Lattice QCD calculation of $0\nu 2\beta$ Decay

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Zi-Yu Wang (王子毓) and Teng Wang (王腾)

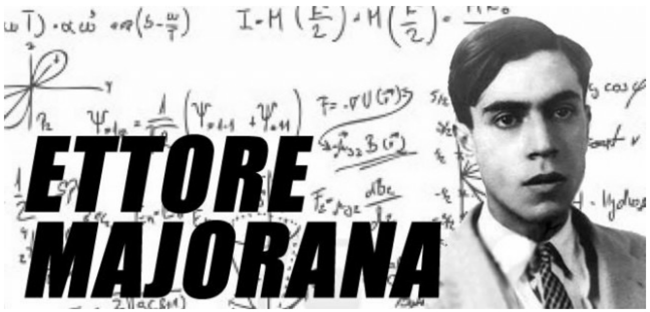
Plan

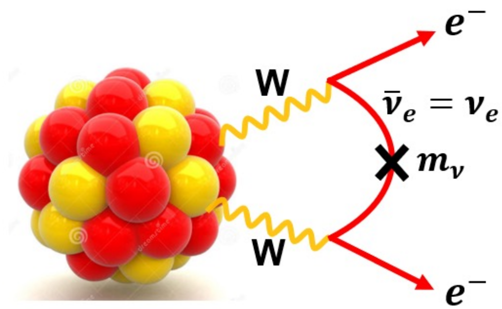
1. Background: Lattice QCD and $0\nu2\beta$ decay
2. Lattice work 1: pionic $0\nu2\beta$ decay
3. Lattice work 2: sterile neutrino contribution
4. Conclusion and outlook

Plan

Why are $0\nu2\beta$ decays so important?

- Test the nature of neutrino: Dirac fermion? Majorana fermion?





1937: Majorana fermion [1]

$$\nu = \bar{\nu}\ ?$$

[1] Ettore Majorana. Nuovo Cim. 1937, 14:171–184

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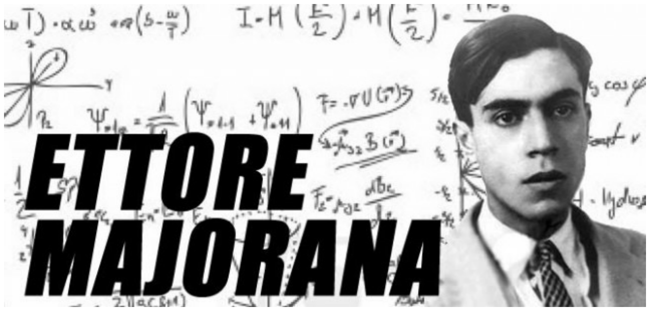
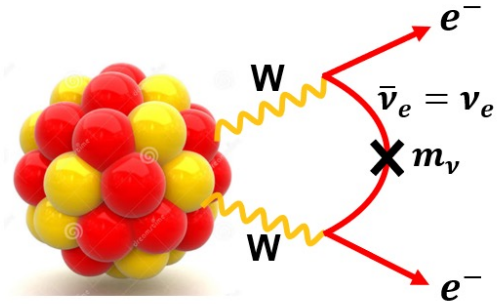


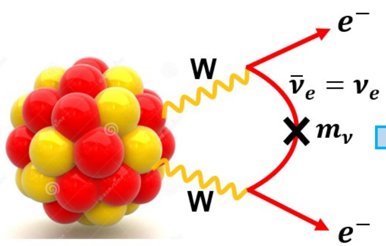
1937 : Majorana fermion [1]

$$\nu = \bar{\nu}\,?$$

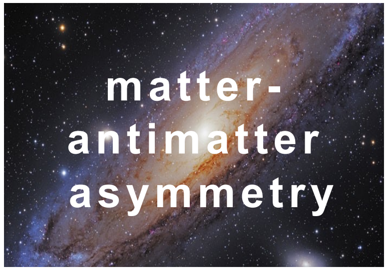


[1] Ettore Majorana. Nuovo Cim. 1937, 14:171–184

- Lepton-number violation: BSM

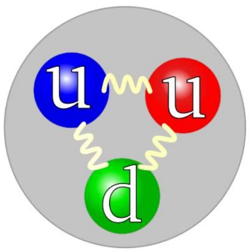


→ **Lepton-number violation** → **Baryon-number violation** →



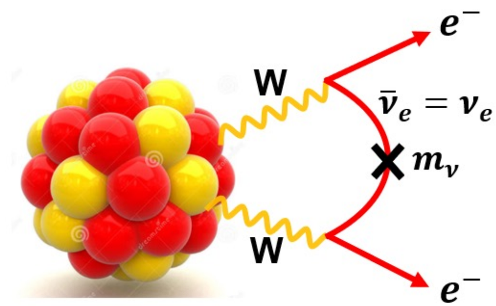
[2] M. A. Luty. Phys Rev. 1992, D45:455–465

Theoretical roadmap



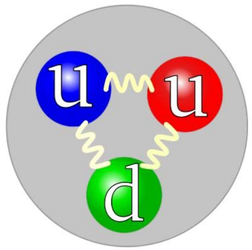
Particle physics

wide energy scales



Nuclear physics

Theoretical roadmap



Particle physics

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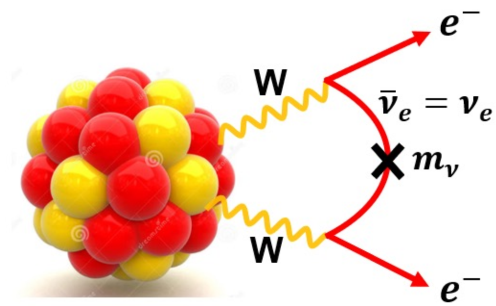


Nuclear physics

Submitted to the Proceedings of the U.S. Community Study
on the Future of Particle Physics (Snowmass 2021)

Neutrinoless Double-Beta Decay:
A Roadmap for Matching Theory to Experiment

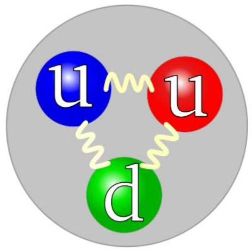
Vincenzo Cirigliano, et al. Snowmass 2021. arxiv:2203.12169

Snowmass points out the theoretical roadmap：

- Effective Field Theories (EFTs)
- Lattice QCD
- nuclear many-body theory

} Cooperation

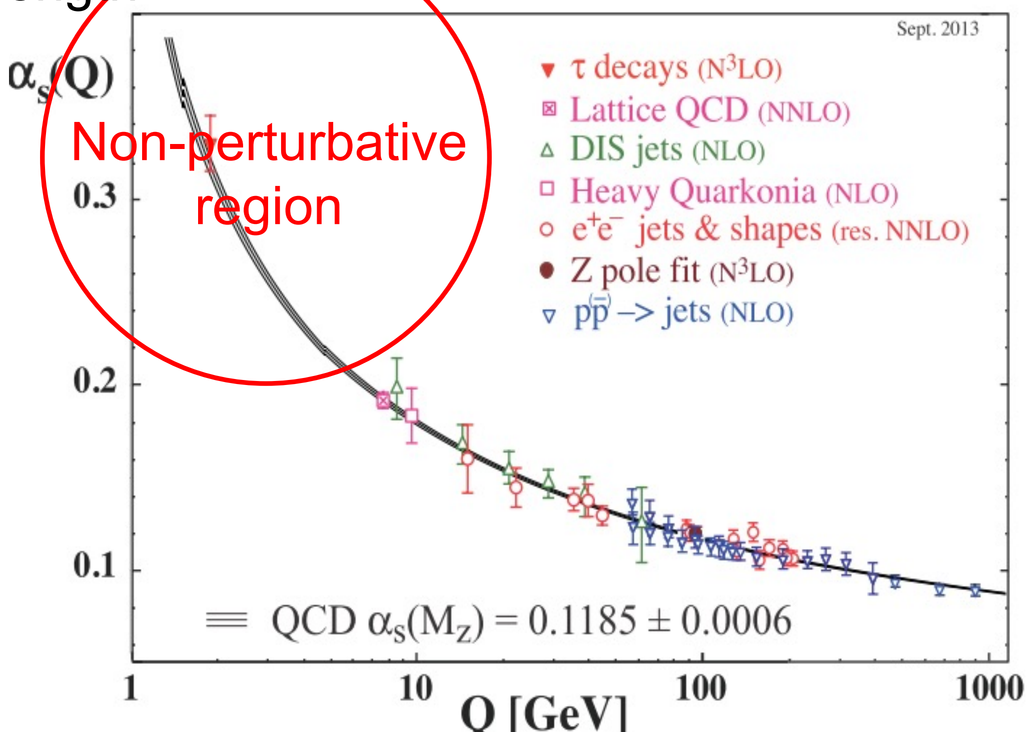
Challenge from low energy QCD



Particle physics

Interaction between quarks and gluons:
Quantum Chromodynamics (QCD)

