核物理中的有效场论和少体方法

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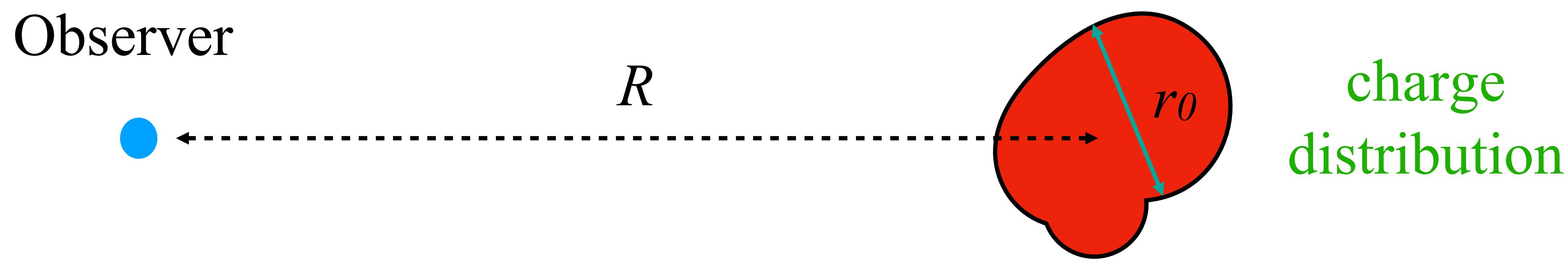
Multipole expansion
A classical example of EFT



- Separation of scales: $R >> r_0$
- Controlled approximation, able to estimate uncertainty

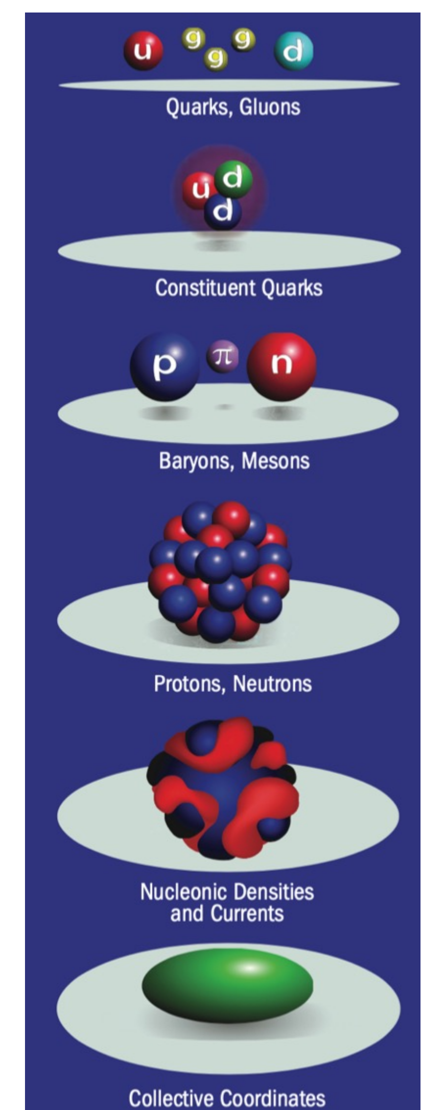
$$V = \frac{q}{R} + \frac{d_i R_i}{R^3} + \frac{Q_{ij} R_i R_j}{R^5} + \cdots$$

- Naturalness $|d_i| \sim q r_0$ $|Q_{ij}| \sim q r_0^2$ $\Rightarrow$ power counting
- What if it is a rod?
  $\Rightarrow$ change power counting

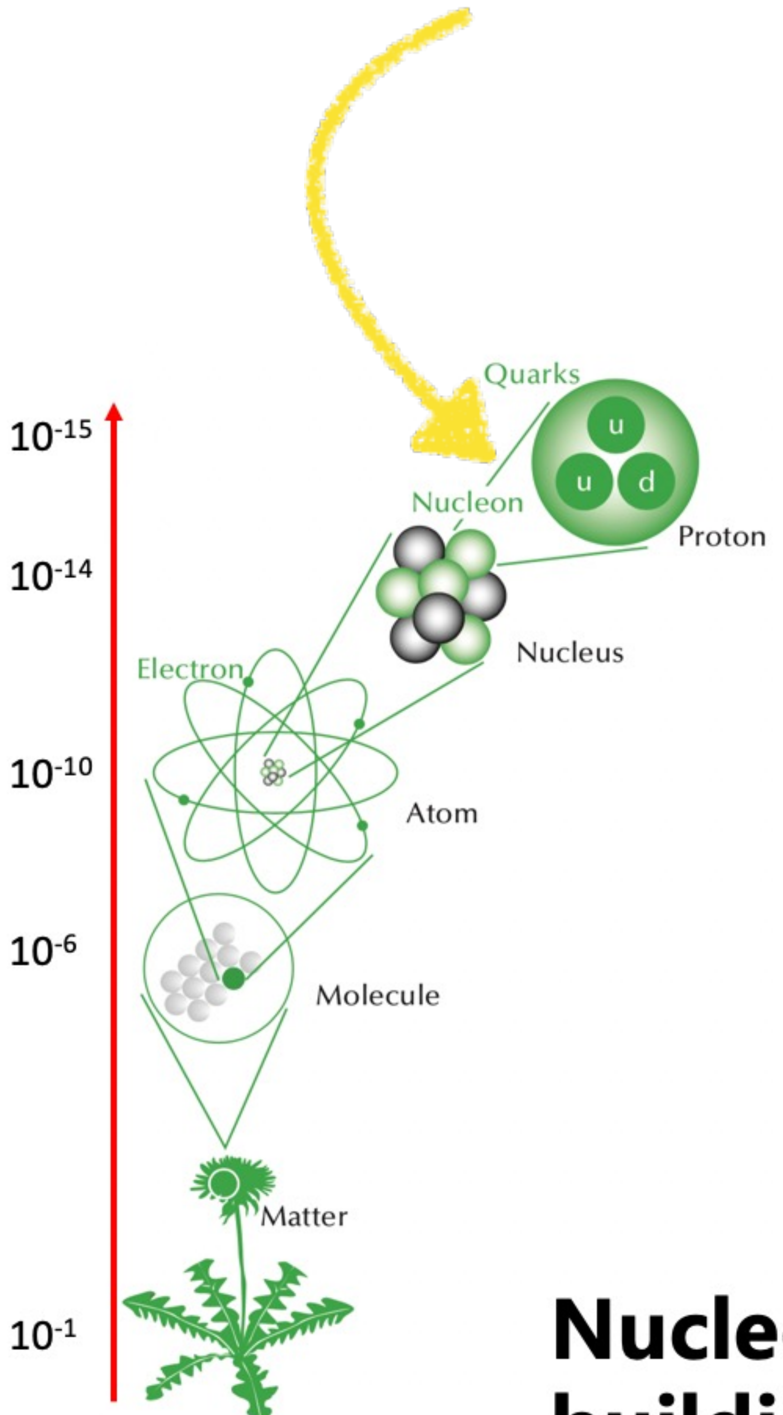
Hierarchy of EFTs

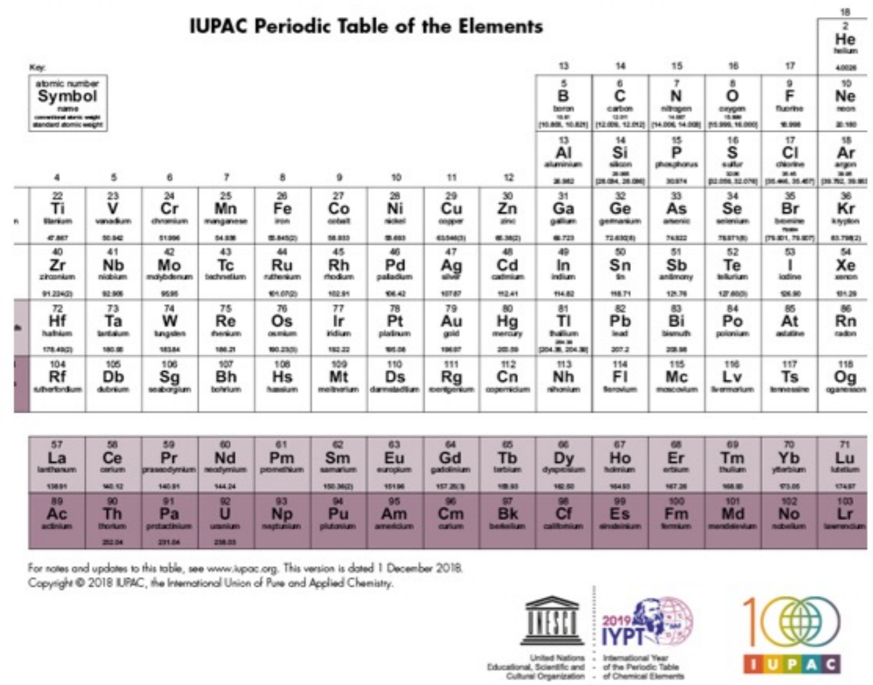
beyond SM		$10^?$ TeV
Standard Model	M_H	100 GeV
	Λ_χ	1 GeV
Chiral EFT	Λ_{nuc}	$\sim$ 100 MeV
	Q_D	45 MeV
Pionless EFT Halo Cluster	$1/a_{1S0}$	8 MeV

