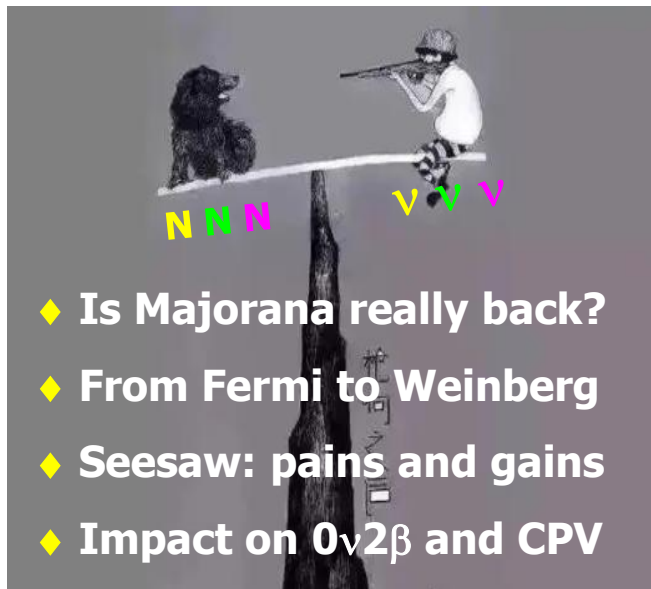


Light + heavy Majorana neutrinos: pains + gains

Zhi-zhong Xing
【IHEP Beijing】



- ◆ Is Majorana really back?
- ◆ From Fermi to Weinberg
- ◆ Seesaw: pains and gains
- ◆ Impact on $0\nu 2\beta$ and CPV

perspective

Nature Physics 5, 614–618 (2009)

Majorana returns

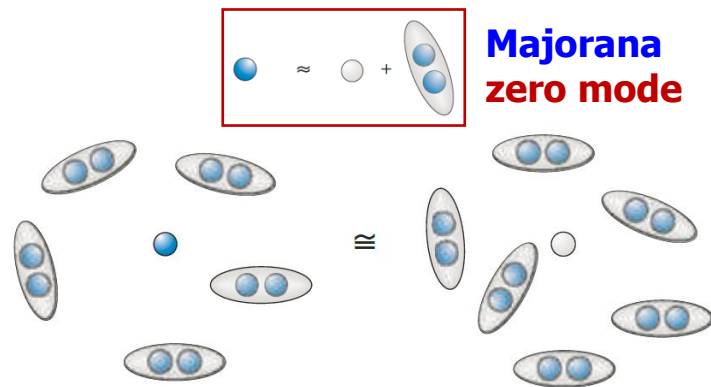
Frank Wilczek

In his short career, Ettore Majorana made several profound contributions. One of them, his concept of 'Majorana fermions' — particles that are their own antiparticle — is finding ever wider relevance in modern physics.

用甜言蜜语哄骗

衍生物、结果

Enrico Fermi had to **cajole** his friend **Ettore Majorana** into publishing his **big idea**: a modification of the **Dirac equation** that would have profound **ramifications** for particle physics. Shortly afterwards, in 1938, **Majorana** mysteriously disappeared, and for 70 years his modified equation remained a rather obscure footnote in **晦涩的** theoretical physics. **Now suddenly, it seems, Majorana's concept is ubiquitous, and his equation is central to recent work not only in neutrino physics, supersymmetry and dark matter, but also on some exotic states of ordinary matter.** **无所不在的、普遍存在的**



A fake signal of $0\nu 2\beta$

2

Modern Physics Letters A, Vol. 16, No. 37 (2001) 2409–2420

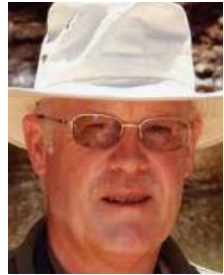
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Citations > 680

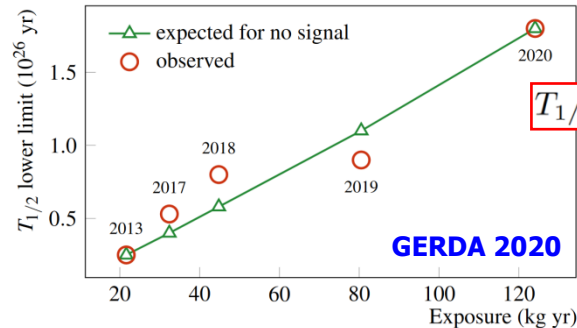
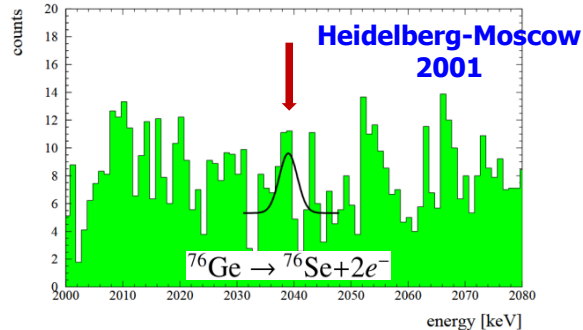
EVIDENCE FOR NEUTRINOLESS DOUBLE BETA DECAY

H. V. KLAPDOR-KLEINGROTHAUS*, A. DIETZ, H. L. HARNEY and I. V. KRIVOSHEINA†

Max-Planck-Institut für Kernphysik, Postfach 103980, D-69029 Heidelberg, Germany



H.V. Klapdor



$T_{1/2} > 1.8 \times 10^{26}$ yr at 90% C.L.

The abstract: First evidence for neutrinoless double beta decay is observed giving first evidence for lepton number violation. The evidence for this decay mode is 97% (2.2σ) with the Bayesian method, and 99.8% c.l. (3.1σ) with the method recommended by the Particle Data Group. The half-life of the process is found with the Bayesian method to be $T_{1/2}^{0\nu} = (0.8 - 18.3) \times 10^{25}$ y (95% c.l.) with a best value of 1.5×10^{25} y. The deduced value of the effective neutrino mass is, with the nuclear matrix elements from ¹, $\langle m \rangle = (0.11 - 0.56)$ eV (95% c.l.), with a best value of 0.39 eV. Uncertainties in the nuclear matrix elements

It is not me!



