Power Corrections to Two-body Hadronic B Decays in QCD Factorization

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第五届重味物理与量子色动力学研讨会，2023/04/21，武汉

Outline

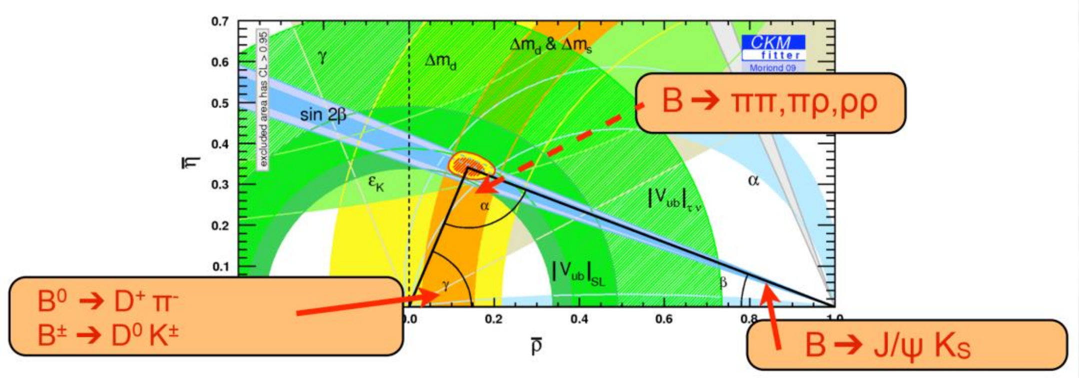
- ☐ Introduction
- ☐ QCD factorization: brief review and NNLO status
- ☐ Data vs SM predictions: some puzzles & possible resolutions
- ☐ Summary

Why hadronic B decays

- ☐ **direct access to the CKM parameters, especially to the three angles of UT.**

- ☐ **further insight into strong-interaction effects involved in hadronic decays.**

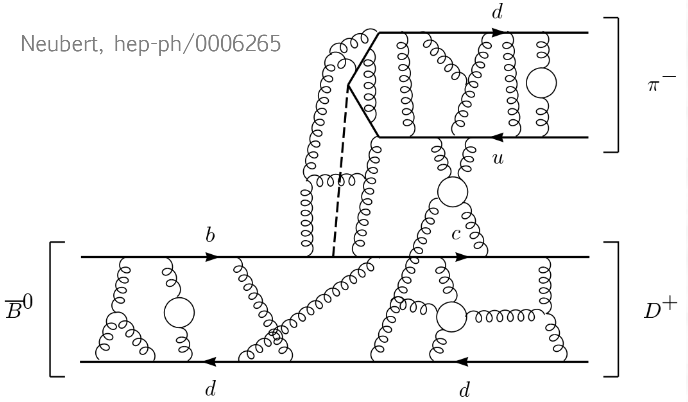
factorization? strong phase origin?...



$B^0 \rightarrow D^+ \pi^-$
$B^\pm \rightarrow D^0 K^\pm$

$B \rightarrow \pi\pi, \pi\rho, \rho\rho$

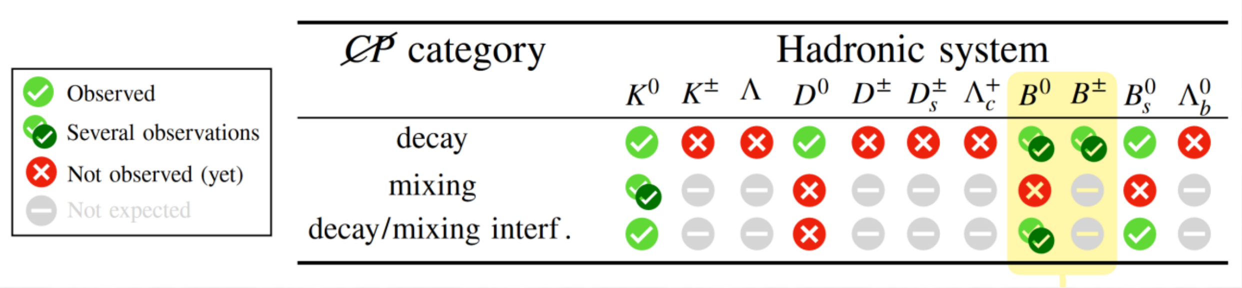
$B \rightarrow J/\psi\, K_S$



Neubert, hep-ph/0006265

- ☐ **deep insight into the hadron structures: especially exotic hadronic states.**

- ☐ **deep our understanding of origin & mechanism of CPV.**



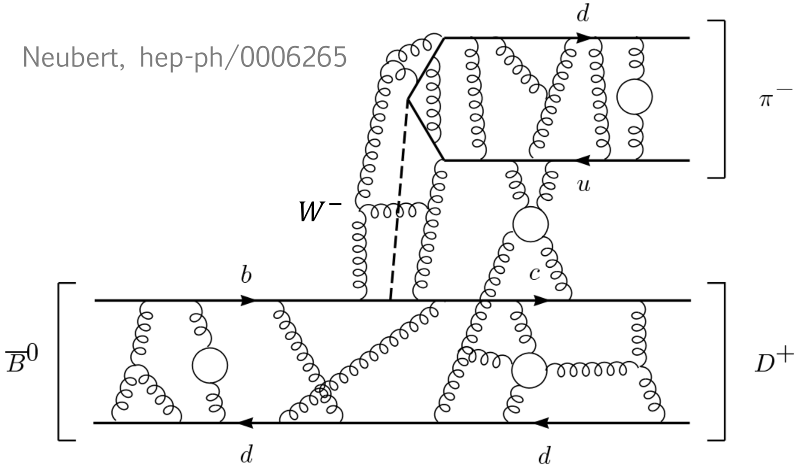
✓ Observed
✓✓ Several observations
✗ Not observed (yet)
⊖ Not expected

CP category	Hadronic system										
	K^0	$K^\pm$	Λ	D^0	$D^\pm$	$D_s^\pm$	Λ_c^+	B^0	$B^\pm$	B_s^0	Λ_b^0
decay	✓	✗	✗	✓	✗	✗	✗	✓✓	✓✓	✓	✗
mixing	✓✓	⊖	⊖	✗	⊖	⊖	⊖	✗	⊖	✗	⊖
decay/mixing interf.	✓	⊖	⊖	✗	⊖	⊖	⊖	✓✓	⊖	✓	⊖

➡ ***very difficult but necessary both theoretically and experimentally!***

Effective Hamiltonian for hadronic B decays

- ☐ **For hadronic B decays: typical multi-scale problem; EFT formalism more suitable!**



Neubert, hep-ph/0006265

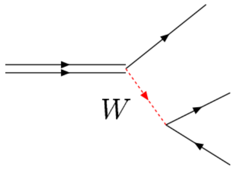
multi-scale problem with highly hierarchical scales!

EW interaction scale	$\gg$	ext. mom'a in B rest frame	$\gg$	QCD-bound state effects
$m_W \sim 80$ GeV $m_Z \sim 91$ GeV	$\gg$	$m_b \sim 5$ GeV	$\gg$	$\Lambda_{\rm QCD} \sim 1$ GeV

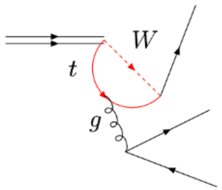
$$\mathcal{L}_{\rm eff} = -\frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb} V^*_{pD} \Big(C_1 \mathcal{O}_1 + C_2 \mathcal{O}_2 + \sum_{i={\rm pen}} C_i \mathcal{O}_{i,{\rm pen}} \Big)$$

- ☐ **Starting point $\mathcal{H}_{\rm eff} = -\mathcal{L}_{\rm eff}$: obtained after integrating out heavy d.o.f. ($m_{W,Z,t} \gg m_b$);**

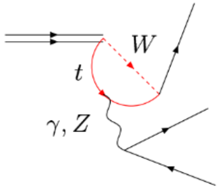
  [Buras, Buchalla, Lautenbacher '96; Chetyrkin, Misiak, Munz '98]



tree



QCD penguin



EW penguin

- ☐ **Wilson coefficients C_i: all physics above m_b; perturbatively calculable & NNLL program now complete;** [Gorbahn, Haisch '04; Misiak, Steinhauser '04]