Interaction strength



Energy

QCD in low energy:

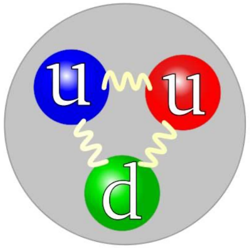
perturbation theory fails

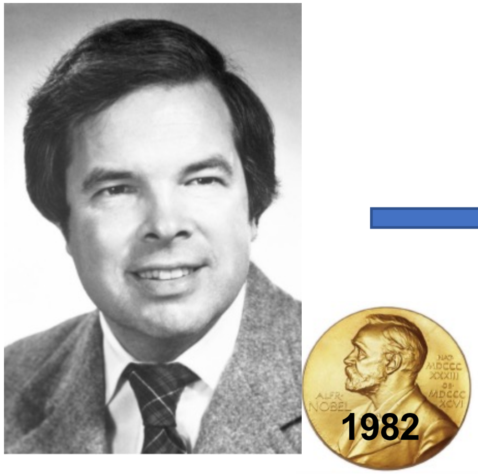
Lattice QCD

Calculation of QCD in non-perturbative region:

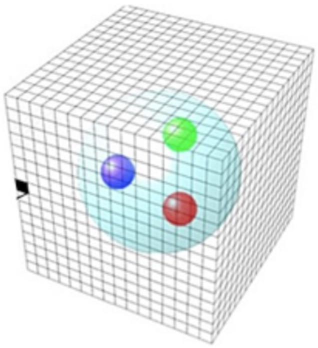
Perturbation theory? ✗

Numerical solution? Lattice QCD ✓





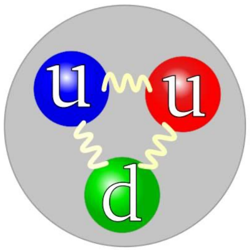
Kenneth G. Wilson



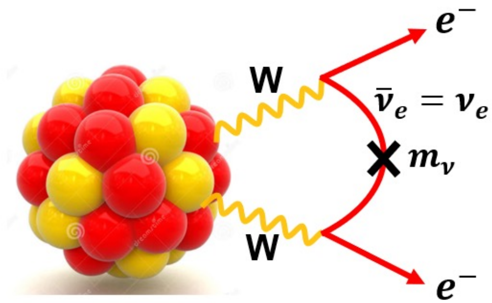
Lattice QCD

High precision calculation on supercomputer

Cooperation between EFTs and LQCD



Particle physics



Nuclear physics

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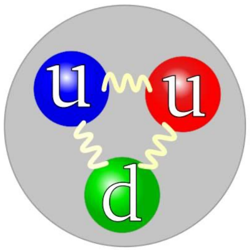
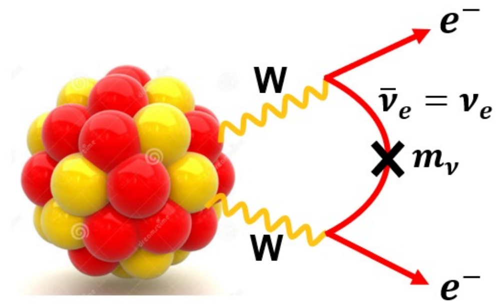
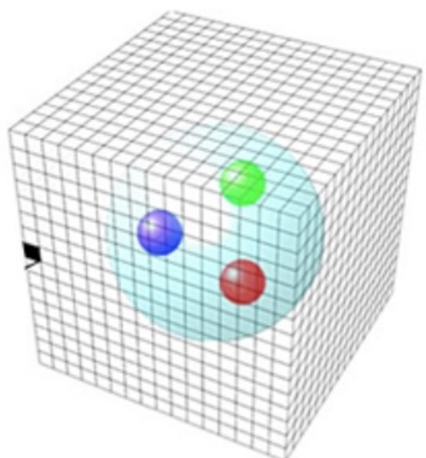


Particle physics



Nuclear physics

First principle
QCD calculation
(SM-EFT)



Lattice QCD

Provide low energy
constants (LECs)
for ChPT/ChEFT

Lattice QCD： bridging theories (EFTs) in different energy scales

Lattice QCD and $0\nu2\beta$ decay

- Coulomb-range contribution:

  hadronic inputs: single-nucleon g_A, scalar charge, tensor charge…

  ⇨ Extracted from lattice QCD Y. Aoki, et al. Eur Phys J C. 2022, 82(10):869

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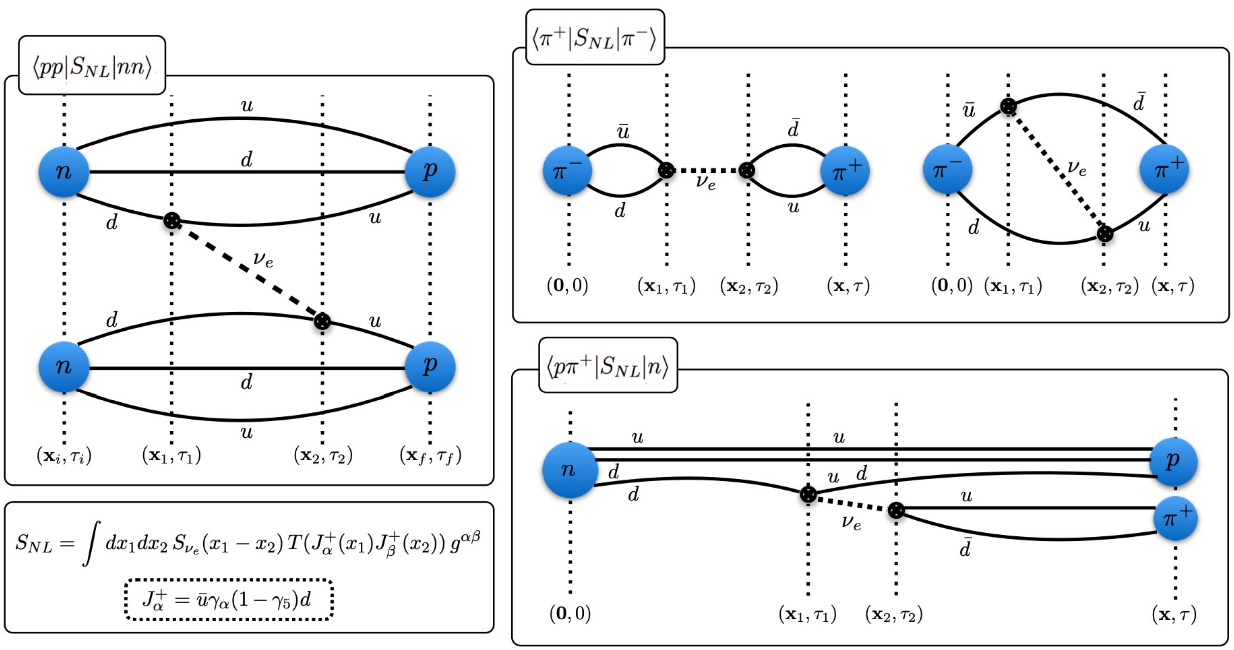
- Short-range contribution: hard neutrino exchange



In ChPT/ChEFT: $g_\nu^{\pi\pi}, g_\nu^{NN}$, ...

