearly point-like stars.

However, it is not possible to follow the orbits of ~10^{11} stars.

Consider the probability of finding a star with given position and velocity

$$dp = f(\vec{x}, \vec{v}; t)\, \mathrm{d}^3\vec{x}\, \mathrm{d}^3\vec{v} \qquad \int f(\vec{x}, \vec{v}; t)\, \mathrm{d}^3\vec{x}\, \mathrm{d}^3\vec{v} = 1$$

f is the distribution function.

$$n(\vec{x}) = N \int f(\vec{x}, \vec{v}; t)\, \mathrm{d}^3\vec{v}$$ is the star number density

Can easily estimate the LOS velocity distribution, average velocity, velocity dispersion, h3, h4, etc., by simple integration od the DF.

[see Binney & Tremaine]

Stellar dynamics

The DF follows the collisionless Boltzmann equation

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \vec{v} \cdot \vec{\nabla} f - \vec{\nabla}\Phi(\vec{x}) \cdot \frac{\partial f}{\partial \vec{v}} = 0 \qquad \nabla^2 \Phi(\vec{x}) = 4\pi G\, \rho(\vec{x})$$

In steady state

$$\vec{v} \cdot \vec{\nabla} f - \vec{\nabla}\Phi(\vec{x}) \cdot \frac{\partial f}{\partial \vec{v}} = 0$$

An integral of motion is a function I of space phase coordinates such that, along any orbit

$$\frac{d}{dt} I[\vec{x}(t), \vec{v}(t)] = \vec{v} \cdot \vec{\nabla} I - \vec{\nabla}\Phi(\vec{x}) \cdot \frac{\partial I}{\partial \vec{v}} = 0$$

examples are energy and components of angular momentum.
Condition for I is similar to condition for f (steady state).
Can demonstrate Strong Jeans Theorem:
DF of steady state stellar system with "regular orbits" (non resonating frequencies) is a function only of 3 integral of motions.

[see Binney & Tremaine]

Jeans equations

How to solve the steady state Boltzmann Equation (BE)?

Two possibilities mainly adopted in the literature

- ⭐ Jeans equations
- ⭐ Schwarzschild orbit superposition method

Jeans equations

Taking the moments of the BE over velocity gives analogs to fluid equations

$$\cancel{\frac{\partial n}{\partial t}} + \frac{\partial (n\bar{v}_i)}{\partial x_i} = 0$$

$$n\cancel{\frac{\partial \bar{v}_j}{\partial t}} - \bar{v}_j \frac{\partial (n\bar{v}_i)}{\partial x_i} + \frac{\partial (n\overline{v_i v_j})}{\partial x_i} = -n\frac{\partial \Phi}{\partial x_j}$$

$\bar{v}_j$ average j velocity

$\overline{v_i v_j}$ velocity dispersion tensor (→anisotropy)

$\overline{v_i v_i} = \sigma_i^2$ i velocity dispersion

incomplete set, knowing potential and density we have 9 unknown functions (3 v_i, 6 symmetric components of tensor v_{ij}) and 4 independent equations. Need assumptions to close the system (e.g. spherical symmetry) but beware that wrong assumption can bring incorrect results (a BH where there is none and viceversa).

[see Binney & Tremaine]

Schwarzschild method

Schwarzschild orbit superposition method

This is a method based on considering many stellar orbits to "reconstruct" the DF, in practice

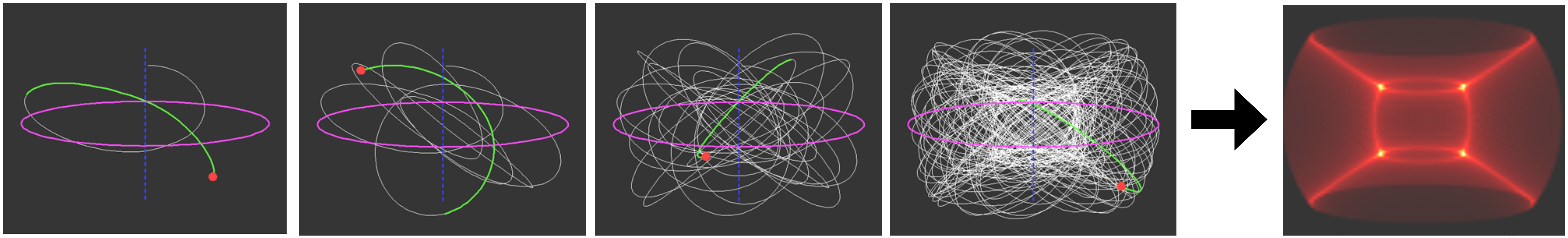
$$f \approx \Sigma_i f_i$$

where f_i is the DF for a single stellar orbit.

The method works as follows

- assume axisymmetry (but can work also for triaxial systems);
- $f = f(E, L_z, I_3)$ (I_3 third integral of motion, not clear physical interpretation);
- derive potential ϕ_{star} from observed surface brightness distribution (for given $M/L = \Upsilon$); then $\phi = \phi_{star} + \phi_{BH} + \phi_{DM}$
- compute a library of several (ten) thousands stellar orbits for different values of integral of motions, and initial conditions;
- reconstruct the galaxy adding up all the orbits; the weight of each orbit is a free parameter (many thousands!);
- compute model surface brightness and kinematics and compare with observations to constrain free parameters and M_{BH}, Υ

Schwarzschild method



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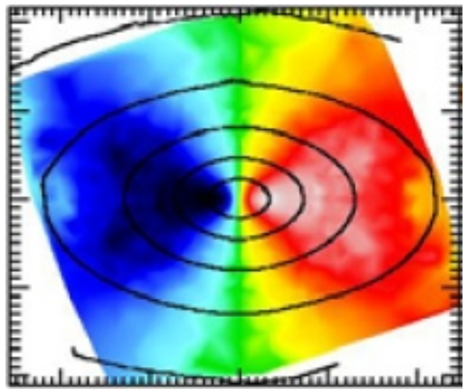
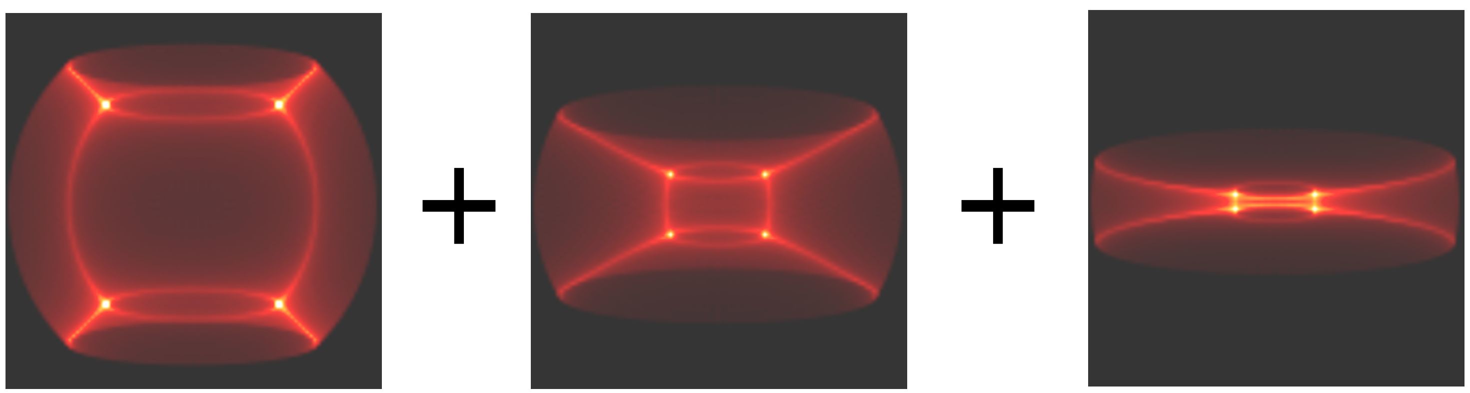
Stellar orbit track

