Decoherence in Neutrino Oscillations

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3 Flavor Oscillations

One of the assumptions in the standard, textbook derivation of neutrino oscillations is clearly wrong

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = U \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix} \quad \Rightarrow \quad |\nu_a\rangle = \sum_\alpha U_{a\alpha} |\nu_\alpha\rangle$$

$$a = e, \mu, \tau, \ \alpha = 1, 2, 3$$

$|\nu_\alpha\rangle$ are eigenstates of the mass matrix

$$|\nu_a(t)\rangle = \sum_\alpha U_{a\alpha} |\nu_\alpha(t)\rangle = \sum_\alpha U_{a\alpha} e^{-iE_\alpha t} |\nu_\alpha\rangle = \sum_{\alpha, a'} U_{a\alpha} e^{-iE_\alpha t} U^\dagger_{\alpha a'} |\nu_{a'}\rangle$$

Here we are assuming that each mass eigenstate is an eigenstate of the Hamiltonian as well

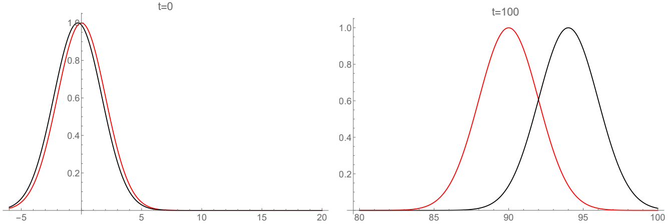
BUT this is an eigenstate of the momentum $\Rightarrow$ plane waves, completely delocalized: they cannot propagate!

Decoherence

In reality, each mass eigenstate can be described using wavepackets, which are localized

$$|\nu\rangle = \int \mathrm{d}p\ f(p)|p\rangle$$

However different masses $\rightarrow$ different velocities: while propagating, the mass eigenstates will be separated; if this distance is larger than the spatial dimension of the wavepackets, there is no interference between their phases and no oscillations anymore



State of The Art

We don't have a solid theoretical model to describe the decoherence in neutrino oscillations, so there is no agreement in the literature, not even whether it is observable at all or not.
The effect of quantum decoherence would be a suppression of neutrino oscillations; however there are many processes that could cause such an effect, such as

- Uncertainty on the baseline (finite dimension of the source, etc...)
- Energy uncertainty: if the energy resolution of the detector is not sufficient to resolve a period, we cannot see the oscillating behavior anymore, just a decreased flux

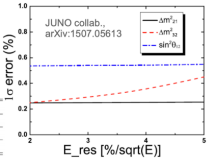
Moreover many of the works in literature rely on strong assumptions, some of which we know are wrong. Estimation of the coherence length could differ by several order of magnitudes

- Decoherence has never been observed, and there is still no solid theoretical description of such an effect
- We are entering in the precision era of neutrino physics: the current or the next generation of experiment will measure most of the mixing parameters to the sub-percent precision. Even a small effect that can modify the oscillation probability could affect the results of experiments; this could be true for JUNO, for example **(Chan,Chu,Tsui,Wong and Xu, Eur. Phys. J. C 76 (2016) no.6, 310)**.

Neutrino oscillation parameters

- Sub-percent accuracy for θ_{12}, Δm^2_{21} and Δm^2_{31}
- Current precision

	Δm^2_{21}	$\lvert\Delta m^2_{31}\rvert$	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	δ
Dominant Exps.	KamLAND	T2K	SNO+SK	Daya Bay	NOνA	T2K
Individual 1σ	2.4%	2.6%	4.5%	3.4%	5.2%	70%
Nu-FIT 4.0	2.4%	1.3%	4.0%	2.9%	3.8%	16%



Yue Meng, Neutrino2020

Our Model

Started by considering very simplified scenarios, increasing gradually the complexity

- 1+1 dimension, real scalar fields, maximal mixing
- Neutrinos created and detected via three-bodies decays

$$S_H \to S_L + \nu_i \qquad D_L + \nu_j \to D_H$$

- Initial state contains all the information on the source particle, it is evolved consistently with QFT using $U(t) = e^{-iHt}$

$$|\Omega(0)\rangle = \int \mathrm{d}q \dots f(q) \dots |q \dots\rangle \qquad \Rightarrow \qquad |\Omega(t)\rangle = e^{-iHt}|\Omega(0)\rangle$$

- Transition amplitude is computed by projecting the time-evolved initial state into a final state $|F\rangle$

$$A = \langle F|\Omega(t)\rangle \qquad \Rightarrow P = |A|^2$$

- Environmental interactions are not considered so far

Tree-Level Approximation

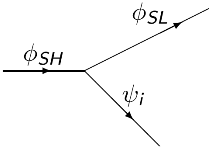
Let us consider just the neutrino creation, using a tree-level approximation (justified when t$\ll \tau$)