Hadronic matrix elements

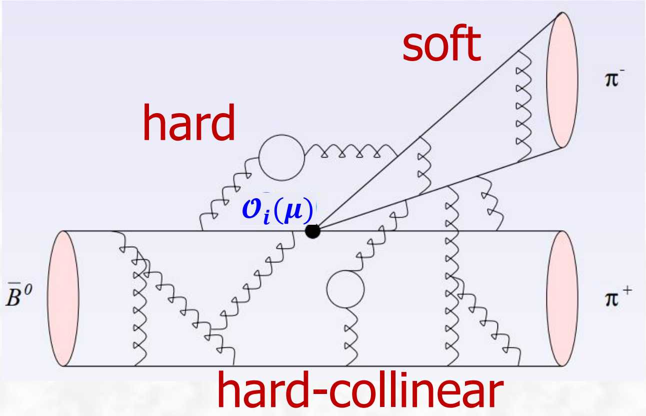
- ☐ **Decay amplitude for a given decay mode:**

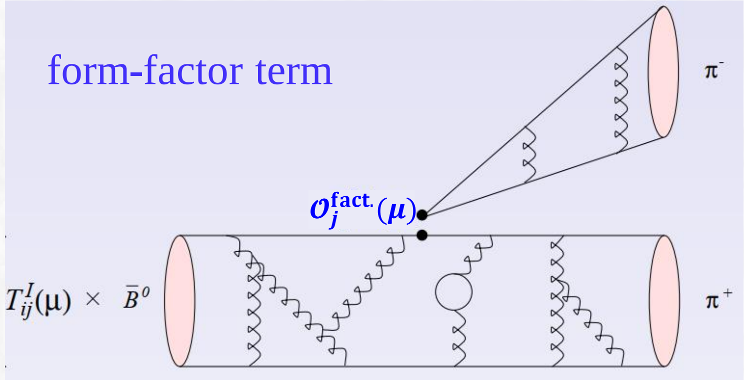
$$\boxed{\mathcal{A}(\bar{B} \to M_1 M_2) = \sum_i [\lambda_{\text{CKM}} \times C_i \times \langle M_1 M_2 | \mathcal{O}_i | \bar{B} \rangle]}$$

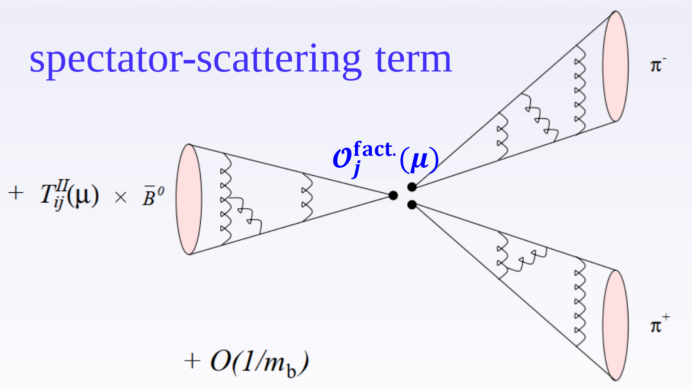
- ☐ $\langle M_1 M_2 | \mathcal{O}_i | \bar{B} \rangle$: depending on spin & parity of $M_{1,2}$; final-state rescattering introduces strong phases, and hence non-zero direct CPV; ⟶ *A quite difficult, multi-scale, strong-interaction problem!*

- ☐ **Different methods proposed for dealing with $\langle M_1 M_2 | \mathcal{O}_i | \bar{B} \rangle$:**
  - Dynamical approaches based on factorization theorems: PQCD, QCDF, SCET, · · · [Keum, Li, Sanda, Lü, Yang '00; Beneke, Buchalla, Neubert, Sachrajda, '00; Bauer, Flemming, Pirjol, Stewart, '01; Beneke, Chapovsky, Diehl, Feldmann, '02]
  - Symmetries of QCD: Isospin, U-Spin, V-Spin, and flavour SU(3) symmetries, · · · [Zeppenfeld, '81; London, Gronau, Rosner, He, Chiang, Cheng *et al.*]

- ☐ **QCDF:** systematic framework to all orders in α_s, but limited by Λ_{QCD}/m_b corrections. [BBNS '99-'03]



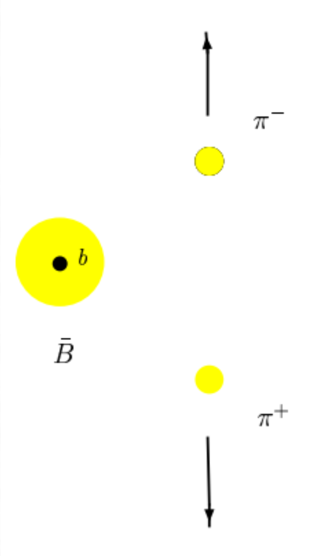


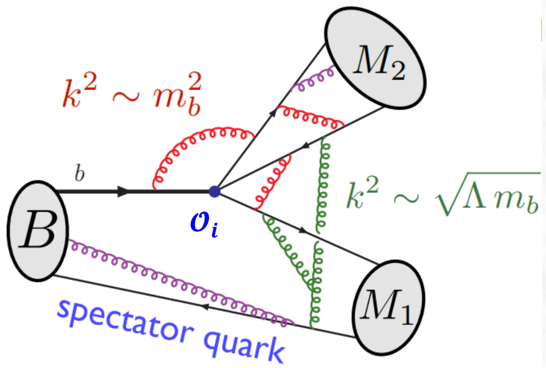


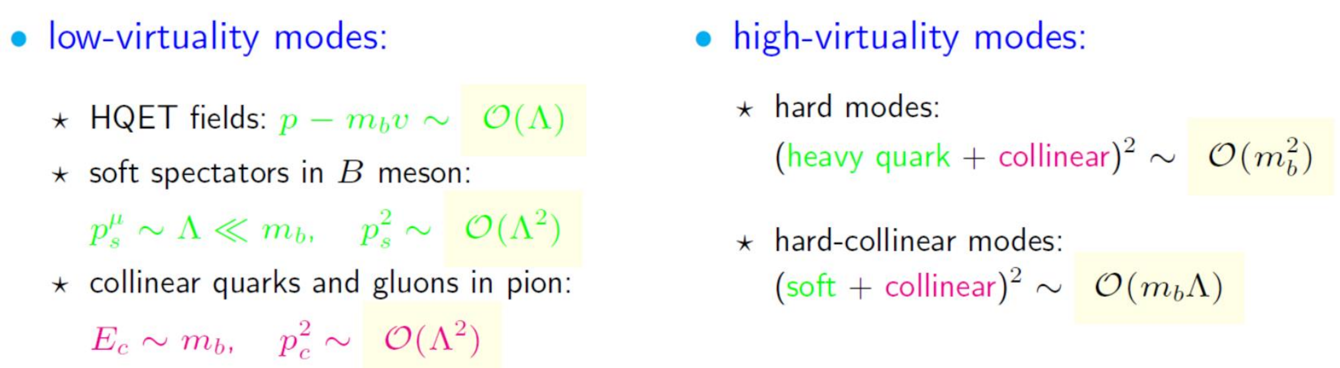
Soft-collinear factorization from SCET

- ☐ **QCDF formula:** based on diagrammatic factorization (method of regions, [Beneke, Smirnov '97] combining $1/m_b$ expansion with light-cone expansion for hard processes); [Lepage, Brodsky '80]

- ☐ **For a two-body decay:** simple kinematics, but complicated dynamics with several typical modes;







- low-virtuality modes:
  - ⋆ HQET fields: $p - m_b v \sim \mathcal{O}(\Lambda)$
  - ⋆ soft spectators in B meson:
    $p_s^\mu \sim \Lambda \ll m_b, \quad p_s^2 \sim \mathcal{O}(\Lambda^2)$
  - ⋆ collinear quarks and gluons in pion:
    $E_c \sim m_b, \quad p_c^2 \sim \mathcal{O}(\Lambda^2)$
- high-virtuality modes:
  - ⋆ hard modes:
    $(\text{heavy quark} + \text{collinear})^2 \sim \mathcal{O}(m_b^2)$
  - ⋆ hard-collinear modes:
    $(\text{soft} + \text{collinear})^2 \sim \mathcal{O}(m_b\Lambda)$