Image of orbit on sky

Observed galaxy image



Observed velocity field

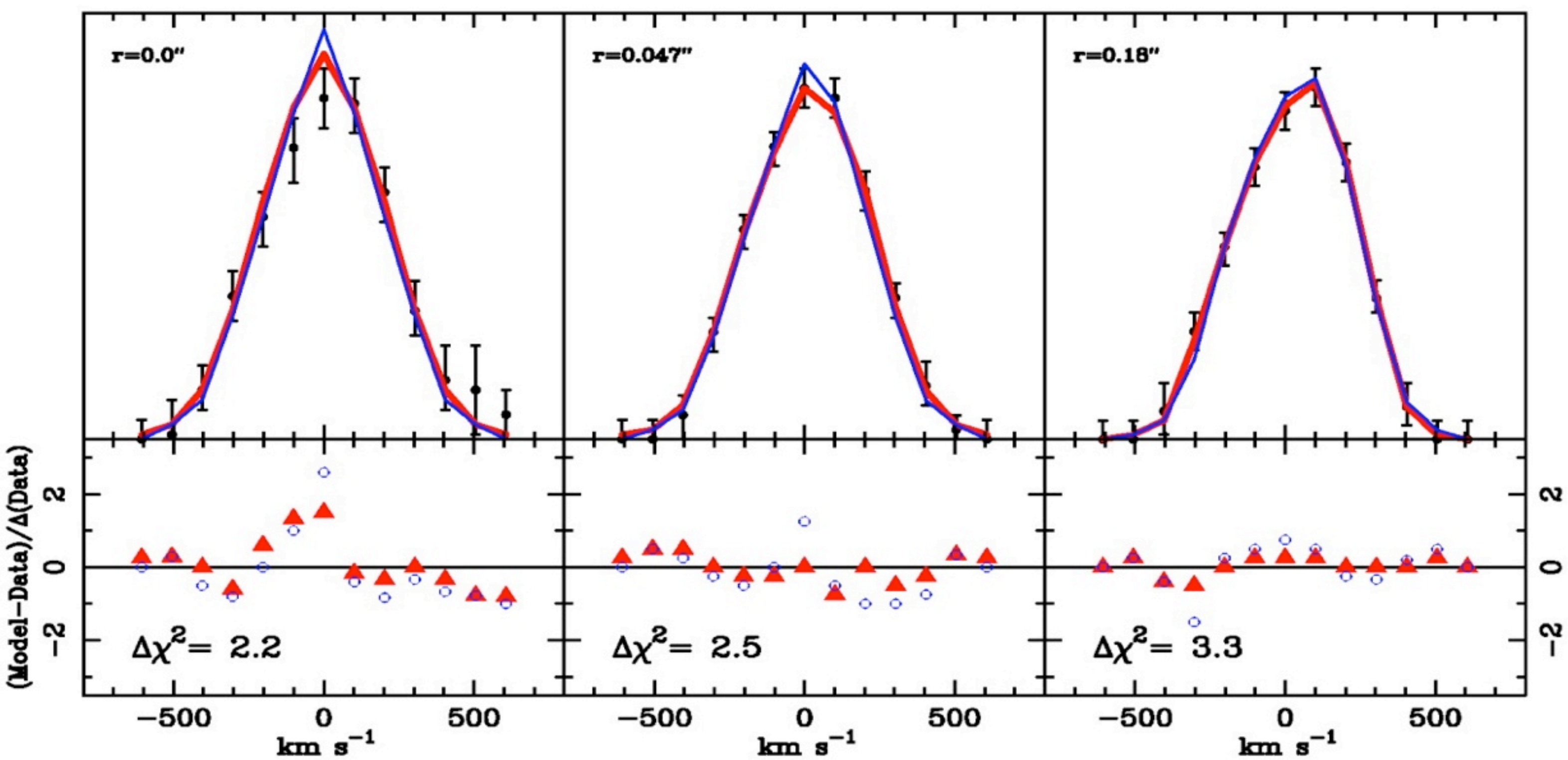


images of model orbits (with weights)
(Cappellari et al. 2004)

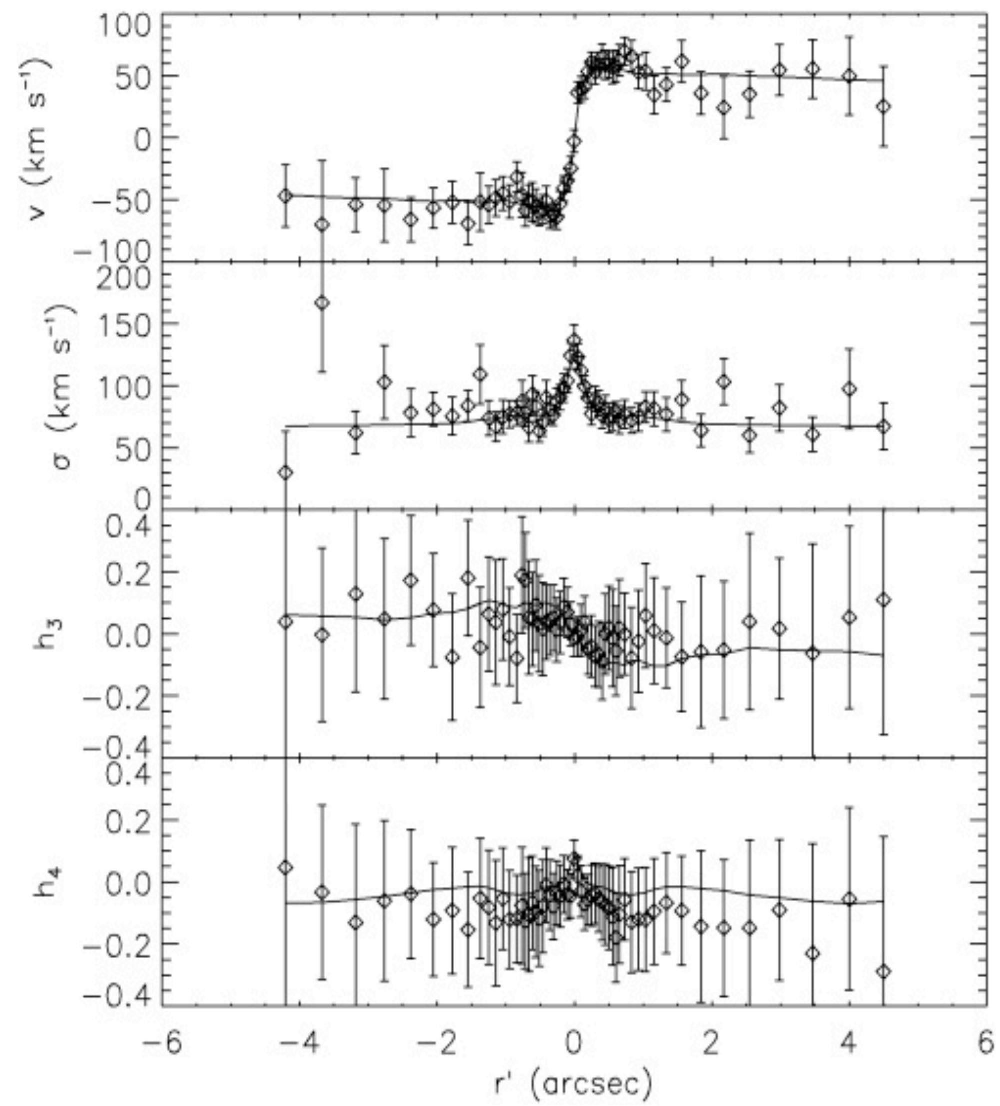
M_{BH}, Υ,
orbital structure

[Courtesy of Michele Cappellari]