Ettore Majorana

RESEARCH

TOPOLOGICAL MATTER

SCIENCE · 21 Jul 2017 · Vol 357, Issue 6348 · pp. 294-299 · DOI: [10.1126/science.aag2792](https://doi.org/10.1126/science.aag2792)

Chiral Majorana fermion modes in a quantum anomalous Hall insulator-superconductor structure

Qing Lin He,^{1*} Lei Pan,^{1†} Alexander L. Stern,³ Edward C. Burks,⁴ Xiaoyu Che,¹ Gen Yin,¹ Jing Wang,^{5,6} Biao Lian,⁶ Quan Zhou,⁶ Eun Sang Choi,⁷ Koichi Murata,¹ Xufeng Kou,^{1,8*} Zhijie Chen,⁴ Tianxiao Nie,¹ Qiming Shao,¹ Yabin Fan,¹ Shou-Cheng Zhang,^{6*} Kai Liu,⁴ Jing Xia,³ Kang L. Wang^{1,2*}

Majorana fermion is a hypothetical particle that is its own antiparticle. We report transport measurements that suggest the existence of one-dimensional chiral Majorana fermion modes in the hybrid system of a quantum anomalous Hall insulator thin film coupled with a superconductor. As the external magnetic field is swept, half-integer quantized conductance plateaus are observed at the locations of magnetization reversals, giving a distinct signature of the Majorana fermion modes. This transport signature is reproducible over many magnetic field sweeps and appears at different temperatures. This finding may open up an avenue to control Majorana fermions for implementing robust topological quantum computing.

过去我们认为有粒子必有其反粒子，正如有天使必有魔鬼。但今天，我们找到了一个没有反粒子的粒子，一个只有天使，没有魔鬼的完美世界——张首晟



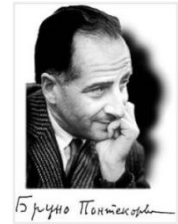
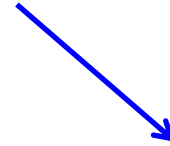
Editorial retraction 2022-11-18

Good news to Majorana hunters

4

Phase A (1930 — 1939):

- ◆ **1930** Wolfgang Pauli, a postcard / letter.
- ◆ **1933** Enrico Fermi, Ric. Sci. 4, 491.
- ◆ **1935** Maria Goeppert-Mayer, Phys. Rev. D 48, 512.
- ◆ **1937** Ettore Majorana, Nuovo Cim. 14, 171.
- ◆ **1939** Wendell Furry, Phys. Rev. D 56, 1184.



$1\nu 1\beta$

$1\nu 1\beta$

$2\nu 2\beta$

$\nu^c = \nu$

$0\nu 2\beta$

Phase B (1957 — 1998):

- ◆ **1956** T.D. Lee, C.N. Yang, Phys. Rev. D 105, 167.
- ◆ **1957** Bruno Pontecorvo, Sov. Phys. JETP 6, 429.
- ◆ **1967** Steven Weinberg, Phys. Rev. Lett. 19, 1264.
- ◆ **1987** S. Elliot, A. Hahn, M. Moe, Phys. Rev. Lett. 59, 2020.
- ◆ **1998** SK collaboration, Phys. Rev. Lett. 81, 1562.

$m_\nu = 0$

$m_\nu \neq 0$

$m_\nu = 0$

$2\nu 2\beta$

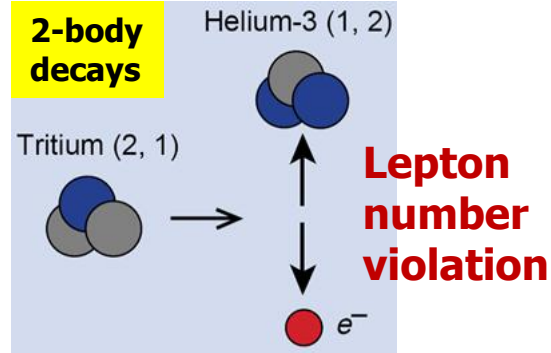
$m_\nu \neq 0$

The observations of $2\nu 2\beta$ decays + ν -oscillations are so encouraging that most hunters believe the existence of $0\nu 2\beta$, a unique signal for the Majorana nature of ν 's.

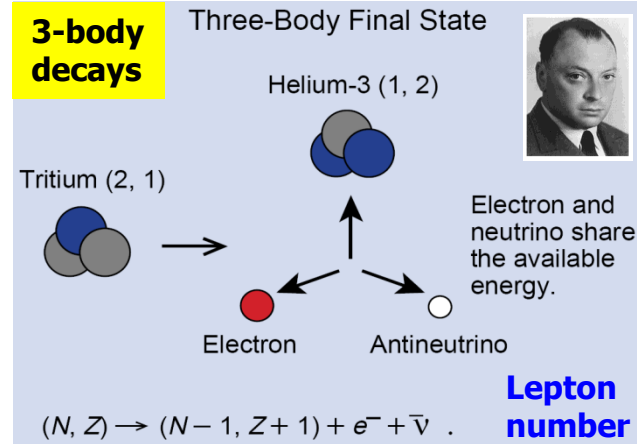
From Pauli to Fermi

The Energy crisis in β decays

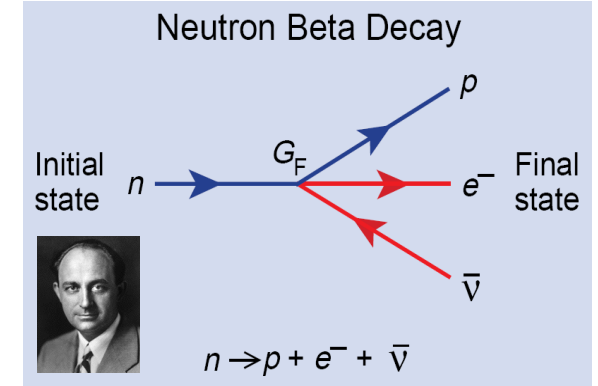
J. Chadwick 1914, C. Ellis 1920~1927



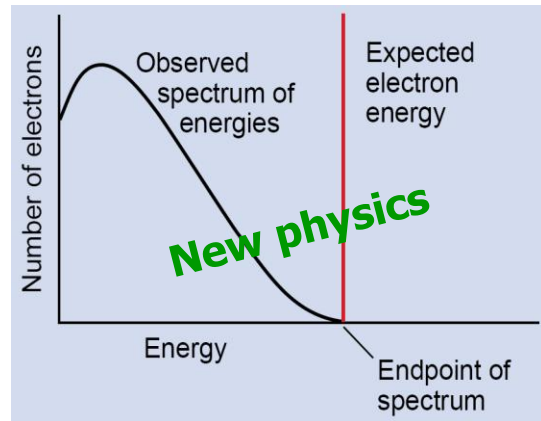
Wolfgang Pauli's new idea (1930)



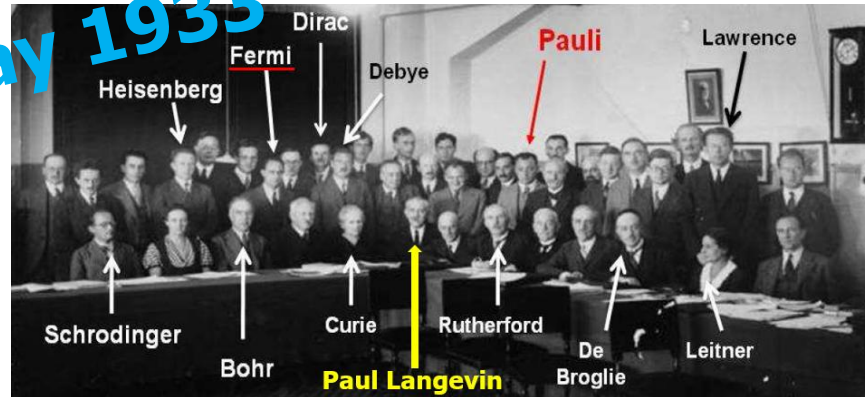
Enrico Fermi's β -theory (1933)



"I will be remembered for this paper!"

