Recent Progress in Nuclear Lattice Effective Field Theory

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(Nuclear Lattice EFT Collaboration)

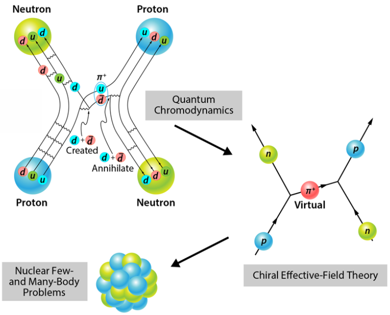
2nd $0\nu\beta\beta$ Workshop, Zhu-Hai, 2023-05-21

Chiral effective field theory

Chiral EFT: The low-energy equivalence of the QCD

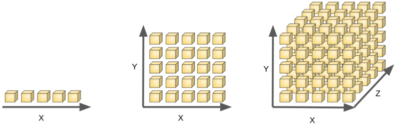
Weinberg (1979,1990,1991), Gasser, Leutwyler (1984,1985)

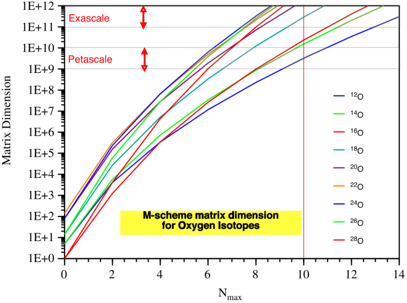
- Proton (uud), neutron (udd), pion ($u\bar{d}$)
- Spontaneously broken chiral symmetry: $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \to \mathrm{SU}(2)_V$
- Goldstone theorem implies a light pion: Long-range part of the nuclear force
- Contact terms: Short-range part of the nuclear force
- Hard scale: $\Lambda_\chi \sim 1$ GeV: Chiral EFT works for momentum $Q \ll \Lambda_\chi$



Quarks confined in nucleons and pions

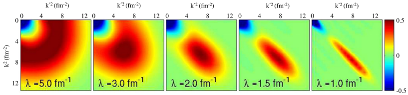
Exponential increase of resources



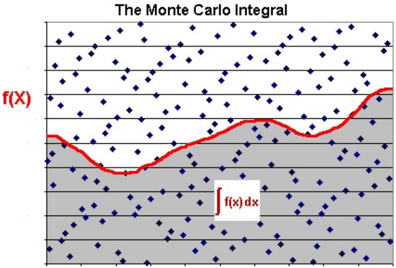


PRC 101, 014318 (2020)

Solution 1: Reduce effective Hilbert space with SRG evolution



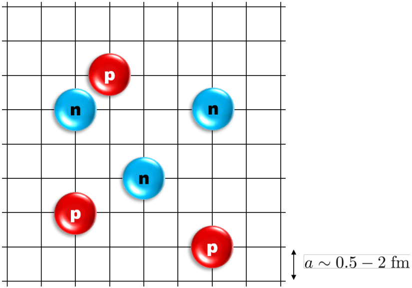
Solution 2: Monte Carlo algorithms



Lattice EFT = Chiral EFT + Lattice + Monte Carlo

Review: Dean Lee, Prog. Part. Nucl. Phys. 63, 117 (2009),
Lähde, Meißner, "Nuclear Lattice Effective Field Theory", Springer (2019)

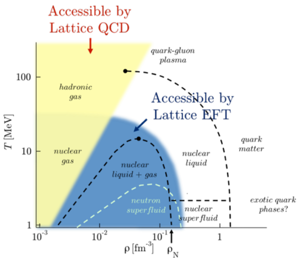
- Discretized chiral nuclear force
- Lattice spacing $a \approx 1$ fm $= 620$ MeV (∼chiral symmetry breaking scale)
- Protons & neutrons interacting via **short-range, δ-like** and **long-range, pion-exchange** interactions
- Exact method, polynomial scaling ($\sim A^2$)

