

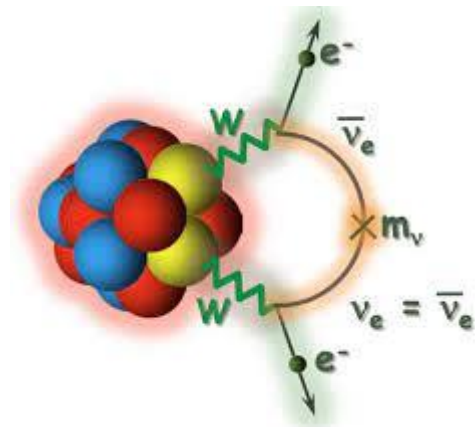
# UV completions of $0\nu\beta\beta$ decay operators at one-loop level

Gui-Jun Ding

University of Science and Technology of China

The 2nd conference on neutrinoless double beta decay and relevant physics, Zhuhai, 20th May 2023

Based on arXiv: 2110.15347, 2301.02503,  
in collaboration with Ping-Tao Chen, Chang-Yuan Yao



# Massive neutrinos: Dirac or Majorana?

$$\nu \neq \bar{\nu}$$



$$\nu = \bar{\nu}$$



**VS.**

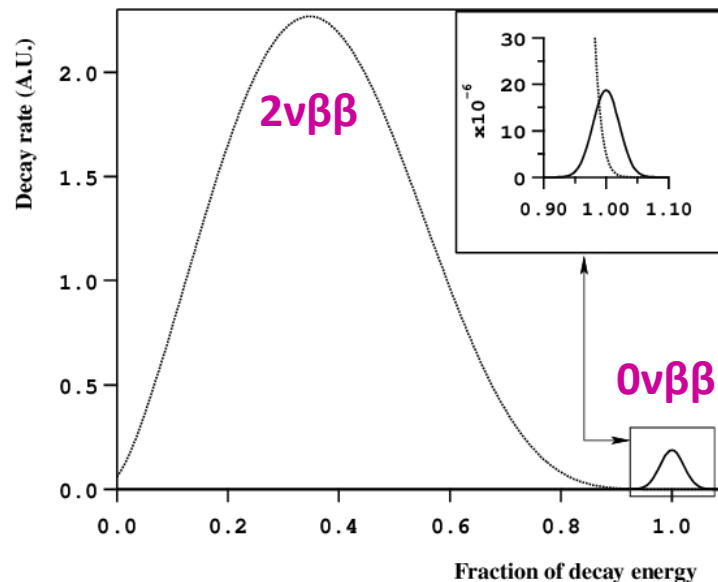
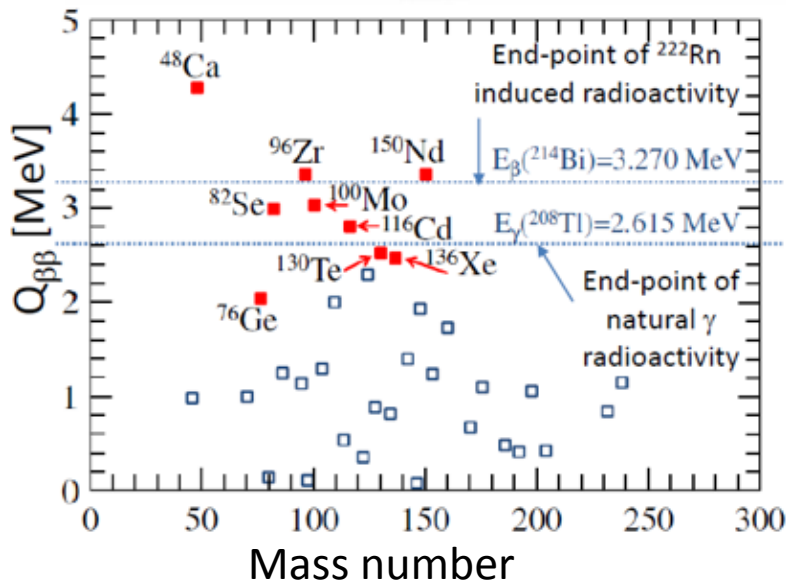
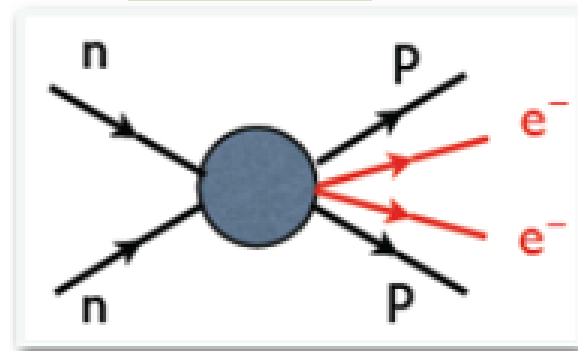
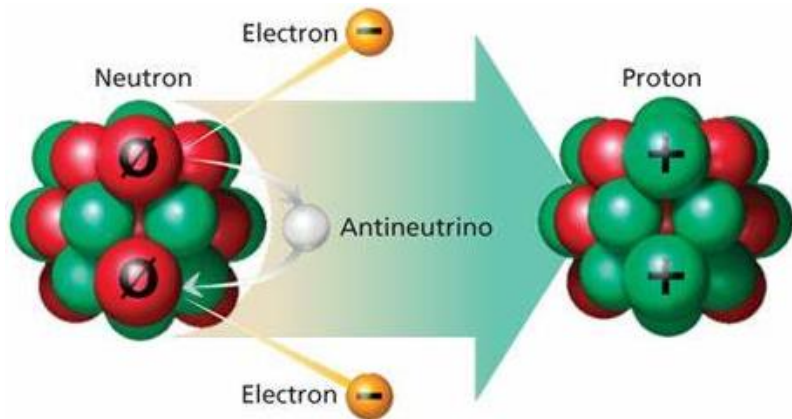
- **neutrinoless double beta decay**
- lepton number violation at collider
- cosmology

# The $0\nu\beta\beta$ decays

$0\nu\beta\beta$  is potentially observable in certain even-even nuclei ( $^{48}\text{Ca}$ ,  $^{76}\text{Ge}$ ,  $^{100}\text{Mo}$ ,  $^{130}\text{Te}$ ,  $^{136}\text{Xe}$ ) for which single beta decay is energetically forbidden. The decay rate is less than **1 event per ton and year**.

$$0\nu\beta\beta: (A, Z) \rightarrow (A, Z + 2) + 2e^-$$

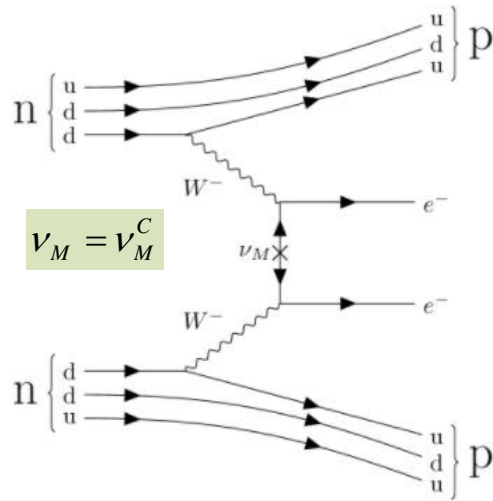
$$T_{1/2} > 10^{25} \text{ yr}$$



# Standard mass mechanism of $0\nu\beta\beta$ decay

$0\nu\beta\beta$  decay is mediated by light and massive Majorana neutrinos (the ones which oscillate) and all other mechanisms potentially leading to  $0\nu\beta\beta$  give negligible or no contribution

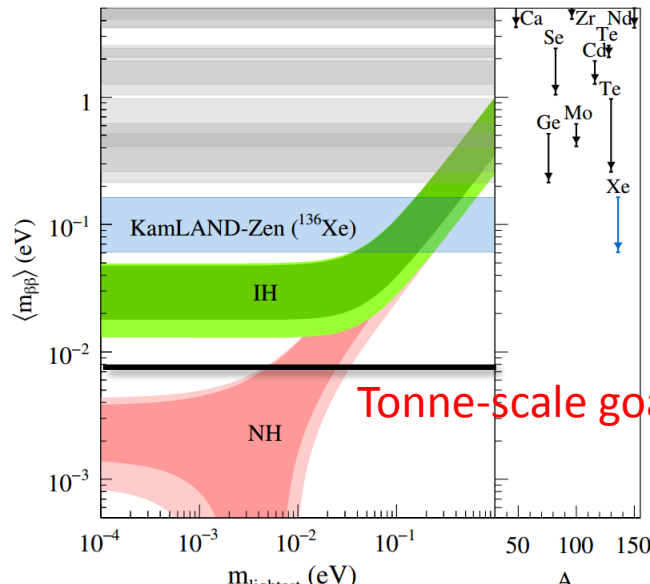
➤ The decay amplitude is proportional to the effective mass



$$|m_{\beta\beta}| = \left| \sum_k U_{ek}^2 m_k \right|$$

$$= \left| c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 e^{i\alpha_{21}} m_2 + s_{13}^2 e^{i\alpha_{31}} m_3 \right|$$

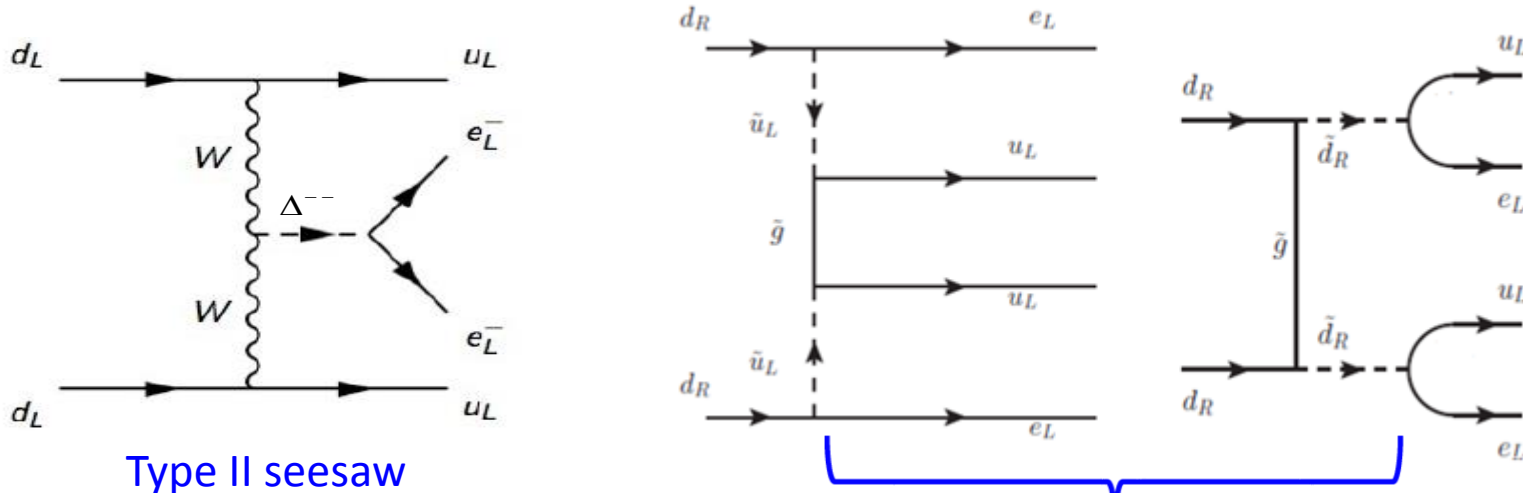
PMNS, neutrino masses, Majorana phases, cosines and sines of the mixing angles



- Unknown: lightest mass, hierarchy and Majorana phases
- Large theoretical uncertainties in the nuclear matrix elements

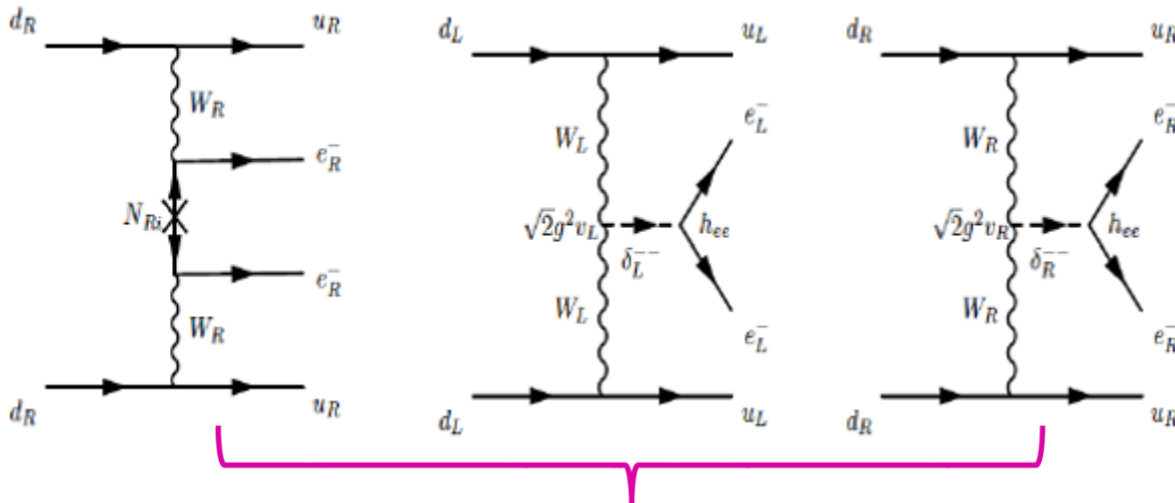
# Possible BSM physics in $0\nu\beta\beta$ decay

