

# Precise predictions for Higgs boson pair production and decay

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# Higgs self-coupling

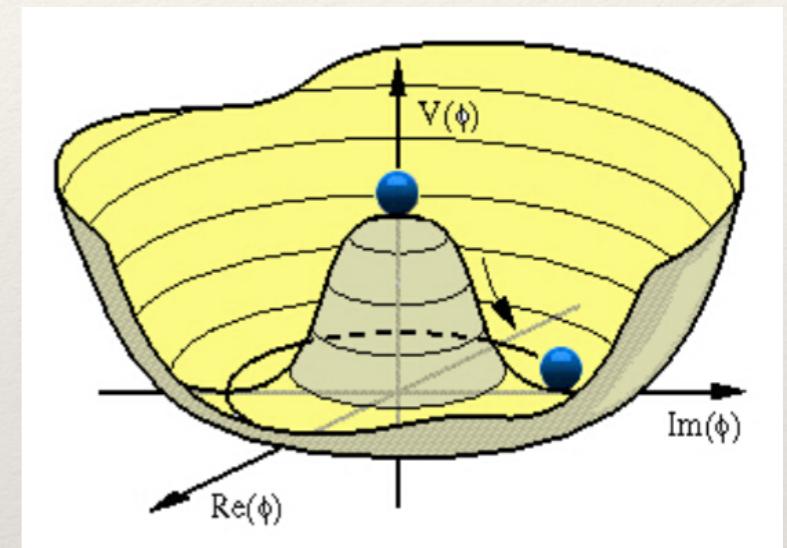
- ★ Mass generations of gauge bosons: Higgs mechanism
- ★ Mass generations of fermions: Higgs mechanism & Yukawa couplings
- ★ Mass generations of scalars?

$$V(\phi) = -m^2|\phi|^2 + \lambda|\phi|^4$$



$$M_H = \sqrt{2}m = \sqrt{2\lambda v}$$

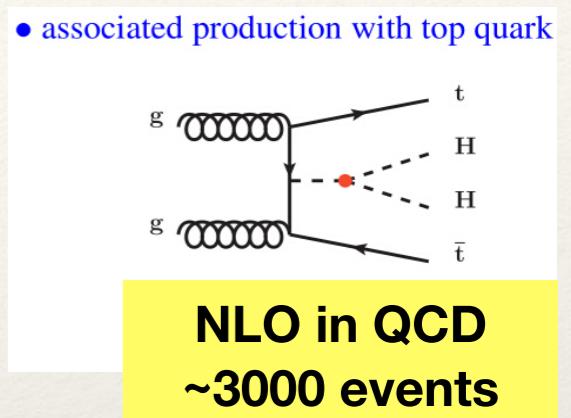
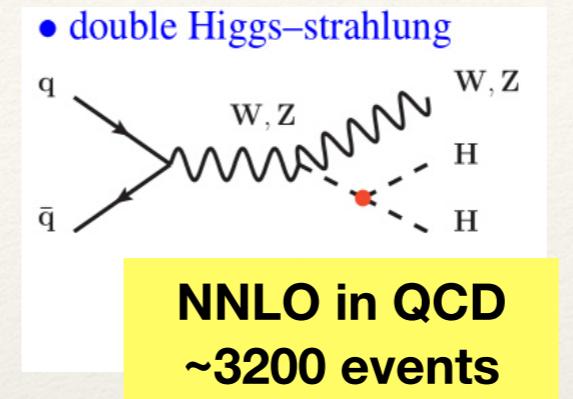
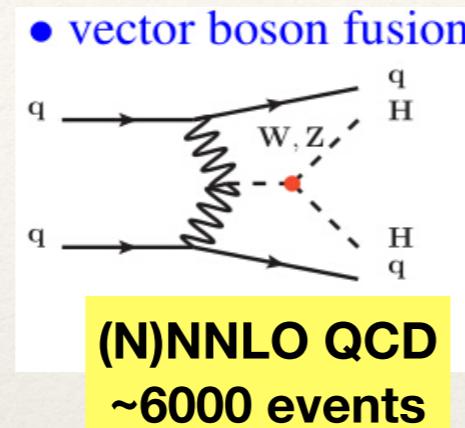
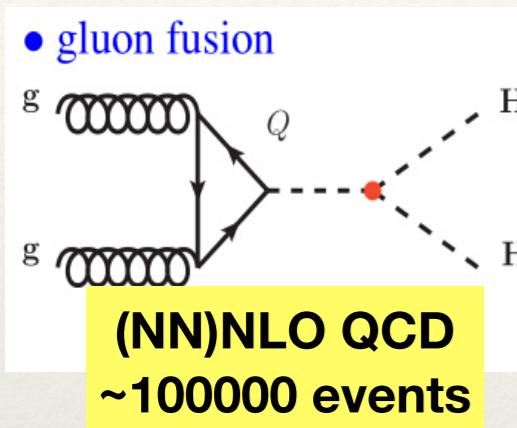
$$\phi = \begin{pmatrix} 0 \\ \frac{v + H(x)}{\sqrt{2}} \end{pmatrix} \Rightarrow V(H) = \frac{1}{2}M_H^2H^2 + \frac{1}{2}\frac{\mathbf{M}_H^2}{v}\mathbf{H}^3 + \frac{1}{8}\frac{M_H^2}{v^2}H^4$$



In some new physics models, the trilinear Higgs self-coupling may change by O(100)%, while the couplings with gauge bosons and fermions are still in agreement with SM.

We need to measure the trilinear self coupling directly.

# Higgs pair production

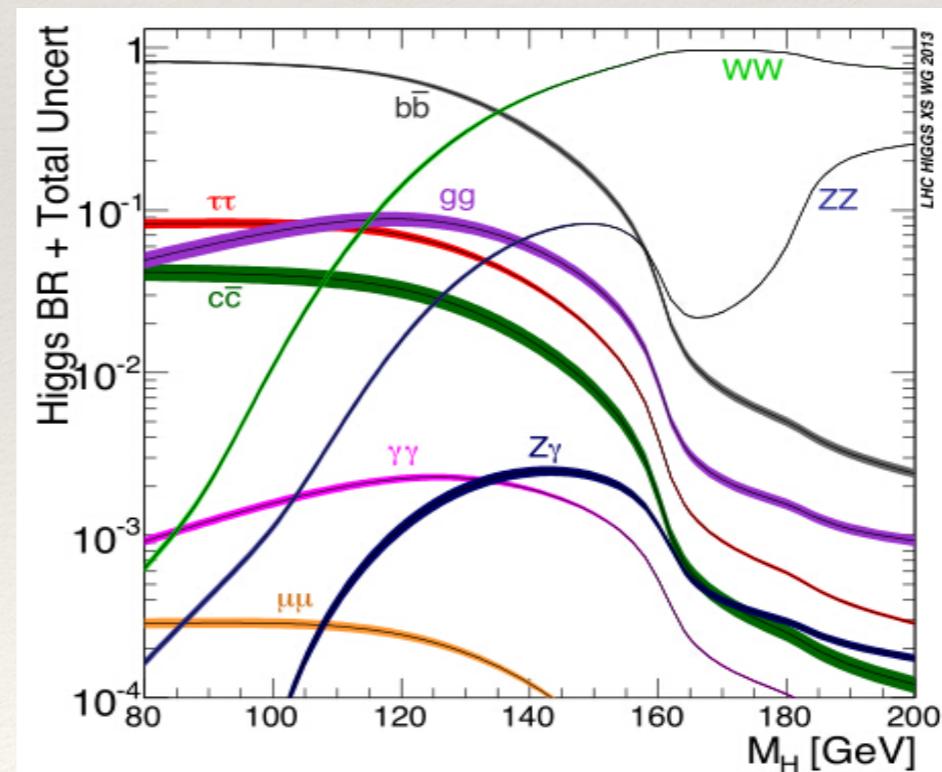
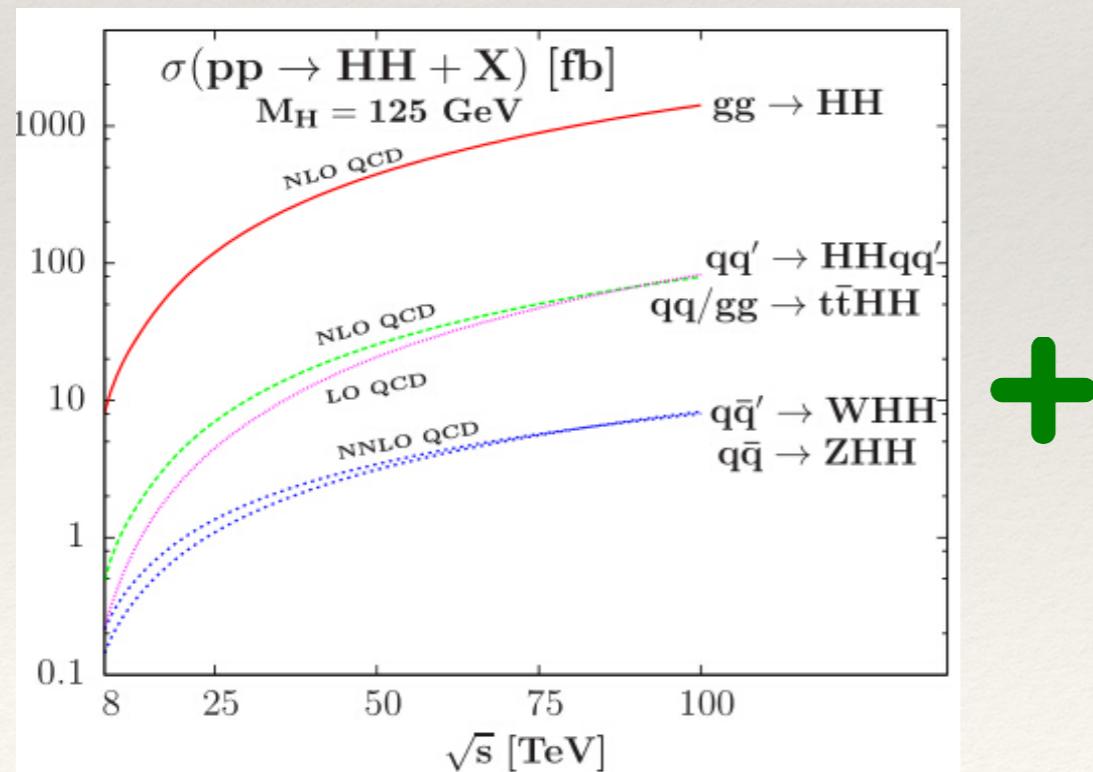


Borowka,et al,PRL117,012001  
 Baglio, et al, EPJC, 79, 459  
 Chen,Li, Shao,**JW**, PLB 803,135292  
 JHEP,2003,072

Ling,Zhang,Ma,Guo,Li,Li,  
 PRD89,073001  
 Dreyer,Karlberg, PRD98,114016,  
 PRD99,074028

Baglio,et al, JHEP1304,151  
 Li,Li,**JW**, PRD97,074026, PLB765,265

Englert,et al, PLB743,93  
 Liu, Zhang, 1410.1855  
 Frederix,et al, PLB 732,142



$b\bar{b}WW$   
 $b\bar{b}\gamma\gamma$   
 $b\bar{b}\tau\tau$   
 $Wb\bar{b}b\bar{b}$   
 $t\bar{t}b\bar{b}b\bar{b}$   
 . . . . .

**CMS**35.9 fb<sup>-1</sup> (13 TeV)*bbVV*Observed 78.6×SM  
Expected 88.8×SM*bbbb*Observed 74.6×SM  
Expected 36.9×SM*bbττ*Observed 31.4×SM  
Expected 25.1×SM*bbγγ*Observed 23.6×SM  
Expected 18.8×SM**Combined**Observed 22.2×SM  
Expected 12.8×SM $gg \rightarrow HH$ 

● Observed

— Median expected

68% expected

95% expected

~20xSM

95% CL on  $\sigma_{HH}/\sigma_{HH}^{\text{SM}}$ 

~7xSM from ATLAS

6 7 8 9 10 20 30 40 50 60 70 100 200 300 400

# Non-resonant HH production at 13 TeV with about $36 fb^{-1}$

Final state	collaboration	allowed $\kappa_\lambda$ interval at 95% CL observed	expected
$b\bar{b}b\bar{b}$	ATLAS	-11 – 20	-12 – 19
	CMS	-23 – 30	-15 – 23
$b\bar{b}\tau^+\tau^-$	ATLAS	-7.3 – 16	-8.8 – 17
	CMS	-18 – 26	-14 – 22
$b\bar{b}\gamma\gamma$	ATLAS	-8.1 – 13	-8.2 – 13
	CMS	-11 – 17	-8.0 – 14
Combined	ATLAS	-5.0 – 12	-5.8 – 12
	CMS	-12 – 19	-7.1 – 14
Our combination	Both experiments	-6.8 – 14	-4.6 – 11

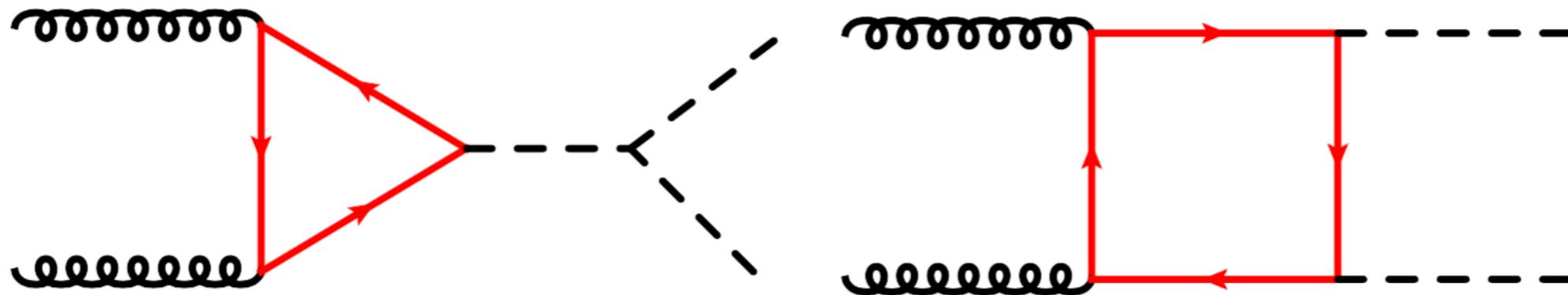
# Why do we need precise prediction?