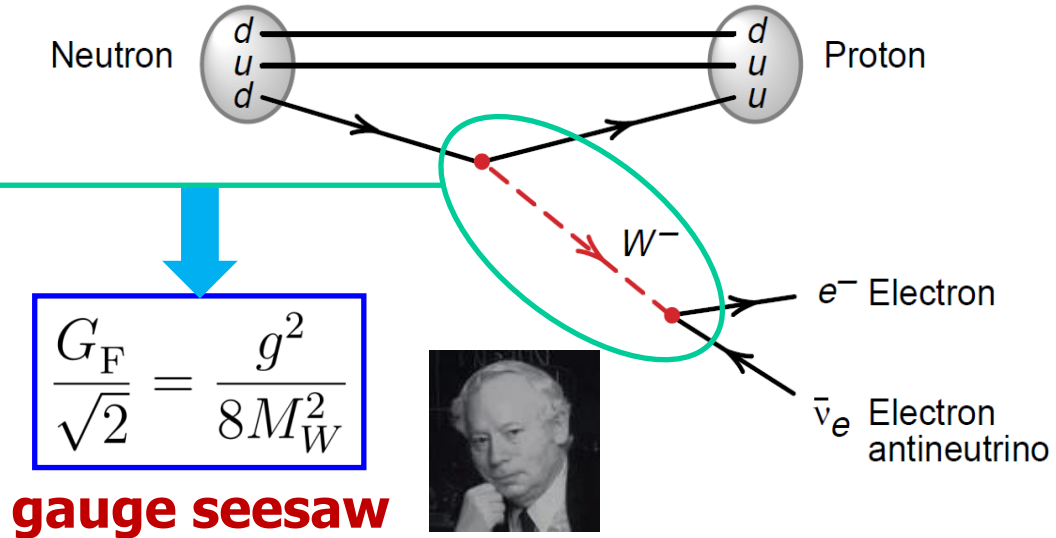
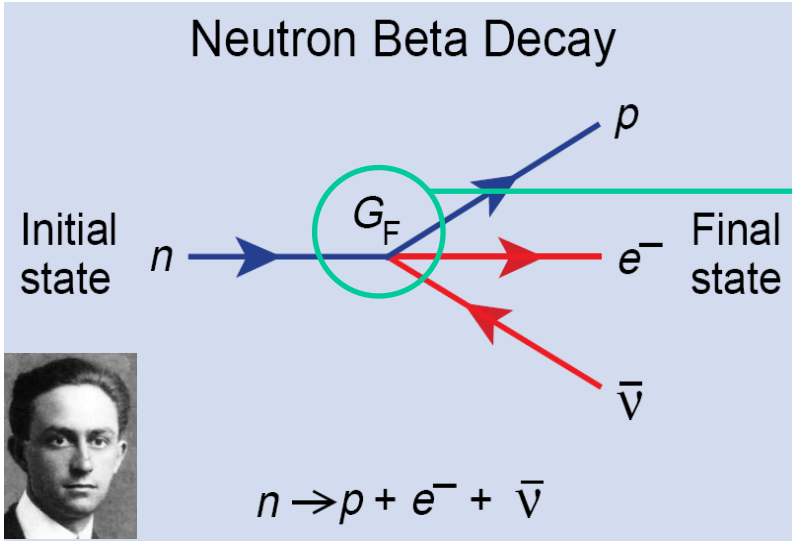


Solvay 1933



Untie Fermi's knot

It's **Steven Weinberg** who untied **Fermi's** knot by introducing new heavy degrees of freedom (**1967**)



Fermi coupling constant

Weak interaction coupling constant

$$G_F \simeq 1.166 \times 10^{-5} \text{ GeV}^{-2}$$

$$g \simeq 0.65 \quad \text{vs} \quad M_W \simeq 80.4 \text{ GeV}$$

A good lesson some effective quantities at low energies are very likely to originate from new heavy degrees of freedom in a more fundamental theory at much higher energy scales.

A MODEL OF LEPTONS*

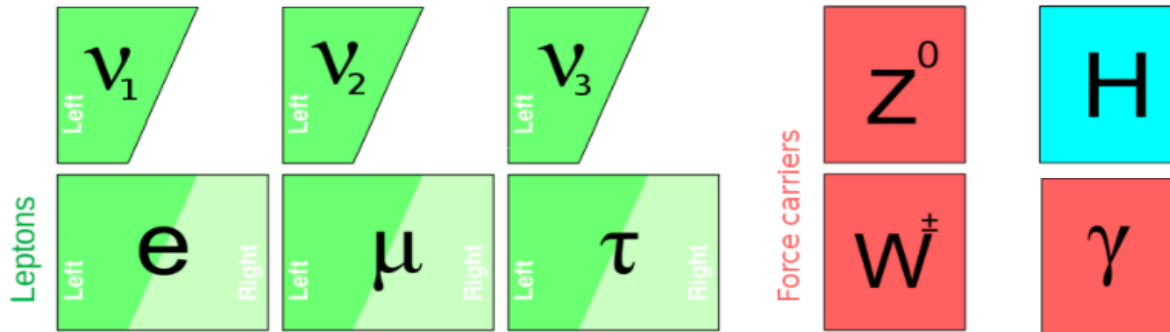
Steven Weinberg†

Laboratory for Nuclear Science and Physics Department,
Massachusetts Institute of Technology, Cambridge, Massachusetts

(Received 17 October 1967)



Citations ~ 14000



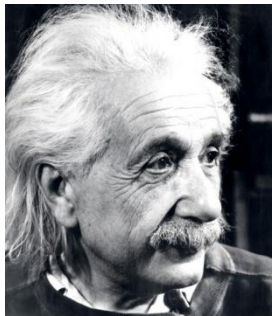
- ◆ Its **theoretical ingredients** are really perfect!
- ◆ Its **particle content** looks very strange: **no** left-handed ν -fields.

