EFFECTIVE FIELD THEORY FOR NEUTRINOLESS DOUBLE BETA DECAY

Jordy de Vries
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EFTs and neutrino mass: an old story

- Neutrinos are formally massless in the SM → but neutrino oscillations ….
- Easy fix: Insert gauge-singlet right-handed neutrino υ_R

$$\mathscr{L} = -y_\nu \bar{L}\tilde{H}\nu_R \qquad\qquad y_\nu \sim 10^{-12} \to m_\nu \sim 0.1\,\mathrm{eV}$$

- Nothing really wrong with this…. **But nothing forbids a Majorana Mass term**

$$\mathscr{L} = -y_\nu \bar{L}\tilde{H}\nu_R - M_R \nu_R^T C \nu_R$$

'Everything that is not forbidden is compulsary'

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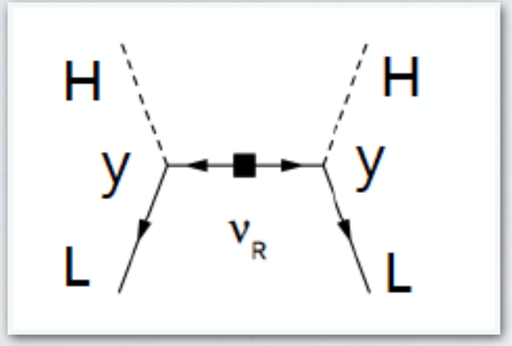
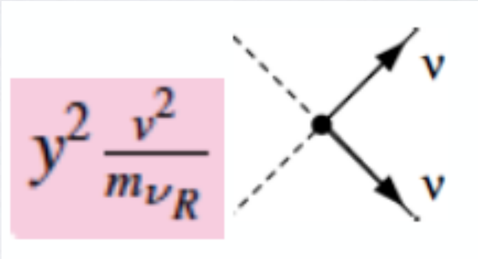
- If M_R is significantly larger than active neutrino masses (< eV) : **see-saw mechanism**
- In case of 1 left and 1 right-handed neutrino: $m_1 \simeq \left| \dfrac{y_\nu^2 v^2}{m_R} \right| \qquad m_2 \simeq m_R$
- The mass eigenstates are Majorana states $\nu_i^c = \nu_i$
- Violation of **lepton number** by two units **—> neutrinoless double beta decay**

EFT point of view

- Integrating out heavy states leads to local operator





- Obtain the single dimension-5 SMEFT operator

Neutrino Majorana mass

Weinberg '79

$$\mathscr{L}_5 = \frac{c_5}{\Lambda}\left(L^T C \tilde{H}\right)(\tilde{H}^T L) \longrightarrow \mathscr{L}_5 = c_5 \frac{v^2}{\Lambda} \nu^T C \nu \qquad c_5 = y_\nu^2$$

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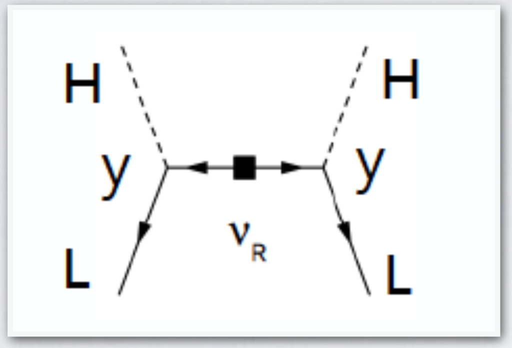

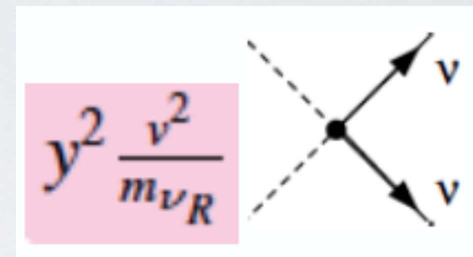

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Dimension-five	Dimension-seven	Dimension-nine
$\mathcal{L}_5 = \frac{c_5}{\Lambda}(L^T C\tilde{H})(\tilde{H}^T L)$ • One operator • Induces Majorana mass	Lehman '14 [illegible] • 12 ΔL=2 operators	Li et al '20 Many many terms 19 4-quark 2-lepton operators after EWSB Graesser et al '17 '18

Applications effective field theory in 0vbb

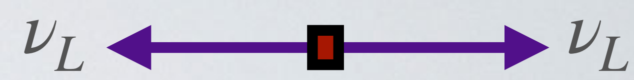
1. **Use EFT to scrutinize and guide nuclear calculations (focus here on Weinberg operator)**

2. **Investigate non-standard mechanisms (beyond Weinberg term)**

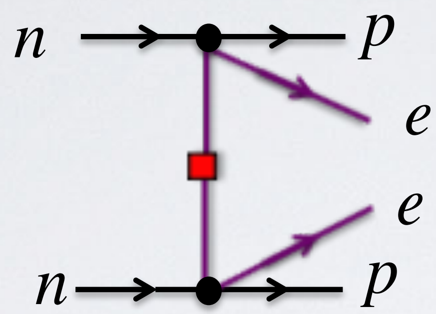
3. Use EFTs in presence of explicit light degrees of freedom (sterile neutrinos)

Leading-order transition currents

- Neutrinos are still degrees of freedom in low-energy chiral EFT



- Leads to ‘long-range’ nn → pp + ee



$$V_\nu \sim \frac{m_{\beta\beta}}{\mathbf{q}^2} \qquad \mathbf{q} \sim k_F \sim m_\pi$$

$$V_\nu = (2G_F^2 m_{\beta\beta})\tau_1^+\tau_2^+\frac{1}{\mathbf{q}^2}\left[(1+2g_A^2) + \frac{g_A^2 m_\pi^4}{(\mathbf{q}^2+m_\pi^2)}\right] \otimes \bar{e}_L e_L^c$$

- Note: the nucleons appear in a bound state and **q** is a loop momentum
- Then insert this into nuclear wave functions (from **nuclear many-body** methods)

$$A_\nu \sim \langle \Psi | V_\nu | \Psi \rangle$$

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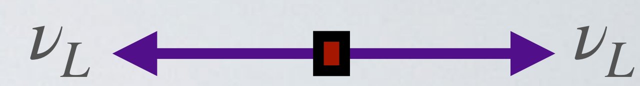

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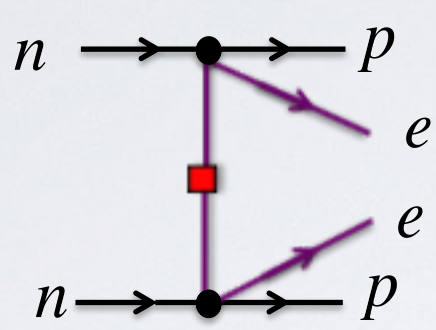

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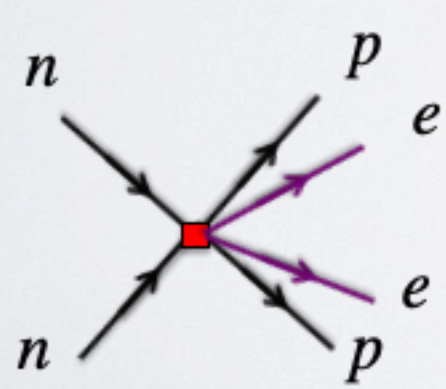

- Contributions from virtual hard neutrinos $\quad \mathbf{q} \sim \Lambda_\chi \sim 1\,\mathrm{GeV}$
- Naive-dimensional analysis tells us this is higher-order

$$V_\nu^{\mathrm{short}} \sim \frac{m_{\beta\beta}}{\Lambda_\chi^2} \ll V_\nu$$

Leading-order transition currents

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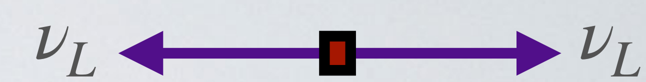

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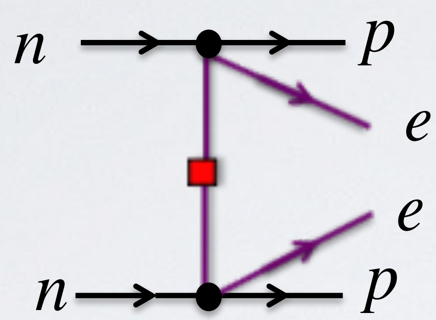
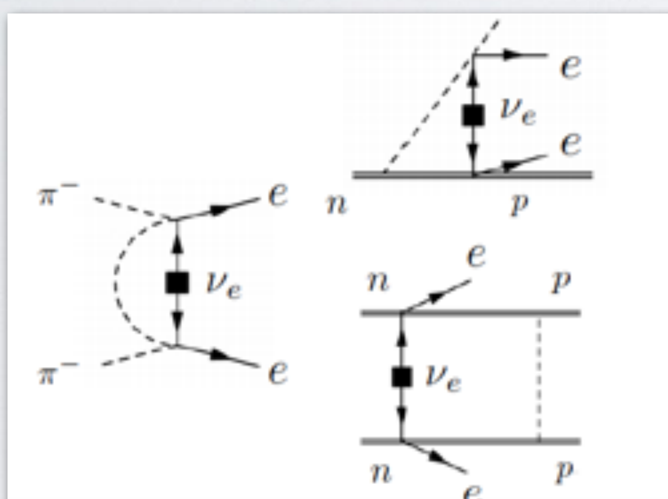


$$V_\nu \sim \frac{m_{\beta\beta}}{\mathbf{q}^2}$$

$$\mathbf{q} \sim k_F \sim m_\pi$$

- More contributions at higher orders in chiral perturbation theory

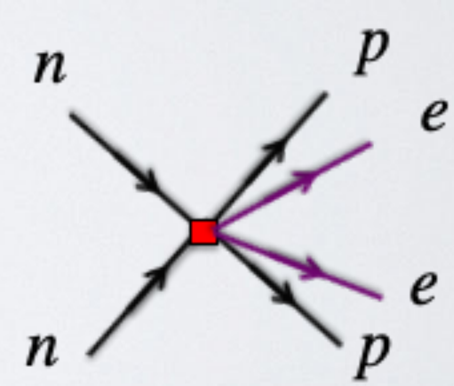


Cirigliano, Dekens, Mereghetti, Walker-Loud '17

- Loops at N2LO are divergent: come with counter terms

$$V_\nu^{\mathrm{N^2LO}} \sim \left(V_{\mathrm{finite}} + V_{UV} \log \frac{m_\pi^2}{\mu^2} + V_{\mathrm{CT}} \right) \otimes \bar{e}_L e_L^c$$

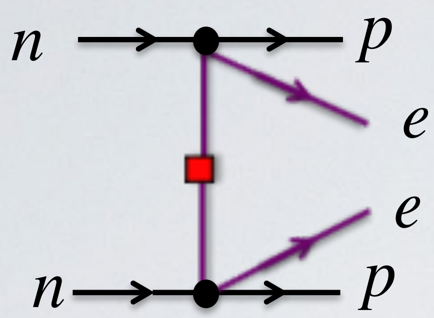
- Divergences absorbed by counter terms



- At higher orders also 'closure corrections' and three-body effects

e.g. Engel et al '18

Leading-order transition currents



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- Leading-order 0vbb current is very simple
- No unknown hadronic input ! Only unknown is $m_{\beta\beta}$

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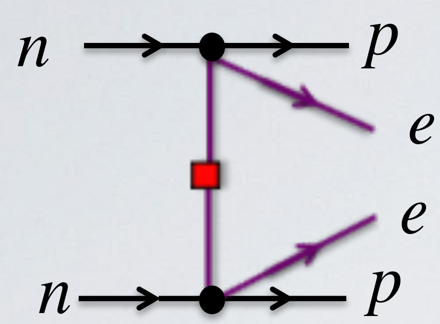

