

Impact of nuclear matrix element calculations on neutrinoless double beta decay searches

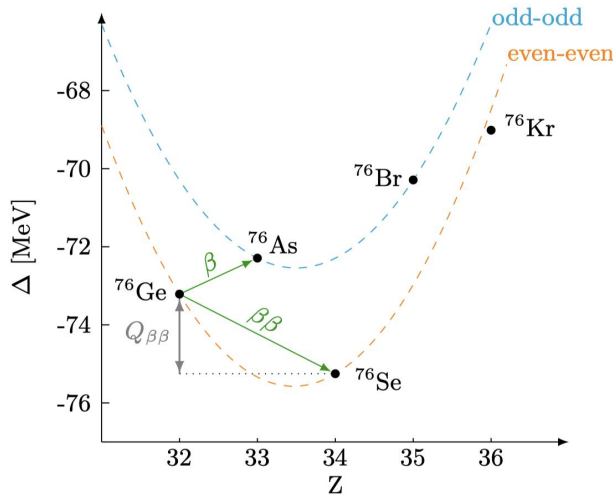
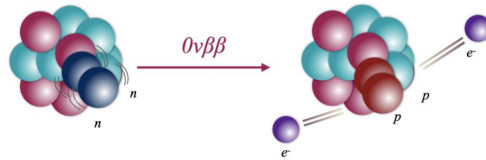
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Based on the work with Prof. Thomas Schwetz and Federica Popma, 2303.10562

第二届“无中微子双贝塔衰变及相关物理研讨会”，2023年5月20日

Brief background



$$(A, Z) \rightarrow (A, Z + 2) + 2e^- + 2\bar{\nu}_e$$

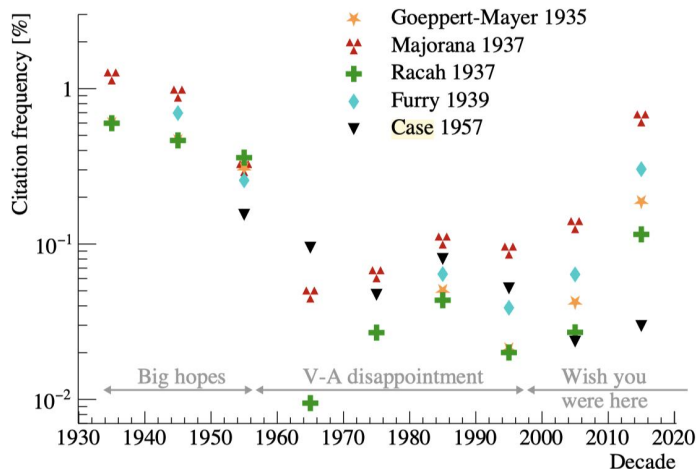
Mayer, 1935; first detected in 1987 by Moe

$$\nu_i^c = \nu_i$$

Majorana, 1937

$$(A, Z) \rightarrow (A, Z + 2) + 2e^-$$

Furry, 1939



Experiments:

background, exposure, energy resolution

Theoretical difficulties:

- The production mechanism
- Calculation of nuclear matrix elements (NME)

Formula (light neutrino exchange mechanism)

$$(T_{1/2}^{-1})_{\alpha} = \tilde{\Gamma}_{\alpha}(m_{\beta\beta}, M_{\alpha i}) = \frac{\Gamma_{\alpha}(m_{\beta\beta}, M_{\alpha i})}{\ln 2} = G_{\alpha} |M_{\alpha i}|^2 m_{\beta\beta}^2$$

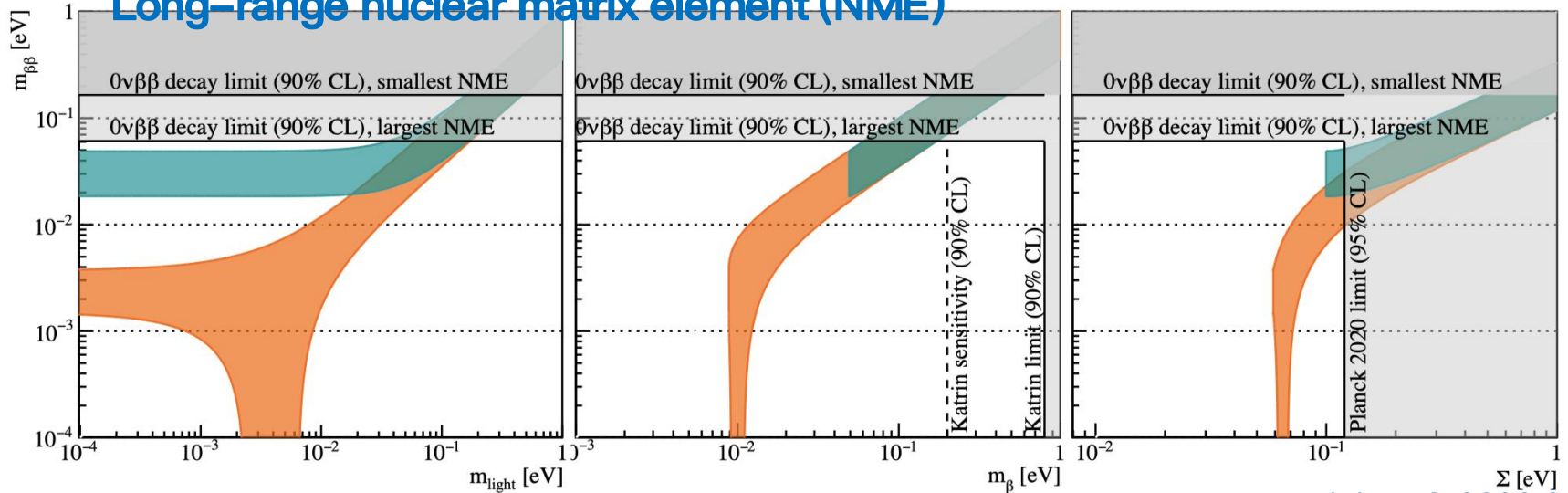
$$m_{\beta\beta} = \left| \sum_j U_{ej}^2 m_j \right|$$

$$M_{\alpha i} = M_{\alpha i}^{\text{long}} + M_{\alpha i}^{\text{short}} = M_{\alpha i}^{\text{long}}(1 + n_{\alpha i}) \quad n_{\alpha i} = \frac{M_{\alpha i}^{\text{short}}}{M_{\alpha i}^{\text{long}}}$$

$$g_A^{\text{eff}} = q g_A^{\text{free}} \quad g_A^{\text{free}} = 1.27$$

Quenching effect: correct the NME by q^2 and the decay rate by q^4

Long-range nuclear matrix element (NME)



Agostini et al. 2202.01787

Motivations:

- Interpreting the constraints/sensitivities on $m_{\beta\beta}$ of current/future 0 $\nu\beta\beta$ experiments
- Checking the possibilities of discriminating NME models in future 0 $\nu\beta\beta$ experiments

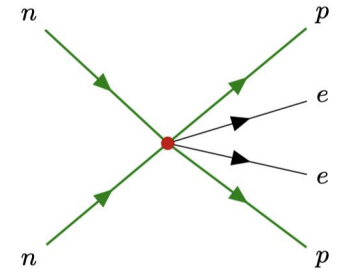
Long-range NME

Nuclear Model	Index	^{76}Ge	^{82}Se	^{100}Mo	^{130}Te	^{136}Xe
NSM	N1	2.89	2.73	-	2.76	2.28
	N2	3.07	2.90	-	2.96	2.45
	N3	3.37	3.19	-	1.79	1.63
	N4	3.57	3.39	-	1.93	1.76
	N5	2.66	2.72	2.24	3.16	2.39
QRPA	Q1	5.09	-	-	1.37	1.55
	Q2	5.26	3.73	3.90	4.00	2.91
	Q3	4.85	4.61	5.87	4.67	2.72
	Q4	3.12	2.86	-	2.90	1.11
	Q5	3.40	3.13	-	3.22	1.18
	Q6	-	-	-	4.05	3.38
EDF	E1	4.60	4.22	5.08	5.13	4.20
	E2	5.55	4.67	6.59	6.41	4.77
	E3	6.04	5.30	6.48	4.89	4.24
IBM	I1	5.14	4.19	3.84	3.96	3.25
	I2	6.34	5.21	5.08	4.15	3.40