Degrees of freedom



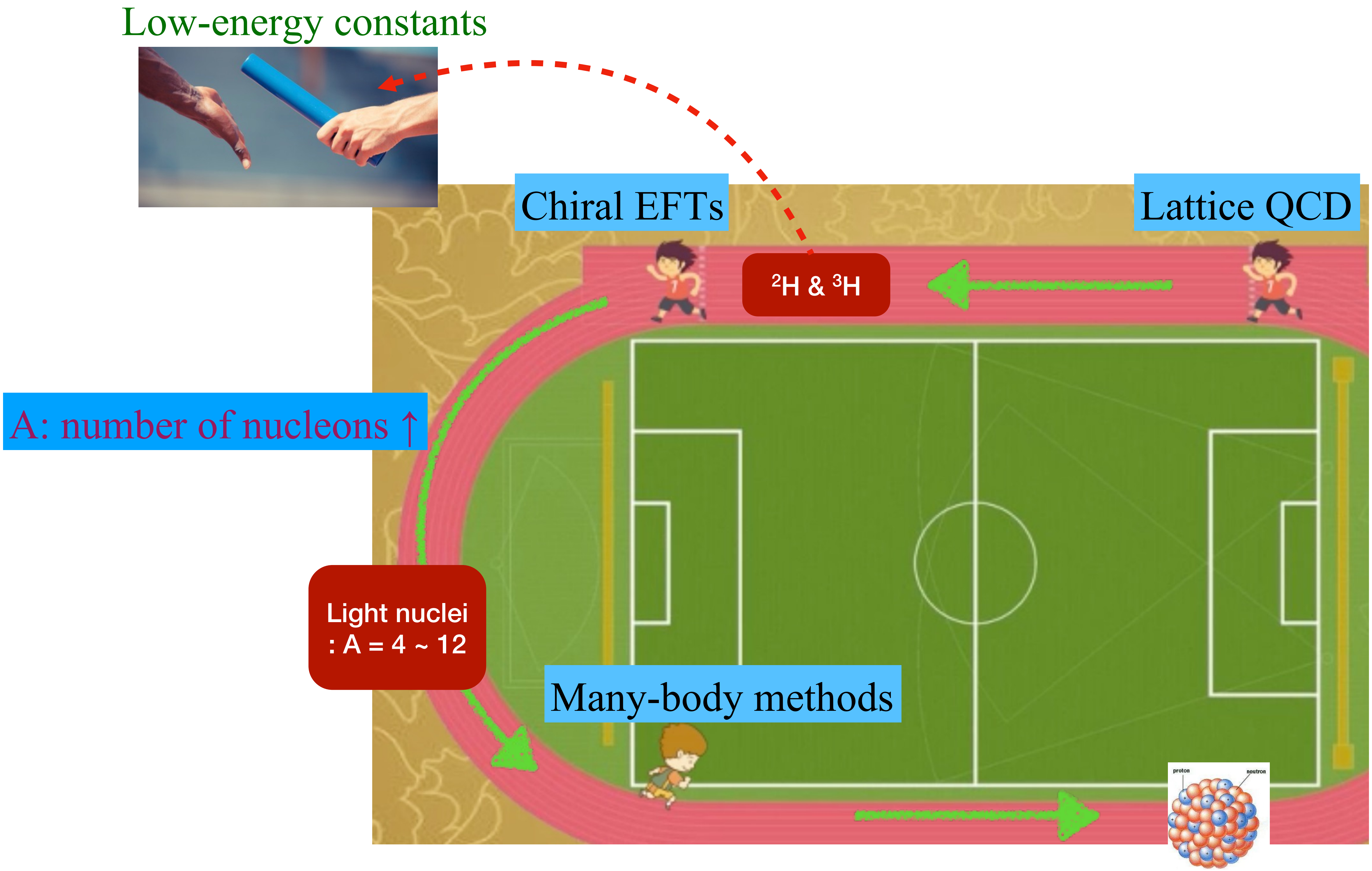
粒子物理与核物理结合，大有可为





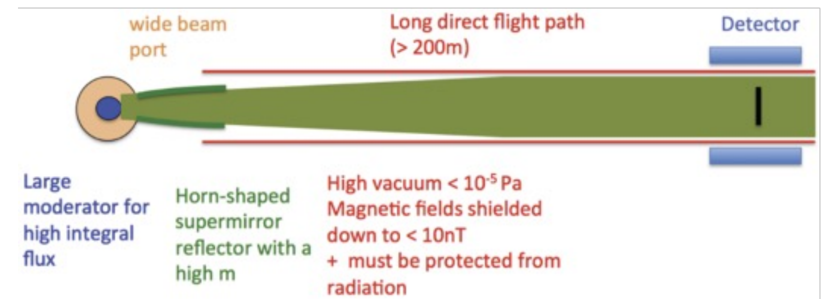
Nucleons are the essential building blocks of Matter!

Relay: from quarks & gluons to Uranium



Neutron-antineutron oscillation

- Some BSM models favor baryon-number violation $|\Delta B| = 2$
- Can explain baryon asymmetry of universe
- Stable nuclei become "unstable"
- Can we relate $\tau_{n\text{-}nbar}$ to Γ_d ?
- EFT helps disentangle different B-violating physics



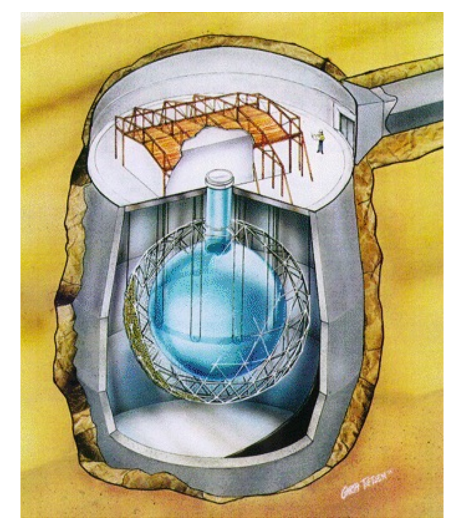
In vacuum (European spallation source)

$$R_d \equiv \Gamma_d^{-1}/\tau_{n\bar{n}}^2$$

Uncertainty

$$R_d = -\left[\frac{m_N}{\kappa}\mathrm{Im}\,a_{\bar{n}p}(1 + 0.40 + 0.20 - 0.13 \pm 0.4)\right]^{-1} = (1.1 \pm 0.3) \times 10^{22}\ \mathrm{s}^{-1}.$$

Oosterhof, BwL, de Vries, van Kolck, Timmermans. PRL'19



Limit on deuteron lifetime (SNO)