- ☐ **SCET:** a very suitable framework for studying factorization and re-summation for processes involving energetic & light particles/jets; [Bauer *et al.* '00; Beneke *et al.* '02]

- ☐ **From SCET point of view:** introduce different fields/modes for different momentum regions;

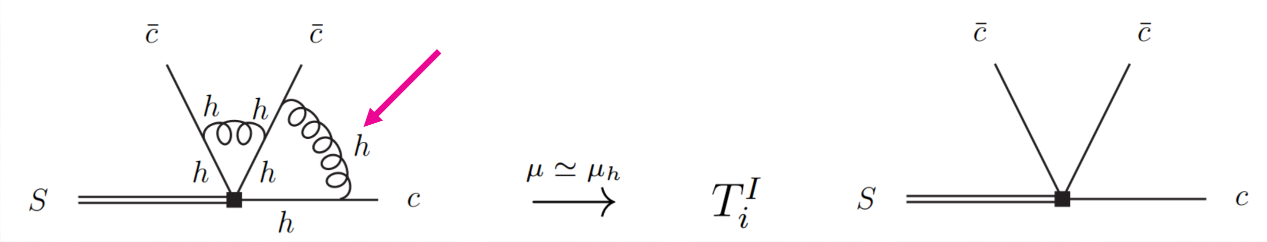
➡ achieve *soft-collinear factorization* via QFT machinery & hence QCDF formula [Beneke, 1501.07374]

Soft-collinear factorization from SCET

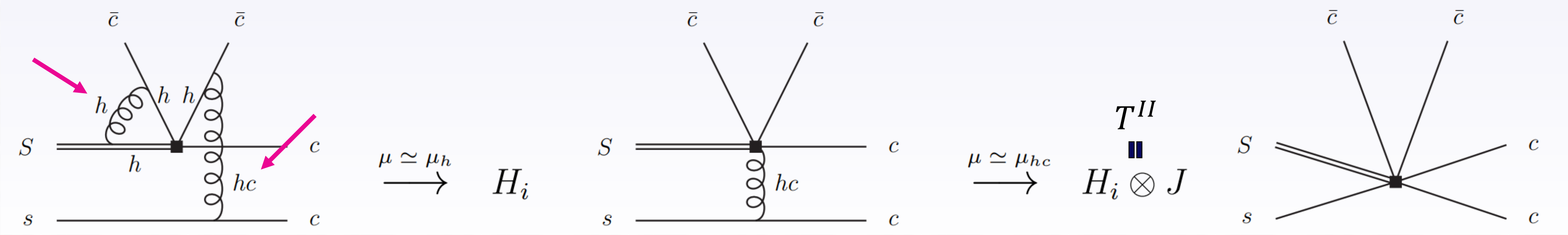
- **SCET diagrams reproduce precisely QCD diagrams in collinear & soft momentum regions;**

→ **QCD - SCET = short-distance coefficients T^I & T^{II}**

- **For hard kernel T^I:** one-step matching from QCD → $\mathrm{SCET_I(hc, c, s)}$!



- **For hard kernel T^{II}:** two-step matching from QCD → $\mathrm{SCET_I(hc, c, s)}$ → $\mathrm{SCET_{II}(c, s)}$!

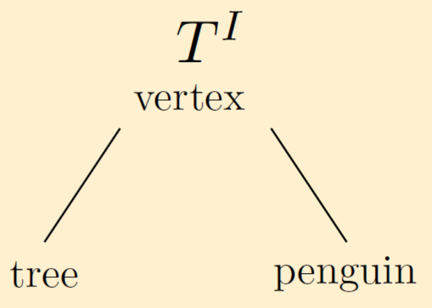


- **SCET formalism reproduces exact QCDF result, but more apparent & efficient;** [Beneke, 1501.07374]

Status of the NNLO calculation of T^I & T^{II}

- ☐ **For each $\mathcal{O}_i$ insertion, both tree & penguin topologies, and contribute to both T^I & T^{II}.**

$$
\begin{aligned}
&\langle M_1 M_2 | \mathcal{O}_i | \bar{B} \rangle \\
&\simeq F^{B\to M_1}\, T_i^I \otimes \phi_{M_2} \\
&+ T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}
\end{aligned}
$$

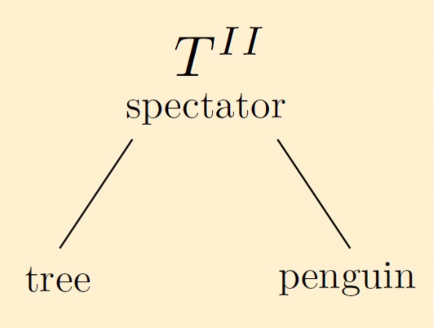


T^I

vertex

tree

penguin

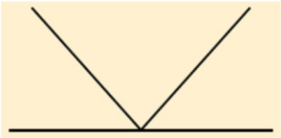
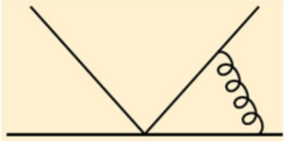
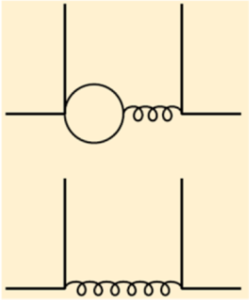
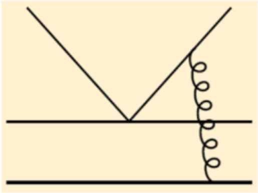
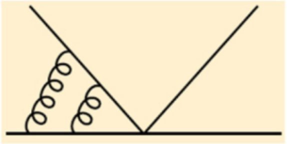
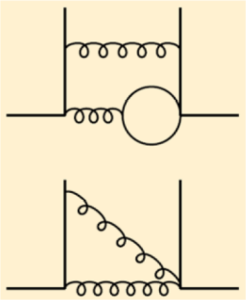
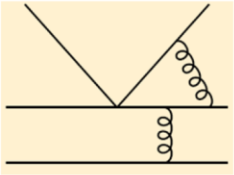
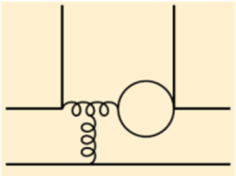


T^{II}

spectator

tree

penguin

	T_i^I, tree	T_i^I, penguin	T_i^{II}, tree	T_i^{II}, penguin
LO: $\mathcal{O}(1)$				
NLO: $\mathcal{O}(\alpha_s)$ BBNS '99-'03				
NNLO: $\mathcal{O}(\alpha_s^2)$	 Bell '07, '09 Beneke, Huber, Li '09 Huber, Krankl, Li '16	 Kim, Yoon '11 Bell, Beneke, Huber, Li '15, '20	 Beneke, Jager '05 Kivel '06, Pilipp '07	 Beneke, Jager '06 Jain, Rothstein, Stewart '07

Status of the NNLO calculation of T^I & T^{II}

- □ **Complete NNLO calculation for T^I & T^{II} at leading power in QCDF/SCET now complete;**
- □ **Soft-collinear factorization at 2-loop level established via explicit calculations;**
- □ **For tree amplitudes, cancellation between T^I & T^{II};**

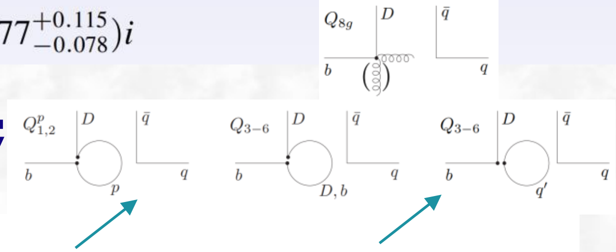
$$\langle M_1 M_2 | \mathcal{O}_i | \bar{B} \rangle \simeq F^{B\to M_1}\, T_i^I \otimes \phi_{M_2} + T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$