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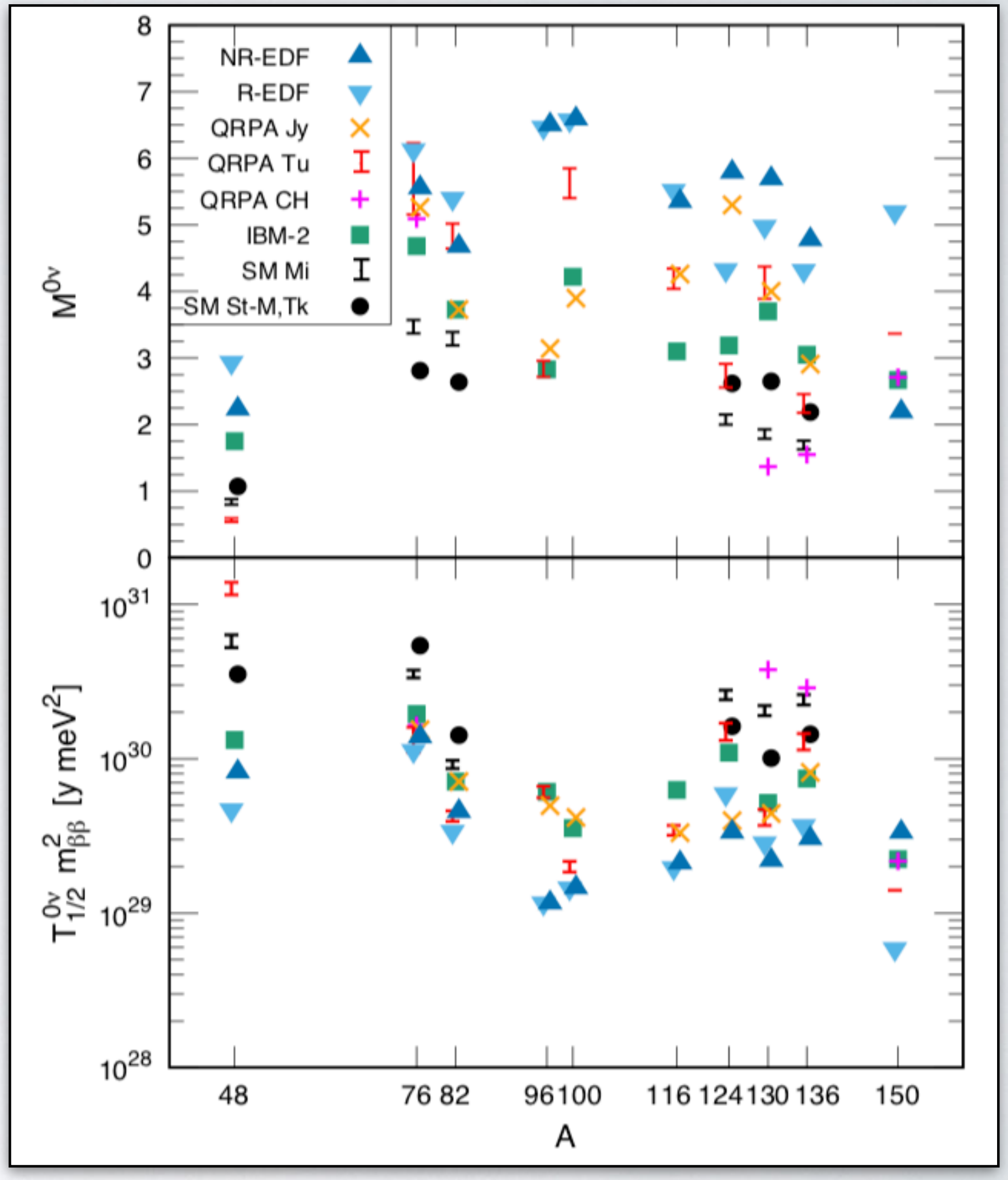
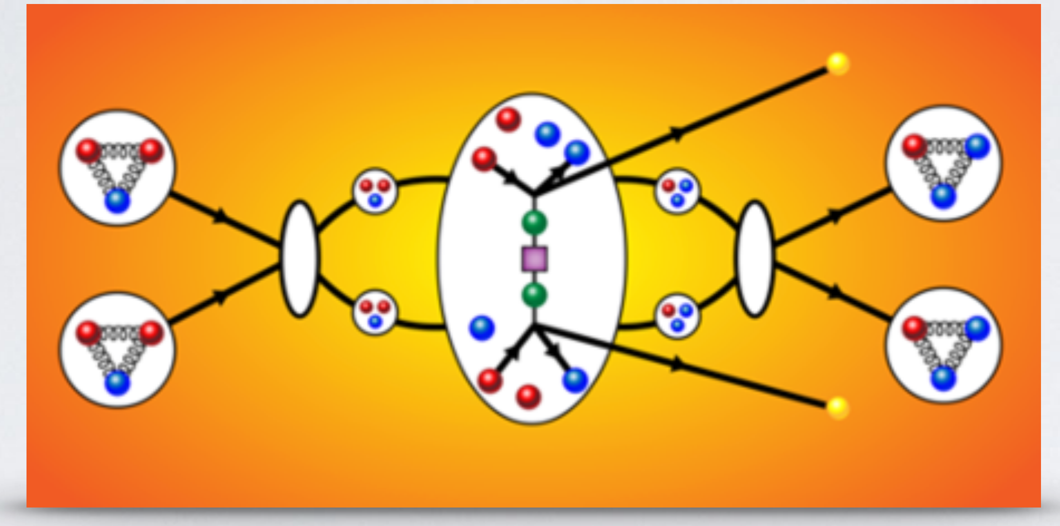
- Leading-order 0vbb current is very simple
- No unknown hadronic input ! Only unknown is $m_{\beta\beta}$
- Many-body methods disagree significantly
- Idea: see what happens for lighter systems
- **Not relevant for experiments but as a theoretical laboratory**



Engel-Menendez '16

Neutron-Neutron → Proton-Proton

- Study simplest nuclear process: nn → pp + ee



Cirigliano, Dekens, JdV, Graesser, Mereghetti, Pastore van Kolck, PRL '18

Cirigliano, Dekens, JdV, Hoferichter, Mereghetti PRL '21

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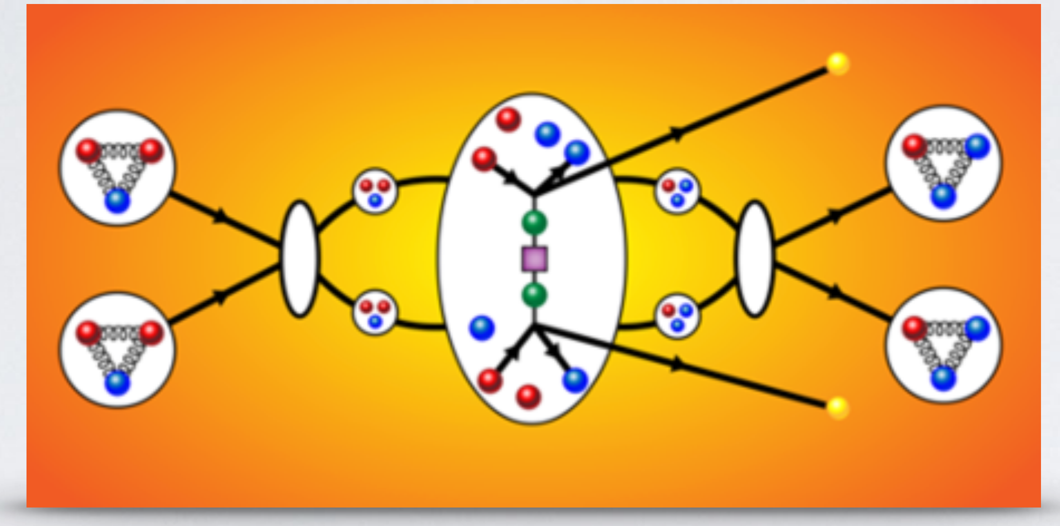
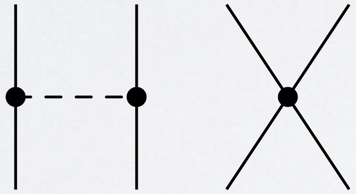


- Derive wave functions from chiral effective field theory $T = V + V G_0 T$

LO



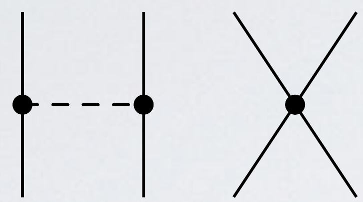
$$V_{\text{strong}} = C_0 - \frac{g_A^2}{4f_\pi^2}\frac{m_\pi^2}{\mathbf{q}^2 + m_\pi^2}$$

Weinberg 90' 91'

- To solve Schrodinger equation need regulator $V_{\text{strong}} \to e^{-p^6/\Lambda^6} \times V_{\text{strong}} \times e^{-p'^6/\Lambda^6}$

- **Dim-reg possible** as well but much more complicated.

Neutron-Neutron → Proton-Proton

LO 

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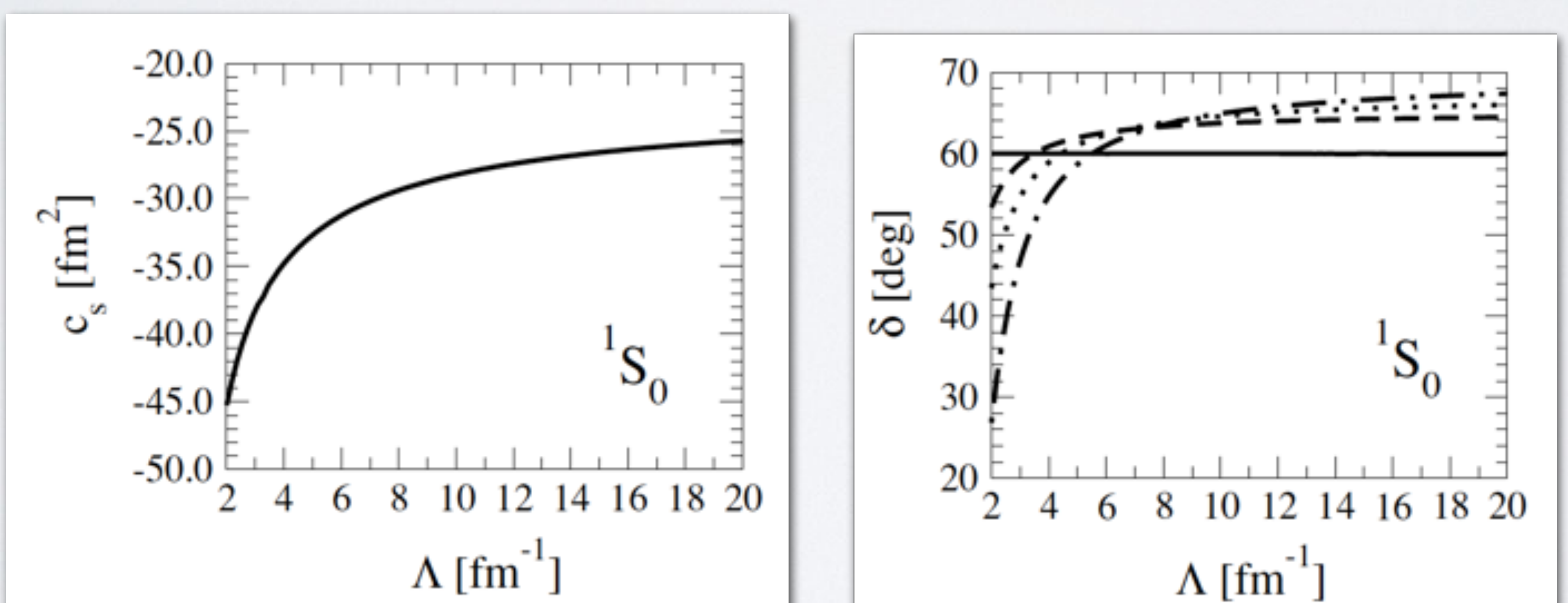
Weinberg 90' 91'

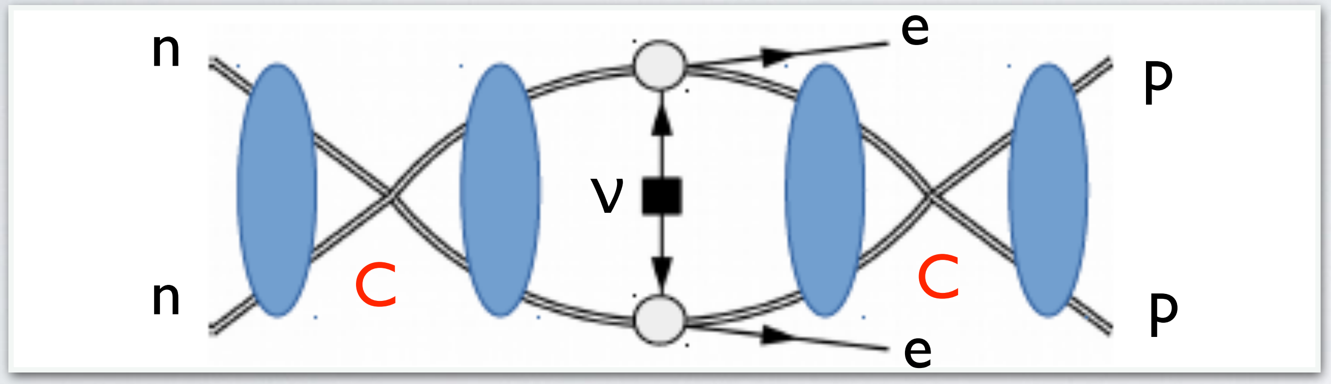
- Fit counter terms to nucleon-nucleon scattering lengths for each Λ
- Predict cross sections (phase shifts) for other energies.