We consider only states were neutrinos are created; the transition amplitude will be a sum of terms like

$$\langle S_L, p-q, \nu_i, q | e^{-iHt} | S_H, p\rangle \propto F(p,q)$$

$$\rightarrow F(p,q) = \frac{e^{-i\mathcal{E}_1 t} - e^{-i\mathcal{E}_0 t}}{\mathcal{E}_1 - \mathcal{E}_0} = \int_0^t \mathrm{d}t_1 e^{-i(\mathcal{E}_0 t_1 + \mathcal{E}_1 (t-t_1))}$$

Time-evolved state is a coherent sum over all the possible creation times



Wavepacekt localized in momentum

$\sigma_\nu \simeq \sigma_{SH}(v_I - v_F) \simeq \sigma_{SH} E_\nu / M_S$

but not in coordinate space (dimension goes like t)

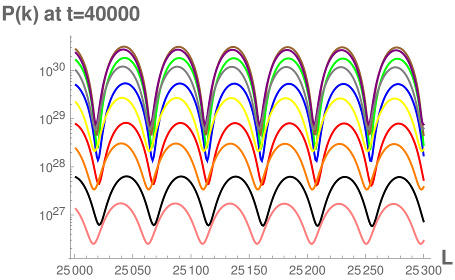
E.C and J. Evslin, EPJC 82 (2022) 12, 1097

Decay and detection in vacuum $\Rightarrow$ final states are not measured $\Rightarrow$ described using plane waves

$|F\rangle = |S_L, k, D_H, l\rangle$ (detector must be considered for baseline!)

Coherent sum over all the possible production times $\Rightarrow$ no decoherence

Even if the mass eigenstates emitted at time t_1 are completely separated when they arrive at the detector, they can still interfere with mass eigenstates created at time $t_1 \pm \epsilon$!



We expect decoherence for $L > 2,000$, but at $L = 25,000$ we still have oscillations!

E.C., H. Mohammed, J. Evslin, EPJC 81 (2021) 4, 325

Localized Final States

However, in reality, environmental interactions would constrain the creation time: **Coherent** sum $\rightarrow$ **Incoherent** sum

W. H. Zurek, Phys. Rev. D 26 (1982) 1862.

But environmental interactions are very difficult to implement in our model, what could be a reasonable approximation?

If the final states are **localized**, kinematic would constrain the region where the neutrino can be created ($G(x,\sigma)$=Gaussian with width σ) $\Rightarrow$ decoherence can emerge! **(E.C and J. Evslin, EPJC 82 (2022) 12, 1097)**

$$|S_L, p-q, \nu_i, q\rangle \rightarrow |F\rangle = \int \mathrm{d}k\mathrm{d}q G(k,\sigma_{SL}) G(q,\sigma_\nu)|S_L, k, \nu_i, q\rangle$$

$$P(\delta L = 0) \propto c - \mathrm{Sin}^2\left(\frac{\Delta m^2 L}{4E}\right) e^{-\delta^2/2} \qquad \delta^2 = \frac{2L^2}{3L_{coh}^2} \qquad L_{coh} = \frac{2E^2\sigma_{\nu,x}}{\Delta m^2}$$

δL is related to the position of SL, it can also be integrated out (but integration would be **incoherent**)

$$P = \int \mathrm{d}\delta L P(\delta L) = \int \mathrm{d}\delta L |A(\delta L)|^2 \qquad \delta_L^2 = \frac{L^2}{3L_{coh}^2} + \frac{(\Delta m^2 \sigma_{\nu,x})^2}{3(4E)^2}$$

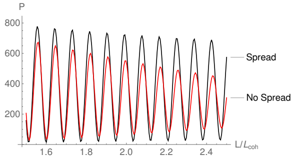
Spread of the Wavepacket

Gaussian wavepacket will spread with time (p-components travel at different velocities!). In order to take this into account, energy expansion up to second order required **(E.C and J. Evslin, EPJC 82 (2022) 12, 1097)**:

$$E(p) \simeq E_0 + v(p - p_0) + v'(p - p_0)^2$$

This is equivalent to rescale σ_ν:

$$\sigma_\nu \to \sigma_\nu \Sigma(L) \qquad \Sigma(L) = \frac{1 - i\sigma_\nu^2 v' L}{1 + (\sigma_\nu^2 v' L)^2}$$



$L \to \infty$, spread of the wavepacket would balance the separation, and decoherence "saturate"

$$\delta^2 \to \frac{L_{sp}^2}{L_{coh}^2} \qquad L_{sp} = \frac{1}{v'\sigma_\nu^2} \qquad v' = \frac{m_i^2}{E^3}$$