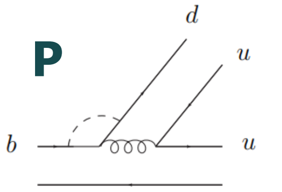
colour-allowed tree α_1 colour-suppressed tree α_2

$$\begin{aligned}\alpha_1(\pi\pi) &= 1.009 + [0.023 + 0.010\,i]_{\rm NLO} + [0.026 + 0.028\,i]_{\rm NNLO} \\ &\quad - \left[\frac{r_{\rm sp}}{0.445}\right]\left\{[0.014]_{\rm LOsp} + [0.034 + 0.027i\,]_{\rm NLOsp} + [0.008]_{\rm tw3}\right\} \\ &= 1.000^{+0.029}_{-0.069} + (0.011^{+0.023}_{-0.050})i\end{aligned}$$

$$\begin{aligned}\alpha_2(\pi\pi) &= 0.220 - [0.179 + 0.077\,i]_{\rm NLO} - [0.031 + 0.050\,i]_{\rm NNLO} \\ &\quad + \left[\frac{r_{\rm sp}}{0.445}\right]\left\{[0.114]_{\rm LOsp} + [0.049 + 0.051i\,]_{\rm NLOsp} + [0.067]_{\rm tw3}\right\} \\ &= 0.240^{+0.217}_{-0.125} + (-0.077^{+0.115}_{-0.078})i\end{aligned}$$

- □ **For QCD penguin amplitude, cancellation between $Q^p_{1,2}$ & $Q_{3-6,8g}$;**





QCD penguins α_4

$$\begin{aligned}a_4^u(\pi\bar{K})/10^{-2} &= -2.87 - [0.09 + 0.09i]_{\rm V_1} + [0.49 - 1.32i]_{\rm P_1} - [0.32 + 0.71i]_{\rm P_2,\,Q_{1,2}} + [0.33 + 0.38i]_{\rm P_2,\,Q_{3-6,8}} \\ &\quad + \left[\frac{r_{\rm sp}}{0.434}\right]\left\{[0.13]_{\rm LO} + [0.14 + 0.12i]_{\rm HV} - [0.01 - 0.05i]_{\rm HP} + [0.07]_{\rm tw3}\right\} \\ &= (-2.12^{+0.48}_{-0.29}) + (-1.56^{+0.29}_{-0.15})i\,,\end{aligned}$$

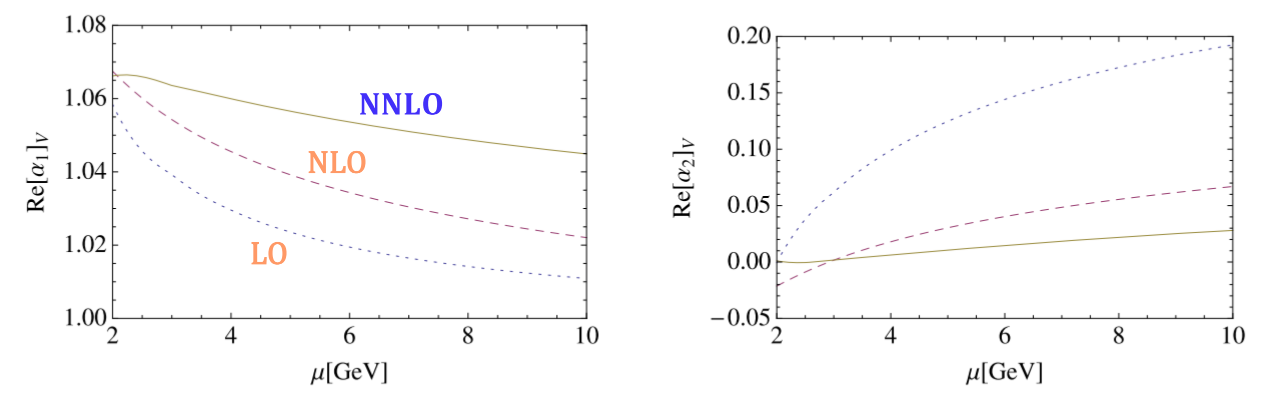
Scale dependence of $a_{1,2}$ and a_4^p

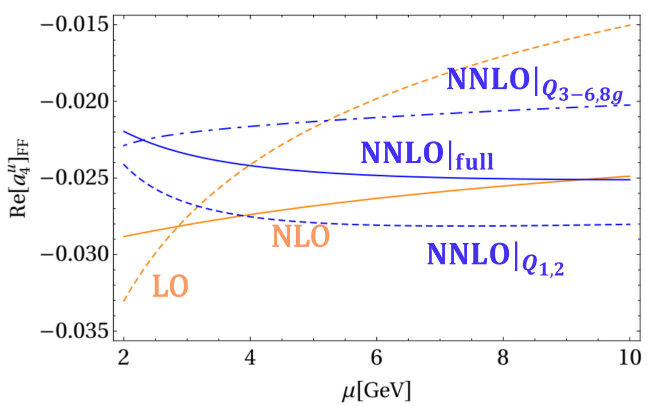
☐ **Phen. no too much changes compared to NLO predictions;**

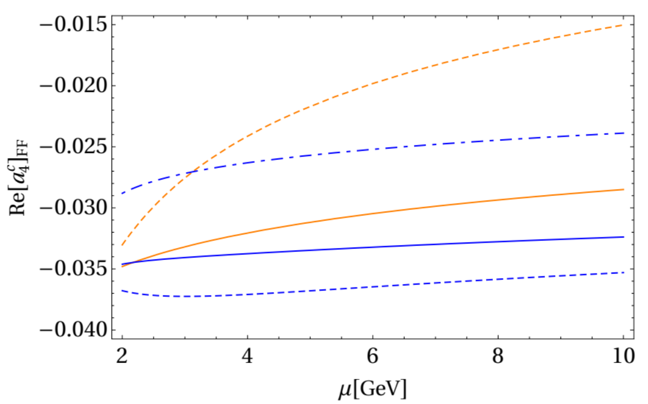
☐ **Scale dependence of $a_{1,2}$: only form-factor term;**

☐ **Scale dependence of a_4^p: only form-factor term;**

- **scale dependence negligible, especially for $\mu > 4$ GeV.**







☐ **More precise than NLO results, and hence welcome oriented at precision measurements @ LHCb & Belle II;**

Factorization also valid? New sources of strong phases?

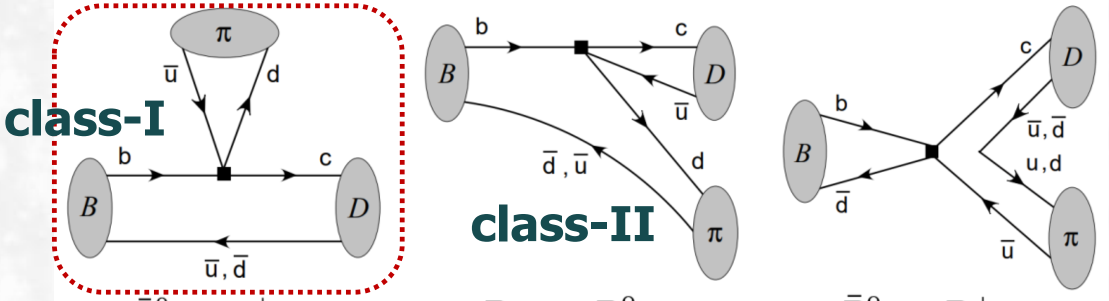
☐ **Main issue in QCDF/SCET:** sub-leading power-corrections $\sim \Lambda_{QCD}/m_b \simeq 0.2$ unknown!

$\bar{B}_q^0 \to D_q^{(*)+} L^-$ decays: class-I

☐ **At quark-level: mediated by $b \to c\bar{u}d(s)$**

all four flavors different from each other,

no penguin operators & no penguin topologies!