My style is usually **not to propose specific models** that will lead to specific experimental predictions, but rather to interpret in a broad way what is going on, and **make very general remarks**, like with the development of the point of view associated with **effective field theory** (**2021@CERN Courier**)

A wrong use of Occam's razor!

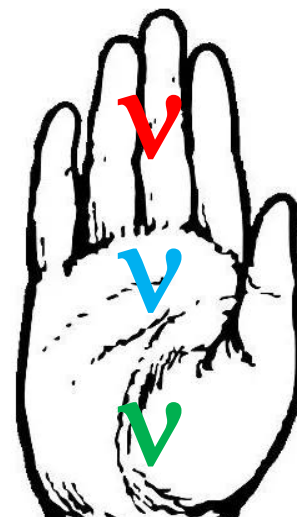
maximal **P** violation

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \begin{matrix} \longleftrightarrow \\ \longleftrightarrow \end{matrix} \begin{matrix} u_R \\ d_R \end{matrix}$$
$$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \begin{matrix} \longleftrightarrow \\ \longleftrightarrow \end{matrix} \begin{matrix} ? \\ e_R \end{matrix}$$



Albert Einstein:

Everything should be made as simple as possible, but not simpler!



Right-handed neutrinos

小刚老师的模型
超越标准模型的
尺度稍大，呵呵

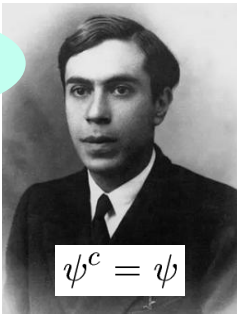
The solution at lowest costs

Dirac or Majorana?

That is a question!

$$-\mathcal{L}_{SS} = \bar{\ell}_L Y_\nu \tilde{H} N_R + \frac{1}{2} \overline{(N_R)^c} M_R N_R + \text{h.c.}$$

1937



Dirac (IQ>90)

Majorana (IQ>130)

What is the Majorana nature?

The antiparticle of a free fermion is itself.

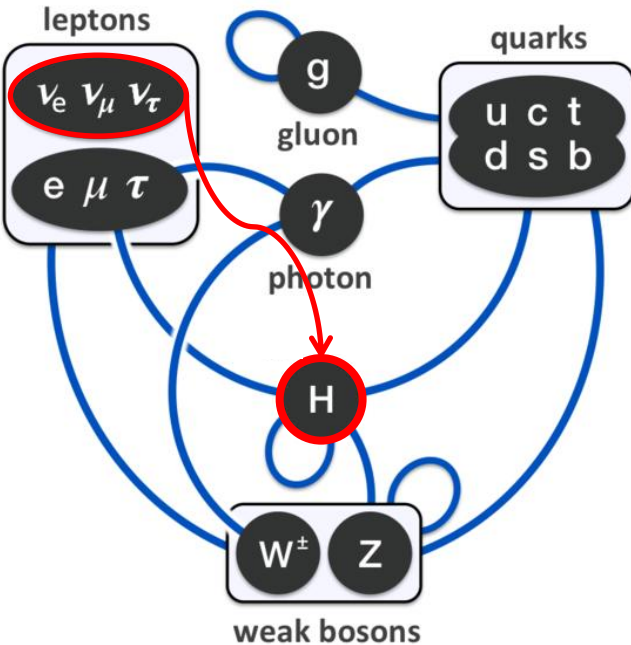
Costs: heavy sterile neutrinos which are inaccessible in today's experiments (and probably the day after tomorrow's).

Rewards: the fundamental structures of the SM are unaffected, seesaw + leptogenesis works (1 stone kills 2 birds)....

Is there a convincing guiding principle to do so?



于是我陷入了沉思



Gell-Mann's totalitarian principle (1956):
Everything not forbidden is compulsory !

Weinberg's 3rd law of progress in TH physics (1983): You may use any degree of freedom you like to describe a physical system, but if you use the wrong ones, you will be sorry.

The **canonical seesaw** mechanism **formally** works **far above** the **Fermi** scale (**ZZX**, 2203.14185, NPB)

Received **2023-01-28**

Revised **2023-01-30**

Accepted **2023-02-01**

$$\begin{aligned}
 -\mathcal{L}_{\text{lepton}} &= \bar{\ell}_L Y_l H l_R + \bar{\ell}_L Y_\nu \tilde{H} N_R + \frac{1}{2} \overline{(N_R)^c} M_R N_R + \text{h.c.} \\
 &= \bar{\ell}_L Y_l l_R \phi^0 + \frac{1}{2} \overline{[\nu_L \quad (N_R)^c]} \begin{pmatrix} \mathbf{0} & Y_\nu \phi^{0*} \\ Y_\nu^T \phi^{0*} & M_R \end{pmatrix} \begin{bmatrix} (\nu_L)^c \\ N_R \end{bmatrix} + \bar{\nu}_L Y_l l_R \phi^+ - \bar{\ell}_L Y_\nu N_R \phi^- + \text{h.c.}
 \end{aligned}$$

The **basis transformation** related to the origin of active **Majorana** neutrino masses **before SSB**:

$$\mathbb{U}^\dagger \begin{pmatrix} \mathbf{0} & Y_\nu \phi^{0*} \\ Y_\nu^T \phi^{0*} & M_R \end{pmatrix} \mathbb{U}^* = \begin{pmatrix} D_\nu & \mathbf{0} \\ \mathbf{0} & D_N \end{pmatrix}$$

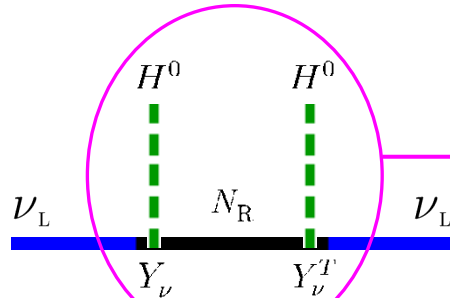
working masses:

$$\begin{cases} D_\nu \equiv \text{Diag}\{m_1, m_2, m_3\} \\ D_N \equiv \text{Diag}\{M_1, M_2, M_3\} \end{cases}$$

SSB

$$\begin{pmatrix} \mathbf{0} & M_D \\ M_D^T & M_R \end{pmatrix}$$

6 x 6 mass matrix



Integrating out the heavy degrees of freedom:

$$-\mathcal{L}_{\text{mass}} = \frac{1}{2} \bar{\nu}_L M_\nu \nu_L^c + \text{h.c.}$$

$$M_\nu \simeq -Y_\nu \frac{\langle H \rangle^2}{M_R} Y_\nu^T$$

consistent with the Weinberg operator (1979)

If you can untie **Weinberg's** knot, you will find new heavy Majorana neutrinos at a superhigh scale.