Vincenzo Cirigliano, et al. Snowmass 2021. arxiv:2203.12169

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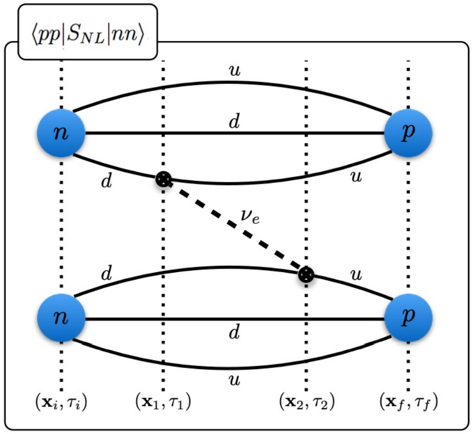
PHYSICAL REVIEW LETTERS **120**, 202001 (2018)

New Leading Contribution to Neutrinoless Double-β Decay

Vincenzo Cirigliano,[1] Wouter Dekens,[1] Jordy de Vries,[2] Michael L. Graesser,[1] Emanuele Mereghetti,[1] Saori Pastore,[1] and Ubirajara van Kolck[3,4]

g_ν^{NN}: additional contact operator at LO



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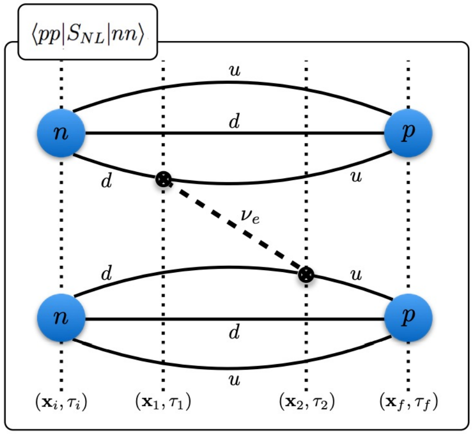
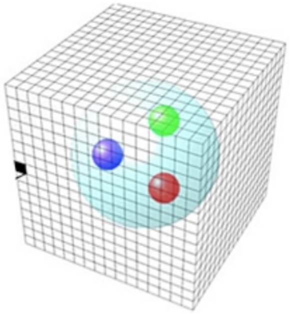


Determination of LEC g_ν^{NN}:

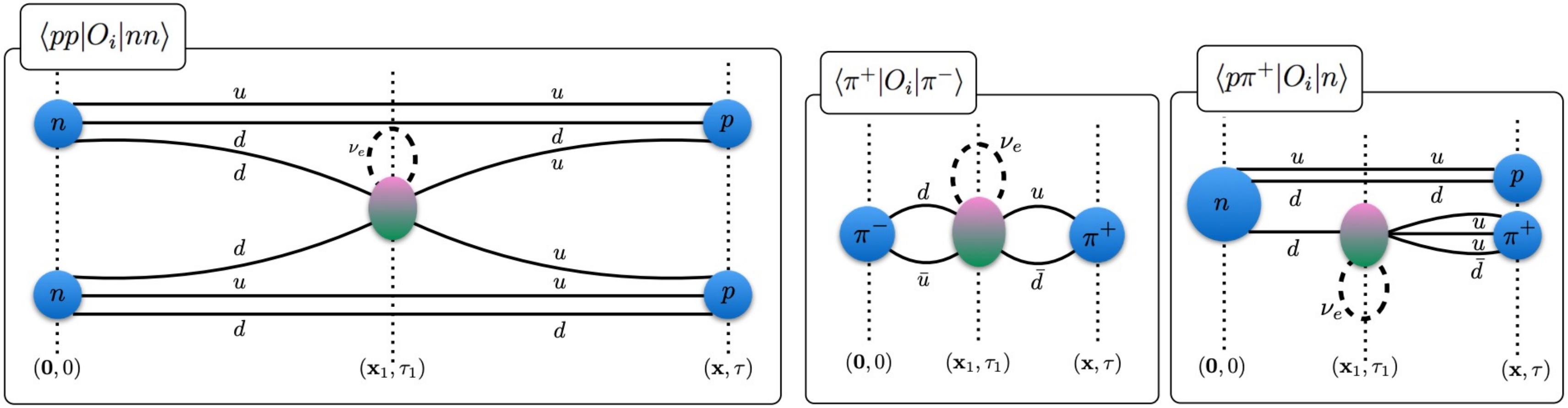
non-perturbative QCD



Lattice calculation

Lattice QCD and $0\nu2\beta$ decay

- Short-range contribution: dimension-9 operators



In ChPT/ChEFT: $g_i^{\pi\pi}$ $(i = 1, \ldots, 5)$, $g_i^{NN}(i = 1, \ldots, 7)$, ...

Vincenzo Cirigliano, et al. Snowmass 2021. arxiv:2203.12169

Lattice QCD and $0\nu 2\beta$ decay

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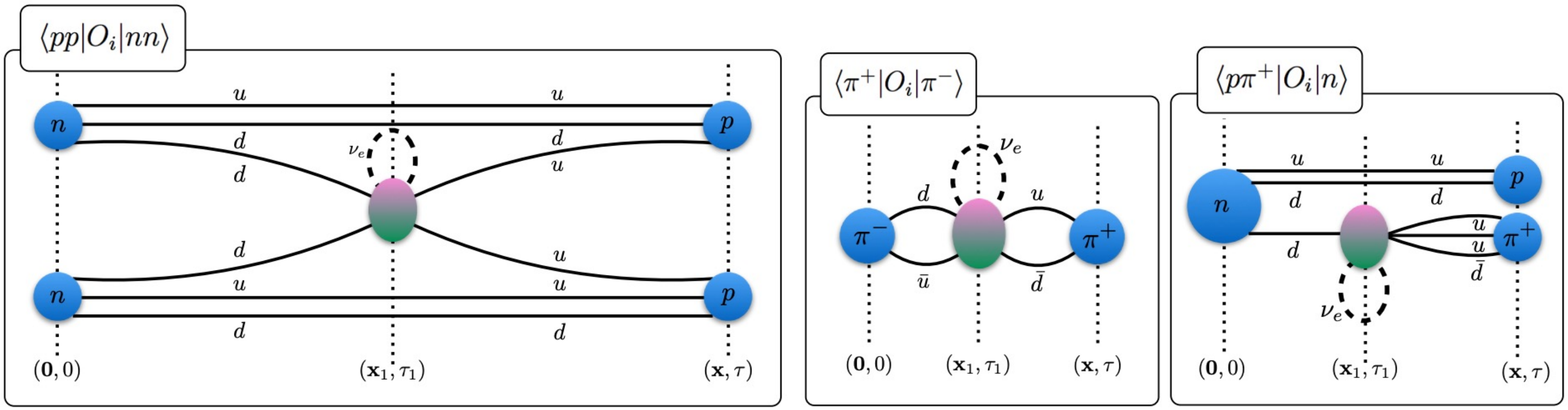
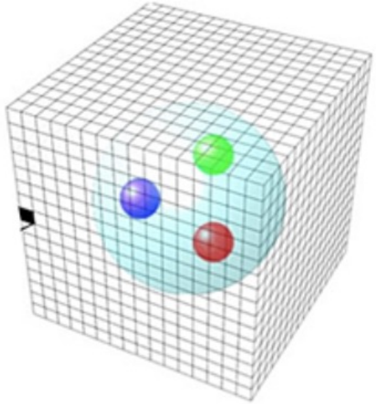


In ChPT/ChEFT: $g_i^{\pi\pi}$ $(i = 1, \ldots, 5), g_i^{NN}(i = 1, \ldots, 7), \ldots$

Short-range contribution
- hard neutrino exchange
- dimension-9 operators

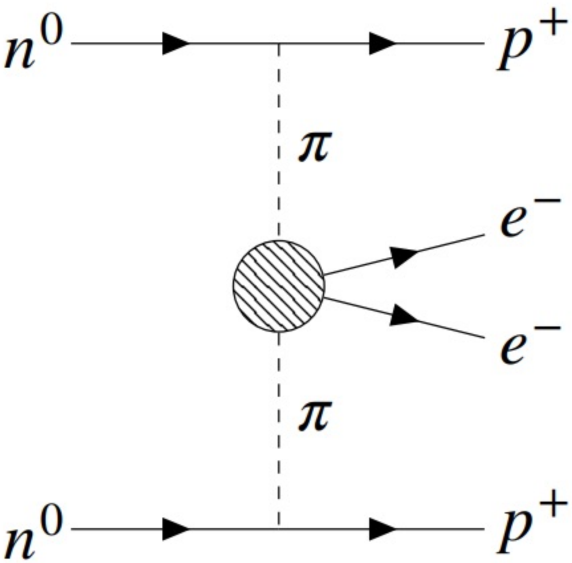


Lattice calculation of LECs

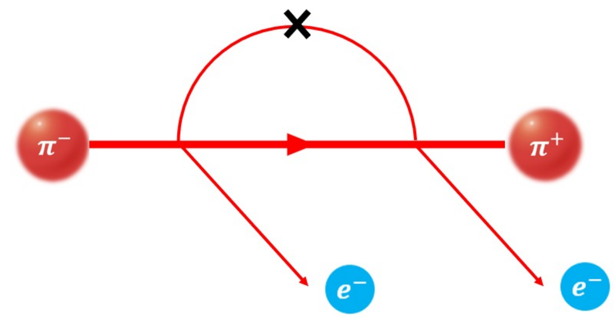
Vincenzo Cirigliano, et al. Snowmass 2021. arxiv:2203.12169