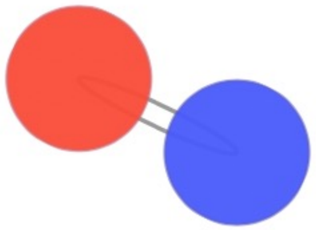
Few-body methods

- Equivalent to Schrodinger eqn
- Precise numerical solutions

Easy

- Two-body: NN, N-cluster, cluster-cluster

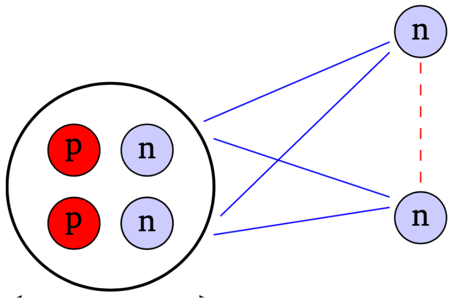
  Lippmann-Schwinger equation



Medium

- Three-body: NNN, 2n-cluster, 3alpha, …

  Faddeev equation



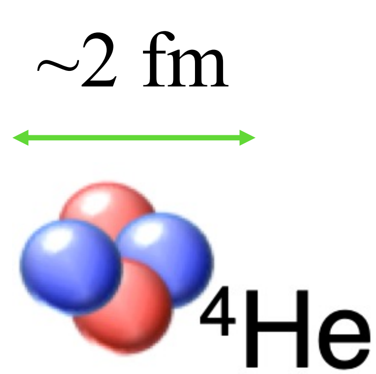
Hard

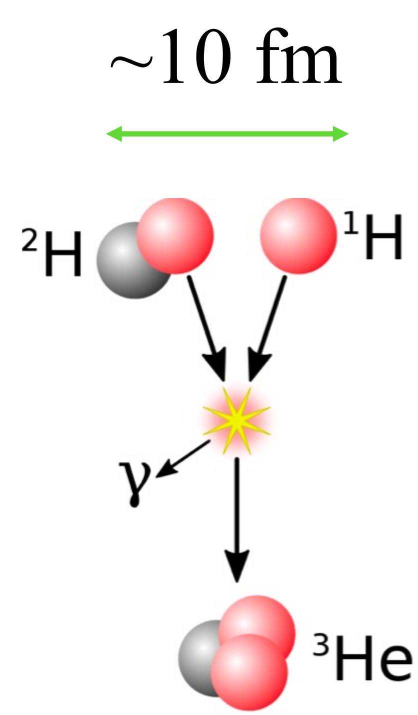
- Four-body: NNNN, 3n-alpha …

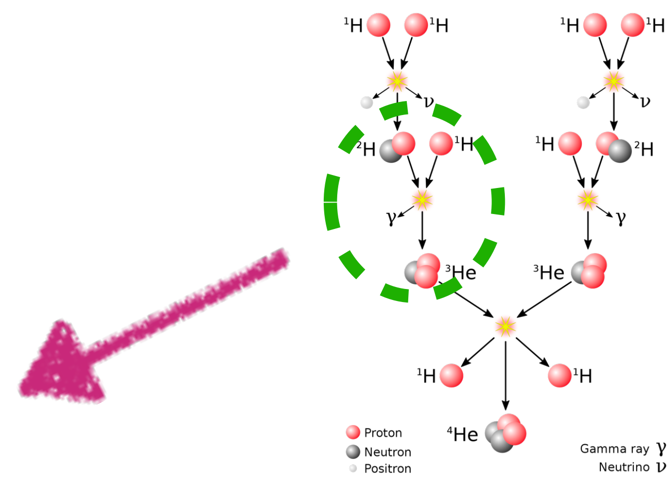
  Faddeev/Yakubowski equation

Ab initio calculations of nuclear reactions

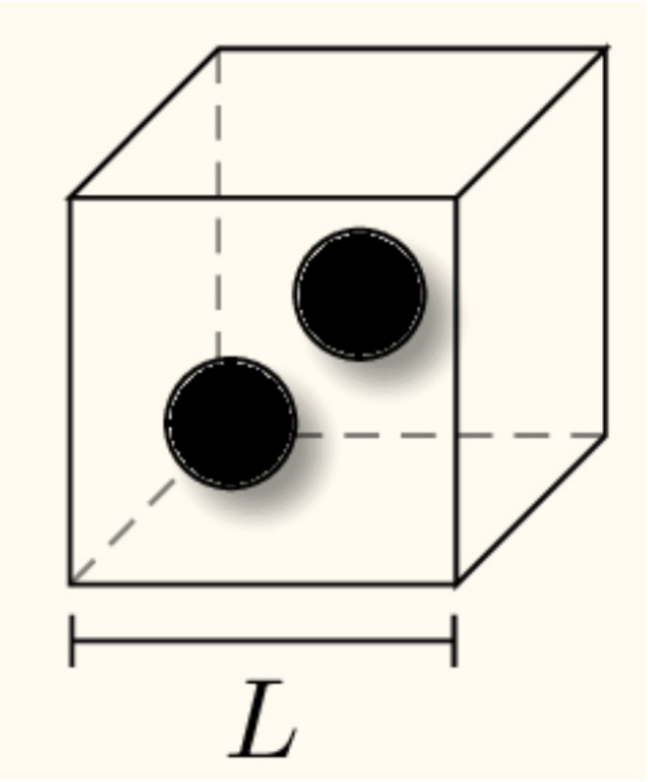
- Ab initio ≈ diagonalizing nuclear Hamiltonian of A-nucleon systems
- Scattering and reactions normally involve larger configuration space







Lesson from LQCD



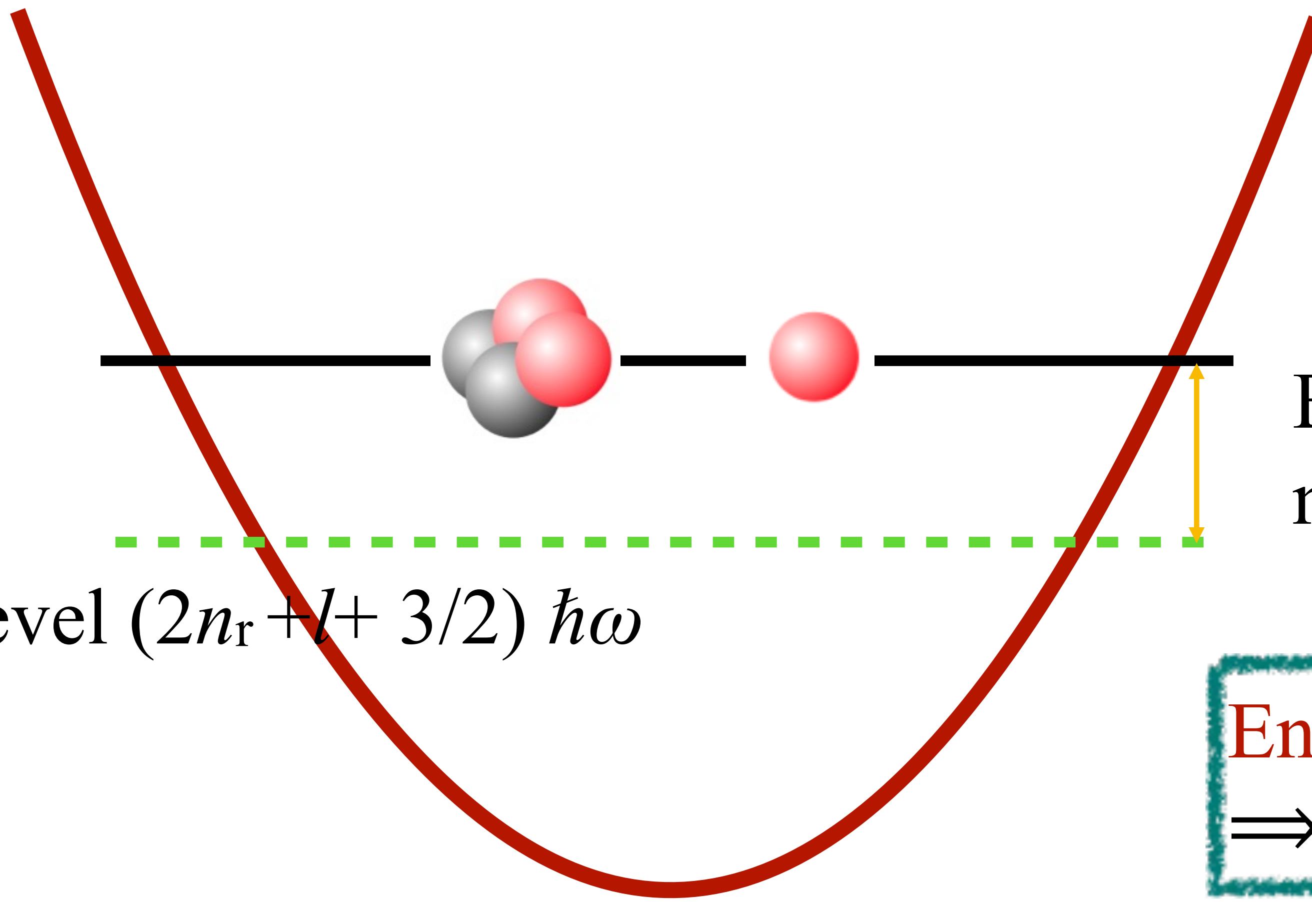
- LQCD ⇒ energy levels of hadrons confined in box
- Luscher's + energy levels ⇒ phase shifts
- Box must be large
- Rotational symmetry broken

Luscher's formula: $$\det[\mathcal{M}^{-1}(E_L) + F^{(P)}(E_L, L)] = 0$$

M. Luscher, Nucl. Phys. B **354**, 531 (1991)

Harmonic-oscillator trap

Busch et al. ’98
Luu, Savage & Vary ’10
Rotureau, et al. ’10 ’12
Xilin Zhang, et al. ’19 ’20



Energies of confined particles
$\Longrightarrow$ Scattering amplitude

- HO potential is isotropic => angular momentum remains good quantum number
- HO w.f. analytically known
- Available software packages

Trick: matching w.f.