Euler-like parameters

$$U_0 = \begin{pmatrix} c_{12}c_{13} & \hat{s}_{12}^*c_{13} & \hat{s}_{13}^* \\ -\hat{s}_{12}c_{23} - c_{12}\hat{s}_{13}\hat{s}_{23}^* & c_{12}c_{23} - \hat{s}_{12}^*\hat{s}_{13}\hat{s}_{23}^* & c_{13}\hat{s}_{23}^* \\ \hat{s}_{12}\hat{s}_{23} - c_{12}\hat{s}_{13}c_{23} & -c_{12}\hat{s}_{23} - \hat{s}_{12}^*\hat{s}_{13}c_{23} & c_{13}c_{23} \end{pmatrix} \begin{matrix} \text{derivational} \\ \text{parameters} \end{matrix}$$

$$c_{ij} \equiv \cos \theta_{ij}$$

$$\hat{s}_{ij} \equiv e^{i\delta_{ij}} \sin \theta_{ij} \quad (\text{for } 1 \leq i < j \leq 6)$$

$$A = \begin{pmatrix} c_{14}c_{15}c_{16} & 0 & 0 \\ -c_{14}c_{15}\hat{s}_{16}\hat{s}_{26}^* - c_{14}\hat{s}_{15}\hat{s}_{25}^*c_{26} & c_{24}c_{25}c_{26} & 0 \\ -\hat{s}_{14}\hat{s}_{24}^*c_{25}c_{26} & & 0 \\ -c_{14}c_{15}\hat{s}_{16}c_{26}\hat{s}_{36}^* + c_{14}\hat{s}_{15}\hat{s}_{25}^*\hat{s}_{26}\hat{s}_{36}^* & -c_{24}c_{25}\hat{s}_{26}\hat{s}_{36}^* - c_{24}\hat{s}_{25}\hat{s}_{35}^*c_{36} & c_{34}c_{35}c_{36} \\ -c_{14}\hat{s}_{15}c_{25}\hat{s}_{35}^*c_{36} + \hat{s}_{14}\hat{s}_{24}^*c_{25}\hat{s}_{26}\hat{s}_{36}^* & -\hat{s}_{24}\hat{s}_{34}^*c_{35}c_{36} & \\ +\hat{s}_{14}\hat{s}_{24}^*\hat{s}_{25}\hat{s}_{35}^*c_{36} - \hat{s}_{14}c_{24}\hat{s}_{34}^*c_{35}c_{36} & & \end{pmatrix}$$

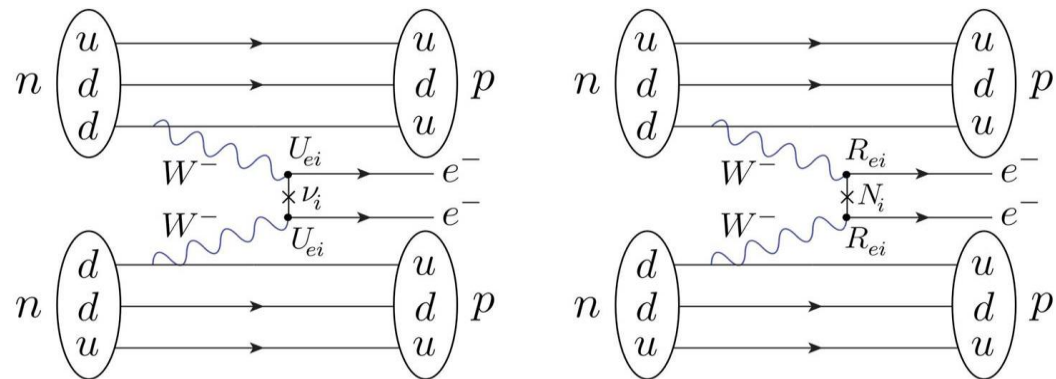
$$R = \begin{pmatrix} \hat{s}_{14}^*c_{15}c_{16} & \hat{s}_{15}^*c_{16} & \hat{s}_{16}^* \\ -\hat{s}_{14}^*c_{15}\hat{s}_{16}\hat{s}_{26}^* - \hat{s}_{14}^*\hat{s}_{15}\hat{s}_{25}^*c_{26} & -\hat{s}_{15}^*\hat{s}_{16}\hat{s}_{26}^* + c_{15}\hat{s}_{25}^*c_{26} & c_{16}\hat{s}_{26}^* \\ +c_{14}\hat{s}_{24}^*c_{25}c_{26} & & \\ -\hat{s}_{14}^*c_{15}\hat{s}_{16}c_{26}\hat{s}_{36}^* + \hat{s}_{14}^*\hat{s}_{15}\hat{s}_{25}^*\hat{s}_{26}\hat{s}_{36}^* & -\hat{s}_{15}^*\hat{s}_{16}c_{26}\hat{s}_{36}^* - c_{15}\hat{s}_{25}^*\hat{s}_{26}\hat{s}_{36}^* & c_{16}c_{26}\hat{s}_{36}^* \\ -\hat{s}_{14}^*\hat{s}_{15}c_{25}\hat{s}_{35}^*c_{36} - c_{14}\hat{s}_{24}^*c_{25}\hat{s}_{26}\hat{s}_{36}^* & +c_{15}c_{25}\hat{s}_{35}^*c_{36} & \\ -c_{14}\hat{s}_{24}^*\hat{s}_{25}\hat{s}_{35}^*c_{36} + c_{14}c_{24}\hat{s}_{34}^*c_{35}c_{36} & & \end{pmatrix}$$

The original seesaw parameters in A+R:
9 angles + 6 phases



Now you can calculate everything that can in principle be measured (see, e.g., [xzz](#), 2305.xxxxx).

The **seesaw**-induced **Majorana** nature of massive neutrinos allows for lepton-number-violating $0\nu 2\beta$ decays to occur, a unique way to hunt for Mr. **Majorana**.



Seesaw + Unitarity:

$$\sum_{i=1}^3 m_i U_{ei}^2 + \sum_{j=1}^3 M_j R_{ej}^2 = 0$$

$$\sum_{i=1}^3 |U_{ei}|^2 + \sum_{j=1}^3 |R_{ej}|^2 = 1$$

A stupid question: which channel is more fundamental?
Smart answer: they are equally fundamental thanks to Yukawa interactions.