The  $0\nu\beta\beta$  decay can also be induced by other  $\Delta L=2$  physics besides the Majorana neutrino mass. There are many possible scenarios:



Type II seesaw

R-parity violating SUSY

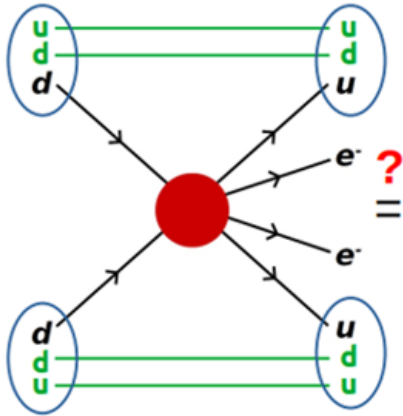


Left-right model

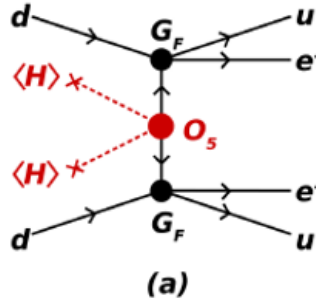
- $0\nu\beta\beta$  decay is connected to TeV scale physics and LFV.
- **A systematical classification is necessary!**

# Classification of $0\nu\beta\beta$ mechanisms from SMEFT

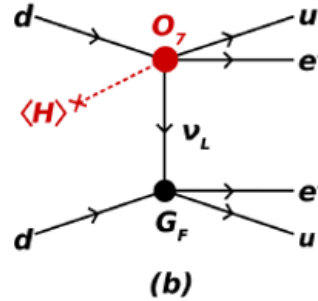
The amplitude of  $0\nu\beta\beta$  decay can be generally divided into: [Pas, Hirsch, Klapdor, Kovalenko, hep-ph/0008182, hep-ph/9804374]



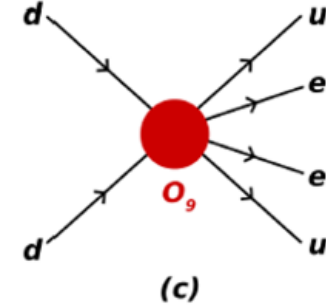
Mass mechanism



"long-range"



"short-range"



EFT:

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C_i^{(5)}}{\Lambda} \mathcal{O}_i^{MM} + \frac{C_i^{(7)}}{\Lambda^3} \mathcal{O}_i^{LR} + \frac{C_i^{(9)}}{\Lambda^5} \mathcal{O}_i^{SR} + \dots$$

[Graesser,1606.04549;

Liao, Ma, 2007.08125; Yu et al, 2007.07899]

See Jiang-Hao Yu's talk

$$\mathcal{O}_1^{MM} = \epsilon^{ik} \epsilon^{jl} (\bar{\ell}_i^c \ell_j) H_k H_l$$

$$\mathcal{O}_1^{LR} = \epsilon^{ik} \epsilon^{jl} (\bar{\ell}_i^c \ell_j) (\bar{d}_R Q_k) H_l,$$

$$\mathcal{O}_2^{LR} = \epsilon^{ik} \epsilon^{jl} (\bar{\ell}_i^c \gamma^{\mu\nu} \ell_j) (\bar{d}_R \gamma_{\mu\nu} Q_k) H_l,$$

$$\mathcal{O}_3^{LR} = \epsilon^{jk} (\bar{\ell}_i^c \ell_j) (\bar{Q}^i u_R) H_k,$$

$$\mathcal{O}_4^{LR} = (\bar{\ell}_i^c \gamma^\mu e_R) (\bar{d}_R \gamma_\mu u_R) \epsilon^{ij} H_j$$

$$\begin{aligned} \mathcal{O}_1^{SR} &= \epsilon_{ij} (\bar{Q}_i \gamma^\mu Q_m) (\bar{u}_R \gamma_\mu d_R) (\bar{\ell}_j \ell_m^c), \\ \mathcal{O}_2^{SR} &= \epsilon_{ij} (\bar{Q}_i \gamma^\mu \lambda^A Q_m) (\bar{u}_R \gamma_\mu \lambda^A d_R) (\bar{\ell}_j \ell_m^c), \\ \mathcal{O}_3^{SR} &= (\bar{u}_R Q_i) (\bar{u}_R Q_j) (\bar{\ell}_i \ell_j^c), \\ \mathcal{O}_4^{SR} &= (\bar{u}_R \lambda^A Q_i) (\bar{u}_R \lambda^A Q_j) (\bar{\ell}_i \ell_j^c), \\ \mathcal{O}_5^{SR} &= \epsilon_{ij} \epsilon_{mn} (\bar{Q}_i d_R) (\bar{Q}_m d_R) (\bar{\ell}_j \ell_n^c), \\ \mathcal{O}_6^{SR} &= \epsilon_{ij} \epsilon_{mn} (\bar{Q}_i \lambda^A d_R) (\bar{Q}_m \lambda^A d_R) (\bar{\ell}_j \ell_n^c), \\ \mathcal{O}_7^{SR} &= (\bar{u}_R \gamma^\mu d_R) (\bar{u}_R \gamma_\mu d_R) (\bar{e}_R e_R^c), \\ \mathcal{O}_8^{SR} &= \epsilon_{ij} (\bar{u}_R \gamma^\mu d_R) (\bar{Q}_i d_R) (\bar{\ell}_j \gamma_\mu e_R^c), \\ \mathcal{O}_9^{SR} &= \epsilon_{ij} (\bar{u}_R \gamma^\mu \lambda^A d_R) (\bar{Q}_i \lambda^A d_R) (\bar{\ell}_j \gamma_\mu e_R^c), \\ \mathcal{O}_{10}^{SR} &= (\bar{u}_R \gamma^\mu d_R) (\bar{u}_R Q_i) (\bar{\ell}_i \gamma_\mu e_R^c), \\ \mathcal{O}_{11}^{SR} &= (\bar{u}_R \gamma^\mu \lambda^A d_R) (\bar{u}_R \lambda^A Q_i) (\bar{\ell}_i \gamma_\mu e_R^c) \end{aligned}$$

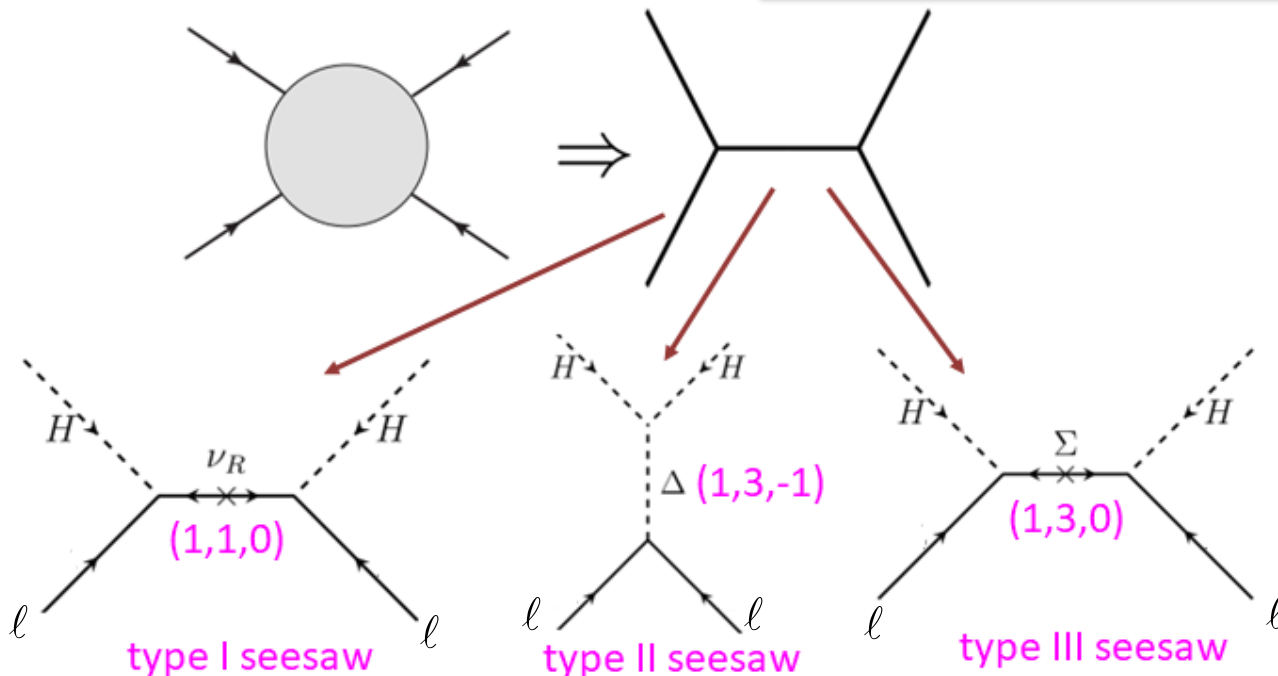
# Decomposing the short-range $0\nu\beta\beta$ operators

A systematic classification (at a given loop order) of the possible realizations is feasible through the following “**recipe**” [Gavela et al,0809.3451; Hirsch et al,1204.5862]

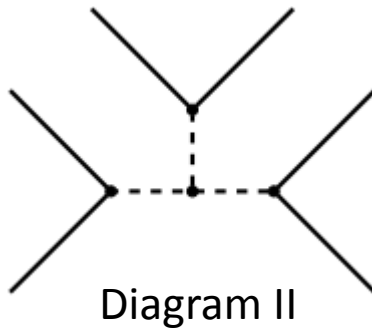
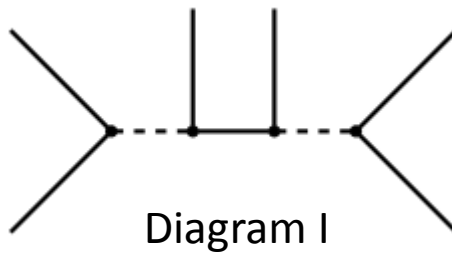
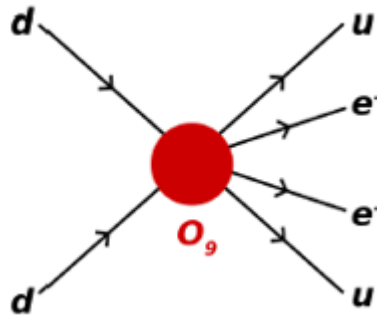
- ① **Topologies**: identify all possible  $L$ -loop connected topologies and 6 external legs
- ② **Diagrams**: assign the fields involved in the concerned operators to the external lines, and specify the Lorentz nature (spinor or scalar) of each internal line.
- ③ **Models**: fix the  $SU(3)_C \times SU(2)_L \times U(1)_Y$  quantum numbers of the internal fields, and each vertex should be invariant under the SM gauge symmetry.