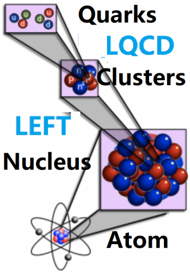


Lattice adapted for nucleus

Comparison to Lattice QCD

	LQCD	LEFT
degree of freedom	quarks & gluons	nucleons and pions
lattice spacing	$\sim$0.1 fm	$\sim$1 fm
dispersion relation	relativistic	non-relativistic
renormalizability	renormalizable	effective field theory
continuum limit	yes	no
Coulomb	difficult	easy
accessibility	high T / low ρ	low T / $\rho_{\rm sat}$
sign problem	severe for $\mu > 0$	moderate





Euclidean time projection

- Get *interacting g. s.* from imaginary time projection:

$$|\Psi_{g.s.}\rangle \propto \lim_{\tau\to\infty} \exp(-\tau H)|\Psi_A\rangle$$

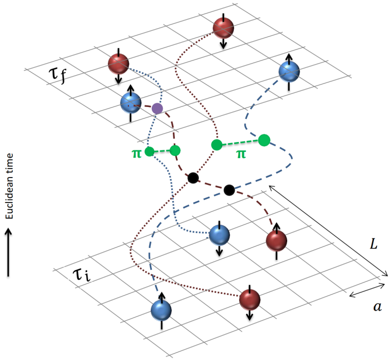
with $|\Psi_A\rangle$ representing A *free* nucleons.

- Expectation value of any operator $\mathscr{O}$:

$$\langle O\rangle = \lim_{\tau\to\infty} \frac{\langle\Psi_A|\exp(-\tau H/2)\mathscr{O}\exp(-\tau H/2)|\Psi_A\rangle}{\langle\Psi_A|\exp(-\tau H)|\Psi_A\rangle}$$

- τ is discretized into time slices:

$$\exp(-\tau H) \simeq \left[:\exp(-\frac{\tau}{L_t}H):\right]^{L_t}$$



All possible configurations in $\tau \in [\tau_i, \quad \tau_f]$ are sampled.
Complex structures like nucleon clustering emerges naturally.

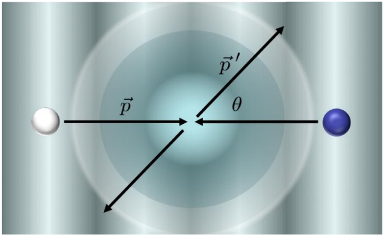
N-N scattering in the center of mass frame

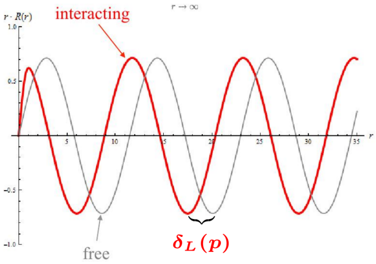
For scattering in the **continuum**:

- Partial wave expansion: $\psi(\boldsymbol{r}) = \sum_{J=0}^{\infty} \psi_J(r) P_J(\cos\theta)$
- Asymptotically ($r > R_{\text{force}}$): $\psi_J(r) \to A h_J^{+}(kr) - B h_J^{-}(kr)$
- Phase shift: $e^{2i\delta} = B/A$

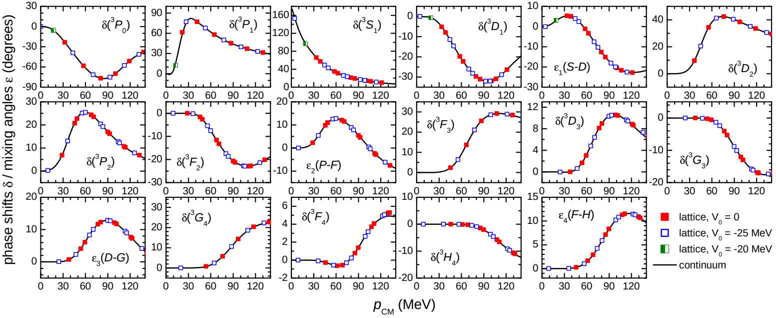
For scattering in a **finite volume**:

- Luescher's formula:
  $e^{2i\delta} = \frac{Z_{00}(1;q^2)+i\pi^{3/2}q}{Z_{00}(1;q^2)-i\pi^{3/2}q}, \quad \boldsymbol{q} = \frac{2\pi \boldsymbol{n}}{L}$
  $Z_{00}(s,q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\boldsymbol{n}} \frac{1}{(\boldsymbol{n}^2-\boldsymbol{q}^2)^s}$
- Standard tool in LQCD
  Beane et al., Int. J. Mod. Phys. E 17(2008) 1517
- Not applicable in LEFT: noisy data, need higher precision