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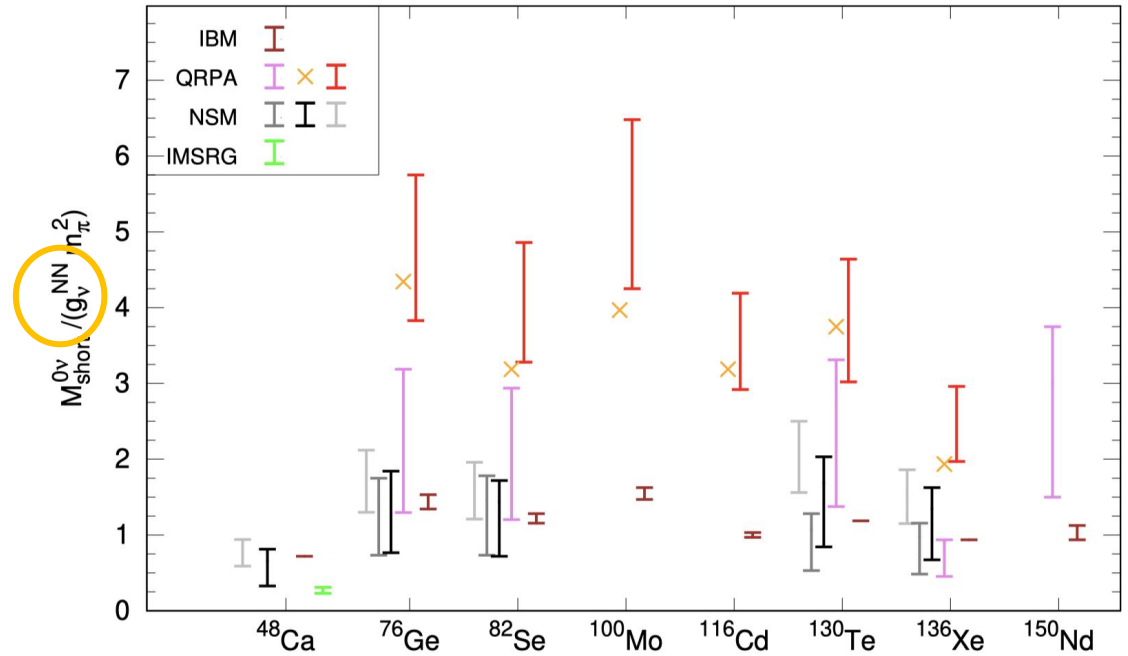
Short-range NME

Cirigliano et al, *Phys.Rev.Lett.* 120 (2018) 20, 202001



Isotope	NSM %	QRPA %
^{76}Ge	15–42	32–73
^{82}Se	15–41	30–70
^{100}Mo	-	49–108
^{130}Te	17–47	34–77
^{136}Xe	17–47	30–70

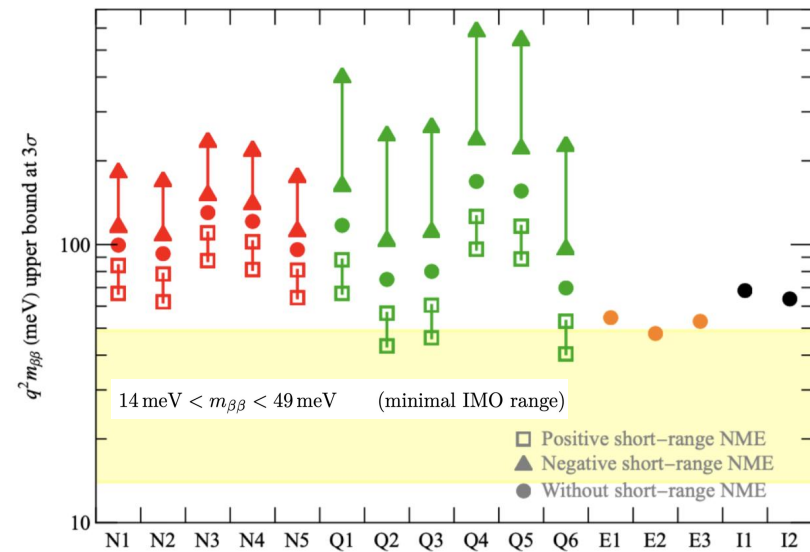
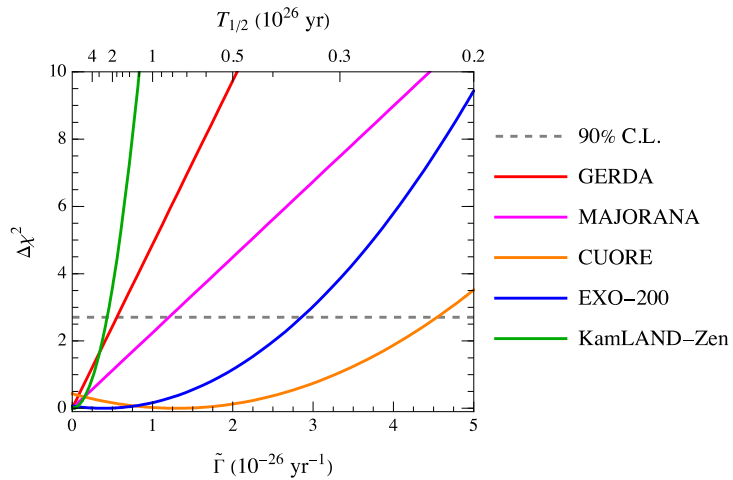
Phys. Lett. B 823 (2021) 136720



Agostini et al. 2202.01787

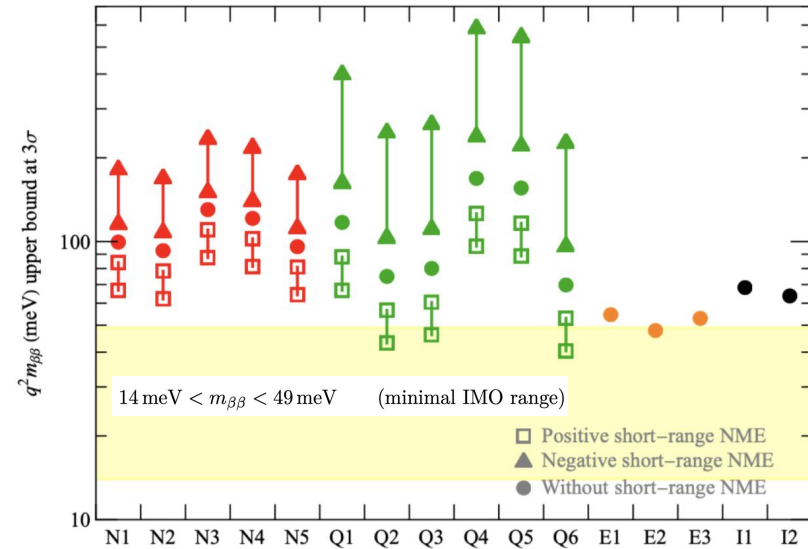
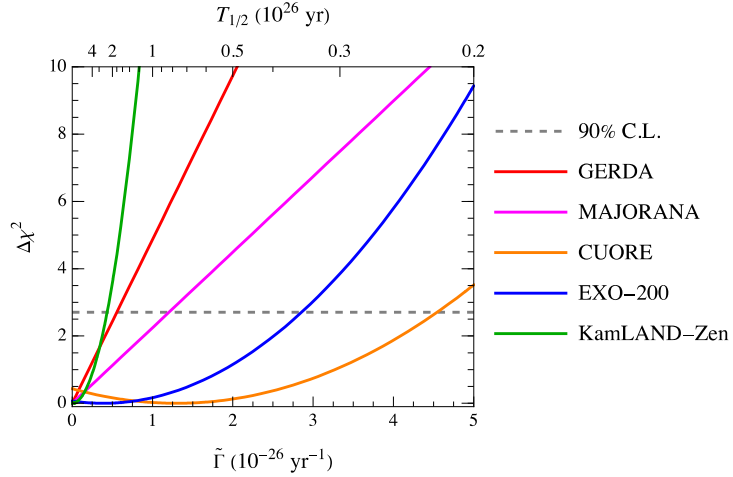
Current constraints:

$$\Delta\chi_r^2(\tilde{\Gamma}_\alpha) = a_r (\tilde{\Gamma}_\alpha)^2 + b_r \tilde{\Gamma}_\alpha + c_r$$



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$$\Delta\chi_r^2(\tilde{\Gamma}_\alpha) = a_r (\tilde{\Gamma}_\alpha)^2 + b_r \tilde{\Gamma}_\alpha + c_r$$



Setups of future experiments:

Experiment	Isotope	ε [mol·yr]	b [events/(mol·y)]	PSF [yr ⁻¹ eV ⁻²]
LEGEND-1000	⁷⁶ Ge	8736	$4.9 \cdot 10^{-6}$	$2.36 \cdot 10^{-26}$
SuperNEMO	⁸² Se	185	$5.4 \cdot 10^{-3}$	$10.19 \cdot 10^{-26}$
CUPID	¹⁰⁰ Mo	1717	$2.3 \cdot 10^{-4}$	$15.91 \cdot 10^{-26}$
SNO+II	¹³⁰ Te	8521	$5.7 \cdot 10^{-3}$	$14.2 \cdot 10^{-26}$
nEXO	¹³⁶ Xe	13700	$4.0 \cdot 10^{-5}$	$14.56 \cdot 10^{-26}$

$$N_{\text{LEGEND-1000}} = \left\{ 0.97 \times \left[\frac{(q^2 m_{\beta\beta})^{\text{True}}}{40 \text{ meV}} \right]^2 \left(\frac{M_{\text{Ge}}^{\text{long}}}{2.66} \right)^2 + 0.04 \right\} \times \frac{T}{1 \text{ yr}}$$

$$N_{\text{SuperNEMO}} = \left\{ 0.09 \times \left[\frac{(q^2 m_{\beta\beta})^{\text{True}}}{40 \text{ meV}} \right]^2 \left(\frac{M_{\text{Se}}^{\text{long}}}{2.72} \right)^2 + 1.0 \right\} \times \frac{T}{1 \text{ yr}}$$

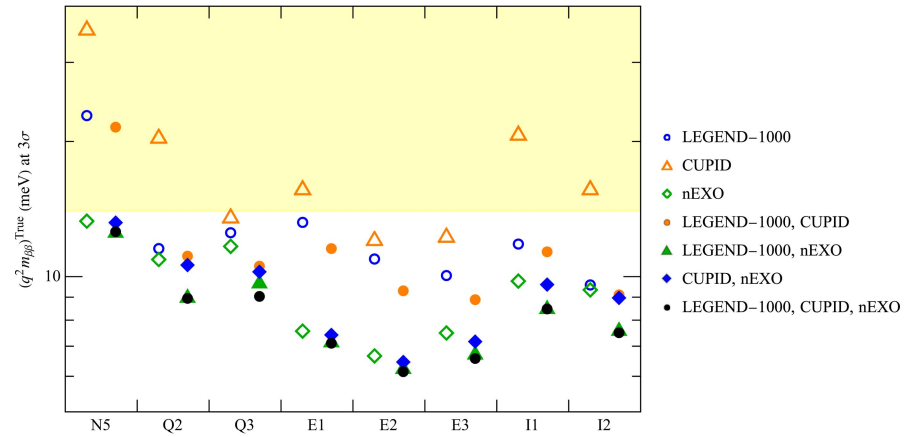
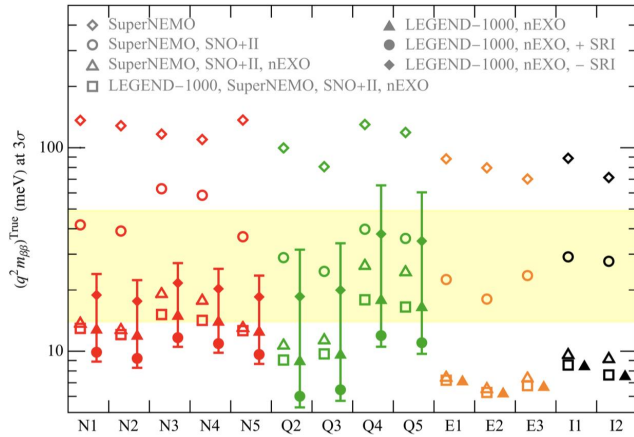
$$N_{\text{nEXO}} = \left\{ 1.64 \times \left[\frac{(q^2 m_{\beta\beta})^{\text{True}}}{40 \text{ meV}} \right]^2 \left(\frac{M_{\text{Xe}}^{\text{long}}}{1.11} \right)^2 + 0.5 \right\} \times \frac{T}{1 \text{ yr}}$$

$$N_{\alpha i} = S_{\alpha i} + B_{\alpha} \quad B_{\alpha} = b_{\alpha} \cdot \varepsilon_{\alpha} \cdot \left(\frac{T}{1 \text{ yr}} \right)$$

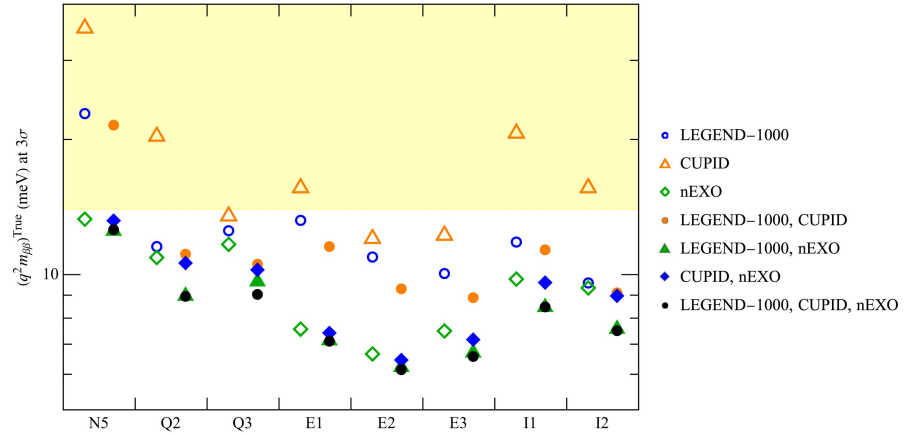
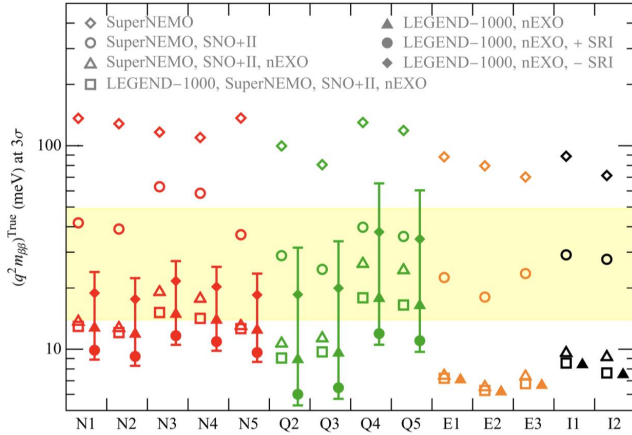
$$S_{\alpha i}(m_{\beta\beta}, M_{\alpha i}) = \ln 2 \cdot N_A \cdot \varepsilon_{\alpha} \cdot \left(\frac{T}{1 \text{ yr}} \right) \cdot \tilde{\Gamma}_{\alpha}(m_{\beta\beta}, M_{\alpha i})$$

$$\Delta\chi_{ij}^2(m_{\beta\beta}, M_{\alpha j}; m_{\beta\beta}^{\text{True}}, M_{\alpha i}^{\text{True}}) = 2 \sum_{\alpha} \left(N_{\alpha j} - N_{\alpha i}^{\text{True}} + N_{\alpha i}^{\text{True}} \ln \frac{N_{\alpha i}^{\text{True}}}{N_{\alpha j}} \right)$$

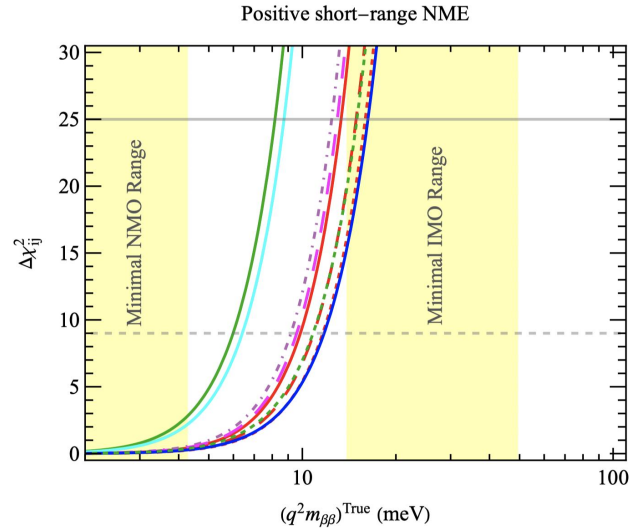
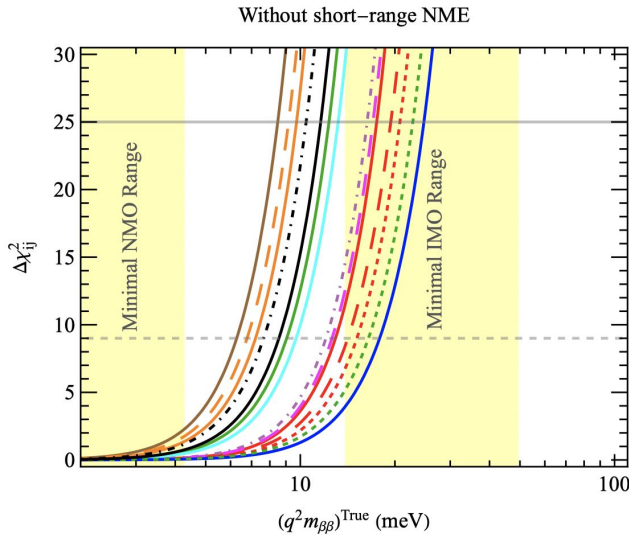
Sensitivities to $(q^2 m_{\beta\beta})_{True}$ at 3σ :