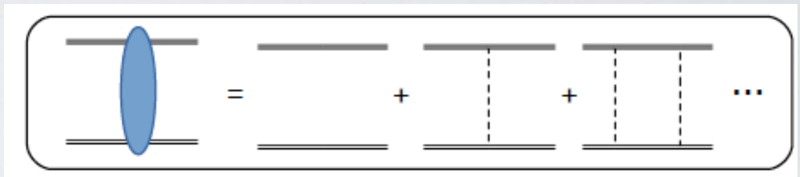


Nogga, Timmermans, van Kolck '05

Leading-order transition currents

- Insert long-distance neutrino exchange into scattering states





$$\sim (1+2g_A^2)\left(\frac{m_N C_0}{4\pi}\right)^2\left(\frac{1}{\epsilon}+\log\frac{\mu^2}{p^2}\right)$$

New divergences

Cirigliano, Dekens, JdV, Graesser, Mereghetti, Pastore, van Kolck PRL '18

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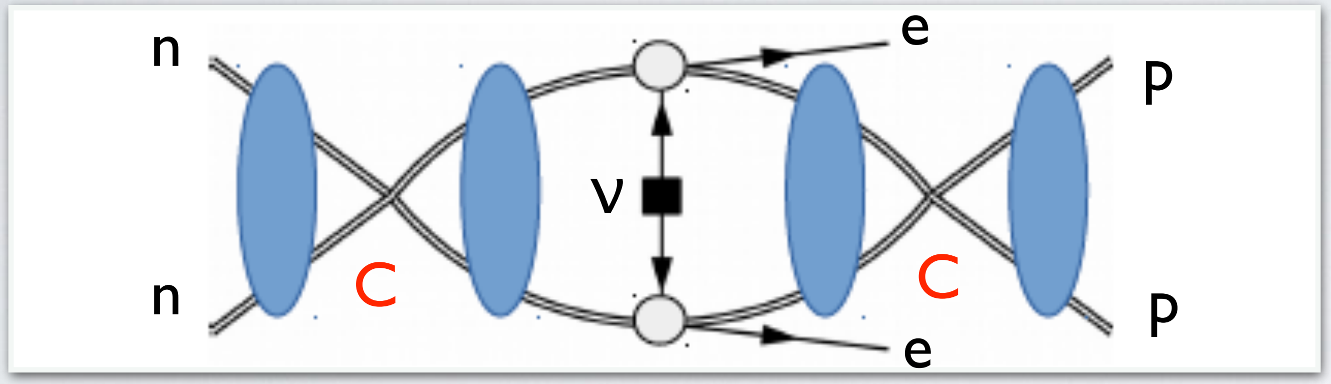
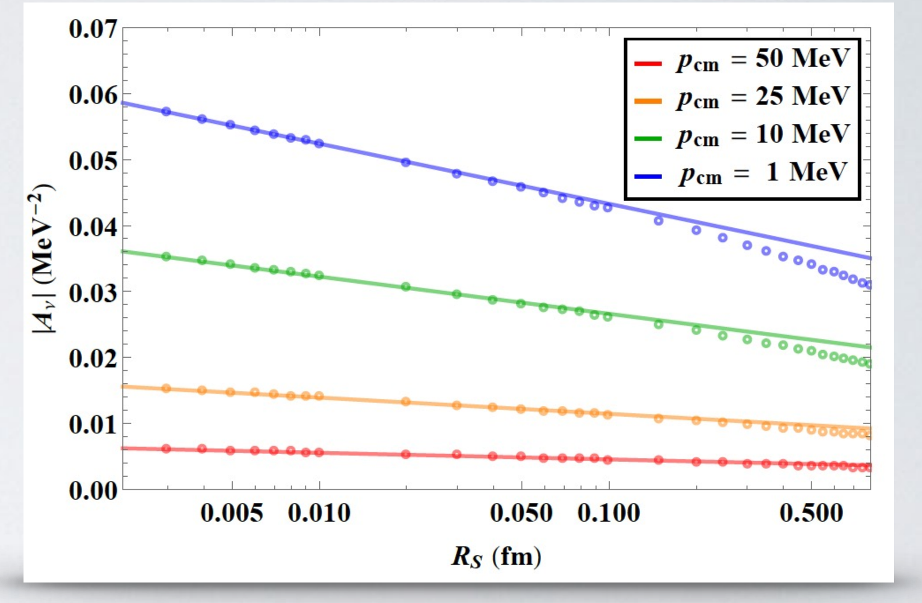




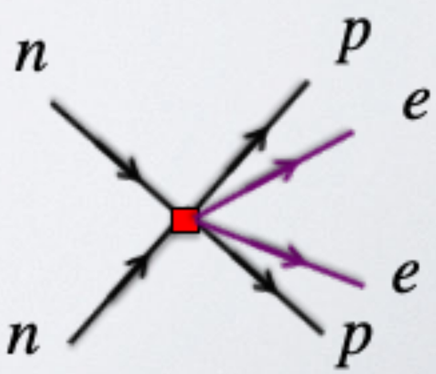
$$\sim (1+2g_A^2)\left(\frac{m_N C_0}{4\pi}\right)^2\left(\frac{1}{\epsilon}+\log\frac{\mu^2}{p^2}\right)$$

New divergences

- **Logarithmic regulator dependence**

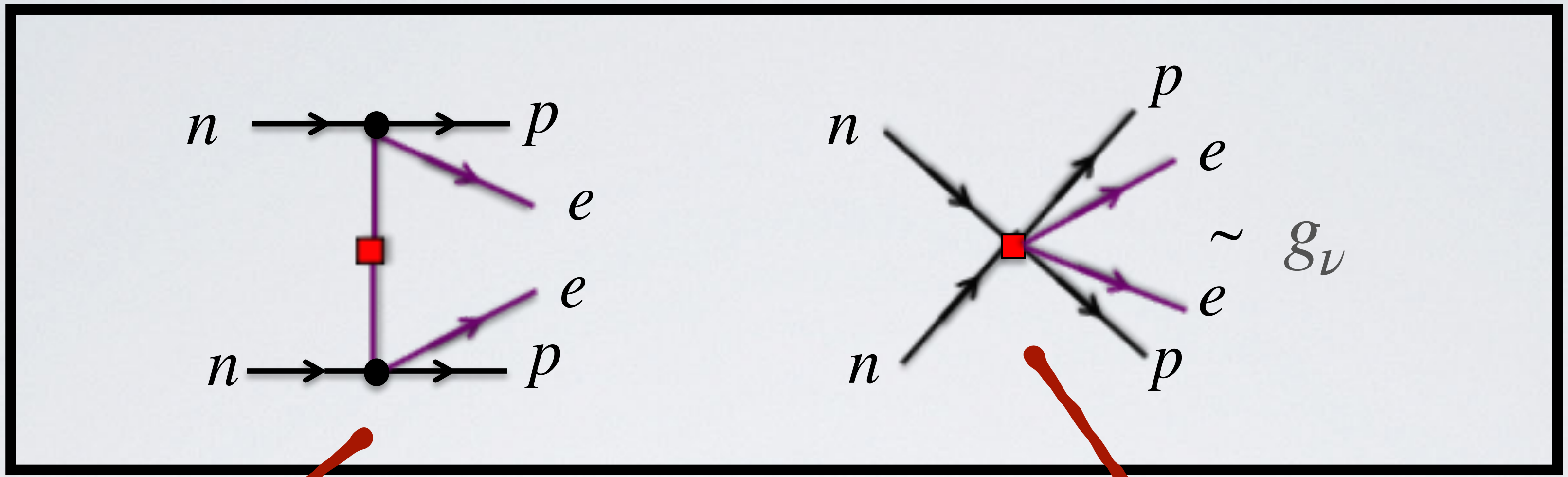
- Divergence indicates sensitivity to short-distance physics (hard-neutrino exchange)

- Requires a counter term: a short-range nn → pp + ee operator



Cirigliano, Dekens, JdV, Graesser, Mereghetti, Pastore, van Kolck PRL '18

A new leading-order contribution



'Long-range' neutrino-exchange

'Short-distance' neutrino exchange
required by renormalization of amplitude

- **Short-distance piece depends on unknown QCD matrix element** g_ν
- How to determine the value of this matrix element ? Obviously no data!
- **Lattice QCD can do this in the future. But not yet….**
- But solved already for the 'toy-problem'

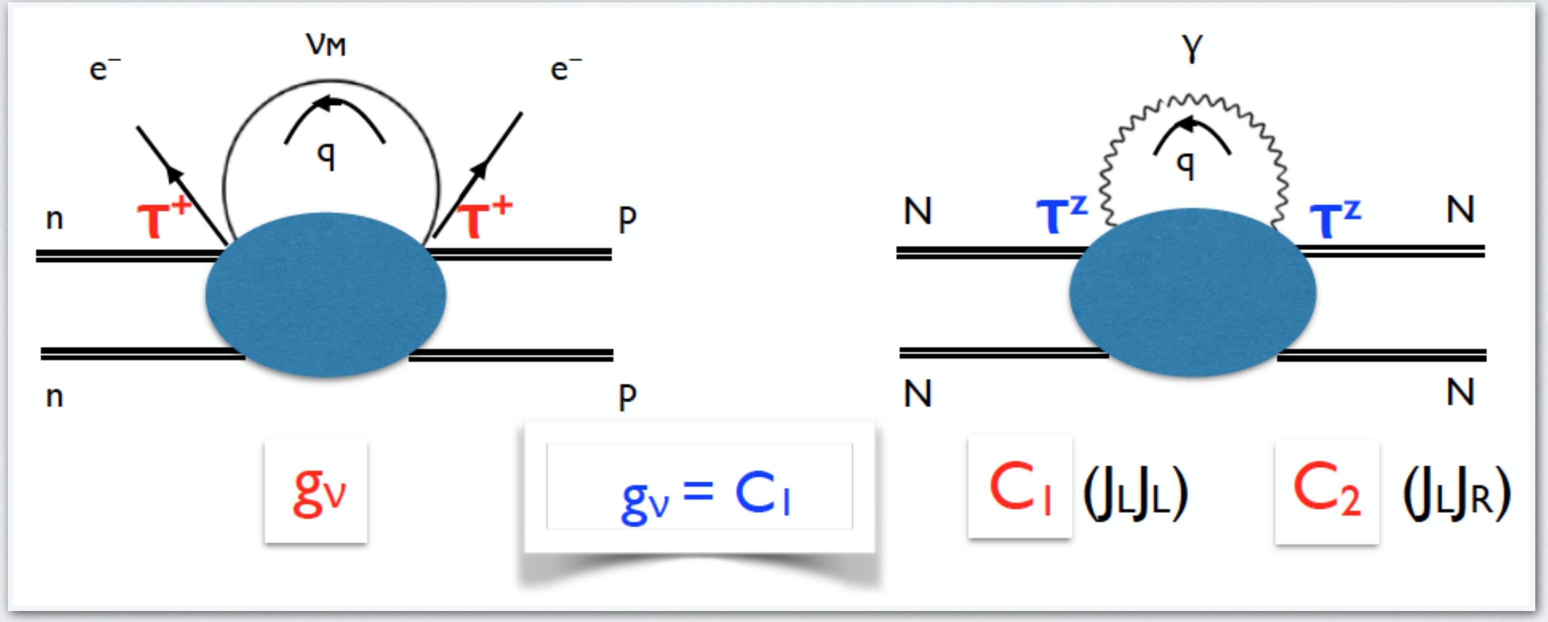
Davoudi, Kadam PRL '21 Briceno et al '19 '20

Tremendous progress for the 'toy-problem' $\pi^- + \pi^- \to e^- + e^-$

Tuo et al. '19; Detmold, Murphy '20 '22

A connection to electromagnetism

- A neutrino-exchange process looks like a photon-exchange process



Cirigliano et al '19

- Isospin-breaking nucleon-nucleon scattering data determines C_1+C_2
- Electromagnetism conserves parity (L + R) coupling and $g_\nu \sim C_1$ only

- Large-Nc arguments indicates $C_1 + C_2 \gg C_1 - C_2$ Richardson, Schindler, Pastore, Springer '21

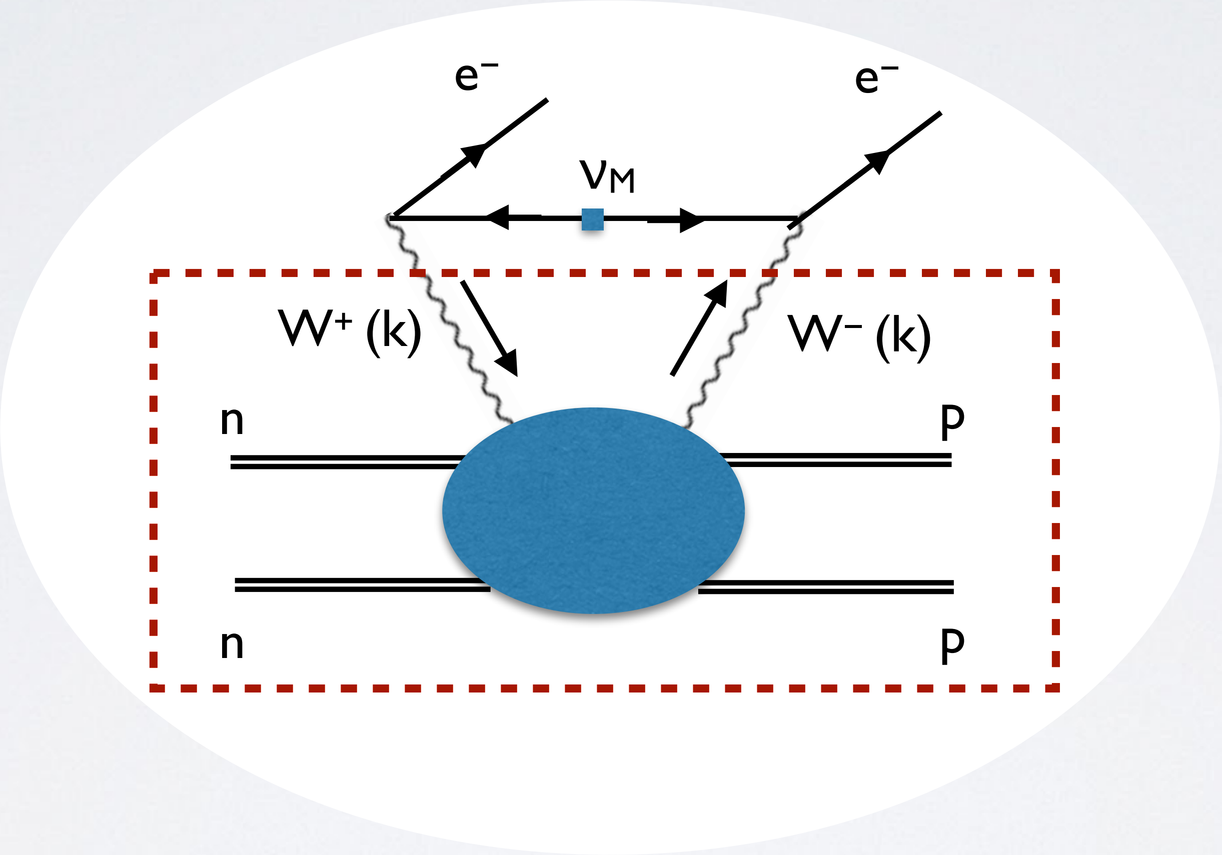
- **In this way, we can extract g_ν from data.**