Auxiliary field method: Restoration of rotational symmetry



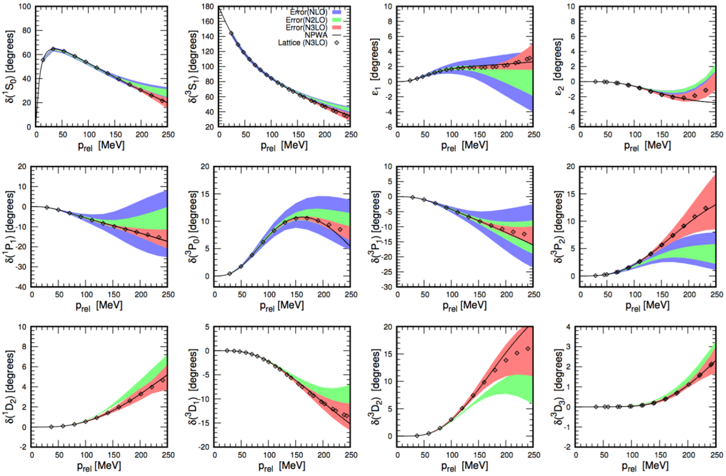
- Phase shifts and mixing angles for a tensor pontential (toy model).

$$V(r) = C\left\{1 + \frac{r^2}{R_0^2}\left[3(\hat{\boldsymbol{r}}\cdot\boldsymbol{\sigma}_1)(\hat{\boldsymbol{r}}\cdot\boldsymbol{\sigma}_2) - \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right]\right\}\exp\left(-\frac{r^2}{2R_0^2}\right)$$

- Continuum results by solving the Lippmann-Schwinger equation.

Auxiliary potential method: Lu et al., Phys. Lett. B 760 (2016) 309;
Generalized to arbitrary number of channels: Bovermann et al., Phys. Rev. C 100, 064001 (2019).

Chiral nuclear force up to N^{3}LO: fit on the lattice



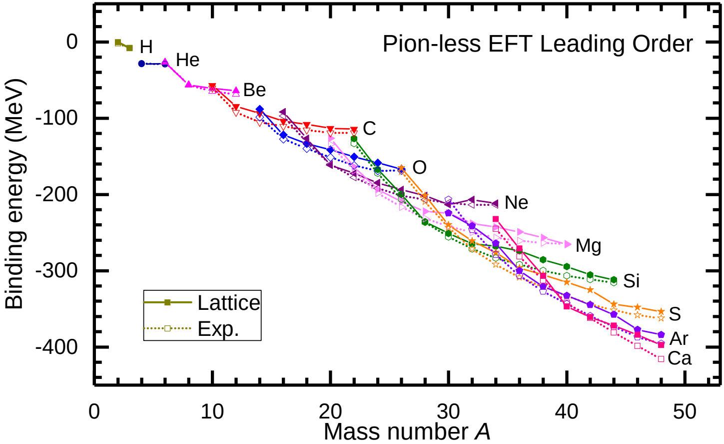
fit to N^{2}LO: Alarcon, Du, Klein, Lahde, Lee, Ning Li, B.L., Luu, Meissner, EPJA 53, 83 (2017)

fit to N^{3}LO: Ning Li, Elhatisari, Epelbaum, Lee, B.L., Meissner, PRC 98, 044002 (2018)

Nuclear binding from a SU(4) nuclear force

Ab initio calculation = **precise nuclear force** + **exactly solving Schrödinger equations**

In full quantum Monte Carlo simulations, **equations are solved exactly**
A simple SU(4) interaction (central force only!) can describe the nuclear binding



"Essential elements for nuclear binding", Lu et al., Phys. Lett. B 797, 134863 (2019)