➤ Example: decompose Weinberg operator

$$\mathcal{O}_W = \frac{C_{\alpha\beta}}{\Lambda} \epsilon^{ik} \epsilon^{jl} (\overline{\ell}_{\alpha i}^c \ell_{\beta j}) H_k H_l$$



# Tree-level decomposition



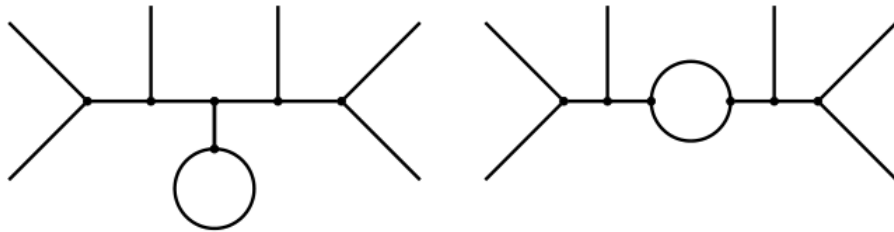
#	Decomposition	Long	Mediator ( $U(1)_{em}, SU(3)_c$ )			Models/Refs./Comments
		Range?	$S$ or $V_\rho$	$\psi$	$S'$ or $V'_\rho$	
1-i	$(\bar{u}d)(\bar{e})(\bar{e})(\bar{u}d)$	(a)	$(+1, \mathbf{1})$	$(0, \mathbf{1})$	$(-1, \mathbf{1})$	Mass mechan., RPV [58,60], LR-symmetric models [39], Mass mechanism with $\nu_S$ [61], TeV scale seesaw, e.g., [62,63,64]
1-ii-a	$(\bar{u}d)(\bar{u})(d)(\bar{e}\bar{e})$		$(+1, \mathbf{8})$ $(+1, \mathbf{1})$	$(0, \mathbf{8})$ $(+5/3, \mathbf{3})$	$(-1, \mathbf{8})$ $(+2, \mathbf{1})$	
1-ii-b	$(\bar{u}d)(d)(\bar{u})(\bar{e}\bar{e})$		$(+1, \mathbf{1})$ $(+1, \mathbf{8})$	$(+4/3, \bar{\mathbf{3}})$ $(+4/3, \bar{\mathbf{3}})$	$(+2, \mathbf{1})$ $(+2, \mathbf{1})$	
2-i-a	$(\bar{u}d)(d)(\bar{e})(\bar{u}\bar{e})$		$(+1, \mathbf{1})$ $(+1, \mathbf{8})$	$(+4/3, \bar{\mathbf{3}})$ $(+4/3, \bar{\mathbf{3}})$	$(+1/3, \bar{\mathbf{3}})$ $(+1/3, \bar{\mathbf{3}})$	
2-i-b	$(\bar{u}d)(\bar{e})(d)(\bar{u}\bar{e})$	(b)	$(+1, \mathbf{1})$ $(+1, \mathbf{8})$	$(0, \mathbf{1})$ $(0, \mathbf{8})$	$(+1/3, \bar{\mathbf{3}})$ $(+1/3, \bar{\mathbf{3}})$	RPV [58,60], LQ [65,66]
2-ii-a	$(\bar{u}d)(\bar{u})(\bar{e})(d\bar{e})$		$(+1, \mathbf{1})$ $(+1, \mathbf{8})$	$(+5/3, \mathbf{3})$ $(+5/3, \mathbf{3})$	$(+2/3, \mathbf{3})$ $(+2/3, \mathbf{3})$	
2-ii-b	$(\bar{u}d)(\bar{e})(\bar{u})(d\bar{e})$	(b)	$(+1, \mathbf{1})$ $(+1, \mathbf{8})$	$(0, \mathbf{1})$ $(0, \mathbf{8})$	$(+2/3, \mathbf{3})$ $(+2/3, \mathbf{3})$	RPV [58,60], LQ [65,66]
2-iii-a	$(d\bar{e})(\bar{u})(d)(\bar{u}\bar{e})$	(c)	$(-2/3, \bar{\mathbf{3}})$ $(-2/3, \bar{\mathbf{3}})$	$(0, \mathbf{1})$ $(0, \mathbf{8})$	$(+1/3, \bar{\mathbf{3}})$ $(+1/3, \bar{\mathbf{3}})$	RPV [58,60] RPV [58,60]
2-iii-b	$(d\bar{e})(d)(\bar{u})(\bar{u}\bar{e})$		$(-2/3, \bar{\mathbf{3}})$ $(-2/3, \bar{\mathbf{3}})$	$(-1/3, \mathbf{3})$ $(-1/3, \bar{\mathbf{6}})$	$(+1/3, \bar{\mathbf{3}})$ $(+1/3, \bar{\mathbf{3}})$	
3-i	$(\bar{u}\bar{u})(\bar{e})(\bar{e})(d\bar{d})$		$(+4/3, \bar{\mathbf{3}})$ $(+4/3, \mathbf{6})$	$(+1/3, \bar{\mathbf{3}})$ $(+1/3, \mathbf{6})$	$(-2/3, \mathbf{3})$ $(-2/3, \mathbf{6})$	only with $V_\rho$ and $V'_\rho$
3-ii	$(\bar{u}\bar{u})(d)(d)(\bar{e}\bar{e})$		$(+4/3, \bar{\mathbf{3}})$ $(+4/3, \mathbf{6})$	$(+5/3, \mathbf{3})$ $(+5/3, \mathbf{3})$	$(+2, \mathbf{1})$ $(+2, \mathbf{1})$	only with $V_\rho$
3-iii	$(d\bar{d})(\bar{u})(\bar{u})(\bar{e}\bar{e})$		$(+2/3, \mathbf{3})$ $(+2/3, \bar{\mathbf{6}})$	$(+4/3, \bar{\mathbf{3}})$ $(+4/3, \bar{\mathbf{3}})$	$(+2, \mathbf{1})$ $(+2, \mathbf{1})$	only with $V_\rho$
4-i	$(d\bar{e})(\bar{u})(\bar{u})(d\bar{e})$	(c)	$(-2/3, \bar{\mathbf{3}})$ $(-2/3, \bar{\mathbf{3}})$	$(0, \mathbf{1})$ $(0, \mathbf{8})$	$(+2/3, \mathbf{3})$ $(+2/3, \mathbf{3})$	RPV [58,60] RPV [58,60]
4-ii-a	$(\bar{u}\bar{u})(d)(\bar{e})(d\bar{e})$		$(+4/3, \bar{\mathbf{3}})$ $(+4/3, \mathbf{6})$	$(+5/3, \mathbf{3})$ $(+5/3, \mathbf{3})$	$(+2/3, \mathbf{3})$ $(+2/3, \mathbf{3})$	only with $V_\rho$ see Sec. 4 (this work)
4-ii-b	$(\bar{u}\bar{u})(\bar{e})(d)(d\bar{e})$		$(+4/3, \bar{\mathbf{3}})$ $(+4/3, \mathbf{6})$	$(+1/3, \bar{\mathbf{3}})$ $(+1/3, \mathbf{6})$	$(+2/3, \mathbf{3})$ $(+2/3, \mathbf{3})$	only with $V_\rho$
5-i	$(\bar{u}\bar{e})(d)(d)(\bar{u}\bar{e})$	(c)	$(-1/3, \mathbf{3})$ $(-1/3, \mathbf{3})$	$(0, \mathbf{1})$ $(0, \mathbf{8})$	$(+1/3, \bar{\mathbf{3}})$ $(+1/3, \bar{\mathbf{3}})$	RPV [58,60] RPV [58,60]
5-ii-a	$(\bar{u}\bar{e})(\bar{u})(\bar{e})(d\bar{d})$		$(-1/3, \mathbf{3})$ $(-1/3, \mathbf{3})$	$(+1/3, \bar{\mathbf{3}})$ $(+1/3, \mathbf{6})$	$(-2/3, \bar{\mathbf{3}})$ $(-2/3, \mathbf{6})$	only with $V'_\rho$
5-ii-b	$(\bar{u}\bar{e})(\bar{e})(\bar{u})(d\bar{d})$		$(-1/3, \mathbf{3})$ $(-1/3, \mathbf{3})$	$(-4/3, \mathbf{3})$ $(-4/3, \mathbf{3})$	$(-2/3, \bar{\mathbf{3}})$ $(-2/3, \mathbf{6})$	only with $V'_\rho$



# Topologies for short-range $0\nu\beta\beta$

**Topologies:** Feynman diagrams where no property of the fields is considered

- (i) All connected topologies with 3- and 4- point vertices and 6 external legs
- (ii) Remove tadpoles and self-energies (**divergent**)

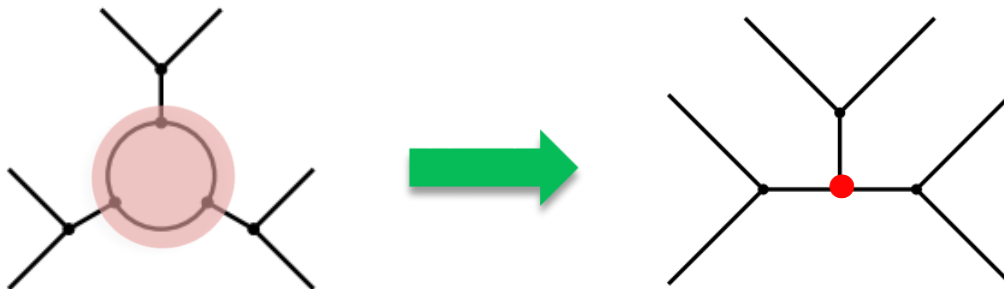


- (iii) Exclude non-renormalizable topologies



The 6 external legs are quark and lepton fields for short-range  $0\nu\beta\beta$

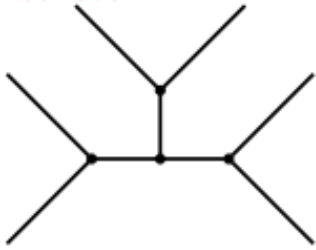
- (iv) Discard topologies with 3-point loop vertices



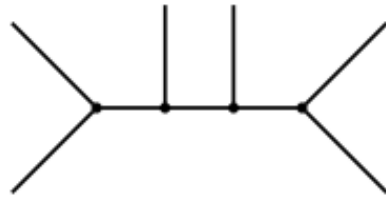
any loop with 3 external legs can be compressible to a renormalizable trilinear vertex

➤ **2 tree + 6 one-loop renormalizable topologies**

Tree level



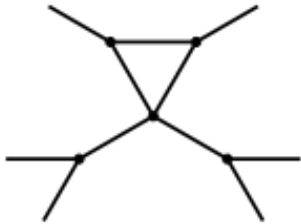
N-0-1



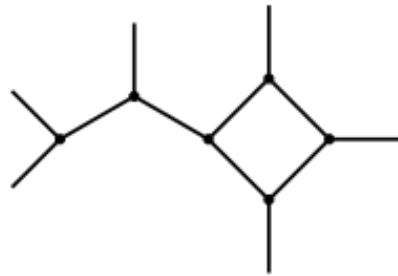
N-0-2

[Chen, Ding, Yao, 2110.15347]

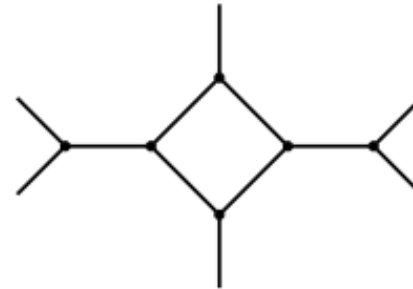
One-loop level



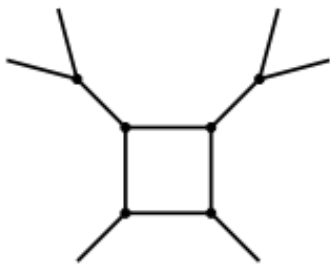
N-1-1



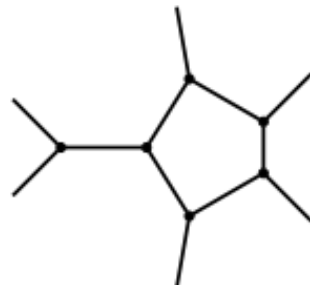
N-1-2



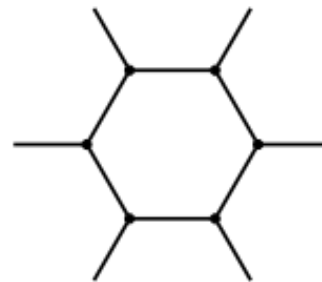
N-1-3



N-1-4



N-1-5

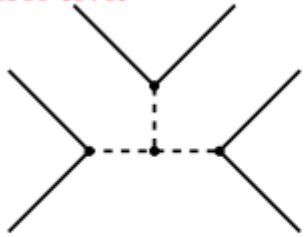


N-1-6

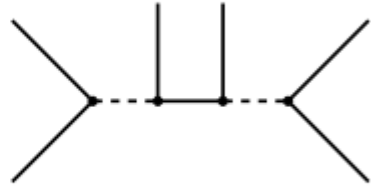
# field insertions: topologies $\rightarrow$ diagrams

Focusing only on fermions and scalar bosons [**Not considering new gauge bosons**]:

Tree level



N-0-1-1



N-0-2-1

➤ Three kinds of renormalizable vertices

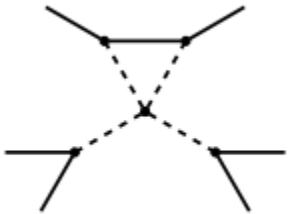
① fermion-fermion-scalar (FFS)

② scalar-scalar-scalar (SSS)

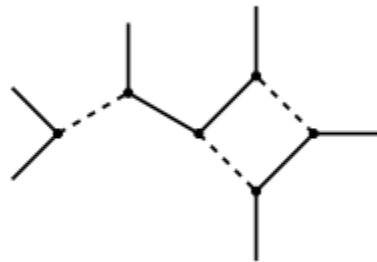
③ scalar-scalar-scalar-scalar (SSSS)

[Chen, Ding, Yao, 2110.15347]