Outside wf : HO trapped

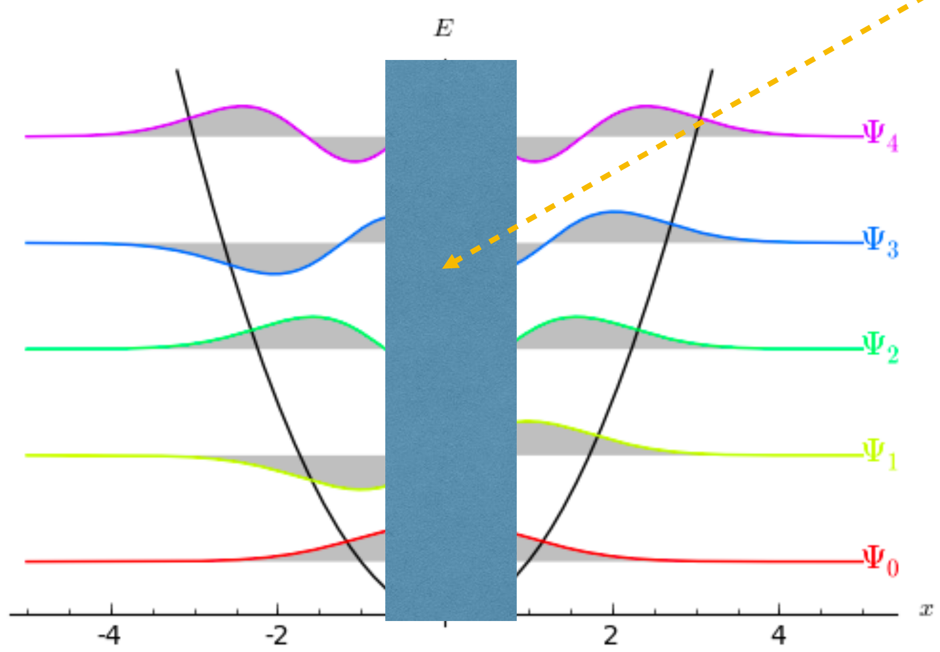
$$\propto aR(r;E) + bY(r;E)$$

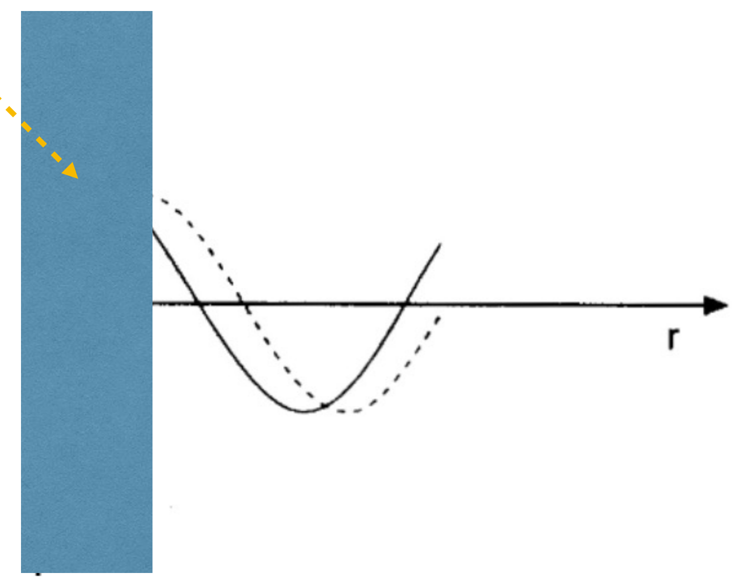
Outside wf : scattering

$$\propto \sin(kr + \delta)$$

Intrinsic potential

R & Y: solutions to HO Schrodinger eq.



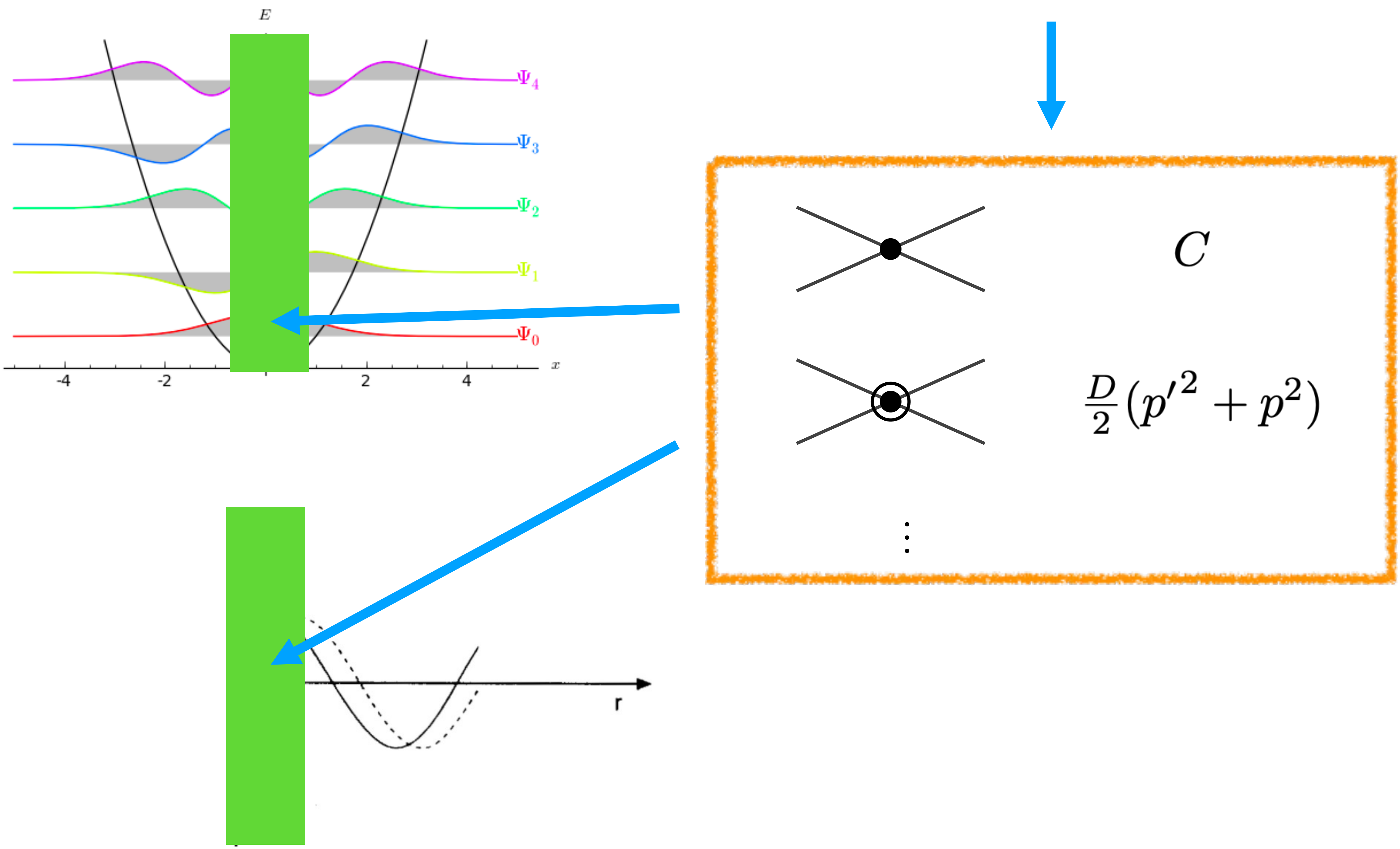


- Both wfs must match at the edge of the intrinsic potential V_i
- To construct outside wfs, detail of V_i does not matter ⇒ use EFT !