Pinhole algorithm: Schematic plot

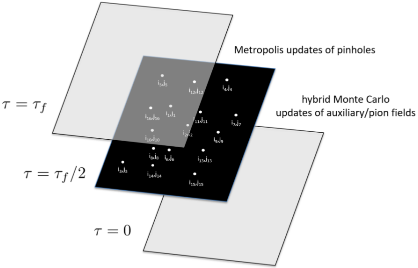
In terms of auxiliary fields, the amplitude Z can be written as a path-integral,

$$Z_{f,i}(i_1, j_1, \cdots, i_A, j_a; \boldsymbol{n}_!, \cdots \boldsymbol{n}_A; L_t)$$
$$= \int \mathscr{D}s\mathscr{D}\pi \langle \Psi_f(s,\pi) | \rho_{i_1,j_1,\cdots,i_A,j_A}(\boldsymbol{n}_1, \cdots, \boldsymbol{n}_A) | \Psi_i(s,\pi) \rangle.$$

We generate a combined probability distribution

$$P(s, \pi, i_1, j_1, \cdots, i_A, j_a; \boldsymbol{n}_!, \cdots \boldsymbol{n}_A) = |\langle \Psi_f(s,\pi) | \rho_{i_1,j_1,\cdots,i_A,j_A}(\boldsymbol{n}_1, \cdots, \boldsymbol{n}_A) | \Psi_i(s,\pi) \rangle|$$

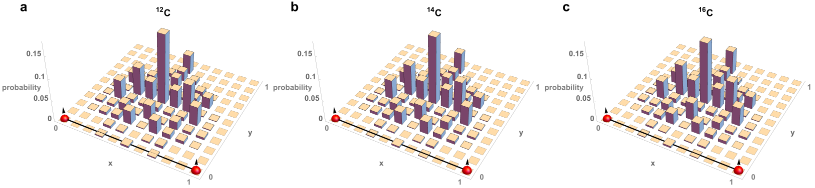
by updating both the auxiliary fields and the pinhole quantum numbers.

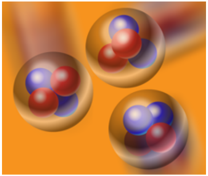


Elhatisari et al., Phys. Rev. Lett. 119, 222505 (2017)

Pinhole algorithm: α-cluster geometry in carbon isotopes

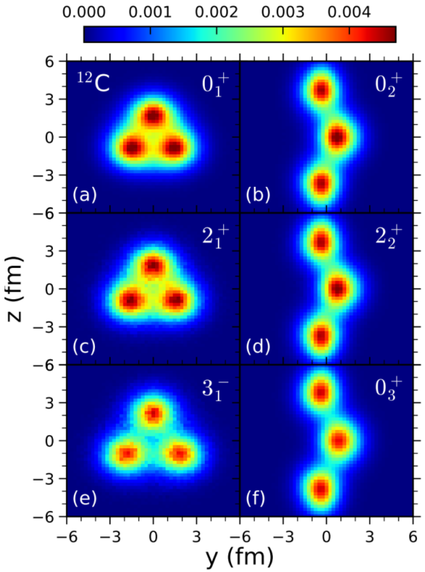
Positions of 3rd proton relative to the other two in ^{12,14,16}C





- **Hoyle state**: Triple-α resonance, essential for creating ^{12}C in stars (Hoyle, 1954). *Fine-tuning for life?*
  Epelbaum et al., Phys. Rev. Lett. 106, 192501 (2011)
- **Perspective:** important many-body correlations, understand **internal structures** of ground and excited states by ***ab initio* calculations**.
- **Next step:** high-precision chiral interaction $\longrightarrow$ EM form factors, shape coexistence, clustering, ...
  Elhatisari et al., Phys. Rev. Lett. 119, 222505 (2017)

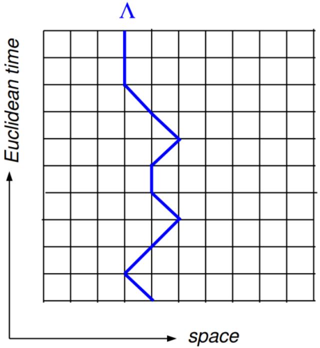
Tomographic scan of ^{12}C

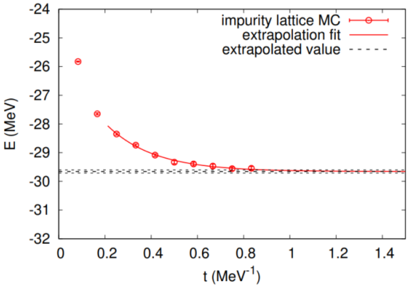


- Structure of ^{12}C states are full of complexity and duality (clustering v.s. mean-field)
- we provide the first model-independent tomographic scan of the three-dimensional geometry of the nuclear states of 12C using the ab initio framework of nuclear lattice EFT.
- 0_1^+: ground state, 0_2^+: Hoyle state