$\bar{B}^0 \to D^+\pi^-$ $B^- \to D^0\pi^-$

$B^- \to D^0\pi^-$ $\bar{B}^0 \to D^0\pi^0$

$\bar{B}^0 \to D^+\pi^-$ $\bar{B}^0 \to D^0\pi^0$

☐ **For class-I decays:** QCDF formula much simpler;

[Beneke, Buchalla, Neubert, Sachrajda '00; Bauer, Pirjol, Stewart '01]

$$\boxed{\begin{aligned} \mathcal{Q}_2 &= \bar{d}\gamma_\mu(1-\gamma_5)u \;\; \bar{c}\gamma^\mu(1-\gamma_5)b \\ \mathcal{Q}_1 &= \bar{d}\gamma_\mu(1-\gamma_5)T^A u \;\; \bar{c}\gamma^\mu(1-\gamma_5)T^A b \end{aligned}}$$

$$\boxed{\begin{aligned} \langle D_q^{(*)+}L^- |\, \mathcal{Q}_i \,|\bar{B}_q^0\rangle = &\sum_j F_j^{\bar{B}_q \to D_q^{(*)}}(M_L^2) \\ &\times \int_0^1 du\, T_{ij}(u)\phi_L(u) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right) \end{aligned}}$$

i) only color-allowed tree amplitude a_1;

ii) spectator & annihilation power-suppressed;

iii) annihilation absent in $\bar{B}^0_{d(s)} \to D^+_{d(s)}K(\pi)^-$;

➡ they are theoretically simpler and cleaner, and used to test factorization theorem

☐ **Hard kernel T:** both NLO and NNLO results known;

[Beneke, Buchalla, Neubert, Sachrajda '00; Huber, Kränkl, Li '16]

$$\boxed{T = T^{(0)} + \alpha_s T^{(1)} + \alpha_s^2 T^{(2)} + O(\alpha_s^3)}$$

Non-leptonic/semi-leptonic ratios

☐ Non-leptonic/semi-leptonic ratios : [Bjorken '89; Neubert, Stech '97; Beneke, Buchalla, Neubert, Sachrajda '01]

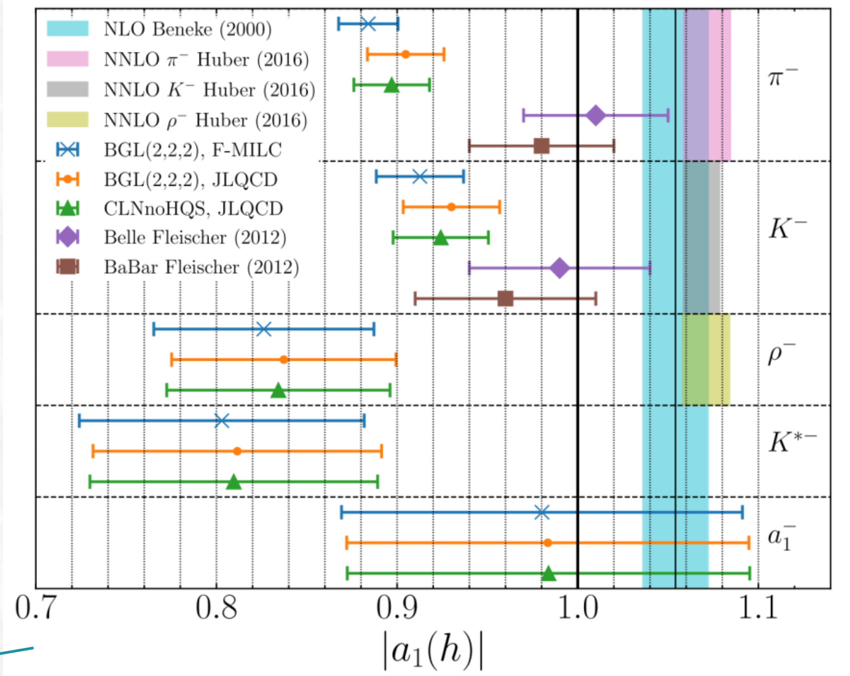
$$R^{(*)}_{(s)L} \equiv \frac{\Gamma(\bar{B}^0_{(s)} \to D^{(*)+}_{(s)} L^-)}{d\Gamma(\bar{B}^0_{(s)} \to D^{(*)+}_{(s)} \ell^- \bar{\nu}_\ell)/dq^2 \,|_{q^2=m_L^2}} = 6\pi^2 \,|V_{uq}|^2 \, f_L^2 \,|a_1(D^{(*)+}_{(s)} L^-)|^2 \, X^{(*)}_L$$

free from uncertainties from V_{cb} & $B_{d,s} \to D^{(*)}_{d,s}$ form factors.

☐ Updated predictions vs data: [Huber, Kränkl, Li '16; Cai, Deng, Li, Yang '21]

$R^{(*)}_{(s)L}$	LO	NLO	NNLO	Exp.	Deviation (σ)
R_π	1.01	$1.07^{+0.04}_{-0.04}$	$1.10^{+0.03}_{-0.03}$	0.74 ± 0.06	5.4
R^*_π	1.00	$1.06^{+0.04}_{-0.04}$	$1.10^{+0.03}_{-0.03}$	0.80 ± 0.06	4.5
R_ρ	2.77	$2.94^{+0.19}_{-0.19}$	$3.02^{+0.17}_{-0.18}$	2.23 ± 0.37	1.9
R_K	0.78	$0.83^{+0.03}_{-0.03}$	$0.85^{+0.01}_{-0.02}$	0.62 ± 0.05	4.4
R^*_K	0.72	$0.76^{+0.03}_{-0.03}$	$0.79^{+0.01}_{-0.02}$	0.60 ± 0.14	1.3
R_{K^*}	1.41	$1.50^{+0.11}_{-0.11}$	$1.53^{+0.10}_{-0.10}$	1.38 ± 0.25	0.6
$R_{s\pi}$	1.01	$1.07^{+0.04}_{-0.04}$	$1.10^{+0.03}_{-0.03}$	0.72 ± 0.08	4.4
R_{sK}	0.78	$0.83^{+0.03}_{-0.03}$	$0.85^{+0.01}_{-0.02}$	0.46 ± 0.06	6.3

☐ Latest Belle data: 2207.00134



$|a_1(\bar{B} \to D^{*+}\pi^-)| = 0.884 \pm 0.004 \pm 0.003 \pm 0.016[1.071^{+0.020}_{-0.016}]$;

15% lower than SM

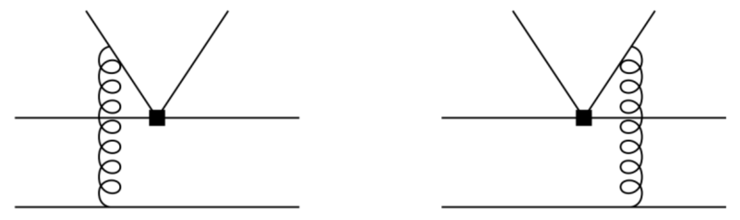
$|a_1(\bar{B} \to D^{*+}K^-)| = 0.913 \pm 0.019 \pm 0.008 \pm 0.013[1.069^{+0.020}_{-0.016}]$;

Power corrections

$$\langle D_q^{(*)+}L^-|\,\mathcal{Q}_i\,|\bar{B}_q^0\rangle = \sum_j F_j^{\bar{B}_q\to D_q^{(*)}}(M_L^2) \times \int_0^1 du\,T_{ij}(u)\phi_L(u) + \boxed{\mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)}$$

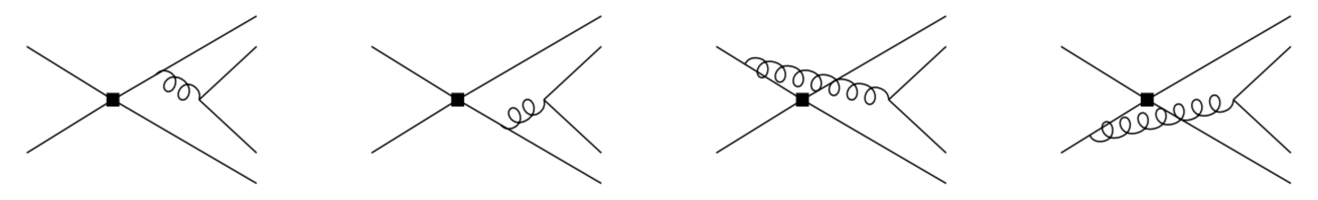
☐ **Sources of sub-leading power corrections:** [Beneke, Buchalla, Neubert, Sachrajda '01; Bordone, Gubernari, Huber, Jung, van Dyk '20]