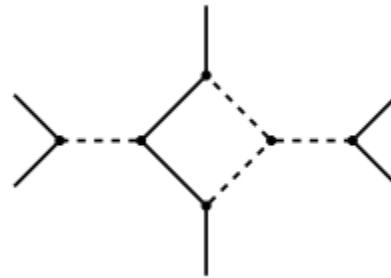
One-loop level



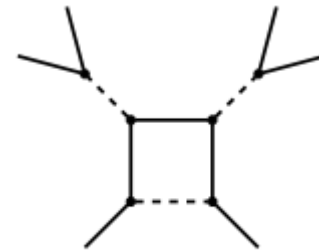
N-1-1-1



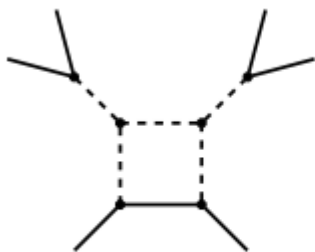
N-1-2-1



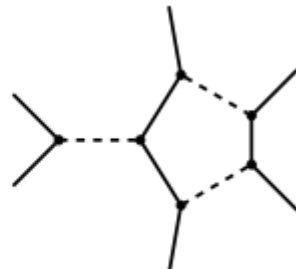
N-1-3-1



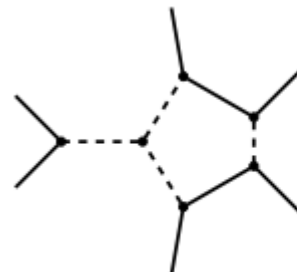
N-1-4-1



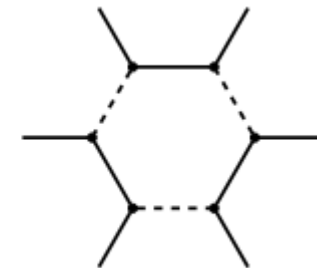
N-1-4-2



N-1-5-1



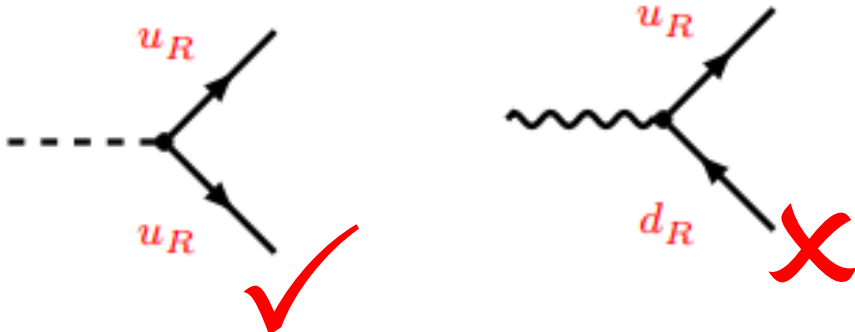
N-1-5-2



N-1-6-1

# ➤ Diagrams continuum: attach external fields

Lorentz invariance fixes the mediator to be **scalar or vector** by chirality of external fermions



The number of possible diagrams

## Classify $0\nu\beta\beta$ operators

Notation	$0\nu\beta\beta$ decay operators	External fields
N1	$\mathcal{O}_1^{SR}, \mathcal{O}_2^{SR}$	$\bar{Q}, Q, \bar{u}_R, d_R, \bar{\ell}, \ell^c$
N2	$\mathcal{O}_3^{SR}, \mathcal{O}_4^{SR}$	$Q, \bar{Q}, \bar{u}_R, \bar{u}_R, \bar{\ell}, \ell^c$
N3	$\mathcal{O}_5^{SR}, \mathcal{O}_6^{SR}$	$\bar{Q}, \bar{Q}, d_R, d_R, \bar{\ell}, \ell^c$
N4	$\mathcal{O}_7^{SR}$	$\bar{u}_R, \bar{u}_R, d_R, d_R, \bar{e}_R, e_R^c$
N5	$\mathcal{O}_8^{SR}, \mathcal{O}_9^{SR}$	$\bar{u}_R, \bar{Q}, d_R, d_R, \bar{\ell}, e_R^c$
N6	$\mathcal{O}_{10}^{SR}, \mathcal{O}_{11}^{SR}$	$\bar{u}_R, \bar{u}_R, Q, d_R, \bar{\ell}, e_R^c$

		$\mathcal{O}_i^{SR}$	N1	N2	N3	N4	N5	N6
Tree	TOPO							
		N-0-1-1	2	2	5	2	2	2
		N-0-2-1	11	6	18	6	11	11
One-loop		N-1-1-1	6	5	12	5	6	6
		N-1-2-1	96	30	54	30	96	96
		N-1-3-1	11	9	21	9	12	12
		N-1-4-1	11	6	18	6	11	11
		N-1-4-2	11	6	18	6	11	11
		N-1-5-1	48	18	30	18	48	48
		N-1-5-2	48	18	30	18	48	48
		N-1-6-1	60	18	18	18	60	60

The redundant diagrams should be removed.

# Determine quantum numbers: diagrams → models

The  $SU(3)_c \times SU(2)_L \times U(1)_Y$  quantum numbers of the mediators fields are fixed by gauge invariance of each interaction vertex

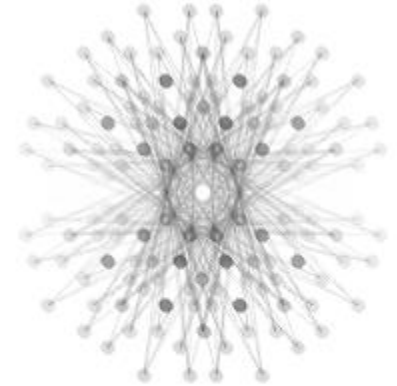
- 3-point vertex:  $\bar{F}_1 F_2 S, S_1 S_2 S_3$

$$\begin{aligned} n_{\bar{F}_1} \otimes n_{F_2} \otimes n_S \supset \mathbf{1}, \quad Y_{\bar{F}_1} + Y_{F_2} + Y_S = 0 \\ n_{S_1} \otimes n_{S_2} \otimes n_{S_3} \supset \mathbf{1}, \quad Y_{S_1} + Y_{S_2} + Y_{S_3} = 0 \end{aligned}$$

$n_X$  denotes the  $SU(2)_L$  or  $SU(3)_c$  representation of the field  $X$

- 4-point vertex:  $S_1 S_2 S_3 S_4$

$$n_{S_1} \otimes n_{S_2} \otimes n_{S_3} \otimes n_{S_4} \supset \mathbf{1}, \quad \sum_i Y_{S_i} = 0$$



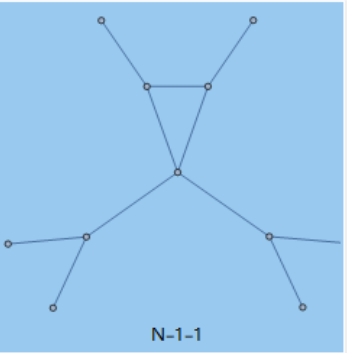
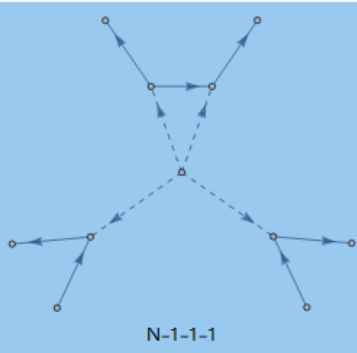
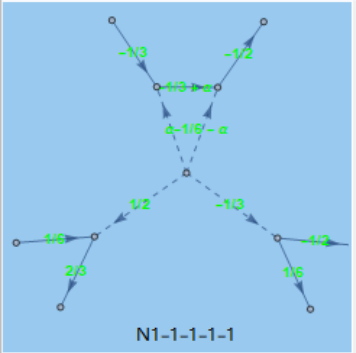
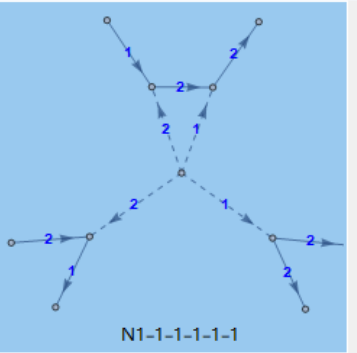
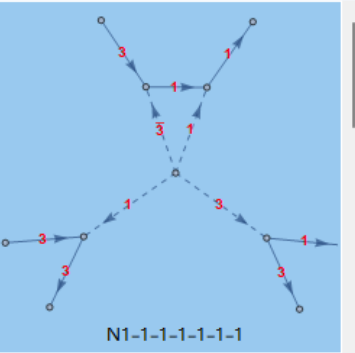
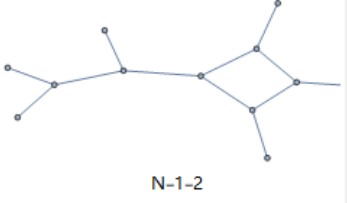
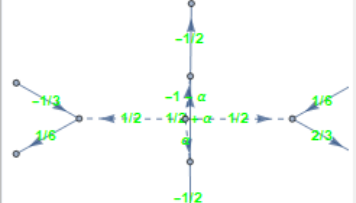
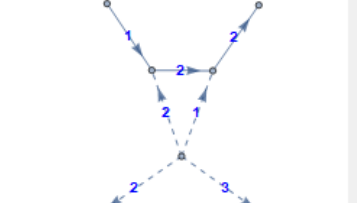
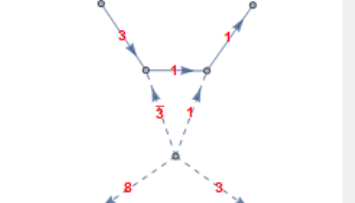
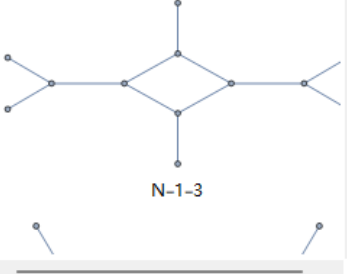
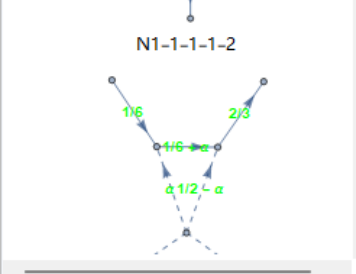
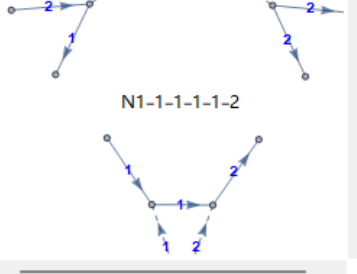
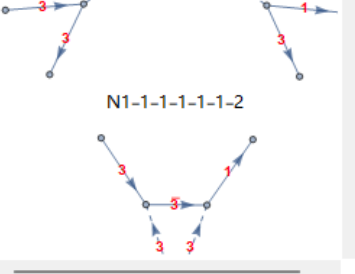
GROUPMATH

- The tensor products of  $SU(3)_c$  representations are a bit complex

$$\begin{aligned} \mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \mathbf{6}, \quad \mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}, \quad \mathbf{3} \otimes \mathbf{6} = \mathbf{8} \oplus \mathbf{10}, \quad \mathbf{3} \otimes \bar{\mathbf{6}} = \bar{\mathbf{3}} \oplus \bar{\mathbf{15}}, \\ \mathbf{3} \otimes \mathbf{8} = \mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{15}, \quad \mathbf{3} \otimes \mathbf{10} = \mathbf{15} \oplus \mathbf{15}, \quad \mathbf{3} \otimes \bar{\mathbf{10}} = \bar{\mathbf{6}} \oplus \bar{\mathbf{24}}, \end{aligned}$$

The Mathematica package **GroupMath** can facilitate the determination of SM quantum numbers.