In most cases, the contributions from heavy **Majorana** neutrinos to $0\nu 2\beta$ are negligibly small in the canonical seesaw mechanism. (**ZZX**, 0907.3014; **W. Rodejohann**, 0912.3388)

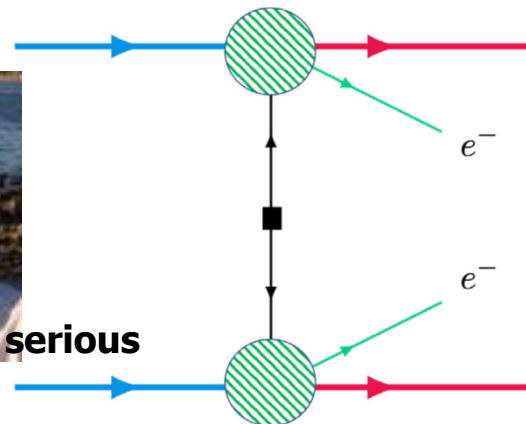
Y.F. Li's talk

But...(e.g., **D.L. Fang**, **Y.F. Li**, **Y.Y. Zhang**, 2112.12779)

Interplay between propagators + NMEs



I'm really serious



J.M. Yao, **J. Meng**, **Y.F. Niu**, **P. Ring**, PPNP 2022

- ◆ The exact seesaw formula — a bridge between the **original** and **derivational** flavor parameters:

$$UD_\nu U^T + RD_N R^T = 0 \longrightarrow M_\nu \equiv U_0 D_\nu U_0^T = (iA^{-1}R) D_N (iA^{-1}R)^T$$

Degrees of freedom (**mass** + **mixing angle** + **CPV phase**): **3 + 3 + 3 (derivational)** ← **3 + 9 + 6 (original)**

- ◆ To calculate the **Jarlskog invariant** of CP violation in the **active** neutrino oscillations, which is defined by

$$\mathcal{J}_\nu \sum_\gamma \varepsilon_{\alpha\beta\gamma} \sum_k \varepsilon_{ijk} = \text{Im} \left[(U_0)_{\alpha i} (U_0)_{\beta j} (U_0)_{\alpha j}^* (U_0)_{\beta i}^* \right]$$



$$\begin{cases} D_\nu \equiv \text{Diag}\{m_1, m_2, m_3\} \\ D_N \equiv \text{Diag}\{M_1, M_2, M_3\} \\ \Delta m_{ij}^2 \equiv m_i^2 - m_j^2 \end{cases}$$

- ◆ On the one hand, we use the **light degrees of freedom** to obtain the relation:

$$\text{Im} \left[\left(M_\nu M_\nu^\dagger \right)_{e\mu} \left(M_\nu M_\nu^\dagger \right)_{\mu\tau} \left(M_\nu M_\nu^\dagger \right)_{\tau e} \right] = \mathcal{J}_\nu \Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2$$

- ◆ On the other hand, we use the **original seesaw-related parameters** to calculate the same quantity *in the leading order approximation of $A^{-1}R$* .

$$A^{-1}R \simeq \begin{pmatrix} \hat{S}_{14}^* & \hat{S}_{15}^* & \hat{S}_{16}^* \\ \hat{S}_{24}^* & \hat{S}_{25}^* & \hat{S}_{26}^* \\ \hat{S}_{34}^* & \hat{S}_{35}^* & \hat{S}_{36}^* \end{pmatrix}$$

But here, we switch off one heavy neutrino for simplicity.

- ◆ The **minimal seesaw** framework with only **2** heavy Majorana neutrinos — a benchmark scenario
- ◆ Consequences: (1) only **2** massive light (active) neutrinos; (2) only **2** CPV phases in the unitary PMNS matrix.
- ◆ Parameter counting: **Original** — **2** masses + **6** mixing angles + **3** CPV phases;
Derivational — **2** masses + **3** mixing angles + **2** CPV phases.

$$\alpha \equiv \delta_{14} - \delta_{15}, \quad \beta \equiv \delta_{24} - \delta_{25}, \quad \gamma \equiv \delta_{34} - \delta_{35}$$

$$\text{Im} \left[(M_\nu M_\nu^\dagger)_{e\mu} (M_\nu M_\nu^\dagger)_{\mu\tau} (M_\nu M_\nu^\dagger)_{\tau e} \right] = C_0 [C_\alpha \sin 2\alpha + C_\beta \sin 2\beta + C_\gamma \sin 2\gamma + C_{\alpha+\beta} \sin(\alpha + \beta) + C_{\beta+\gamma} \sin(\beta + \gamma) + C_{\gamma+\alpha} \sin(\gamma + \alpha) + C_{\alpha-\beta} \sin(\alpha - \beta) + C_{\beta-\gamma} \sin(\beta - \gamma) + C_{\gamma-\alpha} \sin(\gamma - \alpha)] = \mathcal{J}_\nu \Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2$$

◆ Coefficients

$$C_0 = M_1^2 M_2^2 [s_{14}^2 (s_{25}^2 + s_{35}^2) + s_{24}^2 (s_{15}^2 + s_{35}^2) + s_{34}^2 (s_{15}^2 + s_{25}^2) - 2s_{14}s_{15}s_{24}s_{25} \cos(\alpha - \beta) - 2s_{24}s_{25}s_{34}s_{35} \cos(\beta - \gamma) - 2s_{14}s_{15}s_{34}s_{35} \cos(\gamma - \alpha)]$$

$$C_\alpha = M_1 M_2 s_{14}^2 s_{15}^2 (s_{34}^2 s_{25}^2 - s_{24}^2 s_{35}^2) \quad C_{\alpha+\beta} = M_1 M_2 s_{14} s_{15} s_{24} s_{25} (s_{14}^2 s_{35}^2 - s_{34}^2 s_{15}^2 + s_{34}^2 s_{25}^2 - s_{24}^2 s_{35}^2)$$

$$C_\beta = M_1 M_2 s_{24}^2 s_{25}^2 (s_{14}^2 s_{35}^2 - s_{34}^2 s_{15}^2) \quad C_{\beta+\gamma} = M_1 M_2 s_{24} s_{25} s_{34} s_{35} (s_{14}^2 s_{35}^2 - s_{34}^2 s_{15}^2 + s_{24}^2 s_{15}^2 - s_{14}^2 s_{25}^2)$$

$$C_\gamma = M_1 M_2 s_{34}^2 s_{35}^2 (s_{24}^2 s_{15}^2 - s_{14}^2 s_{25}^2) \quad C_{\gamma+\alpha} = M_1 M_2 s_{14} s_{15} s_{34} s_{35} (s_{24}^2 s_{15}^2 - s_{14}^2 s_{25}^2 + s_{34}^2 s_{25}^2 - s_{24}^2 s_{35}^2)$$