Shihang Shen et al., Nat. Commun. 14 (2023) 2777

Impurity Lattice Monte Carlo for Hypernuclei





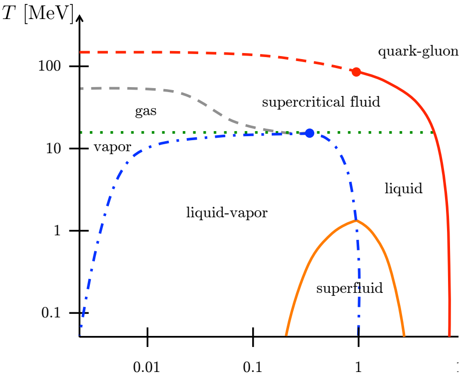
- In hypernuclei the number of hyperons is usually much smaller than the number of nucleons.
- The hyperons can be integrated out and simulated with worldlines.
- Tested for ${}^{5}_{\Lambda}$He with a simplified nucleon-hyperon interaction.
- Perspective: Towards an *ab initio* simulation with realistic *N*-*N* and *N*-Λ interactions of hypernuclei.

"Impurity Lattice Monte Carlo for Hypernuclei", Dillon Frame et al., Eur. Phys. J. A 56:248 (2020)

Towards finite-temperature: *Ab initio* nuclear thermodynamics

Bing-Nan Lu, Ning Li, Serdar Elhatisari, Dean Lee, Joaquín E. Drut, Timo A. Lähde, Evgeny Epelbaum, Ulf-G. Meißner,
Phys. Rev. Lett. 125, 192502 (2020).

Nuclear phase diagram (theoretical)



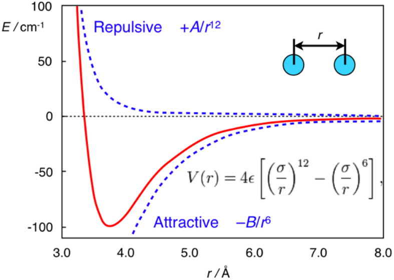
Strong interaction matter

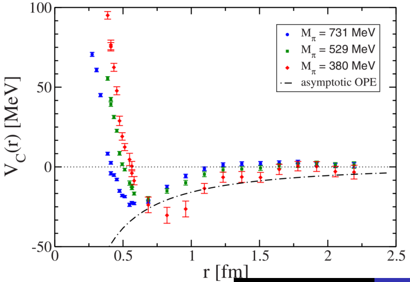
Water liquid-gas phase transition

occurs at order $T \sim 300$ K~0.03 eV

Nuclear liquid-gas phase transition

occurs at order $T \sim 10^{11}$ K~10 MeV





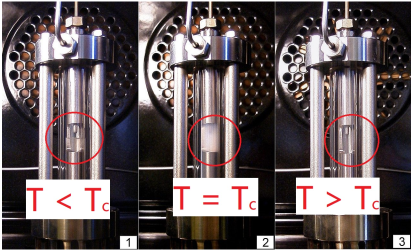
Upper: Alcohol near T_c

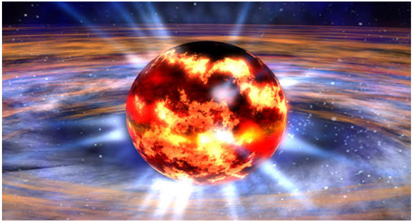
- Critical Opalescence

Lower: Neutron star cooling

- Energies carried off by neutrinos
- Absorption by neutron matter

"In a newly born neutron star, neutrinos are temporarily trapped in the opaque stellar core, but they diffuse out in a matter of seconds, leaving most of their energy to heat the matter in the core to more than **500 billion kelvin**. Over the next million years, the star mainly cools by emitting more neutrinos."
PRL 120, 182701 (2018)