The possible short-range  $0\nu\beta\beta$  models of at 1-loop level are collected in the attachment [http://staff.ustc.edu.cn/~dingji/supplementary\\_materials/0nbb.zip](http://staff.ustc.edu.cn/~dingji/supplementary_materials/0nbb.zip)

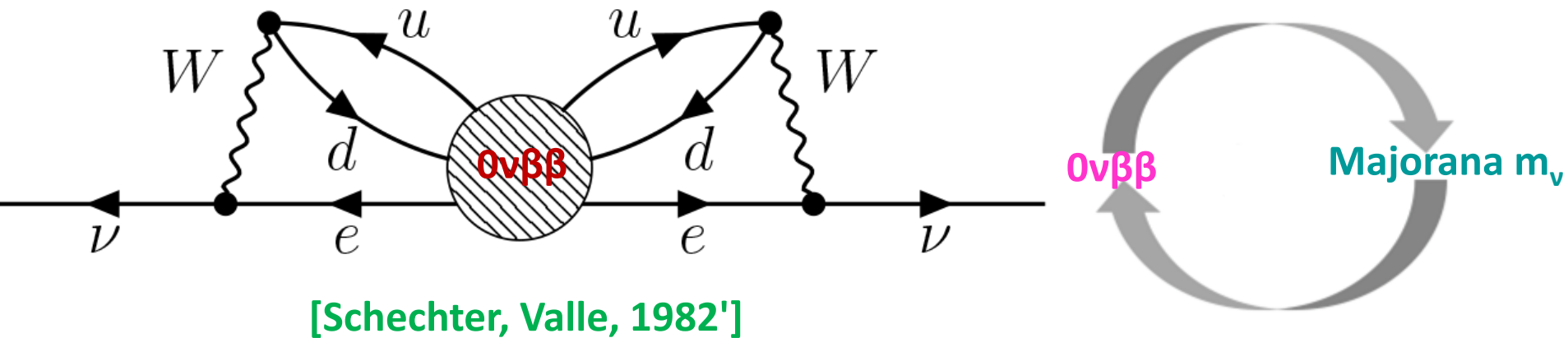
Model Implementations of Neutrinoless Double $\beta$ Decay up to one-loop				
Loop: <input type="checkbox"/> 0 <input checked="" type="checkbox"/> 1	Class: <input checked="" type="checkbox"/> N1 <input type="checkbox"/> N2 <input type="checkbox"/> N3 <input type="checkbox"/> N4 <input type="checkbox"/> N5 <input type="checkbox"/> N6			
Topology	Diagram	External Legs & $U_V(1)$	$SU_L(2)$	$SU_C(3)$
 <p>N-1-1</p>	 <p>N-1-1-1</p>	 <p>N1-1-1-1-1</p>	 <p>N1-1-1-1-1-1</p>	 <p>N1-1-1-1-1-1-1</p>
 <p>N-1-2</p>		 <p>N1-1-1-1-2</p>	 <p>N1-1-1-1-1-2</p>	 <p>N1-1-1-1-1-1-2</p>
 <p>N-1-3</p>		 <p>N1-1-1-1-1-2</p>	 <p>N1-1-1-1-1-1-2</p>	 <p>N1-1-1-1-1-1-1-2</p>

# Genuine models

**Genuine:** A one-loop  $0\nu\beta\beta$  model is called “genuine” if it fulfills the conditions:

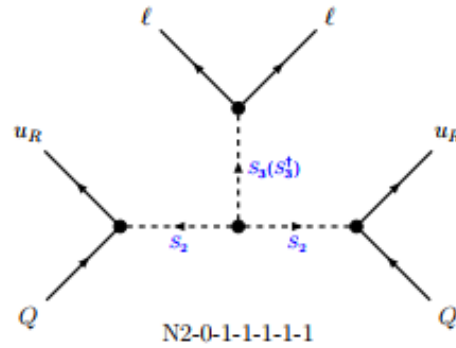
- ① It is renormalizable
- ② The leading contribution to  $0\nu\beta\beta$  decay arises at 1-loop
- ③ No need for extra symmetries beyond those of the SM

- Black box theorem implies that the mass mechanism is always present in  $0\nu\beta\beta$  decay. In  $0\nu\beta\beta$  models, Majorana neutrino masses usually are generated **at less than four-loop order**.

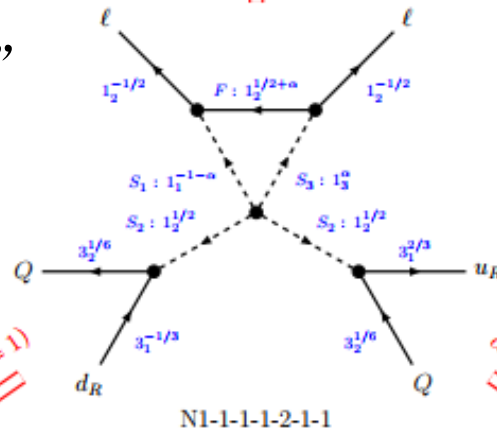


- The short range contribution could **dominate** over the mass mechanism without fine-tuning in some parameter space, if the neutrino mass is generated at least at **two-loop** order.

Certain values of hypercharge is excluded by “genuineness”

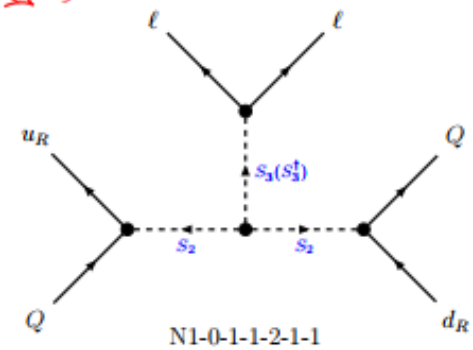
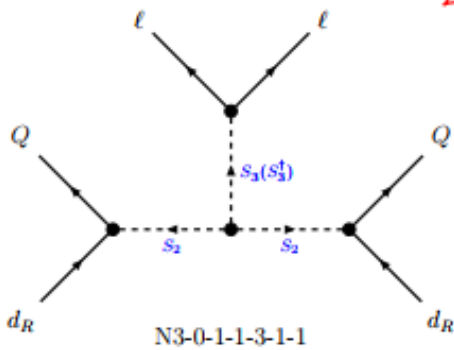


$\alpha = -1 (\alpha = 1)$



$\alpha = -1 (\alpha = 1)$

$\alpha = -1 (\alpha = 1)$



## Filter models

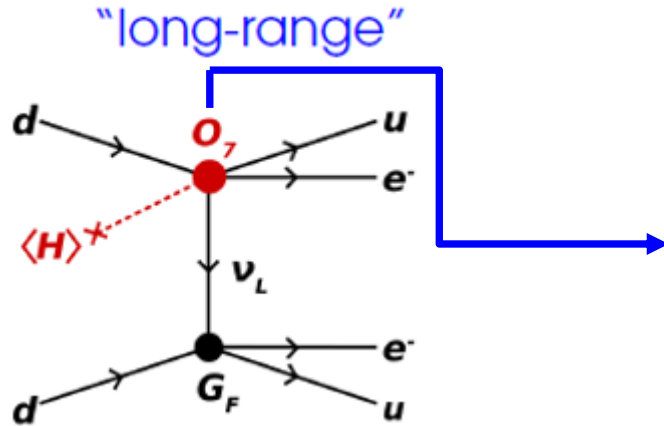




# Decomposing the long-range $0\nu\beta\beta$ operators

➤ Long-range mechanism is **not subject to helicity suppression!**

$\Delta L = 2$



$$\begin{aligned} \mathcal{O}_1^{LR} &= \epsilon^{ik} \epsilon^{jl} (\bar{\ell}_i^c \ell_j) (\bar{d}_R Q_k) H_l, \\ \mathcal{O}_2^{LR} &= \epsilon^{ik} \epsilon^{jl} (\bar{\ell}_i^c \gamma^{\mu\nu} \ell_j) (\bar{d}_R \gamma_{\mu\nu} Q_k) H_l, \\ \mathcal{O}_3^{LR} &= \epsilon^{jk} (\bar{\ell}_i^c \ell_j) (\bar{Q}^i u_R) H_k, \\ \mathcal{O}_4^{LR} &= (\bar{\ell}_i^c \gamma^\mu e_R) (\bar{d}_R \gamma_\mu u_R) \epsilon^{ij} H_j \end{aligned}$$

topologies



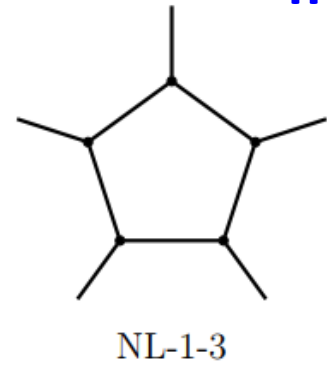
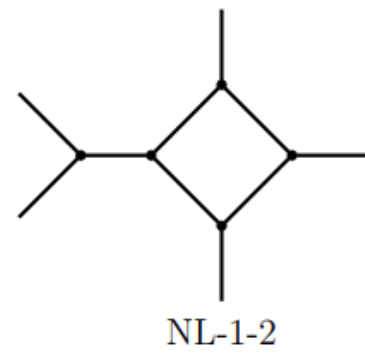
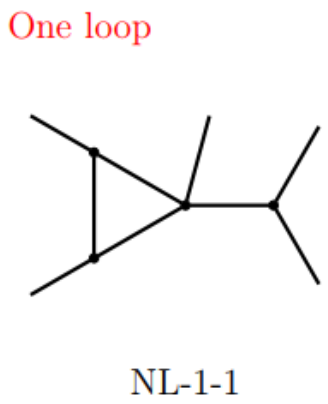
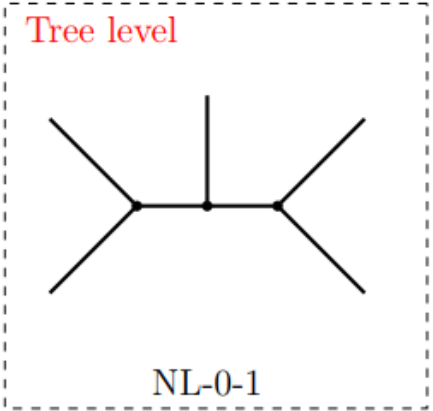
diagrams



models

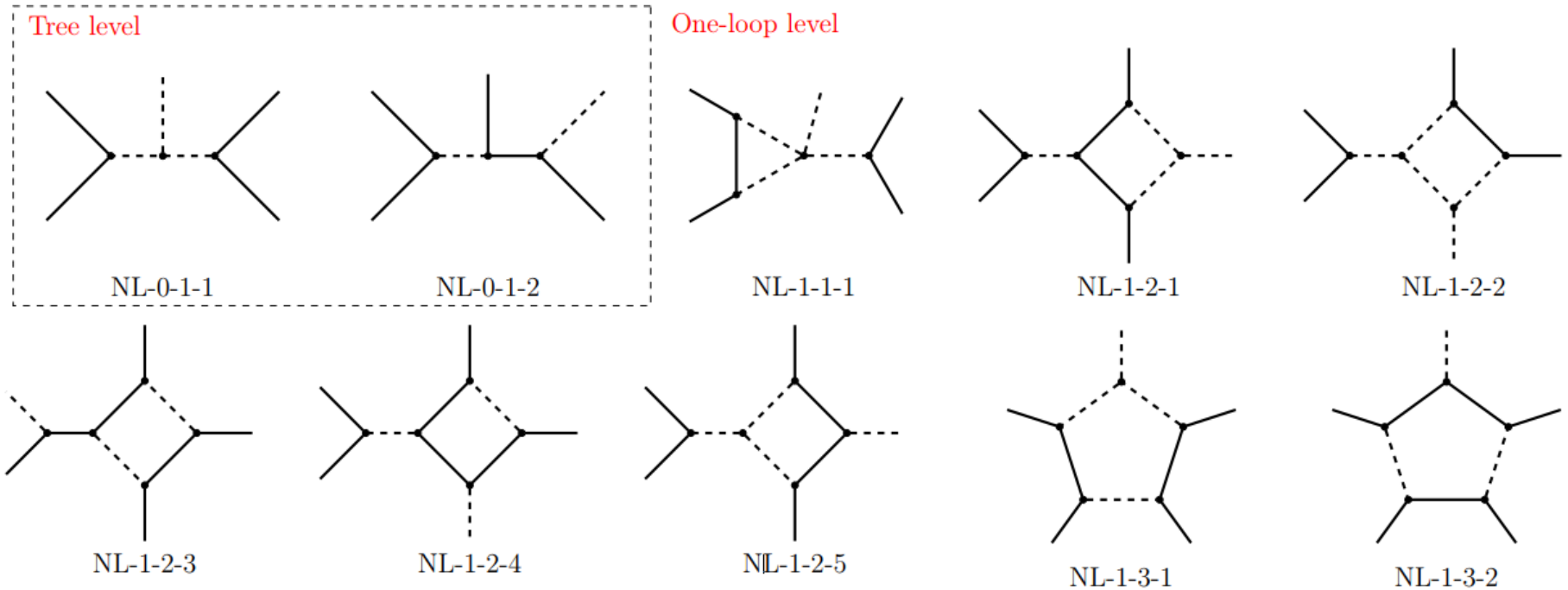
[Babu, Leung, hep-ph/0106054; Helo, Hirsch, Ota, 1602.03362; Lehman, 1410.4193]