◆ Conclusion:

low seesaw scale

$\mathcal{J}_\nu \leftarrow \text{9 ways} \rightarrow \alpha, \beta, \gamma$

$$C_{\alpha-\beta} = [M_1^2 (s_{14}^2 + s_{24}^2 + s_{34}^2) s_{34}^2 - M_2^2 (s_{15}^2 + s_{25}^2 + s_{35}^2) s_{35}^2] s_{14} s_{15} s_{24} s_{25}$$

$$C_{\beta-\gamma} = [M_1^2 (s_{14}^2 + s_{24}^2 + s_{34}^2) s_{14}^2 - M_2^2 (s_{15}^2 + s_{25}^2 + s_{35}^2) s_{15}^2] s_{24} s_{25} s_{34} s_{35}$$

$$C_{\gamma-\alpha} = [M_1^2 (s_{14}^2 + s_{24}^2 + s_{34}^2) s_{24}^2 - M_2^2 (s_{15}^2 + s_{25}^2 + s_{35}^2) s_{25}^2] s_{14} s_{15} s_{34} s_{35}$$

- ◆ The flavor-dependent CP-violating asymmetries in LNV decays of heavy Majorana neutrinos:

$$\varepsilon_{i\alpha} \equiv \frac{\Gamma(N_i \rightarrow \ell_\alpha + H) - \Gamma(N_i \rightarrow \bar{\ell}_\alpha + \bar{H})}{\sum_\alpha [\Gamma(N_i \rightarrow \ell_\alpha + H) + \Gamma(N_i \rightarrow \bar{\ell}_\alpha + \bar{H})]}$$

$$= \frac{1}{8\pi(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu)_{ii}} \sum_{j \neq i} \left\{ \text{Im} \left[(\mathcal{Y}_\nu^*)_{\alpha i} (\mathcal{Y}_\nu)_{\alpha j} (\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu)_{ij} \xi(x_{ji}) + (\mathcal{Y}_\nu^*)_{\alpha i} (\mathcal{Y}_\nu)_{\alpha j} (\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu)_{ij}^* \zeta(x_{ji}) \right] \right\}$$

$$\mathcal{Y}_\nu \equiv Y_\nu U_0'^*$$

$$x_{ji} \equiv M_j^2 / M_i^2$$

where $\xi(x_{ji}) = \sqrt{x_{ji}} \left\{ 1 + 1/(1 - x_{ji}) + (1 + x_{ji}) \ln [x_{ji}/(1 + x_{ji})] \right\}$, $\zeta(x_{ji}) = 1/(1 - x_{ji})$

- ◆ Baryogenesis via leptogenesis in the early Universe

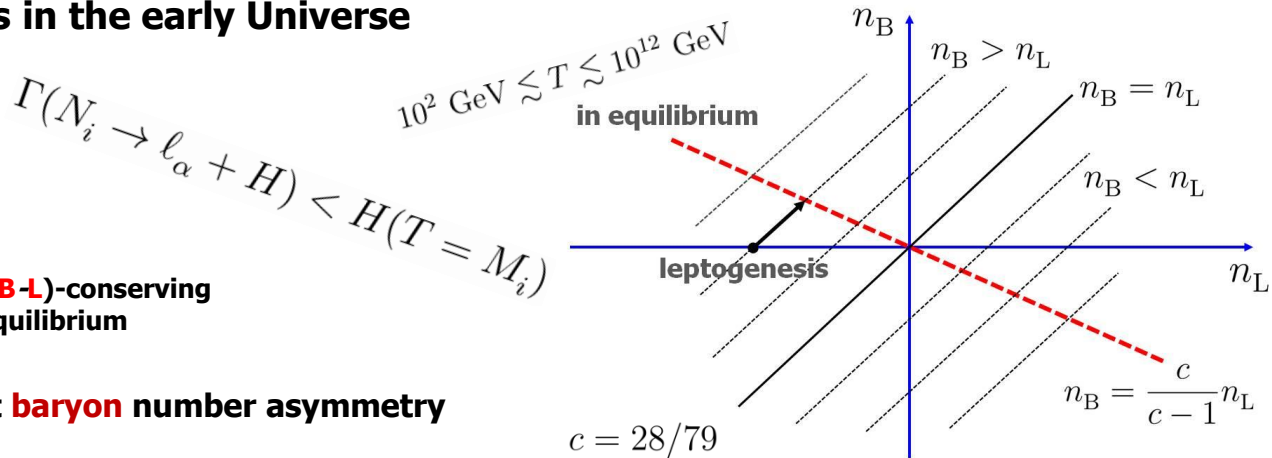
- ◆ A net **lepton** number asymmetry:

$$Y_L \equiv \frac{n_L - n_{\bar{L}}}{s} = \frac{1}{g_*} \sum_{i,\alpha} K_{i\alpha} \varepsilon_{i\alpha}$$

sphaleron-induced (B-L)-conserving process in thermal equilibrium

$$Y_B \equiv \frac{n_B - n_{\bar{B}}}{s} = -c Y_L$$

A net baryon number asymmetry



- ◆ The CP-violating asymmetries in LNV decays of the **first** heavy Majorana neutrino:

$$\varepsilon_{1e} = \frac{M_1^2 s_{14} s_{15}}{8\pi \langle \phi^0 \rangle^2 (s_{14}^2 + s_{24}^2 + s_{34}^2)} \left[x_{21} \xi(x_{21}) \left[s_{14} s_{15} \sin 2\alpha + s_{24} s_{25} \sin(\alpha + \beta) + s_{34} s_{35} \sin(\alpha + \gamma) \right] \right. \\ \left. + x_{21} \zeta(x_{21}) \left[s_{24} s_{25} \sin(\alpha - \beta) + s_{34} s_{35} \sin(\alpha - \gamma) \right] \right]$$

$$\varepsilon_{1\mu} = \frac{M_1^2 s_{24} s_{25}}{8\pi \langle \phi^0 \rangle^2 (s_{14}^2 + s_{24}^2 + s_{34}^2)} \left[x_{21} \xi(x_{21}) \left[s_{14} s_{15} \sin(\alpha + \beta) + s_{24} s_{25} \sin 2\beta + s_{34} s_{35} \sin(\beta + \gamma) \right] \right. \\ \left. + x_{21} \zeta(x_{21}) \left[s_{14} s_{15} \sin(\beta - \alpha) + s_{34} s_{35} \sin(\beta - \gamma) \right] \right]$$

$$\varepsilon_{1\tau} = \frac{M_1^2 s_{34} s_{35}}{8\pi \langle \phi^0 \rangle^2 (s_{14}^2 + s_{24}^2 + s_{34}^2)} \left[x_{21} \xi(x_{21}) \left[s_{14} s_{15} \sin(\alpha + \gamma) + s_{24} s_{25} \sin(\beta + \gamma) + s_{34} s_{35} \sin 2\gamma \right] \right. \\ \left. + x_{21} \zeta(x_{21}) \left[s_{14} s_{15} \sin(\gamma - \alpha) + s_{24} s_{25} \sin(\gamma - \beta) \right] \right]$$