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The significance of observing one positive signal:

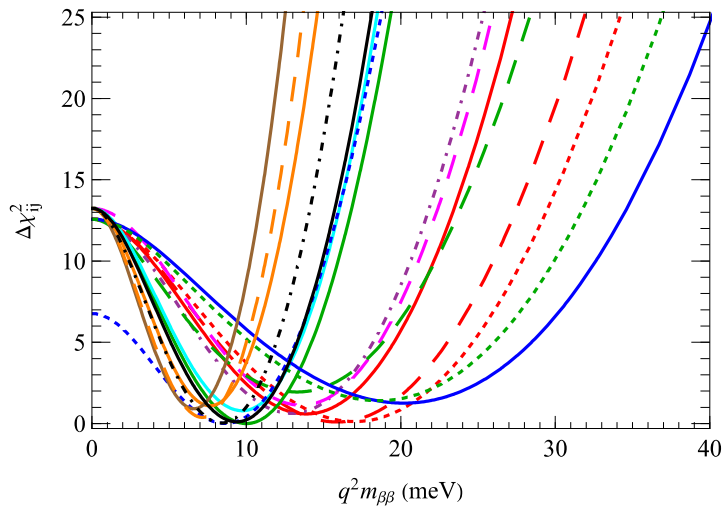
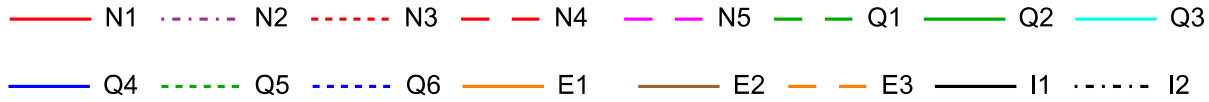


$m_{\beta\beta}=0, T=10 \text{ yr}$

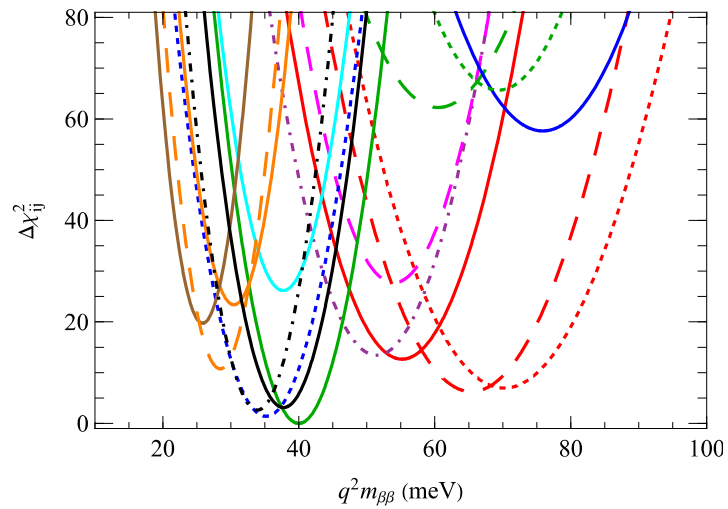
- | | |
|----------|----------|
| — N1 | — Q4 |
| - - - N2 | - - - Q5 |
| - . - N3 | — E1 |
| - - - N4 | — E2 |
| - . - N5 | - - - E3 |
| — Q2 | — I1 |
| — Q3 | - - - I2 |