➤ Topologies



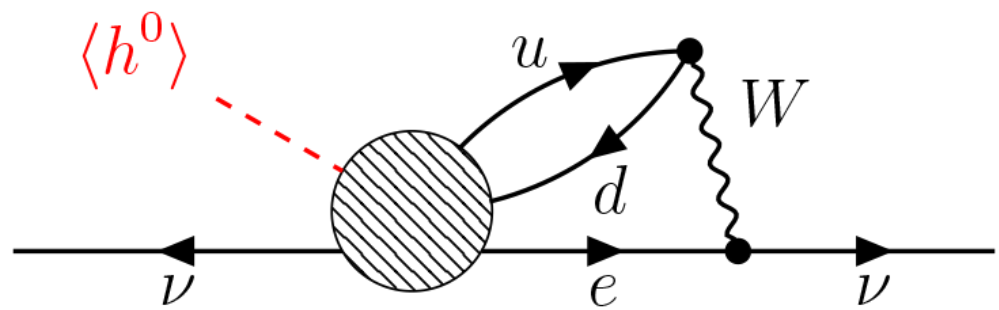
[Chen, Ding, Yao, 2301.02503] 17

➤ Diagrams



➤ Models: large variety of possible realizations accessible at high-energy colliders and high-intensity facilities, all genuine long-range  $0\nu\beta\beta$  models up to 1-loop in the file [http://staff.ustc.edu.cn/~dinggj/supplementary\\_materials/Long\\_range\\_0nbb.zip](http://staff.ustc.edu.cn/~dinggj/supplementary_materials/Long_range_0nbb.zip)

➤ Black box theorem in long-range  $0\nu\beta\beta$  decay:  $\Delta L = 2$  operators  $\rightarrow 0\nu\beta\beta$  &  $\nu$  mass



Majorana neutrino masses are generated at least at the **2-loop order**, regardless of long-range  $0\nu\beta\beta$  operators

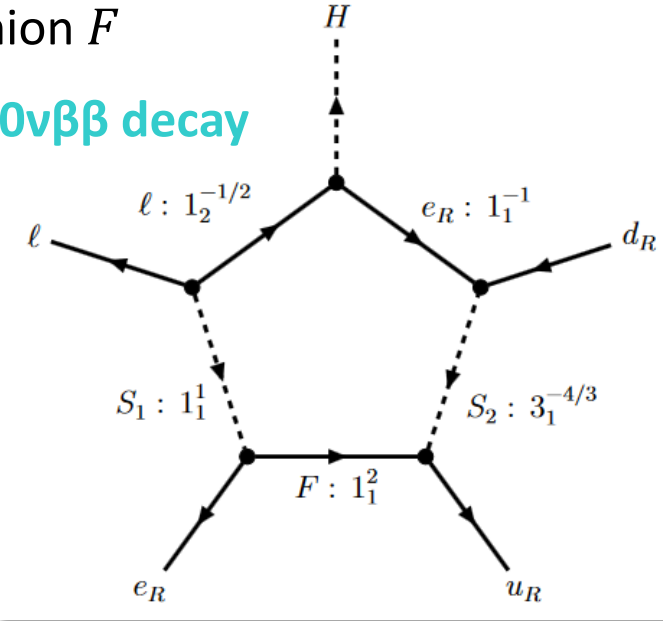
# An example model of long-range $0\nu\beta\beta$ decay

There are many realizations, and a benchmark model is presented here

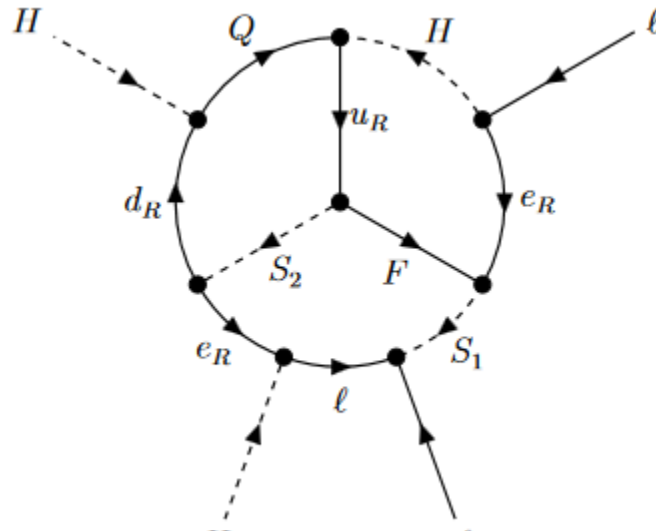
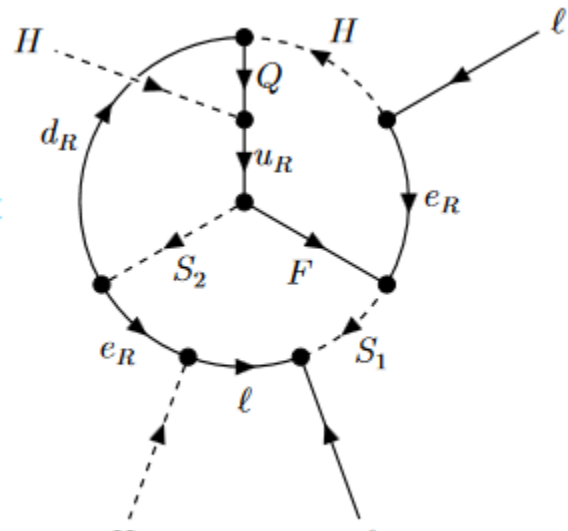
- 3 new fields: two scalars  $S_1, S_2$  and a vector-like fermion  $F$

Fields	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$S_1$	<b>1</b>	<b>1</b>	1
$S_2$	<b>3</b>	<b>1</b>	$-4/3$
$F$	<b>1</b>	<b>1</b>	2

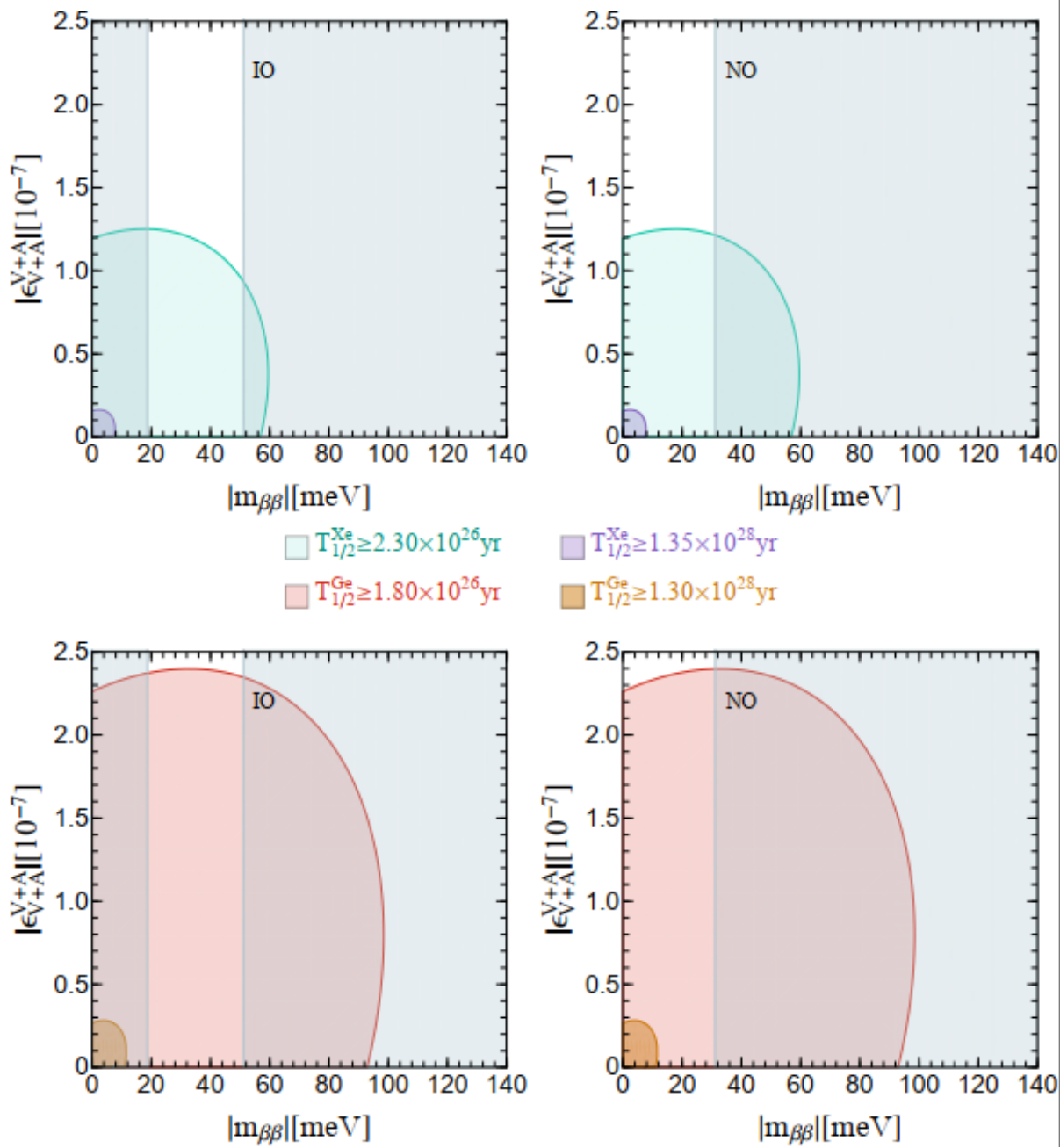
1-loop  $0\nu\beta\beta$  decay

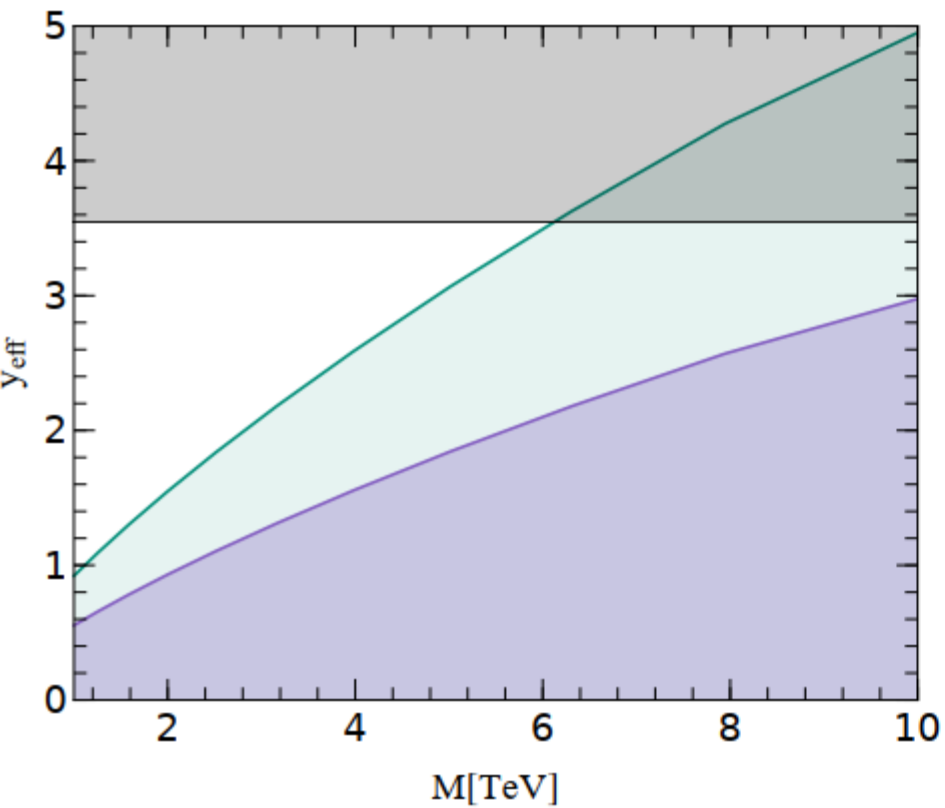


- Majorana neutrino masses generated at **3-loop**



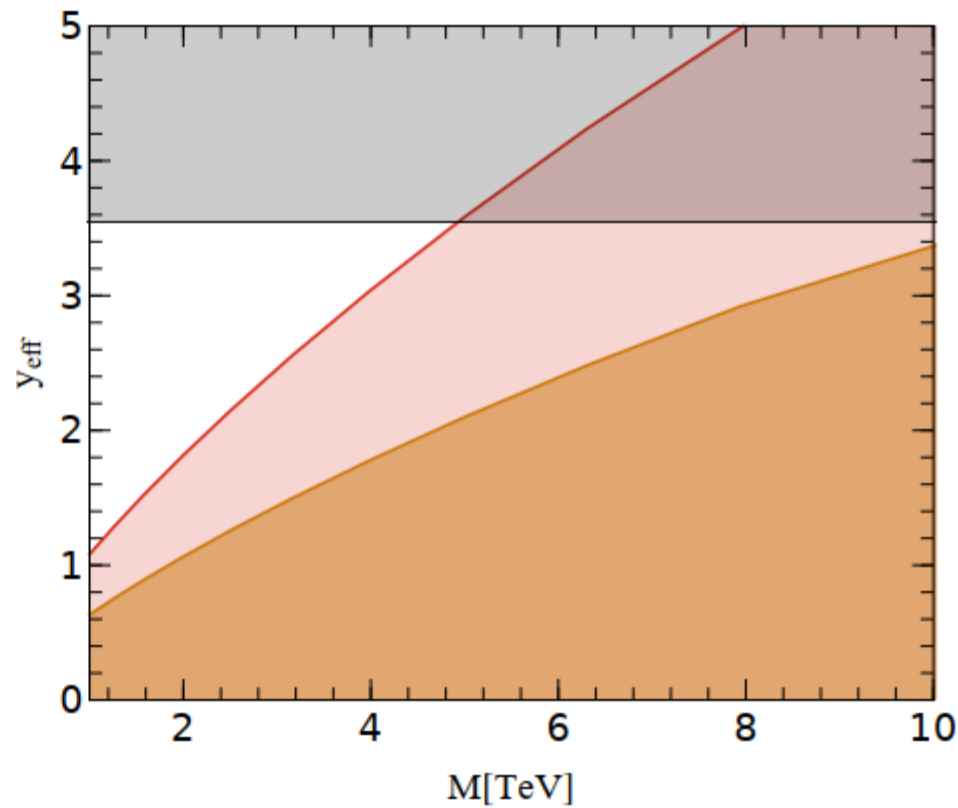
➤ Future ton-scale experiments impose strong constraint on the model and new physics contribution.





