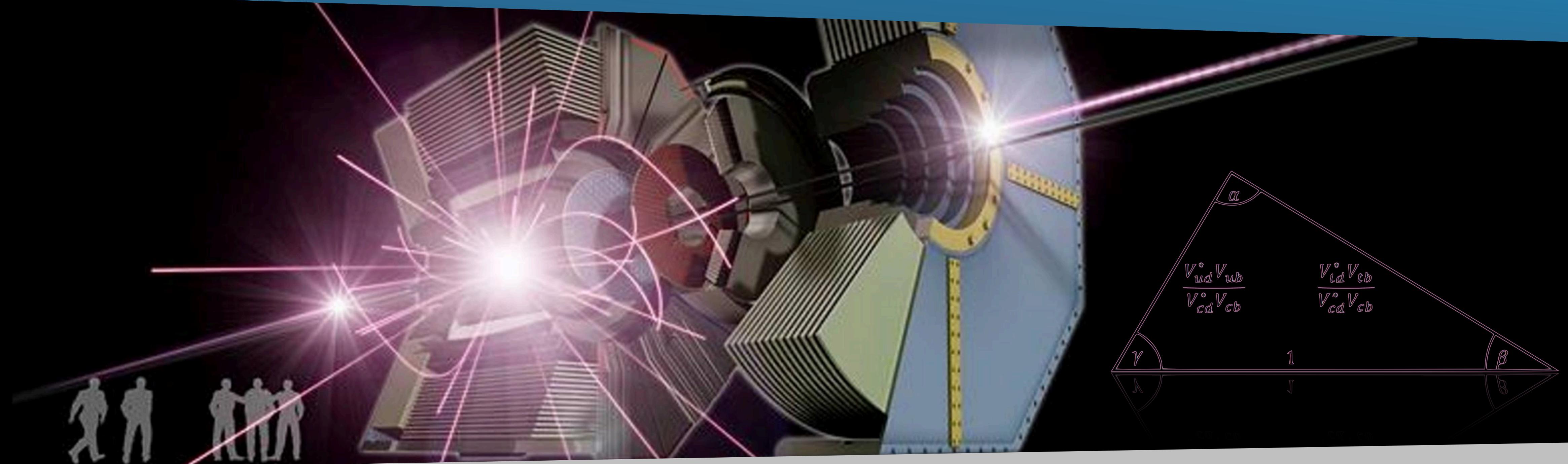


Highlights of recent $|V_{xb}|$ measurements from Belle (II)



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第五届重味物理与量子色动力学研讨会 2023年4月21日



Content

Measurements covered in this talk:

Exclusive $|V_{cb}|$:

- Had. tagged $B^0 \rightarrow D^* \ell \nu$
- Had. tagged $B \rightarrow D^* \ell \nu$ and shapes of key kinematic variables

Exclusive $|V_{ub}|$:

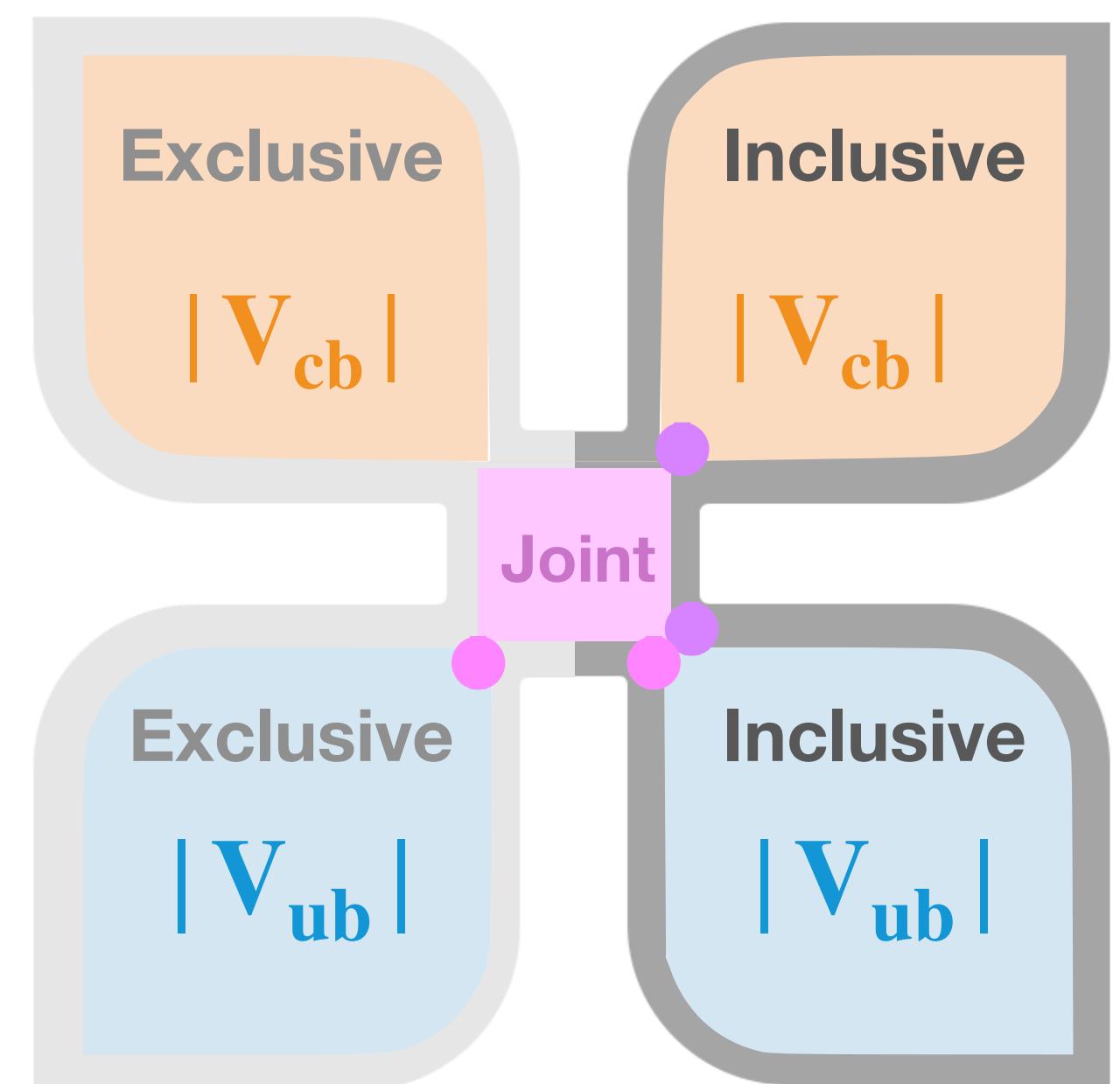
- Untagged $B^0 \rightarrow \pi^- \ell \nu$

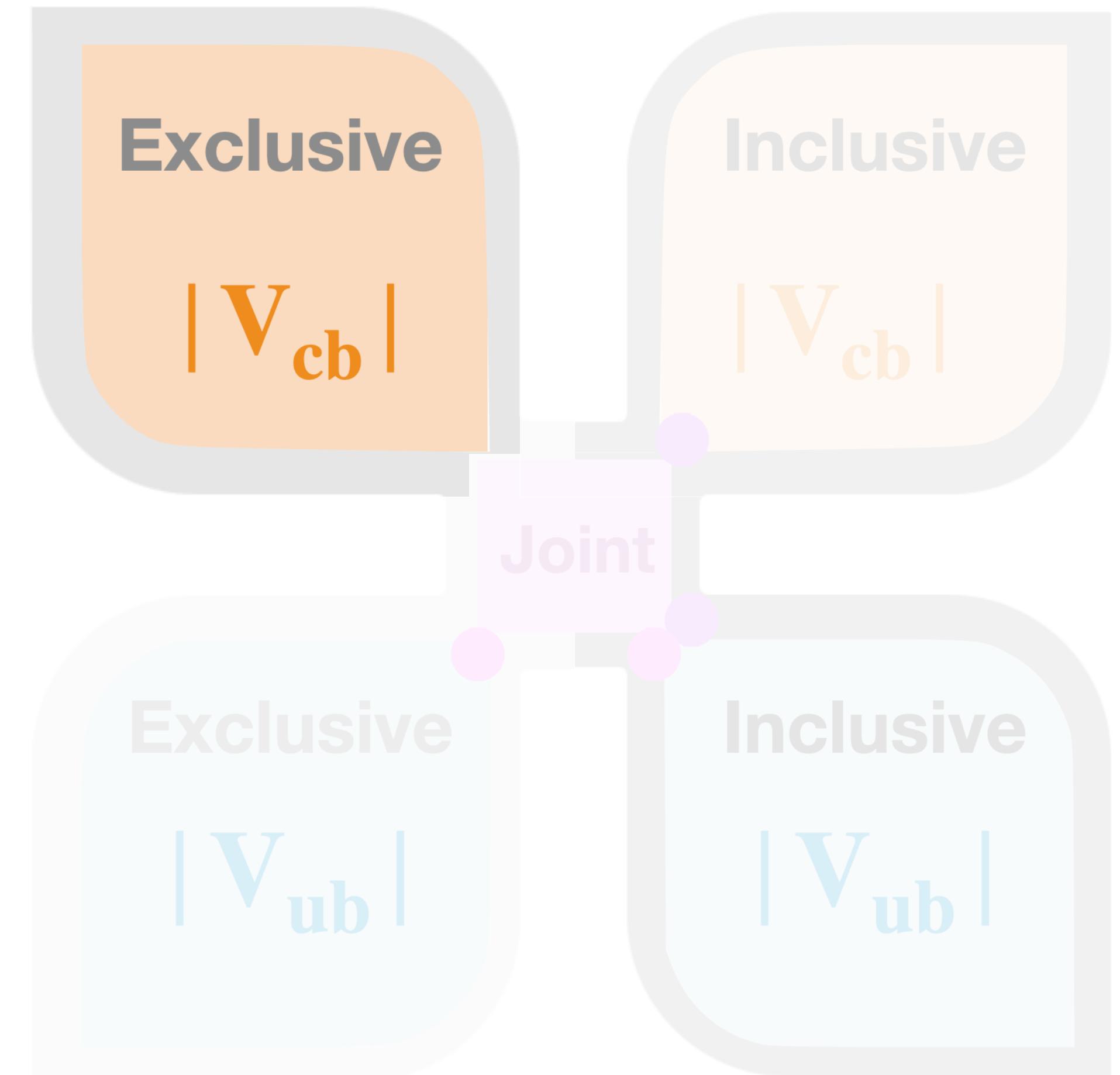
Inclusive $|V_{ub}|$:

- Partial & differential branching fractions of $B \rightarrow X_u \ell \nu$

Combined measurements:

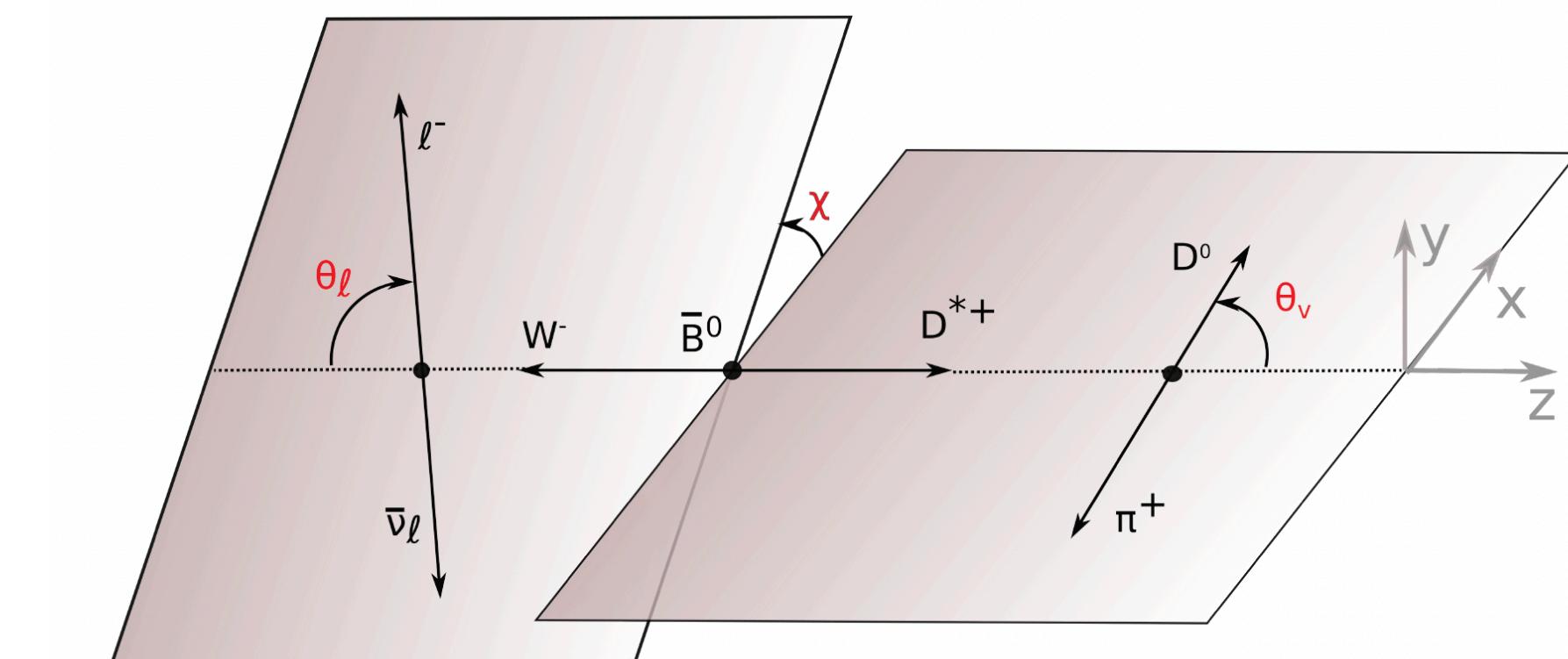
- Excl. $|V_{ub}|$ / incl. $|V_{ub}|$
- Incl. $|V_{ub}|$ / incl. $|V_{cb}|$



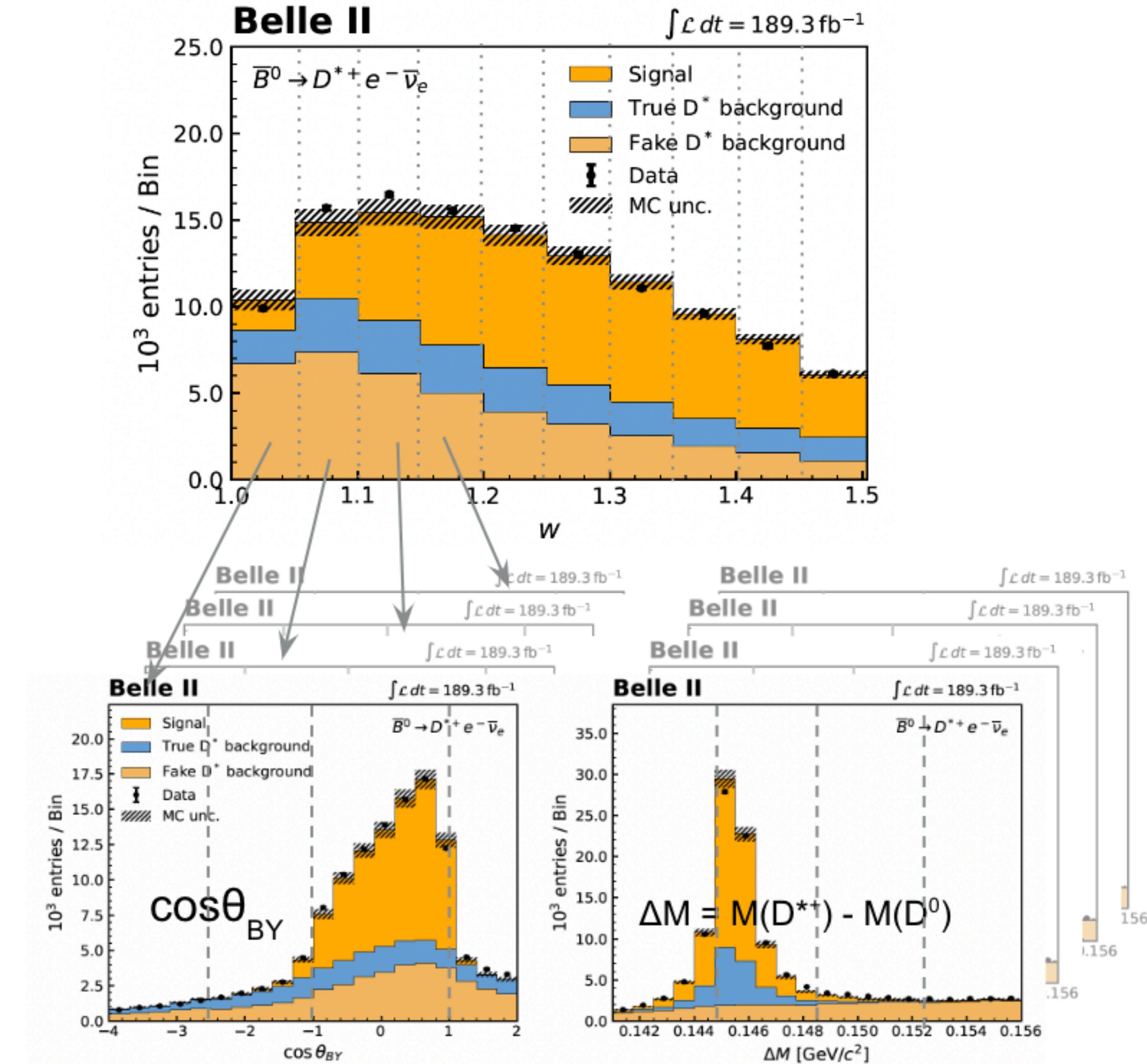


Branching Fraction of $B^0 \rightarrow D^{*+} \ell \nu$ and $|V_{cb}|$

- Decay chain: $B^0 \rightarrow D^{*+} \ell \nu$, $D^{*+} \rightarrow D^0 \pi^+$ _{slow}, $D^0 \rightarrow K^- \pi^+$
- Untagged strategy (higher efficiency than tagged)
- Select energetic signal lepton $p_{CM}^{\ell} > 1.2$ GeV
- $M(D^0) = m_{PDG} \pm 15$ MeV, $M(D^{*+}) - M(D^0) = [0.141, 0.156]$ GeV, $\cos\theta_{BY} = [-4, 2]$
- 2D binned likelihood fit on $(\cos\theta_{BY}, \Delta M)$ for each bin of kinematic variables: recoil parameter w , and angles $\cos\theta_\ell$, $\cos\theta_\nu$, χ
- Systematic shape variations incorporated as bin-wise Nuisance para. for each fit template



$$\cos\theta_{BY} = \frac{2E_B^{CM} E_Y^{CM} - m_B^2 - m_Y^2}{2|\vec{p}_B^{CM}| |\vec{p}_Y^{CM}|}$$

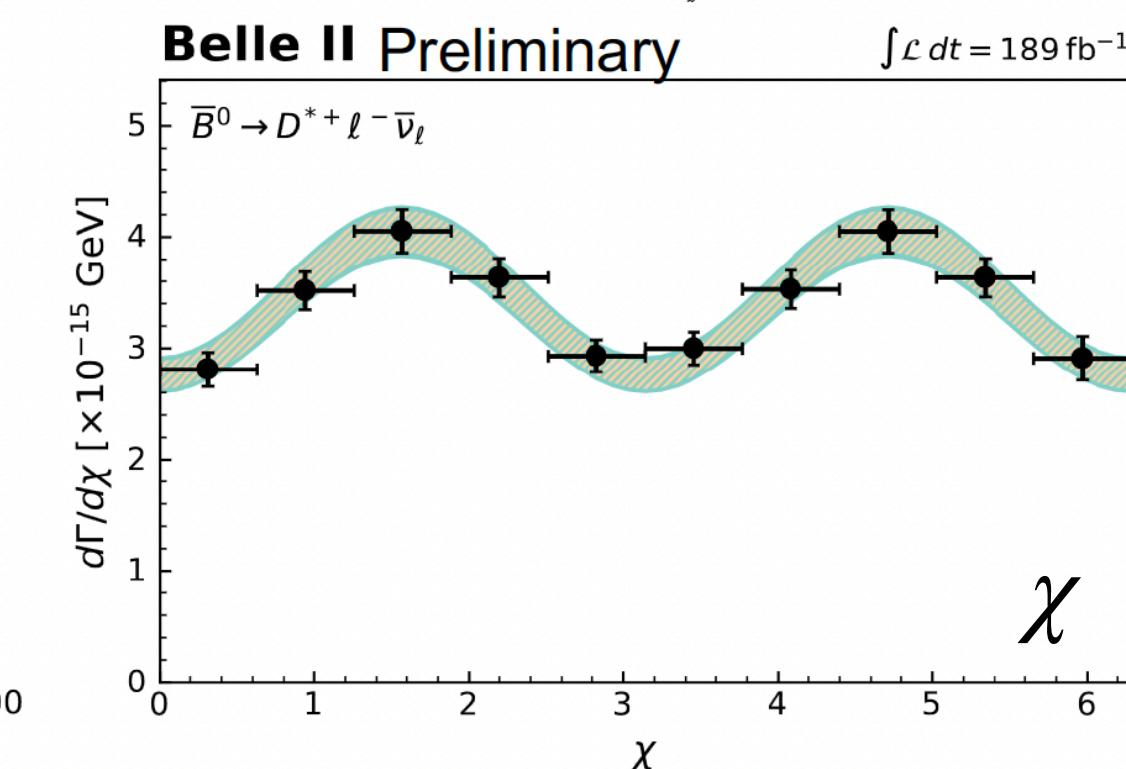
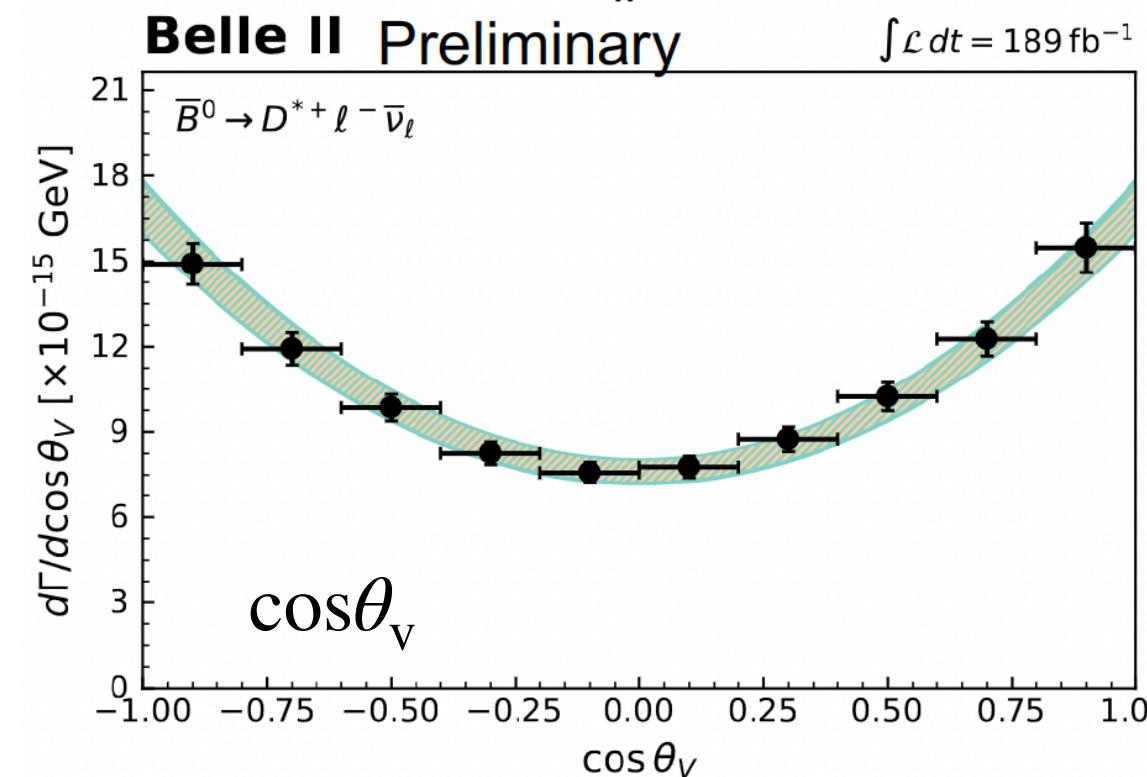
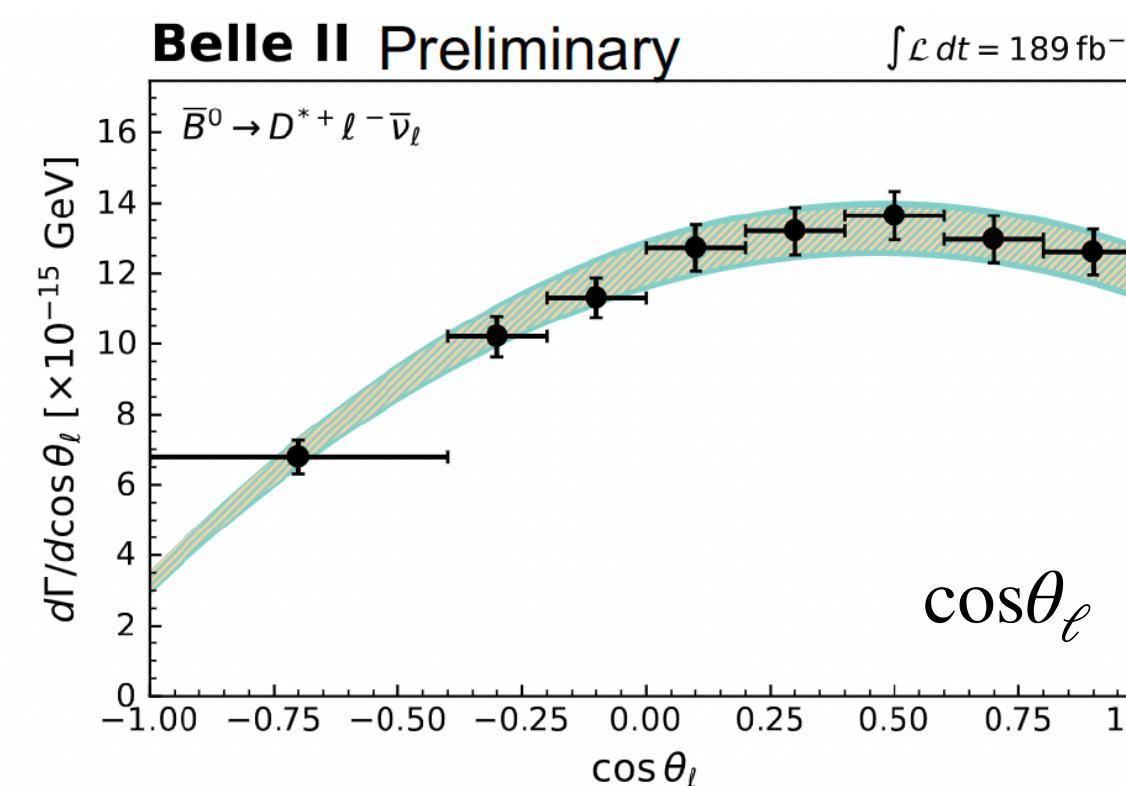
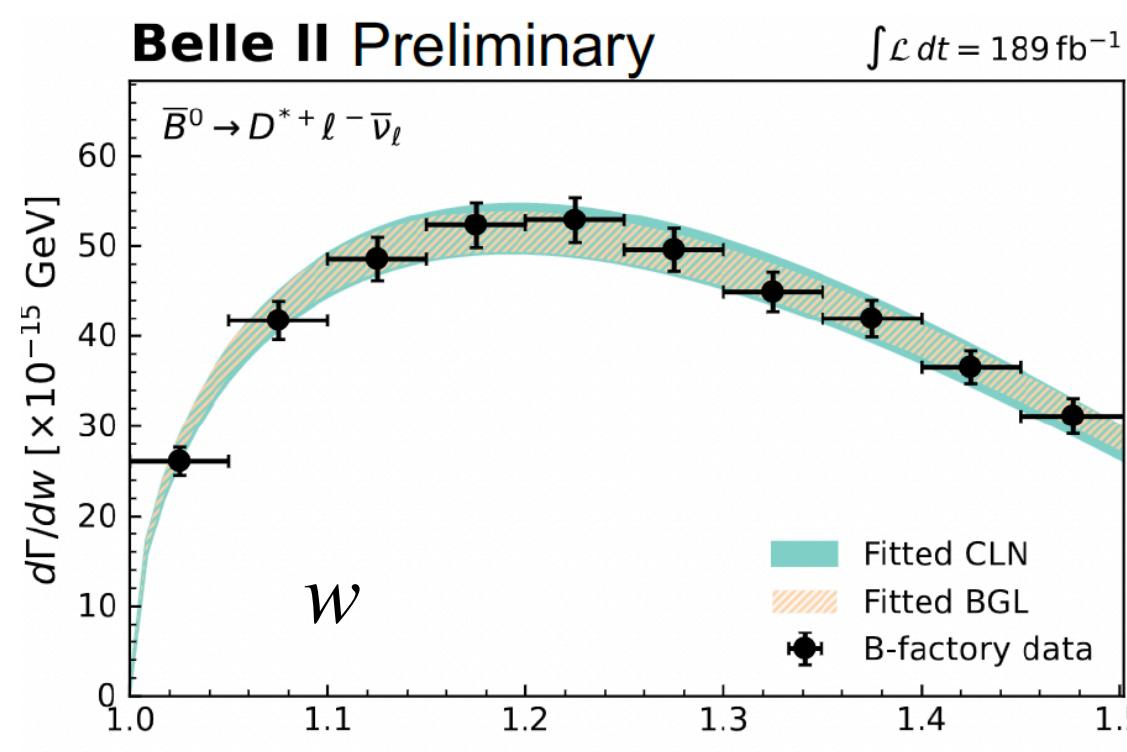


integral projection

Branching Fraction of $B^0 \rightarrow D^* \ell \nu$ and $|V_{cb}|$

Preliminary

- Unfold signal yields using **singular-value-decomposition (SVD)** method within `pyRooUnfold`, regularization para. optimised for low bias & stable result
- Full post-unfolding stat. & syst. covariance propagated into partial decay rate



$$\Delta\Gamma_i = \frac{y_i^{\text{unfolded}}}{\epsilon_i N_{B^0} \mathcal{B}(D^{*+} \rightarrow D^0 \pi^+) \mathcal{B}(D^0 \rightarrow K^- \pi^+) \tau_{B^0}}$$

↑ reco. eff & acc.