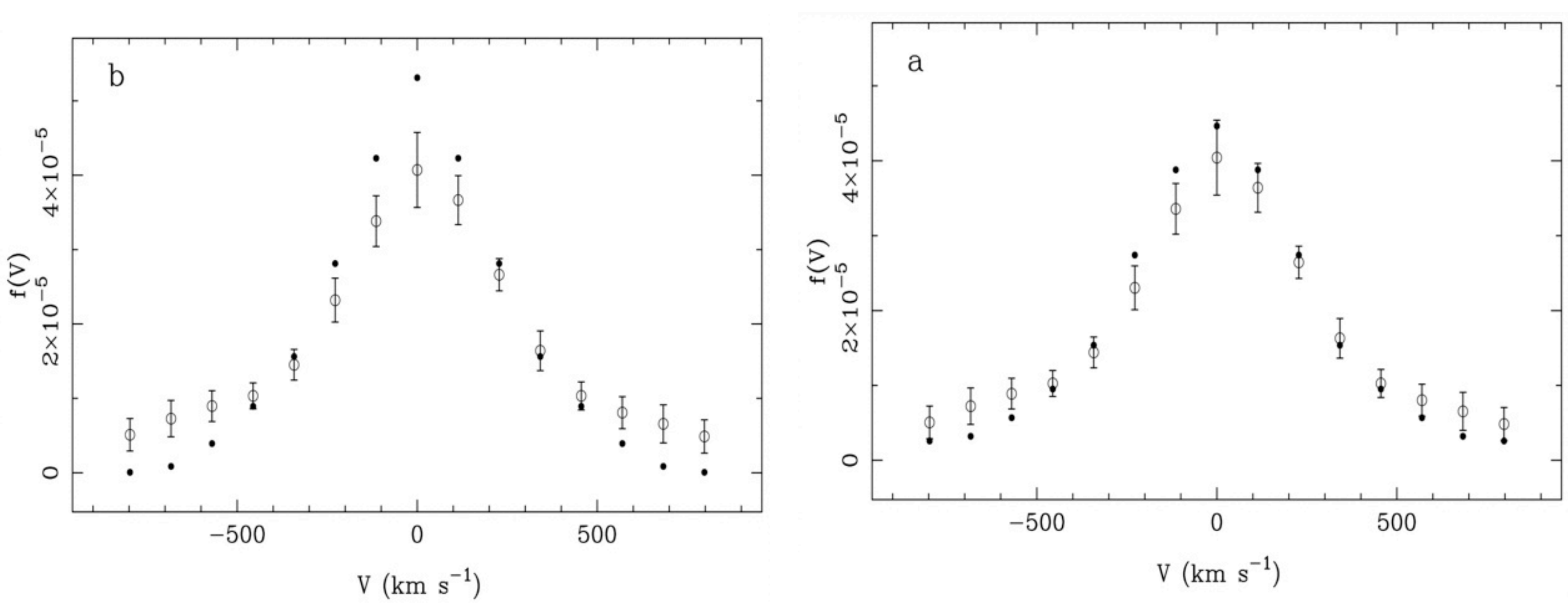
Stellar dynamics (HST/STIS)



NGC 4560: Gebhardt et al. 2003



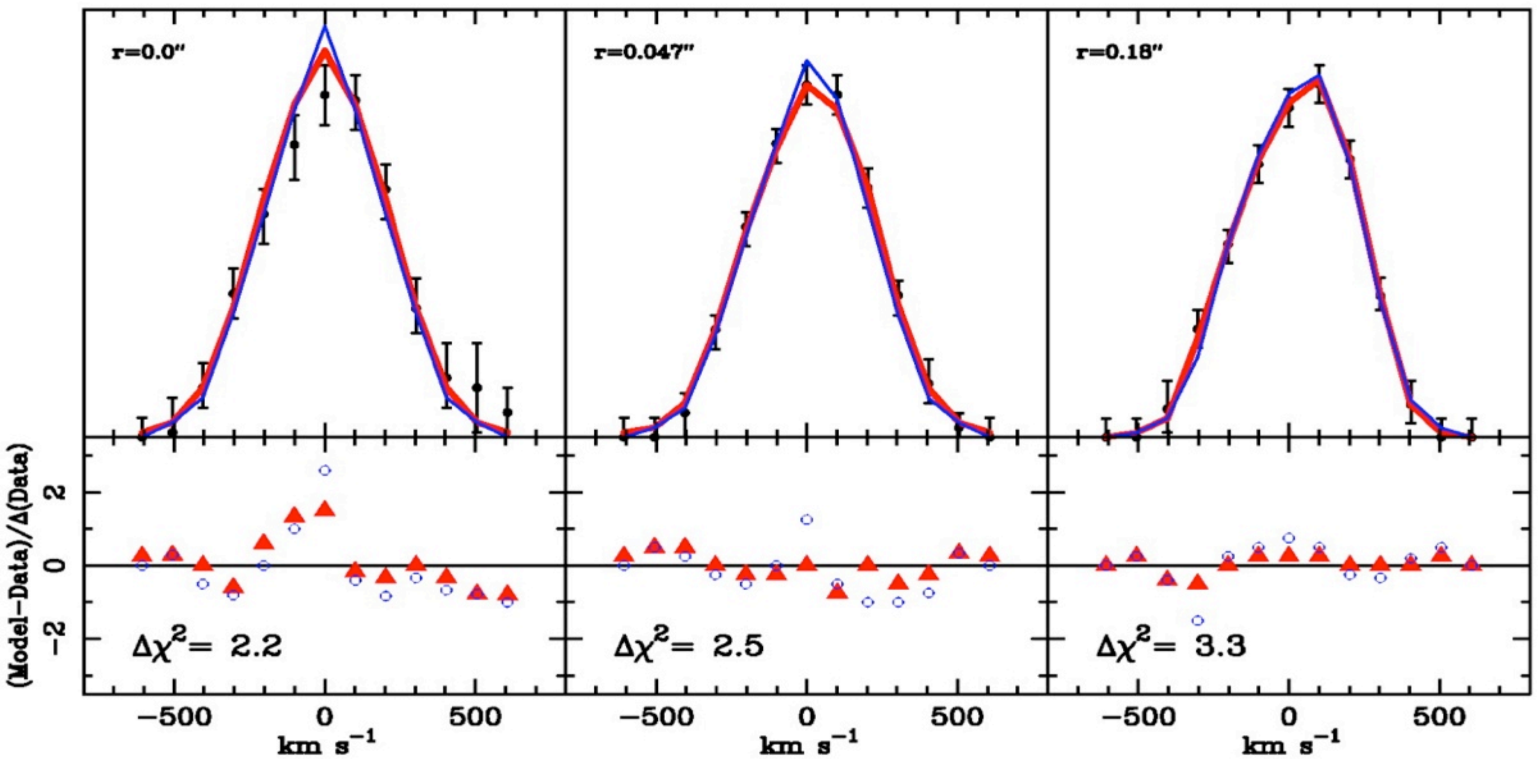
M32: Verolme et al. 2002



NGC 1023: Bower et al. 2001

Line of Sight Velocity Distributions

Stellar dynamics (HST/STIS)