Contact EFT

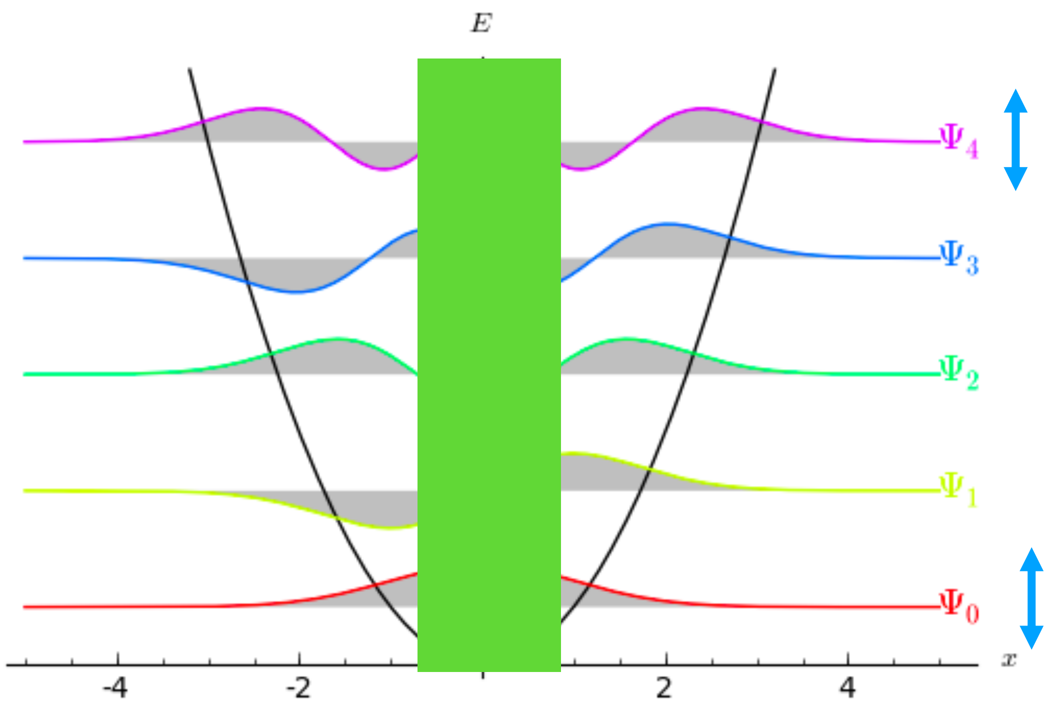
- Use *NN* as example

$$\mathcal{L}_{NN} = -\frac{1}{2}C_s(N^\dagger N)^2 - \frac{1}{2}C_t(N^\dagger\vec{\sigma}N)^2 + \cdots$$



“Manifold” of EFTs

Li, Yu, Peng, Lyu, BwL, PRC 104, 044001



$$\mathcal{L}_{NN} = -\frac{1}{2}C_s(E_4)\,(N^\dagger N)^2 - \frac{1}{2}C_t(E_4)\,(N^\dagger\vec{\sigma}N)^2 + \cdots$$

$$\vdots$$

$$\mathcal{L}_{NN} = -\frac{1}{2}C_s(E_0)\,(N^\dagger N)^2 - \frac{1}{2}C_t(E_0)\,(N^\dagger\vec{\sigma}N)^2 + \cdots$$

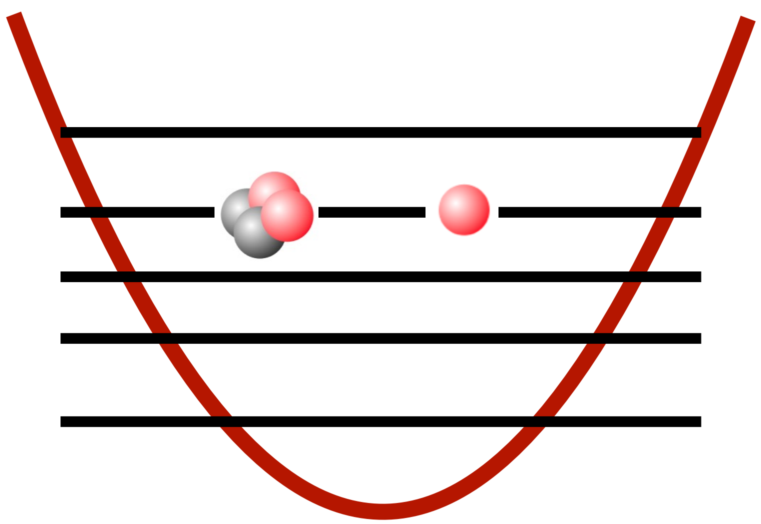
- An EFT for small momentum fluctuations around each eigen energy; no matter how high the energy is!

$$k\cot\delta = \alpha_0(\mathcal{E}_r) + \alpha_1(\mathcal{E}_r)(E-\mathcal{E}_r) + \alpha_2(\mathcal{E}_r)(E-\mathcal{E}_r)^2 + \cdots$$

- Weak predictive power, but it’s OK

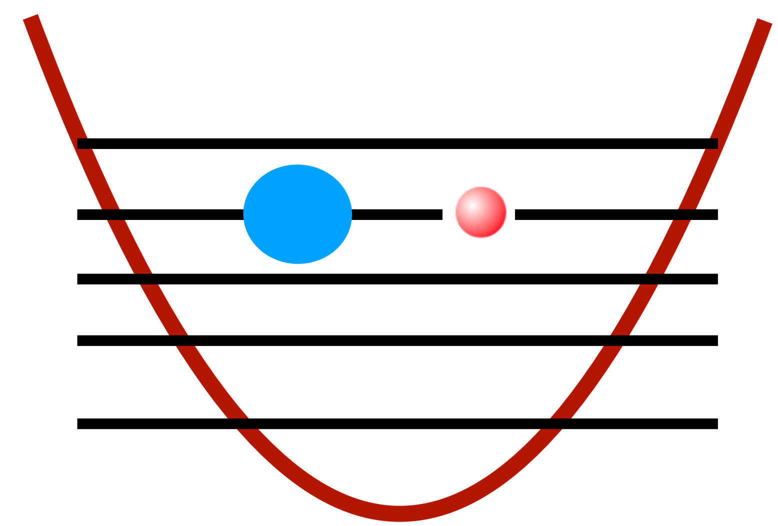
Recipe

V_{real} + ab initio many-body [Hard]



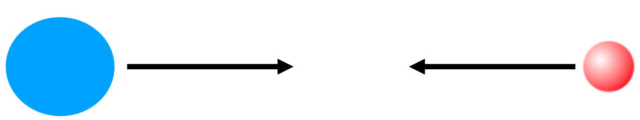
Energy levels ⇒

V_{EFT} two-body bound [Easy]