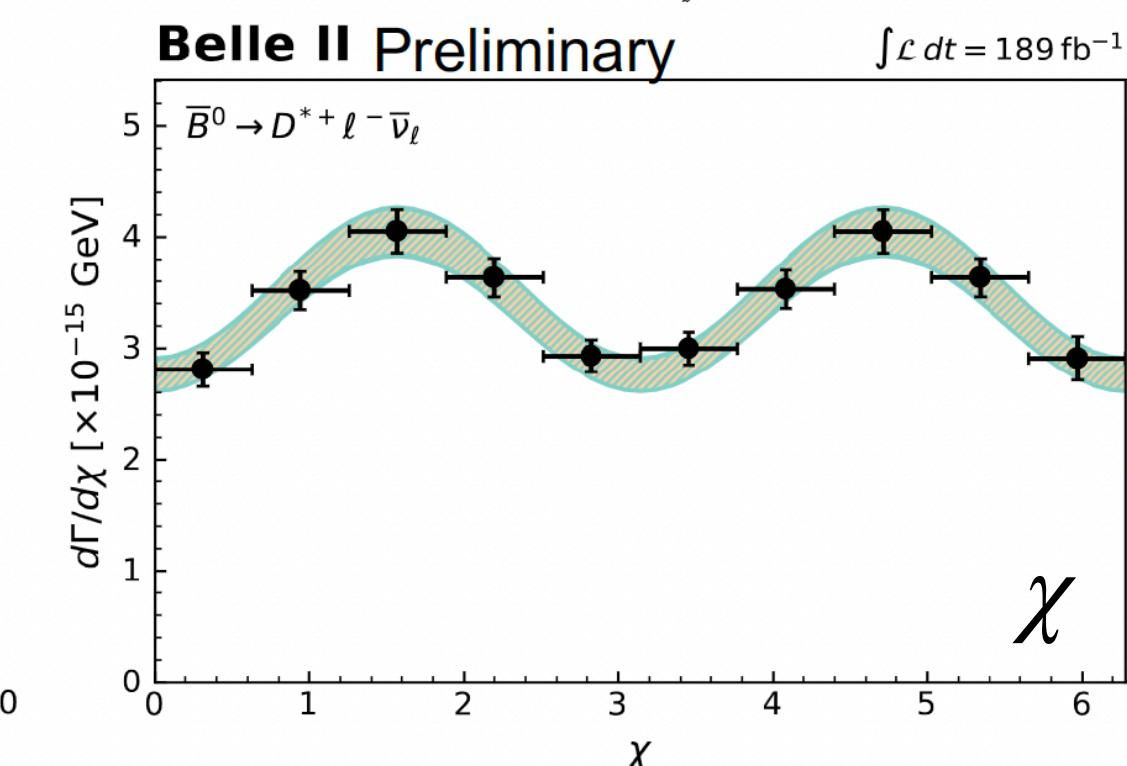
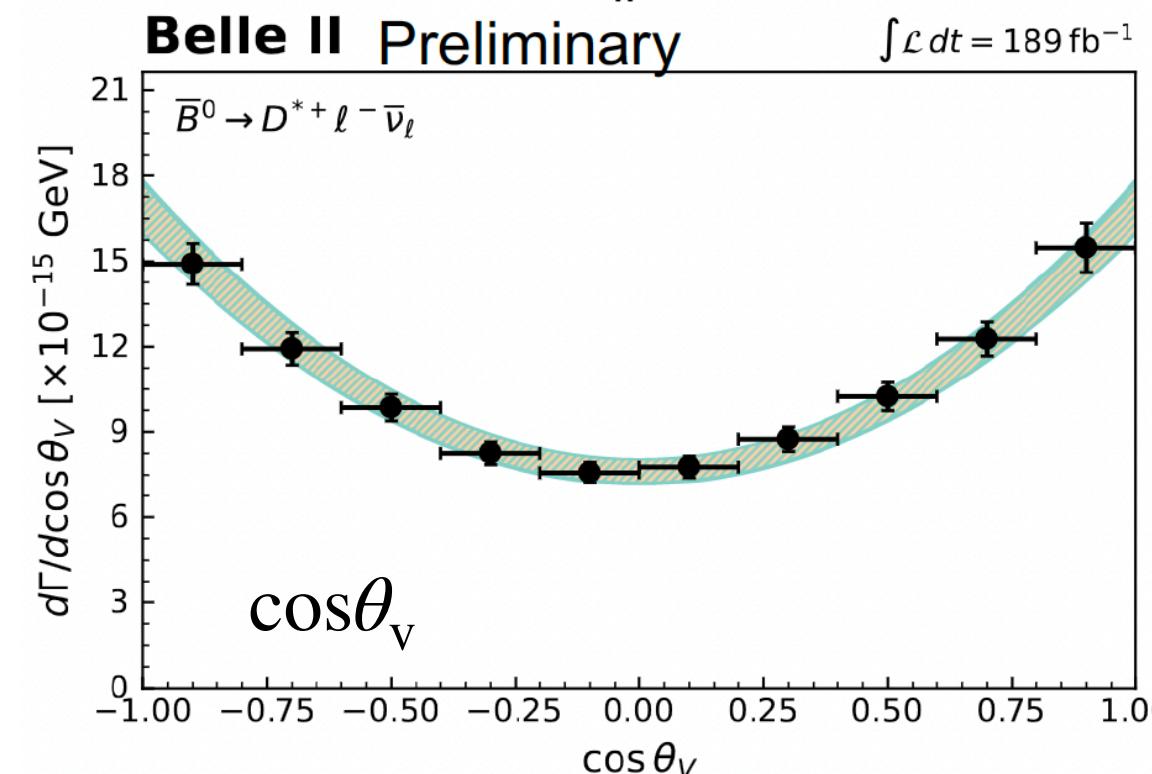
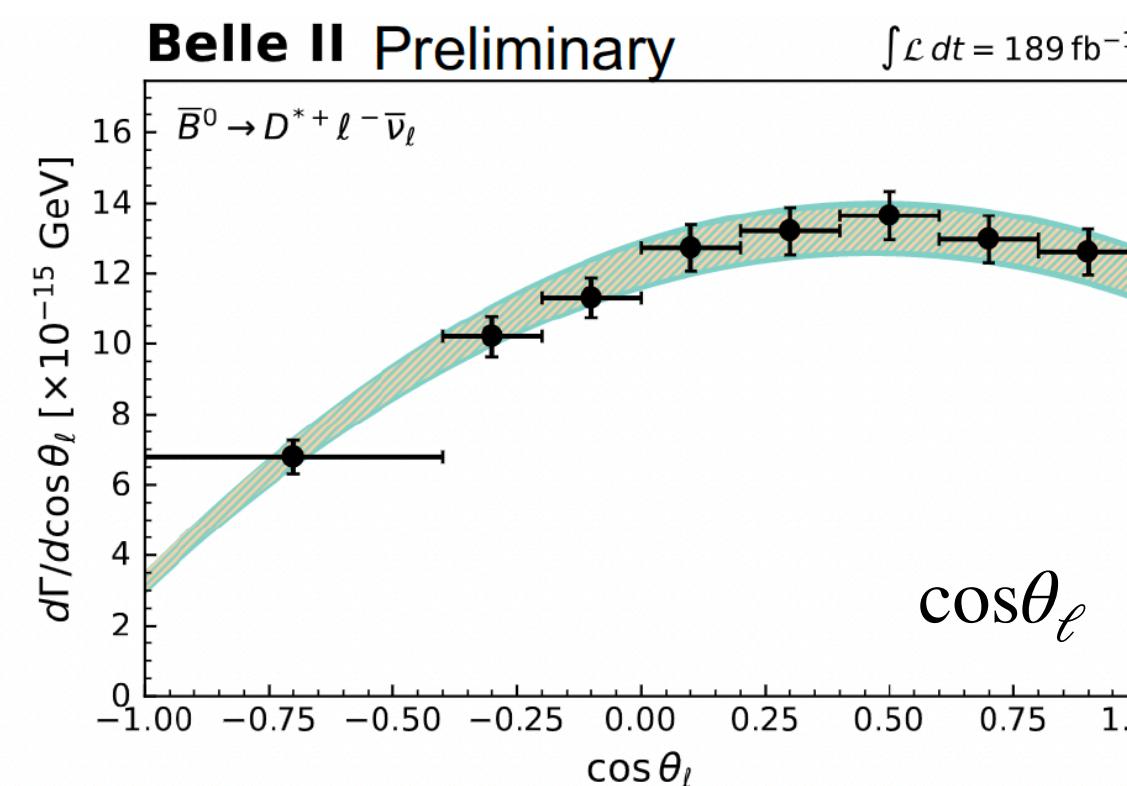
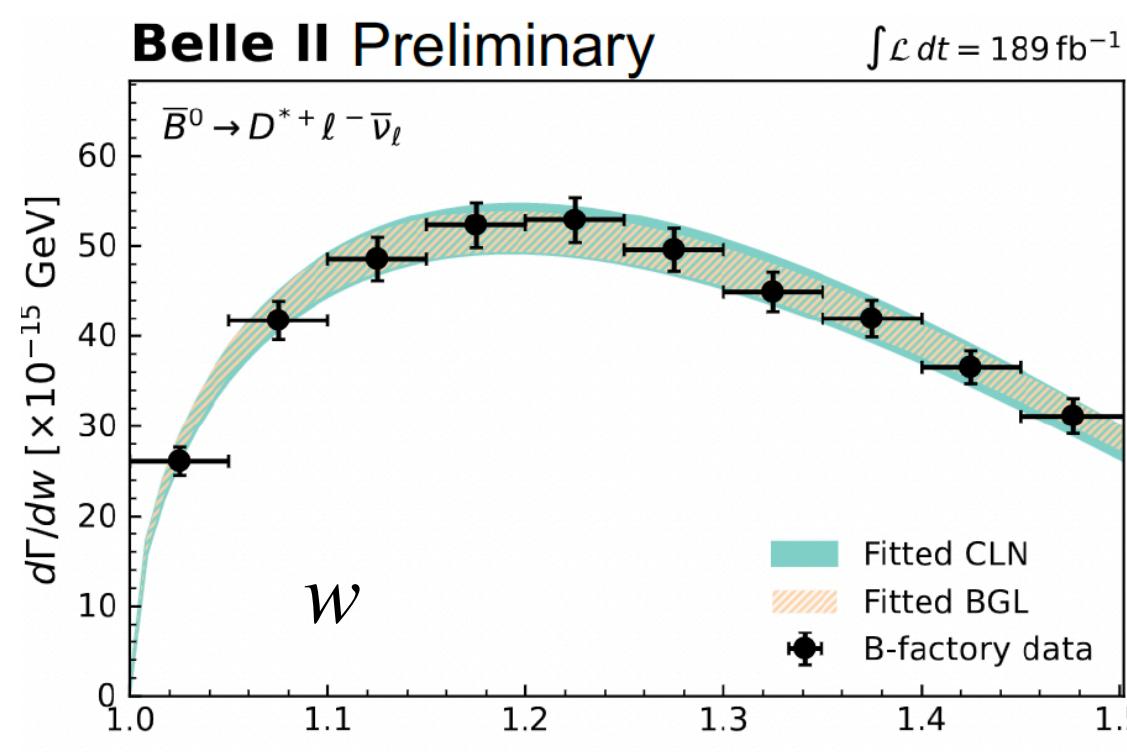
input of PDG2022

$$\Gamma = \left(\sum_{i=1}^{10} \Delta\Gamma_i^w + \sum_{i=1}^8 \Delta\Gamma_i^{\cos\theta_\ell} + \sum_{i=1}^{10} \Delta\Gamma_i^{\cos\theta_V} + \sum_{i=1}^{10} \Delta\Gamma_i^\chi \right) / 4$$

Branching Fraction of $B^0 \rightarrow D^* \ell \bar{\nu}$ and $|V_{cb}|$

Preliminary

- Unfold signal yields using **singular-value-decomposition (SVD)** method within `pyRooUnfold`, regularization para. optimised for low bias & stable result
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input of PDG2022

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Branching fraction extracted by the **total rate** summing over partial decay rates and averaging all kin. variables

e mode: $\mathcal{B}(\bar{B}^0 \rightarrow D^{*+} e^- \bar{\nu}_e) = (4.94 \pm 0.03 \pm 0.22)\%$

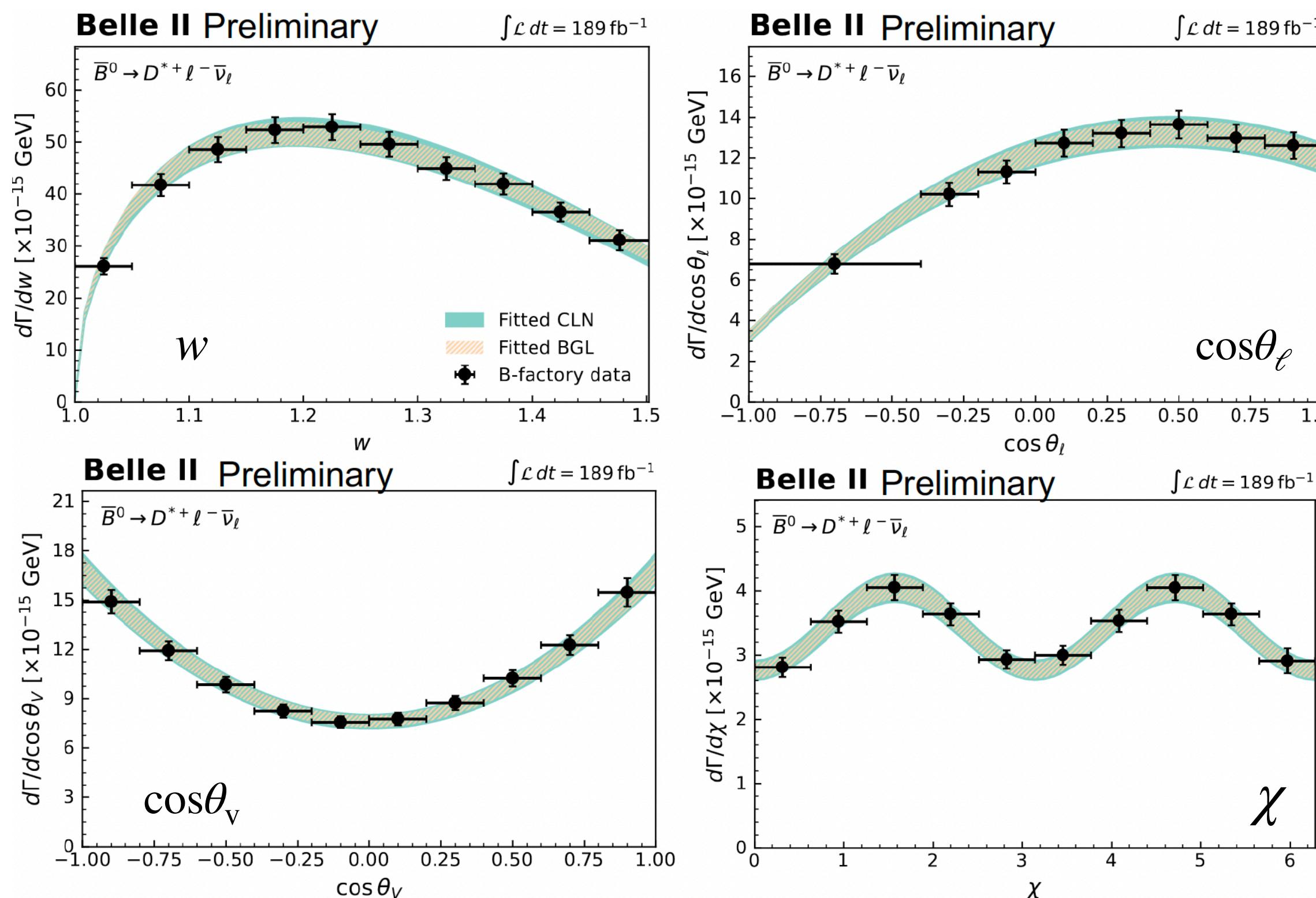
mu mode: $\mathcal{B}(\bar{B}^0 \rightarrow D^{*+} \mu^- \bar{\nu}_\mu) = (4.94 \pm 0.03 \pm 0.24)\%$

average: $\mathcal{B}(\bar{B}^0 \rightarrow D^{*+} \ell^- \bar{\nu}_\ell) = (4.94 \pm 0.02 \pm 0.22)\%$

Branching Fraction of $B^0 \rightarrow D^* \ell \bar{\nu}$ and $|V_{cb}|$

- Include all measured w , $\cos\theta_\ell$, $\cos\theta_v$, χ to extract form factor & $|V_{cb}|$
- Fit with form factor expansion based on **CLN & BGL (truncation tested)**
- Rerdundant degrees of freedom removed by using **normalized partial rates** on each variable together with the **averaged total rate** (ndf = 34+1)

Preliminary



$$\chi^2 = \sum_{i,j}^{34} \left(\frac{\Delta\Gamma_i^{\text{obs}}}{\Gamma_i^{\text{obs}}} - \frac{\Delta\Gamma_i^{\text{pre}}}{\Gamma_i^{\text{pre}}} \right) C_{ij}^{-1} \left(\frac{\Delta\Gamma_j^{\text{obs}}}{\Gamma_j^{\text{obs}}} - \frac{\Delta\Gamma_j^{\text{pre}}}{\Gamma_j^{\text{pre}}} \right) + \frac{(\Gamma^{\text{obs}} - \Gamma^{\text{pre}})^2}{\sigma_\Gamma^2}$$

$$|V_{cb}| \eta_{\text{EW}} \mathcal{F}(1) = \frac{1}{\sqrt{m_B m_{D^*}}} \left(\frac{|\tilde{b}_0|}{P_f(0) \phi_f(0)} \right)$$

$$|V_{cb}|_{\text{BGL}} = (40.9 \pm 0.3 \pm 1.0 \pm 0.6) \times 10^{-3}$$

$$|V_{cb}|_{\text{CLN}} = (40.4 \pm 0.3 \pm 1.0 \pm 0.6) \times 10^{-3}$$

Slow pion eff. plays leading role in syst.

Input from LQCD at zero-recoil $F(1)$

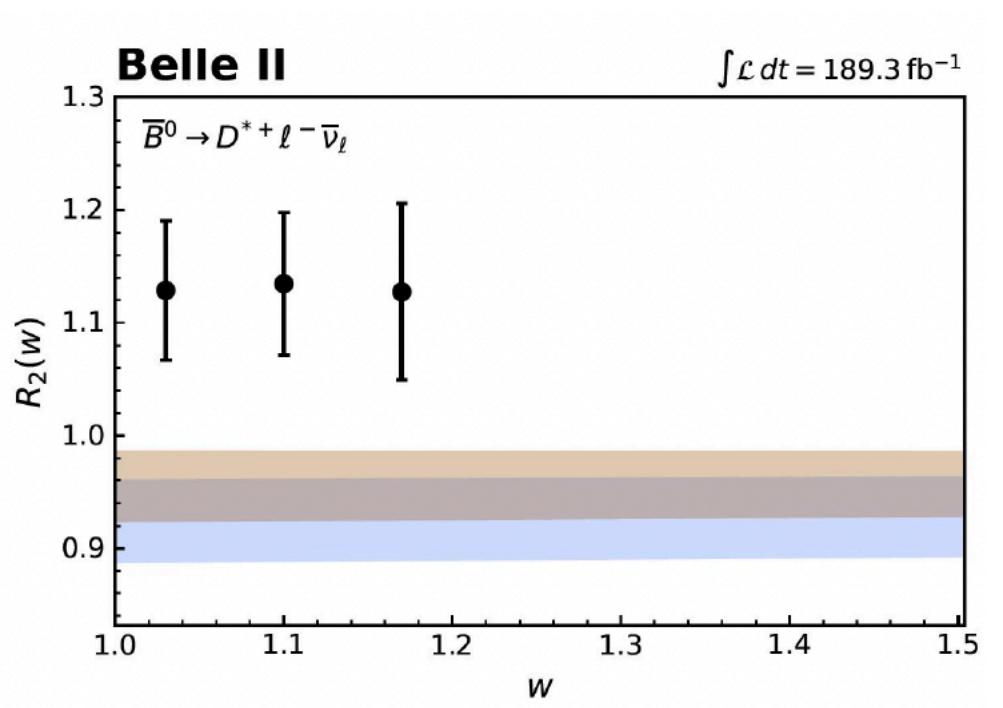
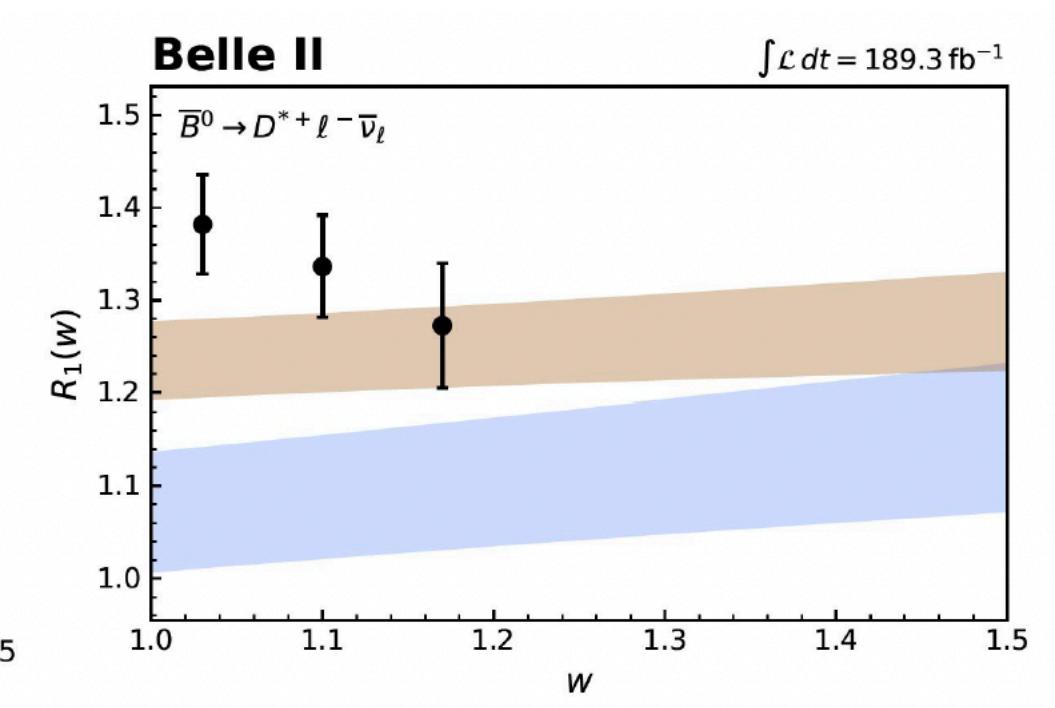
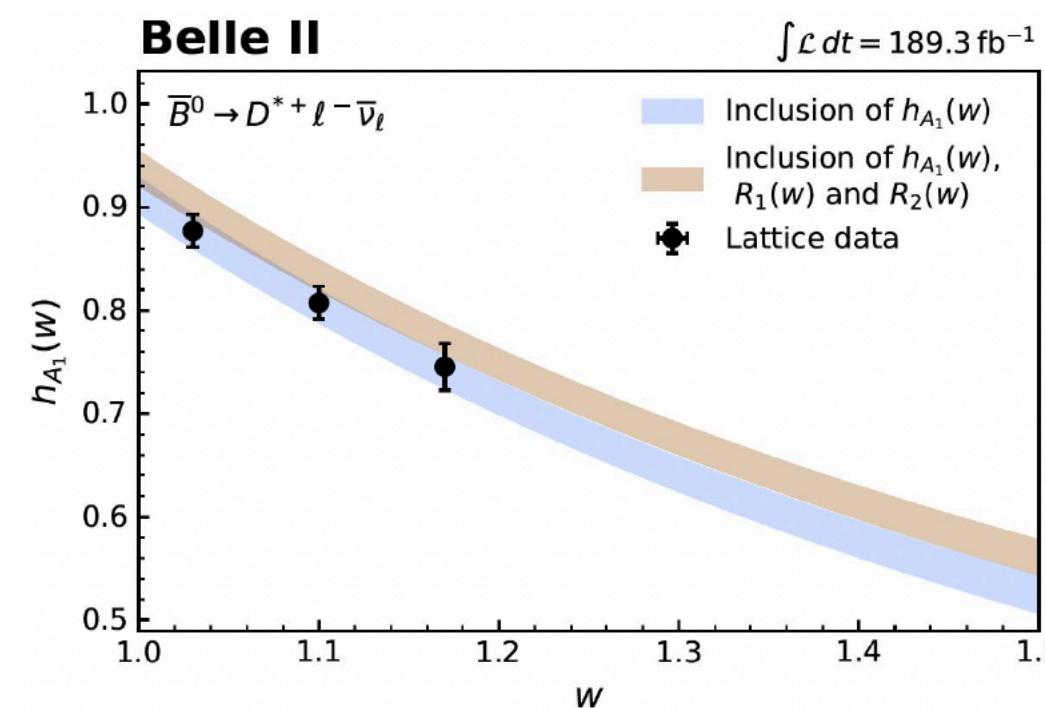
Branching Fraction of $B^0 \rightarrow D^* \ell \bar{\nu}$ and $|V_{cb}|$

- Include all measured w , $\cos\theta_\ell$, $\cos\theta_v$, χ to extract form factor & $|V_{cb}|$
- Fit with form factor expansion based on **CLN & BGL (truncation tested)**
- Rerdundant degrees of freedom removed by using **normalized partial rates** on each variable together with the **averaged total rate** ($\text{ndf} = 34+1$)
- Inclusion of LQCD constraint [arxiv:2105.14019] at beyond zero-recoil ($w = [1.03, 1.10, 1.17]$) in two scenarios

Preliminary

BGL	Constraints on $h_{A_1}(w)$	Constraints on $h_{A_1}(w), R_1(w), R_2(w)$
$a_0 \times 10^3$	21.7 ± 1.4	25.7 ± 0.8
$b_0 \times 10^3$	13.20 ± 0.24	13.58 ± 0.23
$b_1 \times 10^3$	-7 ± 7	2 ± 6
$c_1 \times 10^3$	-1.1 ± 0.8	-0.5 ± 0.8
$ V_{cb} \times 10^3$	40.5 ± 1.2	38.6 ± 1.1
χ^2/ndf	$40/33$	$74/39$
$p\text{-value}$	0.18	0.001

$|V_{cb}|$ shifts when include LQCD full constraints



Consistent with recent Belle (2023) measurement [arXiv:2301.07529]
 ⇒ Both found large disagreements wrt LQCD results on R_2

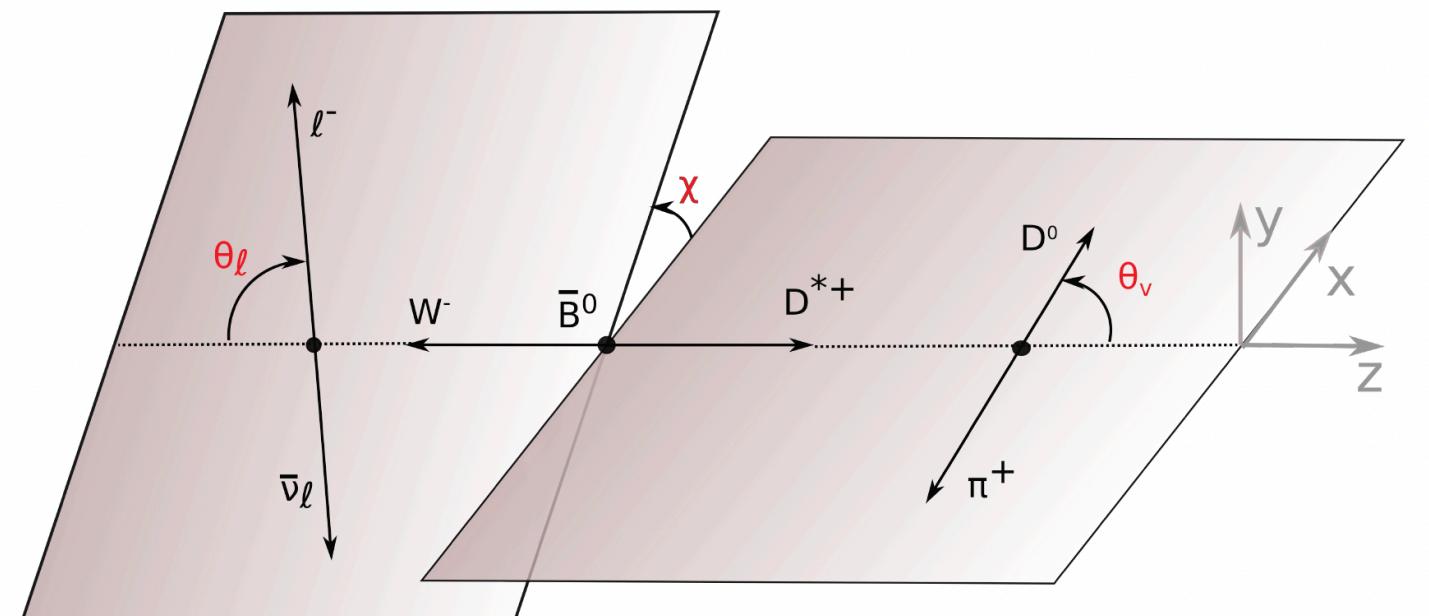
$$\begin{aligned} \chi^2 &= \sum_{i,j}^{34} \left(\frac{\Delta\Gamma_i^{\text{obs}}}{\Gamma_i^{\text{obs}}} - \frac{\Delta\Gamma_i^{\text{pre}}}{\Gamma_i^{\text{pre}}} \right) C_{ij}^{-1} \left(\frac{\Delta\Gamma_j^{\text{obs}}}{\Gamma_j^{\text{obs}}} - \frac{\Delta\Gamma_j^{\text{pre}}}{\Gamma_j^{\text{pre}}} \right) \\ &\quad + \frac{(\Gamma^{\text{obs}} - \Gamma^{\text{pre}})^2}{\sigma_\Gamma^2} \\ &\quad + \sum_{ij} (F_i^{\text{LQCD}} - F_i^{\text{exp}}) C_{ij}^{-1} (F_j^{\text{LQCD}} - F_j^{\text{exp}}) \end{aligned}$$

Branching Fraction of $B^0 \rightarrow D^* \ell \nu$ and $|V_{cb}|$

Preliminary

- Lepton-flavor-universality tested with separate results on e- & mu-mode
- All in **good agreement with SM expectations**

Test on branching fraction ratio: $R_{e/\mu} = 1.001 \pm 0.009 \pm 0.021$



Test on forward-backward asymmetry:

$$\mathcal{A}_{FB} = \frac{\int_0^1 d \cos \theta_\ell d\Gamma / d \cos \theta_\ell - \int_{-1}^0 d \cos \theta_\ell d\Gamma / d \cos \theta_\ell}{\int_0^1 d \cos \theta_\ell d\Gamma / d \cos \theta_\ell + \int_{-1}^0 d \cos \theta_\ell d\Gamma / d \cos \theta_\ell}$$

$$\Delta \mathcal{A}_{FB} = \mathcal{A}_{FB}^\mu - \mathcal{A}_{FB}^e$$

$$\mathcal{A}_{FB}^e = 0.219 \pm 0.011 \pm 0.020,$$

$$\mathcal{A}_{FB}^\mu = 0.215 \pm 0.011 \pm 0.022,$$

$$\Delta \mathcal{A}_{FB} = (-4 \pm 16 \pm 18) \times 10^{-3}$$

Test on D^* longitudinal polarization fraction:

$$\frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta_V} = \frac{3}{2} \left(F_L \cos^2 \theta_V + \frac{1 - F_L}{2} \sin^2 \theta_V \right)$$

$$\Delta F_L = F_L^\mu - F_L^e$$

$$F_L^e = 0.521 \pm 0.005 \pm 0.007$$

$$F_L^\mu = 0.534 \pm 0.005 \pm 0.006$$

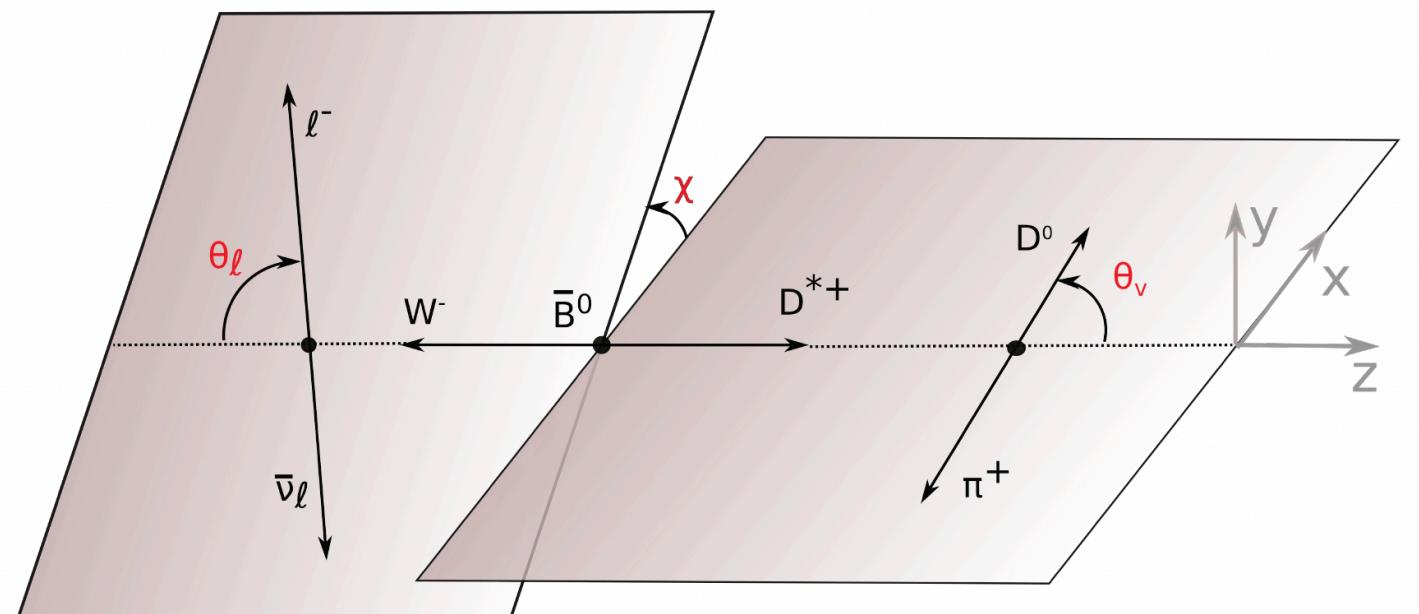
$$\Delta F_L = 0.013 \pm 0.007 \pm 0.007$$

Branching Fraction of $B^0 \rightarrow D^* \ell \nu$ and $|V_{cb}|$

Preliminary

- Lepton-flavor-universality tested with separate results on e- & mu-mode
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$$\Delta F_L = F_L^\mu - F_L^e$$

coming soon...