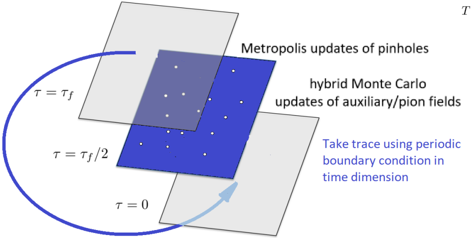
Lattice EFT simulation:
Ma, Yuan-zhuo et al., in preparation

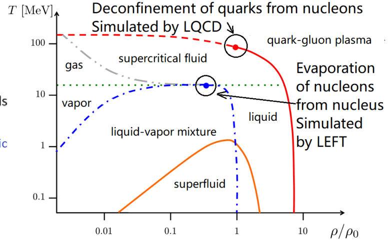
Simulate canonical ensemble with pinhole trace algorithm

- All we need: **partition function** $Z(T,V,A) = \sum_k \langle \exp(-\beta H) \rangle_k$, sum over all othonormal states in Hilbert space $\mathscr{H}(V,A)$.
- The basis states $|\boldsymbol{n}_1, \boldsymbol{n}_2, \cdots, \boldsymbol{n}_A\rangle$ span the whole A-body Hilbert space. $\boldsymbol{n}_i = (\boldsymbol{r}_i, s_i \sigma_i)$ consists of **coordinate, spin, isospin** of i-th nucleon.
- Cannonical partition function can be expressed in this complete basis:

$$Z_A = \mathrm{Tr}_A [\exp(-\beta H)] = \sum_{\boldsymbol{n}_1, \cdots, \boldsymbol{n}_A} \int \mathscr{D}s \mathscr{D}\pi \langle \boldsymbol{n}_1, \cdots, \boldsymbol{n}_A | \exp[-\beta H(s,\pi)] | \boldsymbol{n}_1, \cdots, \boldsymbol{n}_A \rangle$$

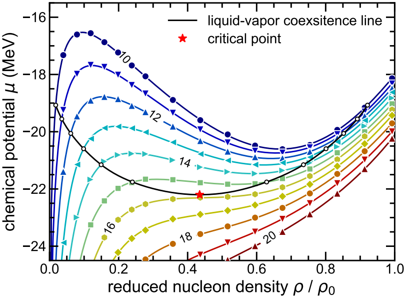
- **Pinhole algorithm** + **periodicity in β** = **Pinhole trace**
- Apply **twisted boundary condition** in 3 spatial dimensions to remove finite volume effects. Twist angle θ averaged with MC.





PRL 125, 192502 (2020)

Critical point: Compare with experiment



Lu et al., Phys. Rev. Lett. 125, 192502 (2020)

- **Pressure** $p = \int \rho \, d\mu$ along every isotherm (Gibbs-Duhem equation).
- Extract T_c, P_c and ρ_c of neutral symmetric nuclear matter by numerical interpolation.
- Uncertainties estimated by adding **noise** and repeat the calculation.
- **Experimental values** and **mean field** results taken from
  Elliott et al., Phys. Rev. C 87, 054622 (2013)

	This work	Exp.	RMF(NLSH)	RMF(NL3)
T_c(MeV)	15.80(3)	17.9(4)	15.96	14.64
P_c(MeV/fm^3)	0.260(3)	0.31(7)	0.26	0.2020
ρ_c(fm^{-3})	0.089(1)	0.06(1)	0.0526	0.0463
ρ_0 (fm^{-3})	0.205(0)	0.132		
ρ_c/ρ_0	0.43	0.45		

Lu et al., Phys. Rev. Lett. 125, 192502 (2020)

Towards realistic interactions: Perturbative Quantum Monte Carlo Method for Nuclear Physics

Bing-Nan Lu, Ning Li, Serdar Elhatisari, Yuan-Zhuo Ma, Dean Lee, Ulf-G. Meißner,

Phys. Rev. Lett. 128, 242501 (2022).