An analytic approach

- The nn → pp + ee amplitude can be represented as an integral expression

$$A_\nu \sim G_F^2 \int \frac{d^4k}{(2\pi)^4} \frac{g_{\mu\nu}}{k^2} \int d^4x e^{ik\cdot x} \langle pp \,|\, T\{J_W^\mu(x) J_W^\nu(0)\} \,|\, nn\rangle$$

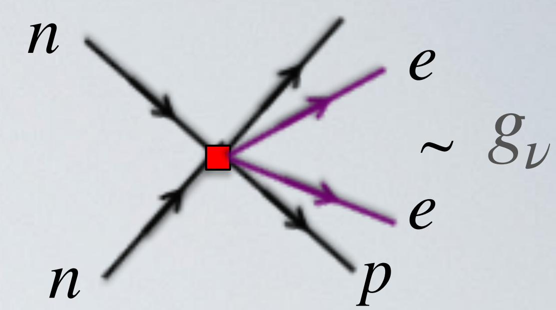
J_W^μ = weak current (V-A)



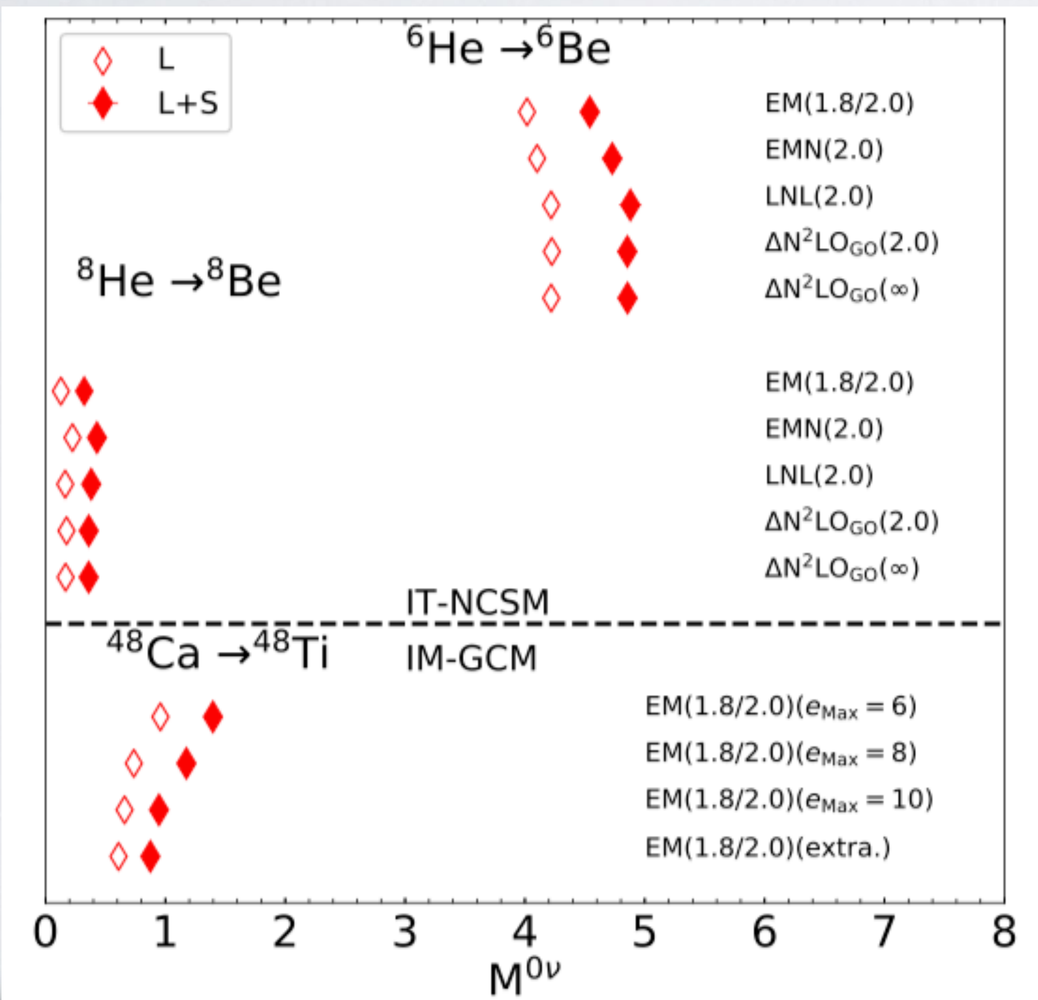
- Can represent the `red box' in regions of the virtual neutrino momentum k
- **Skip the details but analysis agree with CIB extraction**

Cirigliano, Dekens, JdV, Hoferichter, Mereghetti PRL '21

Impact on realistic nucleus



- **First calculations now include this heavier nuclei**
- N3LO chiral nucleon-nucleon potential with certain regulator
- Fit short-distance g_ν to synthetic data
- Similarity Renormalization Group transformation to perform many-body computations



Wirth, Yao, Hergert PRL '21

- Bigger effect for I=2 transitions due to node

$\sim 100\,\%$ for ^{8}He

$\sim 60\,\%$ for ^{48}Ca

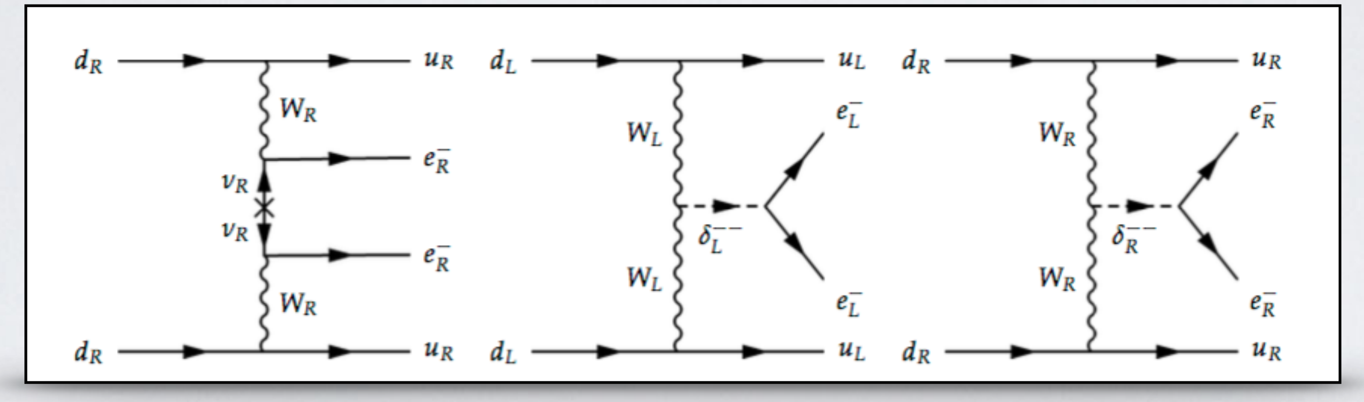
^{48}Ca decay rate increases by factor >2

Enhanced sensitivity to neutrino Majorana mass

Similar enhancements found for ^{76}Ge and ^{136}Xe (Menendez et al '22)

Other mechanism of 0vbb

- **Many beyond-the-SM model induce different 0vbb mechanism**
- Examples: Left-right symmetry, supersymmetry, leptoquarks, …….



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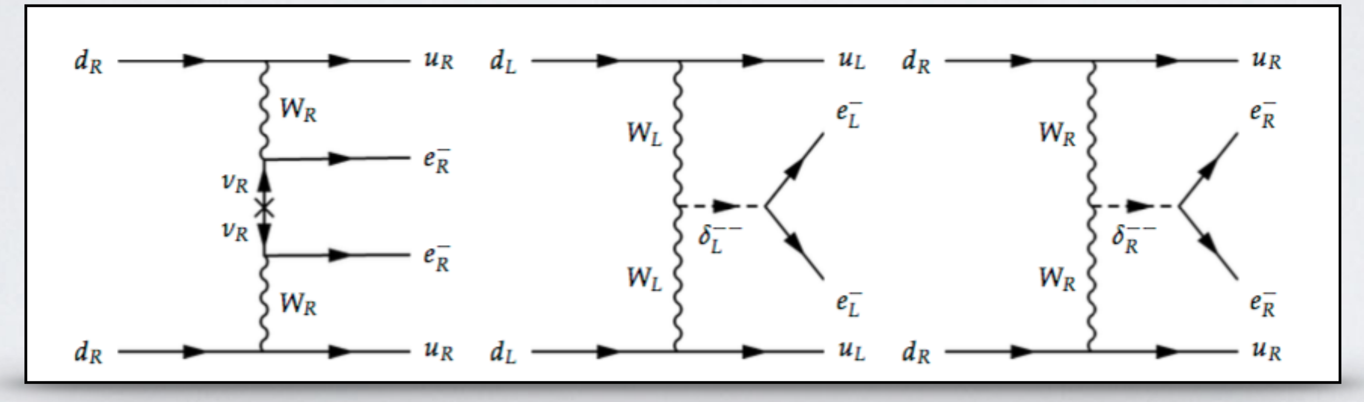
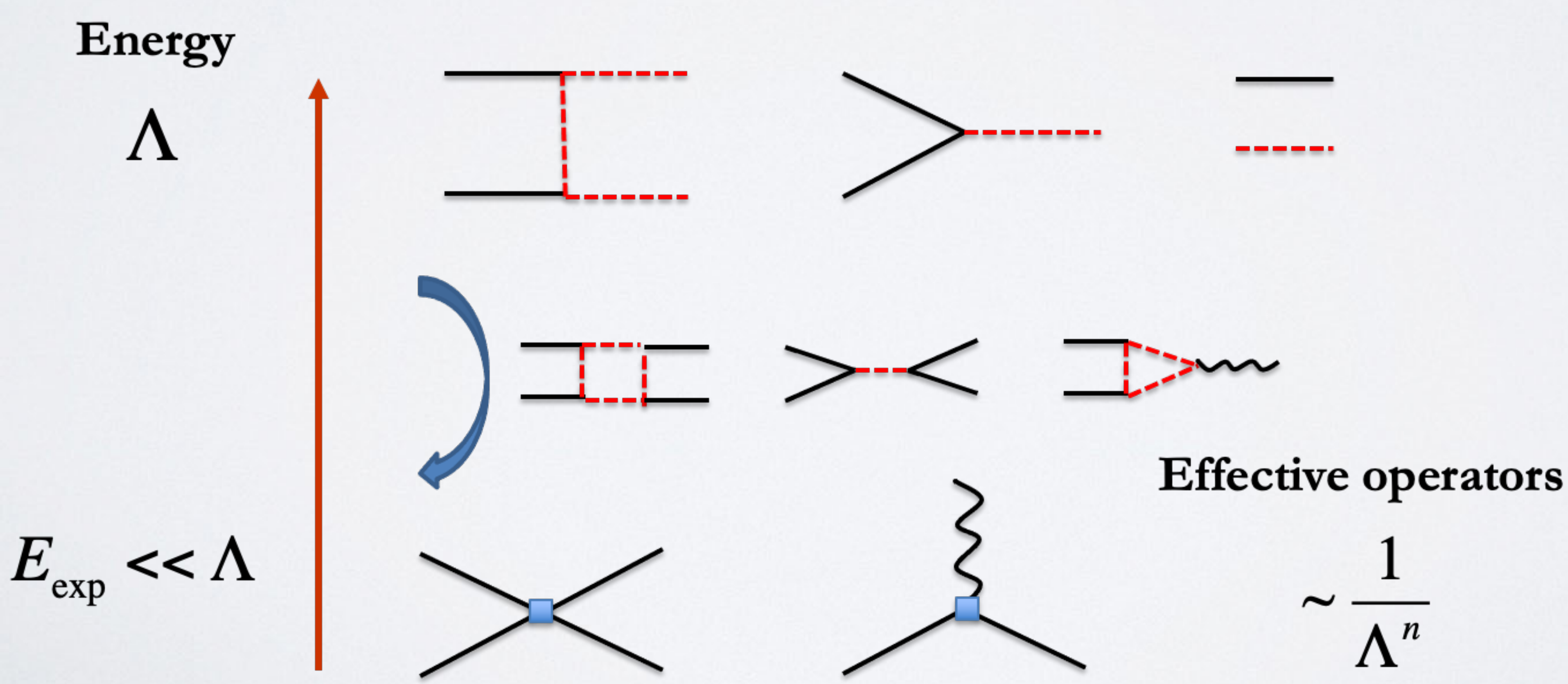
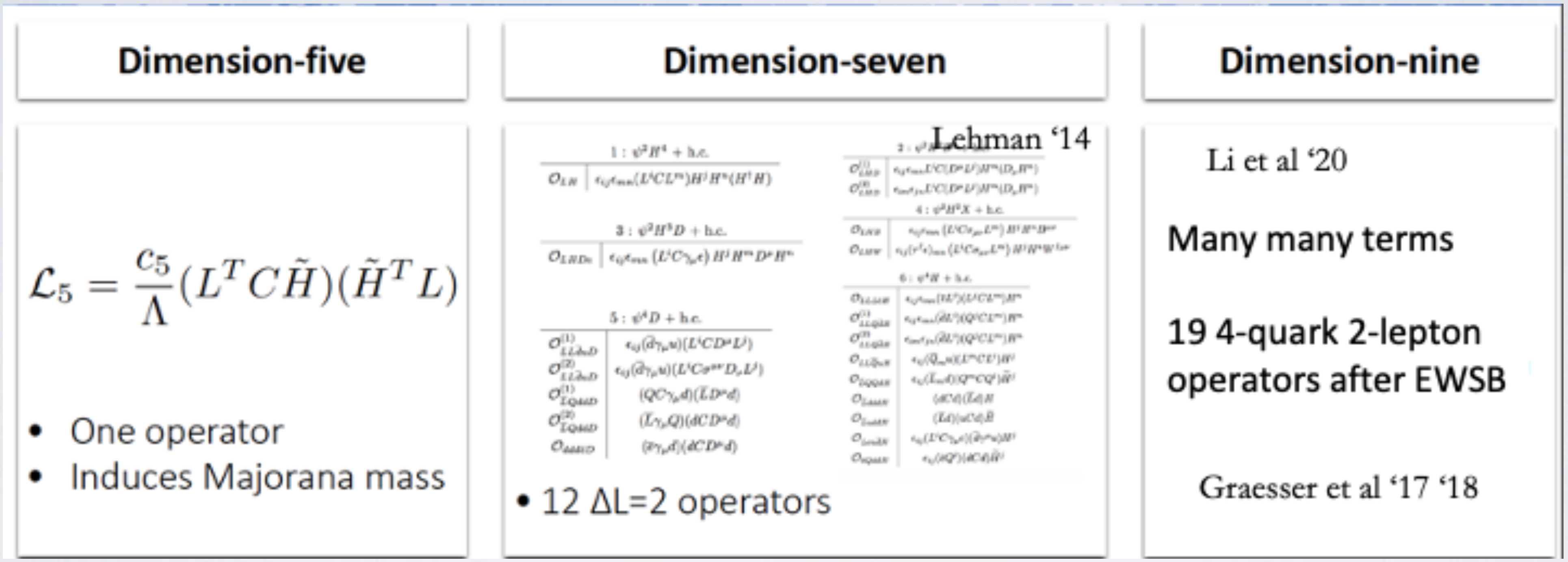