Plan

Lattice work 1: pionic $0\nu 2\beta$ decay



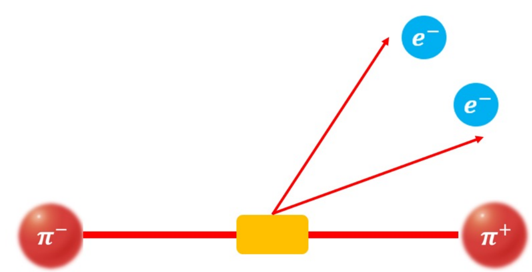
pion exchange contribution



$g_\nu^{\pi\pi}$

hard neutrino exchange

[1] **X. Tuo**, X. Feng, L. Jin, PRD100 (2019) 094511
[2] X. Feng, L. Jin, **X. Tuo**, S. Xia, PRL122 (2019) 022001
[3] W. Detmold, D. Murphy (2020), arxiv:2208.05322



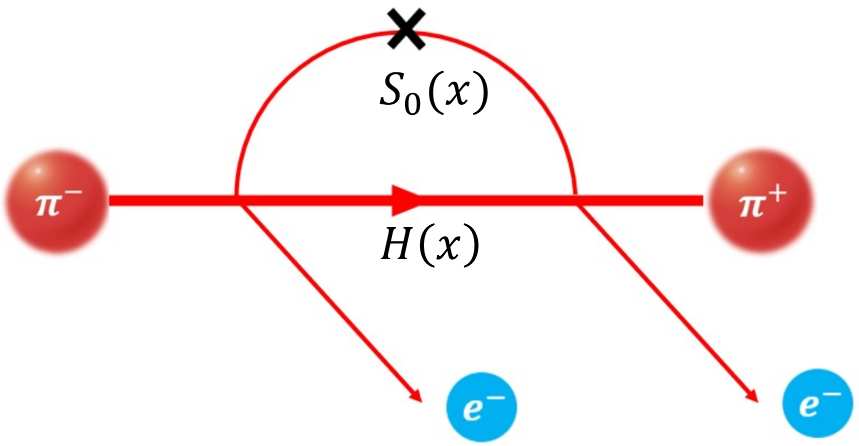
$g_i^{\pi\pi}$
$(i = 1 \dots 5)$

dimension-9 operator

[4] A. Nicholson, et al., PRL121 (2018) 172501
[5] W. Detmold, et al. (2022), arxiv:2208.05322

Challenge: massless neutrino

$$\mathcal{A} = -2T_{lept} \int d^4x \, H(x) S_0(x)$$



$H(x)$: hadronic part, from lattice four-point function

$S_0(x)$: massless neutrino, propagate out of lattice range

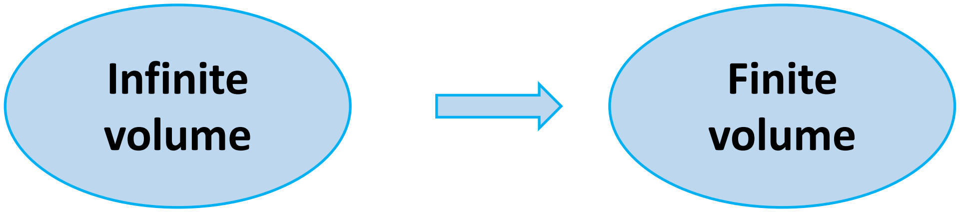
How to combine massless propagator into lattice calculation?

Traditional method: neutrino in finite volume

Z. Davoudi, M. Savage Phys. Rev. D 90, 054503 (2014)

- Lattice data: $H(x)$ in finite volume
- Traditional method: also put neutrino into finite volume
- For example: subtract zero mode (QED_L), massive neutrino

$S_0(x)$:



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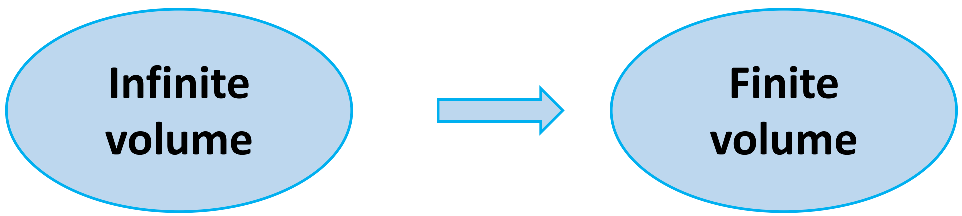
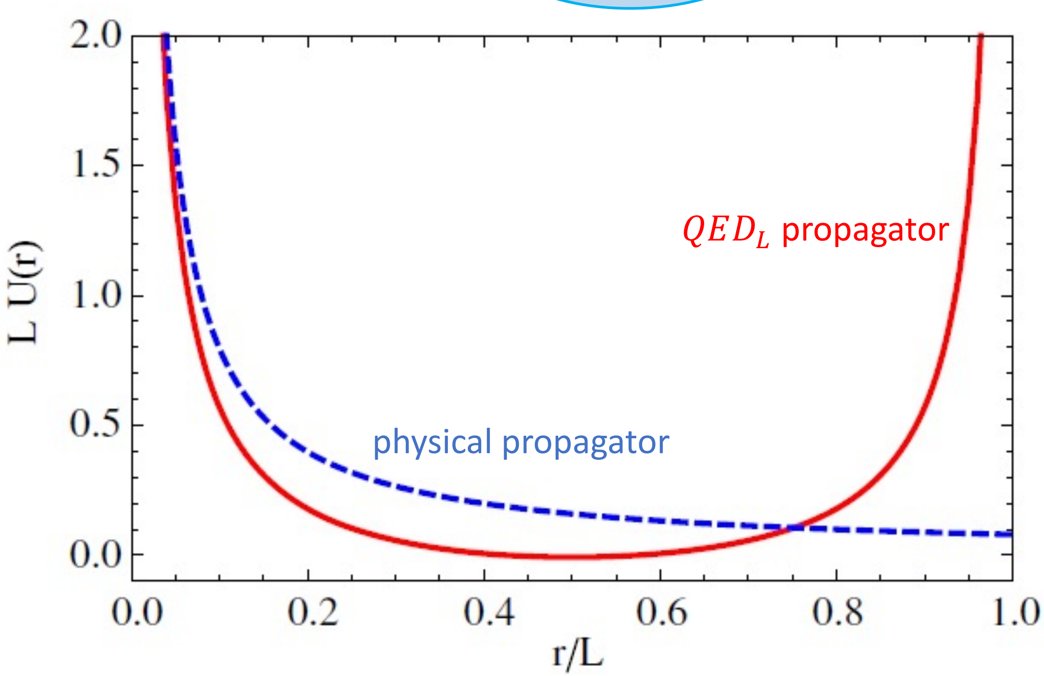
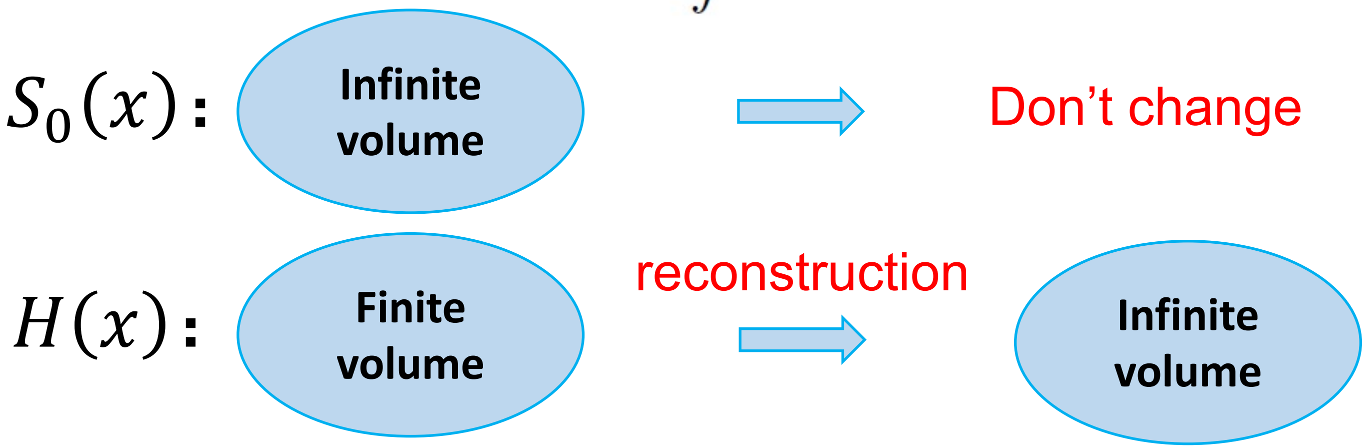




large $O(1/L)$ finite volume errors

New method: neutrino in infinite volume

$$\mathcal{A} = -2T_{lept} \int d^4x \, H(x) S_0(x)$$



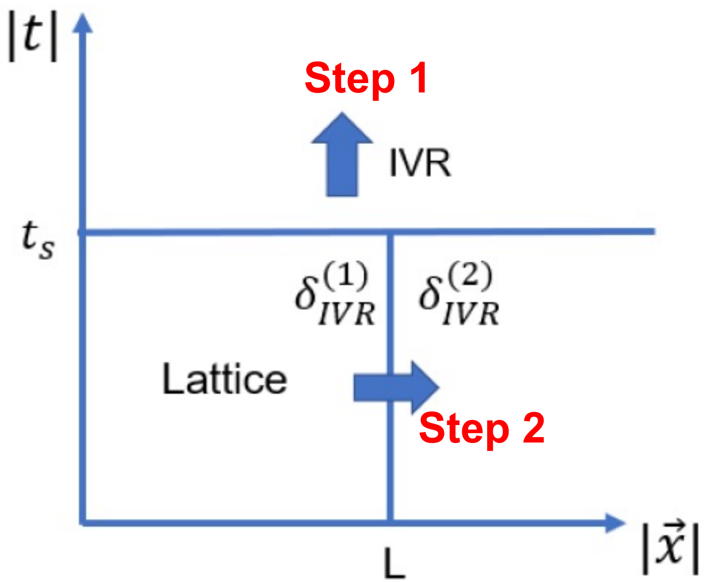
[1] X. Feng, L. Jin, PRD100 (2019) 094509, arXiv:1809.10511
[2] **X. Tuo**, X. Feng, L. Jin, PRD100 (2019) 094511, arXiv:1909.13525