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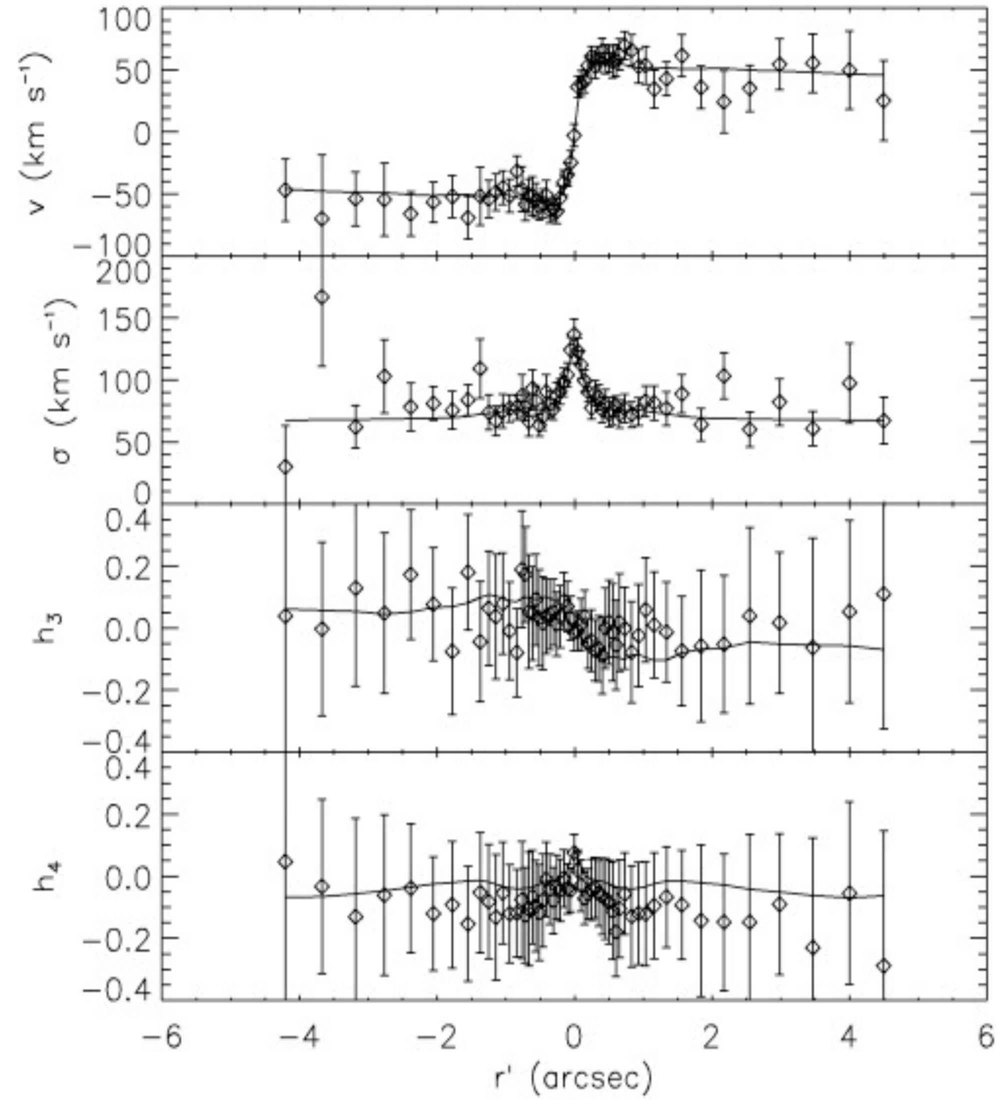

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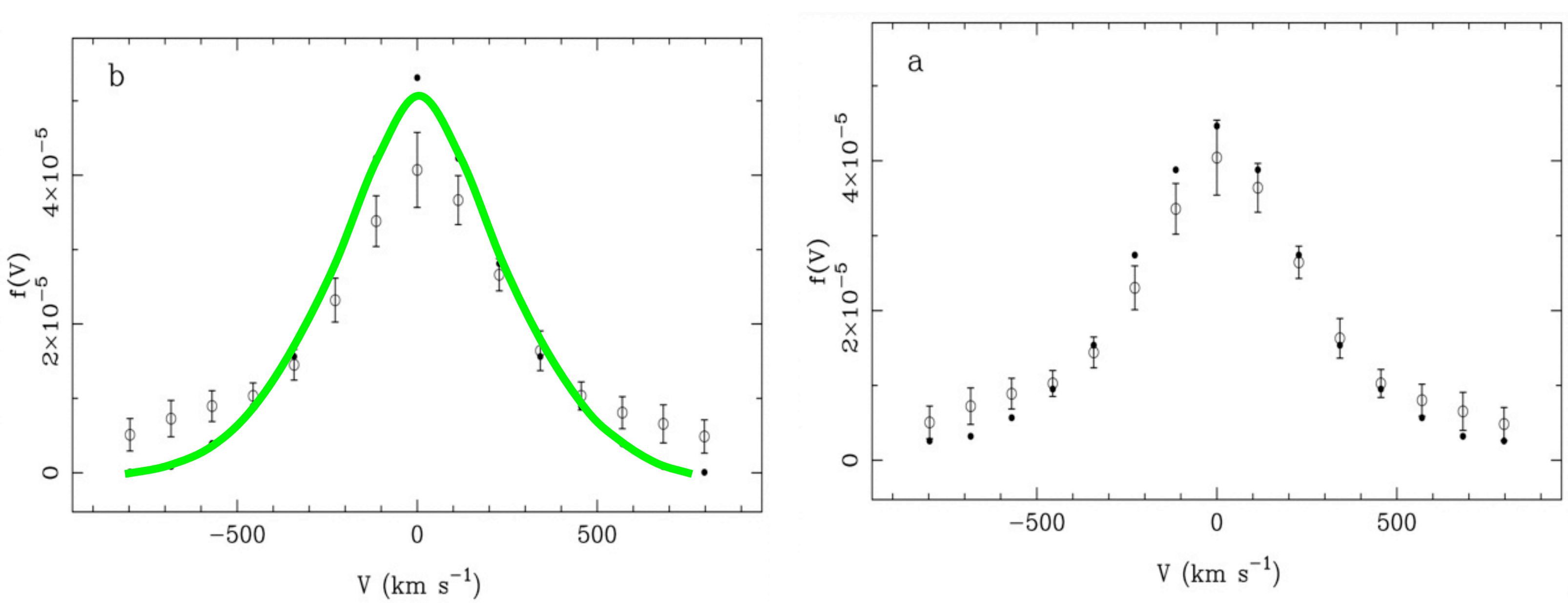

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Line of Sight Velocity Distributions

stars only

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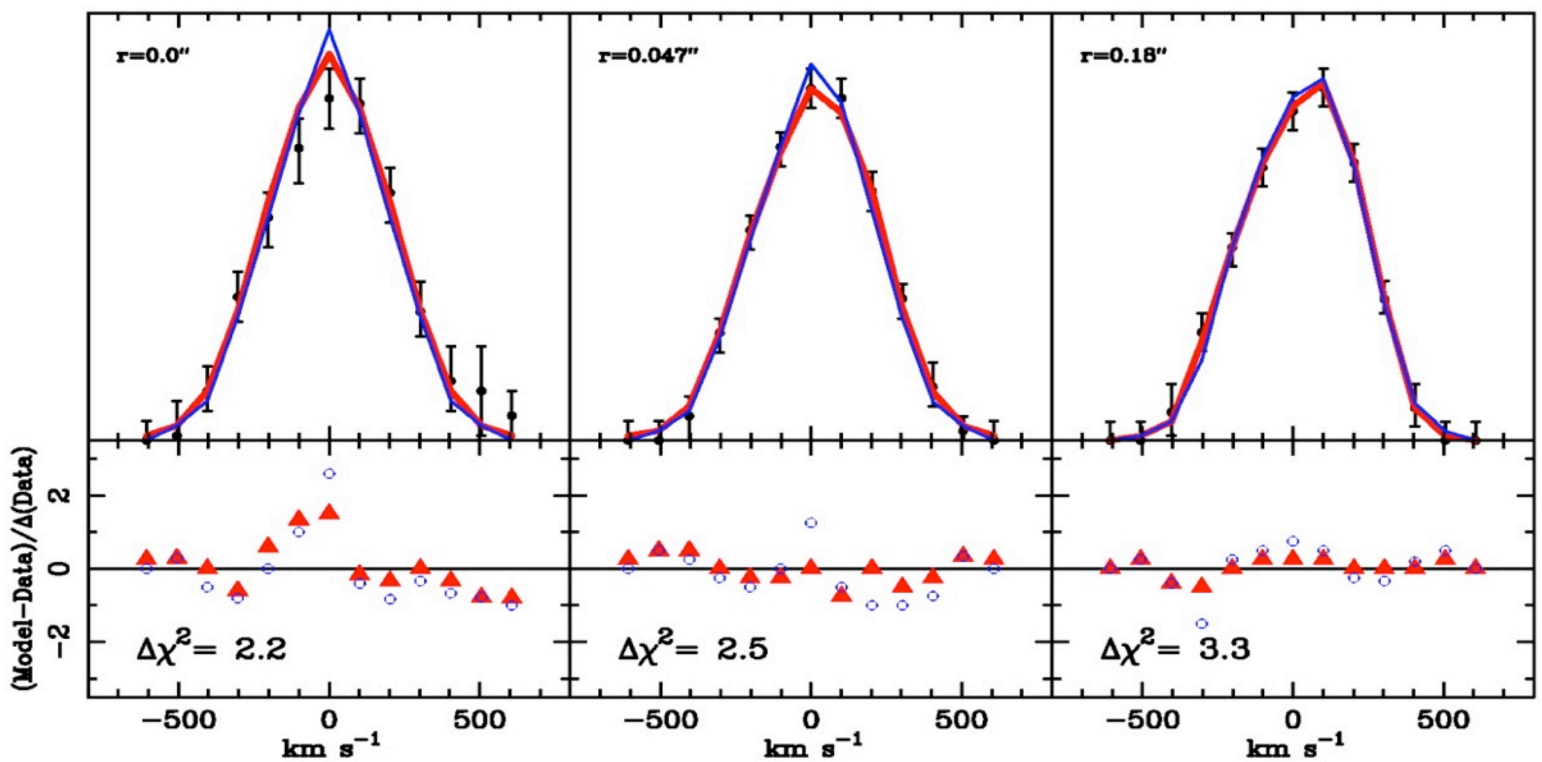