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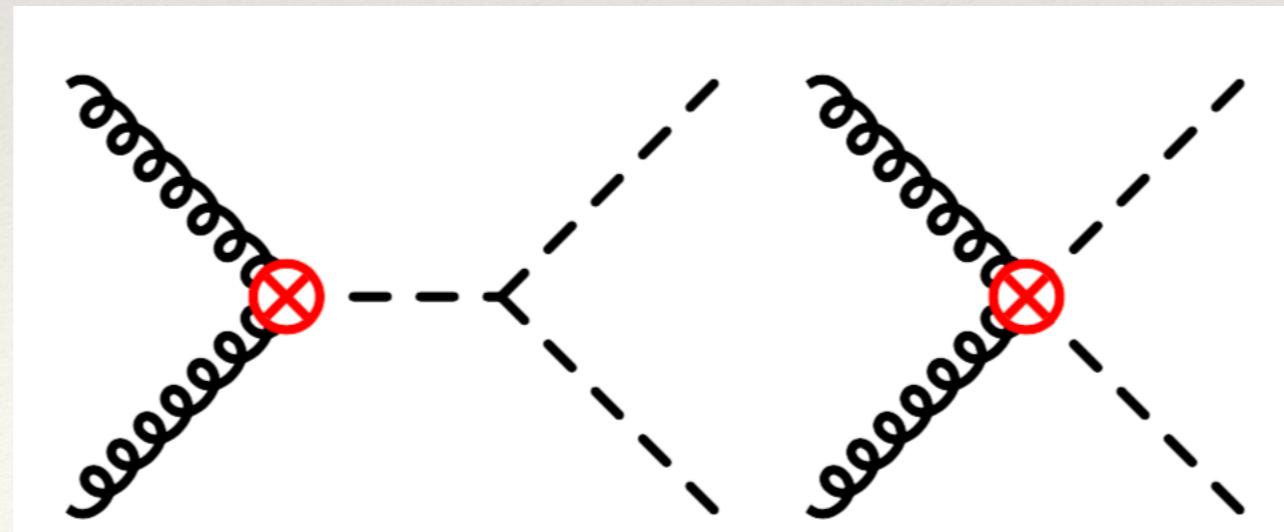
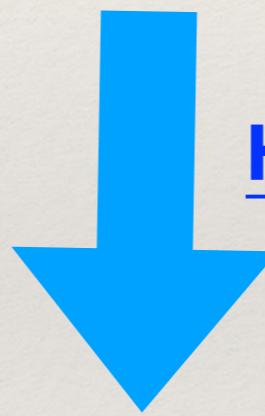
- 1. The measured events numbers do not depend on the theoretical prediction, but the interpretation does.**
- 2. As time goes by, the experimental uncertainties will reduce. Theoretical uncertainties will reduce only after we calculate higher-order corrections.**
- 3. Renormalization and factorization scale uncertainties are sizable, especially for Higgs productions.**

$gg \rightarrow HH$

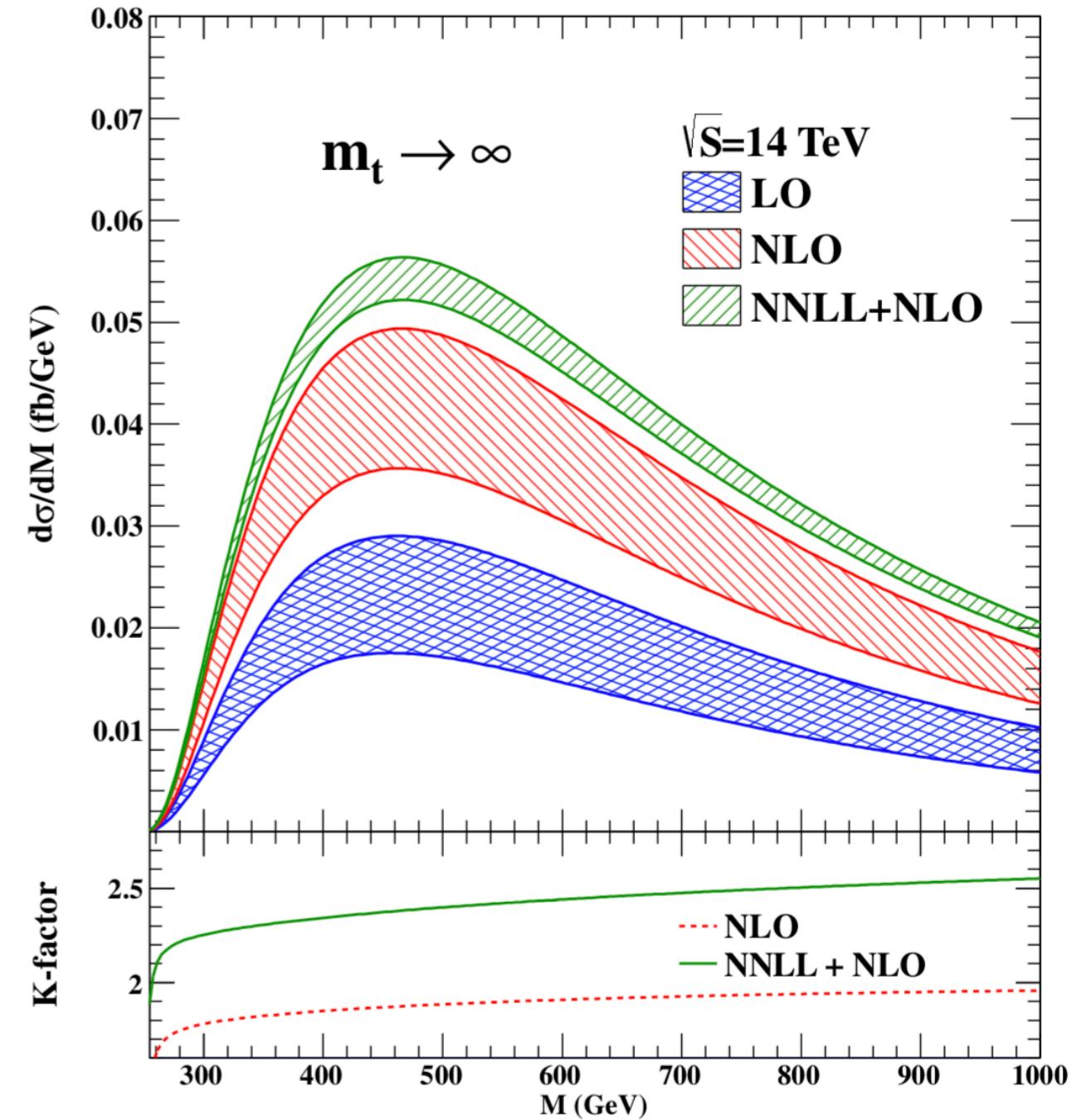
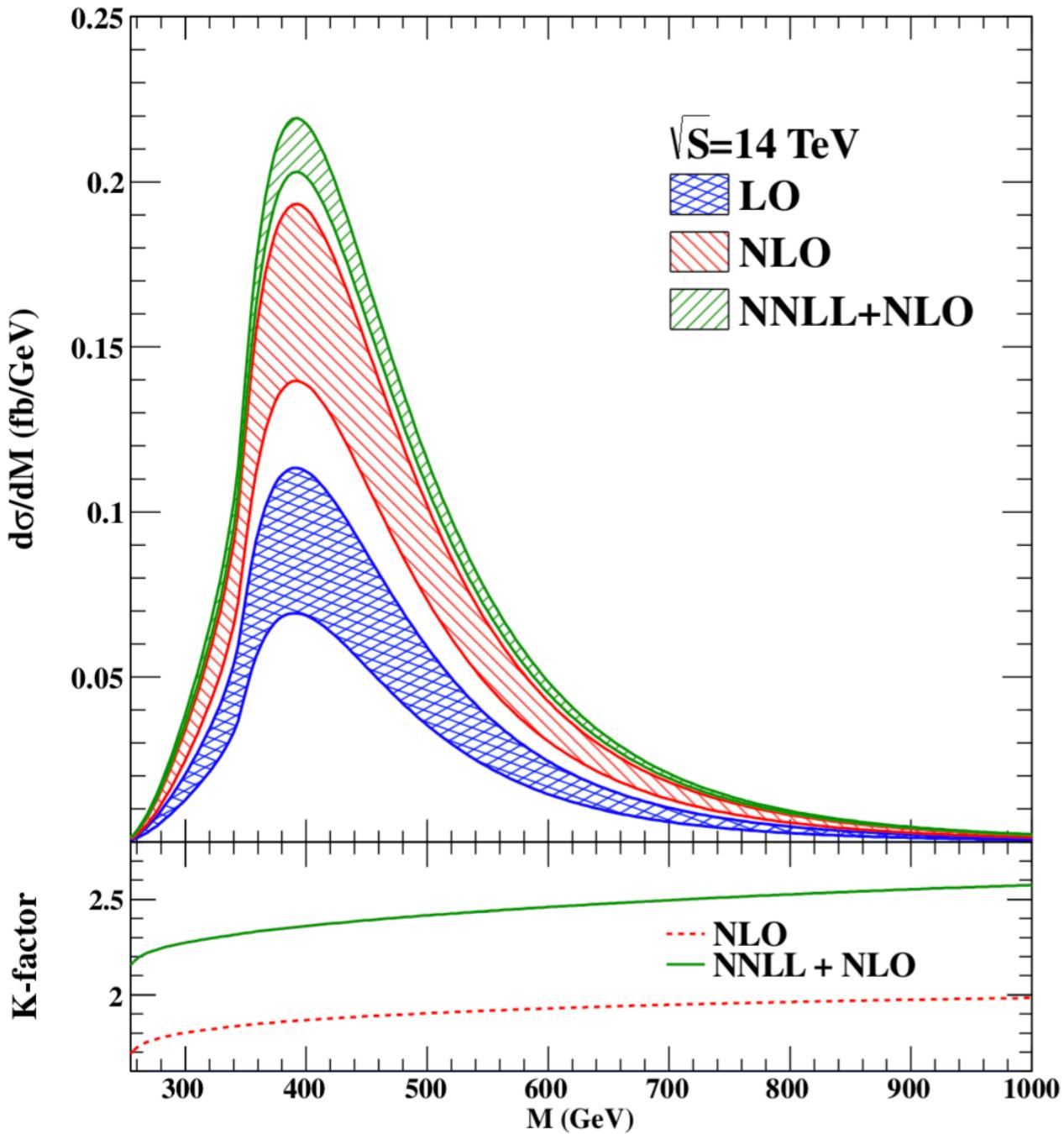
LO in SM



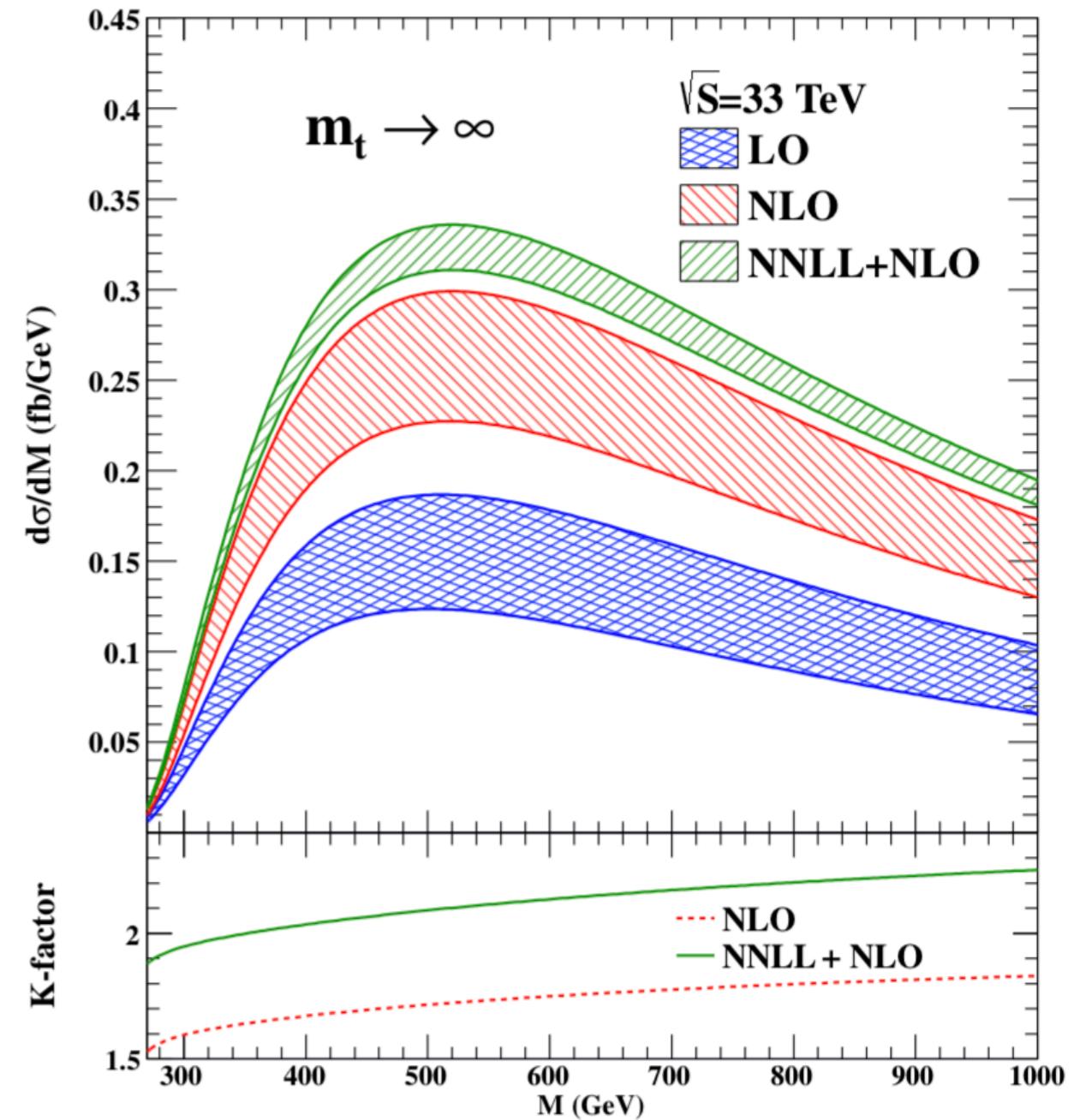
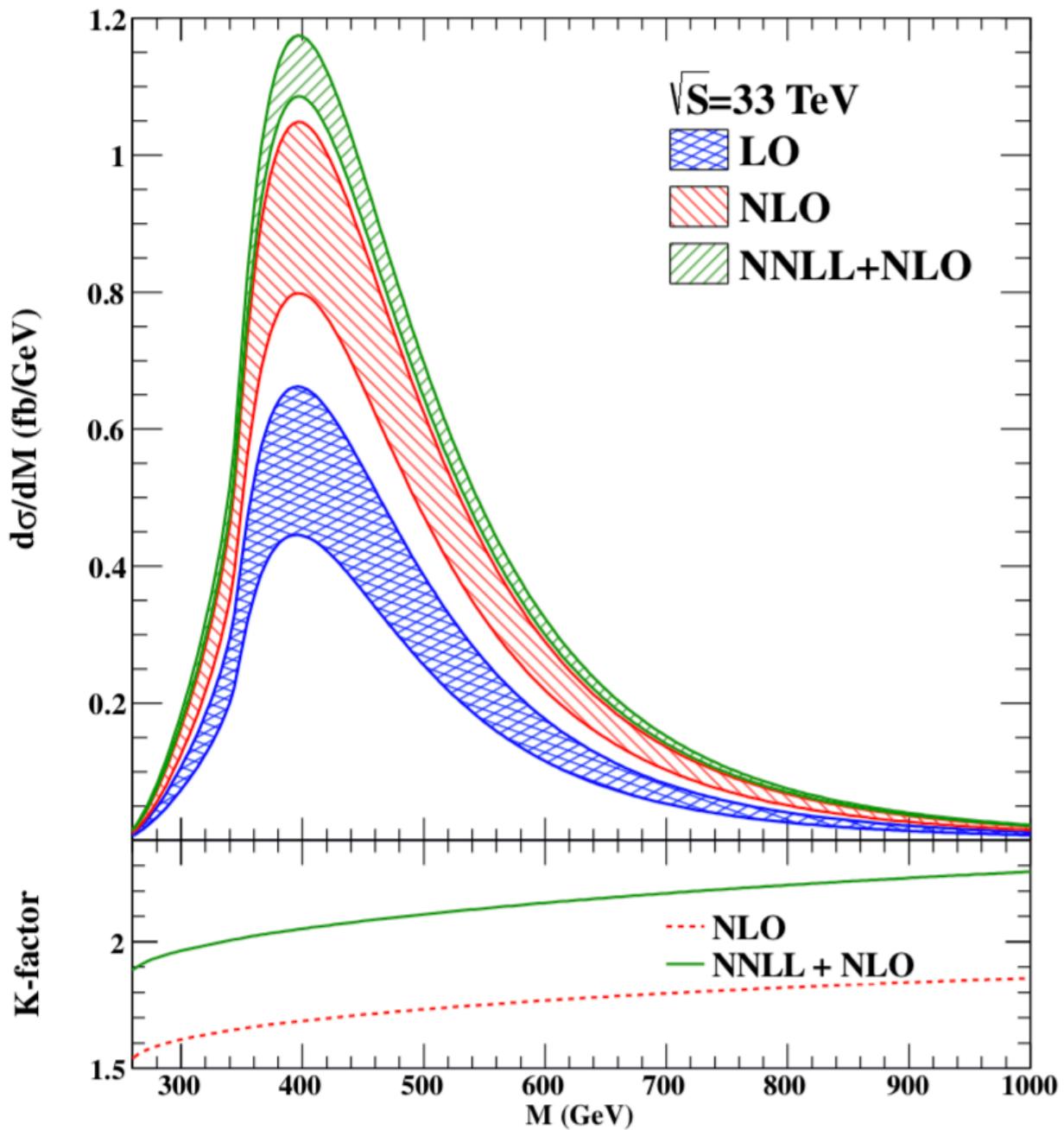
Heavy top quark limit