New method: neutrino in infinite volume

$$\mathcal{A} = -2T_{lept}\int d^4x\, H(x) S_0(x)$$

$S_0(x)$: Infinite volume → Don't change

$H(x)$: Finite volume → reconstruction → Infinite volume



Infinite volume reconstruction (IVR) method

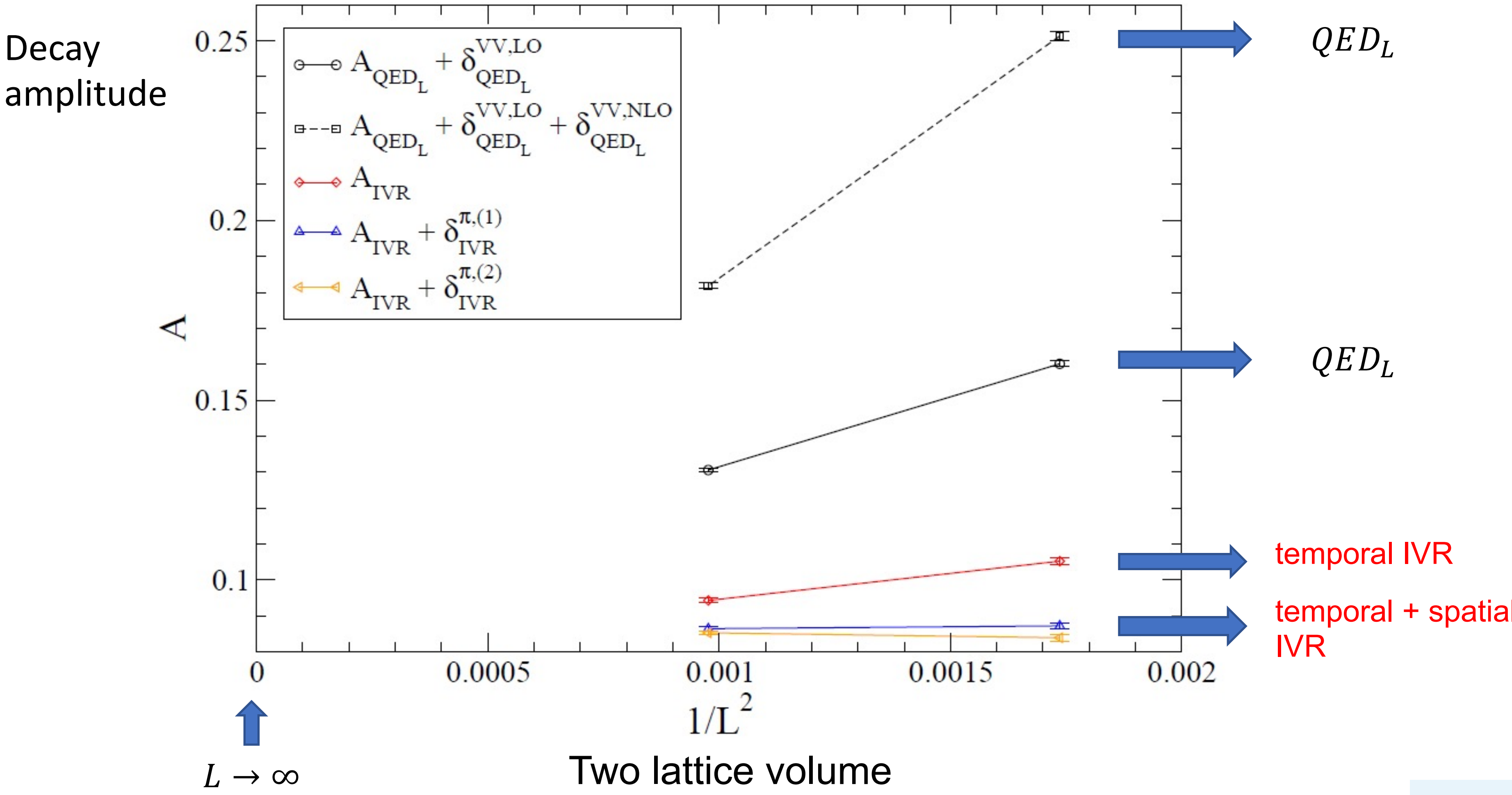
Step 1: reconstruction in temporal direction (model-independent)

Step 2: reconstruction in spatial direction (depend on charge radius $\langle r_\pi^2 \rangle$)

[1] X. Feng, L. Jin, PRD100 (2019) 094509, arXiv:1809.10511
[2] **X. Tuo**, X. Feng, L. Jin, PRD100 (2019) 094511, arXiv:1909.13525

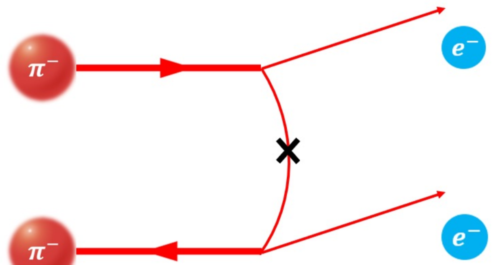
Improving finite volume errors

Benefit of IVR method: $O(e^{-mL})$ FV errors



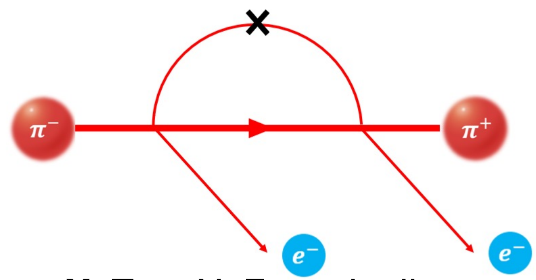
Provide LEC $g_\nu^{\pi\pi}$

Our work



X. Feng, L. Jin, **X. Tuo**, S. Xia,
PRL122 (2019) 022001

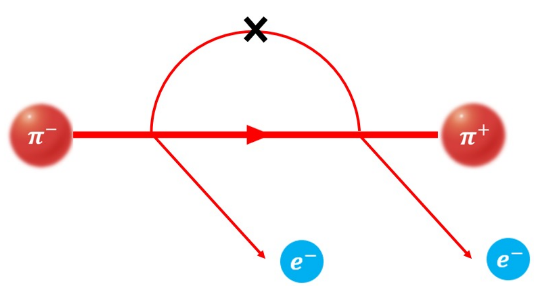
$g_\nu^{\pi\pi} = -12.0(3)_{stat}$



X. Tuo, X. Feng, L. Jin,
PRD100 (2019) 094511

$g_\nu^{\pi\pi} = -10.9(3)_{stat}(7)_{sys}$

NPLQCD

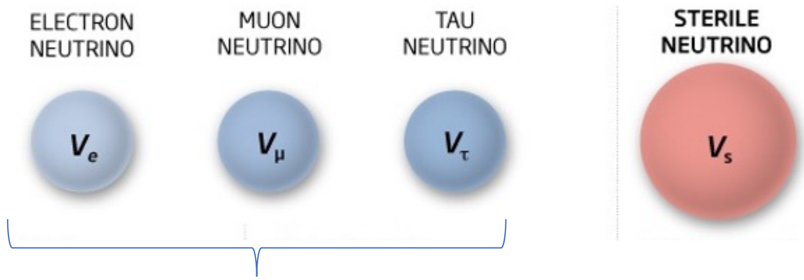


W. Detmold, D. Murphy (2020)
arxiv:2208.05322

$g_\nu^{\pi\pi} = -10.8(1)_{stat}(5)_{sys}$

Plan

Lattice work 2: sterile neutrino contribution

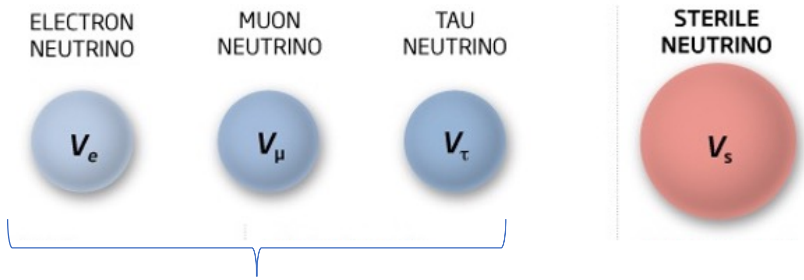


SM neutrinos

Sterile neutrinos

Sterile neutrino: explain the source of tiny mass of neutrino through the seesaw mechanism, the hypothesis of many BSM models