Reyleigh-Schrödinger perturbation theory

For a Hamiltonian $H = H^{(0)} + \lambda V_C$,

- In conventional stationary perturbation theory:

$$E_i = E_i^{(0)} + \lambda \langle \Psi_i^{(0)} | V_C | \Psi_i^{(0)} \rangle + \lambda^2 \sum_{k \neq 0} \frac{\langle \Psi_k^{(0)} | V_C | \Psi_i^{(0)} \rangle}{E_k^{(0)} - E_i^{(0)}} + \mathscr{O}(\lambda^3)$$

$$|\Psi_i\rangle = |\Psi_i^{(0)}\rangle + \lambda \sum_{k \neq 0} \frac{\langle \Psi_k^{(0)} | V_C | \Psi_i^{(0)} \rangle}{E_k^{(0)} - E_i^{(0)}} |\Psi_k^{(0)}\rangle + \mathscr{O}(\lambda^2)$$

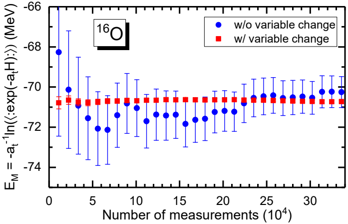
- However, in projection Monte Carlo algorithms,

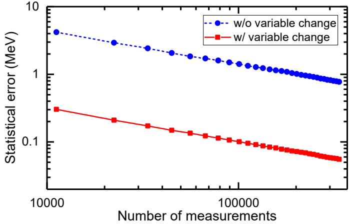
$$E_{\text{g.s.}} = \lim_{\tau \to \infty} \exp(-\tau H) |\Psi_T\rangle$$

  targets the ground states (or low-lying states) directly.

- In projection methods, excited states are very expensive. $\leftarrow$ required for 2nd order energy or 1st order wave function!
- All projection QMC calculations use at most first order perturbation theory.

Perturbative Monte Carlo (ptQMC) algorithm





$$E = E_0 + \delta E_1 + \delta E_2 + \mathscr{O}(V_C^3),$$

where the partial energy contributions

$$E_0 = \langle\Psi_0|(K+V)|\Psi_0\rangle/\langle\Psi_0|\Psi_0\rangle,$$
$$\delta E_1 = \langle\Psi_0|V_C|\Psi_0\rangle/\langle\Psi_0|\Psi_0\rangle,$$
$$\delta E_2 = (\langle\Psi_0|V_C|\delta\Psi_1\rangle - \delta E_1 \mathrm{Re}\langle\delta\Psi_1|\Psi_0\rangle)/\langle\Psi_0|\Psi_0\rangle$$

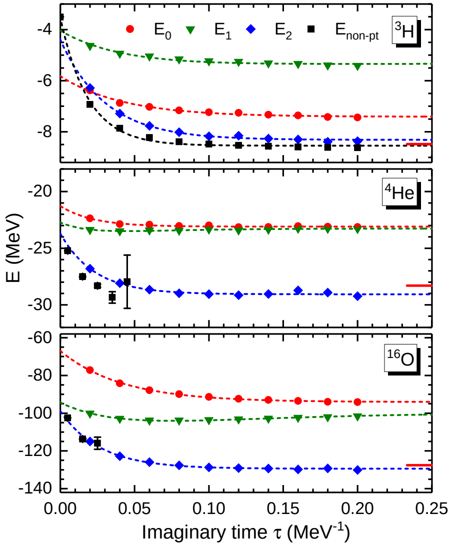
Perturbed amplitude can be transformed into an approximate Gaussian integral with a variable change. Note that

$$\langle\exp(\sqrt{-a_t C}s\rho)\rangle_T \approx \exp(\sqrt{-a_t C}s\langle\rho\rangle_T)$$

$$\mathscr{M}_k(O) = \langle\Psi_T|M_0^{L_t/2}OM_0^{L_t/2-k}MM_0^{k-1}|\Psi_T\rangle$$
$$= \int \mathscr{D}cP(c+\bar{c})\langle\cdots O\cdots M(s_k, c+\bar{c})\cdots\rangle_T$$

Left panel: Test calculation of the transfer matrix energy $E = -\ln\langle:\exp(-a_tH):\rangle/a_t$
Lu *et al.*, PRL 128, 242501 (2022)