- non-factorizable spectator interactions

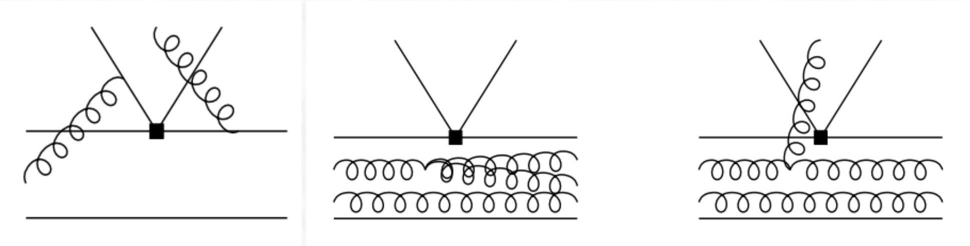


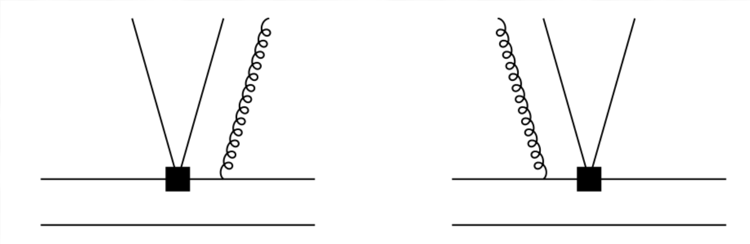
$\frac{\Lambda_{\rm QCD}}{m_b}$

- annihilation topologies



- non-leading higher Fock-state contributions





$\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^2$

☐ **Scaling of the leading-power contribution:** [BBNS '01]

$$\mathcal{A}(\bar{B}_d\to D^+\pi^-) \sim G_F m_b^2\, F^{B\to D}(0)\, f_\pi \sim G_F m_b^2\, \Lambda_{\rm QCD}$$

- All these ESTIMATED to be power-suppressed; not even chirality-enhanced due to (V-A)(V-A)
- Why all measured values of $|a_1(h)|$ several σ smaller than SM?
- *Must consider possible sub-leading power corrections carefully!*

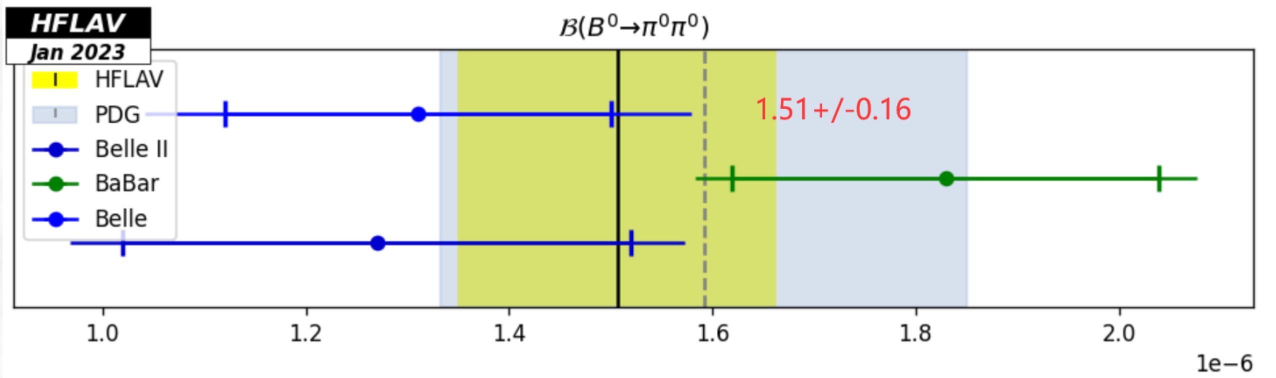
Charmless two-body hadronic B decays

- ☐ **Long-standing puzzles in $\mathrm{Br}(\bar B^0 \to \pi^0\pi^0)$ and $\Delta A_{CP}(\pi K) = A_{CP}(\pi^0 K^-) - A_{CP}(\pi^+ K^-)$:** [HFLAV '23]

$$\mathrm{Br}(B^0 \to \pi^0\pi^0) = (0.3 - 0.9) \times 10^{-6}$$

$$\Delta A_{CP}(\pi K) = (11.5 \pm 1.4)\%$$

differs from 0 by ~8σ



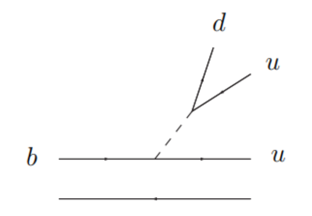
- ☐ **Decay amplitudes in QCDF:**

$$-\mathcal{A}_{\bar B^0\to\pi^0\pi^0} = A_{\pi\pi}\left[\delta_{pu}(\alpha_2-\beta_1)-\hat\alpha_4^p-2\beta_4^p\right]$$

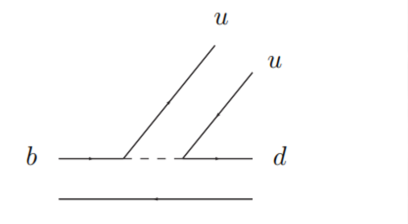
$$\sqrt{2}\,\mathcal{A}_{B^-\to\pi^0K^-} = A_{\pi\bar K}\left[\delta_{pu}\,\alpha_1+\hat\alpha_4^p\right]+A_{\bar K\pi}\left[\delta_{pu}\alpha_2+\delta_{pc}\tfrac{3}{2}\alpha^c_{3,\mathrm{EW}}\right],$$
$$\mathcal{A}_{\bar B^0\to\pi^+K^-} = A_{\pi\bar K}\left[\delta_{pu}\,\alpha_1+\hat\alpha_4^p\right],$$

- ☐ **Dominant topologies:** **LP NNLO known**

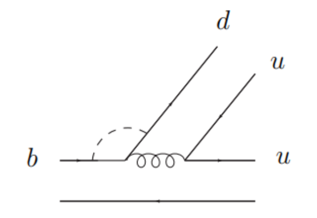
$$A_{\mathrm{CP}}(\pi^0K^\pm)-A_{\mathrm{CP}}(\pi^\mp K^\pm) = -2\sin\gamma\left(\mathrm{Im}(r_C)-\mathrm{Im}(r_T\,r_{\mathrm{EW}})\right)+\dots$$



colour-allowed tree α_1



colour-suppressed tree α_2



QCD penguins α_4

α_2 always plays a key role here!

- ☐ **Find some mechanism to enhance α_2, may we explain both puzzles!**

➢ **necessary & urgent to consider sub-leading power corrections!**

Power-suppressed colour-octet contribution

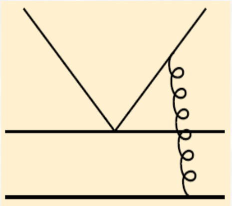
- ☐ **Sub-leading power corrections to a_2: spectator scattering or final-state interactions**

- ☐ **Every four-quark operator in H_{eff} has a colour-octet piece in QCD:**

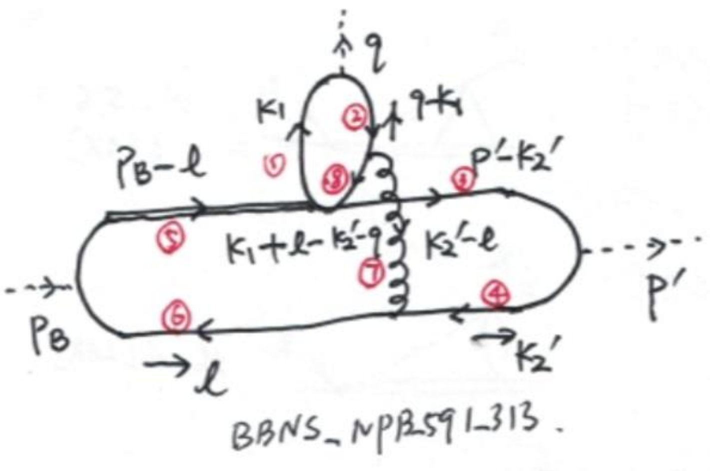
$$\boxed{t^a_{ik} t^a_{jl} = \frac{1}{2}\delta_{il}\delta_{jk} - \frac{1}{2N_c}\delta_{ik}\delta_{jl},}$$

$$Q_1 = (\bar{u}_i b_i)_{V-A} \otimes (\bar{s}_j u_j)_{V-A} = \frac{1}{N_c}(\bar{s}_i b_i)_{V-A} \otimes (\bar{u}_j u_j)_{V-A} + 2(\bar{s}T^A b)_{V-A} \otimes (\bar{u}T^A u)_{V-A}$$

$$Q_2 = (\bar{u}_i b_j)_{V-A} \otimes (\bar{s}_j u_i)_{V-A} = \frac{1}{N_c}(\bar{u}_i b_i)_{V-A} \otimes (\bar{s}_j u_j)_{V-A} + 2(\bar{u}T^A b)_{V-A} \otimes (\bar{s}T^A u)_{V-A}$$



- ☐ **Three-loop correlators with colour-octet operator insertion:**



- ➢ The gluon propagator can be in the hard-collinear region;
  - ➡ hard-spectator scattering contribution;
- ➢ Can also be in the soft region; expected to be $\mathcal{O}(1/m_b)$;
  - ➡ can be non-zero at sub-leading power;
- ➢ Other four regions suppressed by more powers of $1/m_b$;

Soft-exchange effects from emission topology

☐ **Real realization of the mechanism requires study of the three-loop correlators;** [w.i.p.]