Lattice work 2: sterile neutrino contribution



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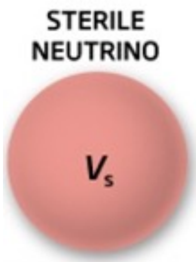
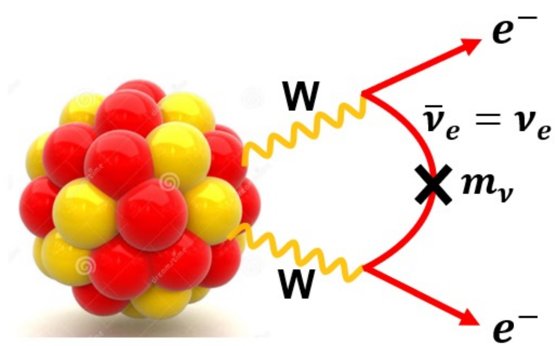


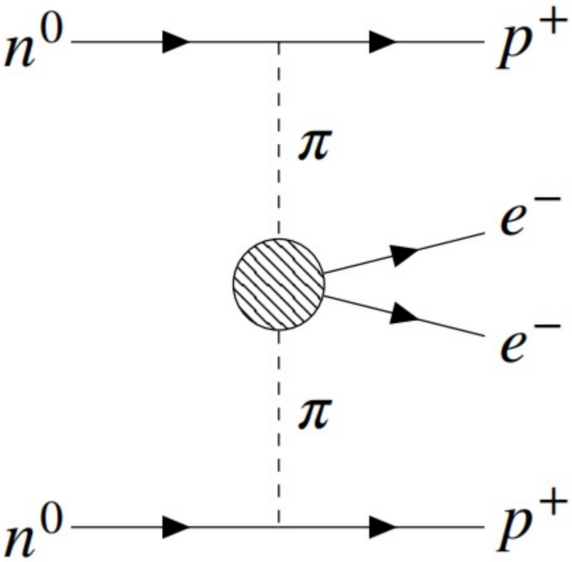
influence $0\nu2\beta$ decay

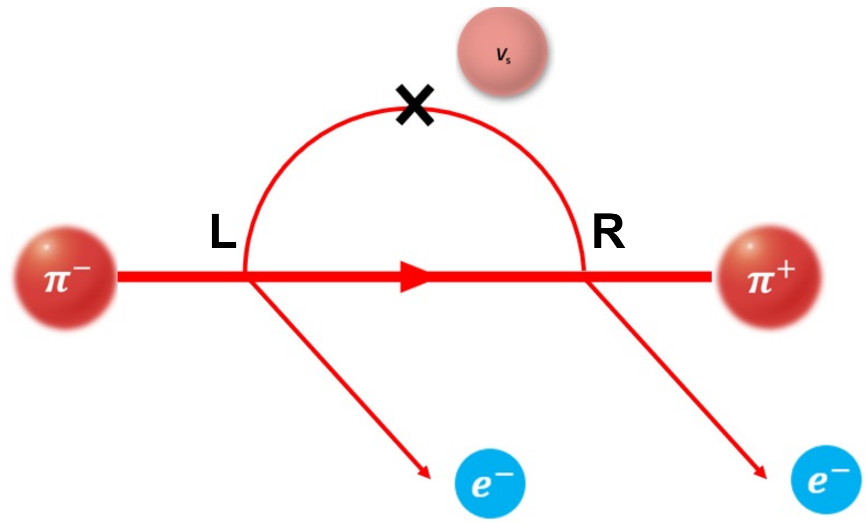


Enhancement due to sterile neutrino

W. Dekens, J. de Vries, K. Fuyuto, E. Mereghetti, and G. Zhou, JHEP 06, 097 (2020)

$0\nu2\beta$ decay can be enhanced by sterile neutrino contribution in pion exchange diagram





Sterile neutrinos contribution
Mixing of left- and right-handed current

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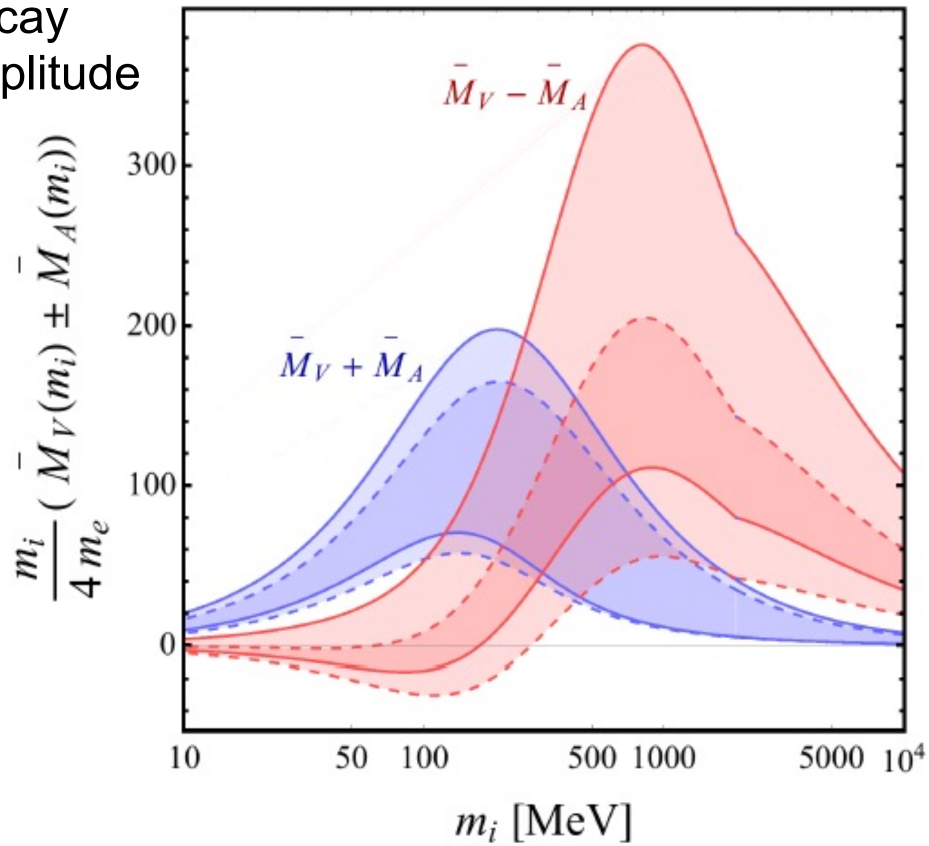
Decay amplitude