$$\alpha \equiv \delta_{14} - \delta_{15}, \quad \beta \equiv \delta_{24} - \delta_{25}, \quad \gamma \equiv \delta_{34} - \delta_{35}$$

$$\varepsilon_1 = + \frac{M_1^2 x_{21} \xi(x_{21})}{8\pi \langle \phi^0 \rangle^2 (s_{14}^2 + s_{24}^2 + s_{34}^2)} \left[s_{14}^2 s_{15}^2 \sin 2\alpha + s_{24}^2 s_{25}^2 \sin 2\beta + s_{34}^2 s_{35}^2 \sin 2\gamma \right. \\ \left. + 2s_{14} s_{15} s_{24} s_{25} \sin(\alpha + \beta) + 2s_{14} s_{15} s_{34} s_{35} \sin(\alpha + \gamma) + 2s_{24} s_{25} s_{34} s_{35} \sin(\beta + \gamma) \right]$$

- ◆ The CP-violating asymmetries in LNV decays of the **second** heavy Majorana neutrino:

$$\begin{aligned} \varepsilon_{2e} = & -\frac{M_2^2 s_{14} s_{15}}{8\pi \langle \phi^0 \rangle^2 (s_{15}^2 + s_{25}^2 + s_{35}^2)} \left[x_{12} \xi(x_{12}) \left[s_{14} s_{15} \sin 2\alpha + s_{24} s_{25} \sin(\alpha + \beta) + s_{34} s_{35} \sin(\alpha + \gamma) \right] \right. \\ & \left. + x_{12} \zeta(x_{12}) \left[s_{24} s_{25} \sin(\alpha - \beta) + s_{34} s_{35} \sin(\alpha - \gamma) \right] \right] \end{aligned}$$

$$\begin{aligned} \varepsilon_{2\mu} = & -\frac{M_2^2 s_{24} s_{25}}{8\pi \langle \phi^0 \rangle^2 (s_{15}^2 + s_{25}^2 + s_{35}^2)} \left[x_{12} \xi(x_{12}) \left[s_{14} s_{15} \sin(\alpha + \beta) + s_{24} s_{25} \sin 2\beta + s_{34} s_{35} \sin(\beta + \gamma) \right] \right. \\ & \left. + x_{12} \zeta(x_{12}) \left[s_{14} s_{15} \sin(\beta - \alpha) + s_{34} s_{35} \sin(\beta - \gamma) \right] \right] \end{aligned}$$

$$\begin{aligned} \varepsilon_{2\tau} = & -\frac{M_2^2 s_{34} s_{35}}{8\pi \langle \phi^0 \rangle^2 (s_{15}^2 + s_{25}^2 + s_{35}^2)} \left[x_{12} \xi(x_{12}) \left[s_{14} s_{15} \sin(\alpha + \gamma) + s_{24} s_{25} \sin(\beta + \gamma) + s_{34} s_{35} \sin 2\gamma \right] \right. \\ & \left. + x_{12} \zeta(x_{12}) \left[s_{14} s_{15} \sin(\gamma - \alpha) + s_{24} s_{25} \sin(\gamma - \beta) \right] \right] \end{aligned}$$

$$\alpha \equiv \delta_{14} - \delta_{15}, \quad \beta \equiv \delta_{24} - \delta_{25}, \quad \gamma \equiv \delta_{34} - \delta_{35}$$

$$\begin{aligned} \varepsilon_2 = & -\frac{M_2^2 x_{12} \xi(x_{12})}{8\pi \langle \phi^0 \rangle^2 (s_{15}^2 + s_{25}^2 + s_{35}^2)} \left[s_{14}^2 s_{15}^2 \sin 2\alpha + s_{24}^2 s_{25}^2 \sin 2\beta + s_{34}^2 s_{35}^2 \sin 2\gamma \right. \\ & \left. + 2s_{14} s_{15} s_{24} s_{25} \sin(\alpha + \beta) + 2s_{14} s_{15} s_{34} s_{35} \sin(\alpha + \gamma) + 2s_{24} s_{25} s_{34} s_{35} \sin(\beta + \gamma) \right] \end{aligned}$$

A summary of the phase dependence

- ◆ The **Jarlskog** invariant and the CPV asymmetries depend on three independent CPV phases:

	2α	2β	2γ	$\alpha + \beta$	$\beta + \gamma$	$\alpha + \gamma$	$\alpha - \beta$	$\beta - \gamma$	$\gamma - \alpha$
\mathcal{J}_ν	✓	✓	✓	✓	✓	✓	✓	✓	✓
ε_{1e}	✓			✓		✓	✓		✓
$\varepsilon_{1\mu}$		✓		✓	✓		✓	✓	
$\varepsilon_{1\tau}$			✓		✓	✓		✓	✓
ε_1	✓	✓	✓	✓	✓	✓			
ε_{2e}	✓			✓		✓	✓		✓
$\varepsilon_{2\mu}$		✓		✓	✓		✓	✓	
$\varepsilon_{2\tau}$			✓		✓	✓		✓	✓
ε_2	✓	✓	✓	✓	✓	✓			

CP 不守恒量之间一般不存在正比关系

- ◆ A more straightforward correlation between the two CPV observables needs some assumptions.

- ◆ The canonical **seesaw** mechanism is the most natural and economical mechanism to produce tiny **Majorana** neutrino masses and interpret the cosmological matter-antimatter asymmetry by thermal **leptogenesis** mechanism, but all these only work **qualitatively**.
- ◆ *For the first time*, we have derived the **generic** + **explicit** expressions of the **Jarlskog** invariant for light neutrino oscillations in terms of the **original** seesaw flavor parameters, and of **CP asymmetries** for heavy neutrino decays based on a **block parametrization** of the active-sterile flavor texture.
- ◆ A full numerical exploration of the **seesaw** parameter space can be done, using a good computer. Testing the **seesaw** mechanism and identifying the **Majorana** nature of massive neutrinos is largely possible in the precision measurement era. The $0\nu 2\beta$ decay will be a **smoking gun**.





Weinberg's Laws of Progress in Theoretical Physics

From: "Asymptotic Realms of Physics" (ed. by Guth, Huang, Jaffe, MIT Press, 1983)

First Law: "The conservation of Information" (*You will get nowhere by churning equations*)

Second Law: "Do not trust arguments based on the lowest order of perturbation theory"

Third Law: "You may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you'll be sorry!"