- **If new fields are heavy, can use effective field theory !**



Higher-dimensional operators

- Effective operators appear at odd dimension (5, 7, 9, …..) Kobach '16



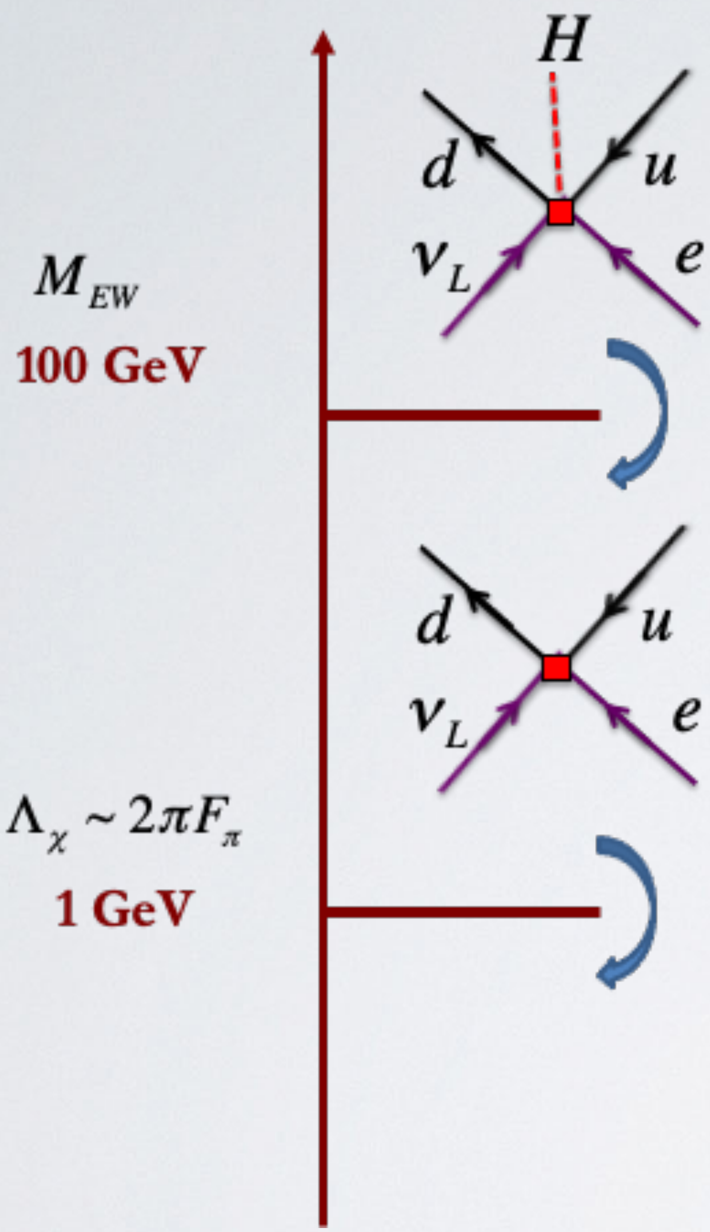
- Higher-dimensional terms only relevant if dim-5 operator are suppressed
- Example: in left-right symmetric models

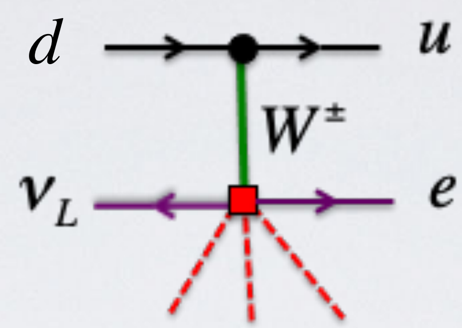
$$c_5 \sim y_e^2 \sim 10^{-10} \qquad c_7 \sim y_e^1 \sim 10^{-5} \qquad c_9 \sim y_e^0 \sim 1$$

- If scale is not too high: $\frac{v^2}{\Lambda^2} \sim y_e \rightarrow \Lambda \simeq (10 - 100)\,\text{TeV}$

- **Dim-7 or dim-9 can dominate low-energy phenomenology !**

Example dim-7 operators



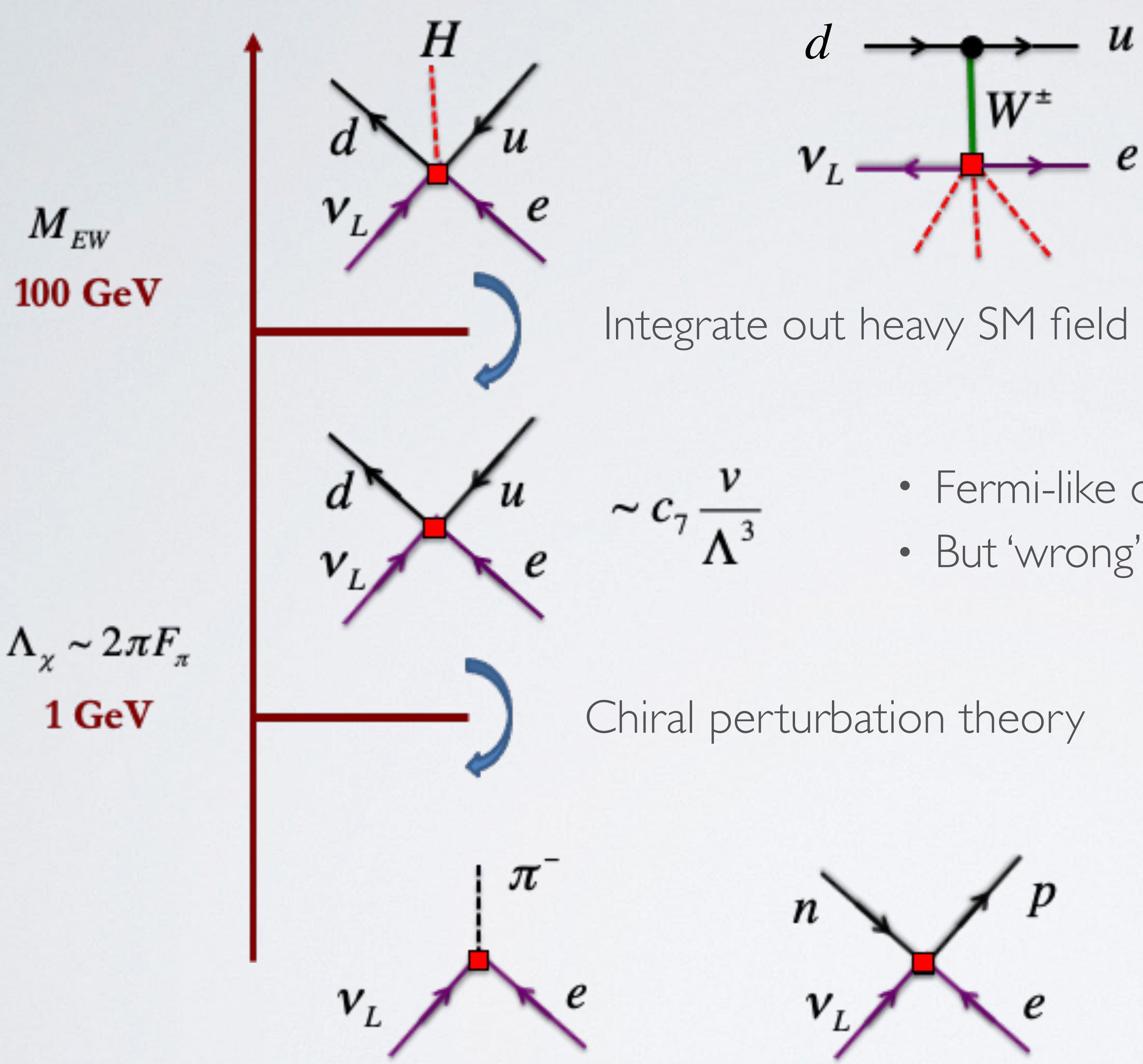


Integrate out heavy SM field and Higgs takes vev

$$\sim c_7 \frac{v}{\Lambda^3}$$

- Fermi-like operator (beta decay)
- But 'wrong' neutrino intstead of anti-neutrino

Example dim-7 operators



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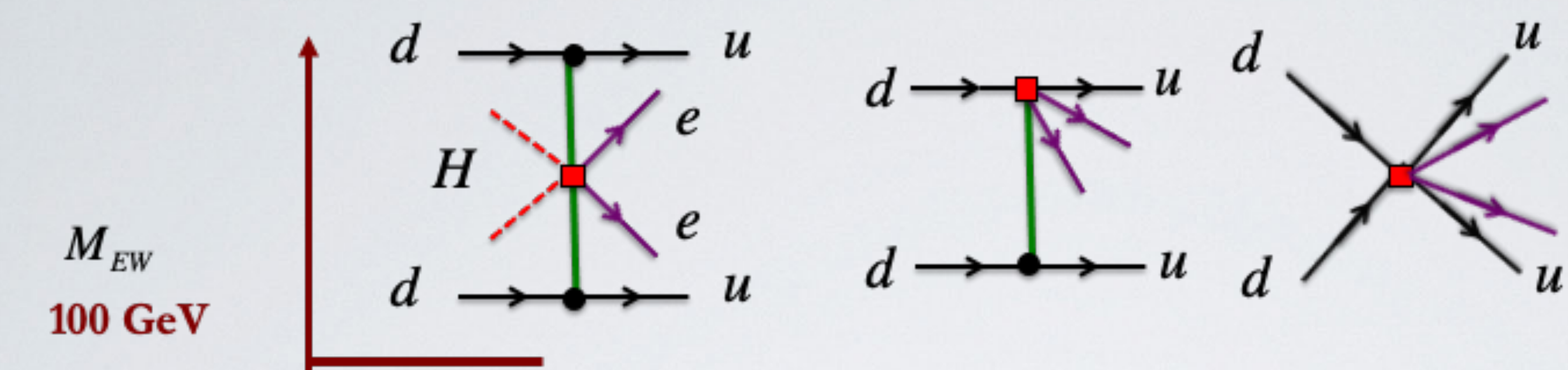
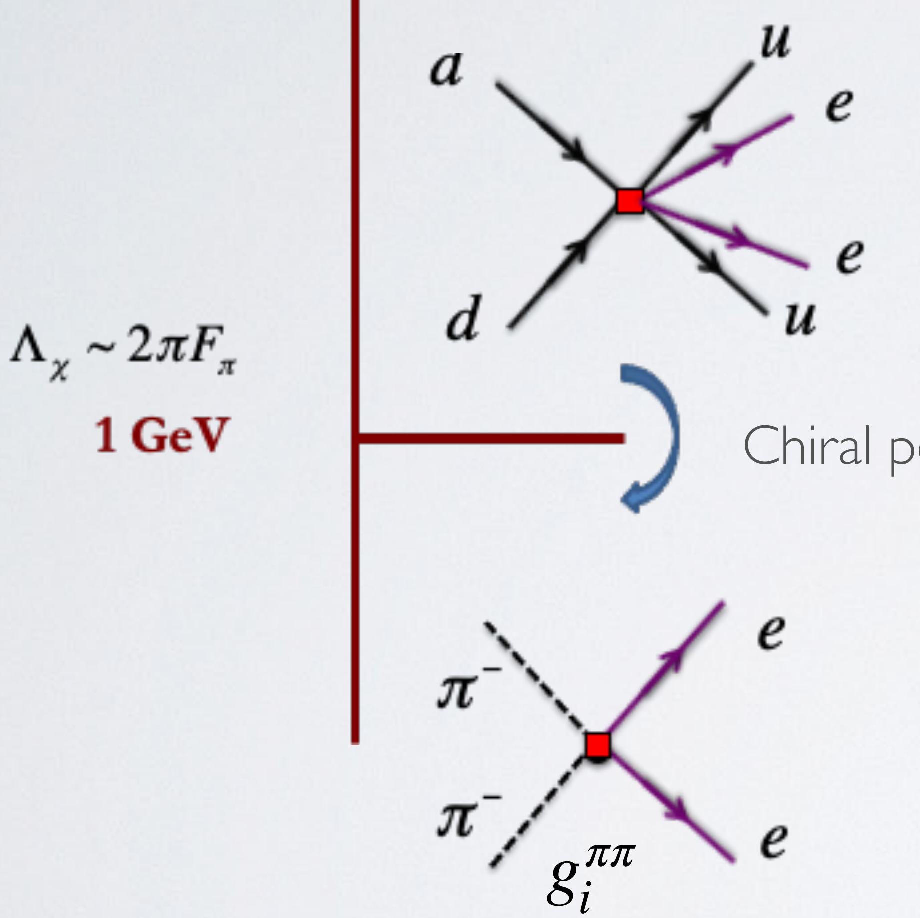
Chiral perturbation theory