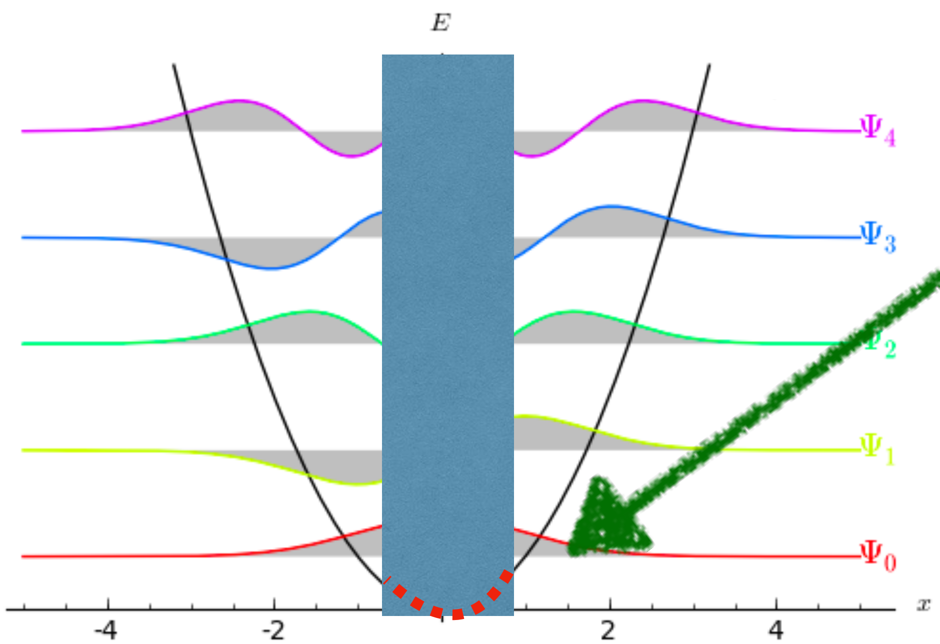


- Fix C_0, C_2, … of EFT

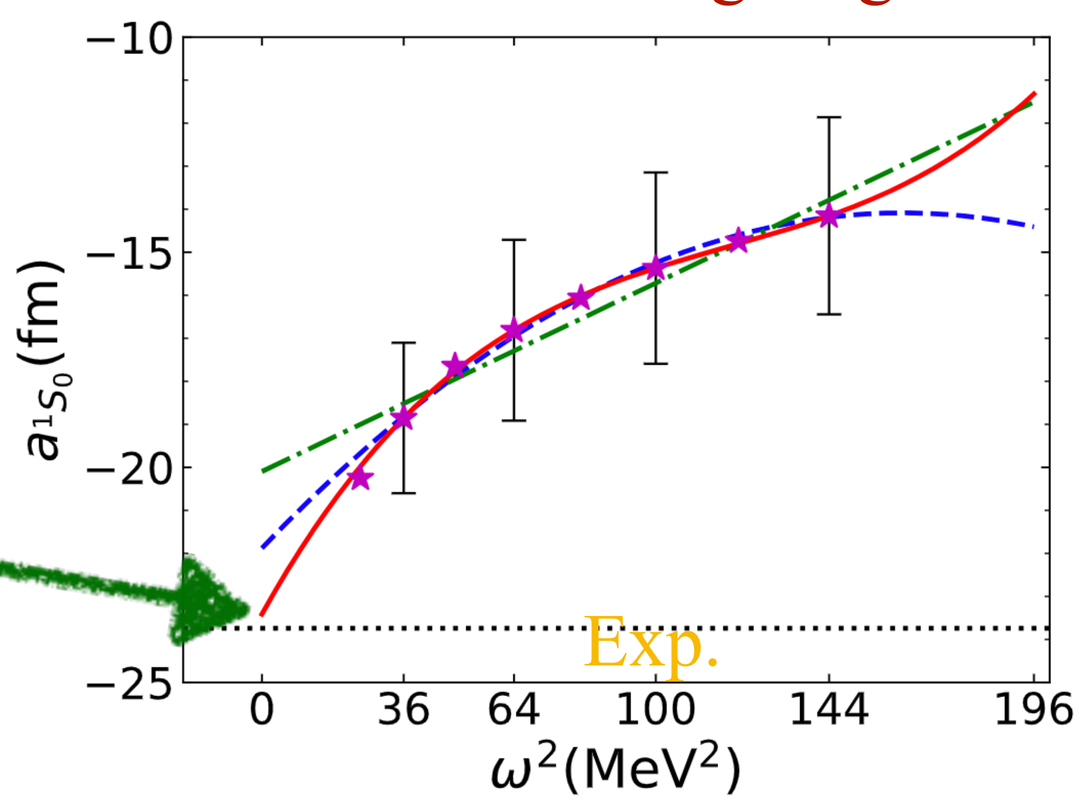
V_{EFT} two-body scattering [Easy]



How EFT helps?



- Previous works exact for $\hbar\omega \rightarrow 0$
  no error estimation for finite $\hbar\omega$
- Systematic approximation of EFT helps extrapolation to $\hbar\omega \rightarrow 0$

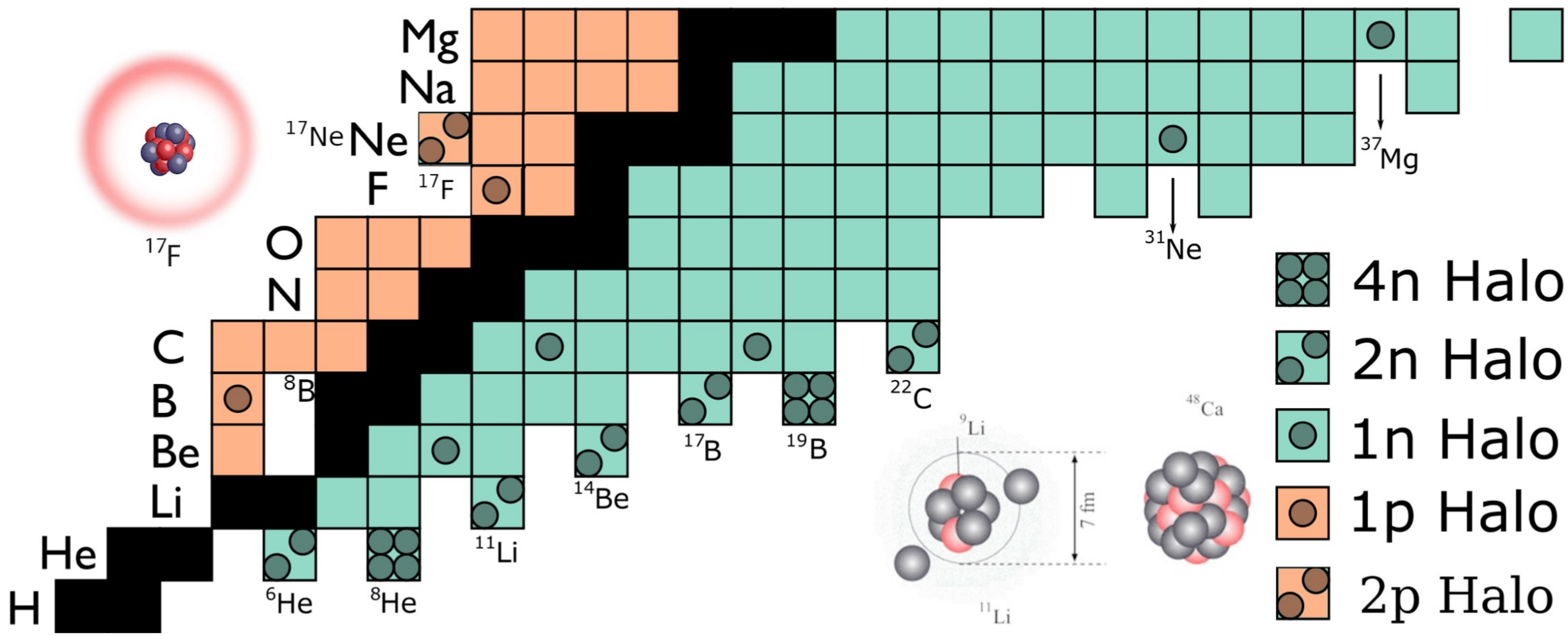


- Scattering amplitude for $\hbar\omega \rightarrow 0$

$$T(E;\omega^2) = T_\infty(E) + c_1\omega^2 + c_2\omega^4 + \cdots ,$$

(Li, Yu, Peng, Lyu, BwL, PRC 104, 044001)

Halo nuclei

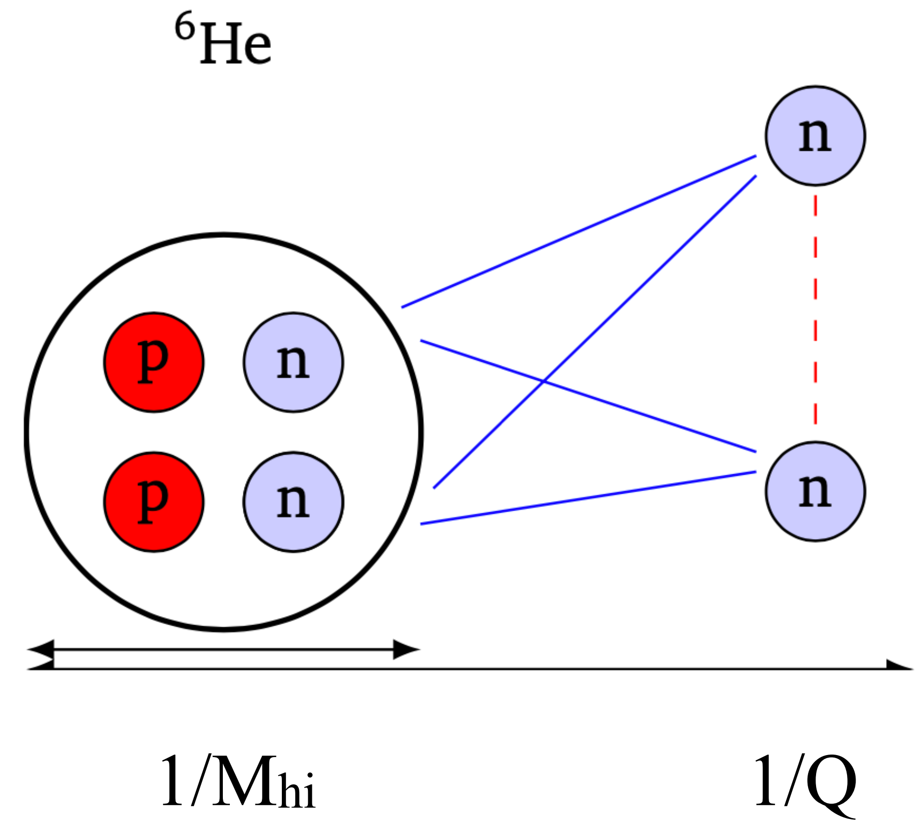


- Important for astrophysical reactions
- Large sizes, difficult for direct ab initio