# Q: How well is the approximation?



# gg>HH@NNLL

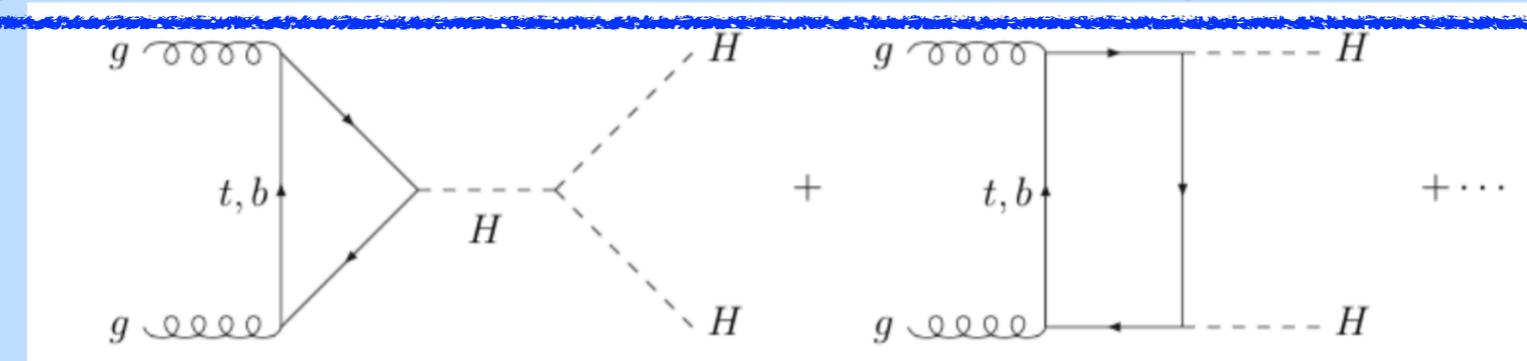


# $gg \rightarrow HH$ @ NLO with full mt dep.

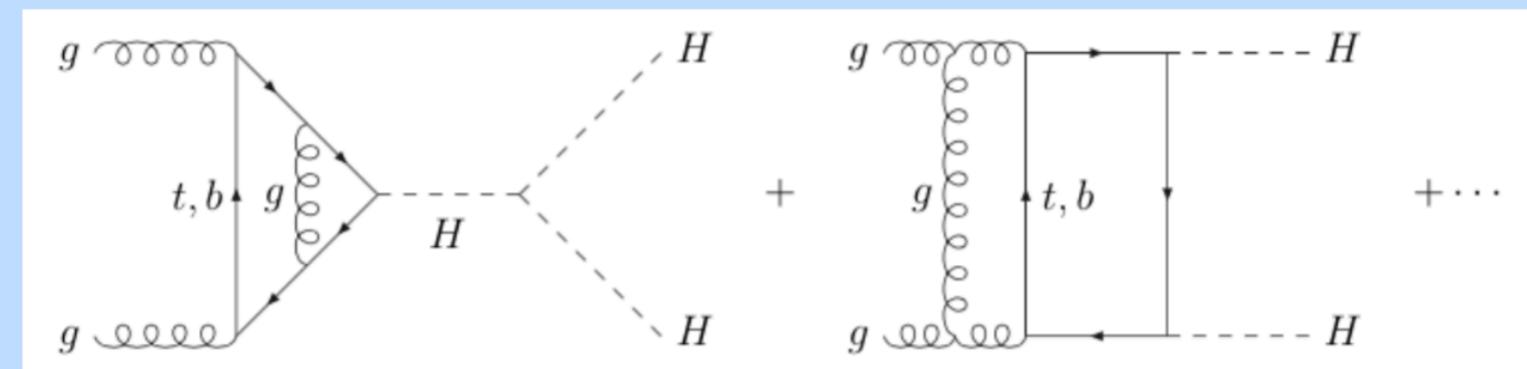
$$\sigma_{\text{NLO}}(pp \rightarrow HH + X) = \sigma_{\text{LO}} + \Delta\sigma_{\text{virt}} + \Delta\sigma_{qq} + \Delta\sigma_{gg} + \Delta\sigma_{q\bar{q}},$$