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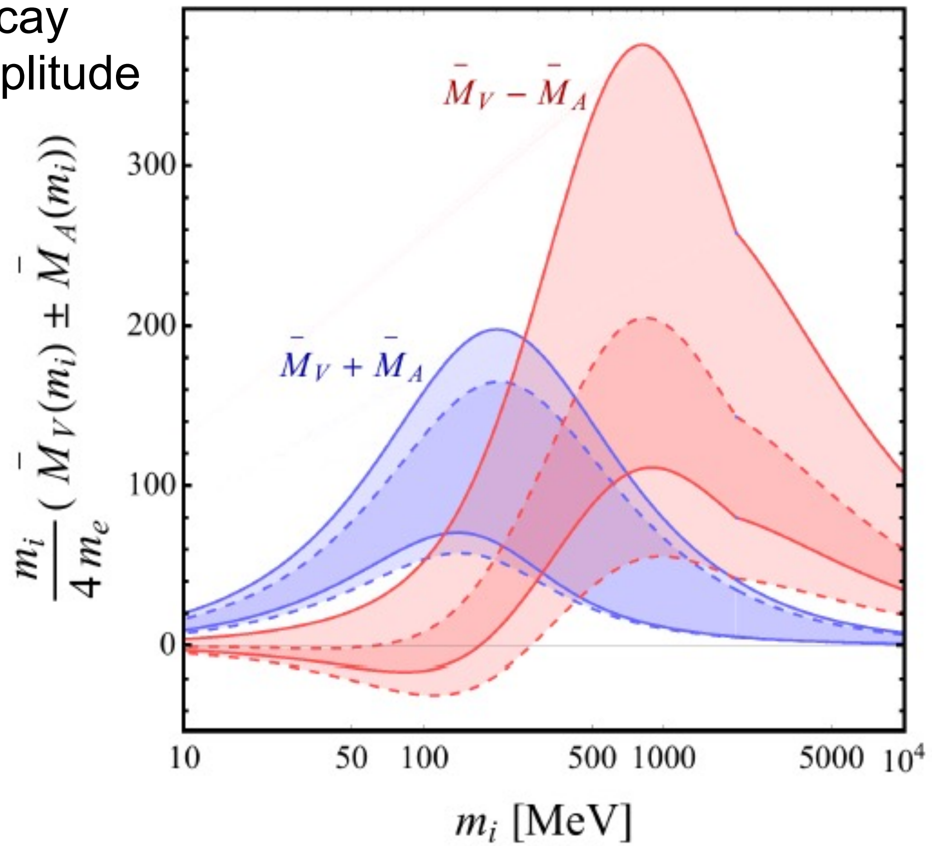
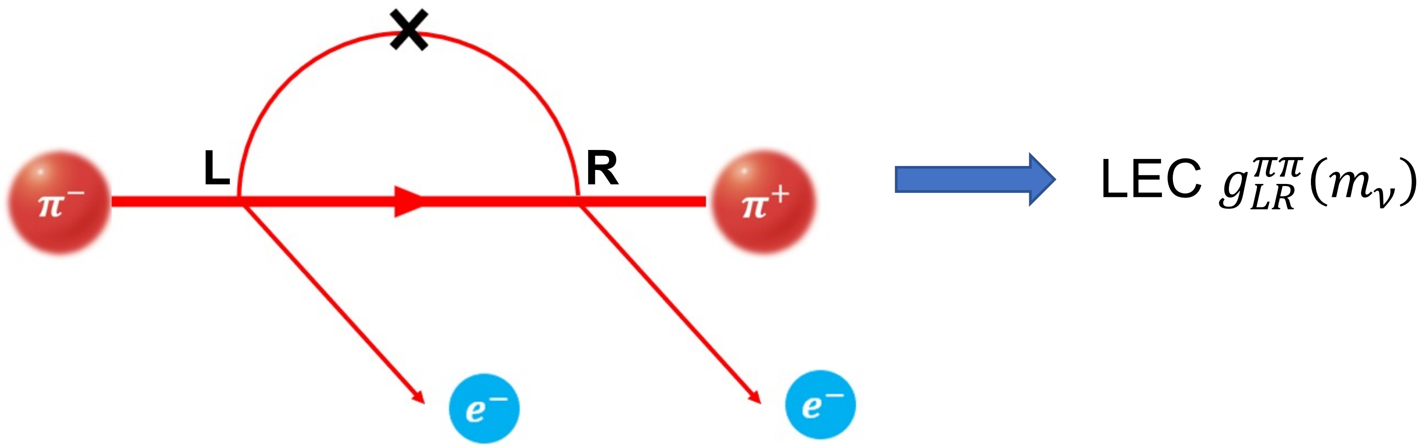


Big error band： poor knowledge of LECs $g_\nu^{NN}, g_{LR}^{NN}, g_{LR}^{\pi\pi}, \ldots$

Lattice QCD can help to reduce uncertainties in LECs

Benefit of our method



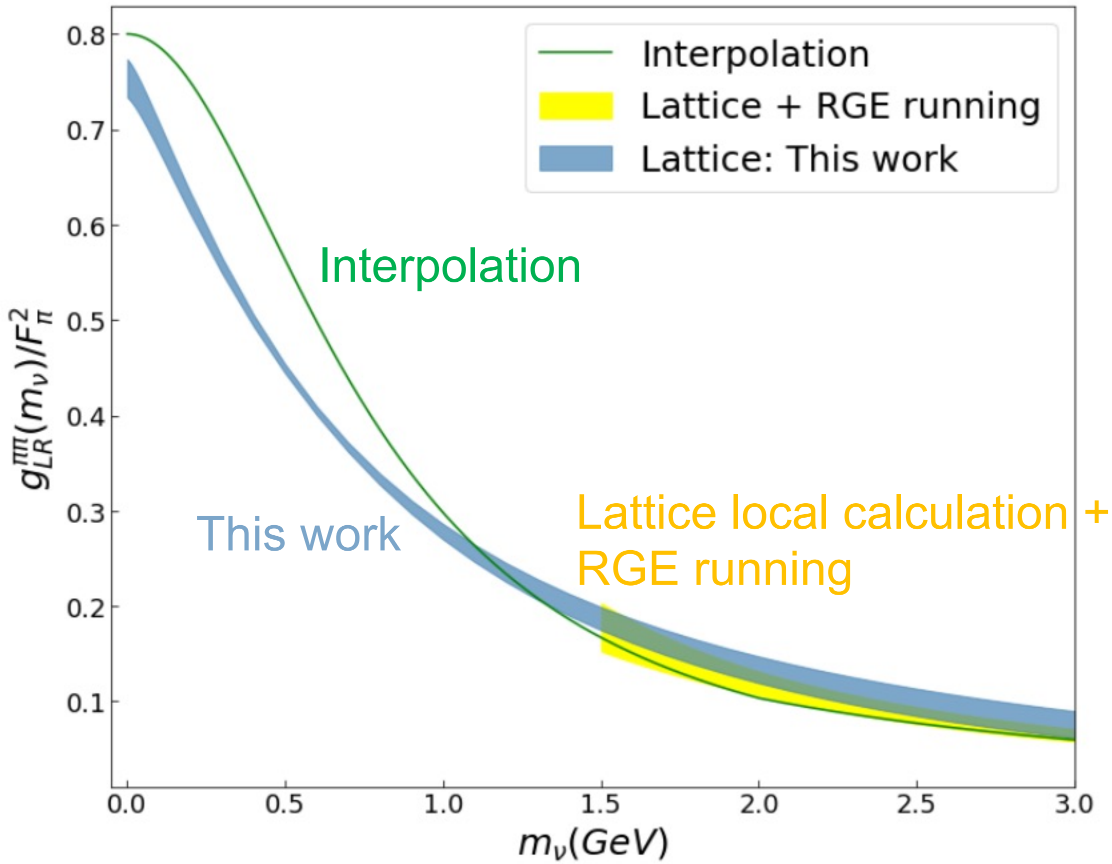
Neutrino: infinite volume version with known analytical form

It is convenient to adjust the neutrino mass and study the mass dependence

$$S_0^E(x) = \int \frac{d^4q}{(2\pi)^4} \frac{e^{-iqx}}{q^2+m_\nu^2} = \frac{m_\nu}{4\pi^2|x|} K_1(m_\nu|x|).$$