$(\Delta\chi_{ij}^2)$ as function of $q^2 m_{\beta\beta}$

$i=Q2$



$(q^2 m_{\beta\beta})_{True} = 10$ meV



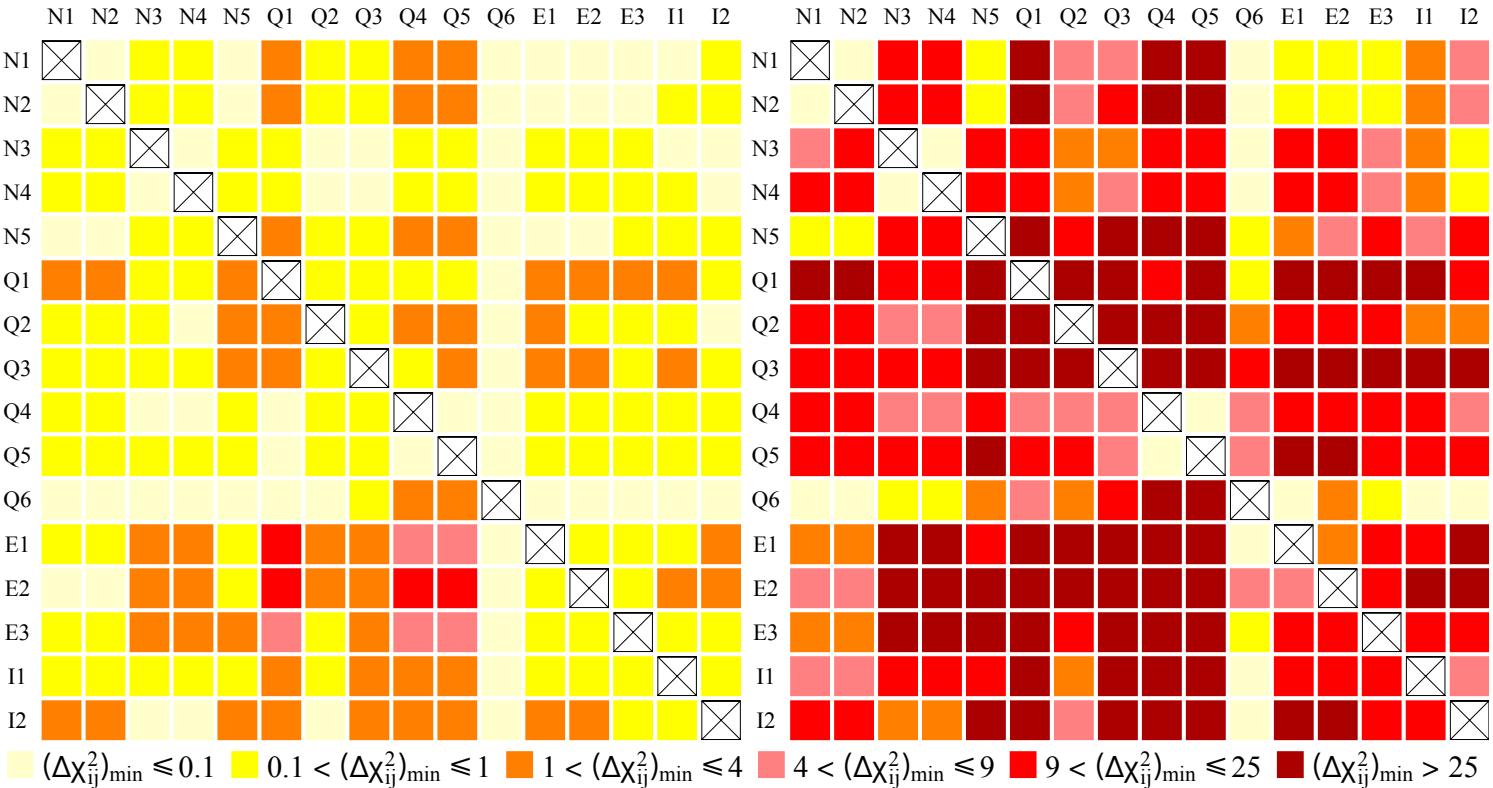
$(q^2 m_{\beta\beta})_{True} = 40$ meV

Discrimination without short-range NME

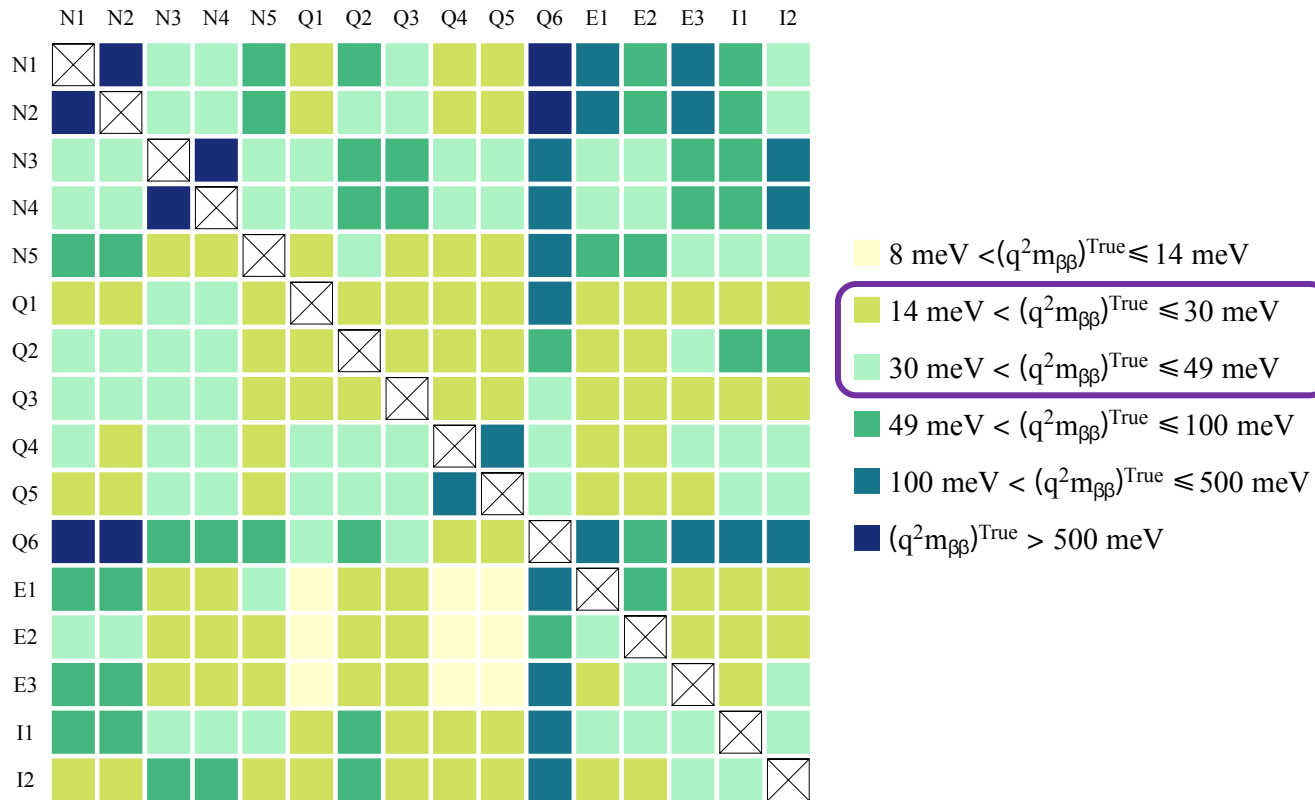
$$(\Delta\chi_{ij}^2)_{\min} = \min_{m_{\beta\beta}} \Delta\chi_{ij}^2(m_{\beta\beta}, M_{\alpha j}; (q^2 m_{\beta\beta})^{\text{True}}, M_{\alpha i}^{\text{True}})$$

$(q^2 m_{\beta\beta})^{\text{True}} = 10 \text{ meV}$

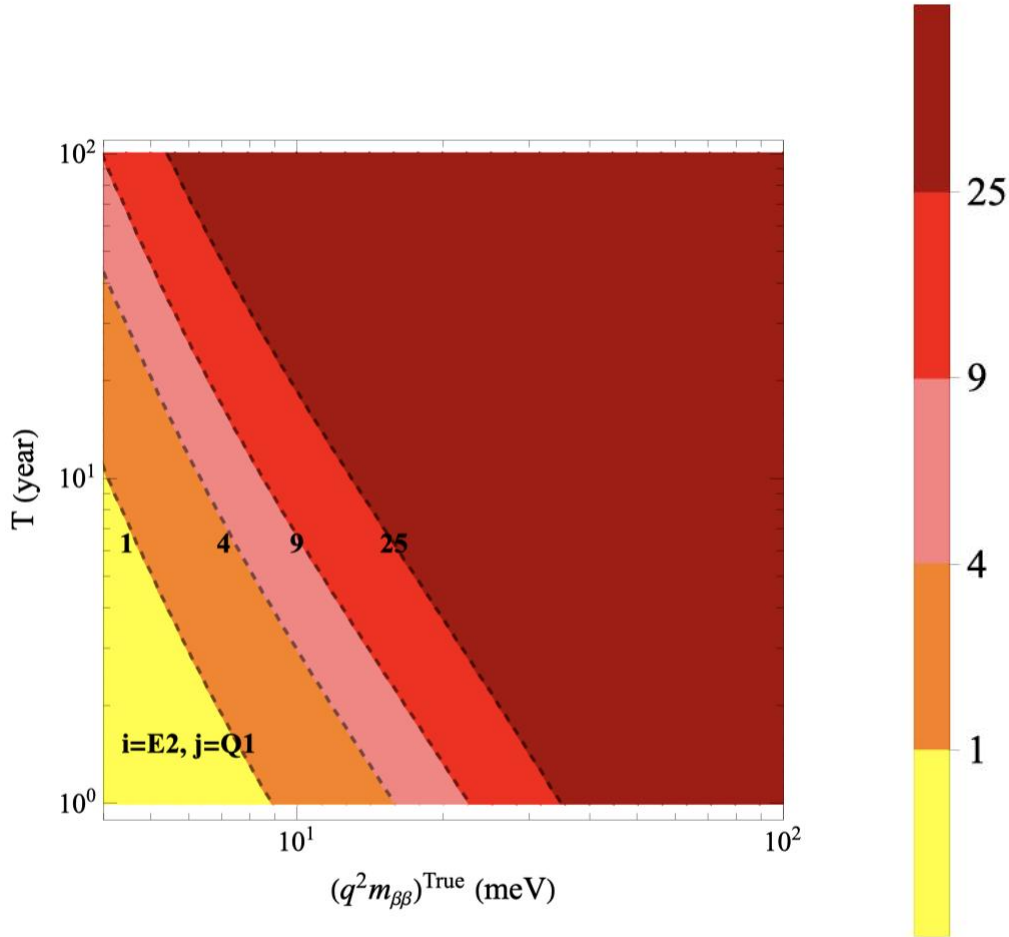
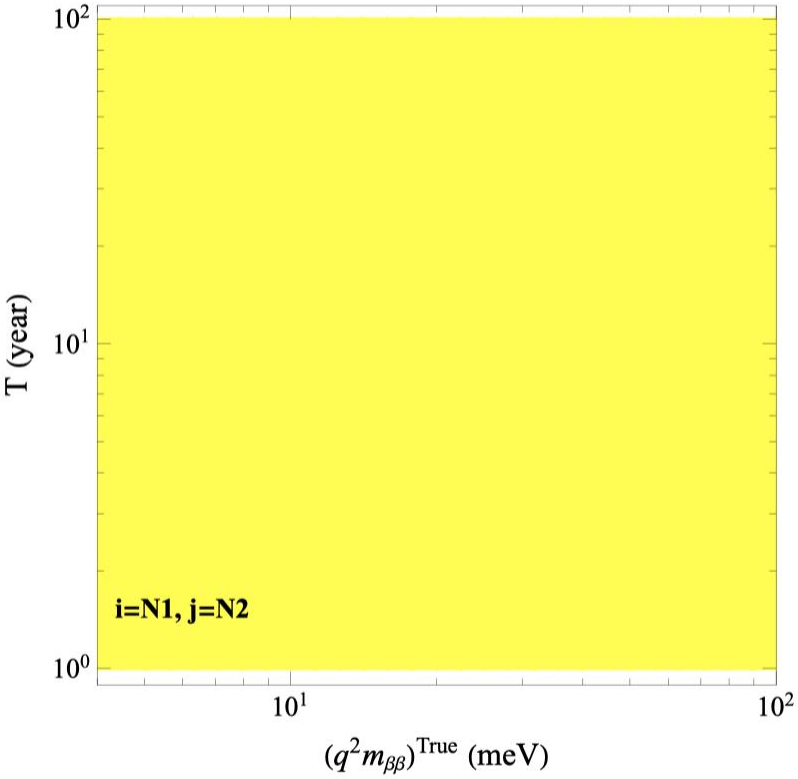
$(q^2 m_{\beta\beta})^{\text{True}} = 40 \text{ meV}$



$m_{\beta\beta}^{True}$ corresponding to discrimination at 3σ (without short-range NME)