Prezeau et al '03
Cirigliano et al '17 '18

Associated low-energy constants well known (nucleon charges $g_{A,S,T,V}$)

Example dim-9 operators

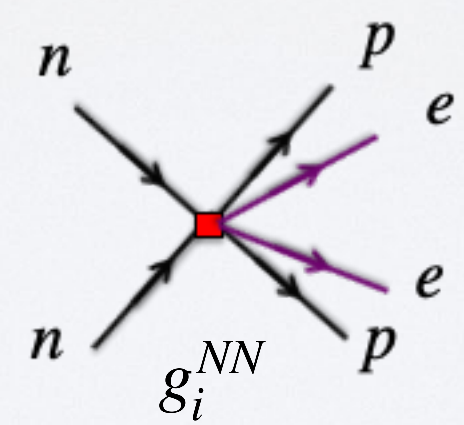




- Four-quark 2-lepton operators
- Neutrinoless interactions

Chiral perturbation theory

Prezeau et al '03

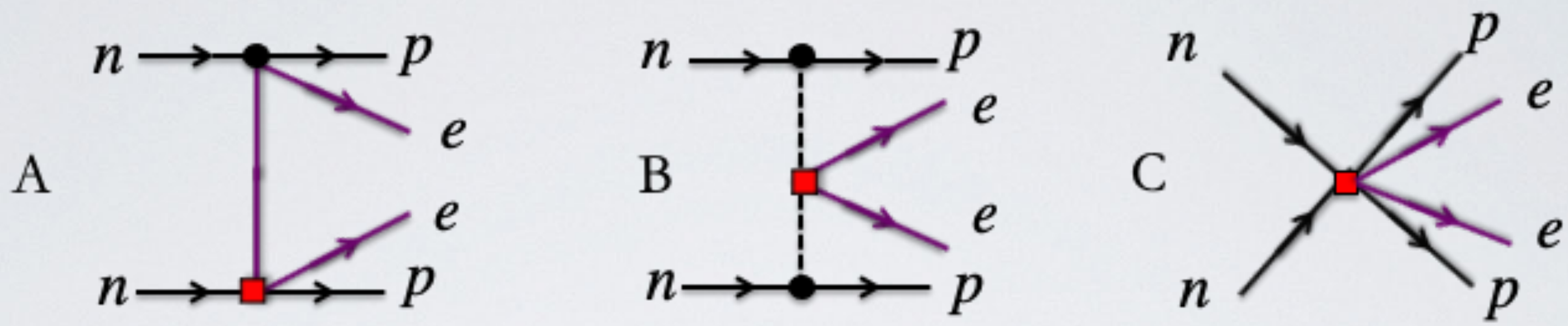


- Often missed in literature which uses factorization methods (O(100) error on decay rate)
- Depend on four-quark matrix elements: great improvements by CalLat and MIT groups

Nicholson et al '18

$$g_4^{\pi\pi} = -(1.9 \pm 0.2)\,\mathrm{GeV}^2 \qquad g_5^{\pi\pi} = -(8.0 \pm 0.6)\,\mathrm{GeV}^2$$

New 0vbb topologies

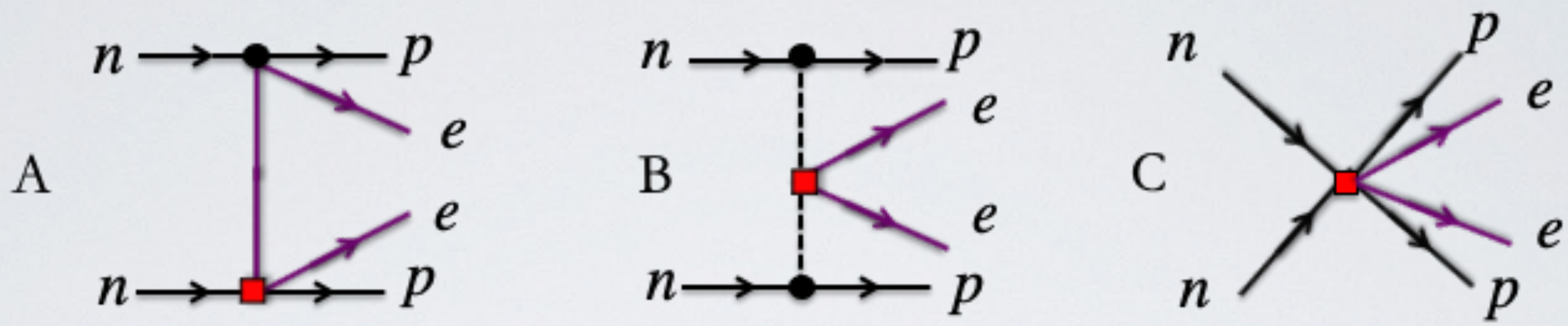


- Straightforward to calculate generalized 0vbb transition current

Cirigliano et al '17 '18

- Need additional nuclear matrix elements (NMEs)

New 0vbb topologies



- Straightforward to calculate generalized 0vbb transition current

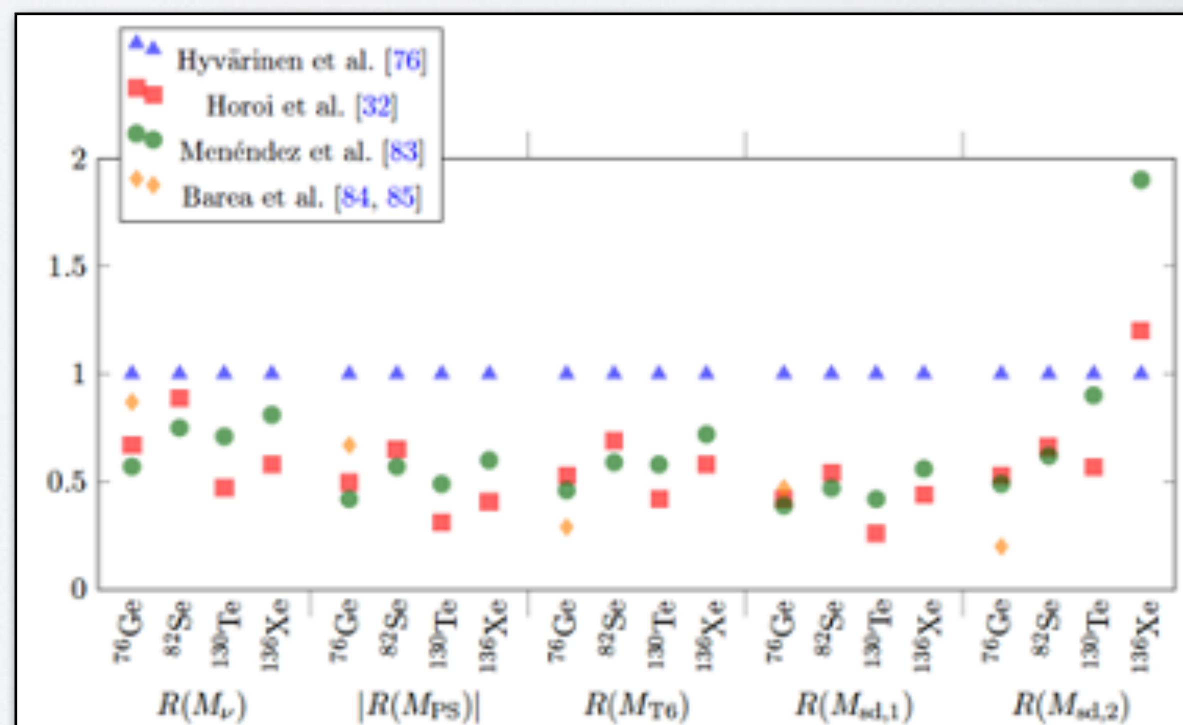
Cirigliano et al '17 '18

- Need additional nuclear matrix elements (NMEs)
- **At leading-order in Chiral-EFT: 15 NMEs (all in literature)**
- Similar uncertainties as before

NMEs	^{76}Ge			
	[74]	[31]	[81]	[82,83]
M_F	-1.74	-0.67	-0.59	-0.68
M_{GT}^{AA}	5.48	3.50	3.15	5.06
M_{GT}^{AP}	-2.02	-0.25	-0.94	
M_{GT}^{PP}	0.66	0.33	0.30	
M_{GT}^{MM}	0.51	0.25	0.22	
M_T^{AA}	–	–	–	
M_T^{AP}	-0.35	0.01	-0.01	
M_T^{PP}	0.10	0.00	0.00	
M_T^{MM}	-0.04	0.00	0.00	

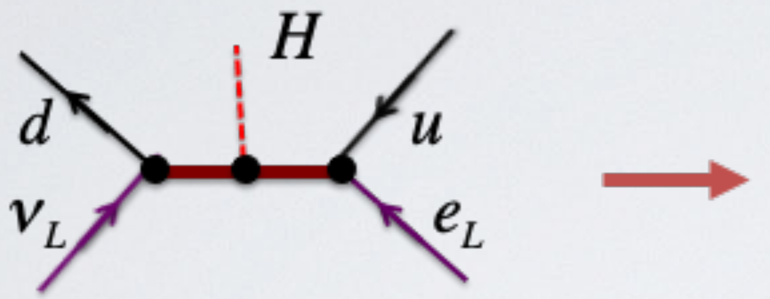
NMEs	^{76}Ge			
$M_{F,sd}$	-3.46	-1.55	-1.46	-1.1
$M_{GT,sd}^{AA}$	11.1	4.03	4.87	3.62
$M_{GT,sd}^{AP}$	-5.35	-2.37	-2.26	-1.37
$M_{GT,sd}^{PP}$	1.99	0.85	0.82	0.42
$M_{T,sd}^{AP}$	-0.85	0.01	-0.05	-0.97
$M_{T,sd}^{PP}$	0.32	0.00	0.02	0.38

Hyvarinen/Suhonen '15
Menendez et al '17 '18
Barea et al '15 '18
Horoi/Neacsu '17



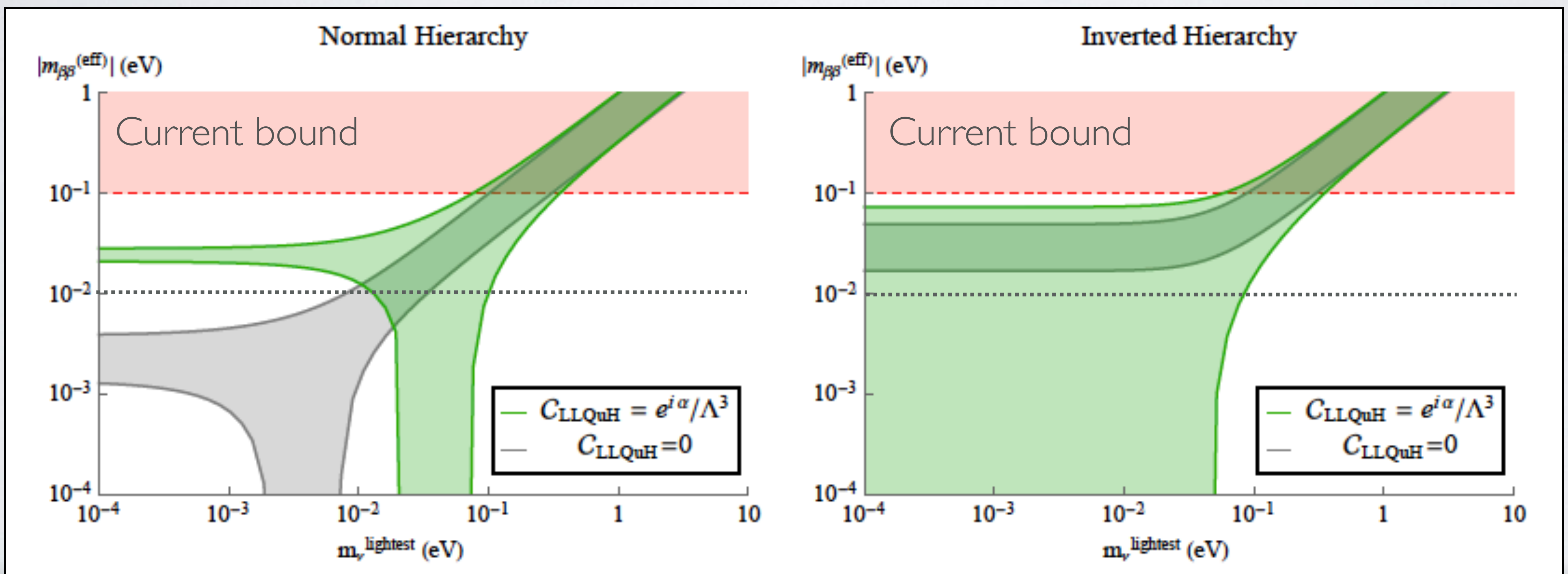
Using the framework

- Example: a model of heavy leptoquarks (LHC probes 1 TeV leptoquarks roughly)