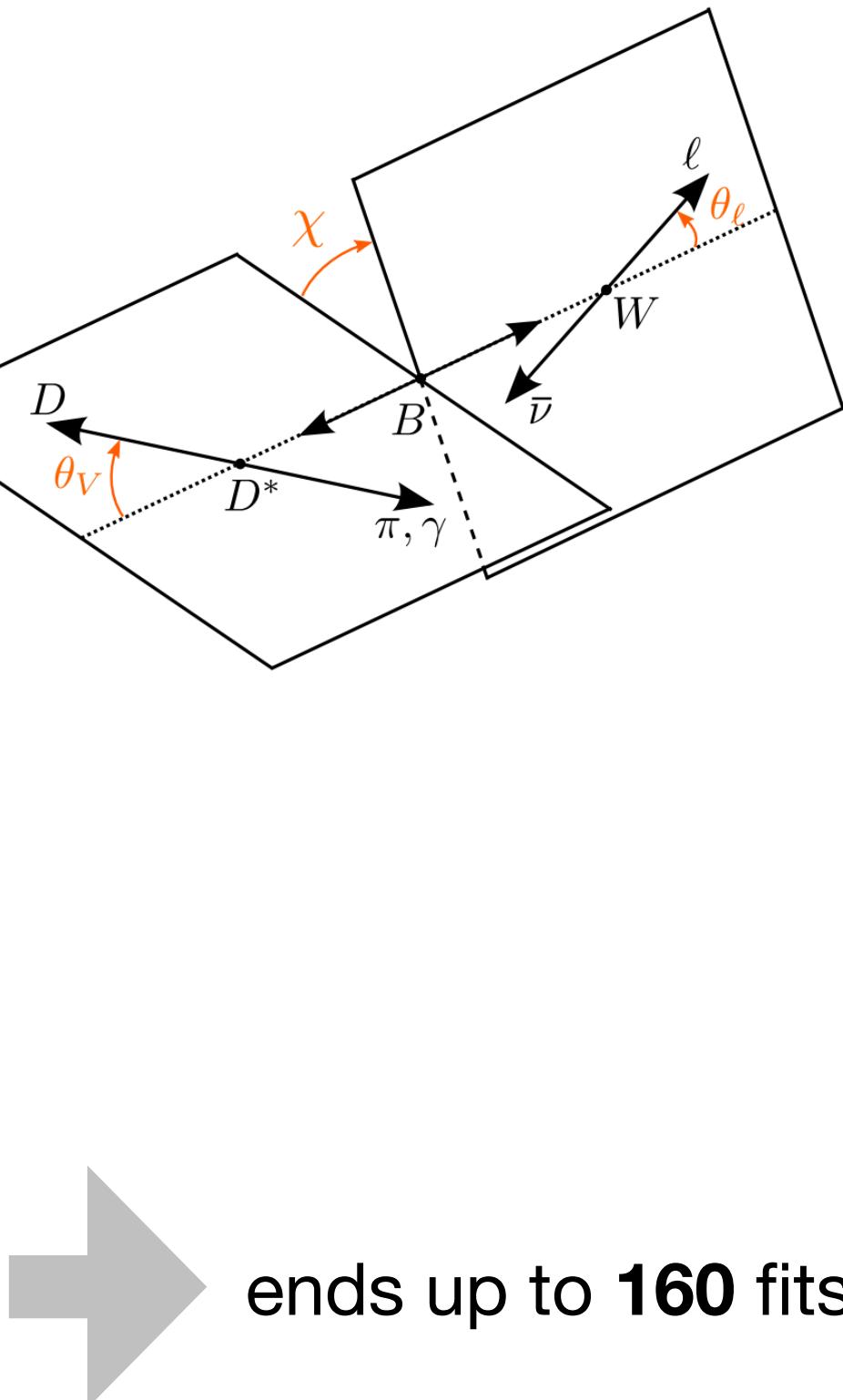
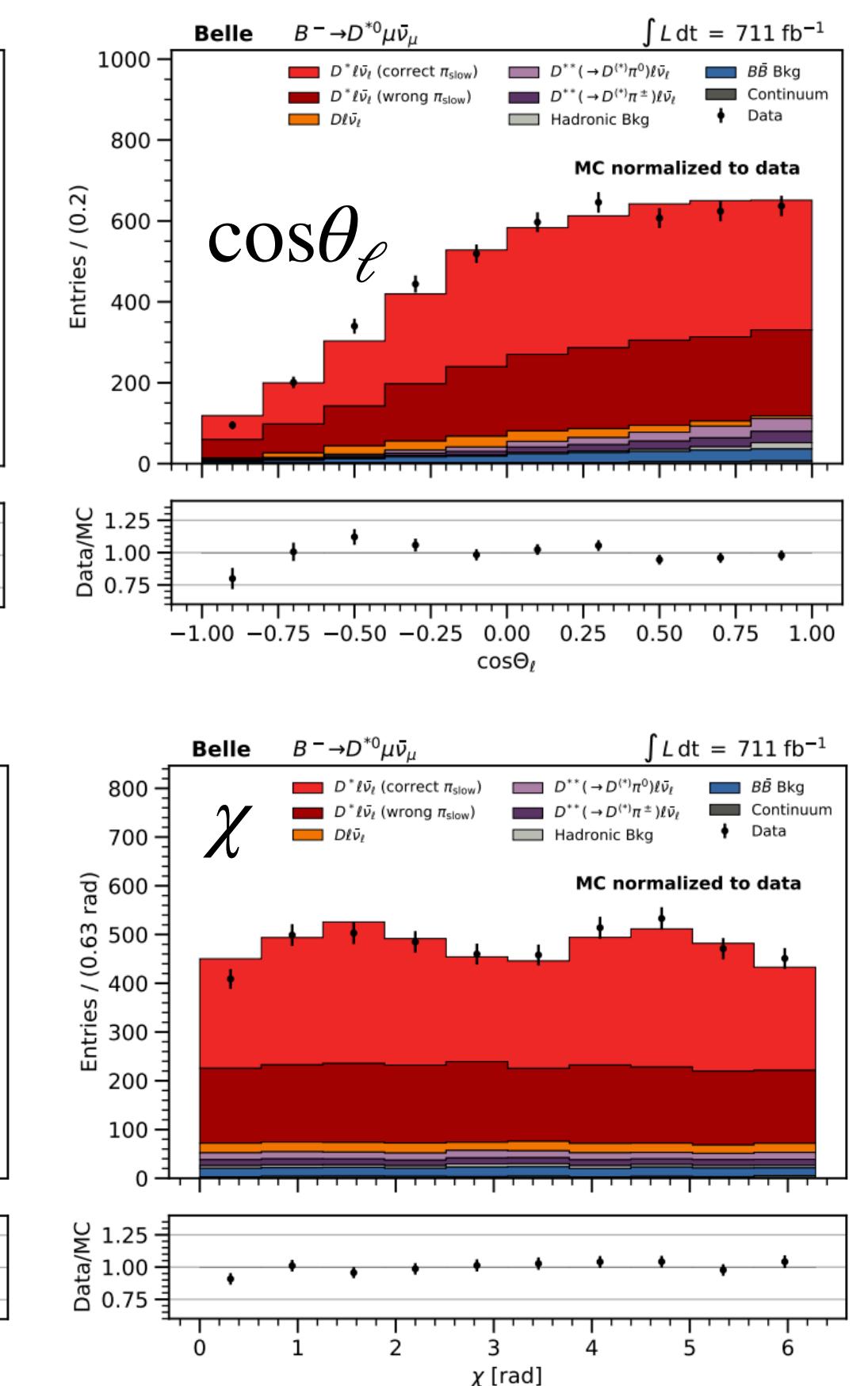
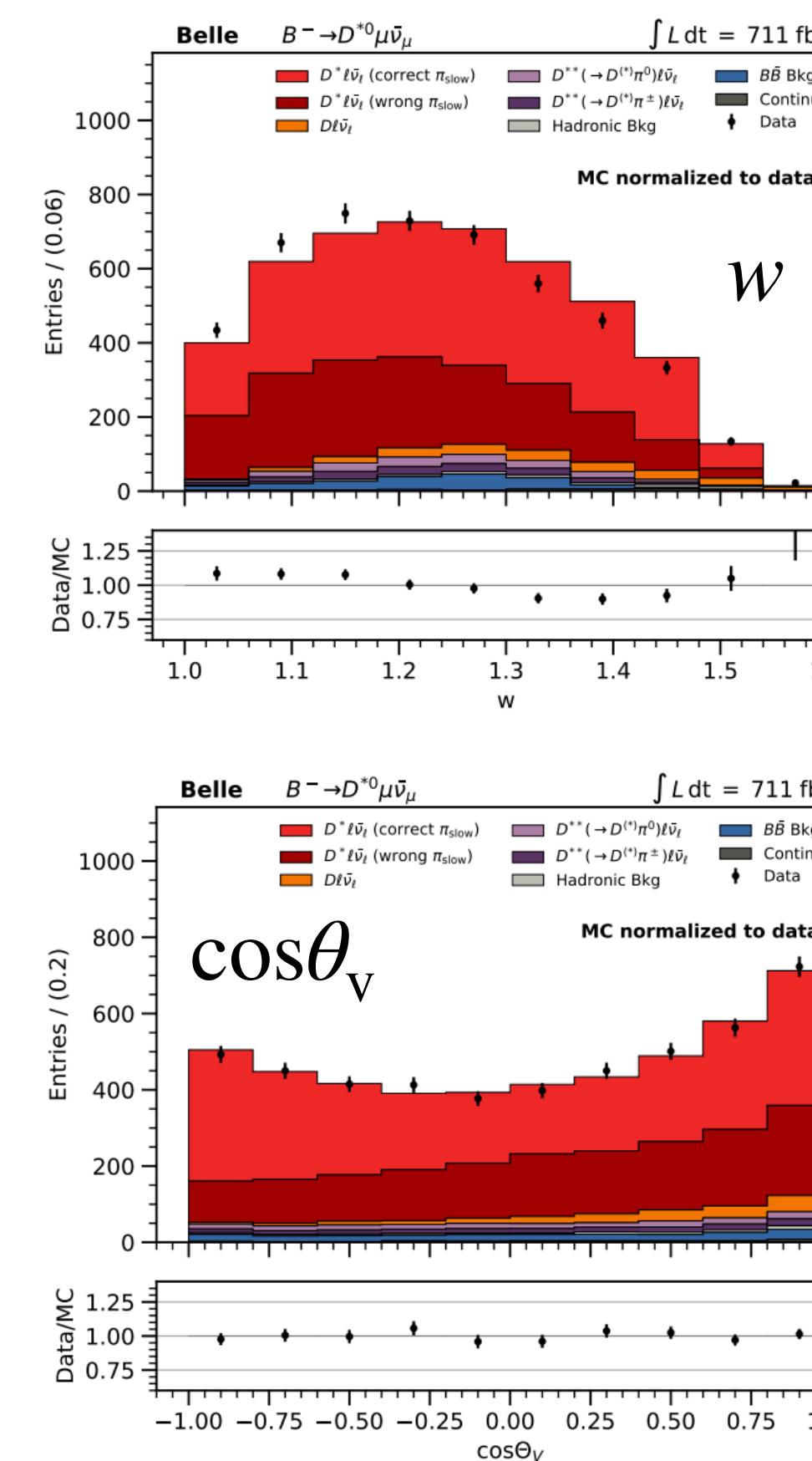
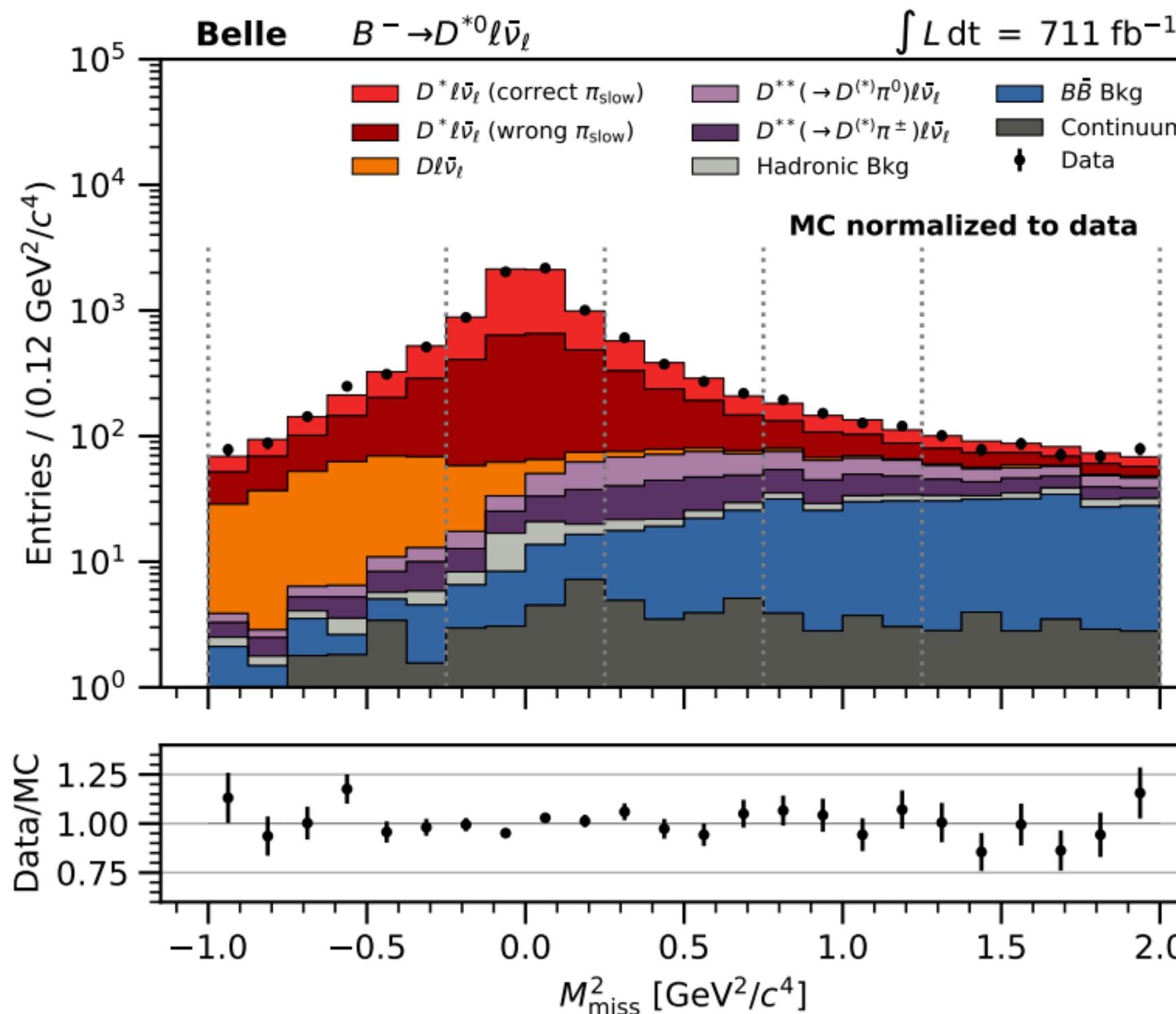
arXiv version of Untagged / Had. tagged
 $B^0 \rightarrow D^* \ell \nu$

$|V_{cb}|$ & Differential Shapes of $B \rightarrow D^* \ell \bar{\nu}_\ell$



arXiv: 2301.07529

- Full Belle data set of 711 fb^{-1} for $B^{\pm,0}, \ell = e, \mu$
- **Hadronic tagging** using Belle II tool (Full Event Interpretation)
- Background subtracted via fitting M_{miss}^2 for bins of $w, \cos\theta_\ell, \cos\theta_v, \chi$ in **each decay mode independently**



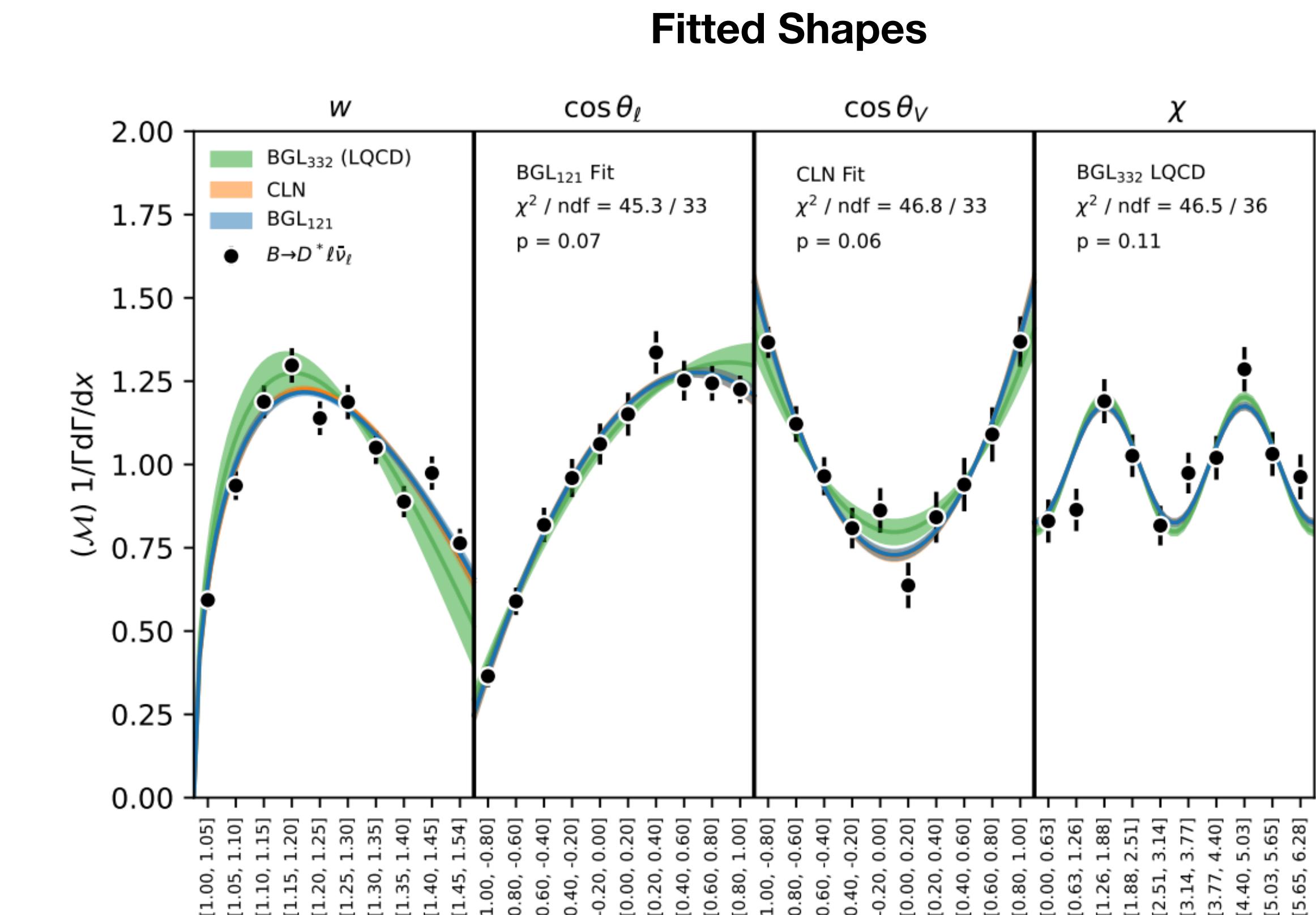
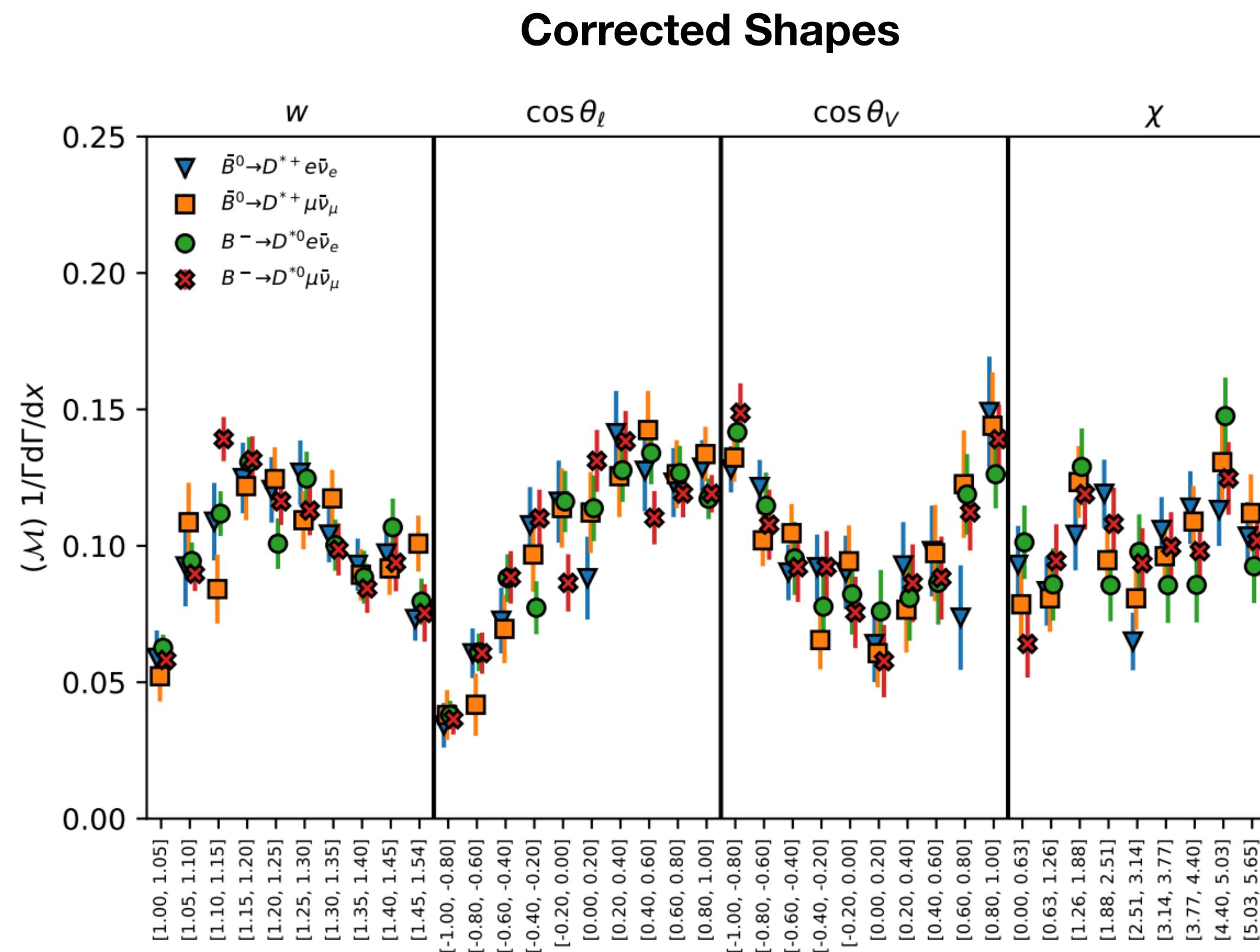
fit stability & consistency
were fully checked and
compatible with expected
uniform behavior

$|V_{cb}|$ & Differential Shapes of $B \rightarrow D^* \ell \nu$

- Signal **shapes** corrected for resolution, reco. efficiency and acceptance effects
- Combined **all kinematic shapes** to extract $|V_{cb}|$ in **BGL/CLN** with external constraints on **branching fractions** (HFLAV) and **LQCD results** (FNAL/MILC)

arXiv: 2301.07529

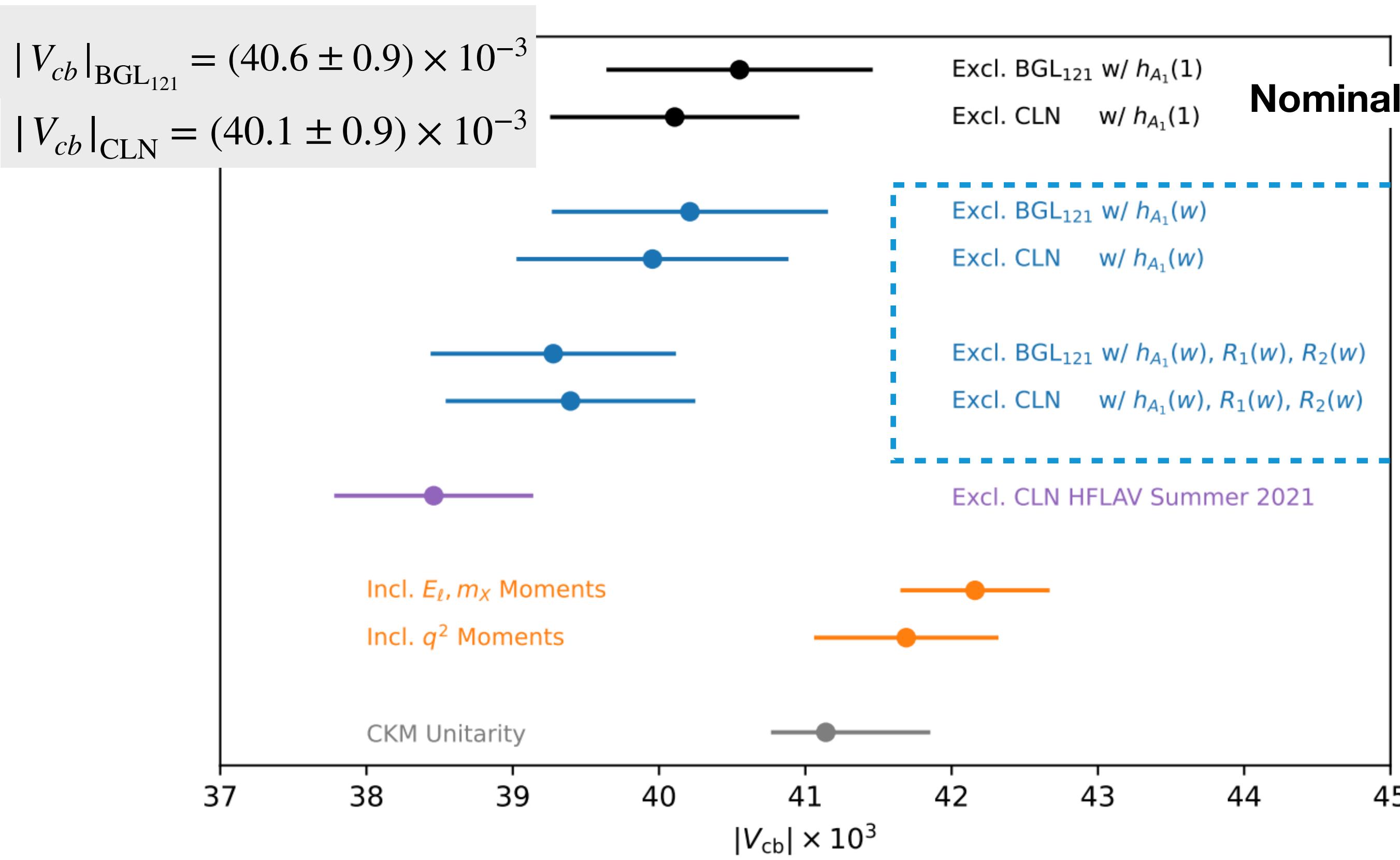
$$\chi^2 = \left(\frac{\Delta \vec{\Gamma}^m}{\Gamma^m} - \frac{\Delta \vec{\Gamma}^p(\vec{x})}{\Gamma^p(\vec{x})} \right) C_{\text{exp}}^{-1} \left(\frac{\Delta \vec{\Gamma}^m}{\Gamma^m} - \frac{\Delta \vec{\Gamma}^p(\vec{x})}{\Gamma^p(\vec{x})} \right)^T + (\Gamma^{\text{ext}} - \Gamma^p(\vec{x}))^2 / \sigma(\Gamma^{\text{ext}})^2 + (h_X - h_X^{\text{LQCD}}) C_{\text{LQCD}}^{-1} (h_X - h_X^{\text{LQCD}})$$