Hammer, Ji & Phillips
JPG 44(2017) 103002

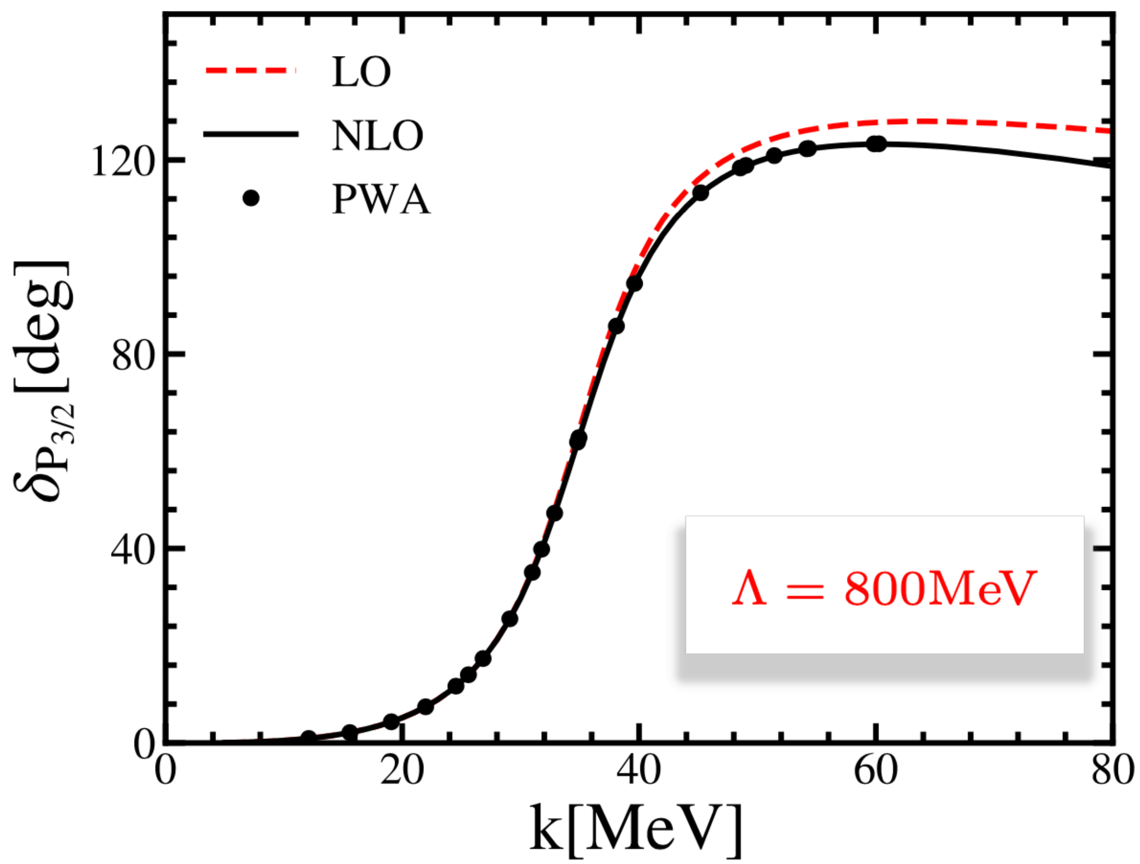
6He - 2n halo

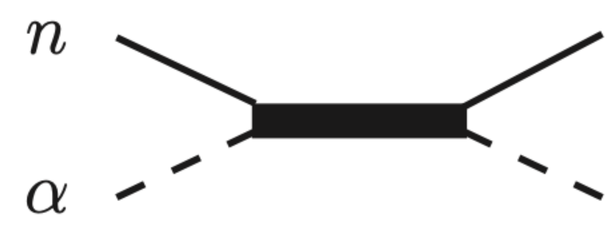
- Degrees of freedom: core + valence nucleons
- Systematic approximation: $(Q/M_{hi})^{\wedge n}$
- However, n-alpha interaction needs reconstruction



C. Ji, C. Elster, and D. R. Phillips, Phys. Rev. C 90, 044004 (2014)

n-alpha P-wave resonance





$$\mathrm{V} = \frac{y^2 p p'}{E + \Delta}$$

[Bertulani, Hammer, van Kolck NPA '02]

- Energy-dependent potential difficult to apply in many-body methods
- Negative-norm states

New EFT for n-alpha system

- Based on non-local potentials

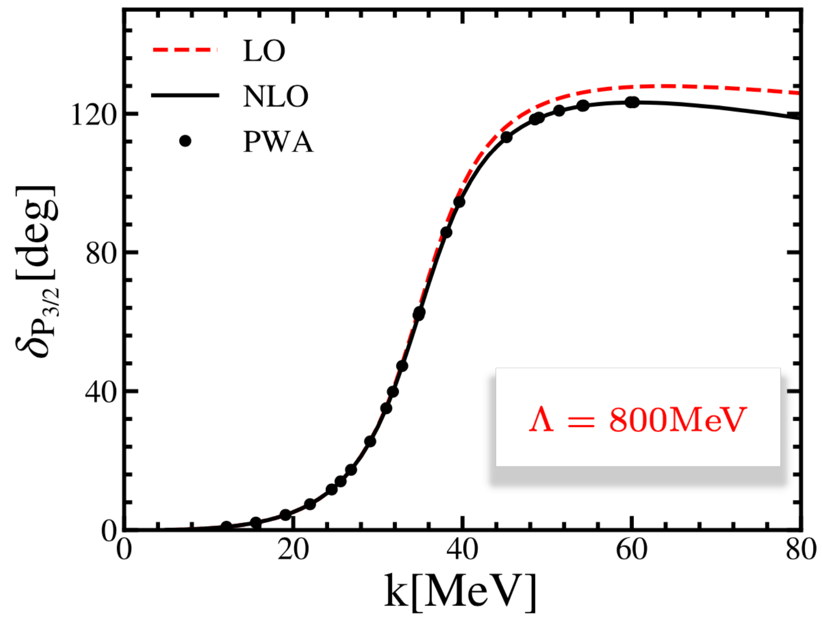
LO pot $$V^{(0)}(p',p) = -\frac{2\pi}{\mu}\frac{\lambda p' p}{\sqrt{p'^2+2\mu\Delta}\sqrt{p^2+2\mu\Delta}}$$

LO amp $$T^{(0)} = \frac{2\pi}{\mu}\frac{k^2}{-1/a_1 + r_1 k^2/2 - ik^3}$$

- Can produce effective-range expansion w/o modeling short-range physics

$$V^{(1)}_{g_2}(p',p) = \frac{2\pi}{\mu}\frac{g_2}{2}\frac{p'p\left(p'^2+p^2\right)}{\sqrt{p'^2+\gamma^2}\sqrt{p^2+\gamma^2}}$$

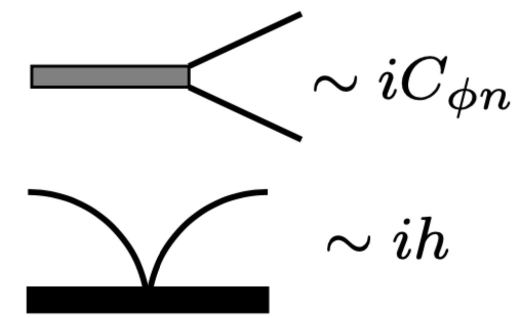
$$T^{(1)}(k) = -\frac{\mu}{2\pi}\frac{\left[T^{(0)}(k)\right]^2}{k^2}\left[\frac{\lambda_R^{(1)}}{\lambda_R^2}\gamma^2 - 2\mu\Delta^{(1)}\left(\lambda_R^{-1}+\frac{3}{2}\gamma\right) - \frac{g_{2R}}{\lambda_R}\gamma^5 + \left(\frac{\lambda_R^{(1)}}{\lambda_R^2} - \frac{g_{2R}}{\lambda_R}\gamma^2\left(\lambda_R^{-1}+\gamma\right)\right)k^2 - \frac{g_{2R}}{\lambda_R^2}k^4\right]$$