Such an effect depends on the absolute mass scale, not $\Delta m^2 \Rightarrow$ possible (in theory) to probe directly m from the oscillations

Other Results

- Some of the assumptions commonly used in literature, such as the covariance of wavepackets (**D.V. Naumov, V.A. Naumov, J. Phys. G 37 (2010) 105014; F. P. An et al., Eur. Phys. J. C 77 (2017) no.9, 606**) are inconsistent, since the time evolution breaks the Lorentz invariance **H. Mohammed, J. Evslin and E.C, Nucl.Phys. B 953 (2020), 114972**
- New quantum effect: in a very short time windows after the first neutrinos arrives, the oscillations have not started yet, if the detector is placed at the oscillation minimum, with sufficient time resolution is should be possible to see the detection probability to go down with time. Most likely the requirements for its observation are well beyond the current technical possibilities, however it is worth of more investigation **E.C, J. Evslin and H. Mohammed,EPJC 81 (2021) no.4, 325**

Summary

- Many models present in literature, crucial parameters must be introduced by hand, leading to different predictions
- We are developing a model where the the fields are treated consistently according to QFT. We started by considering very simplified cases, obtaining nonetheless interesting results; we are working toward more realistic scenarios
- We have shown that the entanglement between environment, source particles and neutrino is crucial for the localization of the wavefunction. In vacuum, no decoherence due to the separation of the wavepackets. Decoherence can emerge if the final states are localized (creation region contrained by kinematic)
- Spread of the wavepacket can affect decoherence (depends on m, not Δm^2)

Thank You!

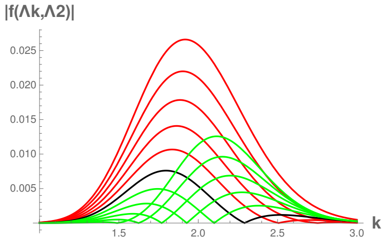
Backup Slides

Covariance

In some works in the literature it is assumed that the neutrino wavefunction is covariant **D.V. Naumov, V.A. Naumov, J. Phys. G 37 (2010) 105014**; such an assumption was used also to compute the Daya Bay constrains on the decoherence parameters **F. P. An et al., Eur. Phys. J. C 77 (2017) no.9, 606**.

$$f(k,p) \propto e^{-(p-k)_\mu (p-k)^\mu / 2\sigma^2}$$

We have shown that such an assumption is inconsistent: even if, at time t_0, the state is covariant, the time evolution would break the Lorentz invariance

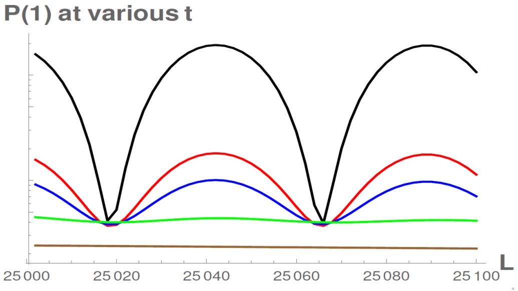


The red, black and green curves correspond to the boosted wavefunction with $\beta < 0$, $\beta = 0$ and $\beta > 0$, respectively

For a very short window of time after the first neutrinos arrive, they have not oscillated yet; if the detector is placed at the oscillation minimum, it would be possible to see the detection probability to increase with time

$$A(k, l) \propto \int \mathrm{d}T [...] e^{-\mu(T-T_0)^2} \rightarrow [...]\delta(T - T_0)$$

To see this effect is equivalent to probe the shape of the Gaussian.



Most likely the requirements for its observation are well beyond the current technical possibilities, however it is worth of more investigation

Quantum Zeno Effect

If the lifetime of the source particles is **smaller** than the timescale of the experiment, the decay probability itself constrain the neutrino production (tree-level approximation no longer valid) To see this effect, we need to compute the non-perturbative transition probability. Calculations similar to the ones for the quantum Zeno Effect (see, for example, **P. Facchi and S. Pascazio, Chaos Solitons Fractals 12 (2001) 2777**)

Quantum Zeno Effect

The decay probability follow an exponential behavior only at intermediate times, at very small (and very large) timescales it behave polynomially $\propto t^n$, with $n > 1$. This means that if we take an unstable system and, over a time T, we measure it n times to check whether or not it is decayed, for $n \to \infty$, $P(n) \to 0$. Usually the transition probability is calculated using the resolvent, namely

$$A(E) = \langle + | \frac{1}{H - E} | + \rangle = \frac{1}{E - \omega_+ - \Sigma(E)}$$