$|V_{cb}|$ & Differential Shapes of $B \rightarrow D^* \ell \nu$

- In $|V_{cb}|$ extraction, tested different BGL truncations, **LQCD constraining scenario** (at or beyond zero-recoil)

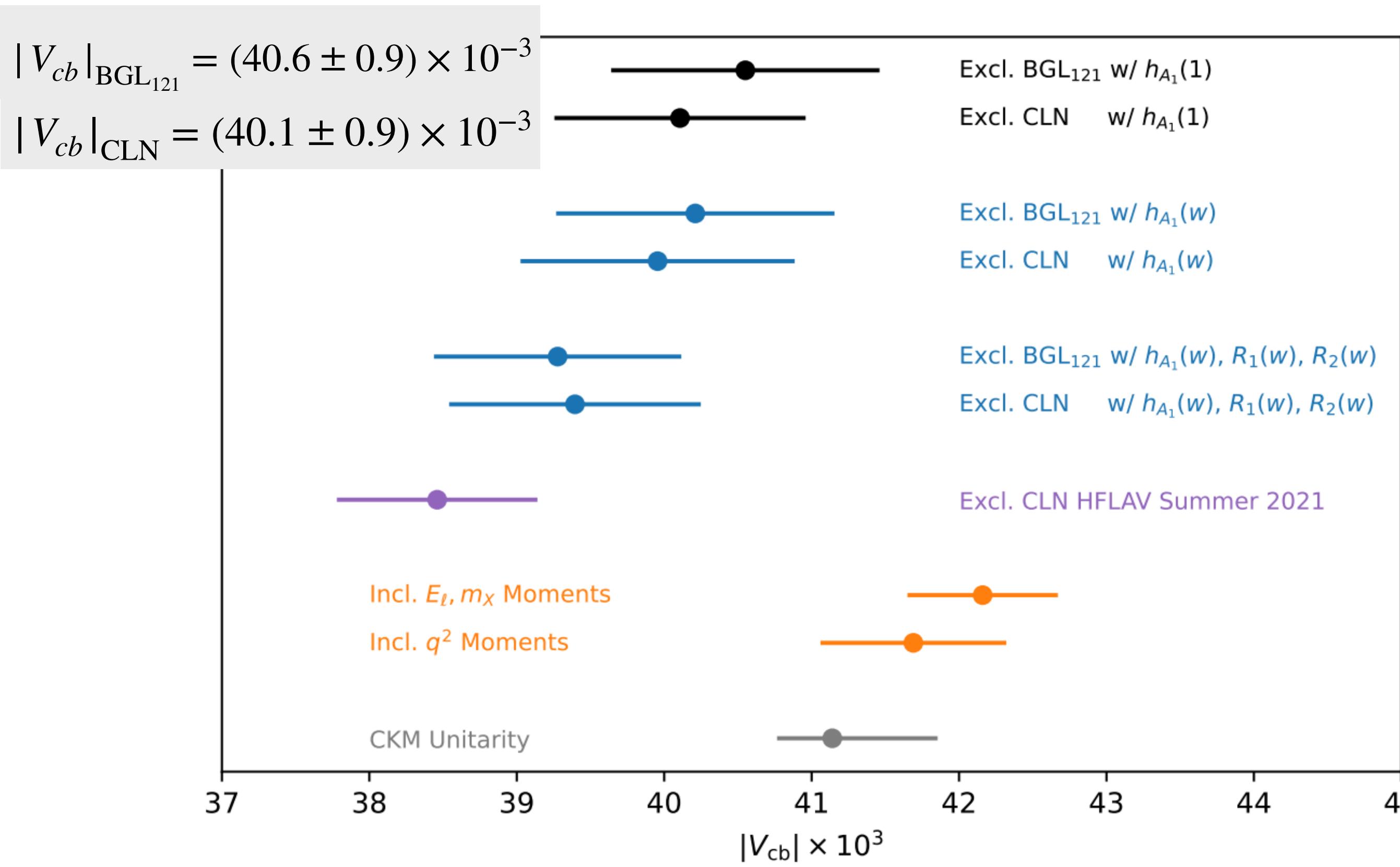
[arXiv: 2301.07529](https://arxiv.org/abs/2301.07529)



$|V_{cb}|$ & Differential Shapes of $B \rightarrow D^* \ell \nu$

[arXiv: 2301.07529](https://arxiv.org/abs/2301.07529)

- In $|V_{cb}|$ extraction, tested different BGL truncations, LQCD constraining scenario (at or beyond zero-recoil)
- Forward-backward asymmetry A_{FB}** and **D^{*} longitudinal polarization fraction $F_L^{D^*}$** and their differences between e, μ also derived. **No significant LFUV found.**

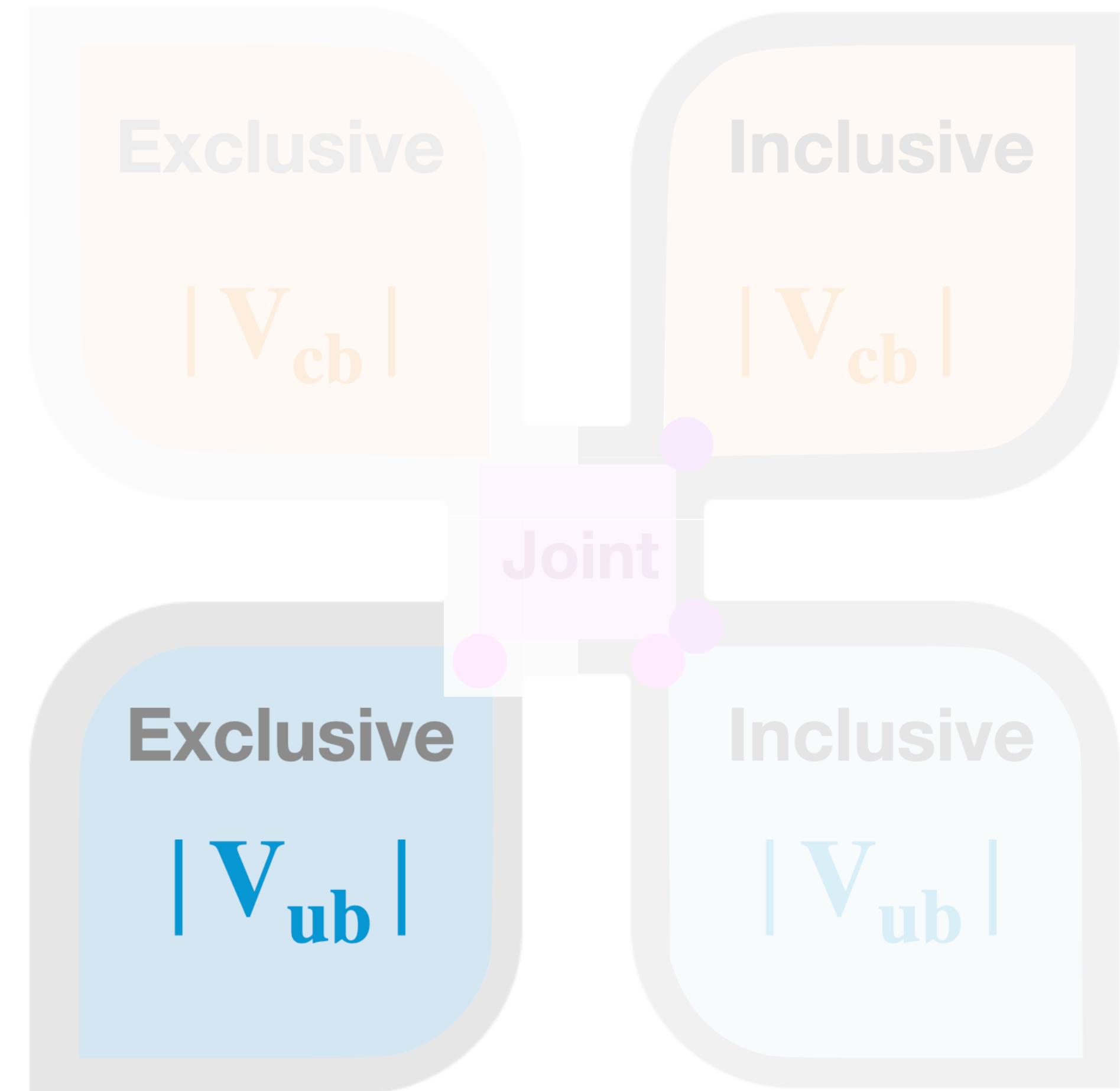


$$A_{FB} = \frac{\int_0^1 d \cos_\ell d\Gamma / d \cos_\ell - \int_{-1}^0 d \cos_\ell d\Gamma / d \cos_\ell}{\int_0^1 d \cos_\ell d\Gamma / d \cos_\ell + \int_{-1}^0 d \cos_\ell d\Gamma / d \cos_\ell}$$

	ΔA_{FB}
$\bar{B}^0 \rightarrow D^{*+} \ell \bar{\nu}_\ell$	$0.062 \pm 0.044 \pm 0.011$
$B^- \rightarrow D^{*0} \ell \bar{\nu}_\ell$	$-0.003 \pm 0.033 \pm 0.009$
$B \rightarrow D^* \ell \bar{\nu}_\ell$	$0.022 \pm 0.026 \pm 0.007$

$$\frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta_V} = \frac{3}{2} \left(F_L \cos^2 \theta_V + \frac{1 - F_L}{2} \sin^2 \theta_V \right)$$

	$\Delta F_L^{D^*}$
$\bar{B}^0 \rightarrow D^{*+} \ell \bar{\nu}_\ell$	$0.032 \pm 0.033 \pm 0.010$
$B^- \rightarrow D^{*0} \ell \bar{\nu}_\ell$	$0.025 \pm 0.035 \pm 0.010$
$B \rightarrow D^* \ell \bar{\nu}_\ell$	$0.034 \pm 0.024 \pm 0.007$

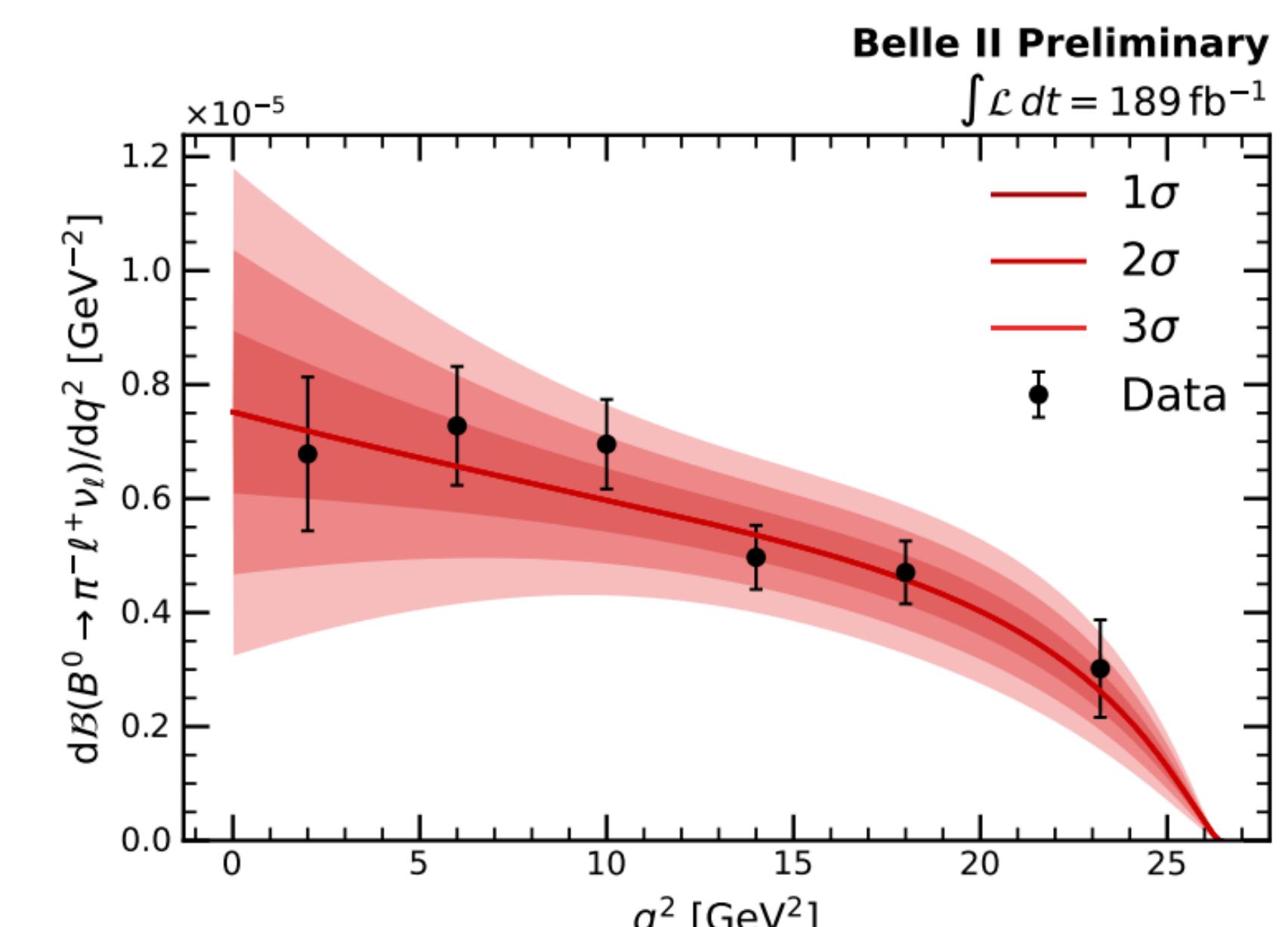
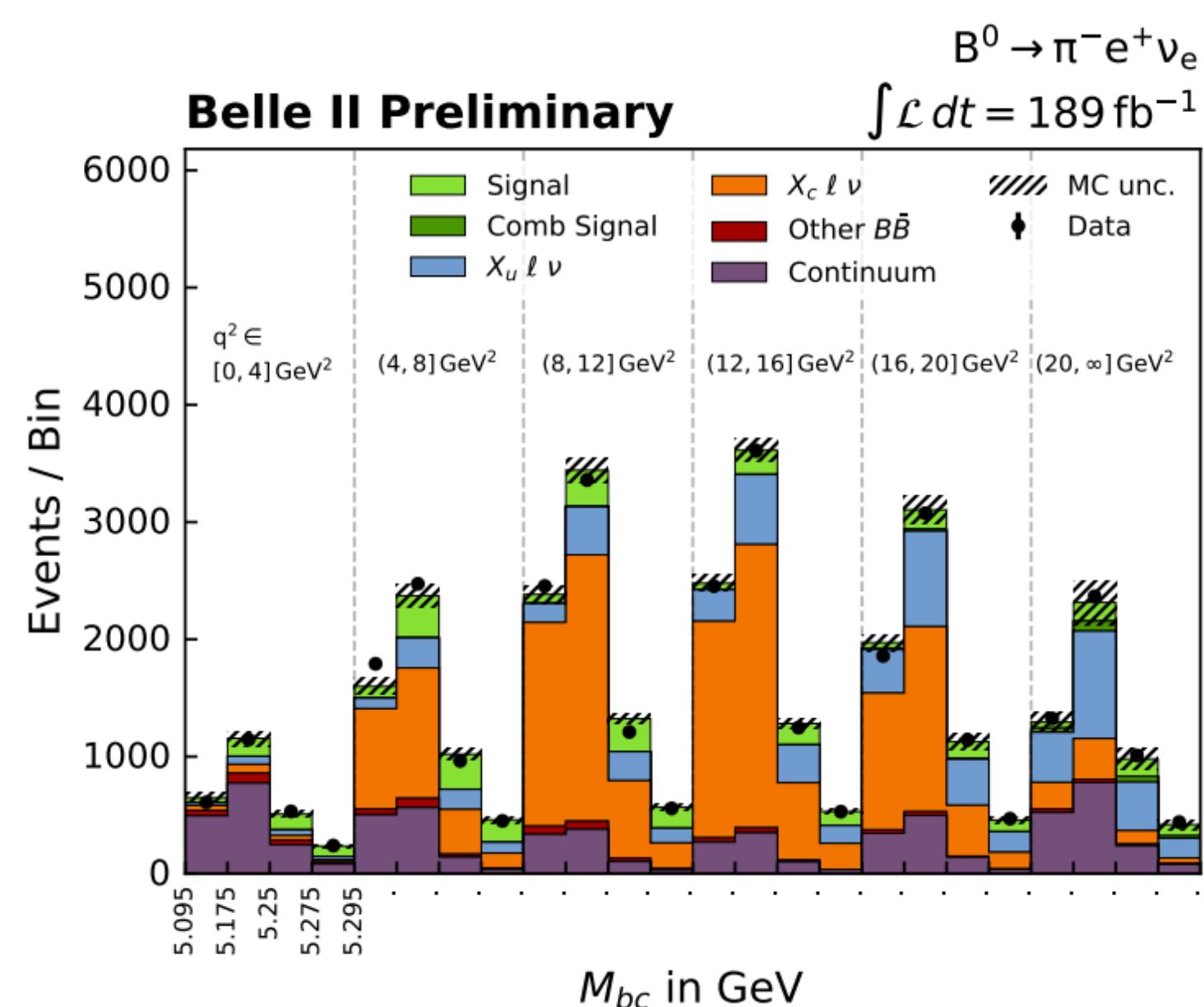
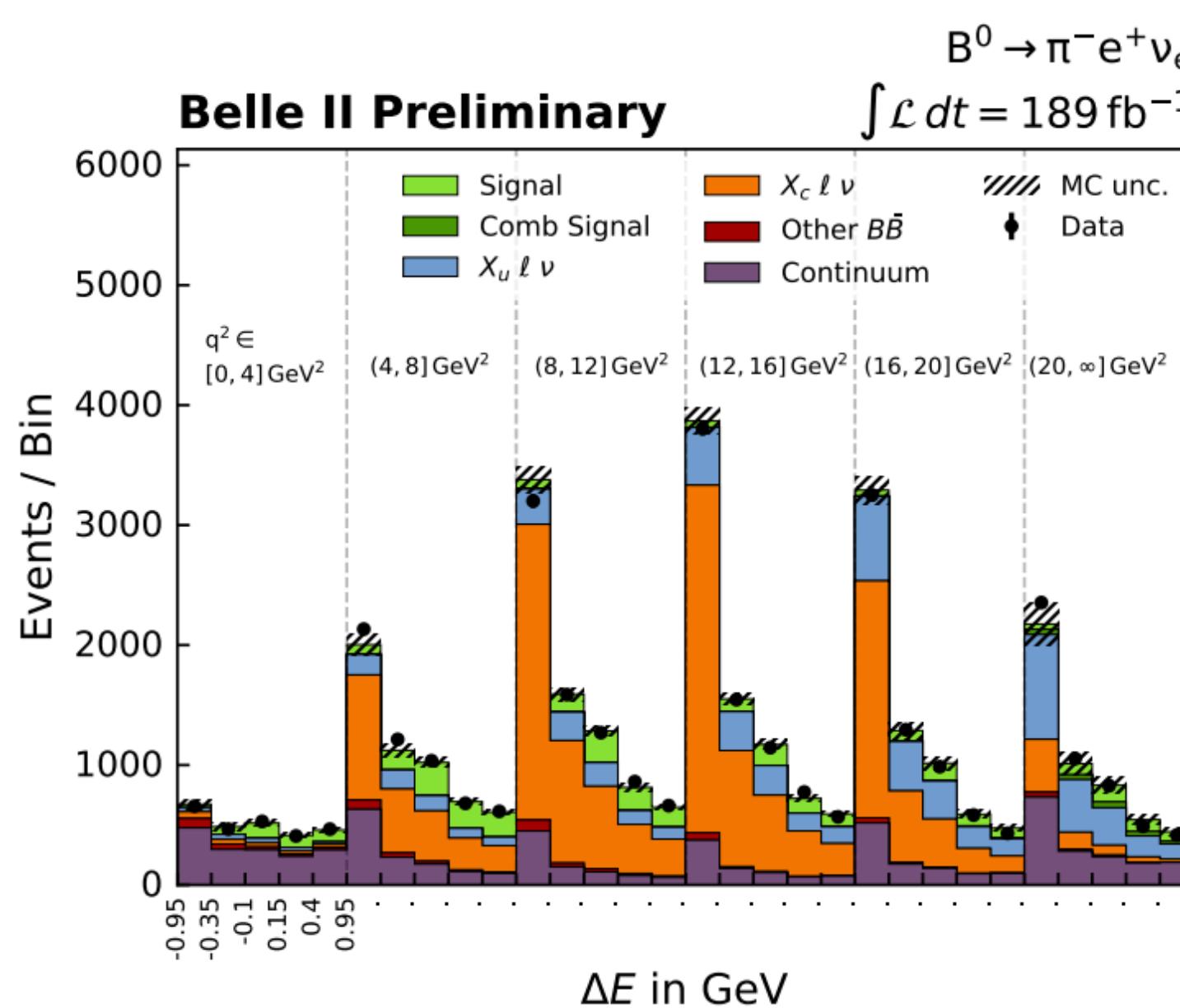


$|V_{ub}|$ in $B^0 \rightarrow \pi^- \ell^+ \nu$ with Belle II data



arXiv: 2210.04224

- Data set of 189.3 fb^{-1} with untagged analysis strategy
- Extract signal in beam-constrained mass M_{bc} and energy difference ΔE for each bin of q^2
- $|V_{ub}|$ fitted with BCL expansion including LQCD constraints (FNAL/MILC)



$$\mathcal{B} = (1.426 \pm 0.12_{\text{stat}} \pm 0.056_{\text{syst}} \pm 0.125_{\text{theo}}) \times 10^{-4}$$

$$\Delta E = E_B^* - E_{\text{beam}}^* = E_B^* - \frac{\sqrt{s}}{2}$$

$$M_{bc} = \sqrt{E_{\text{beam}}^{*2} - |\vec{p}_B^*|^2} = \sqrt{\left(\frac{\sqrt{s}}{2}\right)^2 - |\vec{p}_B^*|^2}$$

$$|V_{ub}| = (3.55 \pm 0.12_{\text{stat}} \pm 0.13_{\text{syst}} \pm 0.17_{\text{theo}}) \times 10^{-3}$$

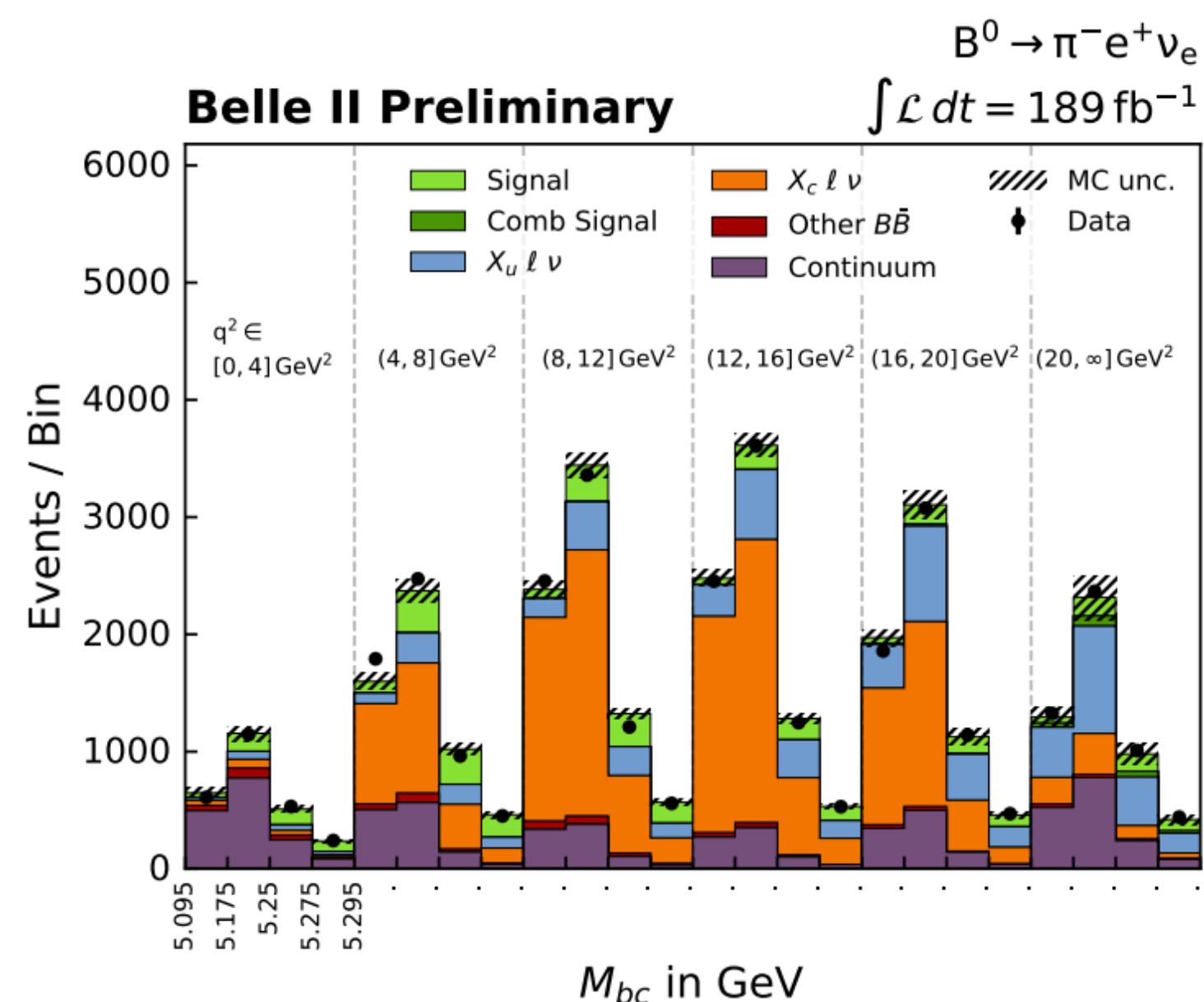
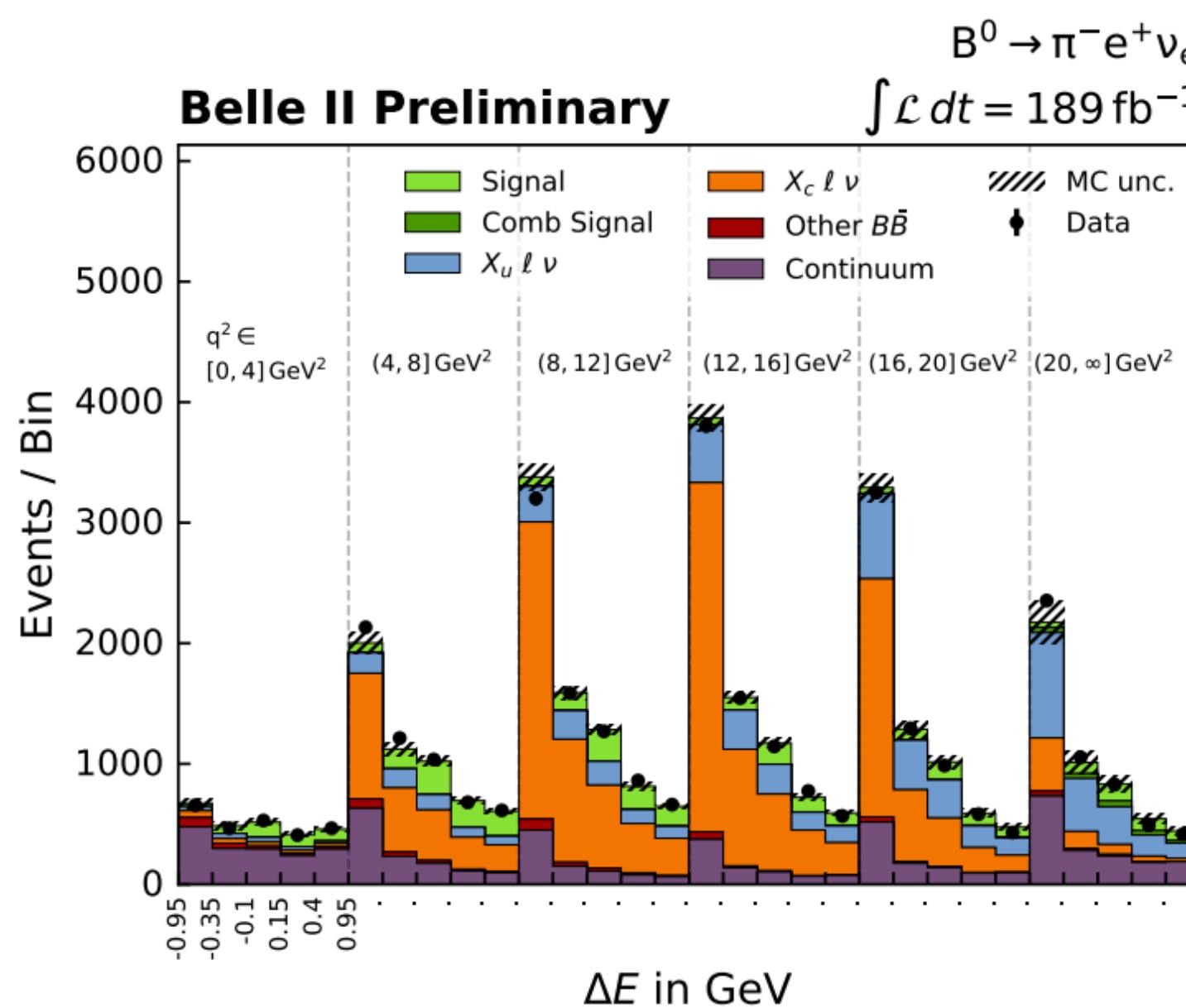
$|V_{ub}|$ in $B^0 \rightarrow \pi^- \ell^+ \nu$ with Belle II data