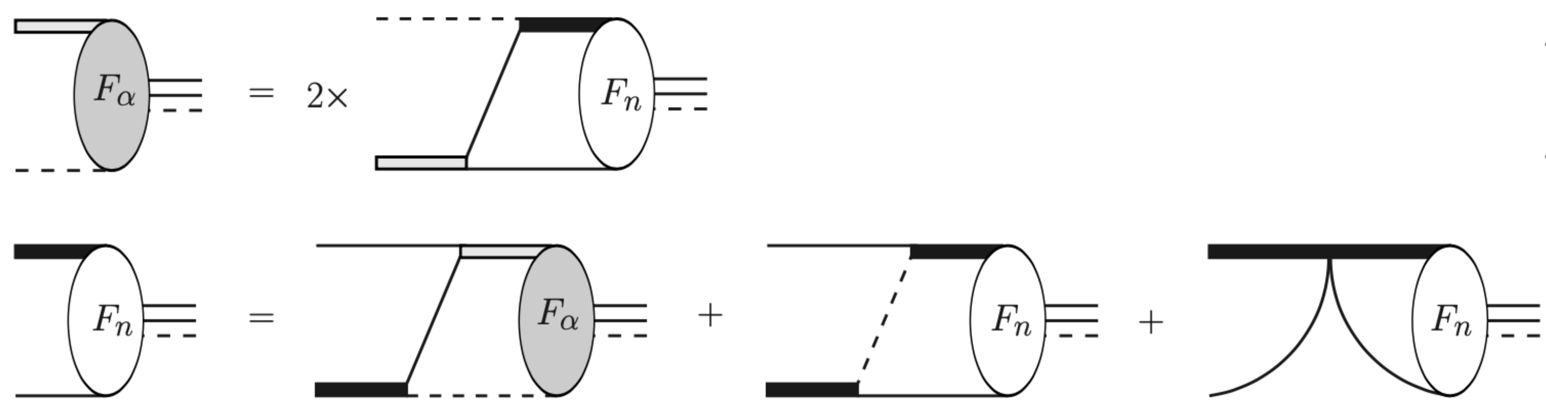


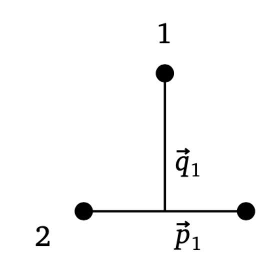
Li, Lyu, Ji & BwL (arxiv 2303.17292)

Faddeev eqn for 6He

- LO: $n-\alpha$ P-wave interaction, nn and $nn\alpha$
- Three-body force at LO
  to eliminate the cutoff dependence
- Solve Faddeev equation







Faddeev components:
F_α: α as spectator
F_n: neutron as spectator

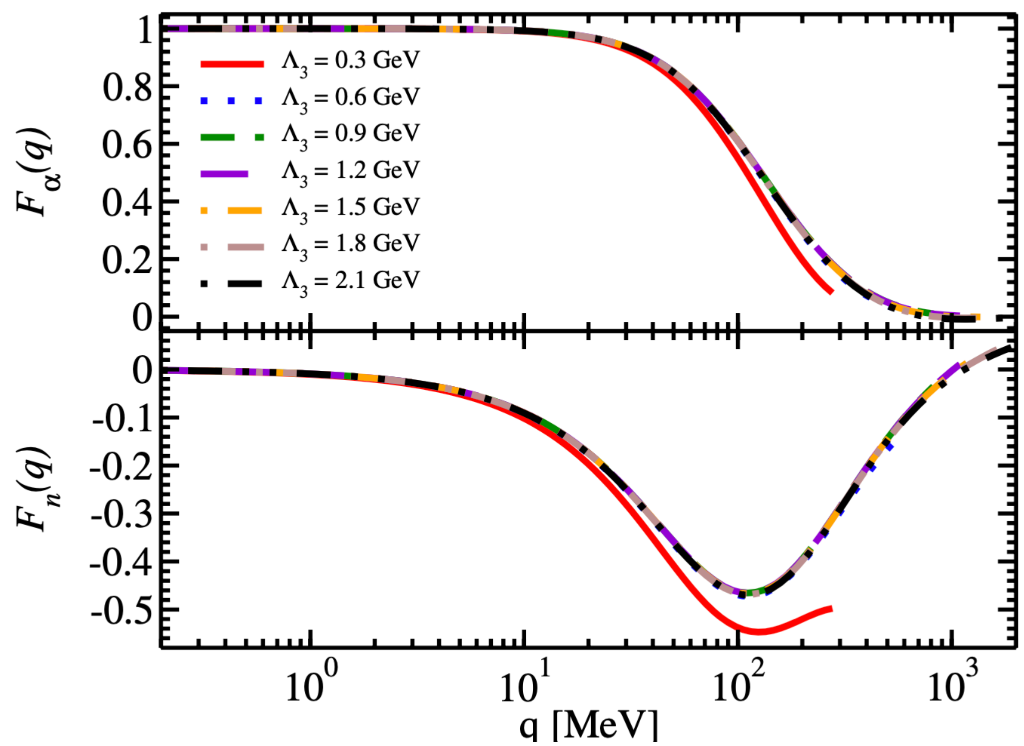
$$F_\alpha(q) = 8\pi \int_0^{\Lambda_3} q'^2 dq' \, X_{n\alpha}(q', q; B_3) \, D_{n\alpha}(\kappa_1) \, F_n(q') \,,$$

$$F_n(q) = 4\pi \int_0^{\Lambda_3} q'^2 dq' \, X_{n\alpha}(q, q'; B_3) \, D_{nn}(\kappa_0) \, F_\alpha(q')$$

$$+ 4\pi \int_0^{\Lambda_3} q'^2 dq' \left[X_{nn}(q, q'; B_3) + \frac{qq'}{\Lambda_3^4} H_0(\Lambda_3) \right] D_{n\alpha}(\kappa_1) \, F_n(q')$$

Wave func comparison

Li, Lyu, Ji & BwL (arxiv 2303.17292)



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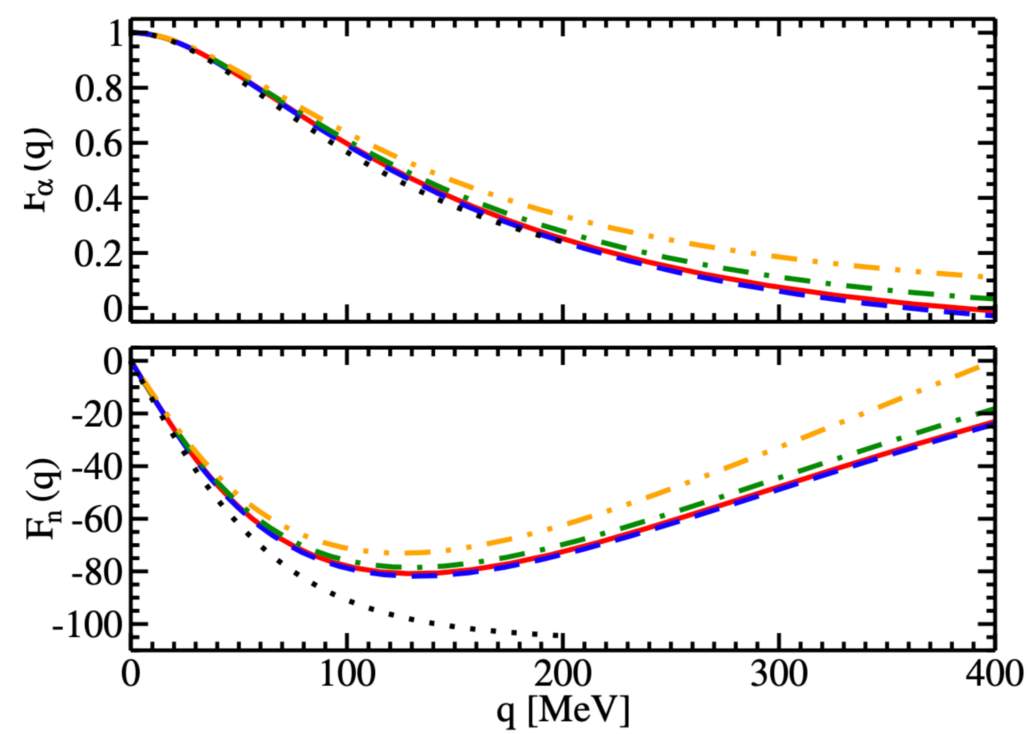

- Rapid convergence w/ momentum cutoff
- Need smaller model space

Summary

- Extracting reaction info from energy levels of trapped nucleons w/ energy-dependent EFT
- Halo EFT He isotope chain → neutron-rich nuclei
- Stage for EW physics in nuclei