**Very difficult to obtain analytical results**

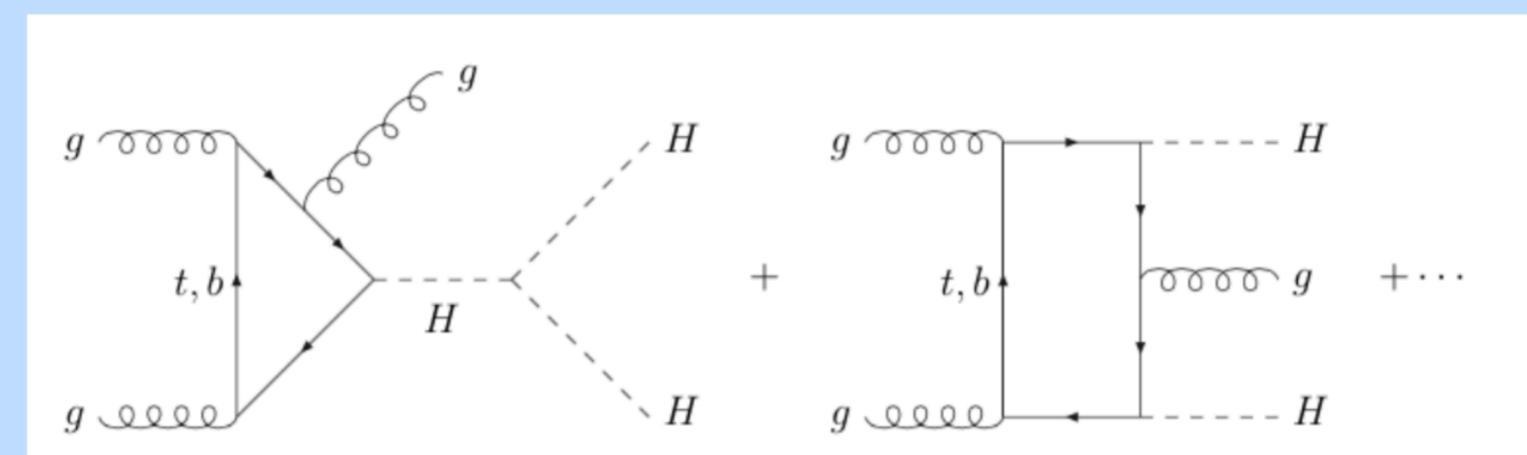
$\sigma_{\text{LO}}:$



$\Delta\sigma_{\text{virt}}:$

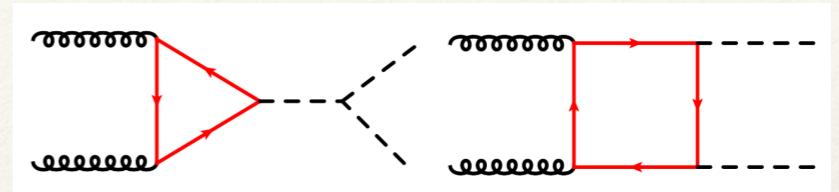


$\Delta\sigma_{ij}:$

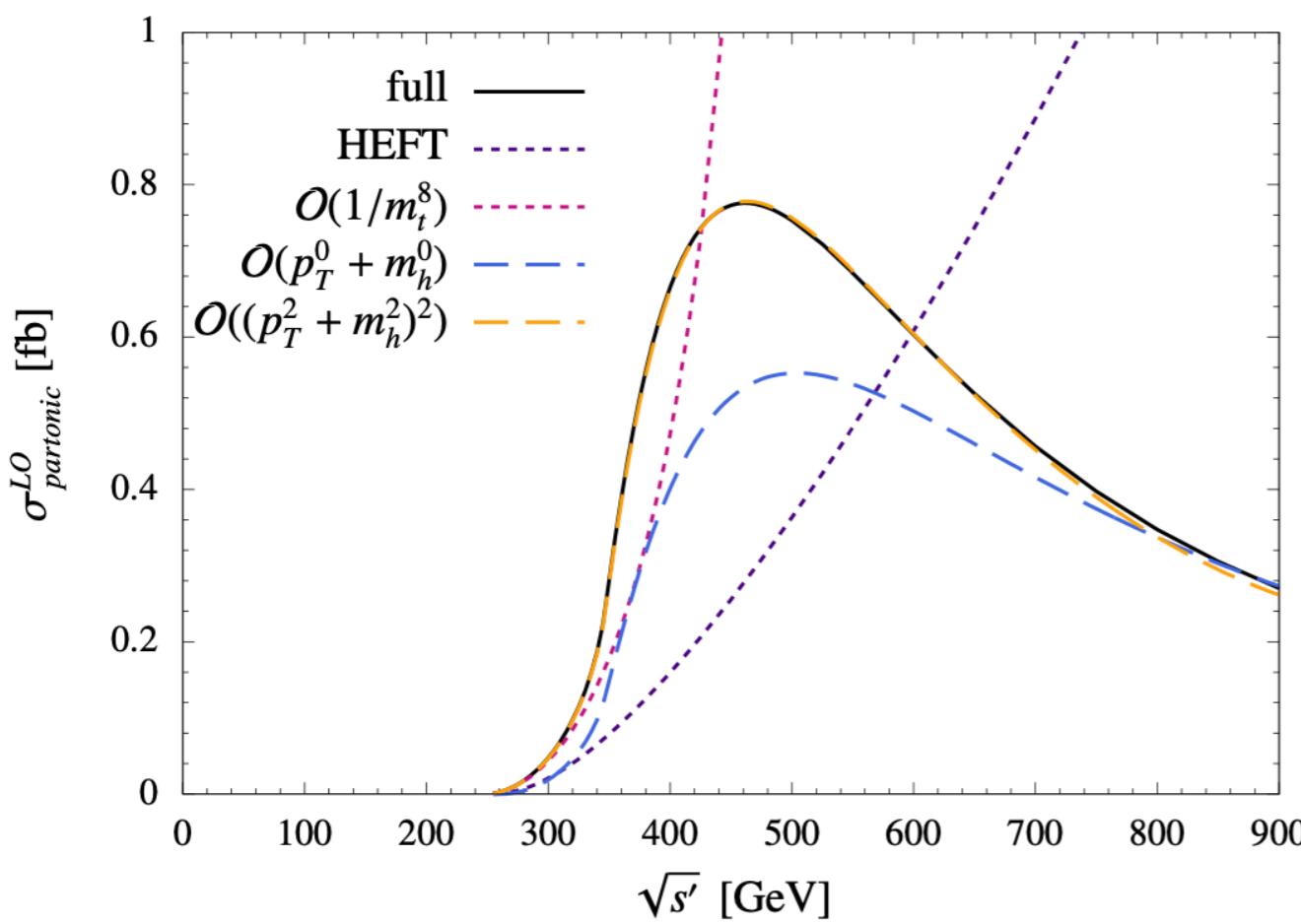


# gg>HH@NLO with full mt dep.

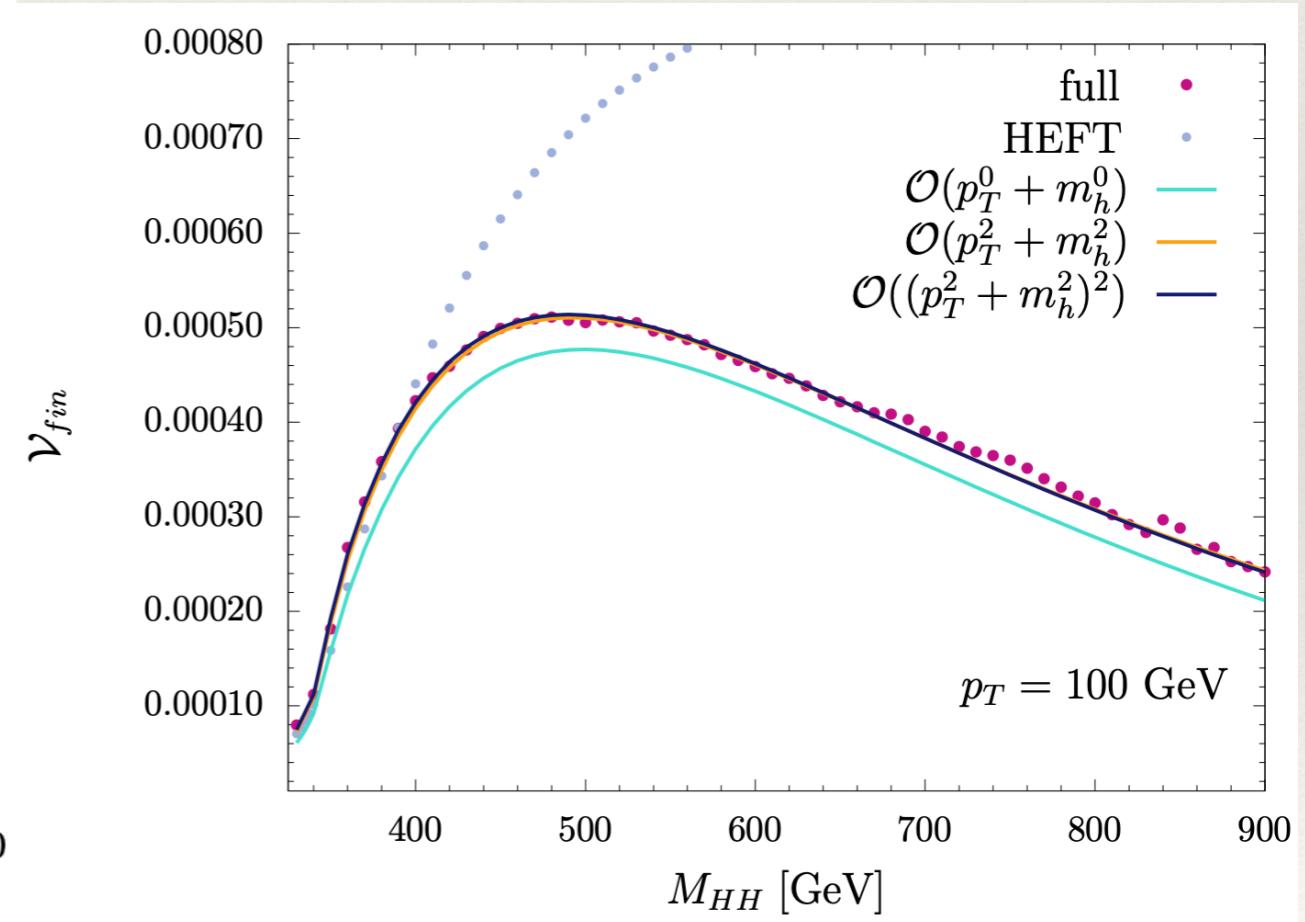
Expand in pT



**One-loop**

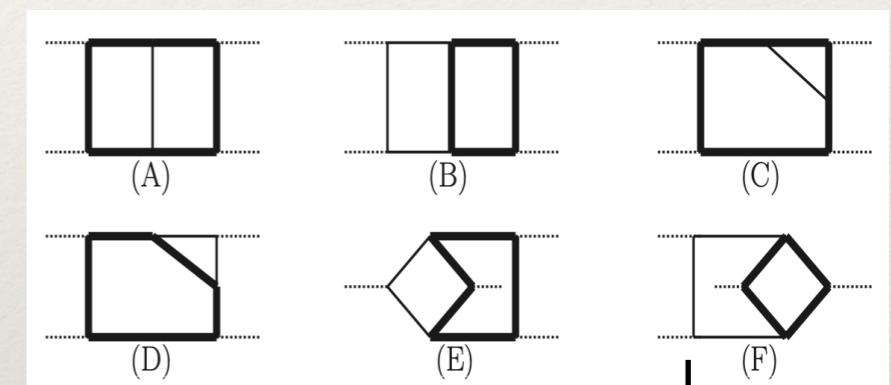
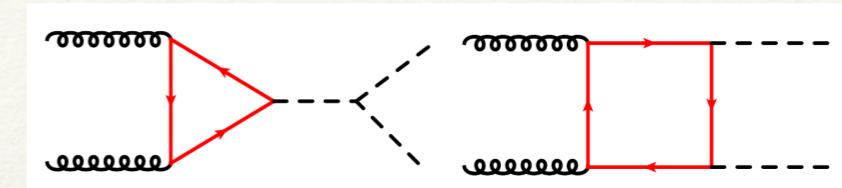
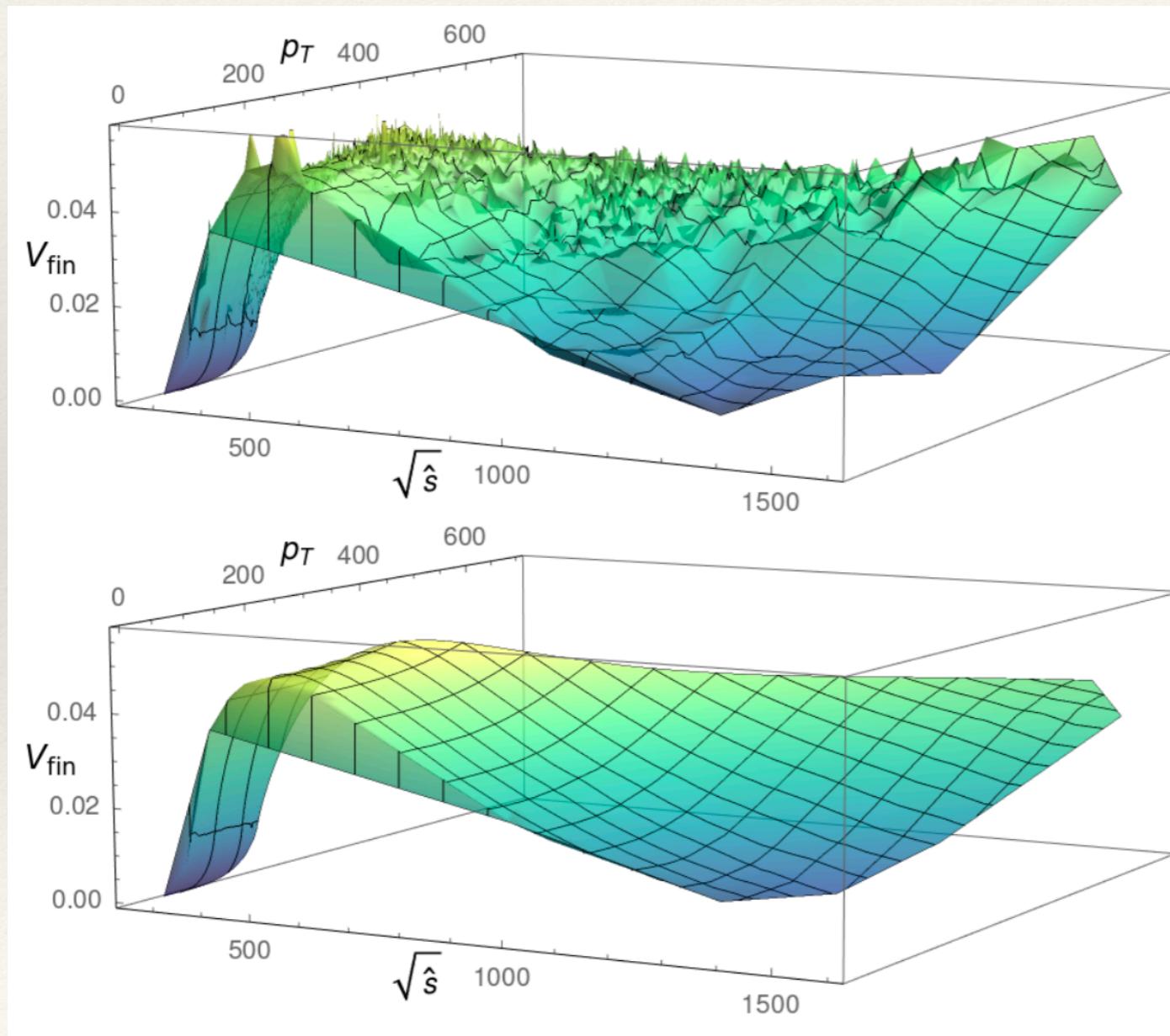


**Two-loop**



# $gg \rightarrow HH @\text{NLO}$ with full mt dep.

Expand in  $mH$



elliptic Feynman integrals

Xu and Yang, JHEP 1901, 211  
Wang, Wang, Xu, Xu, Yang, PRD104,051901

# gg>HH@NLO with full mt dep.

Numerical method: sector decomposition

$$I = \int_0^1 dx \int_0^1 dy x^{-1-\epsilon} y^{-\epsilon} (x + y - xy)^{-1}$$

$$I_1 = \int_0^1 dx x^{-1-\epsilon} \int_0^1 dt t^{-\epsilon} (1 + t - xt)^{-1},$$

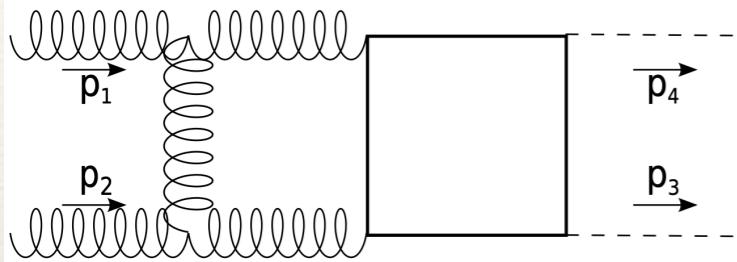
$$I_2 = \int_0^1 dy y^{-1-2\epsilon} \int_0^1 dt t^{-1-\epsilon} (1 + t - yt)^{-1}.$$

The QMC methods were born in the 1950s and 1960s from the desire to achieve faster convergence than the MC rate of  $O(1/n)$ , and can achieve a convergence rate of order  $O(1/n)$  for very smooth functions.

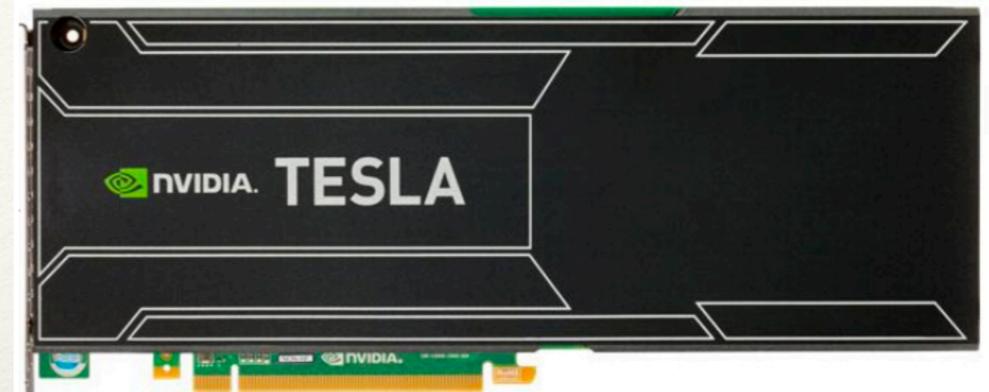
$$x^{-1+n\epsilon} = \frac{\delta(x)}{n\epsilon} + \left(\frac{1}{x}\right)_+ + n\epsilon \left(\frac{\ln x}{x}\right)_+ + \frac{(n\epsilon)^2}{2!} \left(\frac{\ln^2 x}{x}\right)_+ + O(\epsilon^3)$$

Binoth and Heinrich, NPB, 585, 741

# gg>HH@NLO with full mt dep.



$$I_B = e^{-2\epsilon\gamma_E} s^{-3-2\epsilon} \sum_{i=0}^{i=4} \frac{P_i}{\epsilon^i}$$

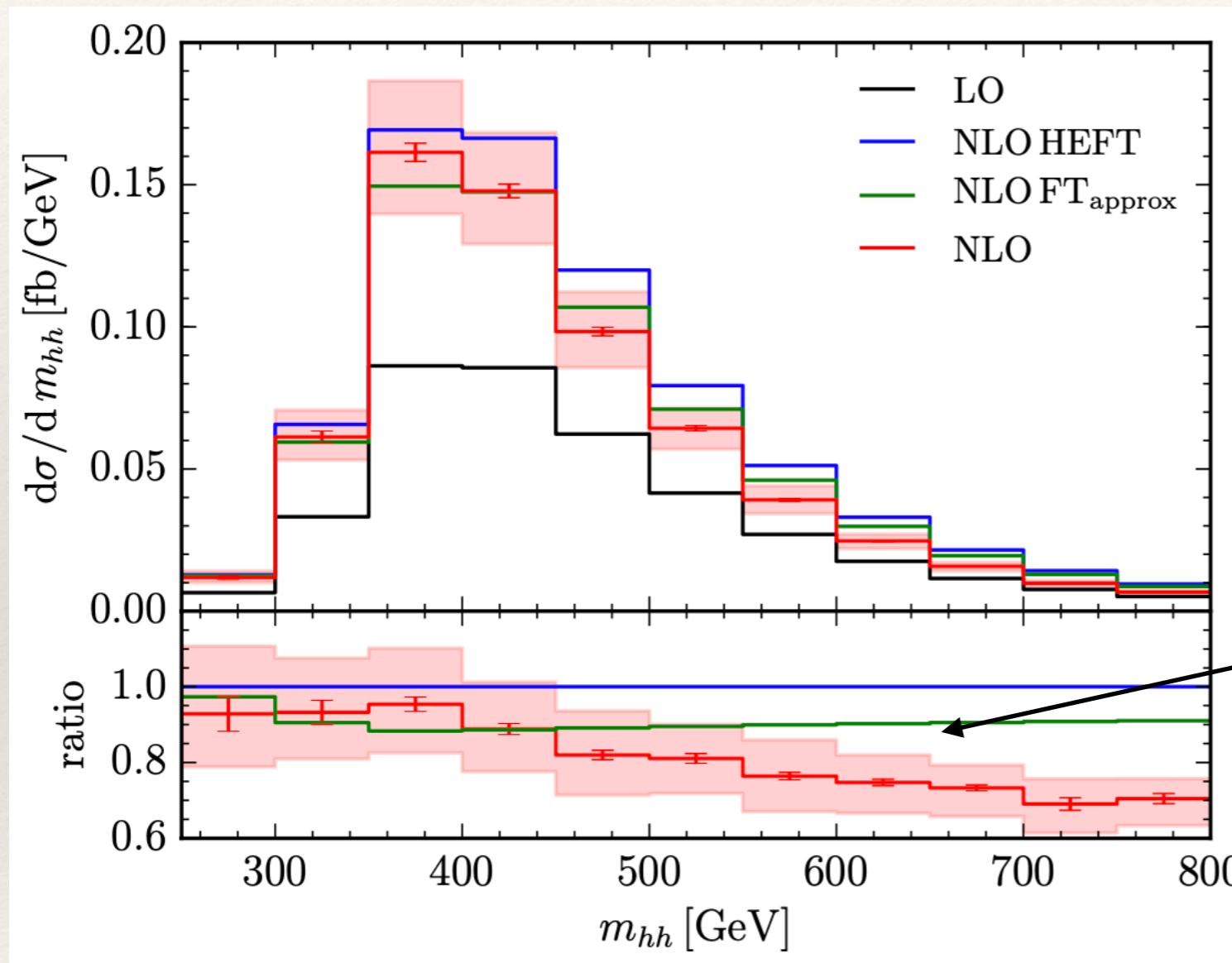


	Vegas/CPU	QMC/GPU
$P_2$	$-7.959 \pm 0.009 - 10.586i \pm 0.009i$	$-7.949 \pm 0.003 - 10.585i \pm 0.005i$
$P_1$	$3.9 \pm 0.1 - 28.1i \pm 0.1i$	$3.831 \pm 0.005 - 28.022i \pm 0.005i$
$P_0$	$-3.9 \pm 0.8 + 92.3i \pm 0.8i$	$-4.63 \pm 0.07 + 92.13i \pm 0.07i$
Time	45540s (12.7h)	19s

Li, JW, Yan, Zhao, *Chin.Phys.C40*, 033103

This method was soon adopted by the SecDec group in their code, and used in various calculations.  
<https://pypi.org/project/pySecDec/>

# $gg \rightarrow HH$ @ NLO with full mt dep.



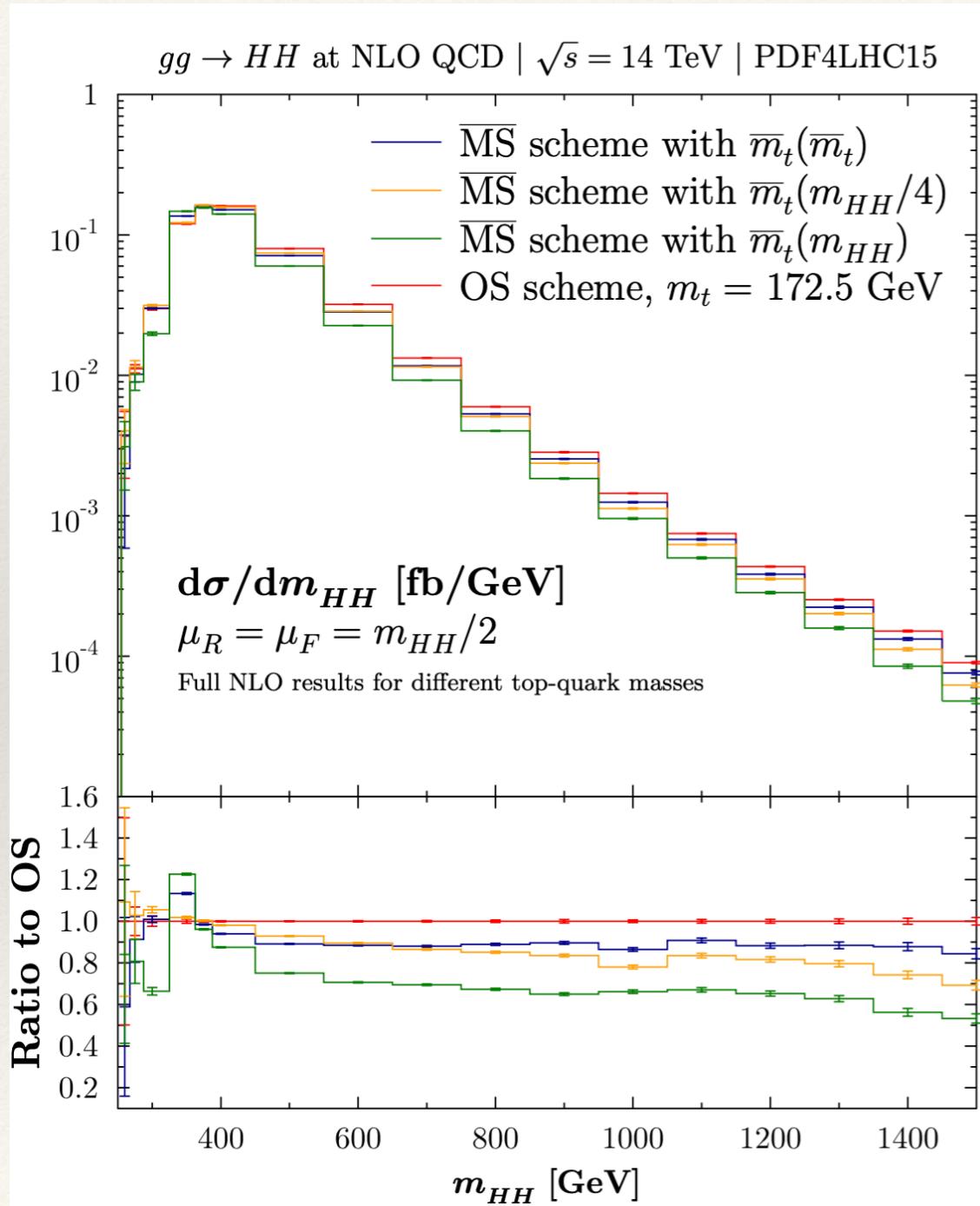
20-30% reduction after including  
top quark mass effects