arXiv: 2210.04224

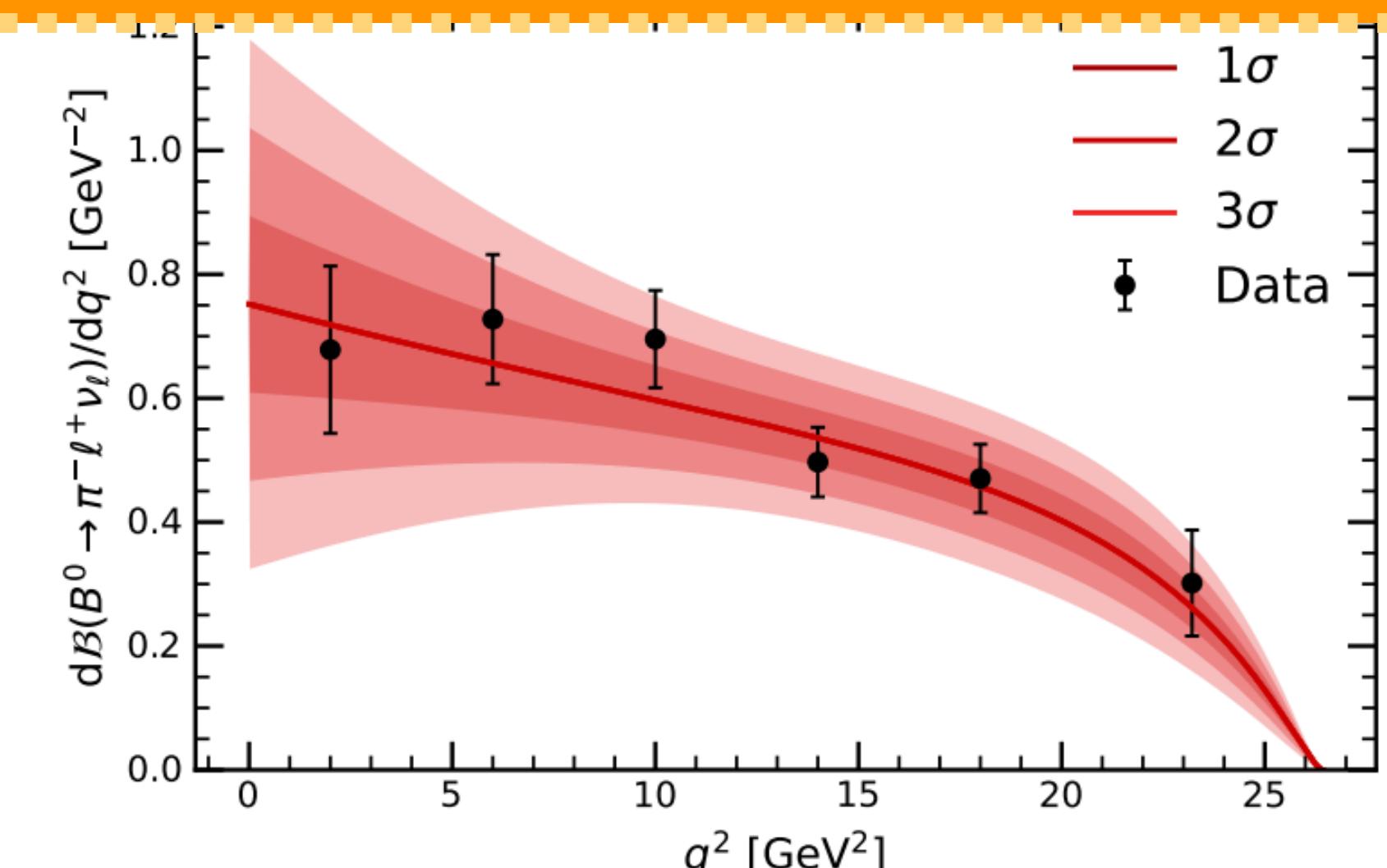
- Data set of 189.3 fb⁻¹ with untagged analysis strategy
- Extract signal in beam-constrained mass M_{bc} and energy difference ΔE for each q^2 bin
- $|V_{ub}|$ fitted with BCL expansion including LQCD constraints (FNAL/MILC)

coming soon...
Untagged / Had. tagged $B \rightarrow (\pi, \rho) \ell \nu$



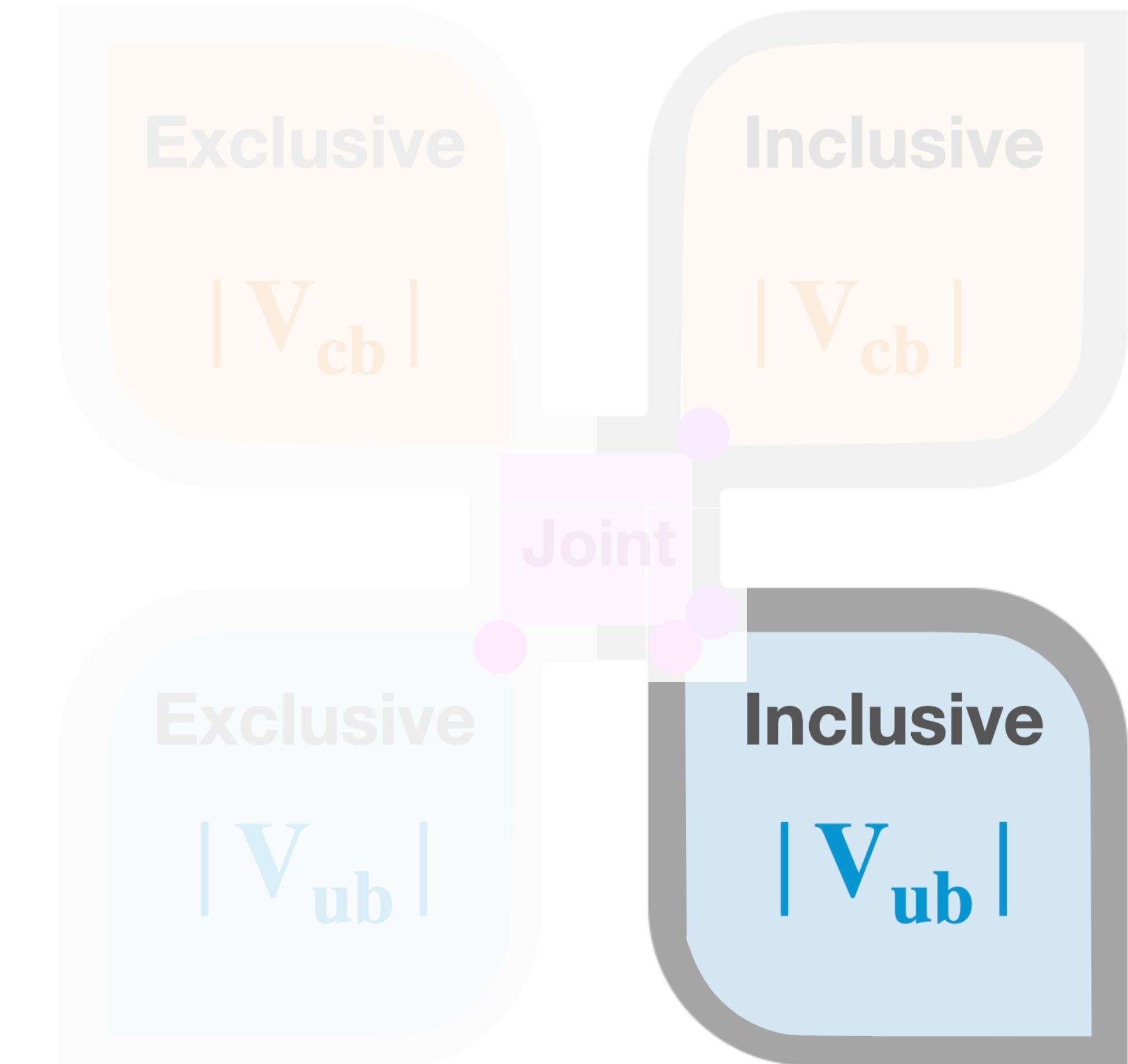
$$\Delta E = E_B^* - E_{\text{beam}}^* = E_B^* - \frac{\sqrt{s}}{2}$$

$$M_{bc} = \sqrt{E_{\text{beam}}^{*2} - |\vec{p}_B^*|^2} = \sqrt{\left(\frac{\sqrt{s}}{2}\right)^2 - |\vec{p}_B^*|^2}$$



$$\mathcal{B} = (1.426 \pm 0.12_{\text{stat}} \pm 0.056_{\text{syst}} \pm 0.125_{\text{theo}}) \times 10^{-4}$$

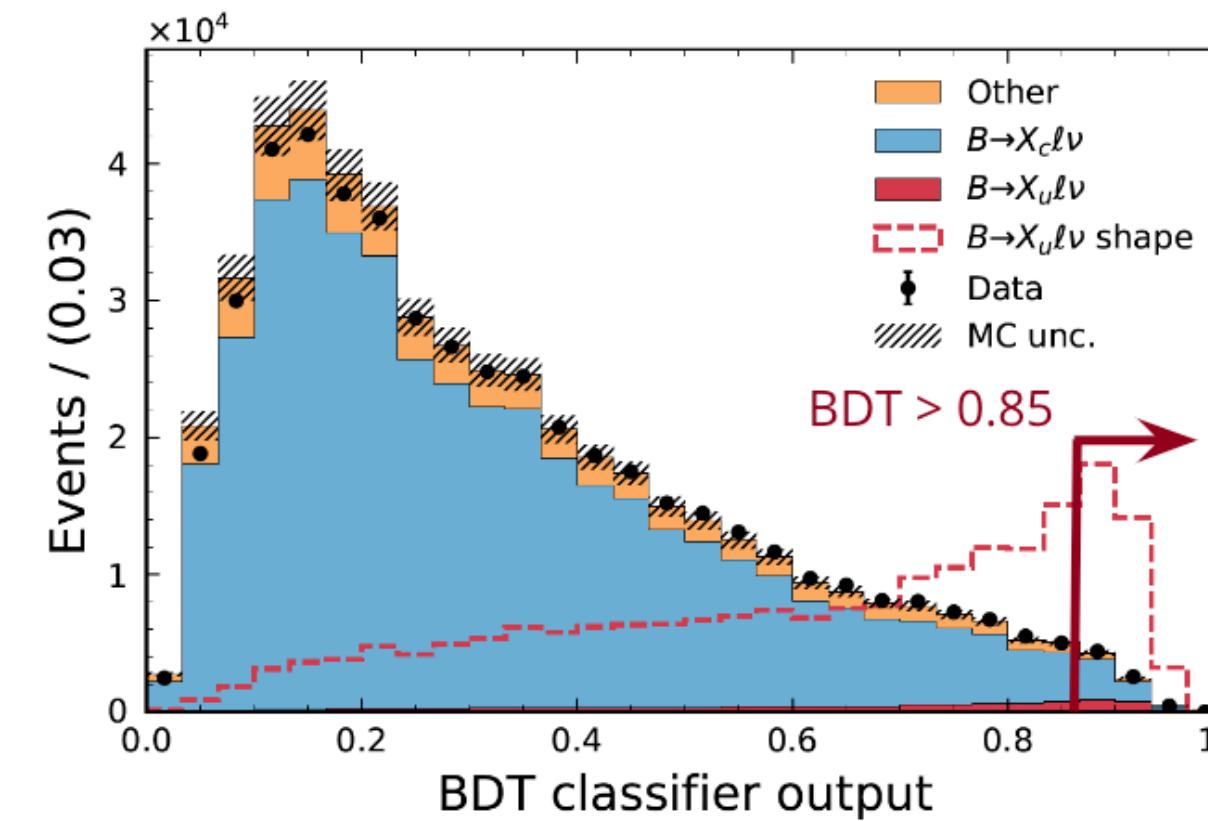
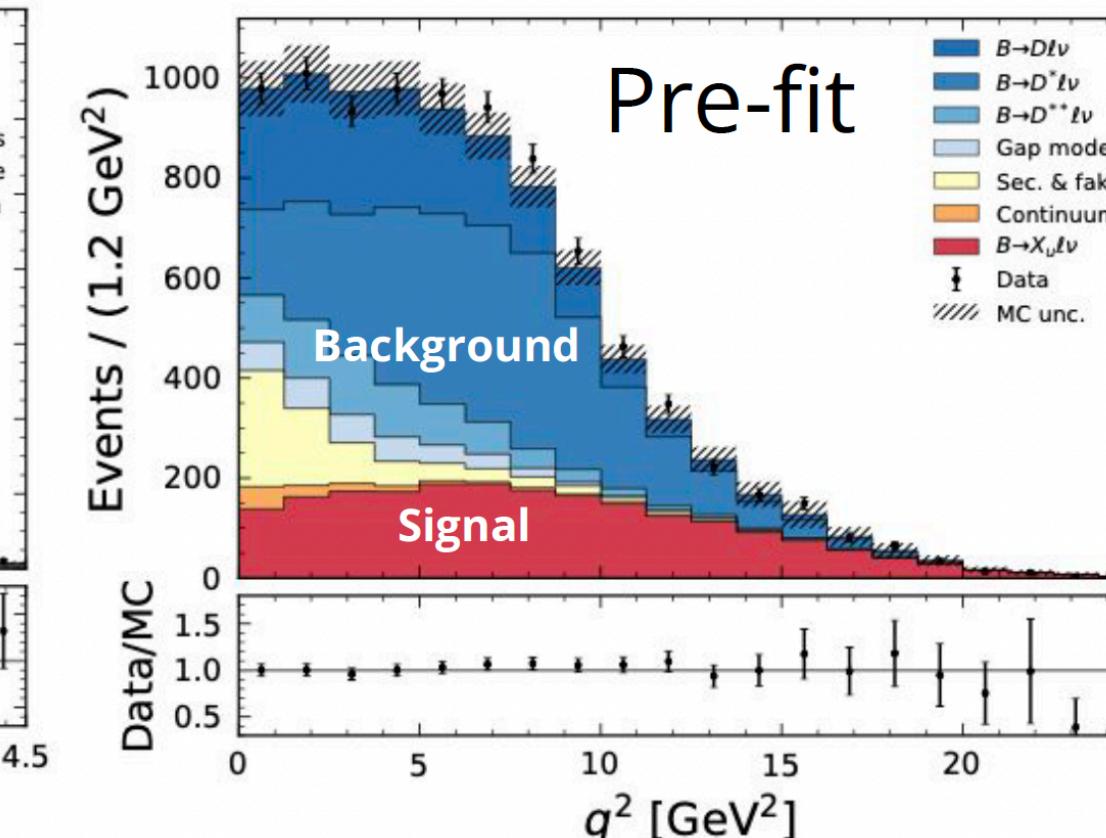
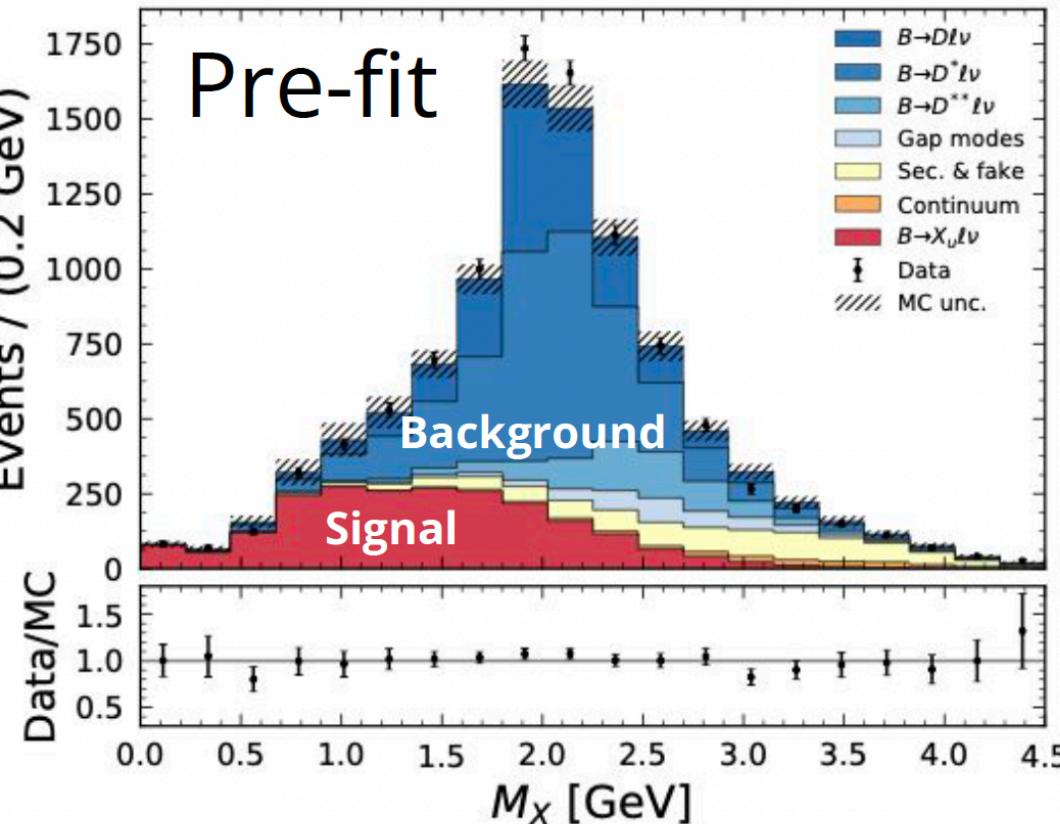
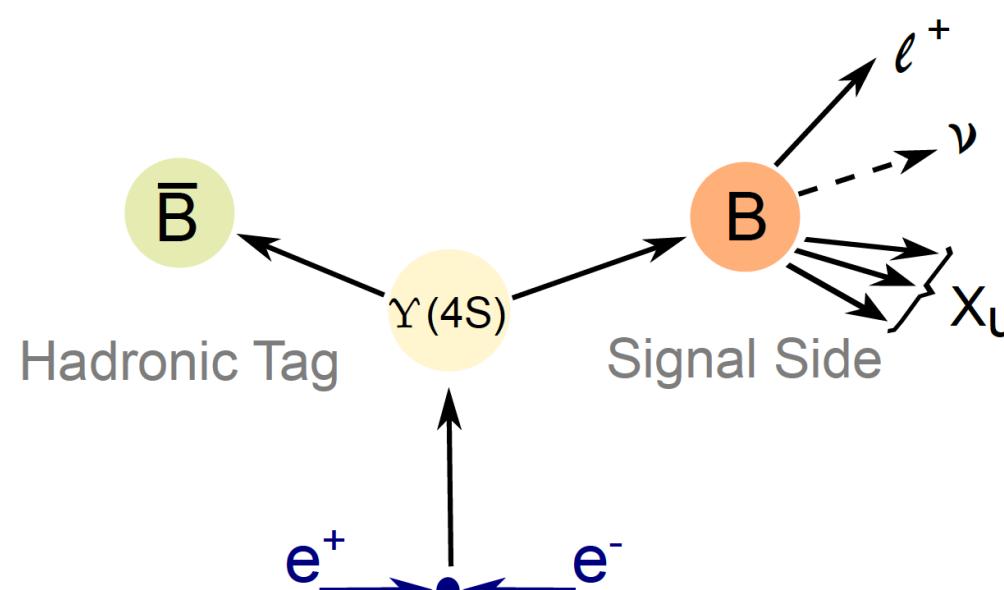
$$|V_{ub}| = (3.55 \pm 0.12_{\text{stat}} \pm 0.13_{\text{syst}} \pm 0.17_{\text{theo}}) \times 10^{-3}$$



Partial Branching Fractions of Inclusive $B \rightarrow X_u \ell \nu$

PRD 104 , 012008 (2021)

- Full Belle dataset with Hadronic tagging
- Use machine learning (BDT) to suppress backgrounds with 11 training features, e.g. MM^2 , $\#K^\pm$, $\#K_s$, etc.



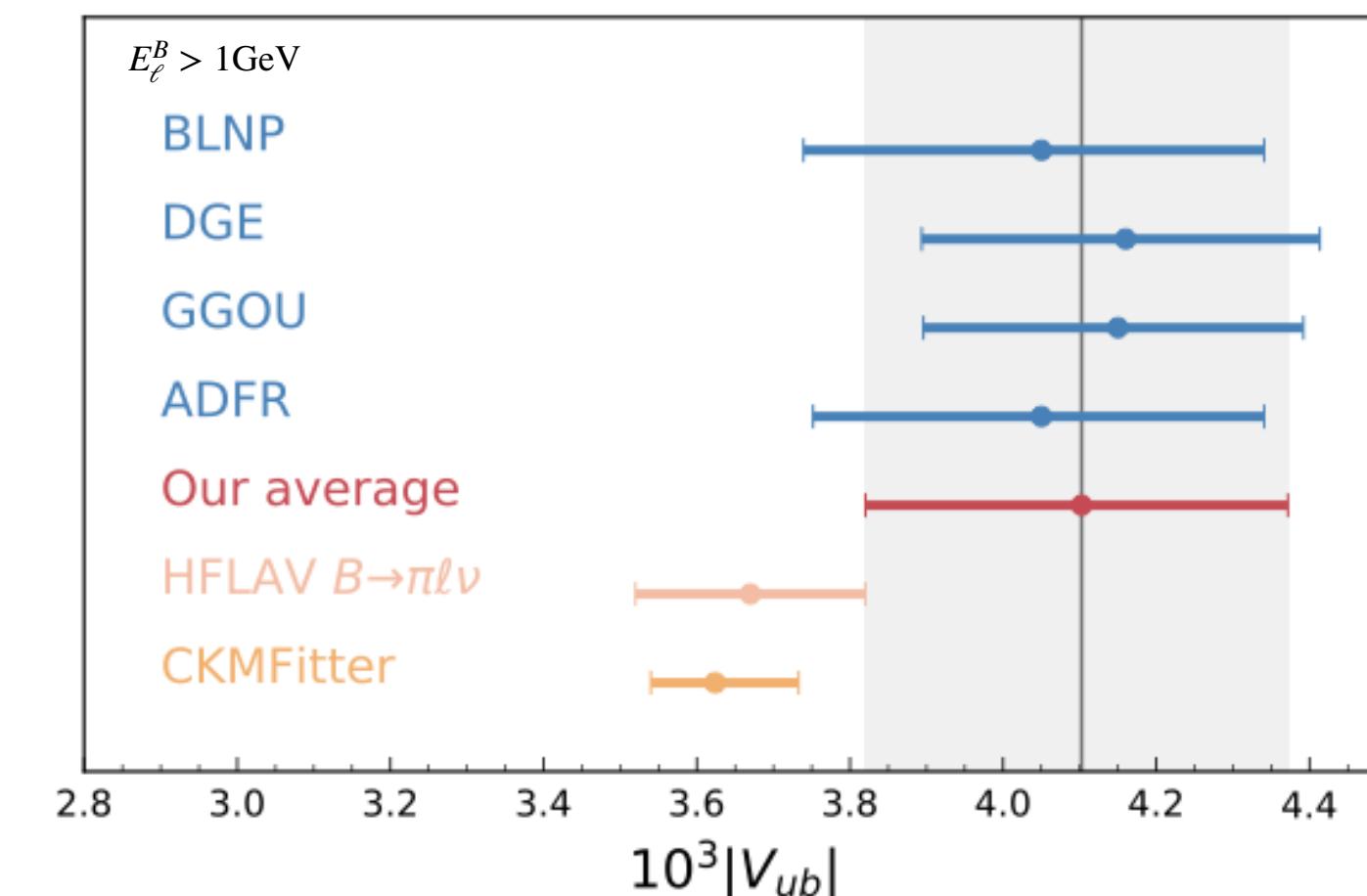
- Extract signal using binned likelihood in **3 phase space (PS) regions:**

- $E_\ell^B > 1 \text{ GeV}$ (covers 86% of available signal PS)
- $E_\ell^B > 1 \text{ GeV}, M_X < 1.7 \text{ GeV}$ (56%)
- $E_\ell^B > 1 \text{ GeV}, M_X < 1.7 \text{ GeV}, q^2 > 8 \text{ GeV}^2$ (31%)

→ Fit either E_ℓ^B , M_X , q^2 or **2D ($M_X : q^2$)**

- Partial BF and inclusive $|V_{ub}|$ derived in each PS

$$\Delta \mathcal{B}(E_\ell^B > 1 \text{ GeV}) = (1.59 \pm 0.07 \pm 0.16) \times 10^{-3}$$



$$|V_{ub}| = \sqrt{\frac{\Delta \mathcal{B}(B \rightarrow X_u \ell \nu)}{\tau_B \cdot \Delta \Gamma(B \rightarrow X_u \ell \nu)}}$$

Arithmetic avr. $|V_{ub}|$ based on various theo. decay rate:

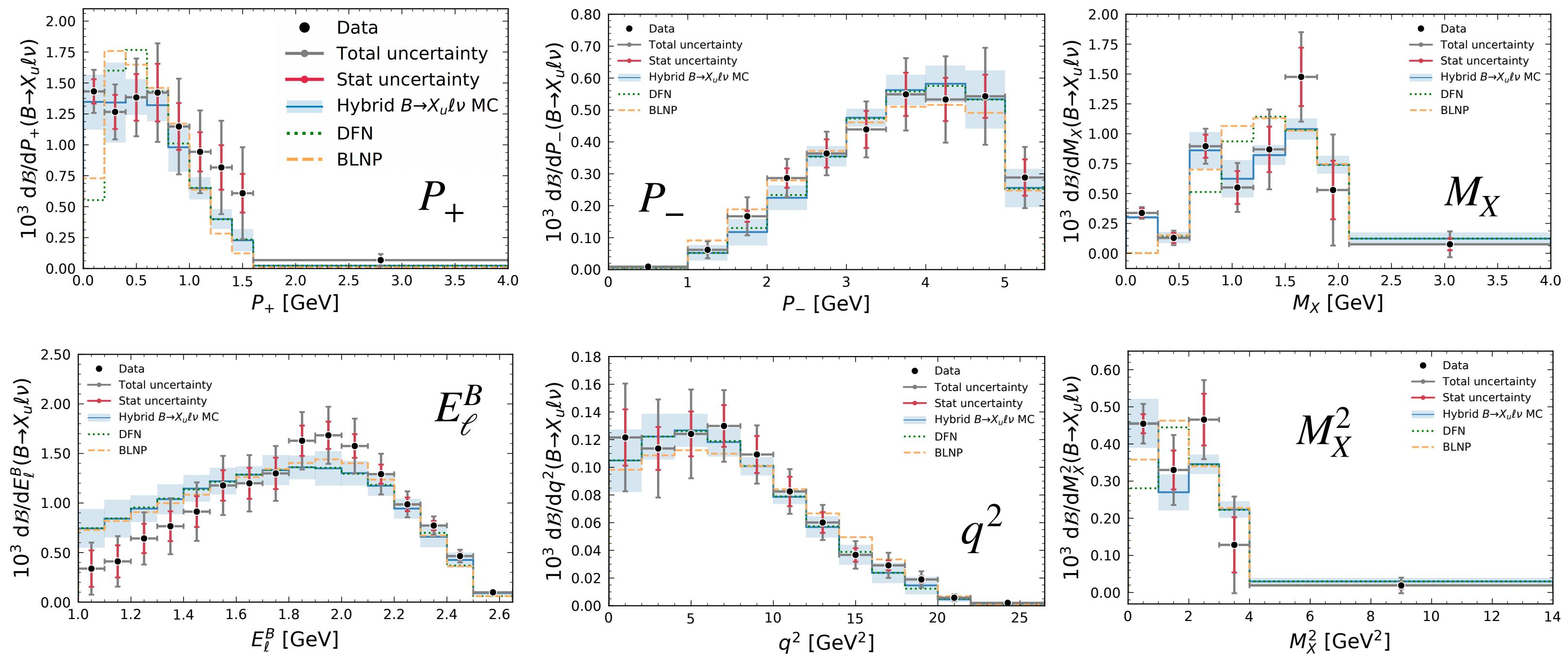
$$(4.10 \pm 0.09_{\text{stat}} \pm 0.22_{\text{sys}} \pm 0.15_{\text{theo}}) \times 10^{-3}$$

compatible with excl. and CKM expectation within 1.3σ and 1.6σ , respectively

First Measurement of Differential Spectra of $B \rightarrow X_u \ell \nu$

PRL 127, 261801 (2021)

- Inherit **same analysis strategy** in the partial BF measurement [PRD 104 , 012008 (2021)]
- Additional selections on $|E_{\text{miss}} - P_{\text{miss}}| < 0.1 \text{ GeV}$ & $M_X < 2.4 \text{ GeV}$ to **improve resolution** and reduce background shape uncertainty
- Background subtraction done via M_X fit, further corrected for efficiency & acceptance effects (phase space: $E_\ell^B > 1 \text{ GeV}$)
- Full experimental covariance, spectra moments, migration matrices etc. available on [HepData](#)



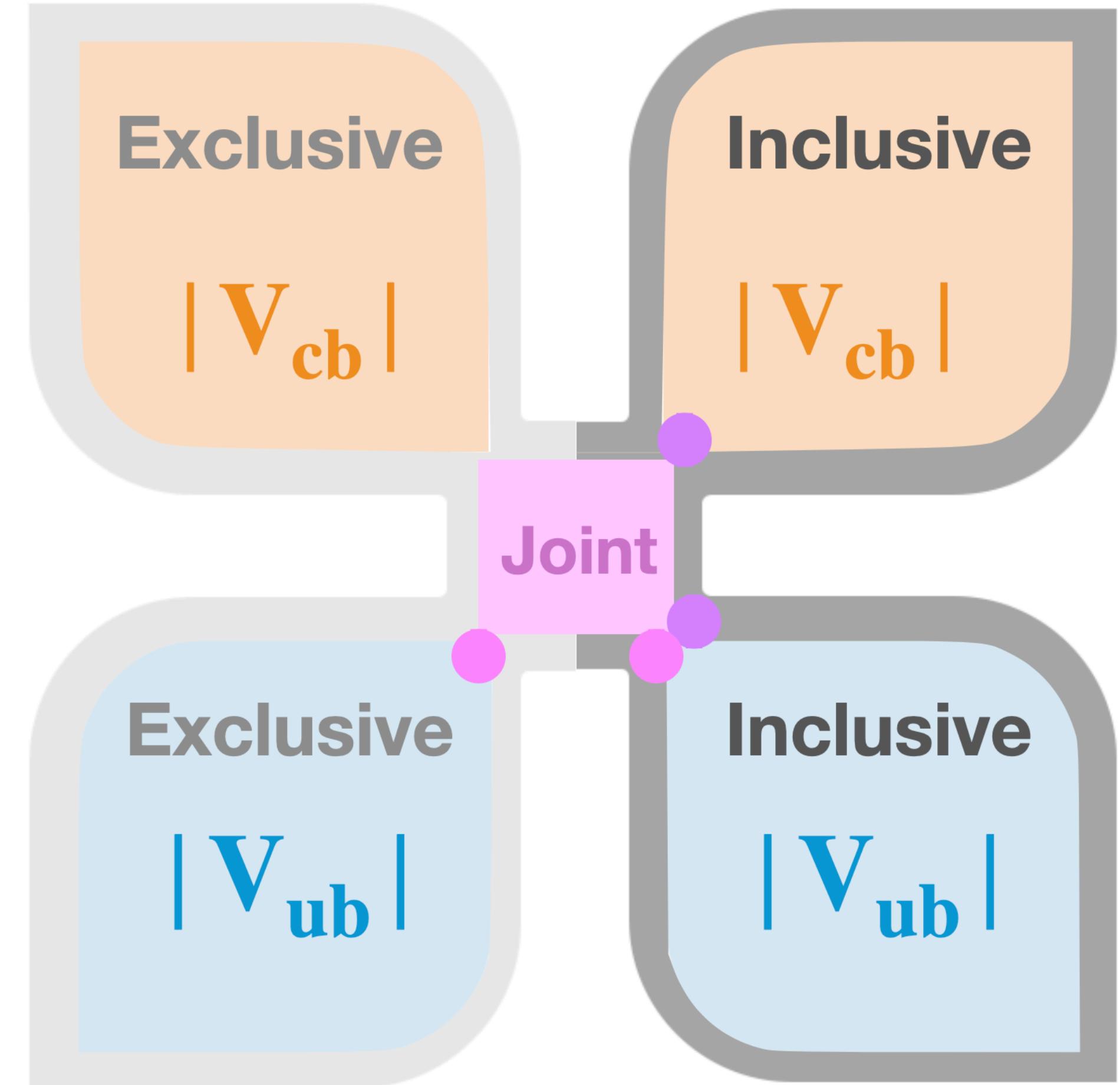
What can we gain for incl. $|V_{ub}|$?