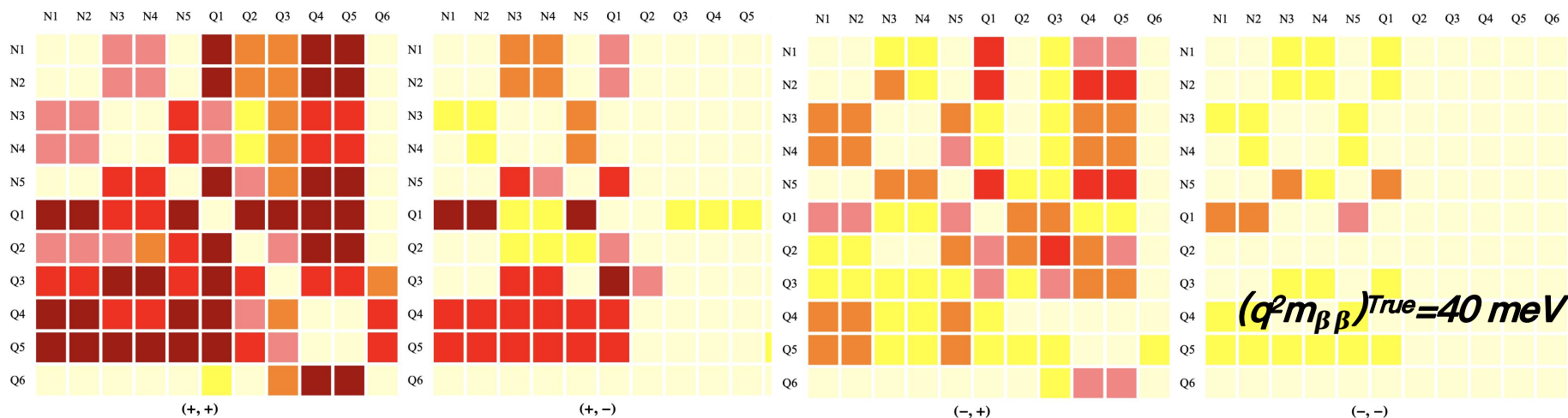
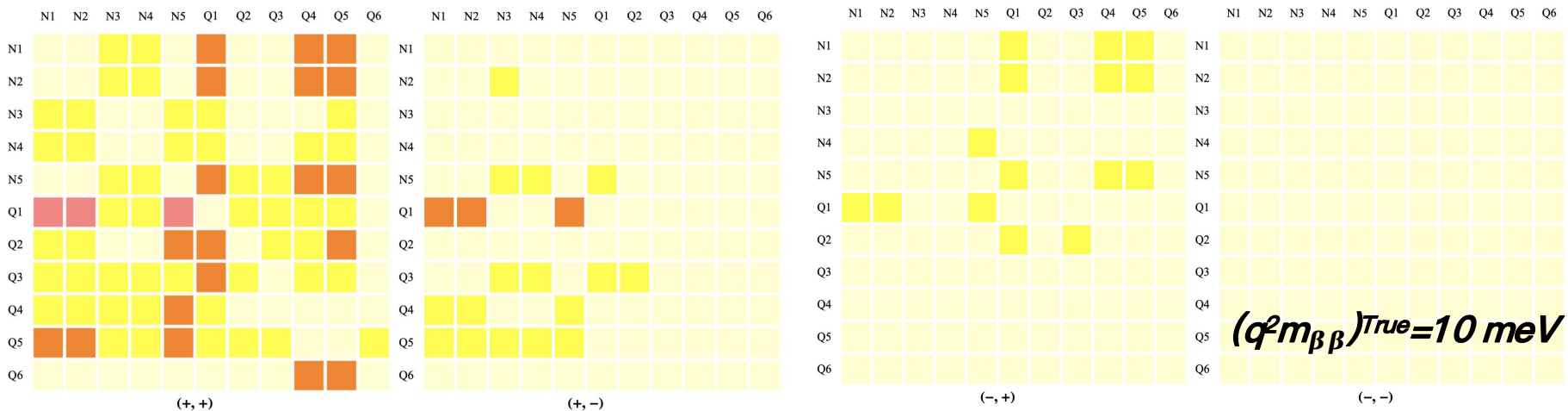
The contours of $(\Delta \chi^2_{ij})_{min}$ as function of the exposure time T and $(q^2 m_{\beta\beta})^{True}$



without short-range NME

Discrimination with short-range NME, $T=10$ yr

$(\Delta\chi_{ij}^2)_{\min} \leq 0.1$
 $0.1 < (\Delta\chi_{ij}^2)_{\min} \leq 1$
 $1 < (\Delta\chi_{ij}^2)_{\min} \leq 4$
 $4 < (\Delta\chi_{ij}^2)_{\min} \leq 9$
 $9 < (\Delta\chi_{ij}^2)_{\min} \leq 25$
 $(\Delta\chi_{ij}^2)_{\min} > 25$



Conclusions and outlook

- NME uncertainties due to the SRI may lead to the bound on $q^2 m_{\beta\beta}$ varying by a factor of order 10
- Promising discrimination of different NMEs if $(q^2 m_{\beta\beta})^{\text{True}} > 40 \text{ meV}$, positive SRI and 10 year exposure

- From $0\nu\beta\beta$ to $m_{\beta\beta}$: improving the calculations of NME
- From $0\nu\beta\beta$ to discriminating NME models: more input of $m_{\beta\beta}$

Which way is faster?
(the first?)

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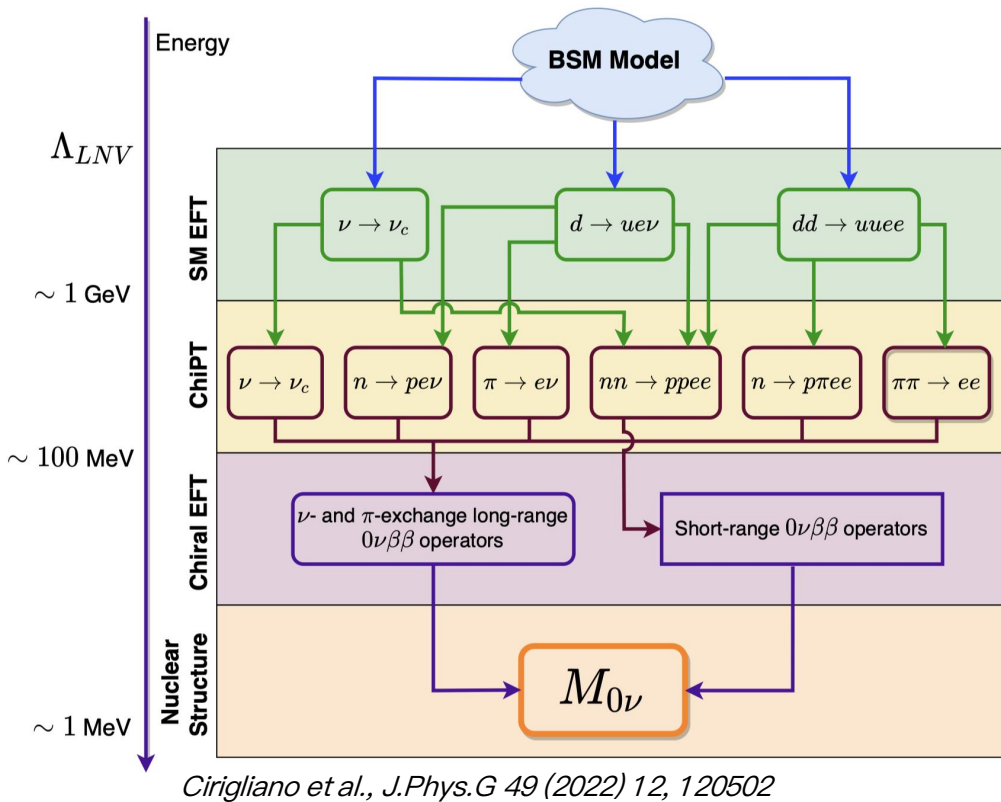
Which way is faster?
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Thanks for your attention!