# gg>HH@NLO: Full mt dependence

	PDF4LHC15	MMHT2014
$\sigma_{LO}$	19.80 fb	23.75 fb
$\sigma_{NLO}^{HTL}$	38.66 fb	39.34 fb
$\sigma_{NLO}$	32.78(7) fb	33.33(7) fb

$$\sigma(gg \rightarrow HH) = 32.78(7)^{+13.5\%}_{-12.5\%} \quad (\text{PDF4LHC15})$$

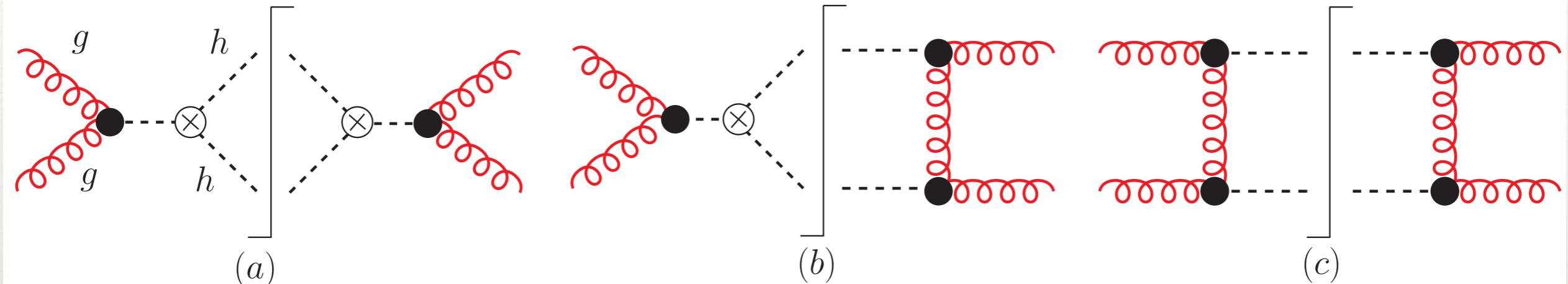
# gg>HH@NLO: Full mt dependence



$$\begin{aligned} \left. \frac{d\sigma_{NLO}}{dQ} \right|_{Q=300 \text{ GeV}} &= 0.02978(7)^{+6\%}_{-34\%} \text{ fb/GeV}, \\ \left. \frac{d\sigma_{NLO}}{dQ} \right|_{Q=400 \text{ GeV}} &= 0.1609(4)^{+0\%}_{-13\%} \text{ fb/GeV}, \\ \left. \frac{d\sigma_{NLO}}{dQ} \right|_{Q=600 \text{ GeV}} &= 0.03204(9)^{+0\%}_{-30\%} \text{ fb/GeV}, \\ \left. \frac{d\sigma_{NLO}}{dQ} \right|_{Q=1200 \text{ GeV}} &= 0.000435(4)^{+0\%}_{-35\%} \text{ fb/GeV}. \end{aligned}$$

$$\begin{aligned} \sqrt{s} = 13 \text{ TeV} : \quad \sigma_{tot} &= 27.73(7)^{+4\%}_{-18\%} \text{ fb}, \\ \sqrt{s} = 14 \text{ TeV} : \quad \sigma_{tot} &= 32.81(7)^{+4\%}_{-18\%} \text{ fb}, \\ \sqrt{s} = 27 \text{ TeV} : \quad \sigma_{tot} &= 127.8(2)^{+4\%}_{-18\%} \text{ fb}, \\ \sqrt{s} = 100 \text{ TeV} : \quad \sigma_{tot} &= 1140(2)^{+3\%}_{-18\%} \text{ fb} \end{aligned}$$

# $gg \rightarrow HH @\text{NNNLO}$



NNNLO

Similar to single Higgs

NNLO

qT subtraction

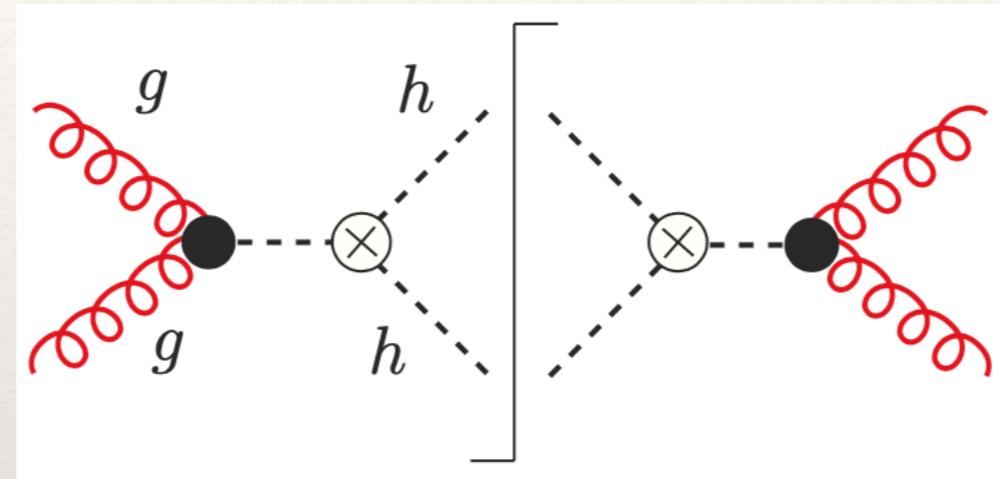
NLO

Standard methods

Many checks:

1. Self consistency (gauge invariance, poles cancellation, RG equations)
2. Reproduce single Higgs xs up to NNLO
3. Reproduce double Higgs xs up to NNLO

# Class-(a)

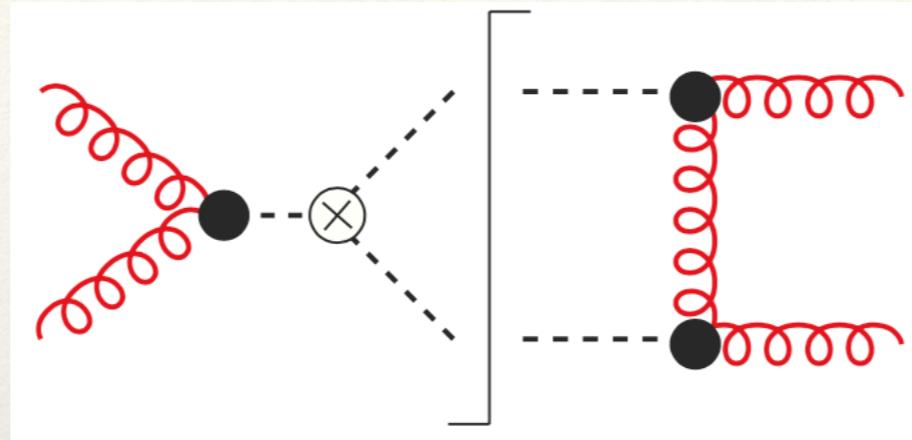


$$\frac{d\sigma_{hh}^a}{dm_{hh}} = f_{h \rightarrow hh} \left( \frac{C_{hh}}{C_h} - \frac{6\lambda_{hhh}v^2}{m_{hh}^2 - m_h^2} \right)^2 \times \left( \sigma_h \Big|_{m_h \rightarrow m_{hh}} \right)$$

$$f_{h \rightarrow hh} = \frac{\sqrt{m_{hh}^2 - 4m_h^2}}{16\pi^2 v^2}$$

Dulat, Lazopoulos, Mistlberger iHixs, 1802.00827

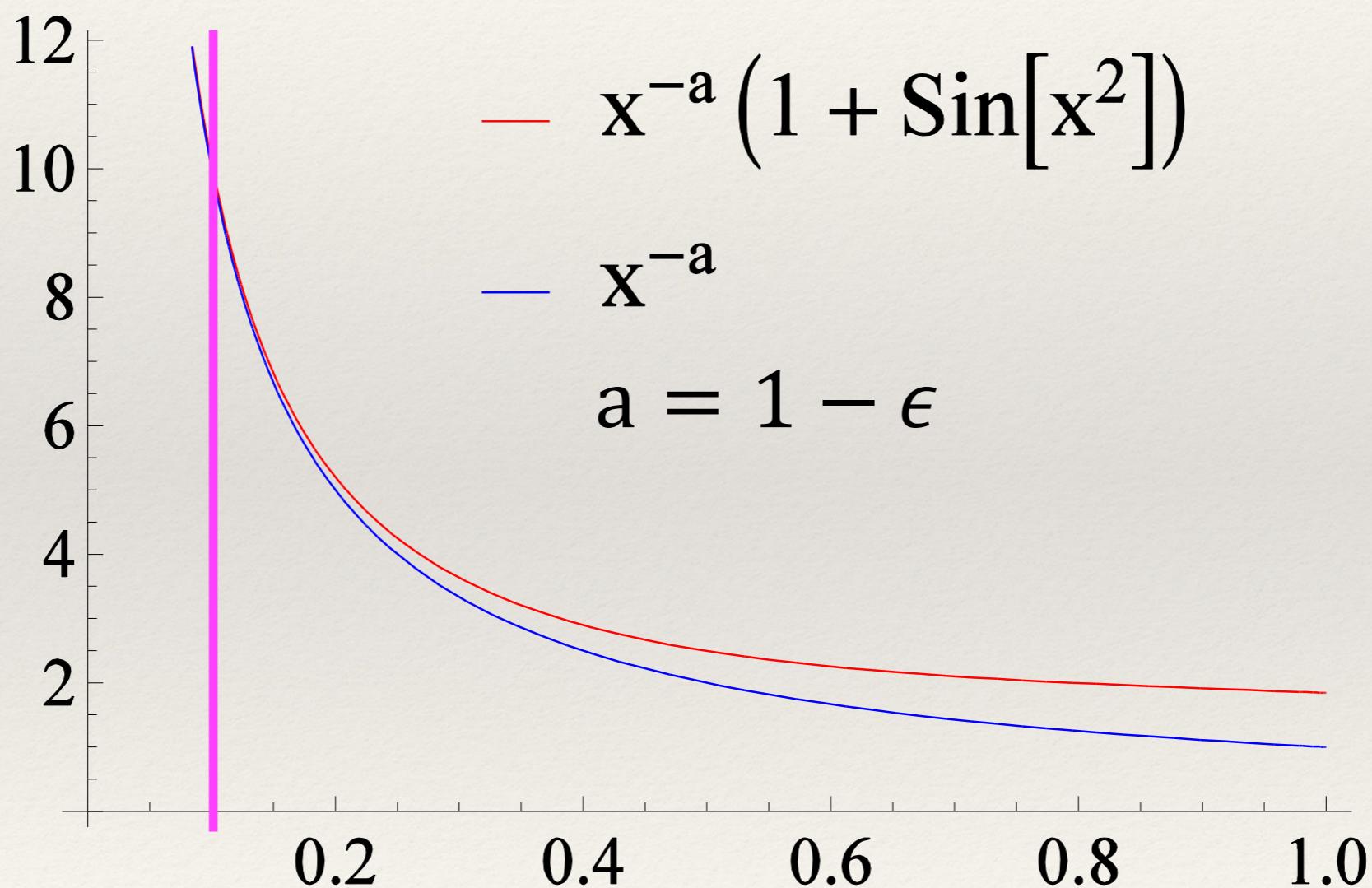
# Class-(b)



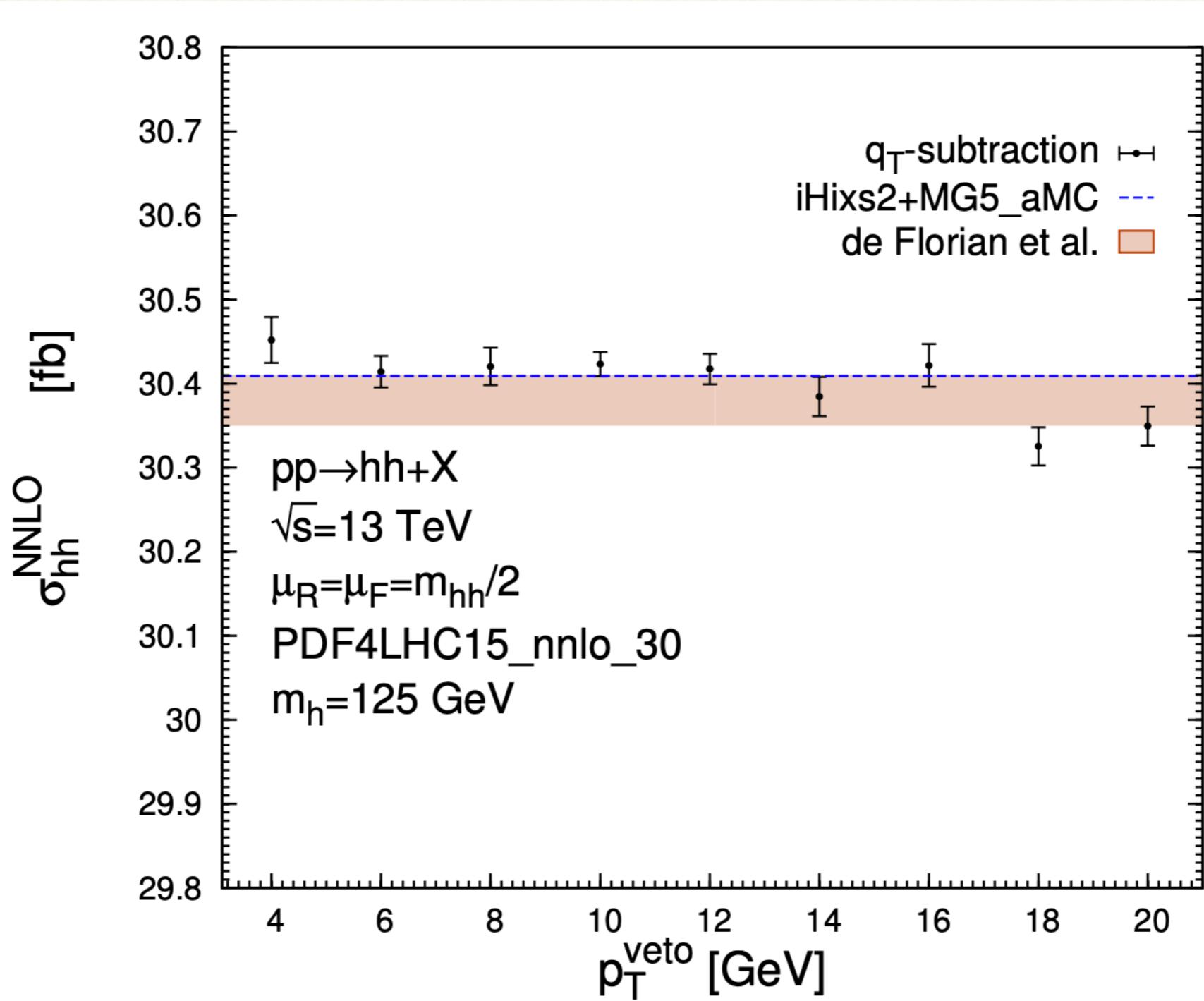
$$d\sigma_{hh}^b = d\sigma_{hh}^b \Big|_{p_T^{hh} < p_T^{\text{veto}}} + d\sigma_{hh}^b \Big|_{p_T^{hh} > p_T^{\text{veto}}}$$

$$\frac{d\sigma_{hh}^b}{dp_T^{hh}} = H^b \otimes B_g \otimes B_g \otimes S \times \left( 1 + \mathcal{O} \left( \frac{(p_T^{hh})^2}{Q^2} \right) \right)$$