Direct & more model-independent extraction

Normalization \Rightarrow Kin. shapes + Normalization

- Allows direct extraction of coefficients for non-perturbative **shape functions** in a global fit and $|V_{ub}|$
- Uncertainty can be further shrunk by including other inclusive B decays, e.g $B \rightarrow X_s \gamma$, $B \rightarrow X_c \ell \nu$ as the shape function in LO is universal
- Methods proposed by [NNVub](#), [SIMBA](#)

All MC shapes are normalised to 1.59×10^{-3} [Belle, PRD 104 , 012008 (2021)]



First Simultaneous Determination of Incl. & Excl. $|V_{ub}|$



arXiv: 2303.17309

Preliminary

- Inherit **same analysis strategy** in the partial BF measurement [PRD 104 , 012008 (2021)]
- Additional selections on thrust of X in c.m.s to **increase significance** of $B \rightarrow \pi\ell\nu$
- Extract signal in $\mathbf{q^2} : \mathbf{N}_{\pi^\pm}$ for $B \rightarrow \pi\ell\nu$ and $B \rightarrow X_u\ell\nu$ simultaneously

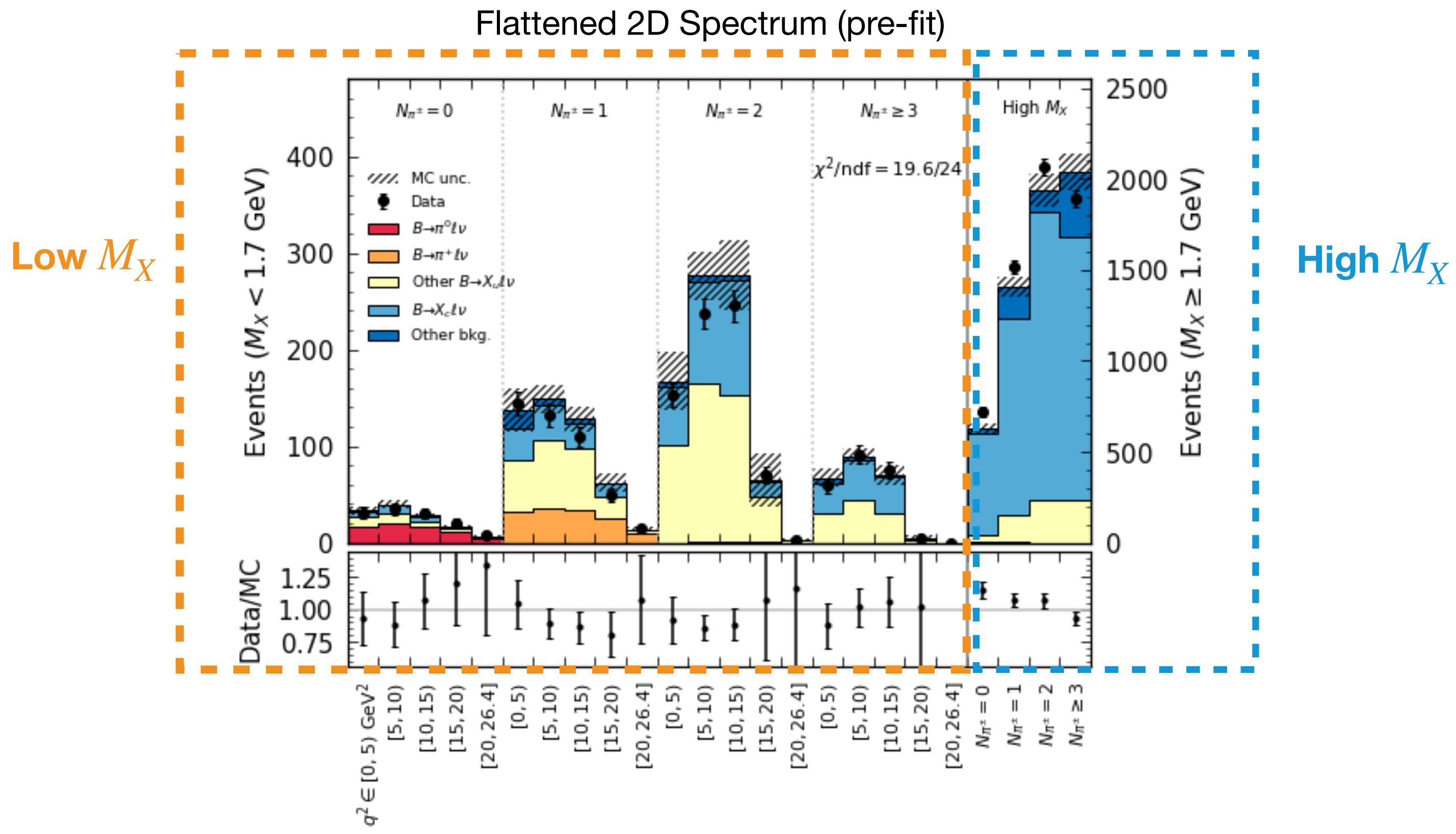
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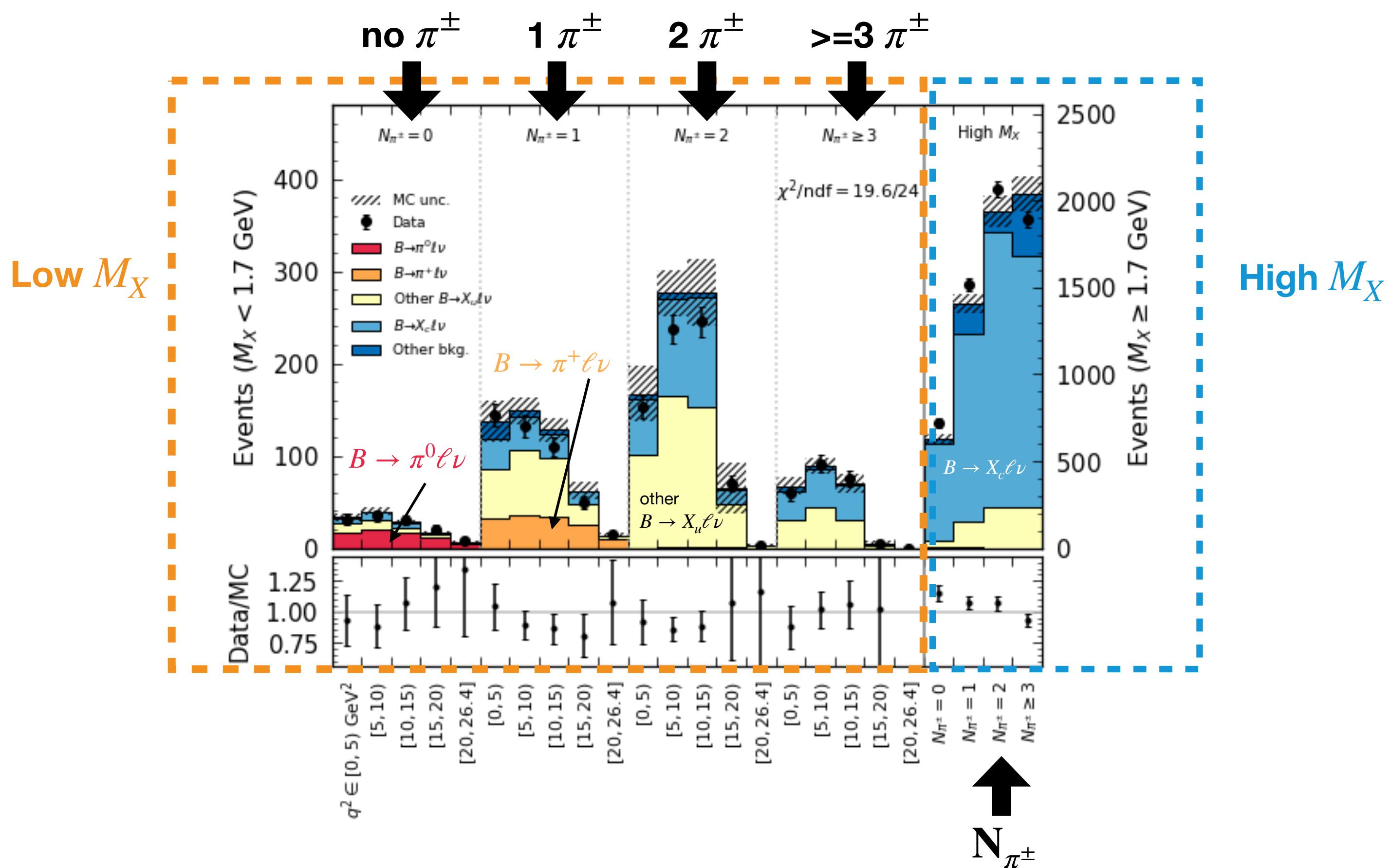


First Simultaneous Determination of Incl. & Excl. $|V_{ub}|$

arXiv: 2303.17309

Preliminary

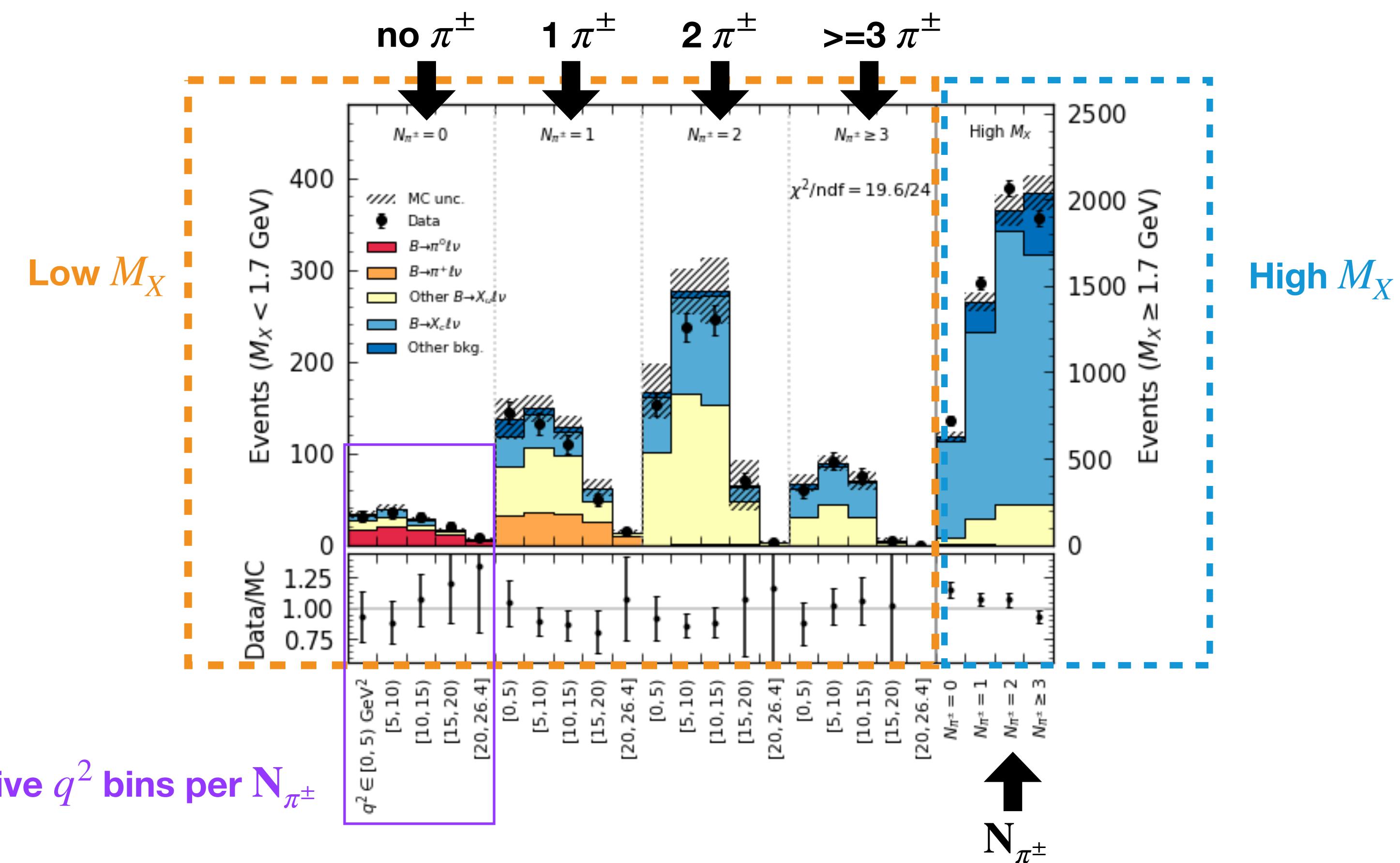
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First Simultaneous Determination of Incl. & Excl. $|V_{ub}|$

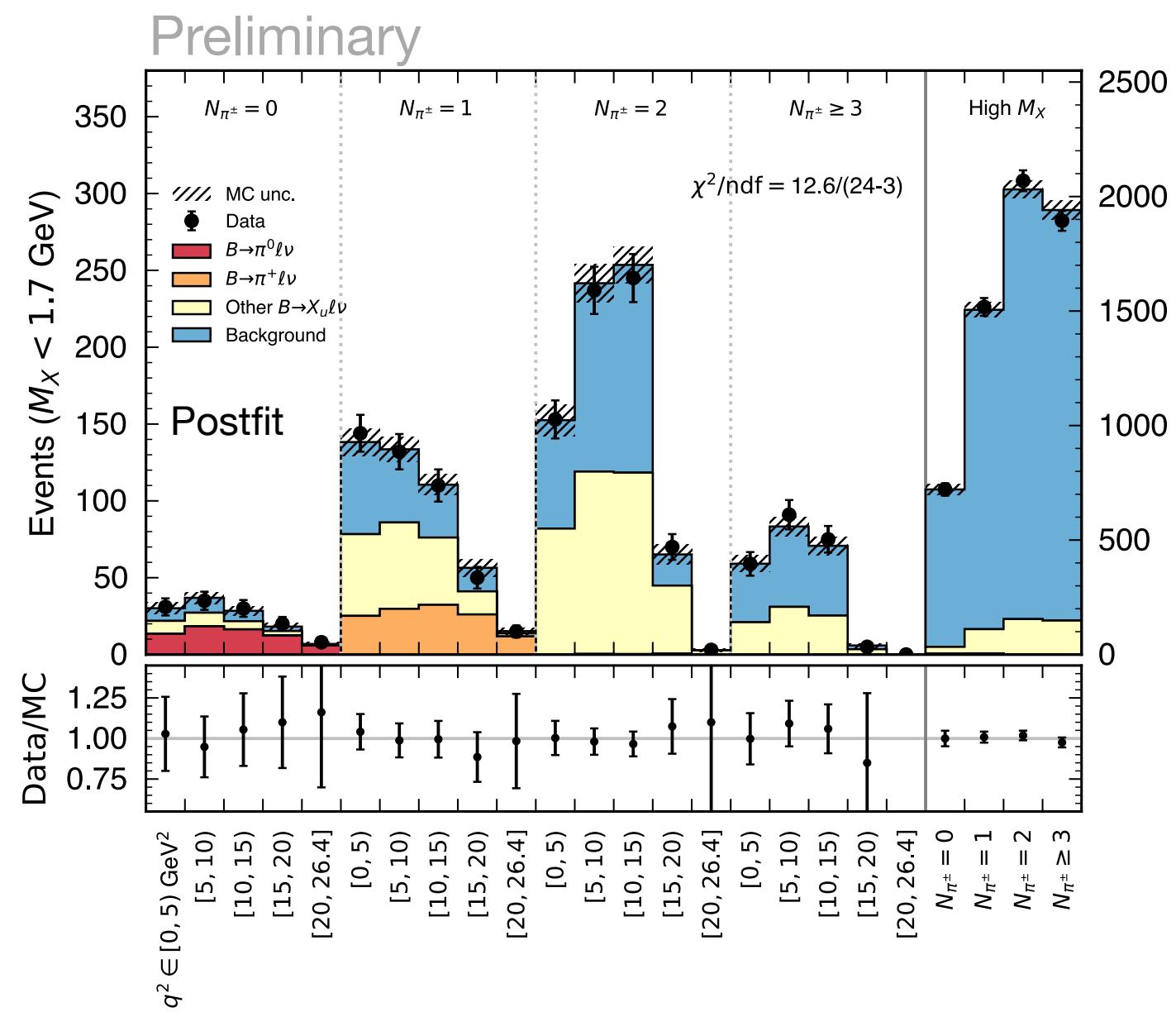


arXiv: 2303.17309

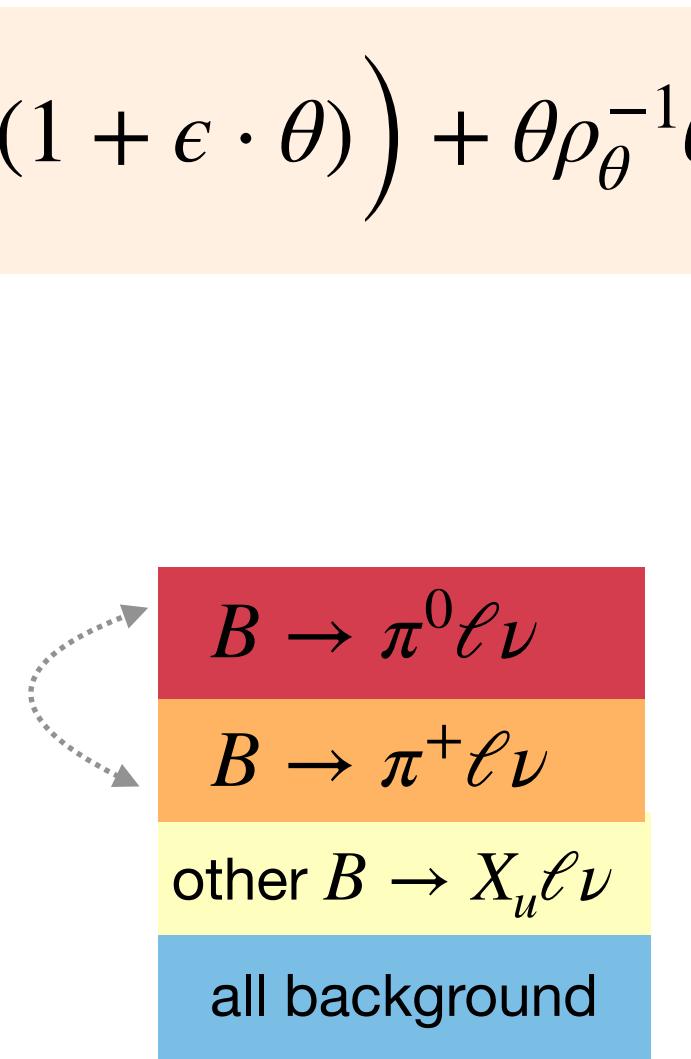
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- Dominant syst. are non-resonant $B \rightarrow X_u \ell \nu$ modelling, fragmentation and reconstruction efficiency (stat. limits $B \rightarrow \pi \ell \nu$)

Preliminary

$$-2 \log \mathcal{L} = -2 \log \prod_i \text{Poisson} \left(\eta_{\text{obs}}, \eta_{\text{pred}} \cdot (1 + \epsilon \cdot \theta) \right) + \theta \rho_{\theta}^{-1} \theta^T + \chi_{\text{FF}}^2$$



Normalizations
can be linked with
isospin relation, or
floating separately
(nominal: linked)



First Simultaneous Determination of Incl. & Excl. $|V_{ub}|$



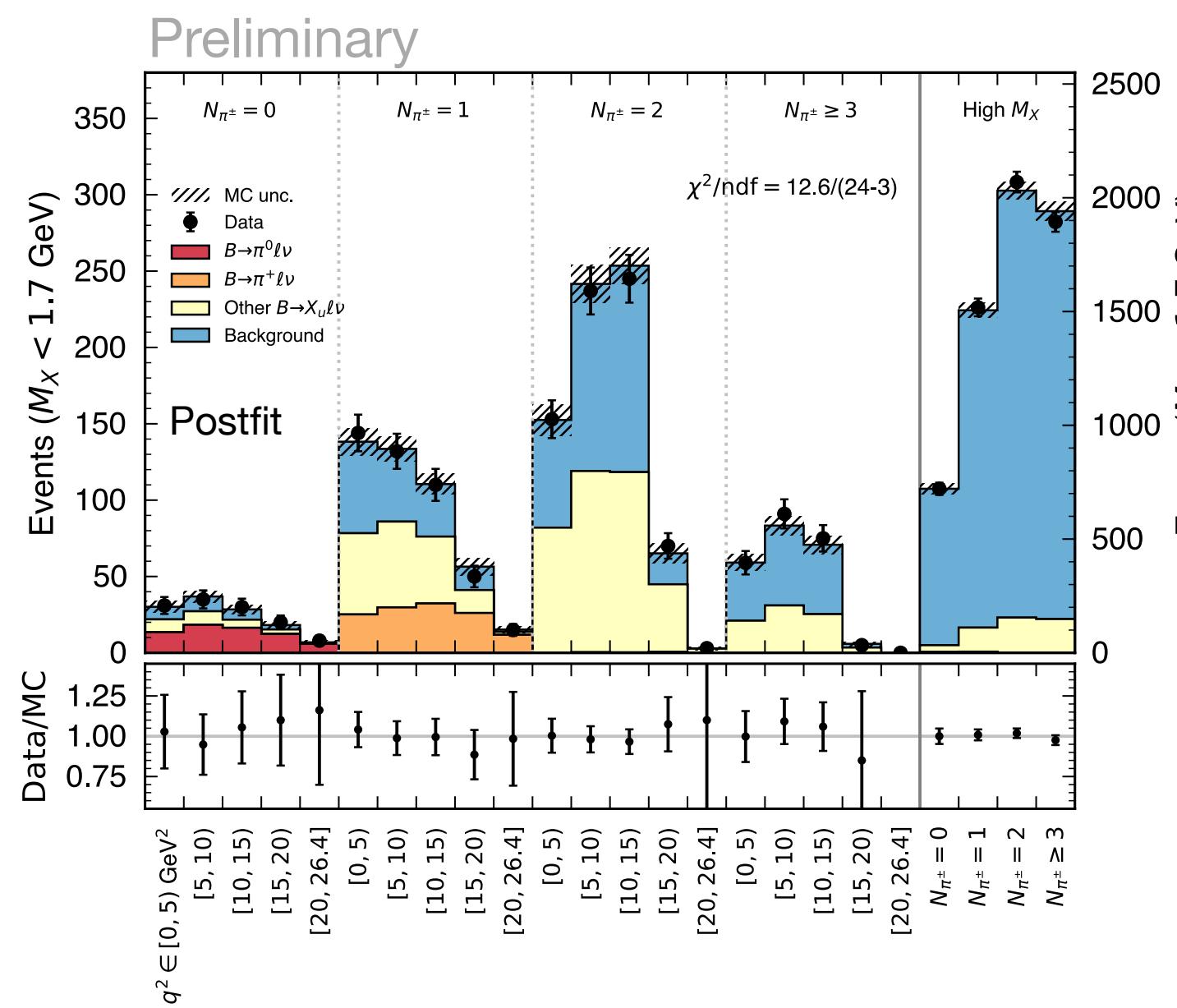
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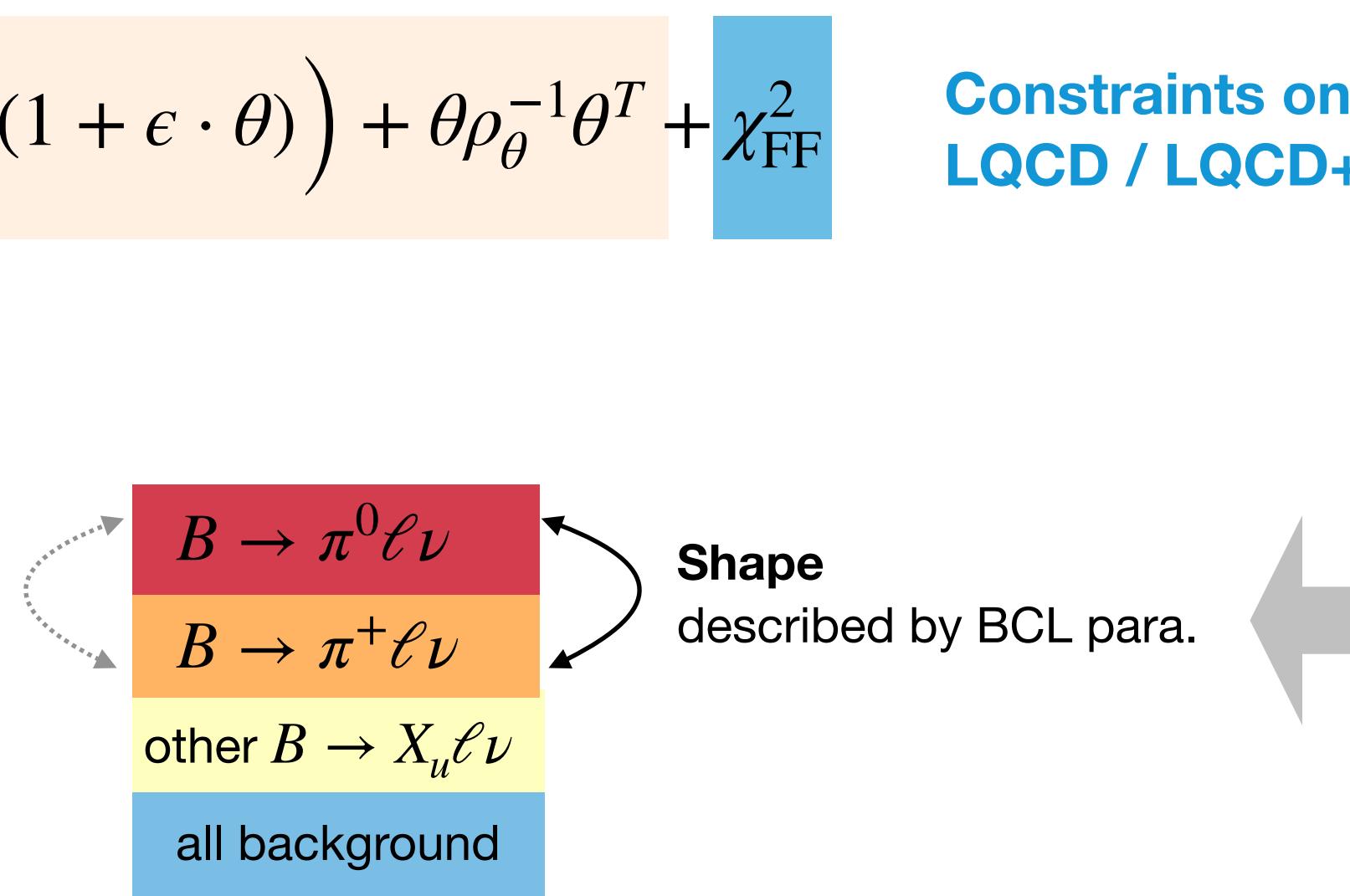
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Constraints on BCL parameters , input taken from LQCD / LQCD+exp fits in [FLAG Review 2021](#)



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Differential decay rates
↓
Acceptance & reco. efficiency
↓
Forward-folding q^2