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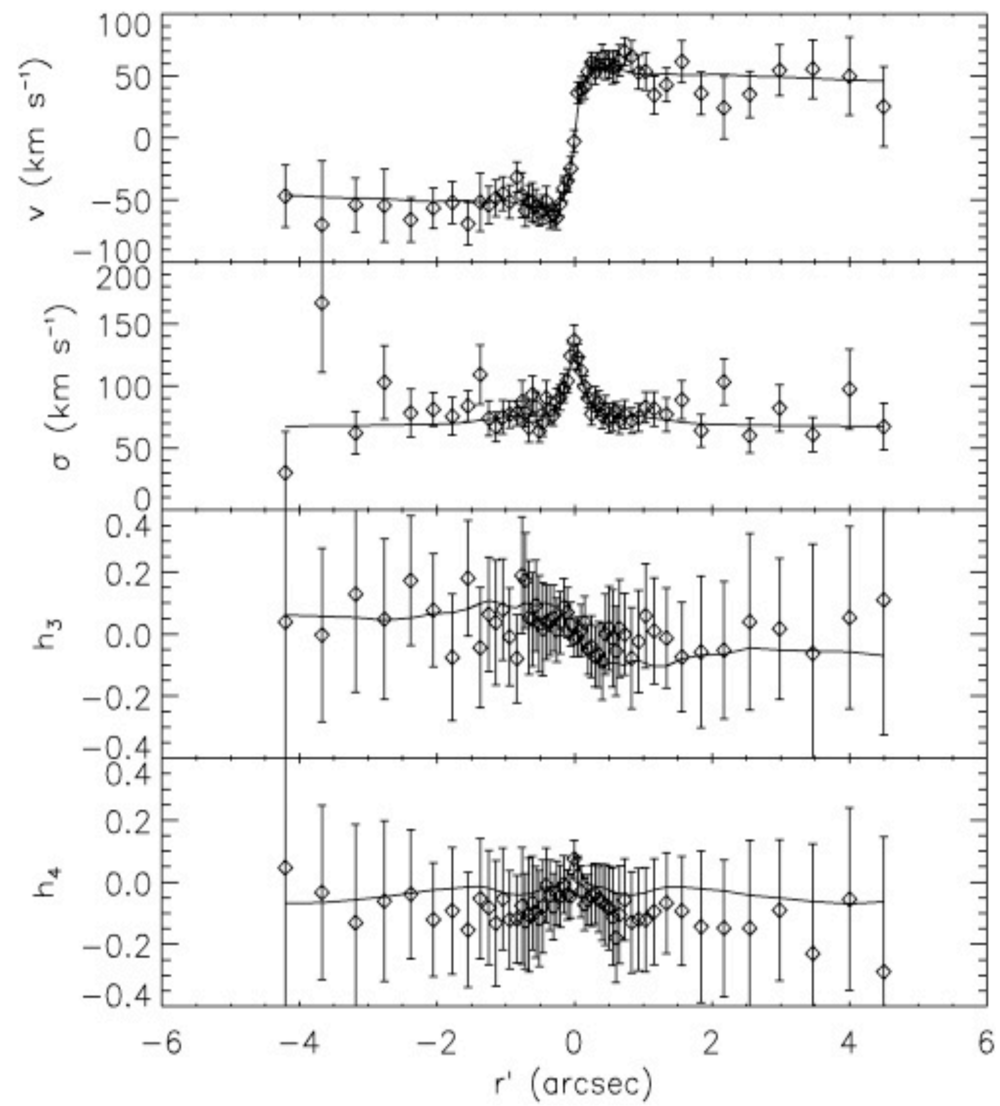

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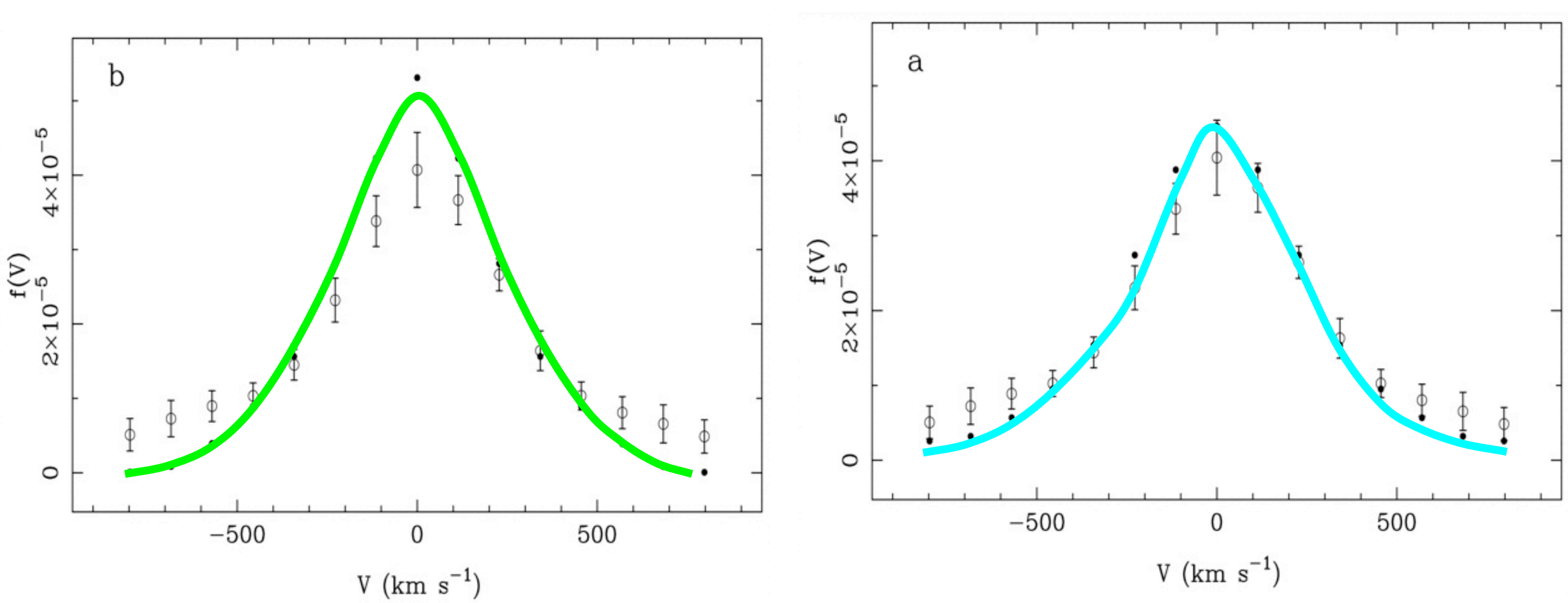


NGC 1023: Bower et al. 2001

Line of Sight Velocity Distributions

stars only

stars + BH

AO/IFU observations of Centaurus A

Moments of LOSVD →

With AO (3" x 3" FOV)