$C_7 \sim (v/\Lambda)^3$

$\Lambda > 400\ TeV$

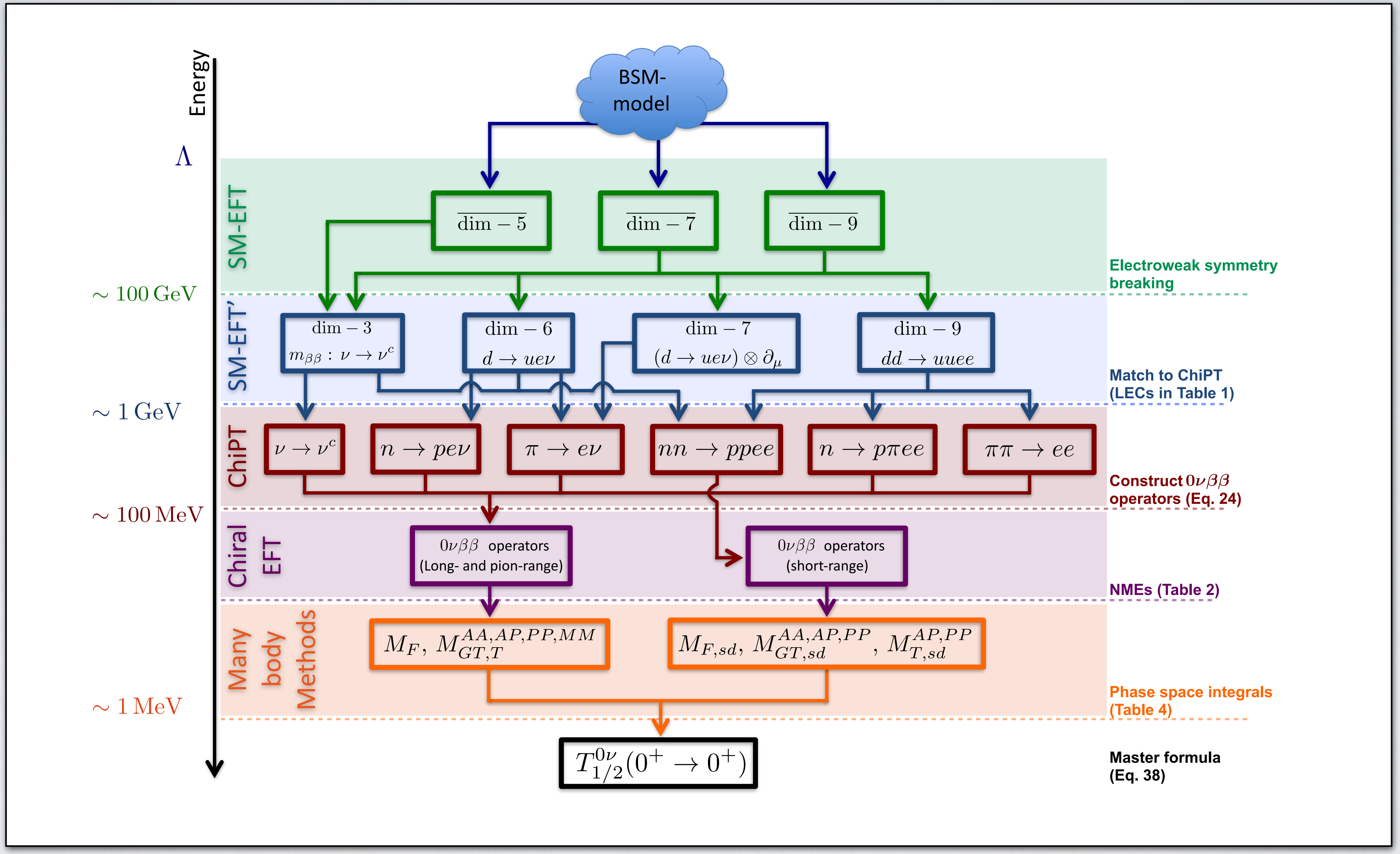


Ton-scale expectations

- Dramatic impact on 0vbb phenomenology !
- Sensitivity to 500-TeV new physics scales

The 0vbb metro map

Cirigliano, Dekens, JdV, Graesser, Mereghetti'18



- Open-access Python tool almost ready to be submitted (Scholer + Graf + JdV)

NuDoBe

- Open-access Python tool automizes the EFT calculations

[Submitted on 11 Apr 2023]

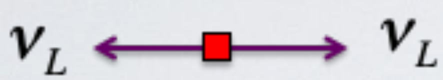
νDoBe -- A Python Tool for Neutrinoless Double Beta Decay

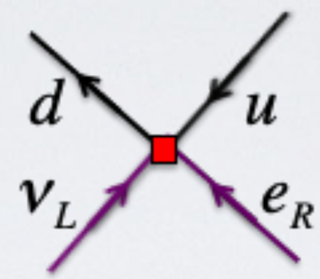
Oliver Scholer, Jordy de Vries, Lukáš Gráf

- User specifies which SM-EFT LNV operators are turned on at which scale
- Code computes 0vbb (differential rates) of all isotopes of interest
- User can vary different results for NMEs/Hadronic LECs etc
- Comes with many built-in plotting options

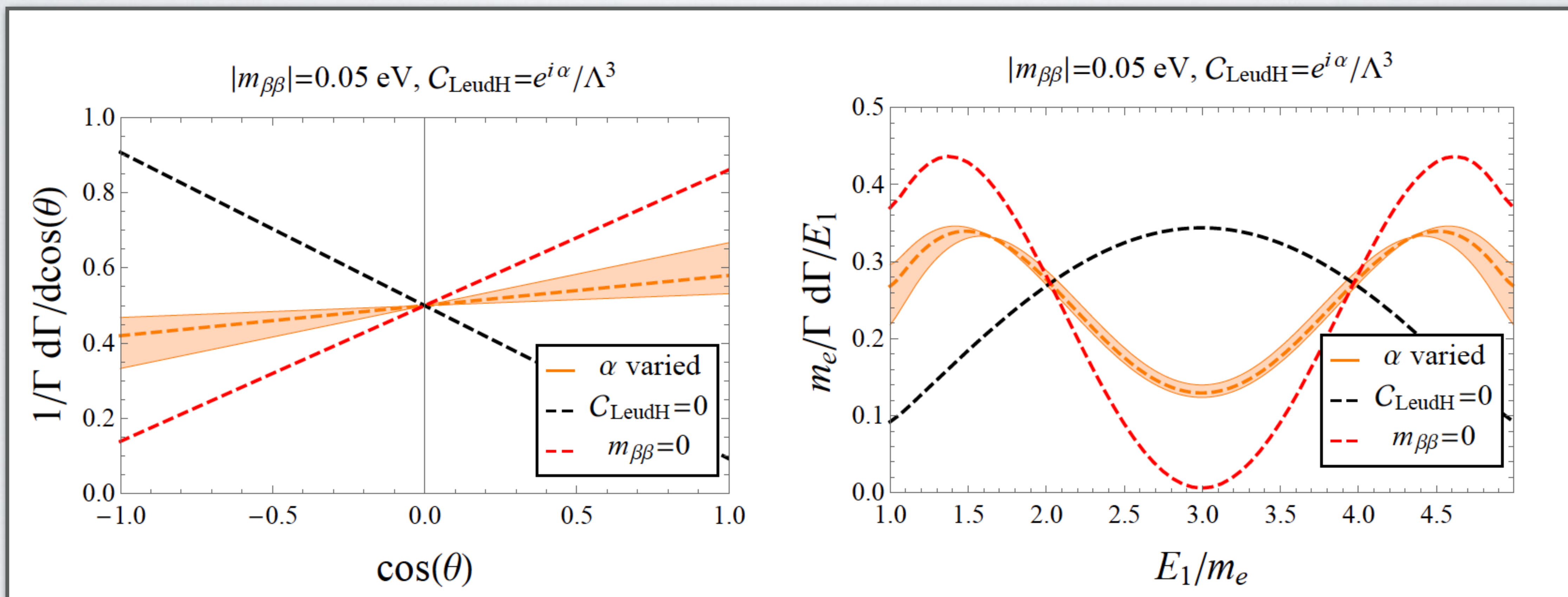
NuDoBe

- **Tool also computes angular&energy electron distributions**





$C_7 \sim (v/\Lambda)^3 e^{i\alpha}$ $\Lambda \sim 50\,\mathrm{TeV}$

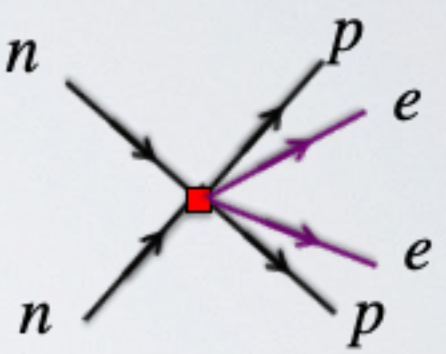


Concluding remarks

- **Neutrinoless double beta decay best way to determine if neutrinos are Majorana states**

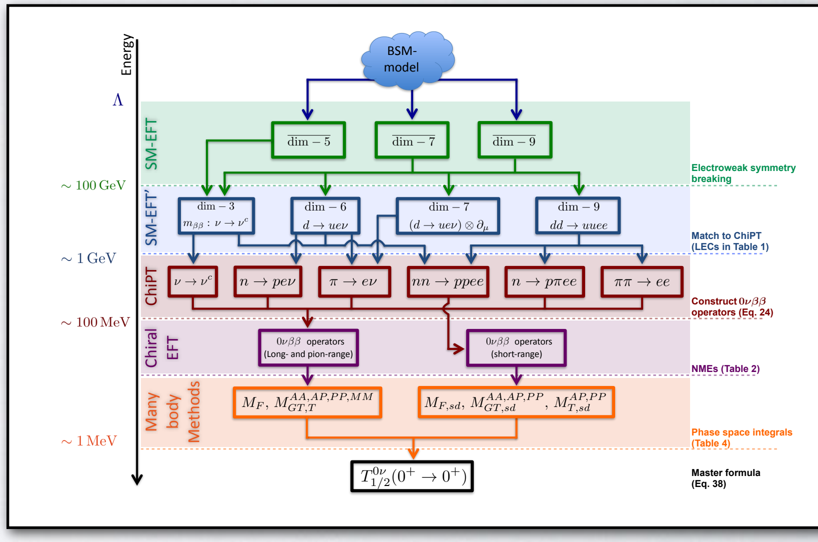
- Heroic experimental effort ! Particle/Hadronic/Nuclear theory needed to interpret data

- Progress from EFT + lattice + nuclear structure



- **New findings:** standard mechanism depends on short-distance physics
Significant impact on NMEs: larger than before.

- End-to-End EFT framework for any LNV source



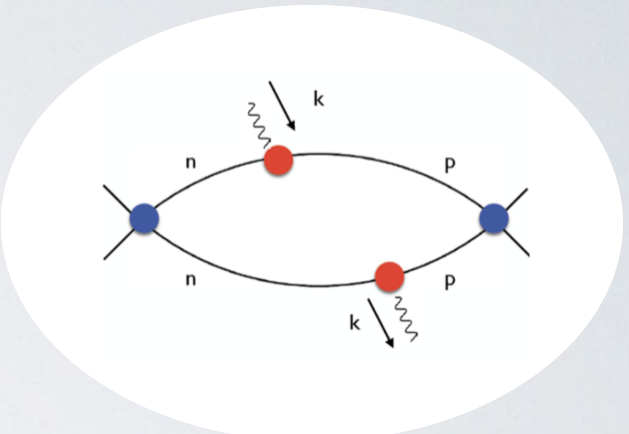
- Automized with Python Tool: **NuDoBE**