First Simultaneous Determination of Incl. & Excl. $|V_{ub}|$



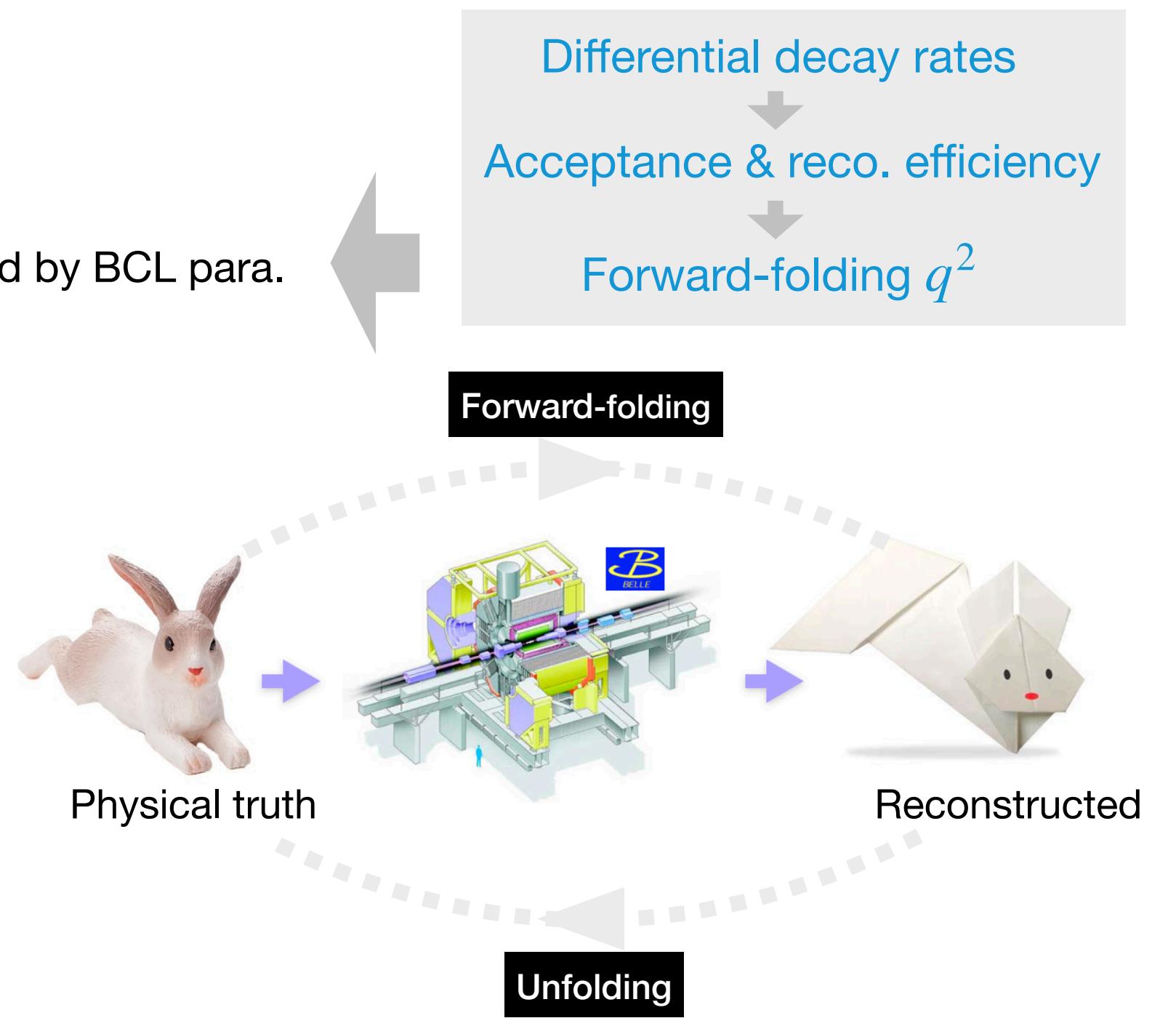
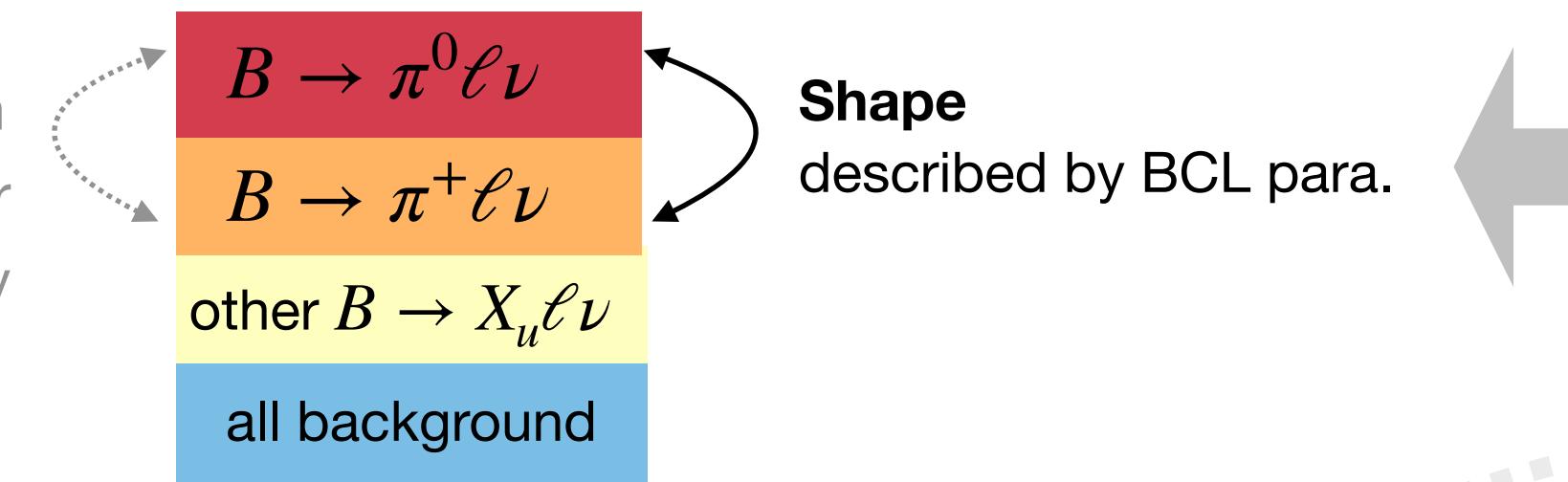
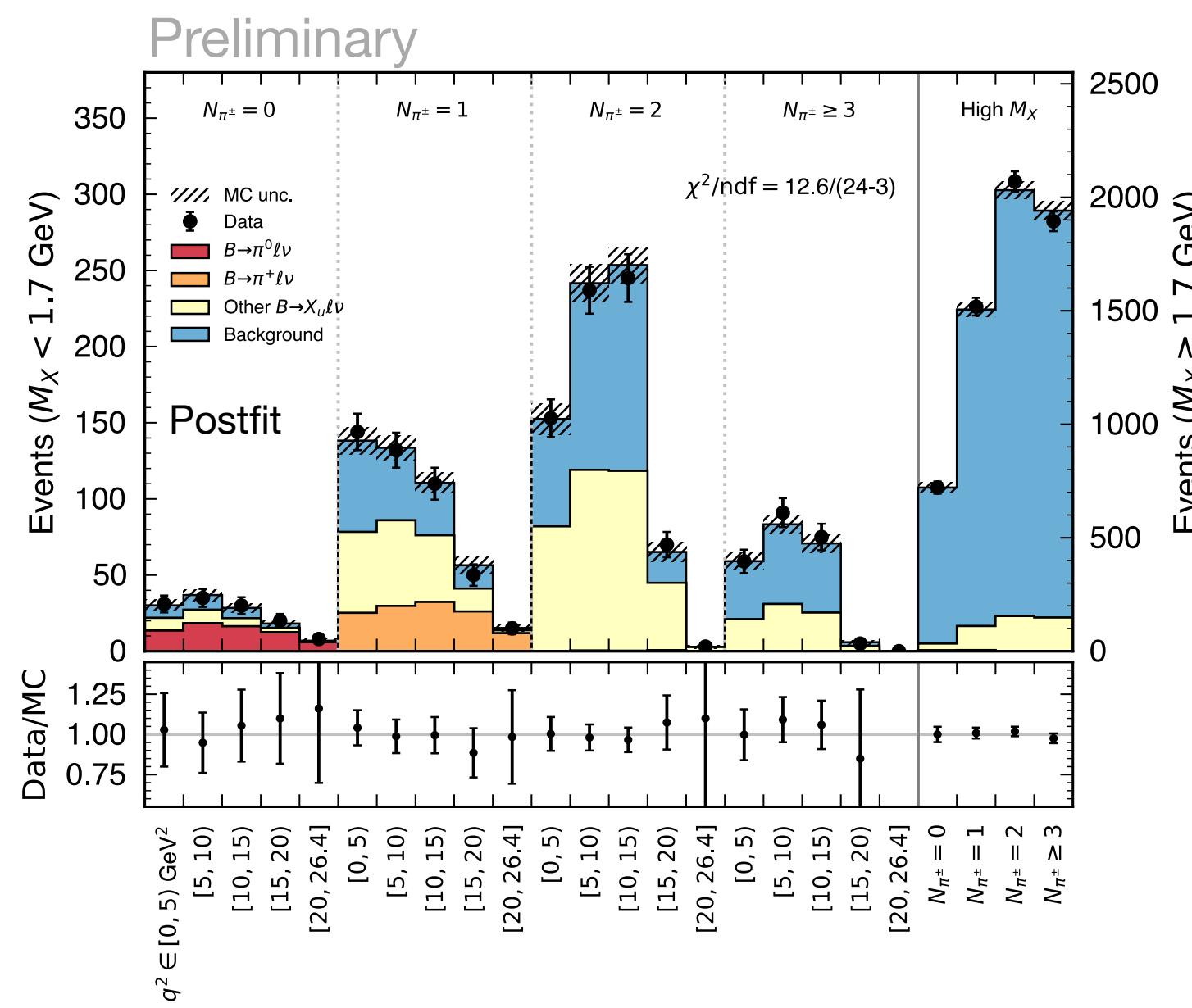
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Preliminary

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First Simultaneous Determination of Incl. & Excl. $|V_{ub}|$



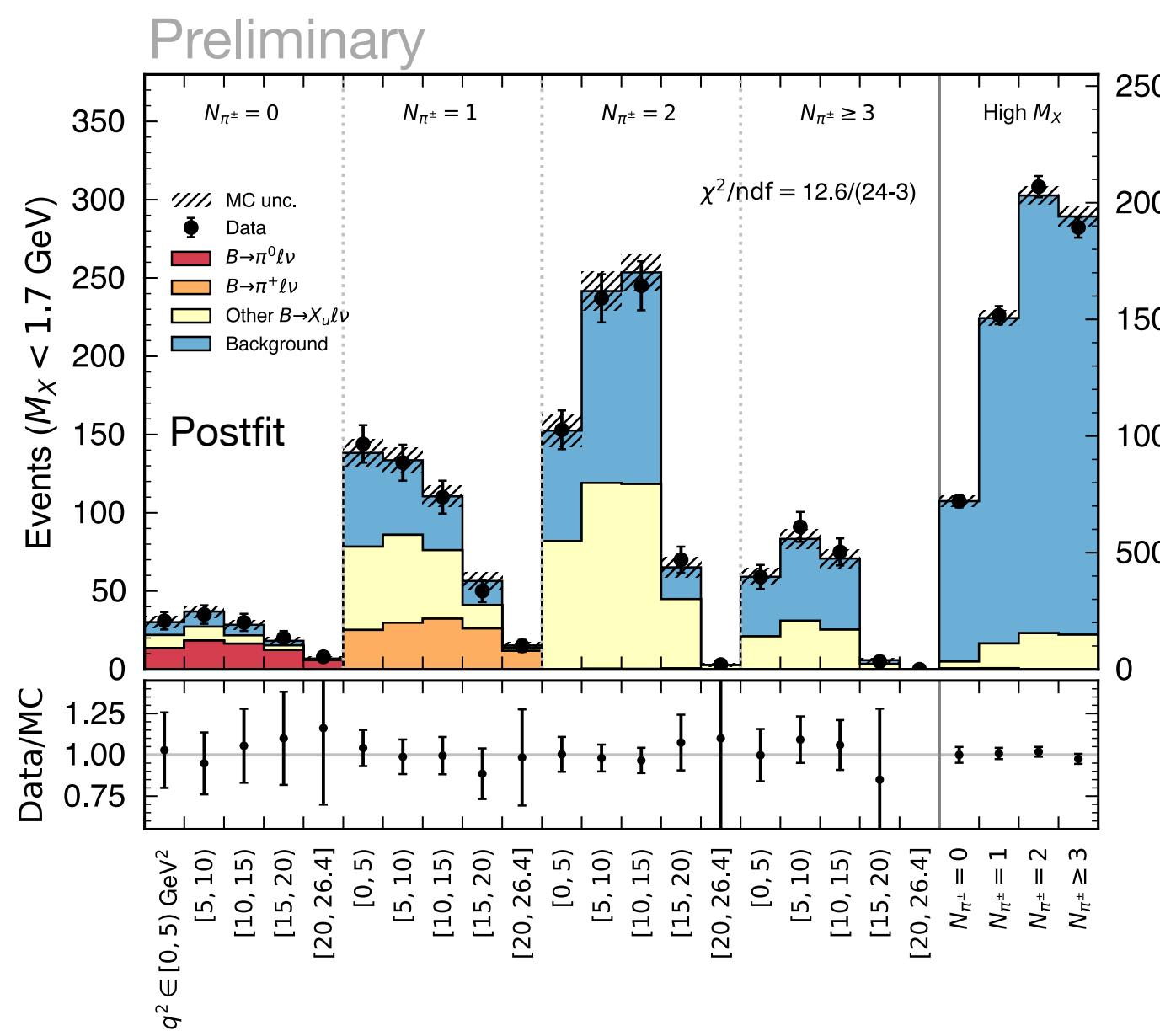
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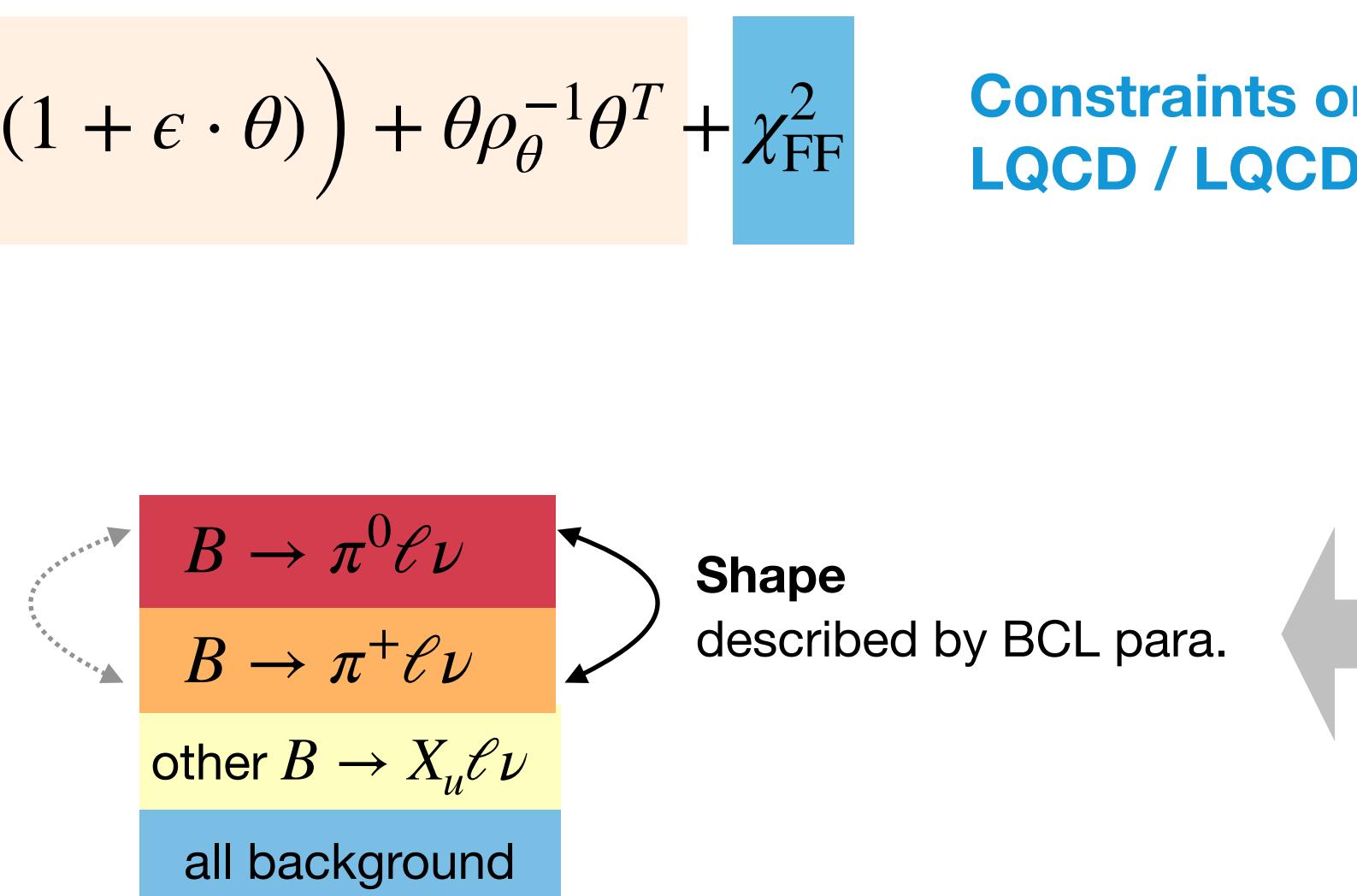
Preliminary

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Constraints on BCL parameters , input taken from LQCD / LQCD+exp fits in [FLAG Review 2021](#)



Normalizations
can be linked with
isospin relation, or
floating separately
(nominal: linked)



$$\mathcal{B}(B \rightarrow X_u \ell \nu) = \mathcal{B}(B \rightarrow \pi^0 \ell \nu) + \mathcal{B}(B \rightarrow \pi^+ \ell \nu) + \mathcal{B}(B \rightarrow X_u^{\text{other}} \ell \nu)$$

$$|V_{ub}^{\text{incl.}}| = \sqrt{\frac{\Delta \mathcal{B}(B \rightarrow X_u \ell \nu)}{\tau_B \cdot \Delta \Gamma_{\text{GGOU}}}}$$

$$|V_{ub}^{\text{excl.}}| = \sqrt{\frac{\mathcal{B}(B \rightarrow \pi \ell \nu)}{\tau_B \cdot \Gamma_{\text{FF}}}}$$

$$\Delta \mathcal{B}(B \rightarrow X_u \ell \nu) = \mathcal{B}(B \rightarrow X_u \ell \nu) \cdot \epsilon_{\Delta \mathcal{B}: E_\ell^B > 1 \text{ GeV}}$$



Fit results provide all \mathcal{B} and
 $B \rightarrow \pi \ell \nu$ FF (decay rate)
=> sufficient to derive exclusive
and inclusive $|V_{ub}|$

First Simultaneous Determination of Incl. & Excl. $|V_{ub}|$



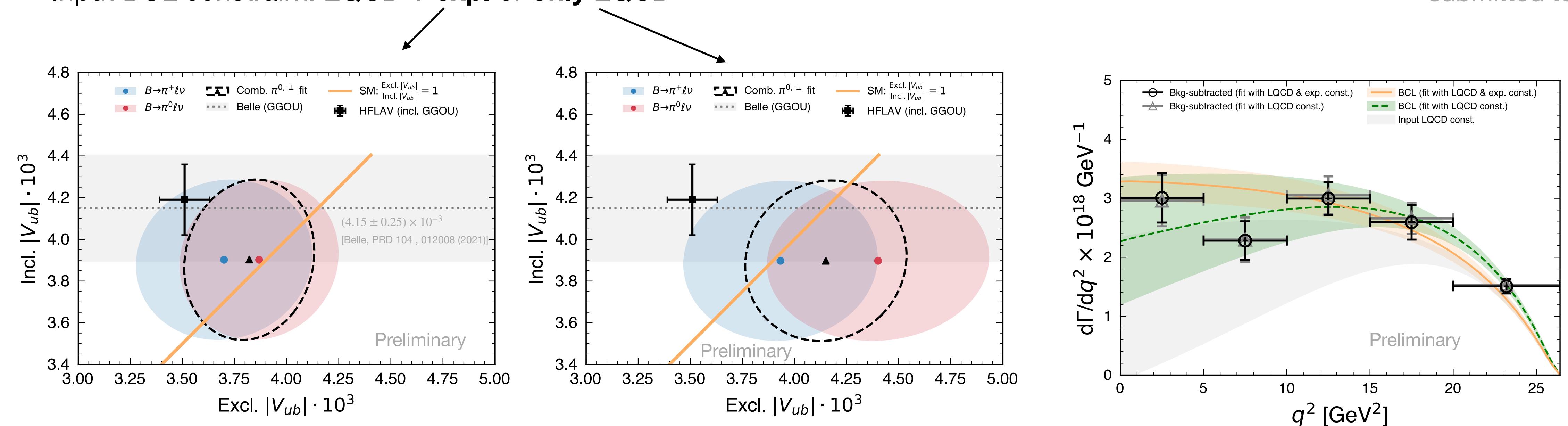
arXiv: 2303.17309

- Various fit scenarios applied:

- Combined or separate $B \rightarrow \pi^+ \ell \nu$, $B \rightarrow \pi^0 \ell \nu$
- Input BCL constraint: LQCD + exp. or only LQCD

Preliminary

submitted to PRL



$|V_{ub}|$ in combined scenario with LQCD+exp const.:

$$\text{Excl. } (3.78 \pm 0.23_{\text{stat}} \pm 0.16_{\text{syst}} \pm 0.14_{\text{theo}}) \times 10^{-3}$$

$$\text{Incl. } (3.90 \pm 0.20_{\text{stat}} \pm 0.32_{\text{syst}} \pm 0.09_{\text{theo}}) \times 10^{-3}$$

$$\text{Ratio } 0.97 \pm 0.12$$

$$\text{Correlation } 0.10$$

Weighted average of excl. & incl.

$$(3.85 \pm 0.26) \times 10^{-3}$$

CKM global fit (w/o $|V_{ub}|$): $(3.64 \pm 0.07) \times 10^{-3}$, compatible within 0.8σ

Ratio of Inclusive $\Delta\mathcal{B}(B \rightarrow X_u \ell \nu)$ and $\Delta\mathcal{B}(B \rightarrow X_c \ell \nu)$

- Full Belle data set with **Hadronic tagging** using Belle II tool (Full Event Interpretation) Preliminary
- **Modified $B \rightarrow X_c \ell \nu$ modeling** using sideband data
- $B \rightarrow X_u \ell \nu$ yields extracted in $q^2 : p_\ell^B$; $B \rightarrow X_c \ell \nu$ yields obtained by subtracting other contributions in total $B \rightarrow X \ell \nu$
- Measured partial phase space region of $p_\ell^B > 1 \text{ GeV}$, $\epsilon_\Delta^u = 86\%$, $\epsilon_\Delta^c = 79\%$

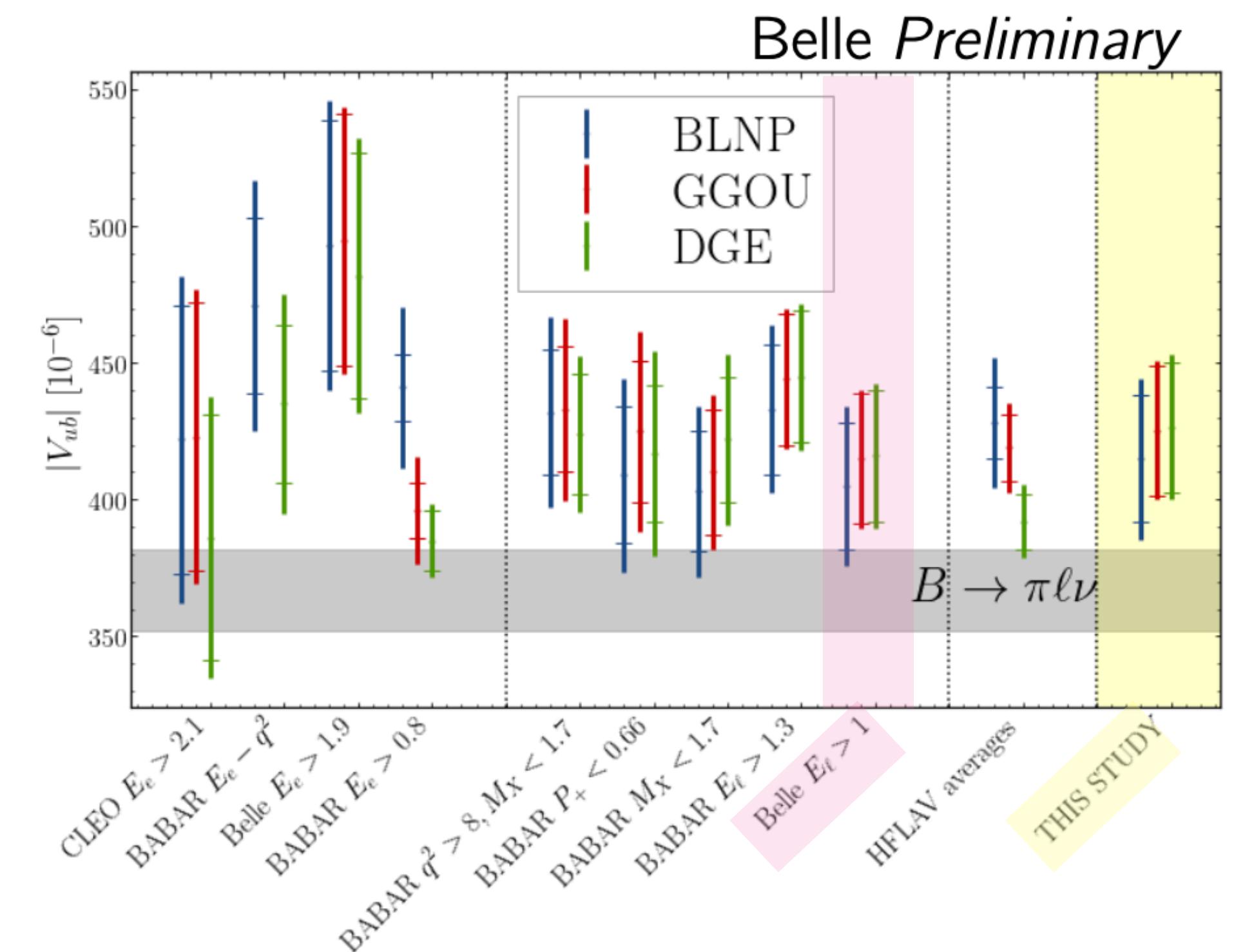
Preliminary

$$\frac{\Delta\mathcal{B}(B \rightarrow X_u \ell \nu)}{\Delta\mathcal{B}(B \rightarrow X_c \ell \nu)} = 1.95(1 \pm 8.4\%_{\text{stat}} \pm 7.2\%_{\text{syst}}) \times 10^{-2}$$

Based on this, one could try the following two quick and naive conversions

$$|V_{ub}| = \sqrt{\frac{1}{\tau_B \Delta\Gamma(B \rightarrow X_u \ell \nu)} \frac{\Delta\mathcal{B}(B \rightarrow X_u \ell \nu)}{\Delta\mathcal{B}(B \rightarrow X_c \ell \nu)}} \boxed{\Delta\mathcal{B}(B \rightarrow X_c \ell \nu)}$$

WA: $(8.55 \pm 0.13)\%$



Consistent with recent Belle result PRD 104 , 012008 (2021)

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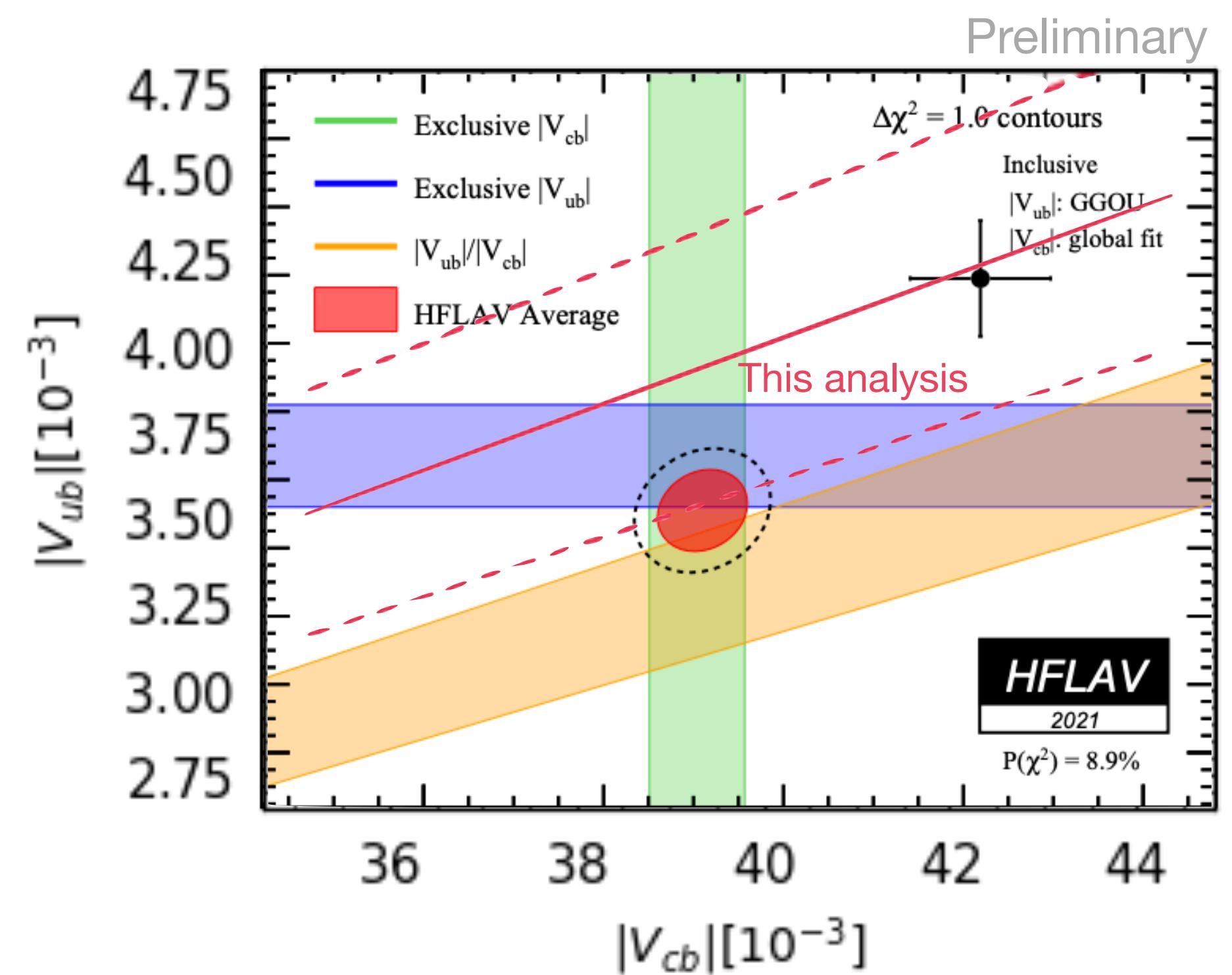
WA: $(8.55 \pm 0.13)\%$

$$\frac{|V_{ub}|}{|V_{cb}|} = \sqrt{\frac{\Delta\mathcal{B}(B \rightarrow X_u \ell \nu)}{\Delta\mathcal{B}(B \rightarrow X_c \ell \nu)} \frac{\Delta\Gamma(B \rightarrow X_c \ell \nu)}{\Delta\Gamma(B \rightarrow X_u \ell \nu)}}$$

Theo. input: $\Delta\Gamma^{\text{GGOU}}(B \rightarrow X_u \ell \nu) = 58.5 \pm 2.7 \text{ ps}^{-1}$

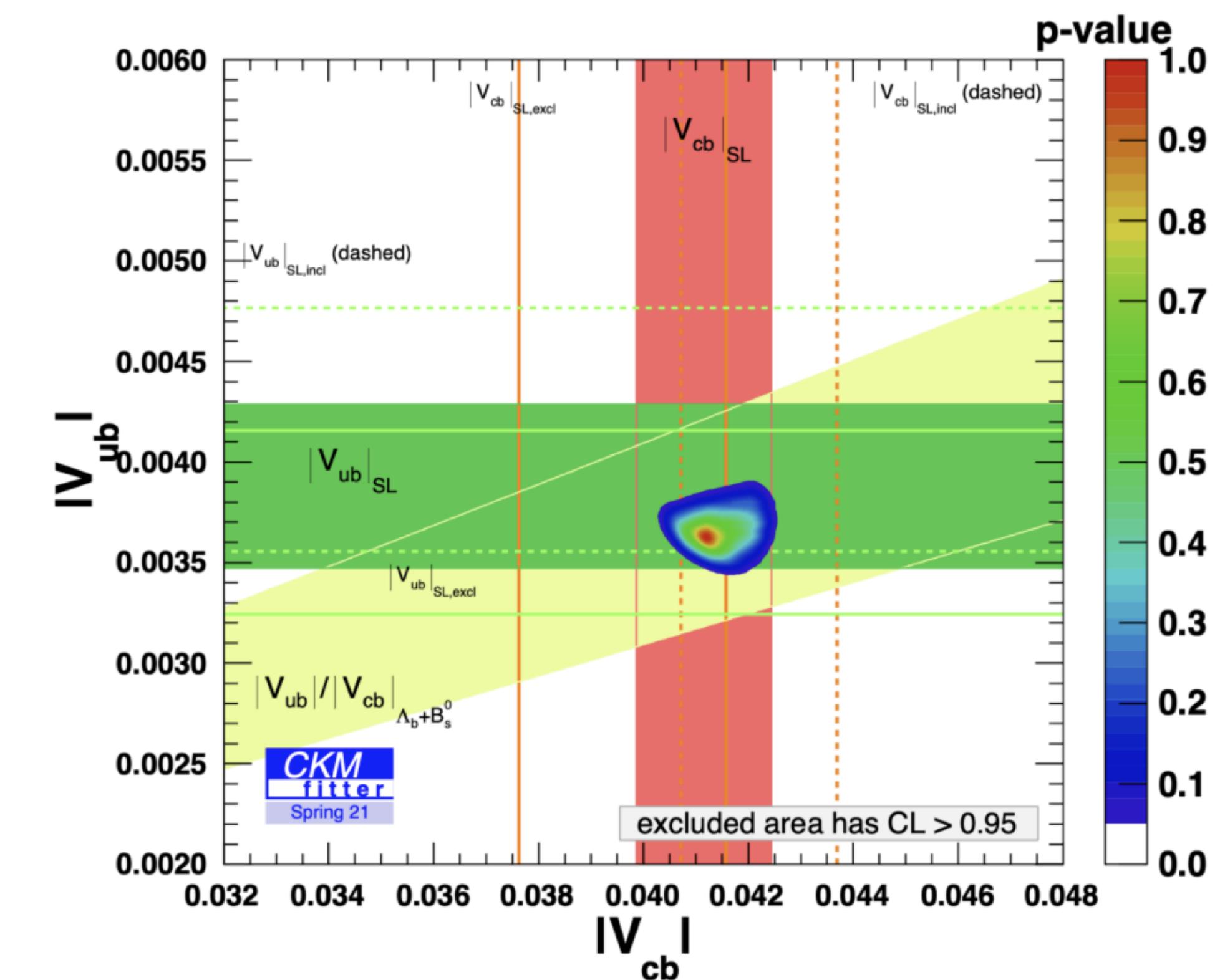
$\Delta\Gamma^{\text{Kin}}(B \rightarrow X_c \ell \nu) = 29.9 \pm 1.2 \text{ ps}^{-1}$