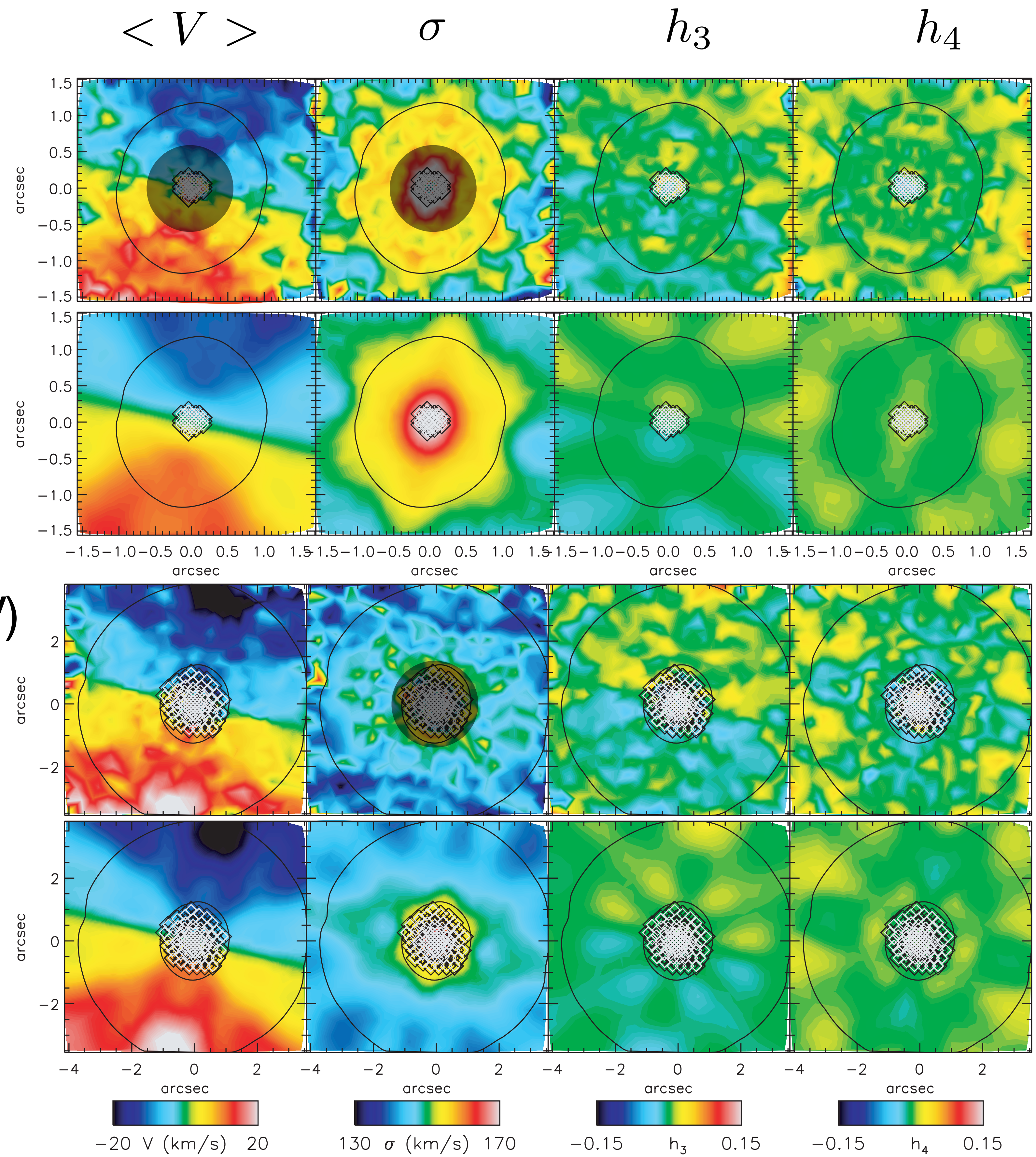
Data

Model

Seeing limited (8" x 8" FOV)

Data

Model



Cappellari et al. 2009

Gas vs Stellar dynamics: summary

Gas:

- high surface brightness, short integration times
- Easy interpretation

- but not in all galaxies
- only if system is a circularly rotating disk

Stars:

- completely gravitational motions
- available in all galaxies

- but interpretation difficult (3D star orbits)
- but observations require long integration times

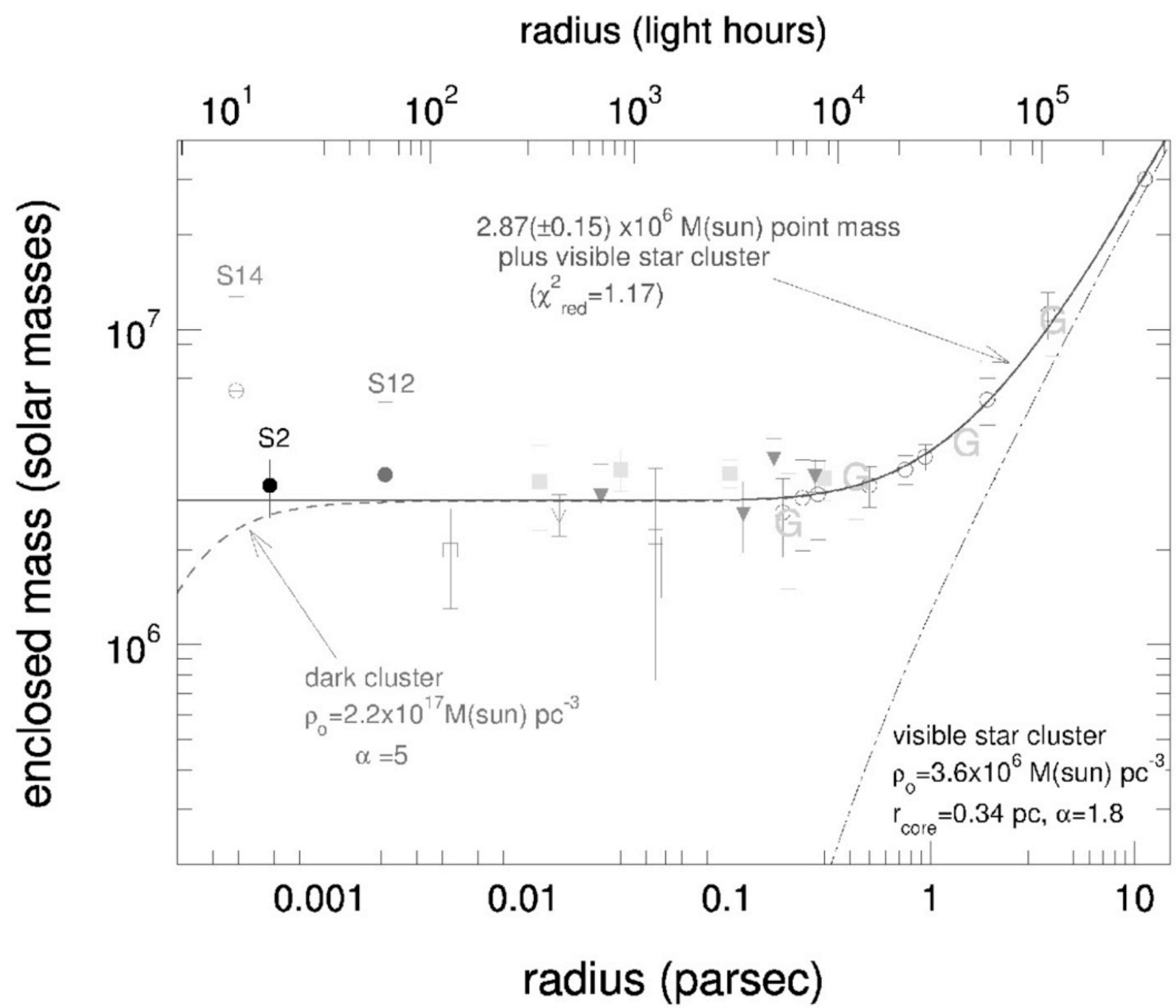
Gas vs Stellar dynamics

Are gas and stellar kinematical BH mass measurements consistent?