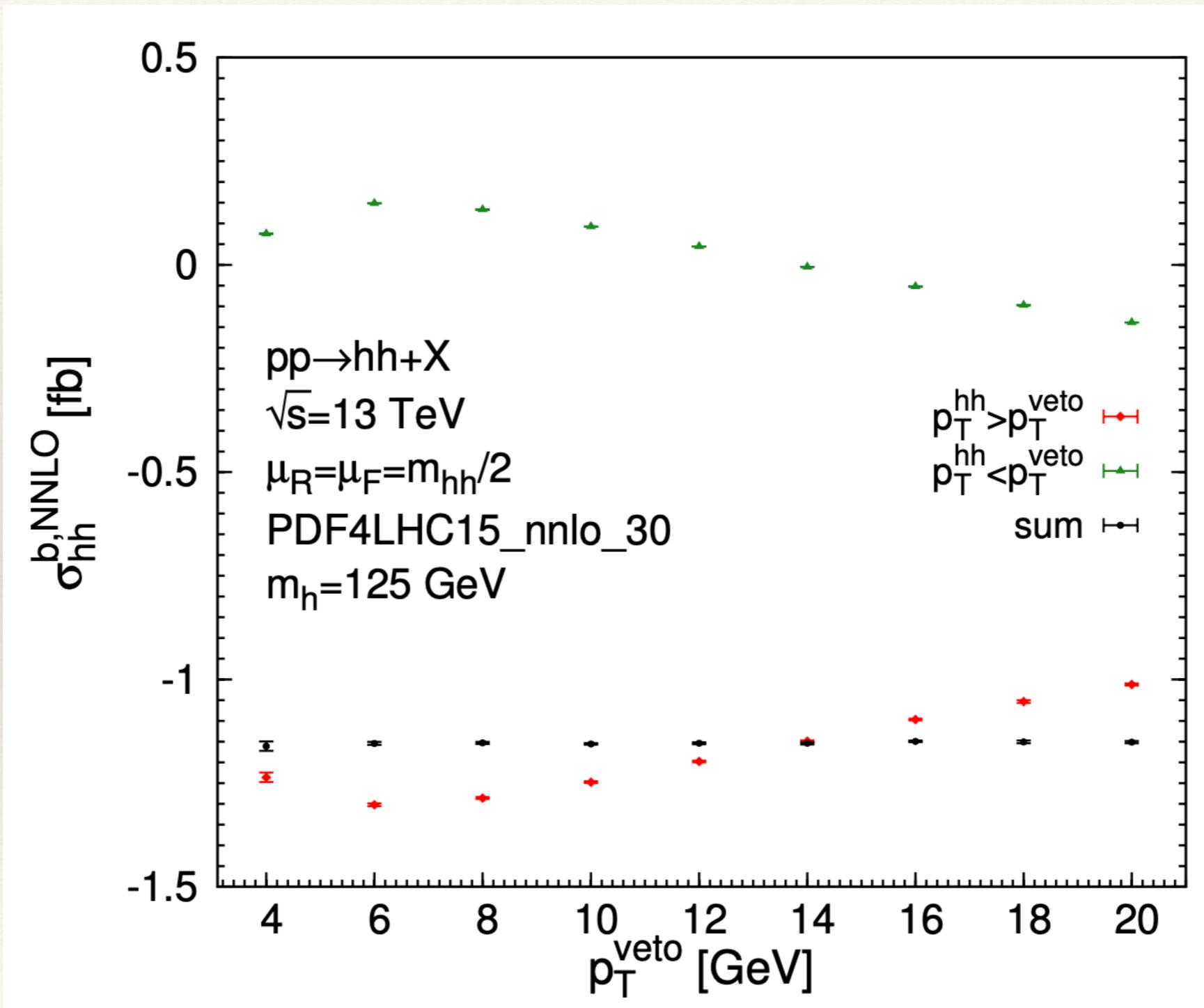
# The idea of $q^T$ subtraction



# Validation of q<sub>T</sub> subtraction



# Validation of qT subtraction



# How large are NNNLO corrections?

$\sqrt{s}$ order	13 TeV	14 TeV	27 TeV	100 TeV
LO	$13.80^{+31\%}_{-22\%}$	$17.06^{+31\%}_{-22\%}$	$98.22^{+26\%}_{-19\%}$	$2015^{+19\%}_{-15\%}$
NLO	$25.81^{+18\%}_{-15\%}$	$31.89^{+18\%}_{-15\%}$	$183.0^{+16\%}_{-14\%}$	$3724^{+13\%}_{-11\%}$
NNLO	$30.41^{+5.3\%}_{-7.8\%}$	$37.55^{+5.2\%}_{-7.6\%}$	$214.2^{+4.8\%}_{-6.7\%}$	$4322^{+4.2\%}_{-5.3\%}$
$N^3LO$	$31.31^{+0.66\%}_{-2.8\%}$	$38.65^{+0.65\%}_{-2.7\%}$	$220.2^{+0.53\%}_{-2.4\%}$	$4438^{+0.51\%}_{-1.8\%}$



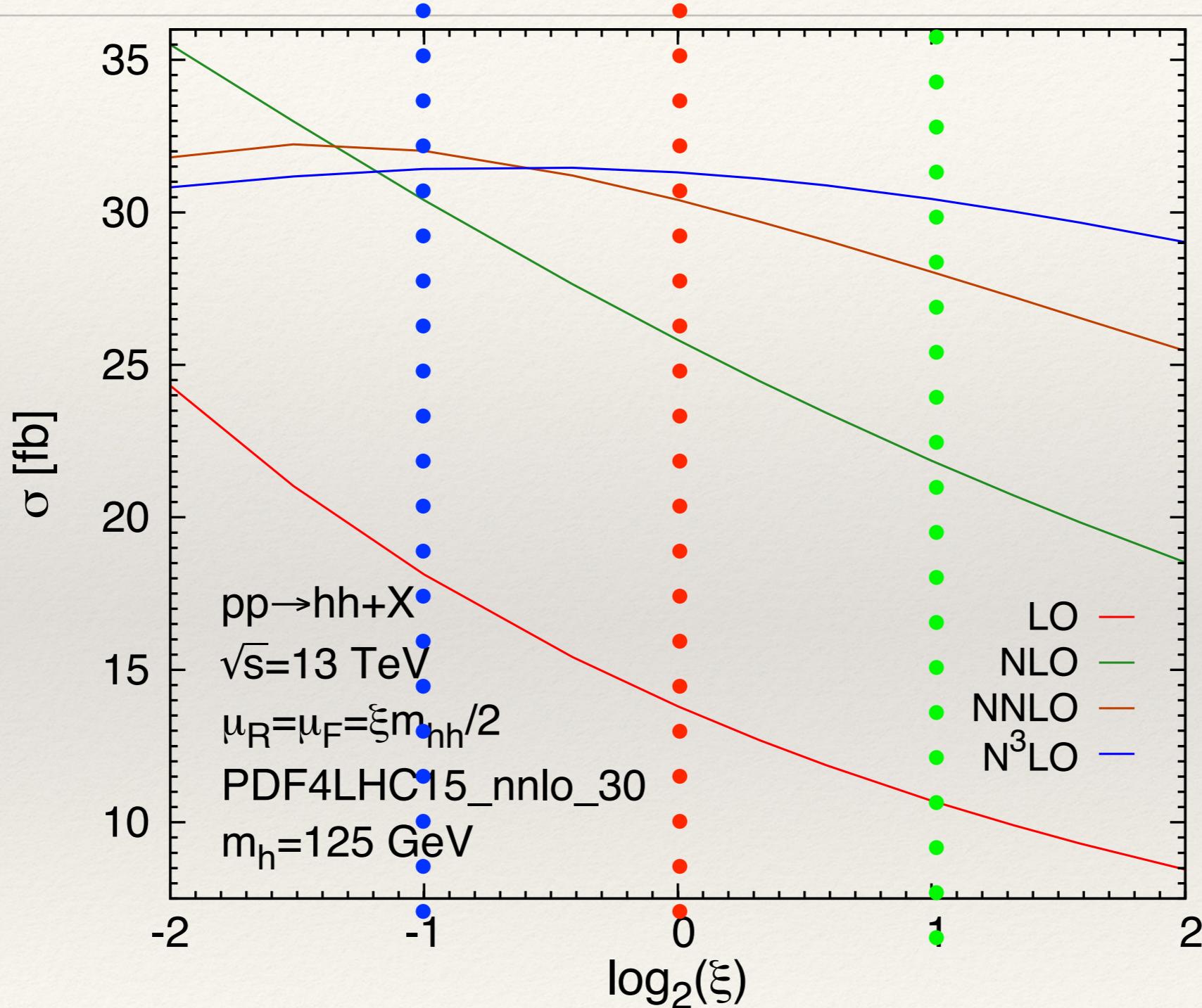
87%

18%

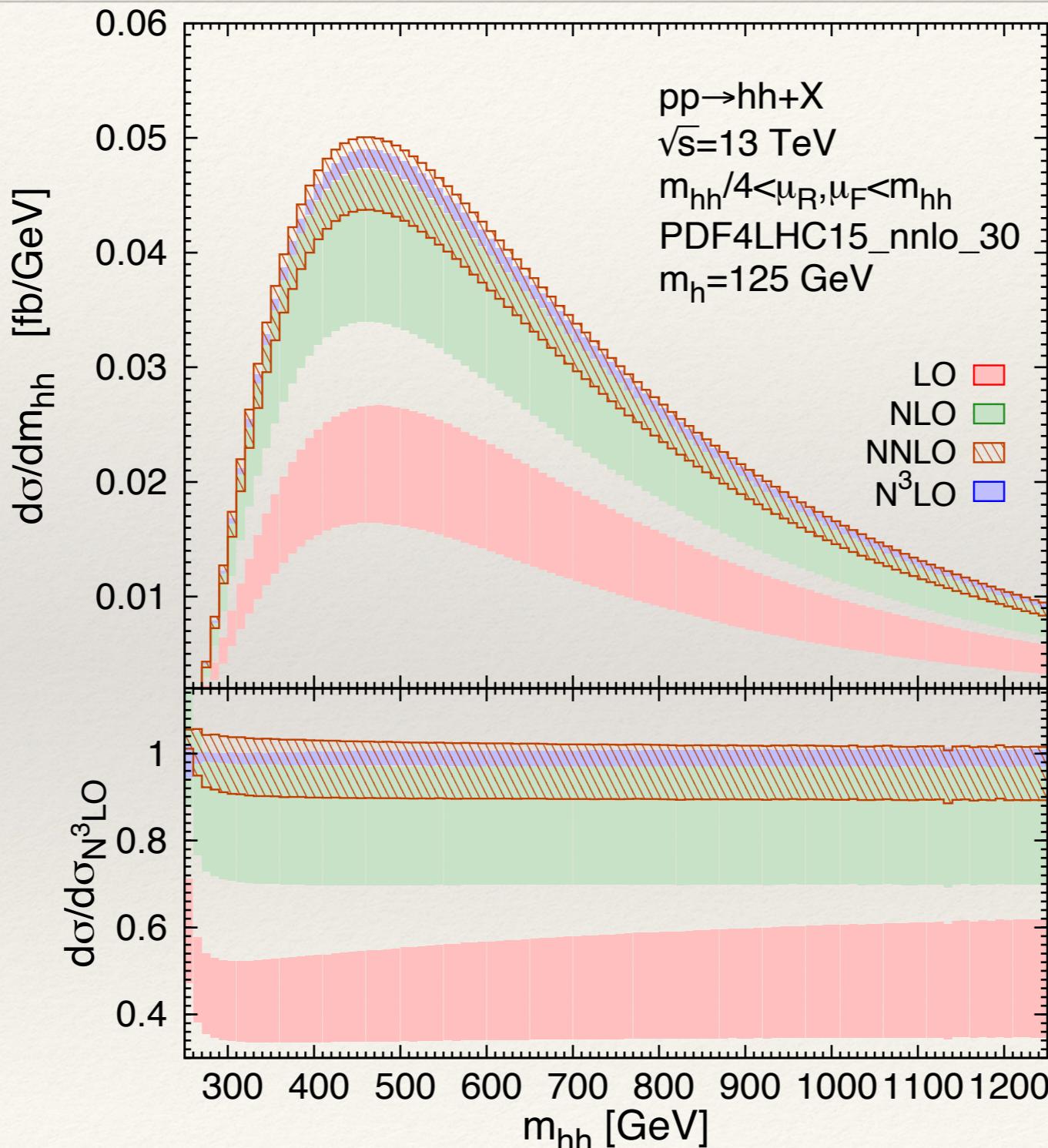
3%

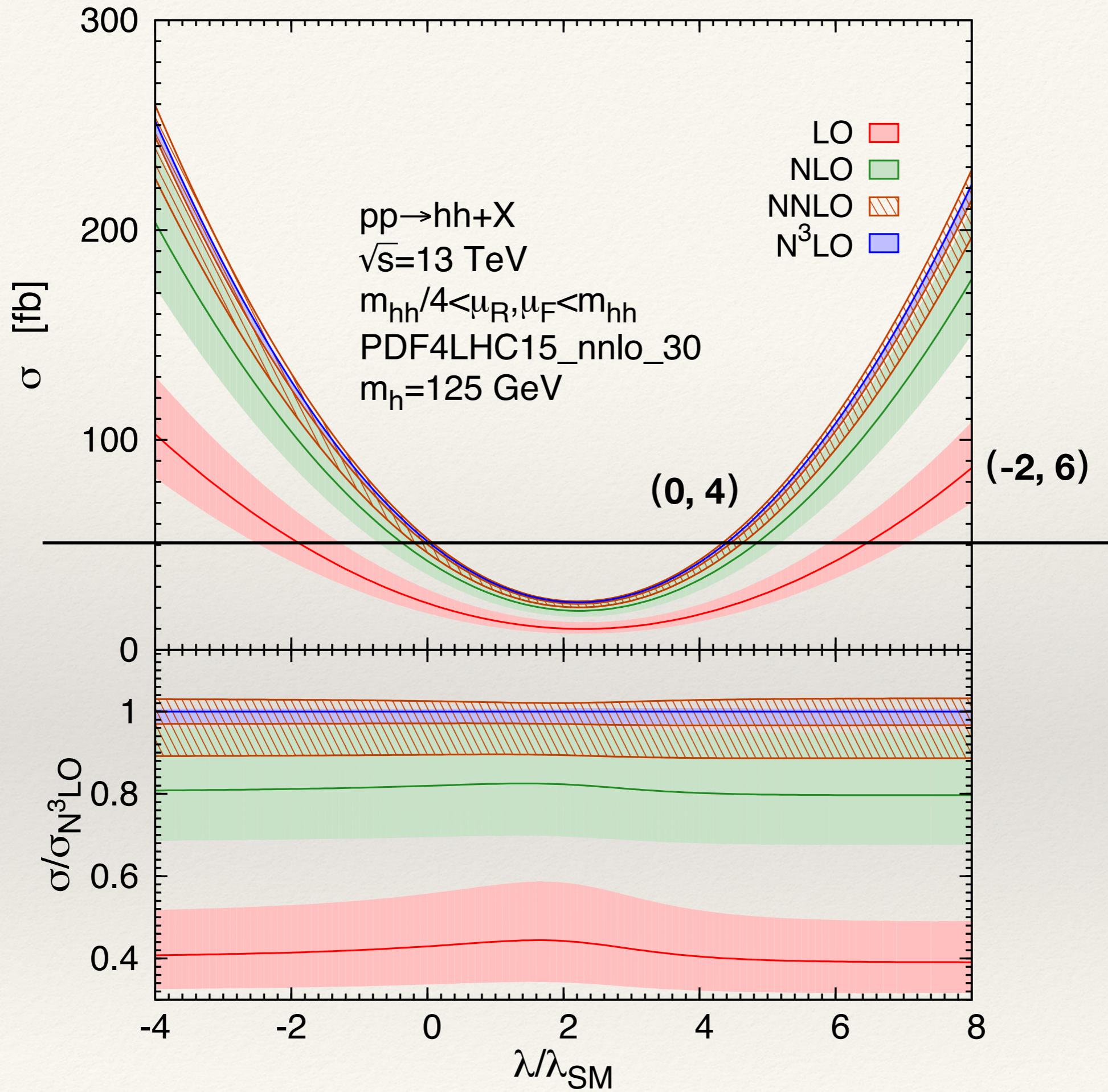
Scale uncer. less than PDF uncer. 3.3% now !