Preliminary



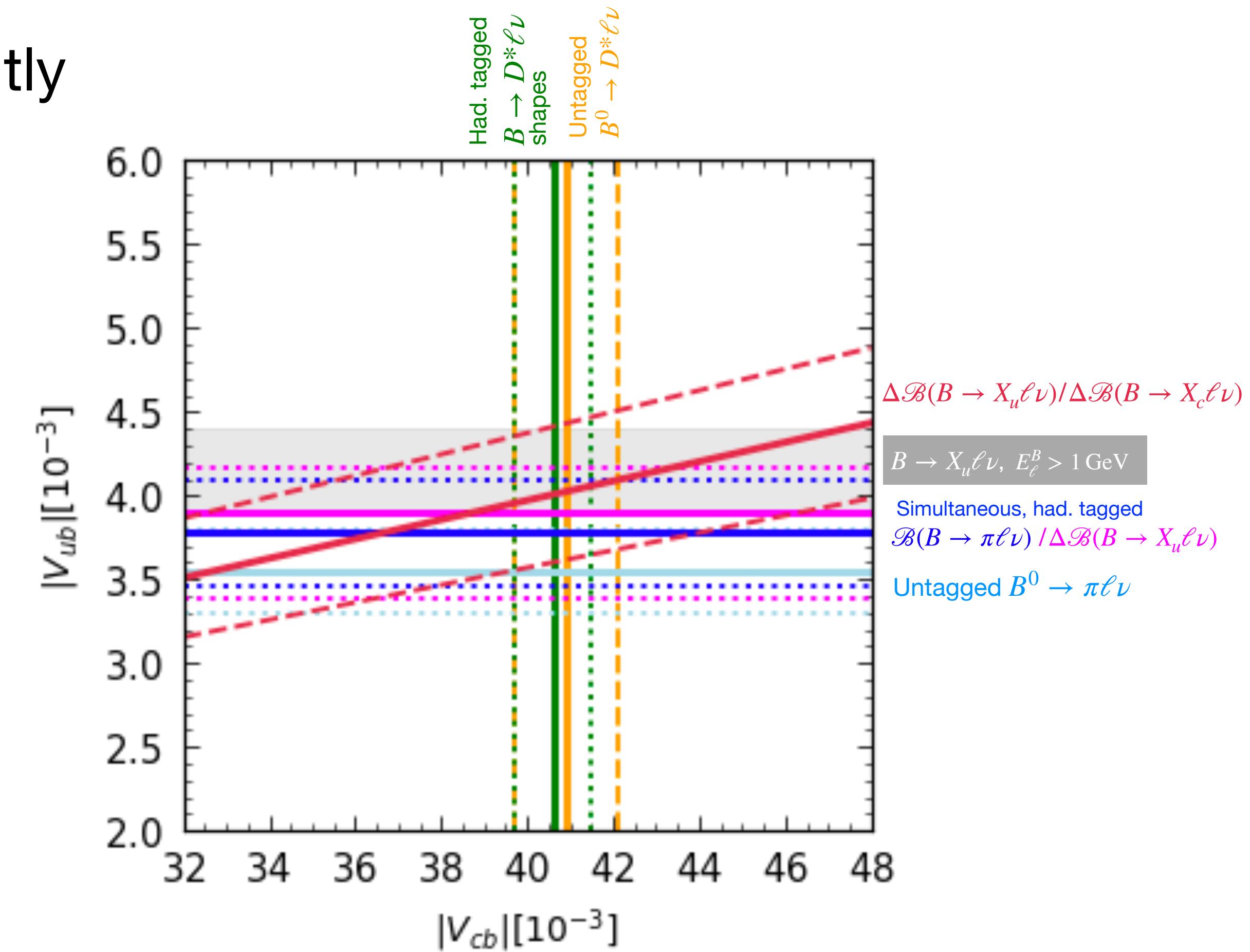
Summary

- Several new results on $|V_{xb}|$ measured recently at Belle and Belle II
- These new results will be very helpful to examine the long-standing $|V_{xb}|$ puzzle
- Continuous efforts from **experiment** and **theory** are still needed
- Beyond these important results, the accumulated knowledge on MC modeling, analysis techniques, etc. will be beneficial for future measurements by e.g. **Belle II** or **LHCb**



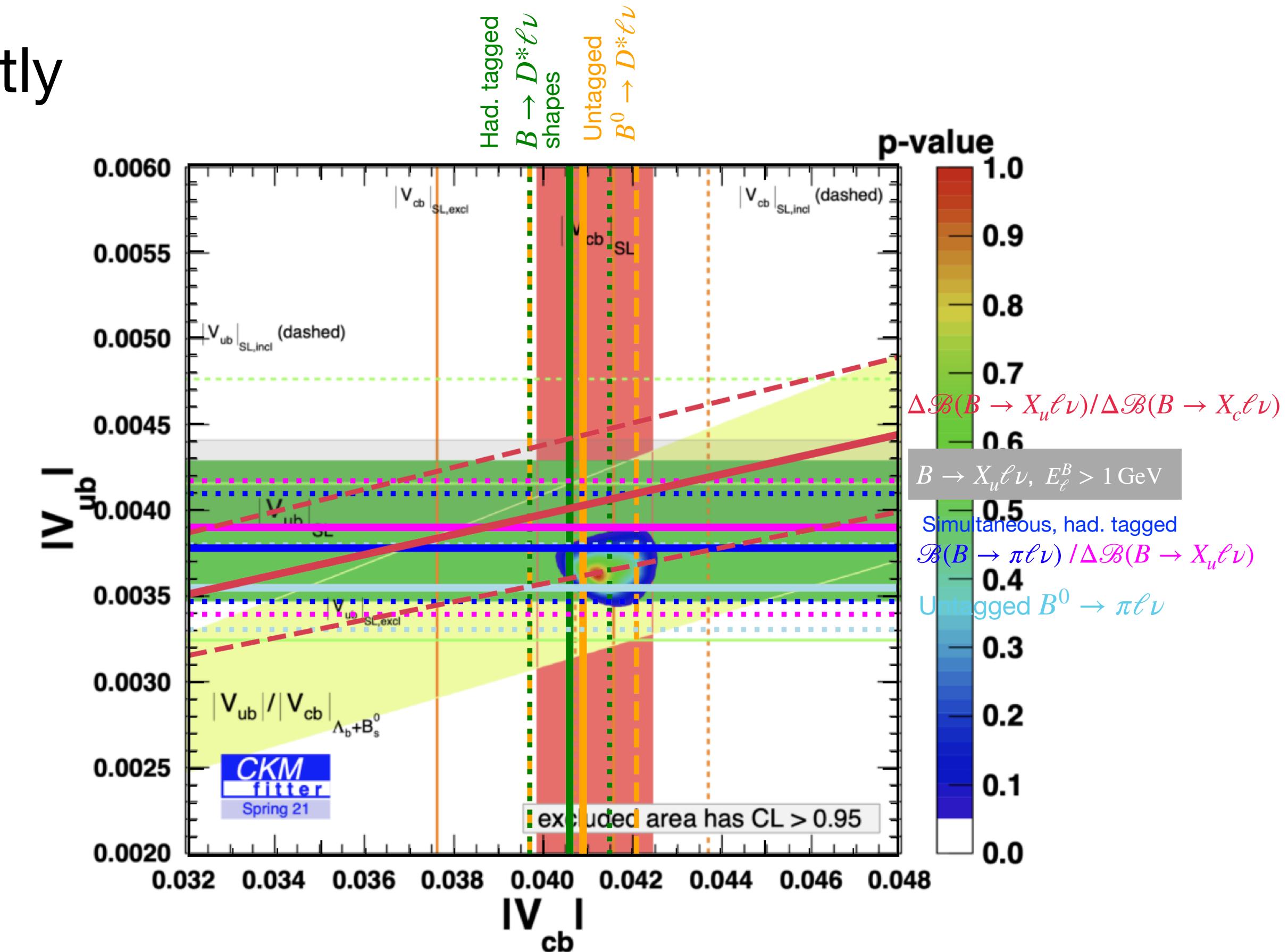
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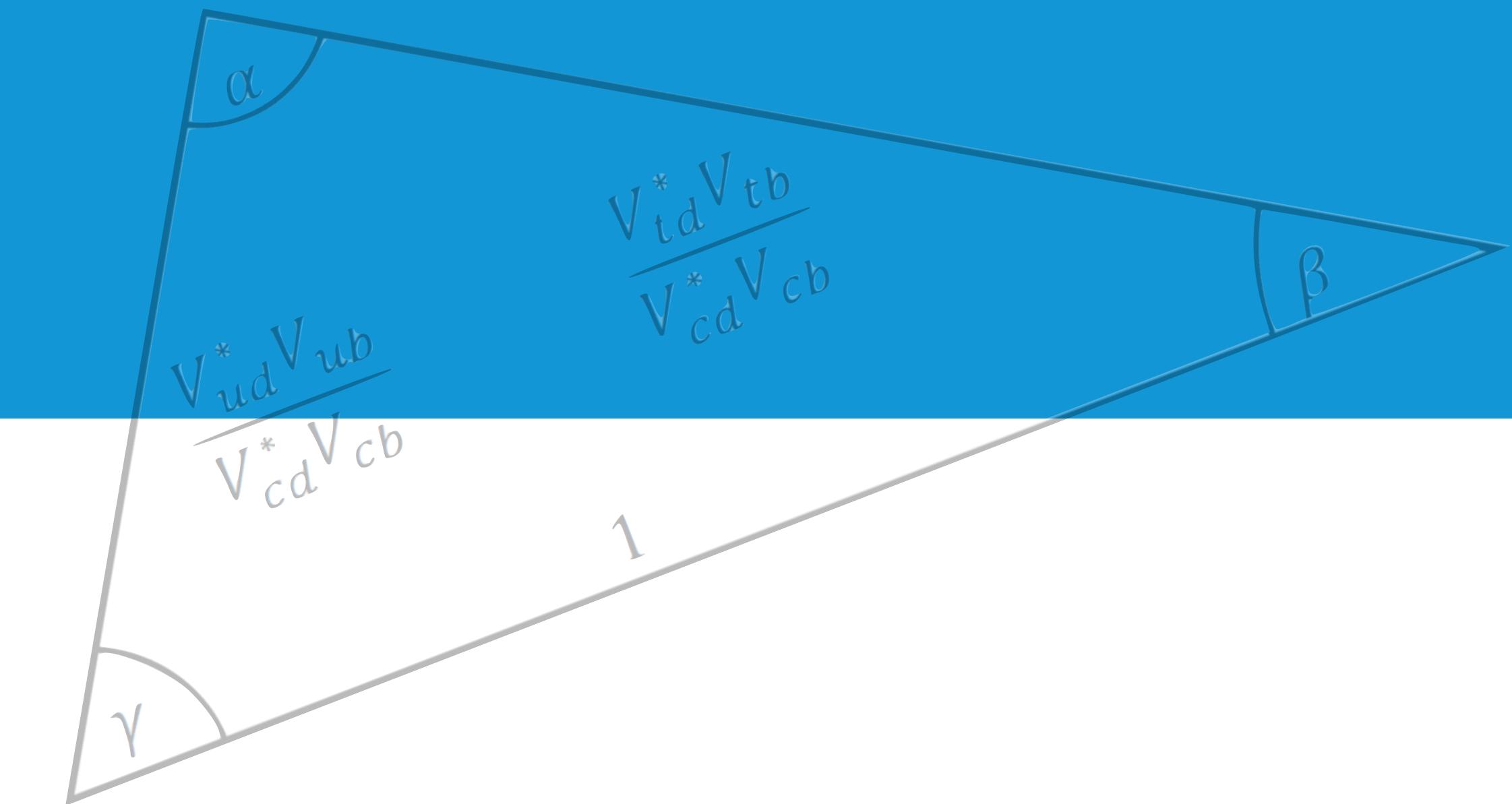


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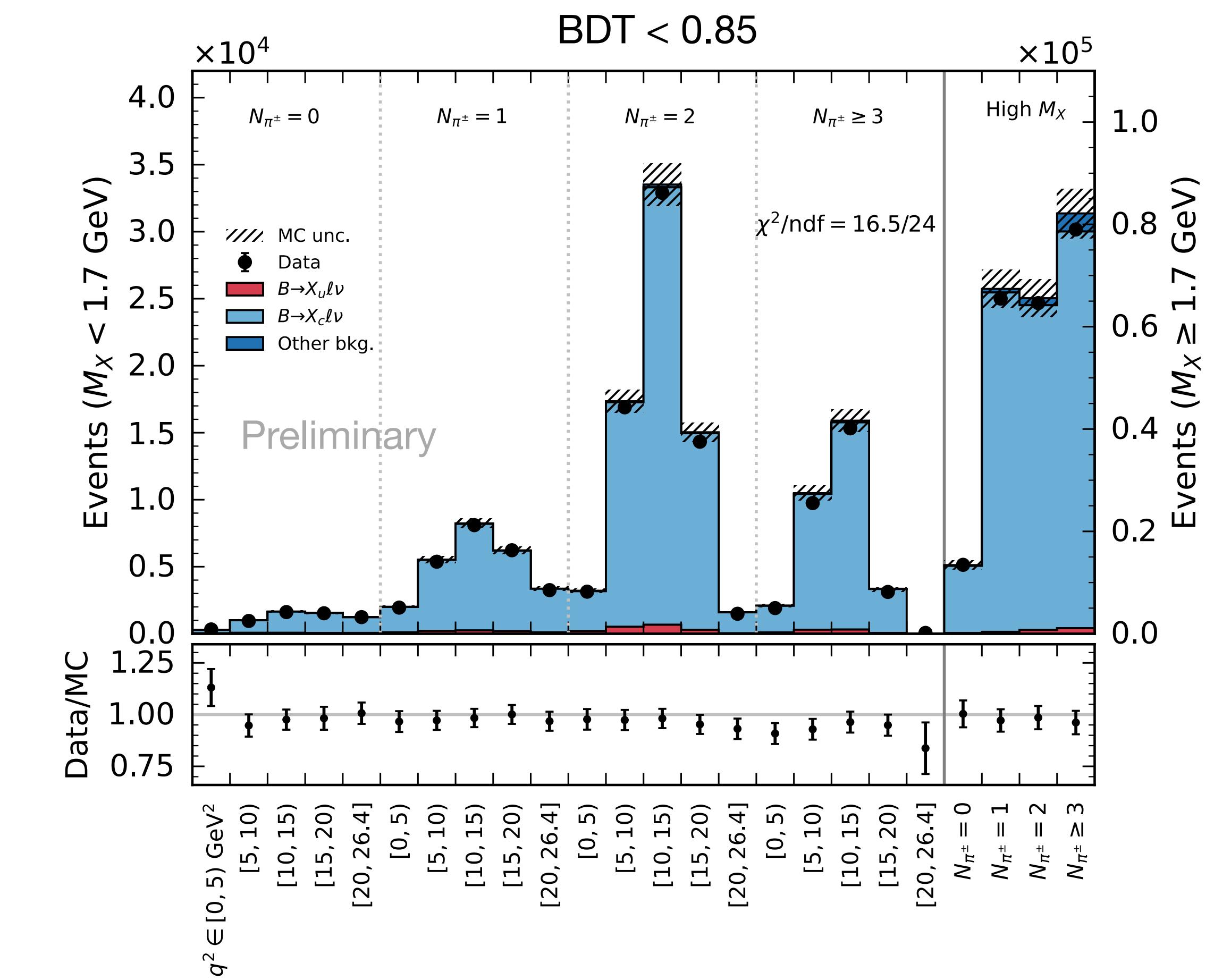
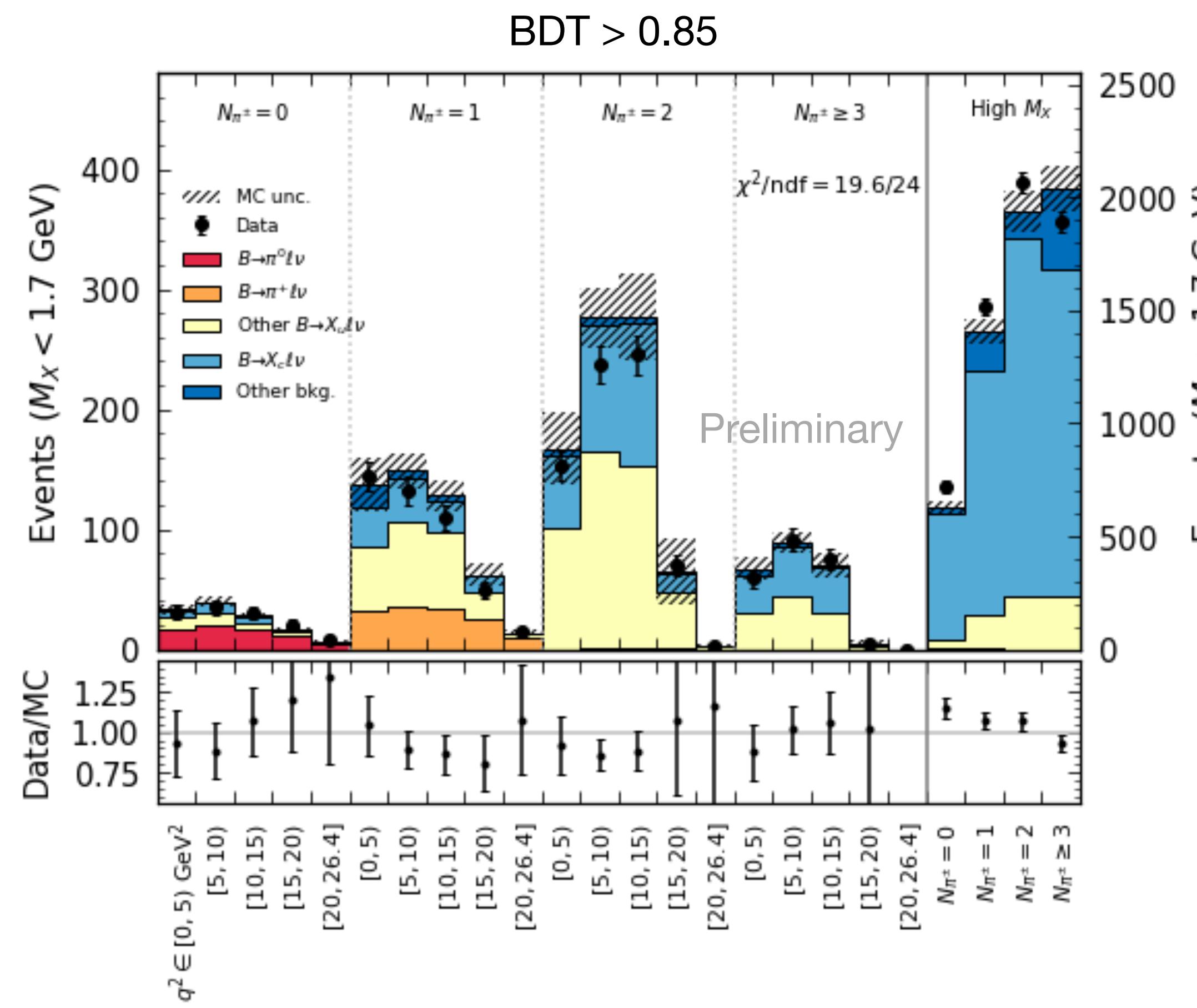
THANK YOU



Backup: First Simultaneous Determination of Incl. & Excl. $|V_{ub}|$

Preliminary

- Prefit distributions



Various Inclusive Decay Rates

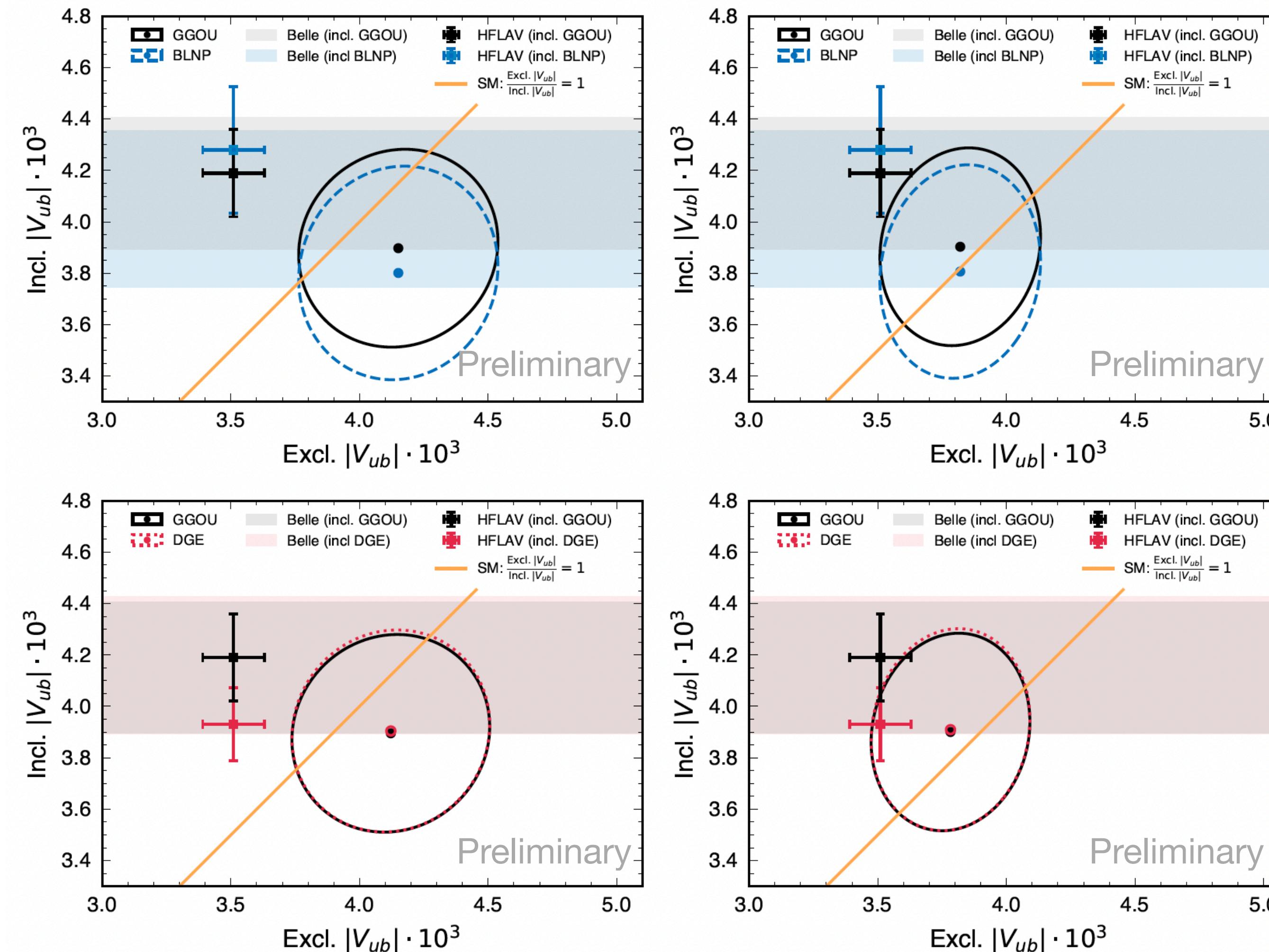


FIG. 4. The $|V_{ub}|$ values obtained using the different theoretical inclusive decay rates are compared: GGOU versus BLNP (up) and GGOU versus DGE (low). The left column shows the fit with only LQCD constraints and the results from combined LQCD-experimental constraints are in the right column.

First Simultaneous Determination of Incl. & Excl. $|V_{ub}|$

- Various fit scenarios applied:

[arXiv: 2303.17309](#)

- Linked or separate $B \rightarrow \pi^+ \ell \nu$, $B \rightarrow \pi^0 \ell \nu$
- Input BCL constraint: LQCD + exp. or only LQCD

Preliminary

$$|V_{ub}^{\text{excl.}}| = (3.78 \pm 0.23_{\text{stat}} \pm 0.16_{\text{syst}} \pm 0.14_{\text{theo}}) \times 10^{-3} \quad (\text{LQCD + exp.})$$

$$\text{Ratio} = 0.97 \pm 0.12$$

Correlation: 0.10

$$|V_{ub}^{\text{incl.}}| = (3.90 \pm 0.20_{\text{stat}} \pm 0.32_{\text{syst}} \pm 0.09_{\text{theo}}) \times 10^{-3} \quad (\text{LQCD + exp.})$$

$$|V_{ub}^{\text{excl.}}| = (4.12 \pm 0.30_{\text{stat}} \pm 0.18_{\text{syst}} \pm 0.16_{\text{theo}}) \times 10^{-3} \quad (\text{LQCD})$$

$$\text{Ratio} = 1.06 \pm 0.14$$

Correlation: 0.07

$$|V_{ub}^{\text{incl.}}| = (3.90 \pm 0.20_{\text{stat}} \pm 0.32_{\text{syst}} \pm 0.09_{\text{theo}}) \times 10^{-3} \quad (\text{LQCD})$$

Weighted average of excl. & incl. :

$$|V_{ub}| = (3.85 \pm 0.26) \times 10^{-3} \quad (\text{LQCD + exp.})$$

CKM global fit (w/o $|V_{ub}|$): $(3.64 \pm 0.07) \times 10^{-3}$,
compatible within 0.8σ and 1.4σ , respectively

$$|V_{ub}| = (4.01 \pm 0.27) \times 10^{-3} \quad (\text{LQCD})$$

Hybrid Model of $B \rightarrow X_u \ell \nu$

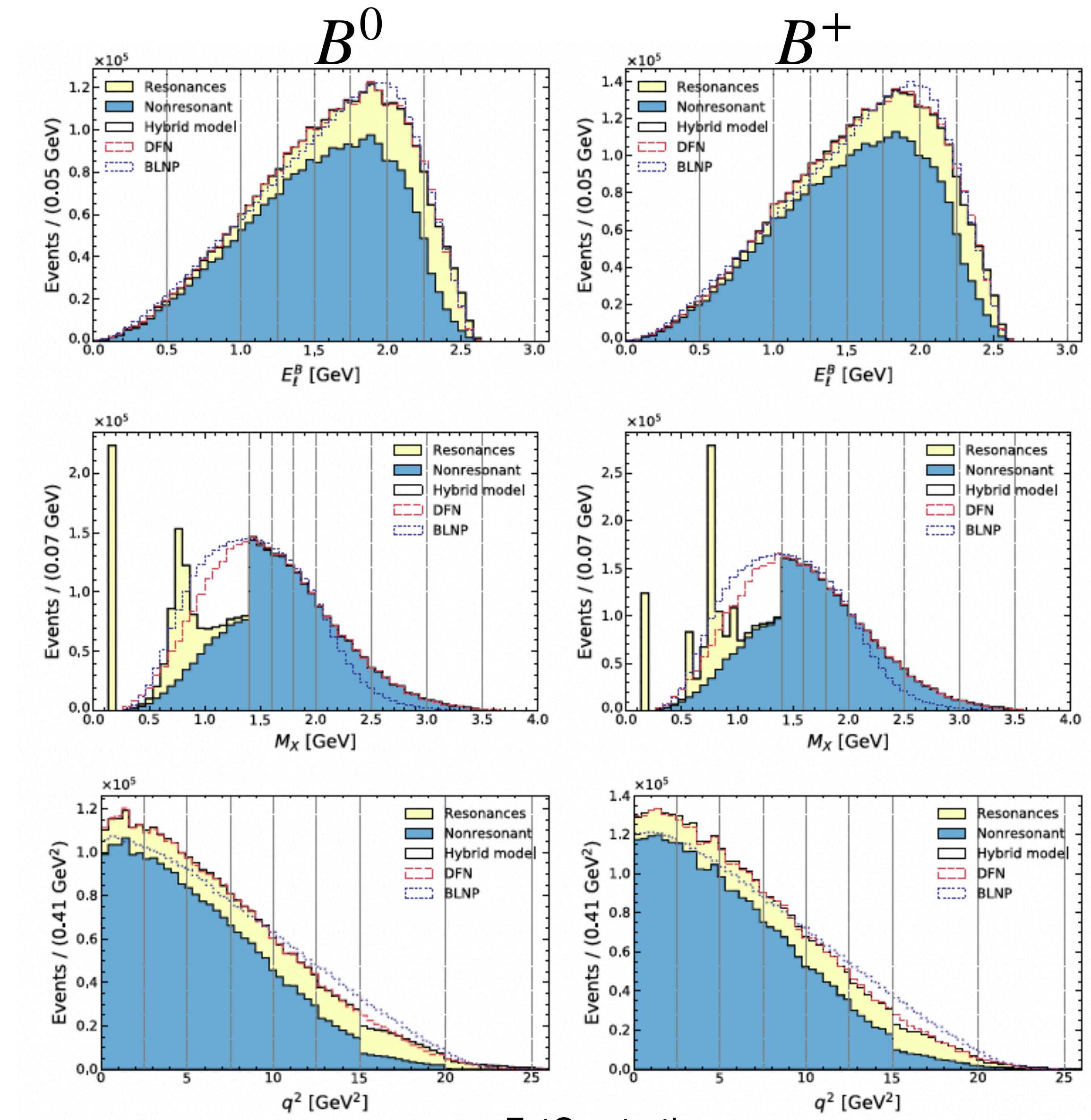
PRD 104 , 012008 (2021)

Hybrid MC is a **combination** of **resonances** (exclusive decays) and **non-resonant** contribution in the inclusive $B \rightarrow X_u \ell \nu$ decays

- EvtGen simulation:
 - (1) exclusive modes $B \rightarrow (\pi, \rho, \omega, \eta^{(\prime)}) \ell \nu$ with latest WA form factors & branching fractions
 - (2) fully inclusive $B \rightarrow X_u \ell \nu$ (only non-resonant shapes, e.g. BLNP, GGOU)
- Calculate hybrid weights to mix resonance & non-res. in **3D binning** of (q^2, E_ℓ^B, M_X) to recover total $\mathcal{B}(B \rightarrow X_u \ell \nu)$ in each bin

$$H_i = R_i + \omega_i N_i$$

- Systematic uncertainties include the impact from exclusive FFs & BRs, total $\mathcal{B}(B \rightarrow X_u \ell \nu)$, inclusive models



EvtGen truth