Comparison have been done only for a few cases and the derived M_{BH} are consistent!

- Galactic Center (Schodel et al. 2003, Genzel & Townes 1987)
- Centaurus A (Marconi et al. 2006, Silge et al. 2005, Neumayer et al. 2007, Cappellari et al. 2007)
- M87 (Macchetto+97, Gebhardt+11)

Need more tests!



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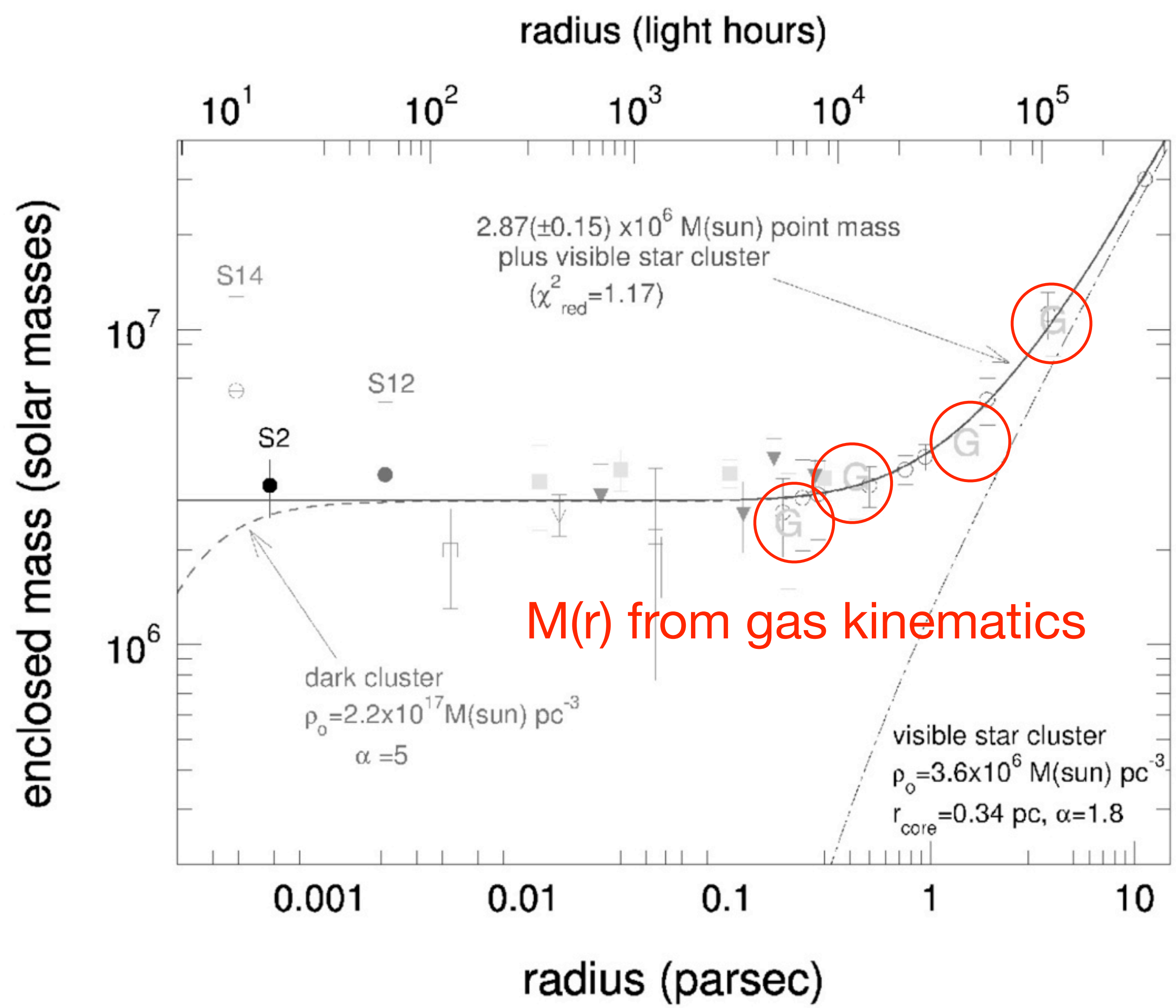

BH Masses and Galaxy distances

To detect a massive BH and measure its mass it is important to "deconvolve" BH gravitational effects on star and/or gas kinematics from those of the galaxy gravitational potential.

Two related key elements:

- spatial resolution of the BH sphere of influence
- signal-to-Noice (S/N) ratio of the kinematical measurements

The required ratio of $R_{BH}/\Delta\theta$ (sp. resolution) depends on S/N and vice-versa.

Assume one needs $\Delta\theta = R_{BH}$ therefore the maximum distances

$$D = 1.8\,\mathrm{Mpc}\left(\frac{M_{BH}}{10^6\,M_\odot}\right)\left(\frac{\sigma_\star}{70\,\mathrm{km/s}}\right)^{-2}\left(\frac{\Delta\theta}{0.1''}\right)^{-1}$$

$$D = 22\,\mathrm{Mpc}\left(\frac{M_{BH}}{10^8\,M_\odot}\right)\left(\frac{\sigma_\star}{200\,\mathrm{km/s}}\right)^{-2}\left(\frac{\Delta\theta}{0.1''}\right)^{-1}$$

$$D = 98\,\mathrm{Mpc}\left(\frac{M_{BH}}{10^9\,M_\odot}\right)\left(\frac{\sigma_\star}{300\,\mathrm{km/s}}\right)^{-2}\left(\frac{\Delta\theta}{0.1''}\right)^{-1}$$

We are really limited to the very nearby universe (D~100-200 Mpc)!

Scales probed

Which are the scales probed in terms of the Schwarzschild radius, i.e. the BH typical *size*?

$$\theta_S = \frac{R_S}{D} = 2\frac{GM_{\rm BH}}{c^2\,D} \simeq 10^{-7}\,{\rm arcsec}\left(\frac{M_{\rm BH}}{10^8\,{\rm M}_\odot}\right)\left(\frac{D}{20\,{\rm Mpc}}\right)$$

The spatial scales we are probing are ~10^6 Schwarschild radii!

Method & Telescope	Scale (R_S)	No. of SBH Detections	$M_\bullet$ Range ($M_\odot$)	Typical Densities ($M_\odot$ pc^{-3})
Fe Kα line (XEUS, ConX)	3–10	0	N/A	N/A
Reverberation mapping (Ground based optical)	600	36	10^6–4×10^8	$\gtrsim 10^{10}$
Stellar proper motion (Keck, NTT, VLT)	1000	1	4×10^6	4×10^{16}
H_2O megamasers (VLBI)	10^4	1	4×10^7	4×10^9
Gas dynamics (optical) (Mostly *HST*)	10^6	11	7×10^7–4×10^9	$\sim 10^5$
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Review on BHs in galaxies by Ferrarese & Ford 2005

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Are they really Black Holes?

- The fact that we are probing scales ~10^6 R_S can cast doubts on whether we are really detecting black holes.
- Unambiguous proof of existence of BHs, as defined by GR, requires determination of ϕ_{BH} on scales of the event horizon.
- Ideally one should follow motions of test particles close to R_S
- This is has not been possible until now, not even in the Galactic Center
- To lower degree of confidence, one can show that ϕ_{BH} is due to a central mass condensation of non-stellar origin, which must be a BH because all other possible configurations are more extended, not stable or produce more light (Genzel et al. 2010)

Are they really Black Holes?

BHs are in reality *Massive Dark Objects*, i.e. dark matter objects unresolved at the spatial resolution of observations.

Maoz (1997) use simple physical considerations to derive the maximum possible lifetime of a dark cluster (brown dwarfs, Jupiters, white dwarfs, neutron stars, stellar black holes, etc.)

He computes the lifetime of the cluster against collapse to a supermassive BH. Main physical processes which lead to collapse are:

- Evaporation (objects in the tail of Maxwellian distribution can escape gravitational attraction of cluster; cluster readjusts itself and contracts for energy loss)
- Collisions

Are they really Black Holes?