$T_{1/2}^{\text{Xe}} \geq 2.30 \times 10^{26} \text{ yr}$

$T_{1/2}^{\text{Xe}} \geq 1.35 \times 10^{28} \text{ yr}$



$T_{1/2}^{\text{Ge}} \geq 1.80 \times 10^{26} \text{ yr}$

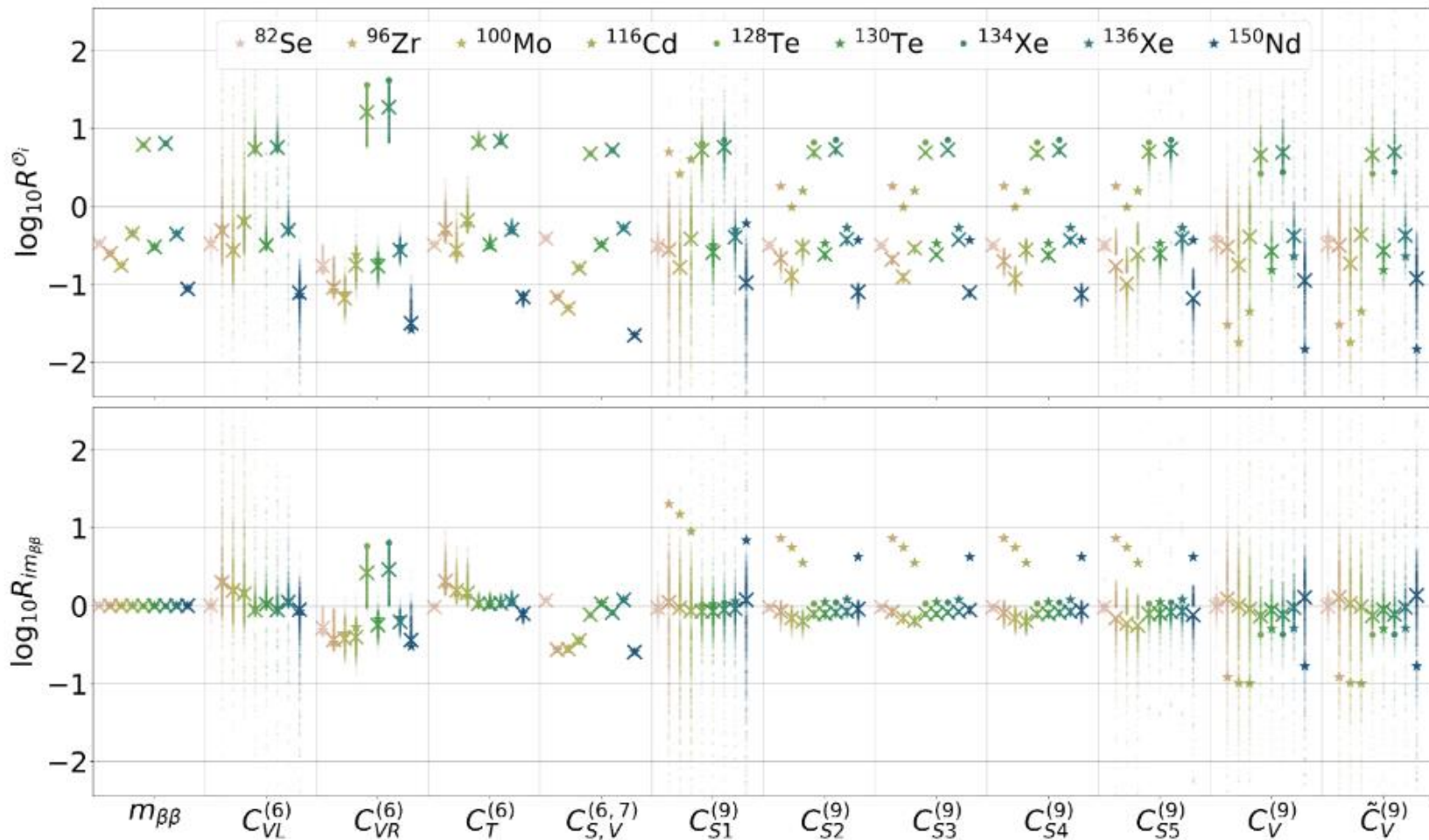
$T_{1/2}^{\text{Ge}} \geq 1.30 \times 10^{28} \text{ yr}$

# Distinguishing different $0\nu\beta\beta$ mechanisms

- Comparison of the decay rates obtained using different isotopes

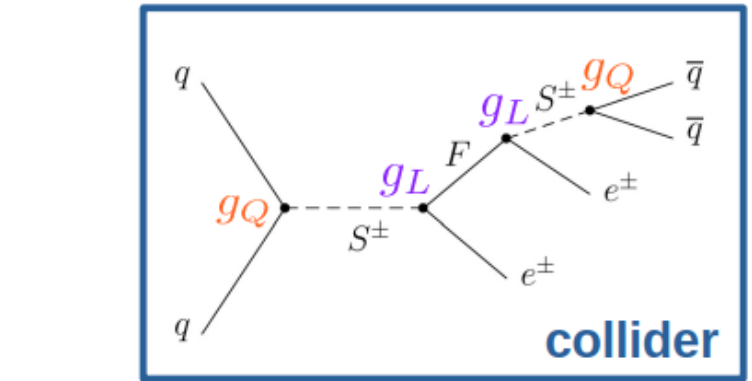
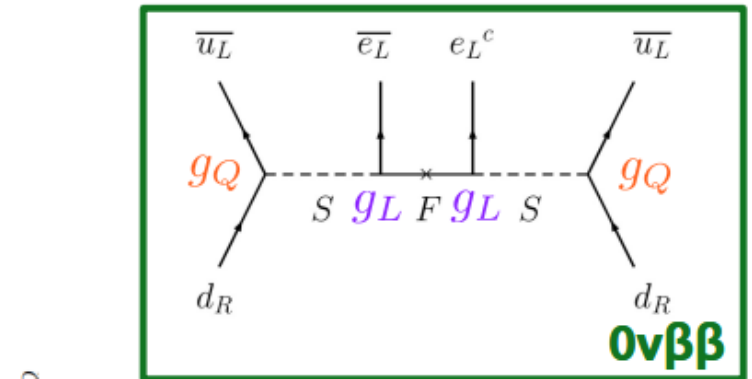
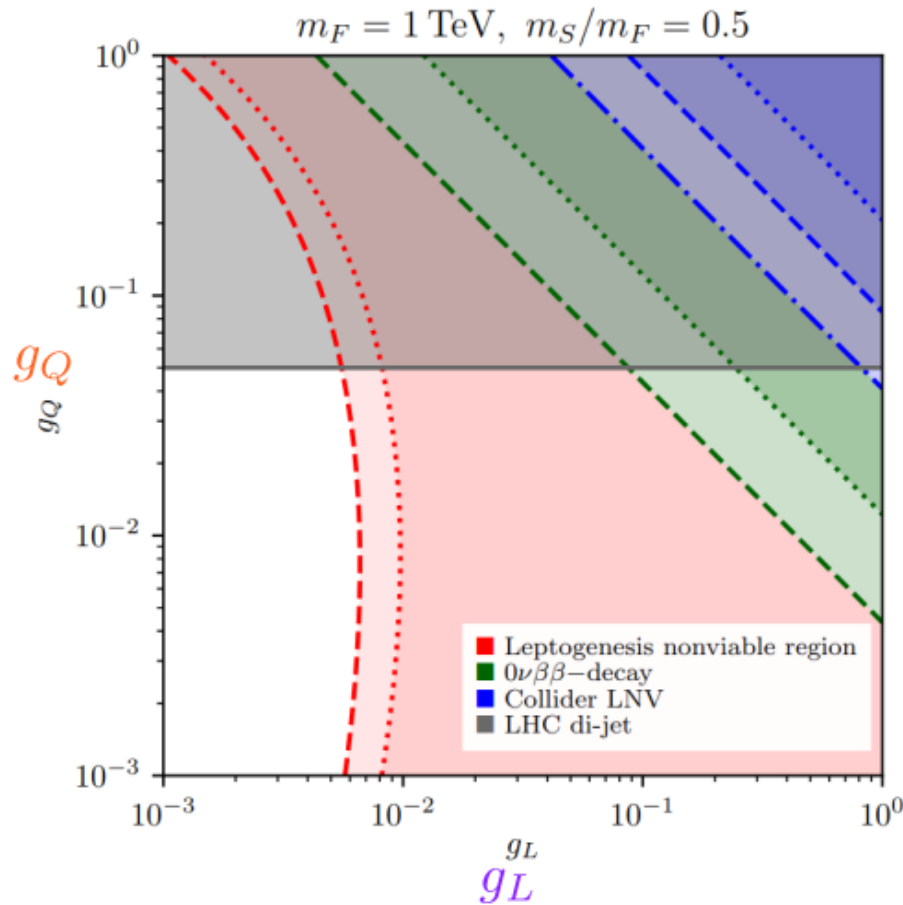
$$R^{O_i(A X)} \equiv \frac{T_{1/2}^{O_i(A X)}}{T_{1/2}^{O_i(^{76}\text{Ge})}} = \frac{\sum_j |\mathcal{M}_j^{O_i(^{76}\text{Ge})}|^2 G_j^{O_i(^{76}\text{Ge})}}{\sum_k |\mathcal{M}_k^{O_i(A X)}|^2 G_k^{O_i(A X)}}$$

[Graf,Lindner,Scholer,2204.10845]



➤ Combination of  $0\nu\beta\beta$  decay, collider measurement and cosmology

$$\mathcal{L} \supset g_Q \bar{Q} S d_R + g_L \bar{L} (i\tau^2) S^* F + \lambda_{HS} (S^\dagger H)^2 + \text{h.c.}$$



See Michael's Talk

[Harz, Ramsey-Musolf, Shen, Urrutia-Quiroga, 2106.10838;  
 Graesser, Li, Ramsey-Musolf, Shen, Urrutia-Quiroga, 2202.01237]

# Summary

---

- $0\nu\beta\beta$  is the most sensitive probe to the Majorana nature of neutrinos. However, there are many possible underlying physics mechanisms which can be divided into **mass mechanism**, **short-range mechanism** and **long-range mechanism**.
- We have performed systematic decomposition of both  $d=7$  short-range and  $d=9$  long-range  $0\nu\beta\beta$  operators, the possible renormalizable UV realizations are presented. An example model is given.
- Many open questions: the new  $0\nu\beta\beta$  models in future colliders and LFV searches, implications in cosmology and leptogenesis....
- Further theoretical and experimental efforts are needed to find out whether  $0\nu\beta\beta$  exists and what is the underlying mechanism.