# How to choose a scale?



# Invariant mass of Higgs pair





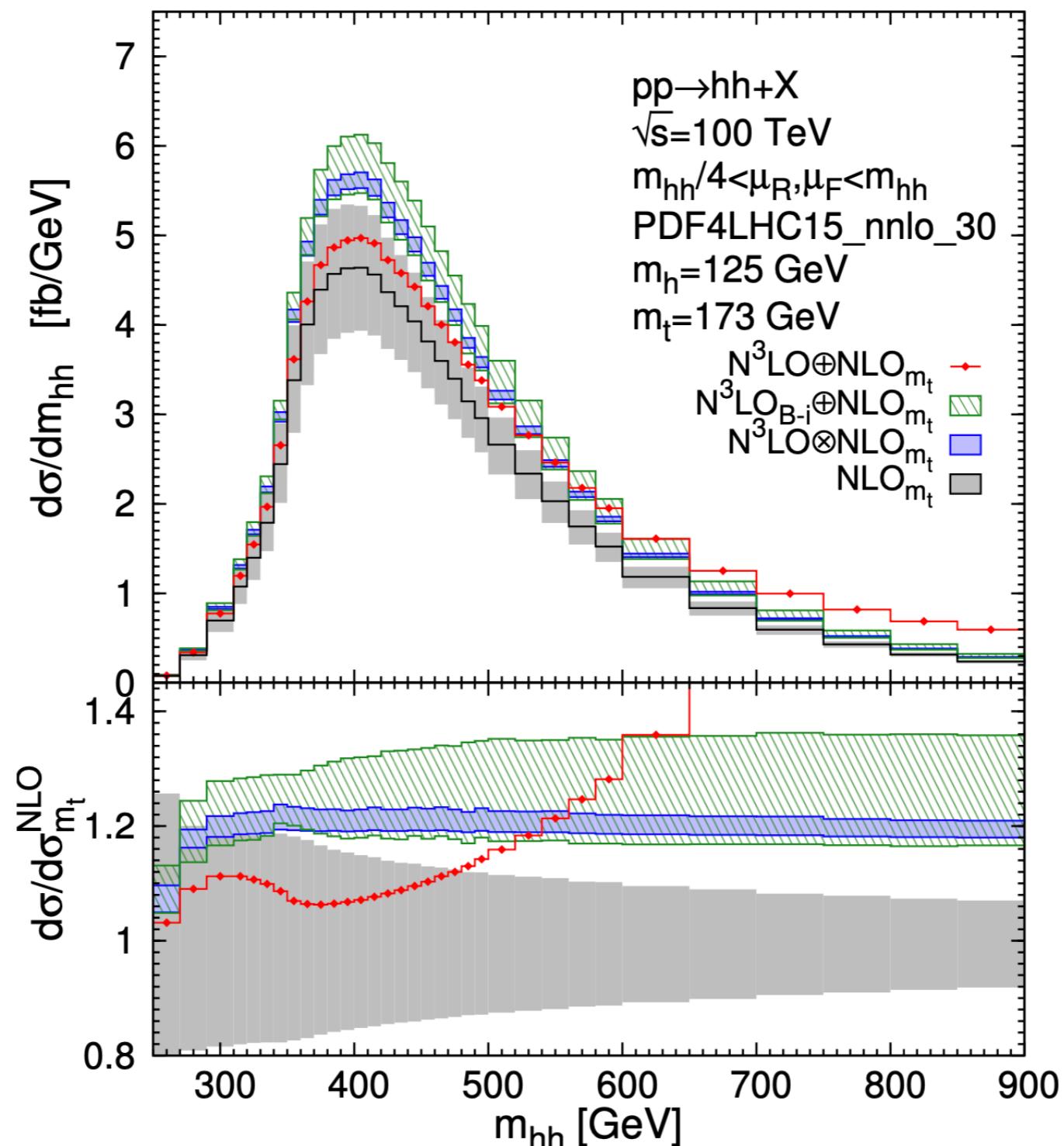
# Top quark mass effects

$$d\sigma^{\mathbf{N}^k \mathbf{LO} \oplus \mathbf{N}^l \mathbf{LO}_{\mathbf{m}_t}} = d\sigma_{m_t}^{\mathbf{N}^l \mathbf{LO}} + \Delta\sigma_{m_t \rightarrow \infty}^{k,l}$$

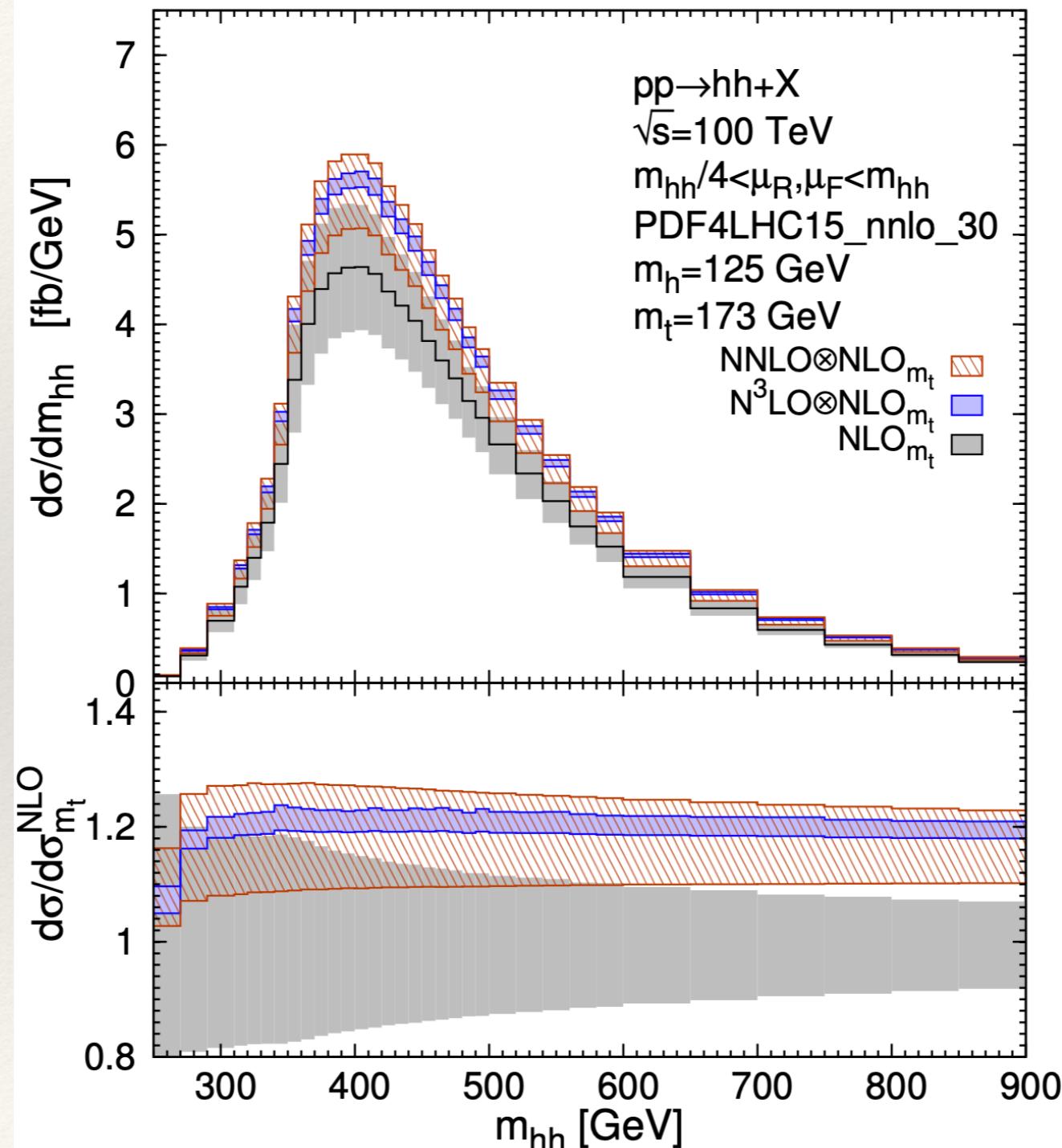
$$d\sigma^{\mathbf{N}^k \mathbf{LO}_{\mathbf{B}-\mathbf{i}} \oplus \mathbf{N}^l \mathbf{LO}_{\mathbf{m}_t}} = d\sigma_{m_t}^{\mathbf{N}^l \mathbf{LO}} + \Delta\sigma_{m_t \rightarrow \infty}^{k,l} \frac{d\sigma_{m_t}^{\mathbf{LO}}}{d\sigma_{m_t \rightarrow \infty}^{\mathbf{LO}}}$$

$$d\sigma^{\mathbf{N}^k \mathbf{LO} \otimes \mathbf{N}^l \mathbf{LO}_{\mathbf{m}_t}} = d\sigma_{m_t}^{\mathbf{N}^l \mathbf{LO}} \frac{d\sigma_{m_t \rightarrow \infty}^{\mathbf{N}^k \mathbf{LO}}}{d\sigma_{m_t \rightarrow \infty}^{\mathbf{N}^l \mathbf{LO}}} = d\sigma_{m_t}^{\mathbf{N}^l \mathbf{LO}} + \Delta\sigma_{m_t \rightarrow \infty}^{k,l} \frac{d\sigma_{m_t}^{\mathbf{N}^l \mathbf{LO}}}{d\sigma_{m_t \rightarrow \infty}^{\mathbf{N}^l \mathbf{LO}}}$$

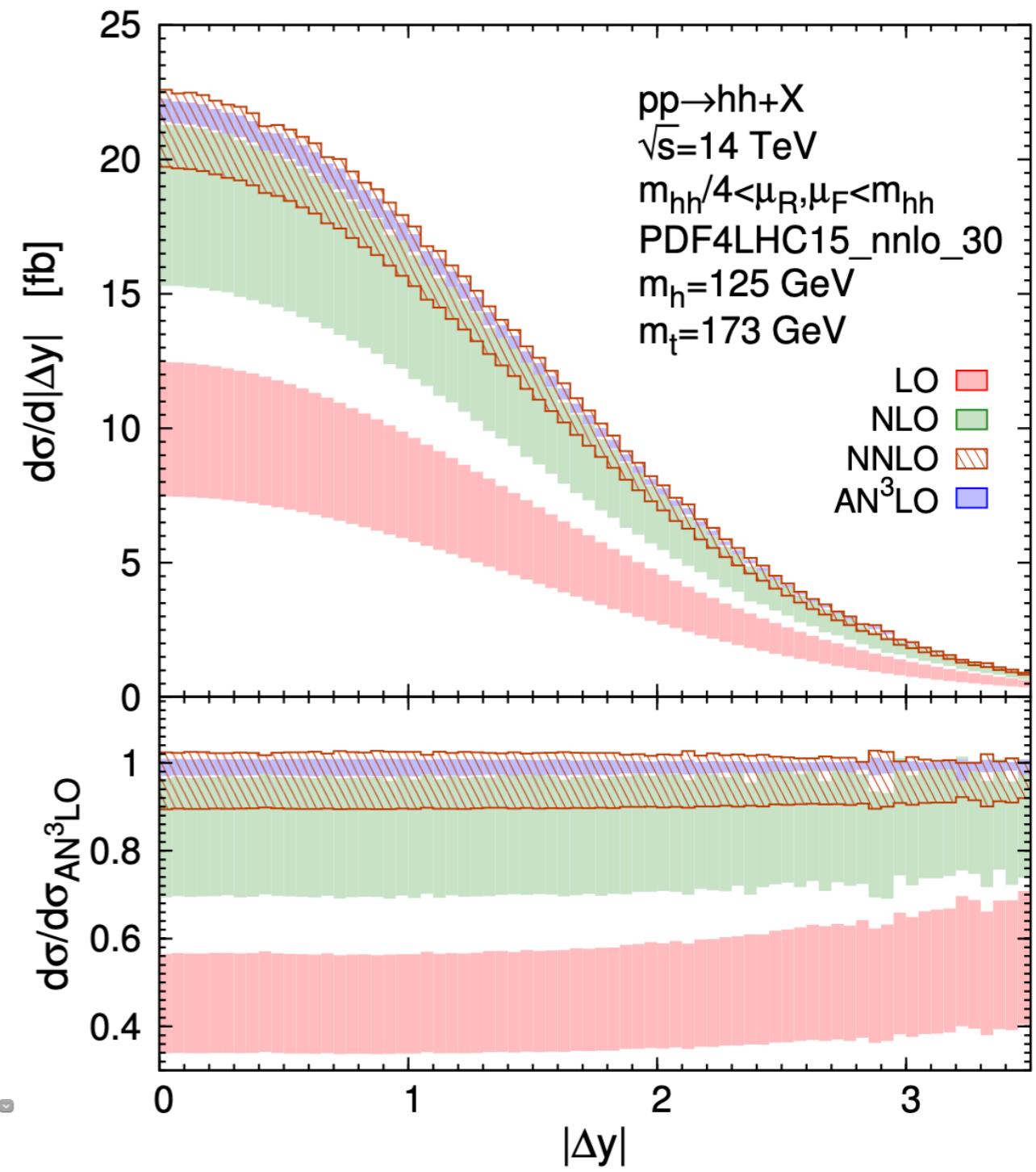
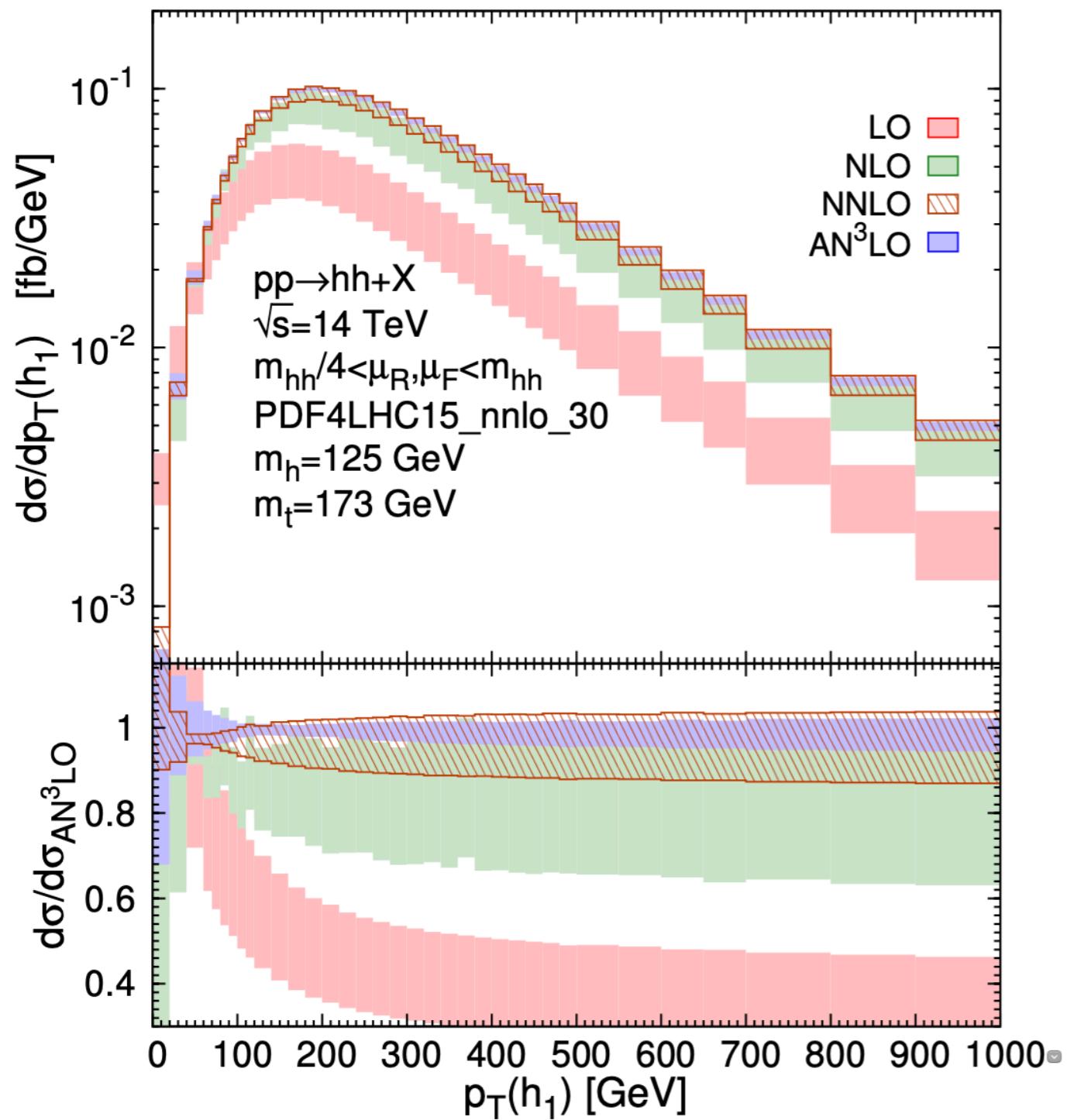
# $gg \rightarrow HH @\text{NNNLO}$