☐ **Matching from QCD to $\mathrm{SCET_I}$:**

$$Q_1 \to H_1(u) \otimes [\bar{u}_c h_v]_{\Gamma_1} [\bar{s}_{\bar{c}} u_{\bar{c}}]_{\Gamma_2}(u) + H_2(u) \otimes \frac{1}{N_c} [\bar{s}_c h_v]_{\tilde{\Gamma}_1} [\bar{u}_{\bar{c}} u_{\bar{c}}]_{\tilde{\Gamma}_2}(u) + H_3(u) \otimes 2 \underline{\left[\bar{s}_c T^A h_v\right]_{\tilde{\Gamma}_1} \left[\bar{u}_{\bar{c}} T^A u_{\bar{c}}\right]_{\tilde{\Gamma}_2}(u)}$$

$$\begin{aligned} Q_2 &= [\bar{u}_i b_j]_{\Gamma_1} [\bar{s}_j u_i]_{\Gamma_2} = [\bar{s} b]_{\tilde{\Gamma}_1} [\bar{u} u]_{\tilde{\Gamma}_2} \\ &\to H_1(u) \otimes [\bar{s}_c h_v]_{\tilde{\Gamma}_1} [\bar{u}_{\bar{c}} u_{\bar{c}}]_{\tilde{\Gamma}_2}(u) + H_2(u) \otimes \frac{1}{N_c} [\bar{u}_c h_v]_{\Gamma_1} [\bar{s}_{\bar{c}} u_{\bar{c}}]_{\Gamma_2}(u) \\ &\quad + H_3(u) \otimes 2 \underline{\left[\bar{u}_c T^A h_v\right]_{\Gamma_1} \left[\bar{s}_{\bar{c}} T^A u_{\bar{c}}\right]_{\Gamma_2}}(u), \end{aligned}$$

colour-octet $\mathrm{SCET_I}$ operators

- $H_i(u)$: hard matching coefficients; at tree-level, $H_i(u) = 1$;

☐ **How to implement $\langle M_1 M_2 \left| [\bar{u}_c T^A h_v]_{\Gamma_1} [\bar{s}_{\bar{c}} T^A u_{\bar{c}}]_{\Gamma_2} \right| \bar{B} \rangle$: function of u, depending on $M_{1,2}$ & $\bar{B}$**

- For colour-singlet $\mathrm{SCET_I}$ operators:

$$\langle M_1 M_2 | [\bar{u}_c h_v]_{\Gamma_1} [\bar{s}_{\bar{c}} u_{\bar{c}}]_{\Gamma_2}(u) | \bar{B} \rangle = c\, \hat{A}_{M_1 M_2} \phi_{M_2}(u), \text{ with } \hat{A}_{M_1 M_2} = i\, m_B^2 F^{B \to M_1}(0) f_{M_2}$$

- For colour-octet $\mathrm{SCET_I}$ operators: normalized to the naïve factorizable amplitude

$$\langle M_1 M_2 | [\bar{u}_c T^A h_v]_{\Gamma_1} [\bar{s}_{\bar{c}} T^A u_{\bar{c}}]_{\Gamma_2}(u) | \bar{B} \rangle = \hat{A}_{M_1 M_2} \mathfrak{F}_{M_2}^{B M_1}(u), \text{ with } \mathfrak{F}_{M_2}^{B M_1}(u) \text{ an arbitrary function}$$

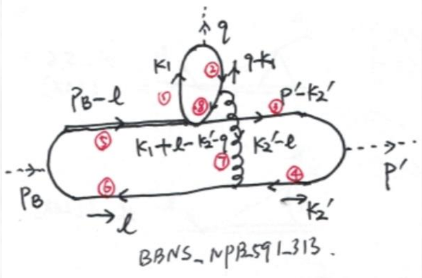
Soft-exchange effects from emission topology

☐ **To have predictive power, make the following two approximations:**

- Working to lowest order in the hard QCD → SCET$_{\text{I}}$ matching, then $H_i(u) = 1$

$$\Rightarrow \mathfrak{F}_{M_2}^{BM_1} = \int_0^1 du\, \mathfrak{F}_{M_2}^{BM_1}(u)$$

- When gluon propagator is soft, the propagator 8 is anti-hard-collinear;



⇒ The SCET$_{\text{I}}$ operator naively factorizes after matching to SCET$_{\text{II}}$:

$$\mathfrak{F}_{M_2}^{BM_1}(u) = \frac{1}{\hat{A}_{M_1M_2}} \frac{f_{M_2}\phi_{M_2}(u)}{8N_c u\bar{u}} \times (-1)\int_0^\infty ds \left\langle M_1 \middle| \left[\bar{u}_c T^A h_v\right]_{\Gamma_1} \epsilon_{\mu\nu\alpha\beta} n_+^\nu g_s G^{A,\alpha\beta}(-sn_+) \middle| \bar{B} \right\rangle$$

$$= \frac{1}{\hat{A}_{M_1M_2}} \frac{f_{M_2}\phi_{M_2}(u)}{8N_c u\bar{u}} \times (-i) F^{B\to M_1}(0) g_{\Gamma_1}^{BM_1} = \frac{\phi_{M_2}(u)}{8N_c u\bar{u}} g_{\Gamma_1}^{BM_1}$$

Indep. of M_2

- With the asymptotic $\phi_{M_2}(u) = 6u\bar{u}$, we have:

$$\boxed{\mathfrak{F}_{M_2}^{BM_1} = \int_0^1 du\, \mathfrak{F}_{M_2}^{BM_1}(u) = \frac{1}{4} g_{\Gamma_1}^{BM_1}}$$

Soft-exchange effects from emission topology

- ☐ **The usual colour-allowed & colour-suppressed tree amplitudes now changed to:**

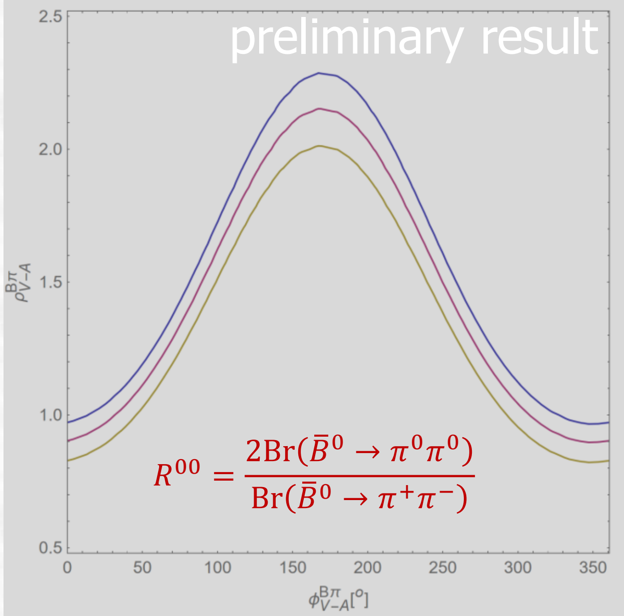
$$\alpha_1(M_1M_2) = C_1 + C_2\left[\frac{1}{N_c} + \frac{g_{V-A}^{BM_1}}{2}\right]$$