Backups

From EFT to Nuclear Matrix Element (NME)

Naive Dimensional Analysis(NDA) problem

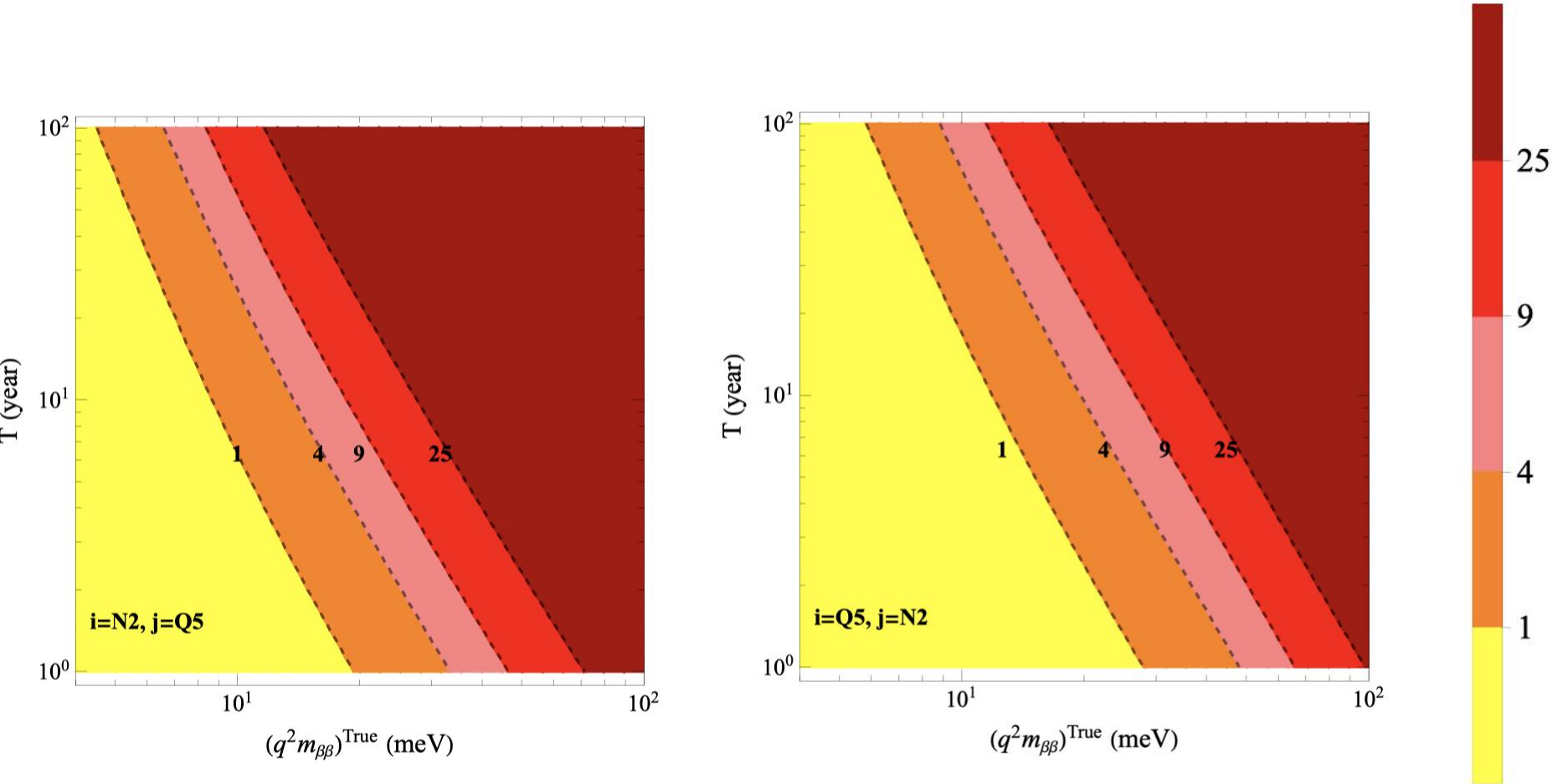


- | D. B. Kaplan, M. J. Savage, and M. B. Wise, *Nucl. Phys.* **B478**, 629 (1996).
- | S. R. Beane, P. F. Bedaque, M. J. Savage, and U. van Kolck, *Nucl. Phys.* **A700**, 377 (2002).
- | A. Nogga, R. G. E. Timmermans, and U. van Kolck, *Phys. Rev. C* **72**, 054006 (2005).
- | B. Long and C.-J. Yang, *Phys. Rev. C* **86**, 024001 (2012).
- | M. Pavón Valderrama and D. R. Phillips, *Phys. Rev. Lett.* **114**, 082502 (2015).

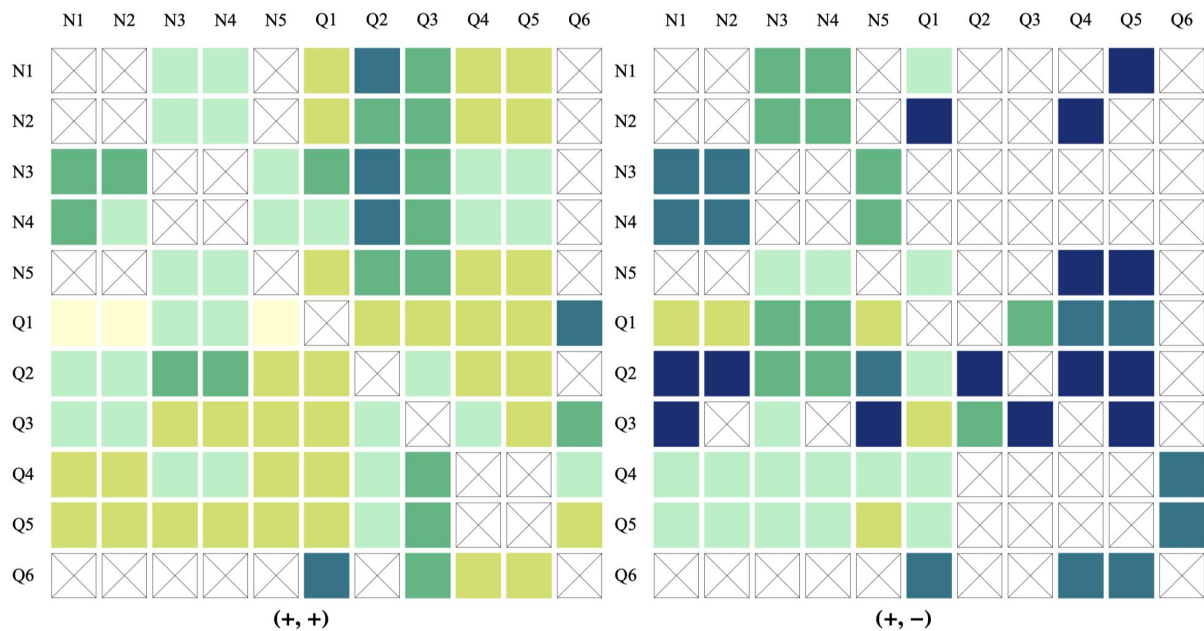
and $F_\pi = 92.2 \text{ MeV}$ is the pion decay constant. However, it is known that Weinberg's power counting leads to inconsistent results in nucleon-nucleon scattering [34–37] and nuclear processes mediated by external currents [38], due to a conflict between naive dimensional analysis and nonperturbative renormalization. We therefore investigate the scaling of g_ν^{NN} by studying the amplitude $\mathcal{A}(nn \rightarrow p p e e) \equiv \mathcal{A}_{\Delta L=2}$ with strong interactions H_{strong} included nonperturbatively.

Cirigliano et al, Phys.Rev.Lett. 120 (2018) 20, 202001

The contours of $(\Delta \chi_{ij}^2)_{min}$ as function of the exposure time T and $(q^2 m_{\beta\beta})^{True}$



$m_{\beta\beta}^{True}$ corresponding to discrimination at 3σ (with SRI)