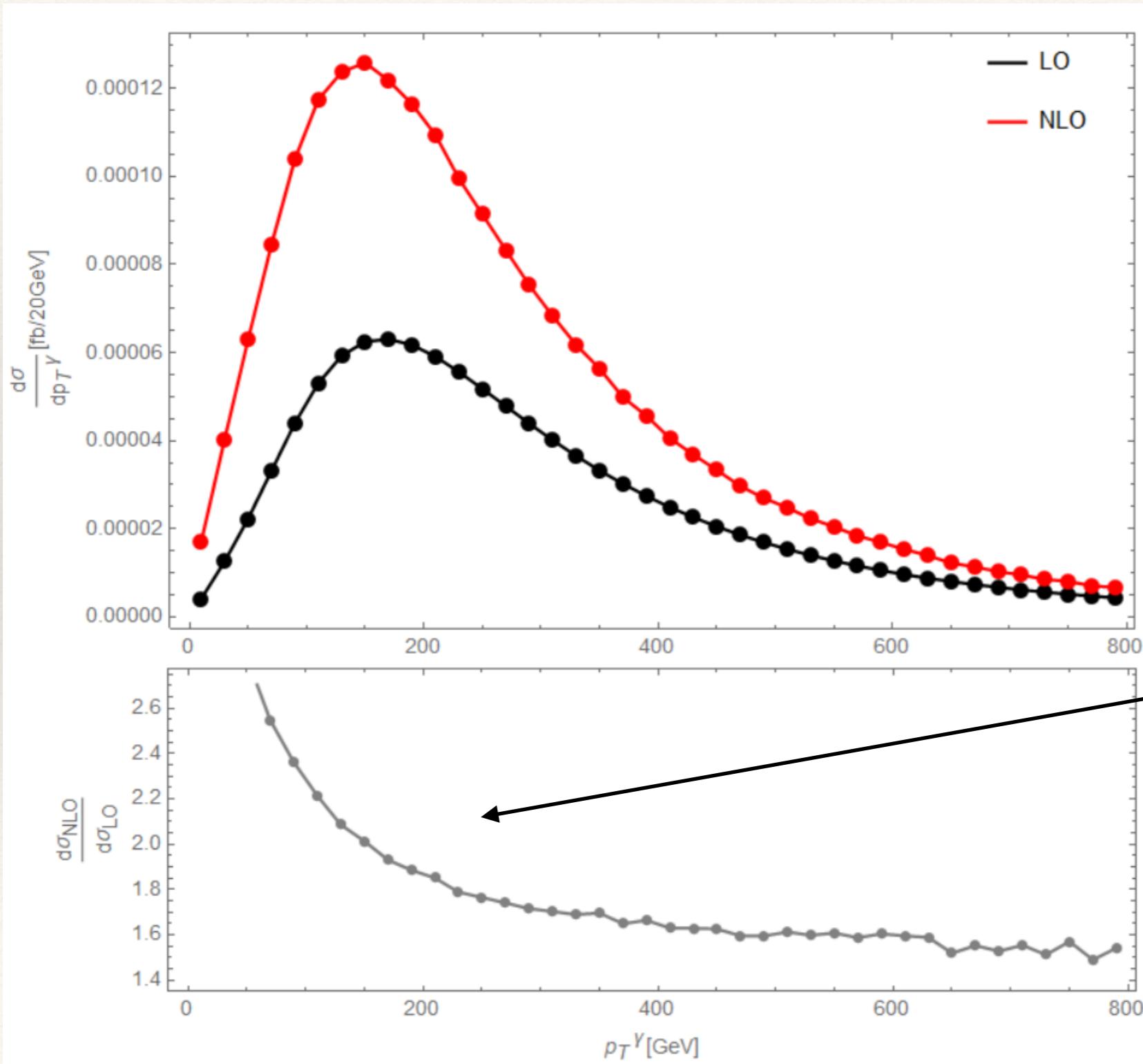
# $gg \rightarrow HH @\text{NNNLO}$



# $gg \rightarrow HH @NNNLO$



# $gg \rightarrow HH$ including decay

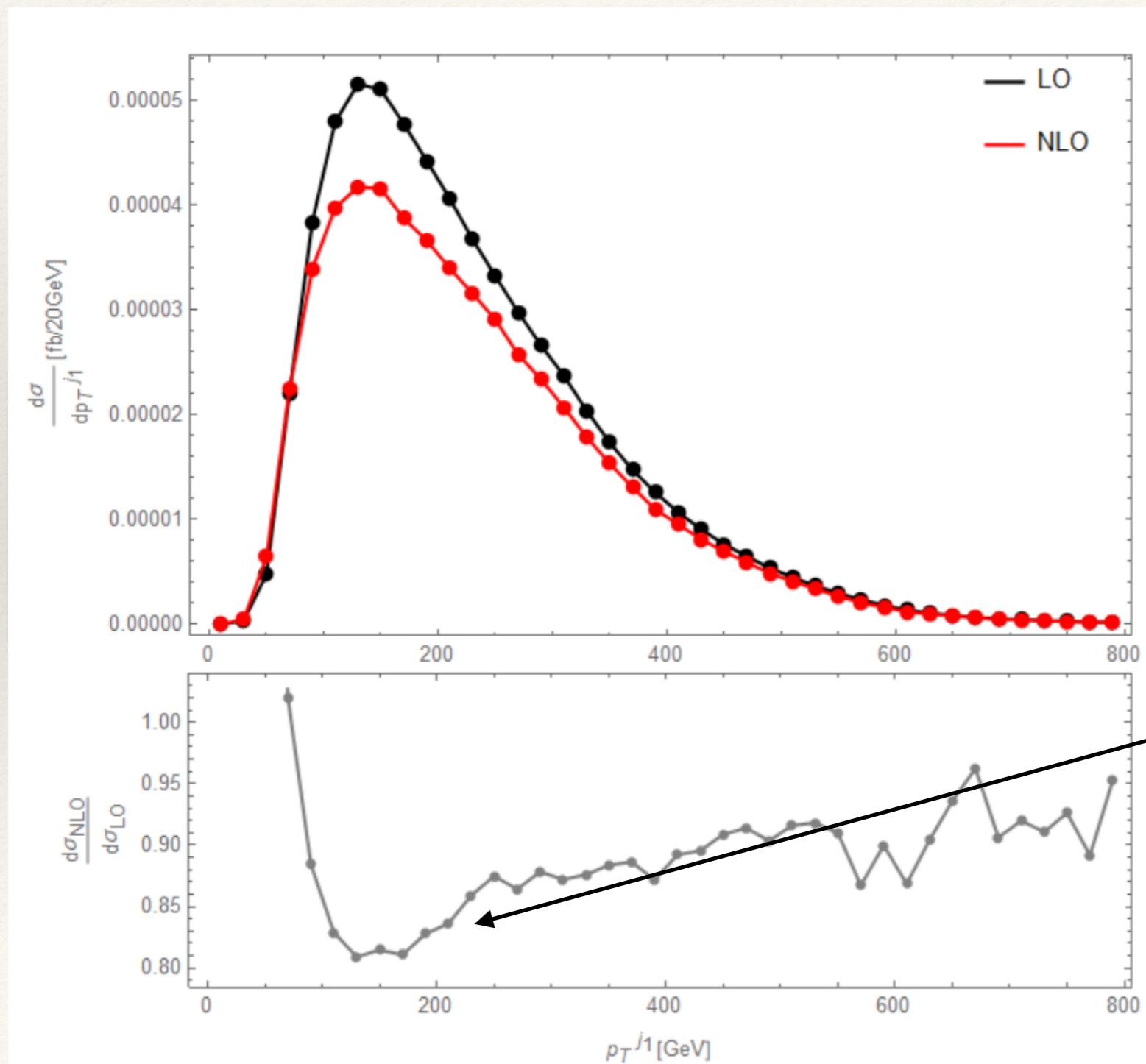


**NLO prod. & LO decay**

**Non-constant enhancement**

# $gg \rightarrow HH$ including NLO decay

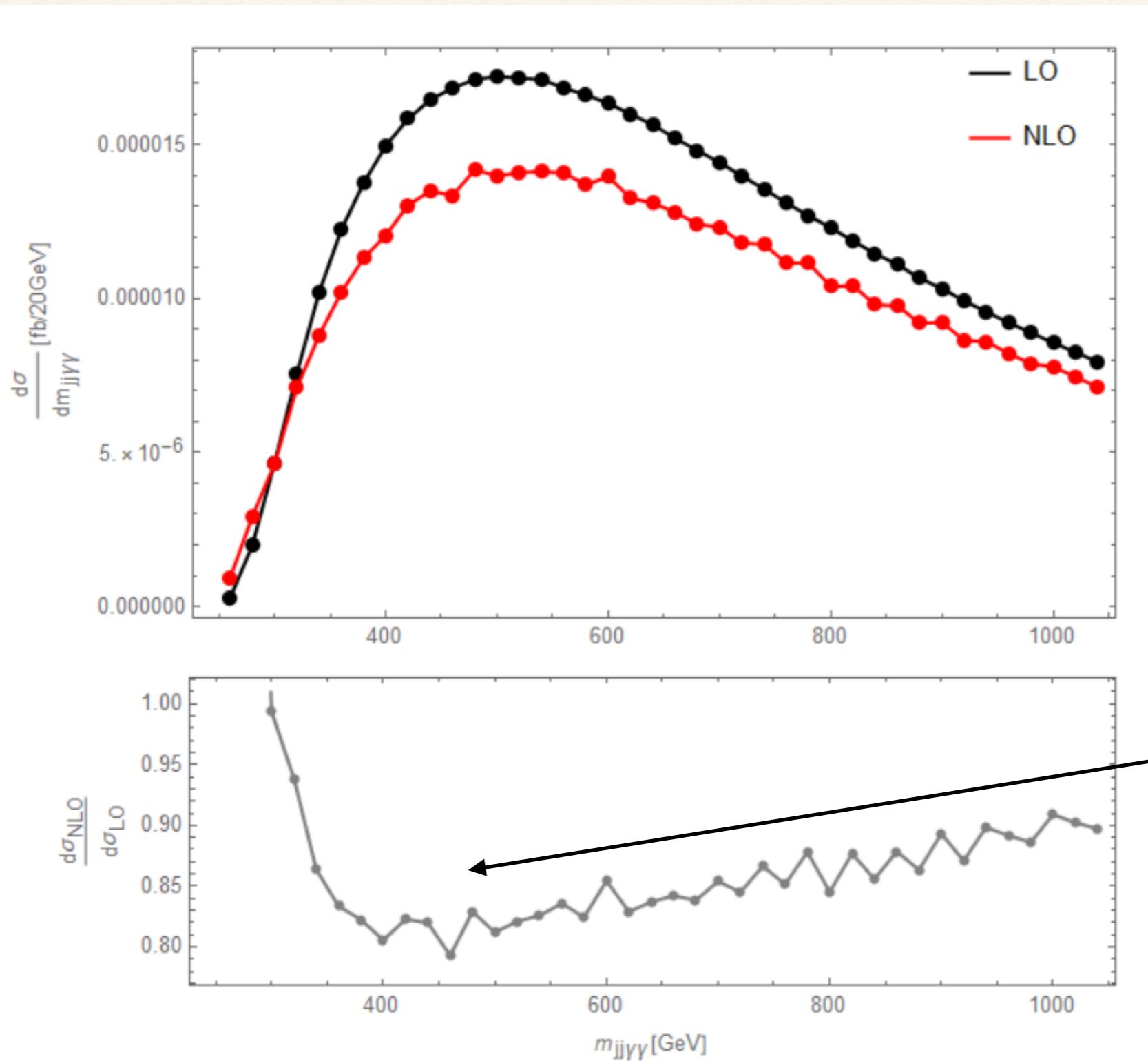
**LO prod. & NLO decay**



$p_j^T, p_\gamma^T > 25$  GeV  
 $| \eta_j |, | \eta_\gamma | < 2.5$   
 $\Delta R_{jj, \gamma j, \gamma\gamma} > 0.4$   
90GeV <  $m_{jj}$  < 190GeV

**Up to 20% reduction**

# $gg \rightarrow HH$ including decay



**LO prod. & NLO decay**

$p_j^T, p_\gamma^T > 25$  GeV  
 $| \eta_j |, | \eta_\gamma | < 2.5$   
 $\Delta R_{jj,\gamma j,\gamma\gamma} > 0.4$   
 $90\text{GeV} < m_{jj} < 190\text{GeV}$

**Up to 20% reduction**

# Conclusion

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- Measuring Higgs self-couplings is of great importance in the future.
- Precise theoretical prediction is needed to properly interpret the data.
- The dominant channel  $gg \rightarrow HH$  has been calculated up to NLO/NNNLO in the finite/infinite  $m_t$  scheme.
- The higher order effects in decay should also be considered for a detailed study.



Thank you!