Lattice calculation of $g_{LR}^{\pi\pi}(m_\nu)$

X. Tuo, X. Feng, L. Jin, PRD106 (2022) 074510, arXiv:2206.00879



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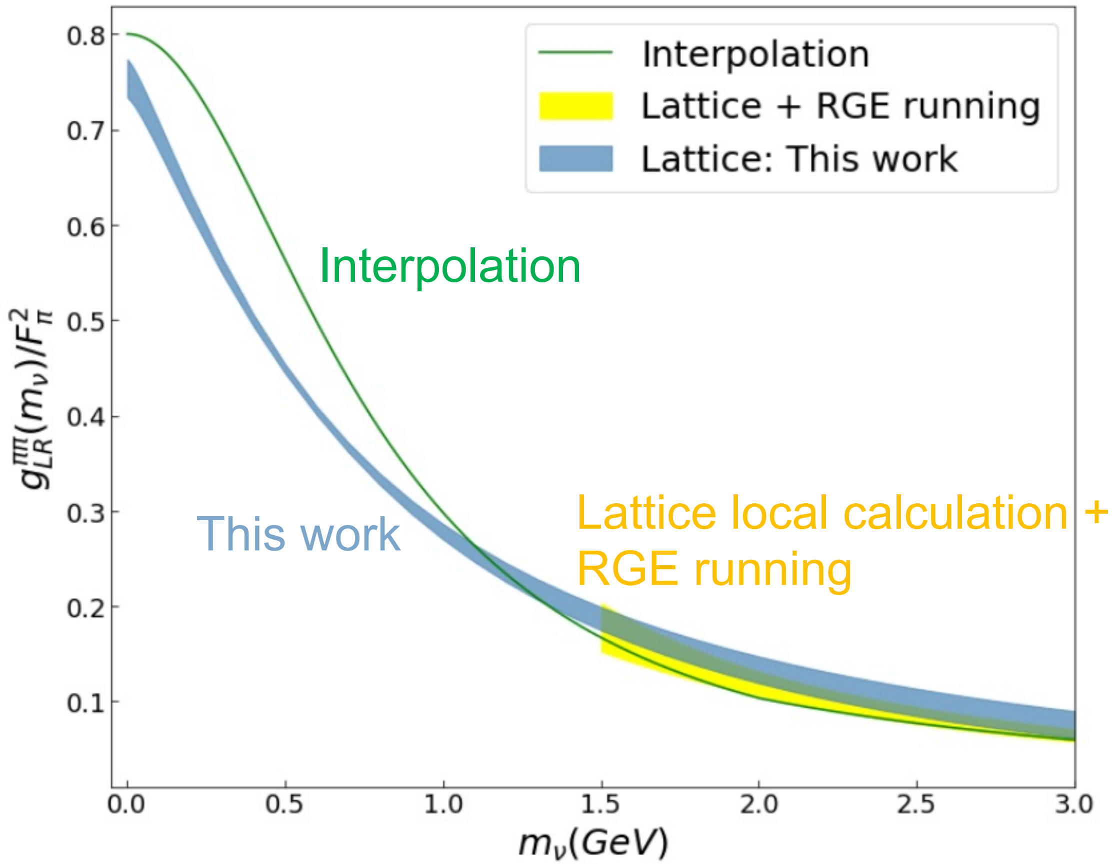
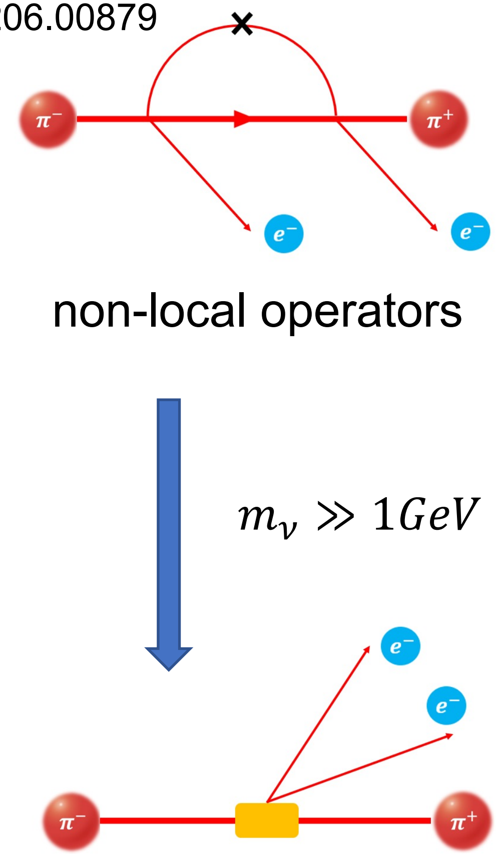
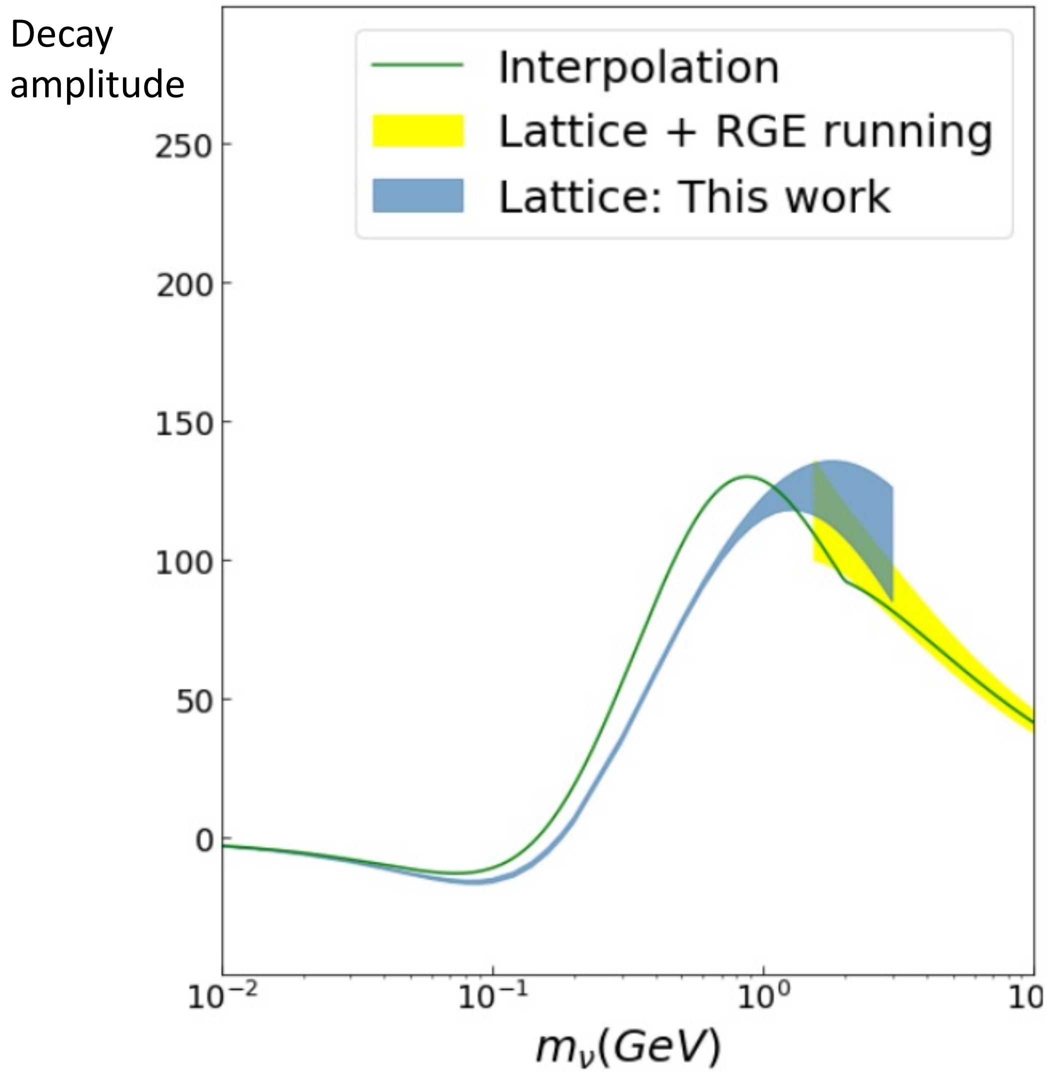




local operators
A. Nicholson, et al., PRL121 (2018) 172501

Nontrivial consistency check

Enhancement due to $g_{LR}^{\pi\pi}(m_\nu)$



Help to reduce the uncertainties from LEC $g_{LR}^{\pi\pi}(m_\nu)$ and determine the peak shape

Plan

1 Background: Lattice QCD and $0\nu 2\beta$ decay

2 Lattice work 1: pionic $0\nu 2\beta$ decay

3 Lattice work 2: sterile neutrino contribution

4 Conclusion and outlook

Conclusion

1. Infinite volume reconstruction method solves the finite volume effects caused by massless neutrino.

2. Determine LEC related to pionic $0\nu 2\beta$ decay $g_{\nu}^{\pi\pi} = 10.9(3)(7)$

   [1] X. Feng, L. Jin, **X. Tuo**, S. Xia, PRL122 (2019) 022001

   [2] **X. Tuo**, X. Feng, L. Jin, PRD100 (2019) 094511

3. Study the mass Dependence of LEC $g_{LR}^{\pi\pi}(m_{\nu})$ related to sterile neutrino Contributions

   [3] **X. Tuo**, X. Feng, L. Jin, PRD106 (2022) 074510

Outlook: nucleon sector g_ν^{NN}

[1] Zohreh Davoudi, et al. Report of the Snowmass 2021 Topical Group on Lattice Gauge Theory[C]. Snowmass 2021.

Three stages:

1. Calculation of two-nucleon spectra and elastic scattering
2. Calculation of two-nucleon $0\nu2\beta$ matrix elements
3. Relating lattice calculation to physical quantities

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Stages 1–2: Challenging due to signal-to-noise problem, main goal of future lattice QCD study

Stage 3 → [2] Xu Feng, Lu-Chang Jin, Zi-Yu Wang, Zheng Zhang. Phys Rev D. 2021, 103(3):034508
[3] Zohreh Davoudi, Saurabh V. Kadam. Phys Rev Lett. 2021, 126(15):152003

Zi-Yu Wang (王子毓), talk on 5.22, 16:10-16:30