- Not discussed today: light sterile neutrinos and EFT

An analytic approach

- The nn → pp + ee amplitude can be represented as an integral expression

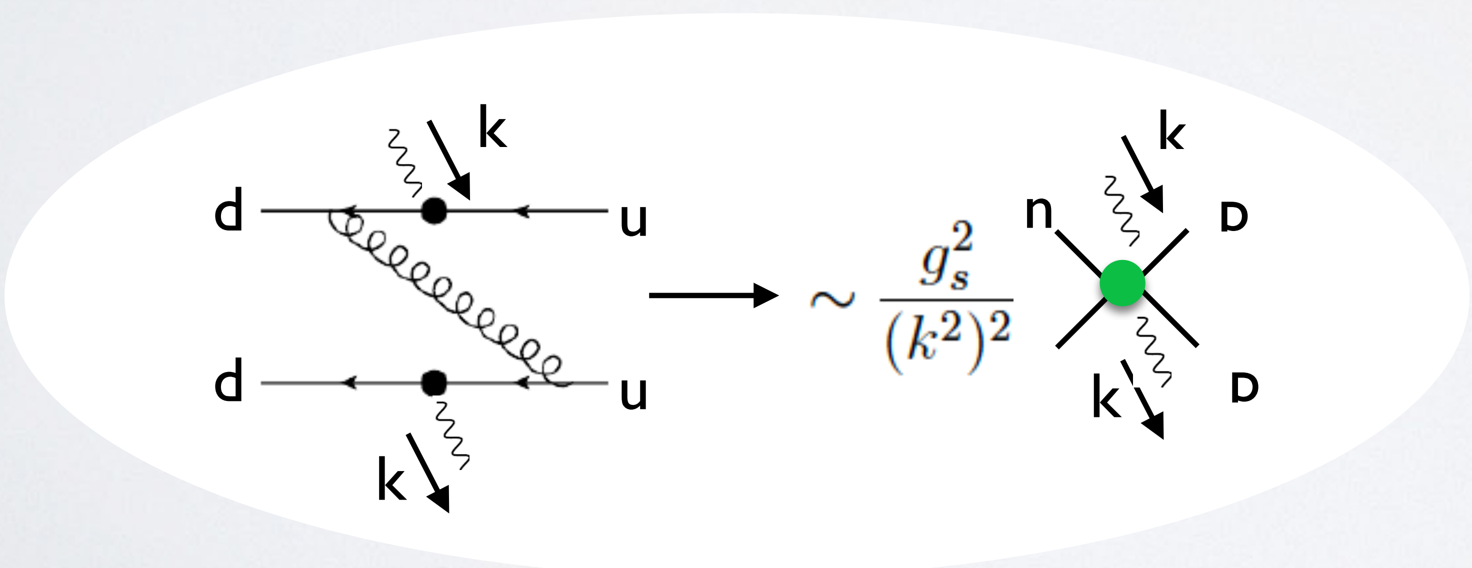
$$A_\nu \sim G_F^2 \int \frac{d^4k}{(2\pi)^4} \frac{g_{\mu\nu}}{k^2} \int d^4x e^{ik\cdot x} \langle pp \,|\, T\{J_W^\mu(x) J_W^\nu(0)\} \,|\, nn \rangle$$



- At small virtual momentum: NLO chiral EFT

- Intermediate momentum: (**model-dependent**) resonance contributions to nucleon form factors and to NN scattering

- Large momentum: Perturbative QCD + Operator Product Expansion



Small dependence on local 4-quark matrix elements

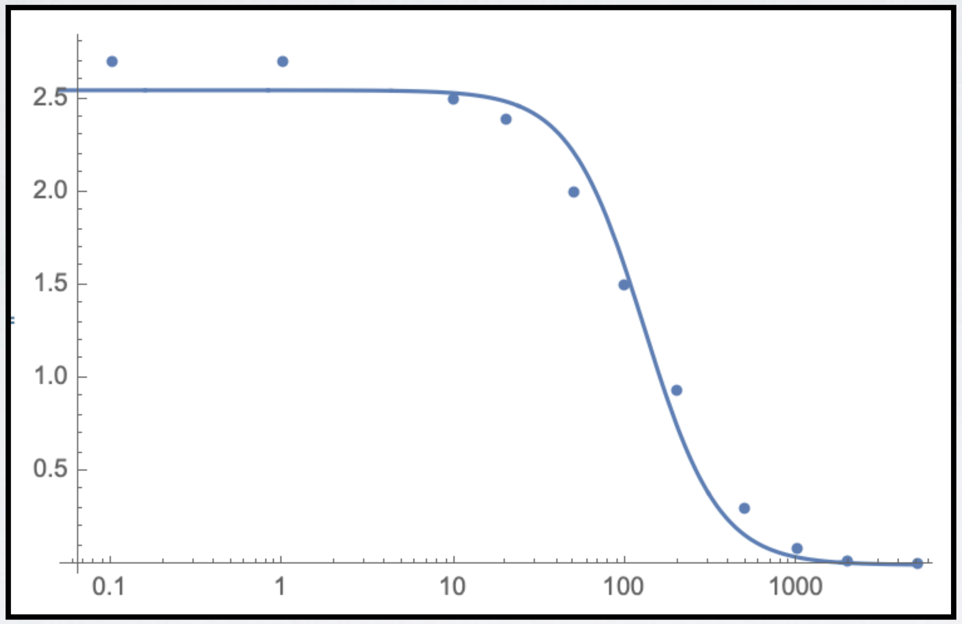
- **Analytic and CIB approach within uncertainties (~30-40%)**
- Analytic approach agrees with nucleon-nucleon CIB data

Current procedure in literature

- Compute nuclear matrix element computations for different neutrino masses

$M(m_4)$

Shell model Xe
(Menendez)



$m_4(\mathrm{MeV})$

$$M(m_4) \sim \frac{1}{\langle p^2 \rangle + m_4^2}$$

$$\langle p^2 \rangle \simeq (100\,\mathrm{MeV})^2$$

$$A_\nu \sim \sum_{i=1}^{3} U_{ei}^2 m_i \frac{1}{\langle p^2 \rangle} + U_{e4}^2 m_4 \frac{1}{\langle p^2 \rangle + m_4^2}$$

$m_4 \gg 100\,\mathrm{MeV}$ →

$$A_\nu \sim \sum_{i=1}^{3} U_{ei}^2 m_i \frac{1}{\langle p^2 \rangle} + \frac{U_{e4}^2}{m_4}$$

$m_4 \ll 100\,\mathrm{MeV}$ →

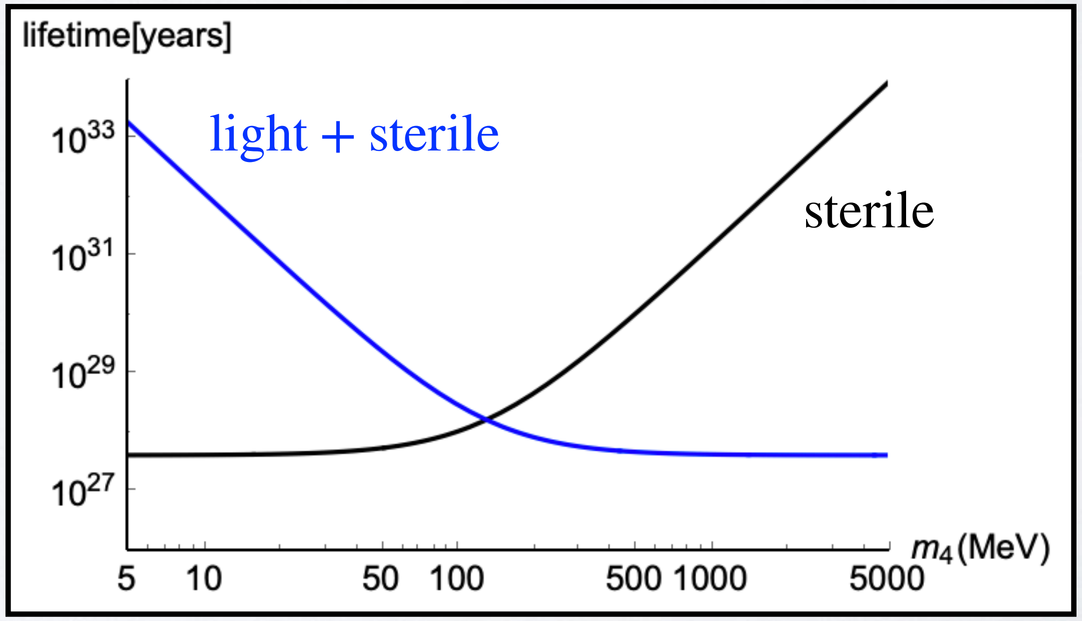
$$A_\nu \sim \sum_{i=1}^{4} U_{ei}^2 m_i \frac{1}{\langle p^2 \rangle}$$

Revisit the light regime

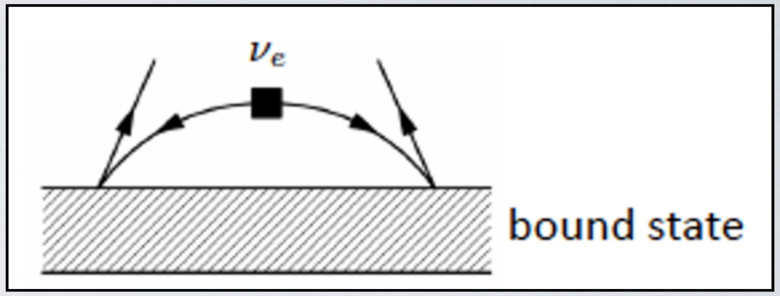
$$A_\nu \sim \sum_{i=1}^{3} U_{ei}^2 m_i \frac{1}{\langle p^2 \rangle} + U_{e4}^2 m_4 \frac{1}{\langle p^2 \rangle + m_4^2} \xrightarrow{m_4 \ll 100\,\text{MeV}} A_\nu \sim \sum_{i=1}^{4} U_{ei}^2 m_i \frac{1}{\langle p^2 \rangle} + \mathcal{O}\left(\frac{m_i^3}{\langle p^2 \rangle^2}\right)$$

- The first term depends on $\sum_{i=1}^{4} U_{ei}^2 m_i = M_{ee} = 0$ $\quad M = \begin{pmatrix} 0 & v y_\nu \\ v y_\nu & M_R \end{pmatrix}$
- **The 'GIM' mechanism for neutrinos !** (only valid if all steriles are light)
- The amplitude is strongly suppressed $A_\nu \sim \sum_{i=1}^{4} U_{ei}^2 m_i^3$ Blennow et al '10 JHEP
- Example in 3+1model
- Cancellation between light + sterile contributions leads to

$$\tau_{1/2} \sim m_4^4$$



Light extra neutrinos

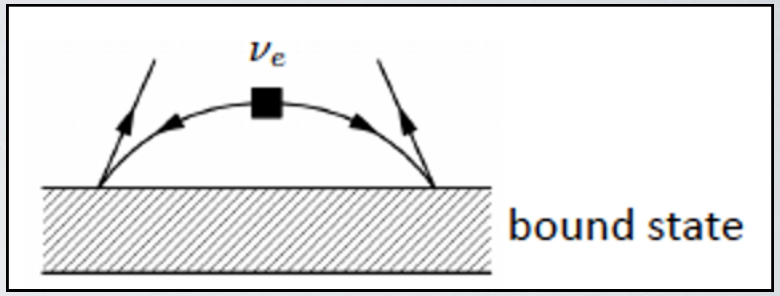


- Is there a way to avoid the GIM mechanism ?
- There are additional contributions from 'ultra-soft' neutrinos

$$\sum_n \langle f | J_\mu | n \rangle \langle f | J^\mu | i \rangle \times \int \frac{d^3k}{(2\pi)^3} \frac{1}{E_\nu [E_\nu + (E_n - E_0) - i\epsilon]} \qquad E_\nu = \sqrt{k^2 + m_i^2}$$

- Depends on nuclear excited states. Normally these are tiny effects (5%)
- But become dominant in the GIM mechanism ! $\sim U_{ei}^2 m_i^3$
- For m_4 ~ MeV we get new contributions $\sim U_{ei}^2 m_i^2$
- For m_4 << MeV we get new contributions $\sim U_{ei}^2 m_i^3 \log \frac{(E_n - E_0)^2}{m_i^2}$
- These effects are not considered in any analysis of neutrinoless double beta decay
- Javier Menendez computed for us the necessary matrix elements

Light extra neutrinos



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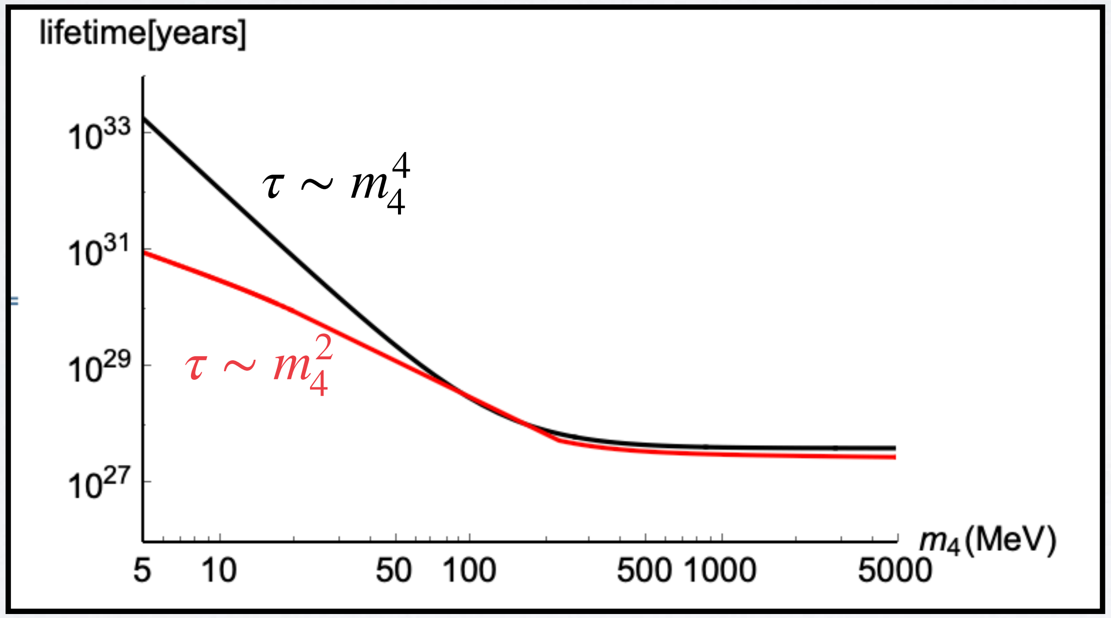

100x larger decay rates

Still small ;)

- These effects are not considered in any analysis of neutrinoless double beta decay
- **Work in progress: compute these corrections for realistic models**
- Other issues (not today) $g_\nu^{NN}(m_i)$? NME's don't make sense for m_i >> GeV …