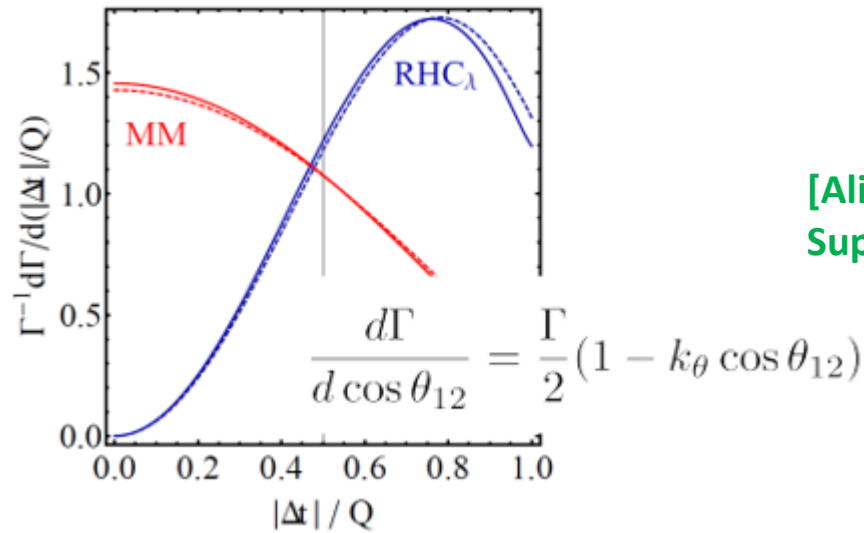
***Thank you for your attention!***



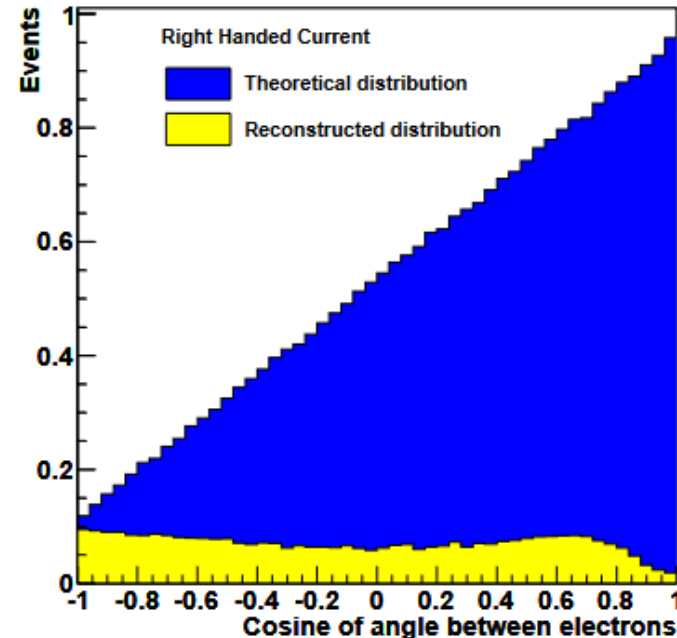
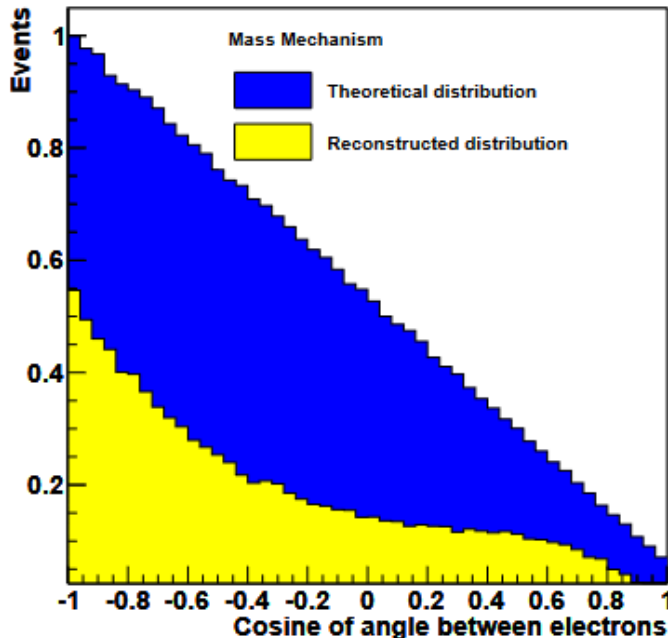
# Backup

# determine $0\nu\beta\beta$ mechanisms

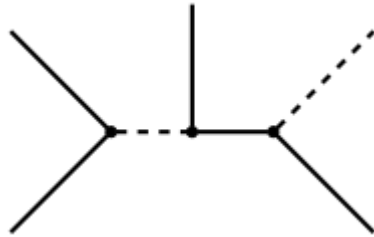
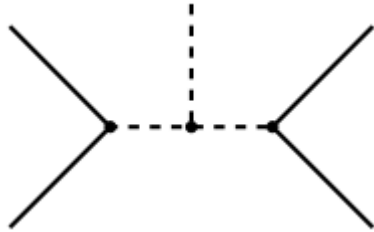
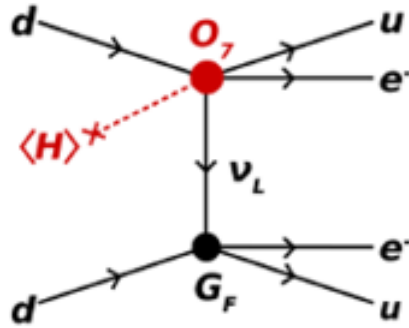
- measure the angular and energy distributions of electron



[Ali, Borisov, Zhuridov, 0706.4165;  
SuperNEMO Collaborarion,1005.1241]



# tree-level decomposition of long-range $0\nu\beta\beta$ operators



#	Decompositions	Mediators	Projection to the basis ops.	$m_\nu$ @tree	$m_\nu$ @1loop	$m_\nu$ @2loop	
#1	$(L_\alpha L_\beta)(H)(\overline{d}_R Q)$	$S(1,1)_{+1}$	$S'(1,2)_{+\frac{1}{2}}$	$-\mathcal{O}_{3a}(\alpha, \beta)$	—	$T\nu$ I-ii w. $\overline{\ell}_R L S^\dagger$	$T2_4^B(\alpha \neq \beta)$ $\mathcal{O}_3^7$ in [38]
		$S(1,3)_{+1}$	$S'(1,2)_{+\frac{1}{2}}$	$-\mathcal{O}_{3b}(\alpha, \beta) - \mathcal{O}_{3b}(\beta, \alpha)$	type II		
#2	$(L_\alpha Q)(H)(\overline{d}_R L_\beta)$	$S(\overline{3},1)_{+\frac{1}{3}}$	$S'(\overline{3},2)_{-\frac{1}{6}}$	$\frac{1}{2}\mathcal{O}_{3b}(\alpha, \beta) - \frac{1}{2}\mathcal{O}_{3b}^{\text{ten.}}(\alpha, \beta)$	—	$T\nu$ I-ii [53] $\mathcal{O}_3^8$ in [38]	[14, 68]
		$S(\overline{3},3)_{+\frac{1}{3}}$	$S'(\overline{3},2)_{-\frac{1}{6}}$	$\frac{1}{2}\mathcal{O}_{3a}(\alpha, \beta) - \frac{1}{2}\mathcal{O}_{3a}^{\text{ten.}}(\alpha, \beta)$ $-\frac{1}{2}\mathcal{O}_{3b}(\beta, \alpha) - \frac{1}{2}\mathcal{O}_{3b}^{\text{ten.}}(\beta, \alpha)$	—	$T\nu$ I-ii [53] $\mathcal{O}_3^9$ in [38]	[14]
#3	$(L_\alpha L_\beta)(Q)(\overline{d}_R H)$	$S(1,1)_{+1}$	$\psi_{L,R}(\mathbf{3},2)_{-\frac{5}{6}}$	$-\mathcal{O}_{3a}(\alpha, \beta)$	—	$T\nu$ I-ii [53] w. $S^\dagger H H'$	$T2_1^B(\alpha \neq \beta)$ $\mathcal{O}_3^1$ in [38]
		$S(1,3)_{+1}$	$\psi_{L,R}(\mathbf{3},2)_{-\frac{5}{6}}$	$-\mathcal{O}_{3b}(\alpha, \beta) - \mathcal{O}_{3b}(\beta, \alpha)$	type II		
#4	$(L_\alpha H)(Q)(\overline{d}_R L_\beta)$	$\psi_R(\mathbf{1},1)_0$	$S(\overline{3},2)_{-\frac{1}{6}}$	$\frac{1}{2}\mathcal{O}_{3b}(\beta, \alpha) + \frac{1}{2}\mathcal{O}_{3b}^{\text{ten.}}(\beta, \alpha)$	type I		
		$\psi_R(\mathbf{1},3)_0$	$S(\overline{3},2)_{-\frac{1}{6}}$	$-\frac{1}{2}\mathcal{O}_{3a}(\alpha, \beta) + \frac{1}{2}\mathcal{O}_{3a}^{\text{ten.}}(\alpha, \beta)$ $-\frac{1}{2}\mathcal{O}_{3b}(\alpha, \beta) + \frac{1}{2}\mathcal{O}_{3b}^{\text{ten.}}(\alpha, \beta)$	type III		
#5	$(L_\alpha L_\beta)(\overline{d}_R)(QH)$	$S(1,1)_{+1}$	$\psi_{L,R}(\mathbf{3},1)_{+\frac{2}{3}}$	$\mathcal{O}_{3a}(\alpha, \beta)$	—	$T\nu$ I-ii [53] w. $S^\dagger H H'$	$T2_2^B(\alpha \neq \beta)$ $\mathcal{O}_3^2$ in [38]
		$S(1,3)_{+1}$	$\psi_{L,R}(\mathbf{3},3)_{+\frac{2}{3}}$	$-\mathcal{O}_{3b}(\alpha, \beta) - \mathcal{O}_{3b}(\beta, \alpha)$	type II		
#6	$(L_\alpha Q)(\overline{d}_R)(L_\beta H)$	$S(\overline{3},1)_{+\frac{1}{3}}$	$\psi_R(\mathbf{1},1)_0$	$-\frac{1}{2}\mathcal{O}_{3b}(\alpha, \beta) + \frac{1}{2}\mathcal{O}_{3b}^{\text{ten.}}(\alpha, \beta)$	type I		
		$S(\overline{3},3)_{+\frac{1}{3}}$	$\psi_R(\mathbf{1},3)_0$	$\frac{1}{2}\mathcal{O}_{3a}(\alpha, \beta) - \frac{1}{2}\mathcal{O}_{3a}^{\text{ten.}}(\alpha, \beta)$ $-\frac{1}{2}\mathcal{O}_{3b}(\beta, \alpha) - \frac{1}{2}\mathcal{O}_{3b}^{\text{ten.}}(\beta, \alpha)$	type III		
#7	$(L_\alpha Q)(L_\beta)(\overline{d}_R H)$	$S(\overline{3},1)_{+\frac{1}{3}}$	$\psi_{L,R}(\mathbf{3},2)_{-\frac{5}{6}}$	$\frac{1}{2}\mathcal{O}_{3b}(\alpha, \beta) - \frac{1}{2}\mathcal{O}_{3b}^{\text{ten.}}(\alpha, \beta)$	—	$T\nu$ I-iii $\mathcal{O}_3^4$ in [38]	
		$S(\overline{3},3)_{+\frac{1}{3}}$	$\psi_{L,R}(\mathbf{3},2)_{-\frac{5}{6}}$	$\frac{1}{2}\mathcal{O}_{3a}(\alpha, \beta) - \frac{1}{2}\mathcal{O}_{3a}^{\text{ten.}}(\alpha, \beta)$ $-\frac{1}{2}\mathcal{O}_{3b}(\beta, \alpha) - \frac{1}{2}\mathcal{O}_{3b}^{\text{ten.}}(\beta, \alpha)$	—	$T\nu$ I-iii $\mathcal{O}_3^5$ in [38]	
#8	$(\overline{d}_R L_\alpha)(L_\beta)(QH)$	$S(\mathbf{3},2)_{+\frac{1}{6}}$	$\psi_{L,R}(\mathbf{3},1)_{+\frac{2}{3}}$	$-\frac{1}{2}\mathcal{O}_{3a}(\alpha, \beta) - \frac{1}{2}\mathcal{O}_{3a}^{\text{ten.}}(\alpha, \beta)$	—	—	$T2_2^B(m_\nu)_{\alpha \neq \beta}$ $\mathcal{O}_3^3$ in [38], [43]
		$S(\mathbf{3},2)_{+\frac{1}{6}}$	$\psi_{L,R}(\mathbf{3},3)_{+\frac{2}{3}}$	$\frac{1}{2}\mathcal{O}_{3b}(\alpha, \beta) + \frac{1}{2}\mathcal{O}_{3b}^{\text{ten.}}(\alpha, \beta)$ $+\frac{1}{2}\mathcal{O}_{3b}(\beta, \alpha) - \frac{1}{2}\mathcal{O}_{3b}^{\text{ten.}}(\beta, \alpha)$	—	$T\nu$ I-iii $\mathcal{O}_3^6$ in [38]	
#9	$(L_\alpha H)(L_\beta)(\overline{d}_R Q)$	$\psi_R(\mathbf{1},1)_0$	$S(1,2)_{+\frac{1}{2}}$	$\mathcal{O}_{3b}(\beta, \alpha)$	type I		
		$\psi_R(\mathbf{1},3)_0$	$S(1,2)_{+\frac{1}{2}}$	$\mathcal{O}_{3a}(\alpha, \beta) + \mathcal{O}_{3b}(\alpha, \beta)$	type III		