Perturbative Monte Carlo with realistic chiral interaction



- We split $H = H_0 + (H - H_0)$ and perform perturbative calculations
- E_0 is the ground state of H_0
- $E_1 = E_0 + \delta E_1$ is the first order corrected energy
- $E_2 = E_1 + \delta E_2$ is the second order corrected energy
- $E_{\mathrm{non-pt}}$ is the exact solution ($\sim$infinite order)
- Red bars on the right: Experiments
  Lu *et al.*, PRL 128, 242501 (2022)

For ^{4}He and ^{16}O, sign problem prevent us from going to large τ, resulting in large statistical errors. But no need to worry,

Perturbation theory can save us!

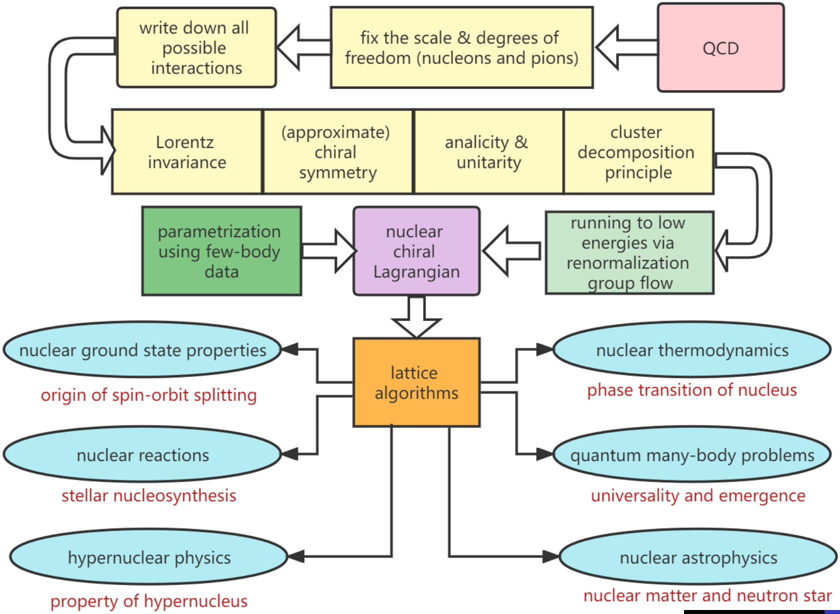
Numerical results for several light nuclei

Table: The nuclear binding energies at different orders calculated with the ptQMC. $E_{\rm exp}$ is the experimental value. All energies are in MeV. We only show statistical errors from the MC simulations.

	E_0	δE_1	E_1	δE_2	E_2	$E_{\rm exp}$
^{3}H	−7.41(3)	+2.08	−5.33(3)	−2.99	−8.32(3)	−8.48
^{4}He	−23.1(0)	−0.2	−23.3(0)	−5.8	−29.1(1)	−28.3
^{8}Be	−44.9(4)	−1.7	−46.6(4)	−11.1	−57.7(4)	−56.5
^{12}C	−68.3(4)	−1.8	−70.1(4)	−18.8	−88.9(3)	−92.2
^{16}O	−94.1(2)	−5.6	−99.7(2)	−29.7	−129.4(2)	−127.6
^{16}O†	−127.6(4)	+24.2	−103.4(4)	−24.3	−127.7(2)	−127.6
^{16}O‡	−161.5(1)	+56.8	−104.7(2)	−22.3	−127.0(2)	−127.6

Realistic N^{2}LO chiral Hamiltonian fixed by few-body data + perturbative quantum MC simulation = nice agreement with the experiments
Excellent predicative power $\Longrightarrow$ Demonstration of both **nuclear force model** and **many-body algorithm**

Summary of lattice EFT



Perspective: Open problems

- Can we have a general renormalization scheme for non-relativistic many-body systems?
- Can we design a sign-problem-free Monte Carlo algorithm for realistic nuclear interactions?
- Can we understand the universality and emergence in nuclear physics from *ab initio* calculations?
- Can we connect the many-body EFT to the underlying theory of QCD? (e.g., nuclear physics at different m_π)

THANK YOU FOR YOUR ATTENTION