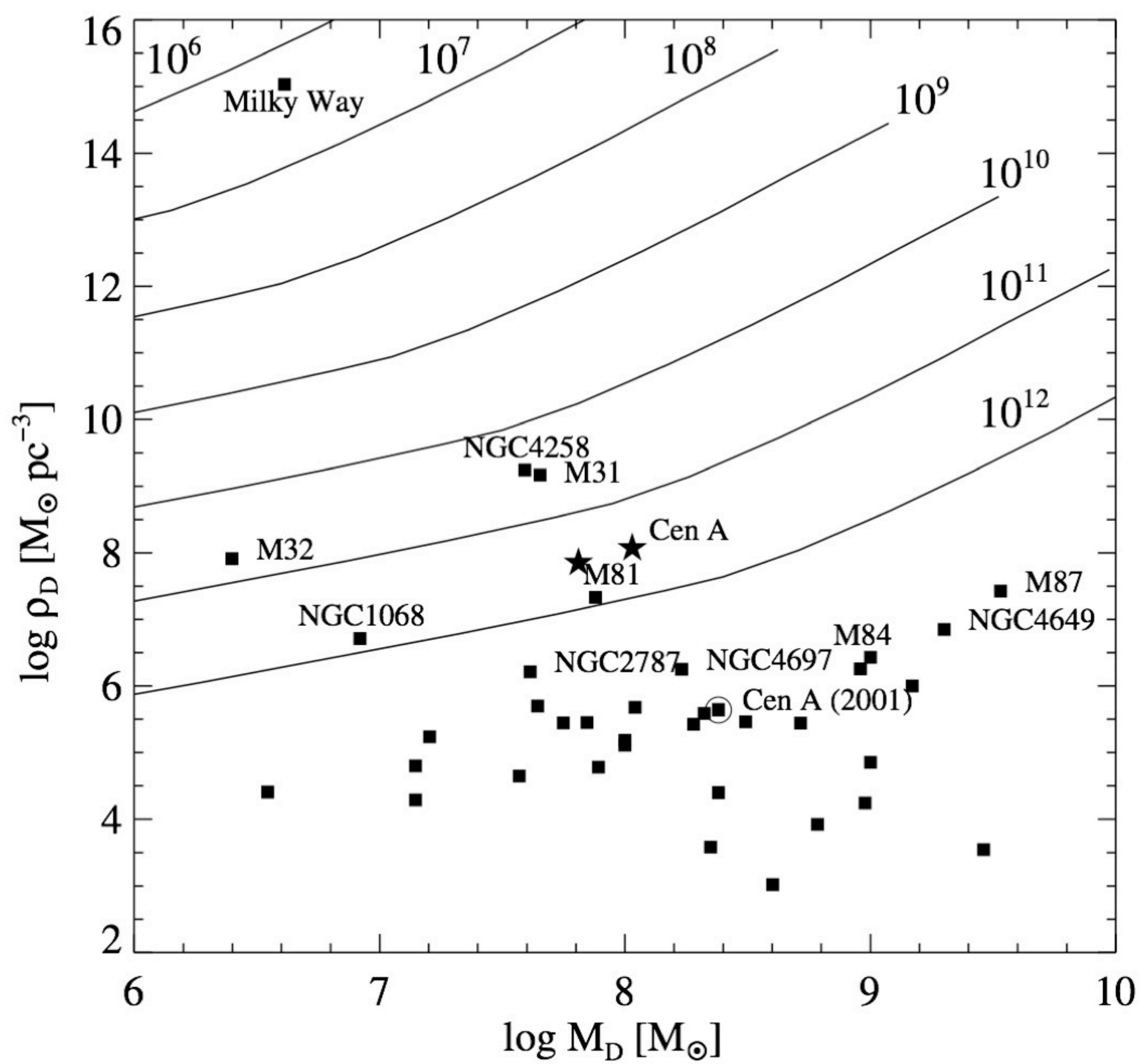
M_D from observations

Cluster size from spatial resolution of observations (FWHM~2 R_D)

Estimate average density of star cluster ρ_D

Only in the case of the Milky Way the cluster lifetime << age of the universe

Boson star is the only possible alternative to a BH (?)



adapted from Maoz 1998 (MW updated)

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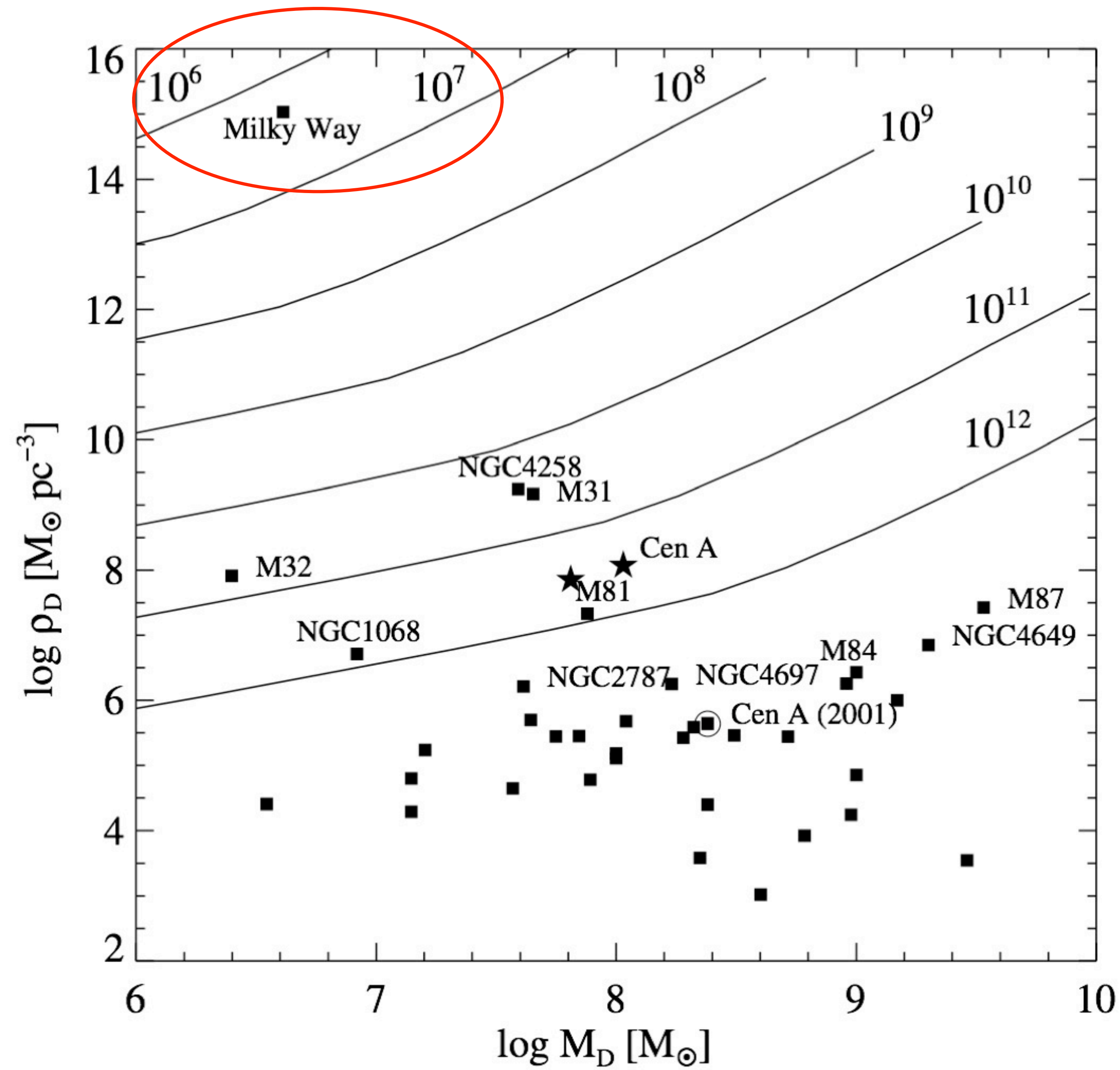

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