$12 \text{ meV} < (q^2 m_{\beta\beta})^{True} \leq 14 \text{ meV}$

 $14 \text{ meV} < (q^2 m_{\beta\beta})^{True} \leq 30 \text{ meV}$

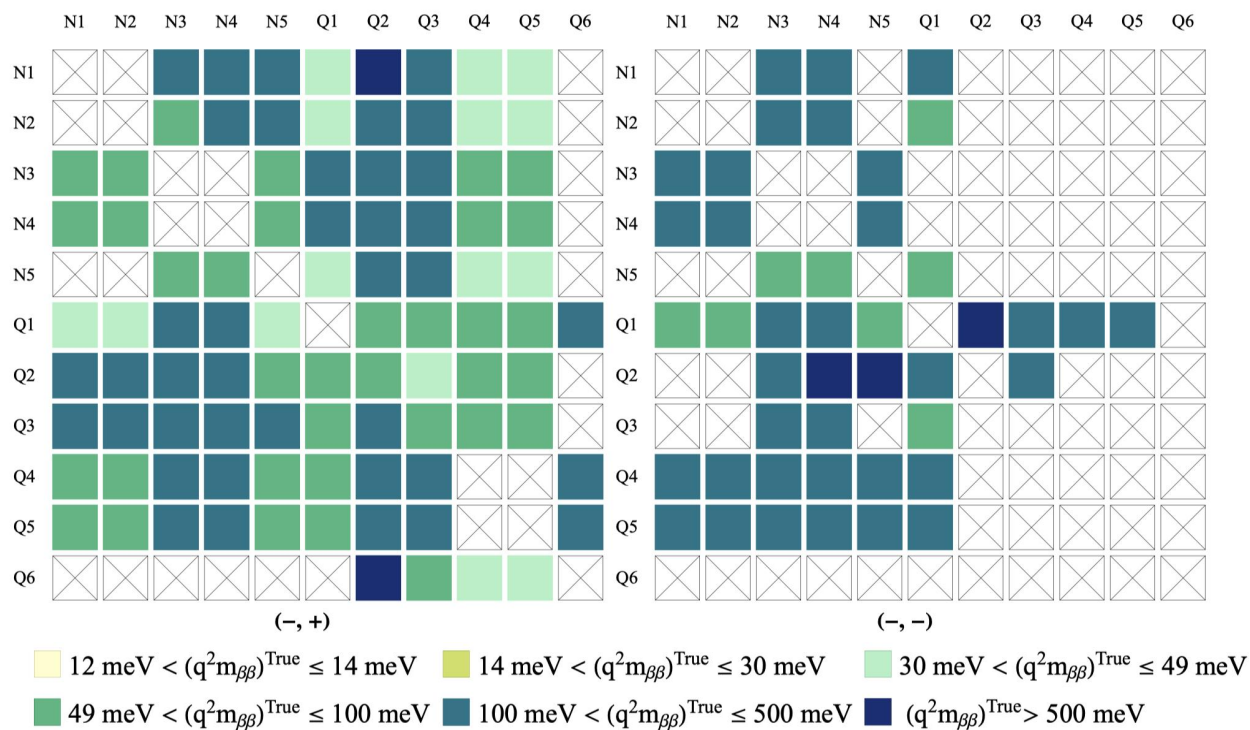
 $30 \text{ meV} < (q^2 m_{\beta\beta})^{True} \leq 49 \text{ meV}$

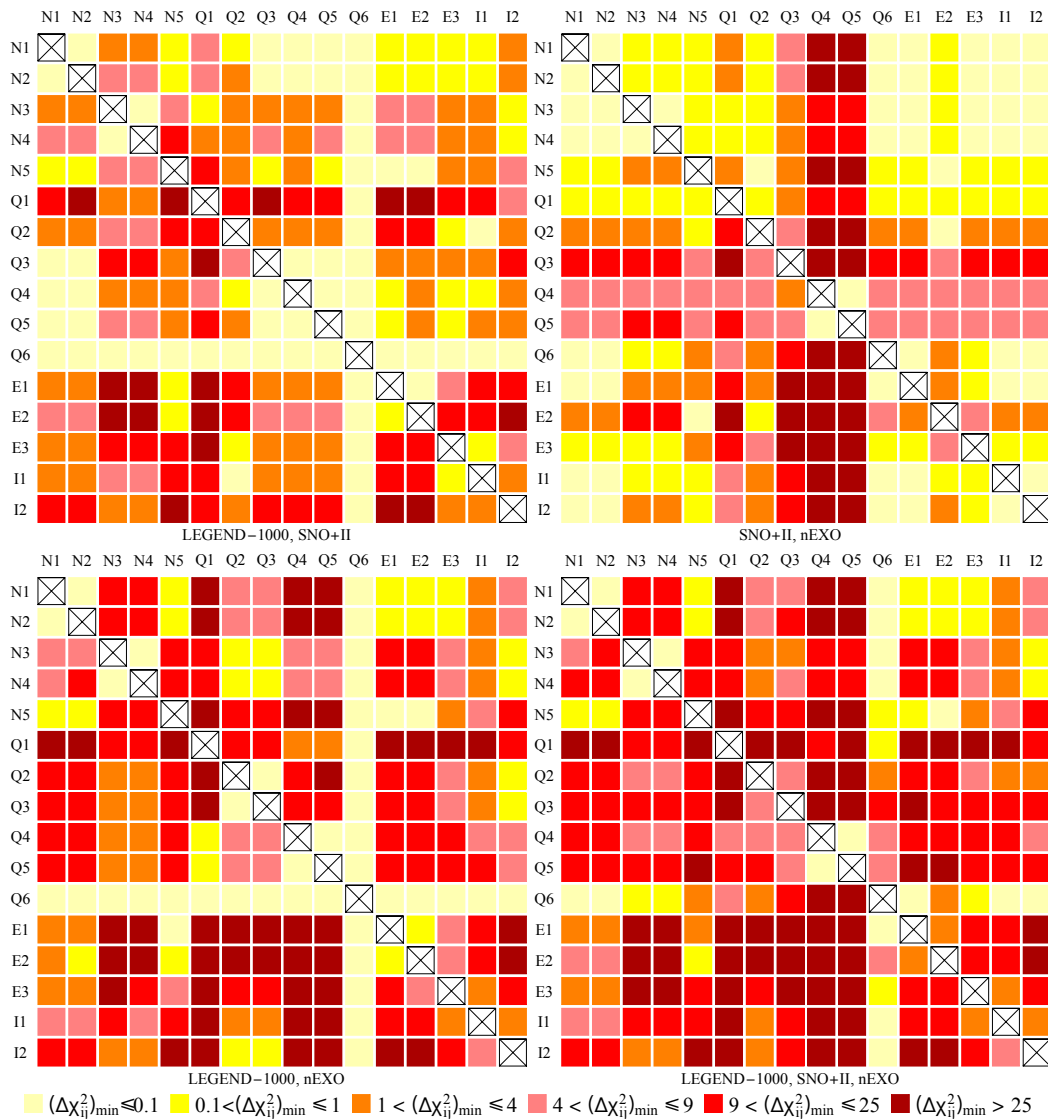
 $49 \text{ meV} < (q^2 m_{\beta\beta})^{True} \leq 100 \text{ meV}$

 $100 \text{ meV} < (q^2 m_{\beta\beta})^{True} \leq 500 \text{ meV}$

 $(q^2 m_{\beta\beta})^{True} > 500 \text{ meV}$

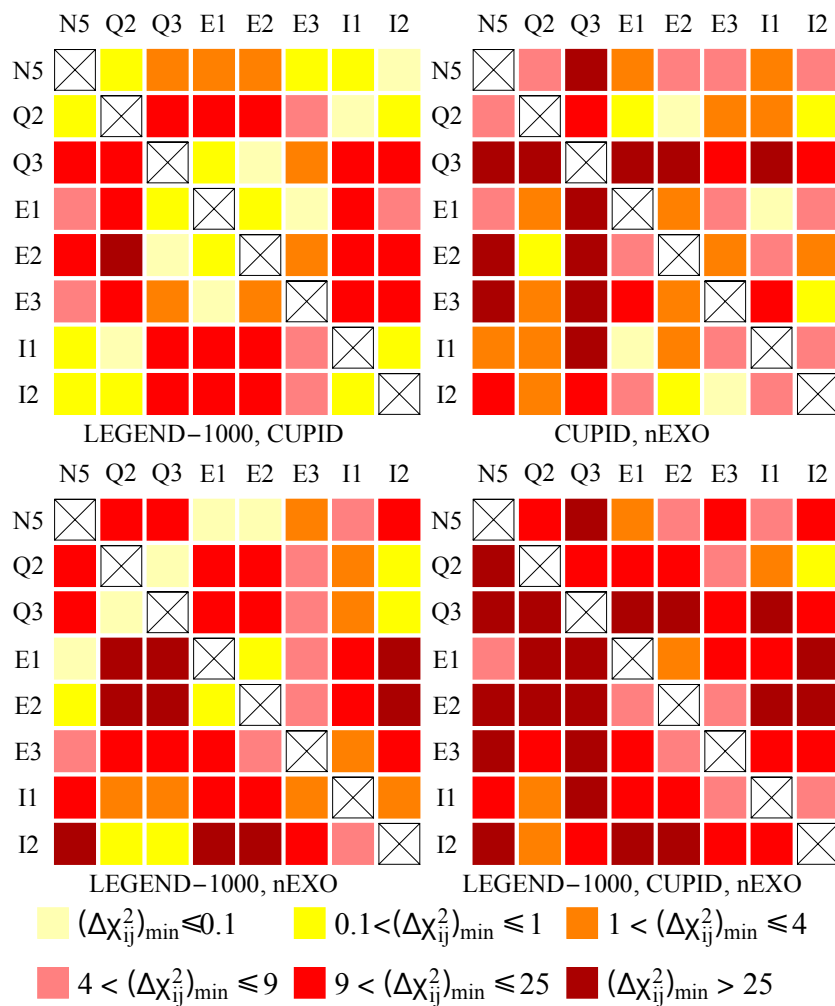
$m_{\beta\beta}^{True}$ corresponding to discrimination at 3σ (with SRI)





nEXO and LEGEND-1000 dominate

$$m_{\beta\beta}^{True} = 40 \text{ meV}$$



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