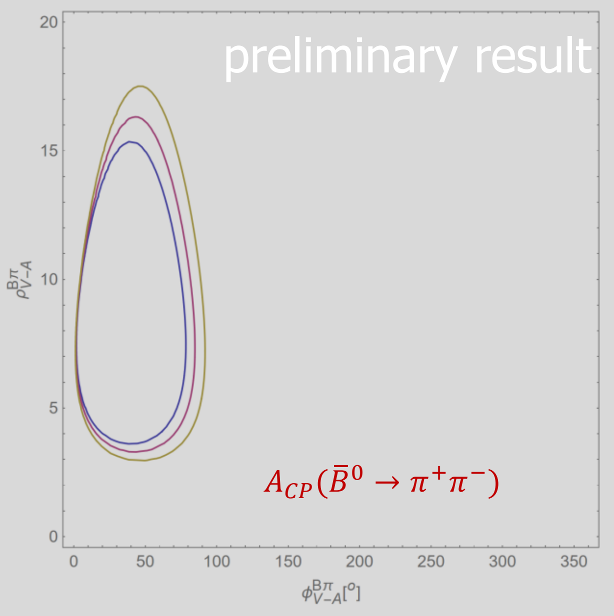
$$\alpha_2(M_1M_2) = C_2 + C_1\left[\frac{1}{N_c} + \frac{g_{V-A}^{BM_1}}{2}\right]$$

$g_{V-A}^{BM_1}$ can be complex in general ! $\qquad g_{V-A}^{BM_1} = \rho_{V-A}^{BM_1} e^{i\phi_{V-A}^{BM_1}}$

- ☐ **Taking $g_{V-A}^{BM_1}$ as free parameter, we can at least fit it from the current data;**

[Cheng, Chu '09; Lu, Yang '22; Wang Yang '22]





- ➢ With only soft-exchange effect from emission topology, it is impossible to explain both Br and ACP data;
- ➢ We need to take into account other power-suppressed contributions!

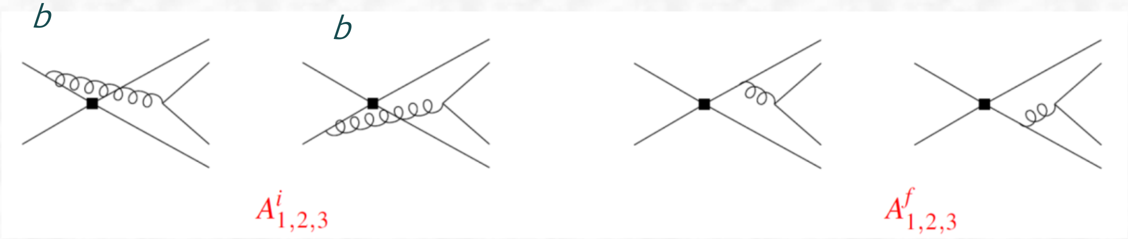
Pure annihilation B decays

- ☐ **Two typical pure annihilation decay modes:** $\bar{B}_s^0 \to \pi^+\pi^-$ vs $\bar{B}^0 \to K^+K^-$

$$\mathcal{A}(\bar{B}_s \to \pi^+\pi^-) = B_{\pi\pi}\left[\delta_{pu} b_1 + 2b_4^p + \frac{1}{2} b^p_{4,\mathrm{EW}}\right]$$

$$\mathcal{A}(\bar{B}_d \to K^+K^-) = A_{\bar{K}K}\left[\delta_{pu}\beta_1 + \beta_4^p + b^p_{4,\mathrm{EW}}\right] + B_{K\bar{K}}\left[b_4^p - \frac{1}{2} b^p_{4,\mathrm{EW}}\right]$$

$$= A_{\bar{K}K}\left[\delta_{pu}\beta_1 + \beta_4^p\right] + B_{K\bar{K}}\left[b_4^p\right]$$



- ☐ **Both involve the building blocks $b_1 = \frac{C_F}{N_c^2} C_1 A_1^i$ & $b_4^p = \frac{C_F}{N_c^2}[C_4 A_1^i + C_6 A_2^i]$:**

A_1^i: $(\mathrm{V}-\mathrm{A}) \otimes (V-A)$

A_2^i: $(\mathrm{V}-\mathrm{A}) \otimes (V+A)$

$$A_1^i(M_1M_2) = \pi\alpha_s \int_0^1 dxdy \left\{\Phi_{M_2}(x)\,\Phi_{M_1}(y)\left[\frac{1}{y(1-x\bar{y})} + \frac{1}{\bar{x}^2 y}\right] + r_\chi^{M_1} r_\chi^{M_2}\,\Phi_{m_2}(x)\,\Phi_{m_1}(y)\,\frac{2}{\bar{x}y}\right\},$$

$$A_2^i(M_1M_2) = \pi\alpha_s \int_0^1 dxdy \left\{\Phi_{M_2}(x)\,\Phi_{M_1}(y)\left[\frac{1}{\bar{x}(1-x\bar{y})} + \frac{1}{\bar{x}y^2}\right] + r_\chi^{M_1} r_\chi^{M_2}\,\Phi_{m_2}(x)\,\Phi_{m_1}(y)\,\frac{2}{\bar{x}y}\right\},$$

- ☐ **With the asymptotic LCDAs, we have $A_1^i = A_2^i$:** [BBNS '99-'03]

$$A_1^i(M_1M_2) = \pi\alpha_s\left\{18X_A - 18 - 6(9-\pi^2) + r_\chi^{M_1} r_\chi^{M_2}\left(2X_A^2\right)\right\},$$

$$A_2^i(M_1M_2) = \pi\alpha_s\left\{18X_A - 18 - 6(9-\pi^2) + r_\chi^{M_1} r_\chi^{M_2}\left(2X_A^2\right)\right\},$$

$$X_A = \left(1+\varrho_A e^{i\varphi_A}\right)\ln\left(m_B/\Lambda_h\right),$$

$\Lambda_h = 0.5\mathrm{GeV}$, $\varrho_A \le 1$ and an arbitrary phase φ_A

Ways to improve the modelling of annihilations

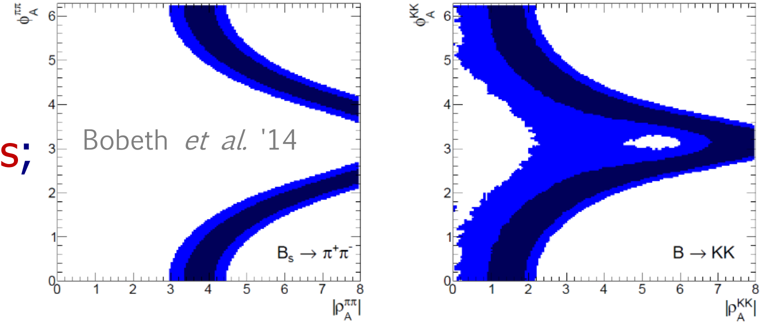
☐ With universal X_A and different scenarios, we have:

[BBNS '03]

Mode	Theory	S1 (large γ)	S2 (large a_2)	S3 ($\varphi_A = -45^{\circ}$)	S4 ($\varphi_A = -55^{\circ}$)	Exp.
$\bar{B}_s^0 \to \pi^+\pi^-$	$0.024^{+0.003+0.025+0.000+0.163}_{-0.003-0.012-0.000-0.021}$	0.027	0.032	0.149	0.155	0.671 ± 0.083
$\bar{B}^0 \to K^-K^+$	$0.013^{+0.005+0.008+0.000+0.087}_{-0.005-0.005-0.000-0.011}$	0.007	0.014	0.079	0.070	0.0803 ± 0.0147

☐ Large SU(3) flavor symmetry breaking or flavor-dependent $A_{1,2}^{i}$?

[Wang, Zhu '03; Bobeth *et al.* '14; Chang, Sun *et al.* '14-15]



Bobeth *et al.* '14

FIGURE 5.8: 68% and 95% CRs for the complex parameter $\rho_A^{\pi^+\pi^-}$ and $\rho_A^{K^+K^-}$ obtained from a branching-ratio fit assuming the SM.

☐ How to improve the situation:

- Including higher Gegenbauer moments to include SU(3)-breaking effects;

$$\Phi_M(x,\mu) = 6x\bar{x}\left[1 + \sum_{n=1}^{\infty} a_n^M(\mu)\, C_n^{(3/2)}(2x-1)\right]$$

due to G-parity, $a_{odd}^{\pi} = 0$, but $a_{odd}^{K} \neq 0$

- Including the difference between the chirality factors to include SU(3)-breaking effects;

$$r_\chi^\pi(1.5\text{GeV}) = \frac{2m_\pi^2}{m_b(\mu)\big(m_u(\mu) + m_d(\mu)\big)} \simeq 0.86, \qquad r_\chi^K(1.5\text{GeV}) = \frac{2m_K^2}{m_b(\mu)\big(m_u(\mu) + m_s(\mu)\big)} \simeq 0.91$$

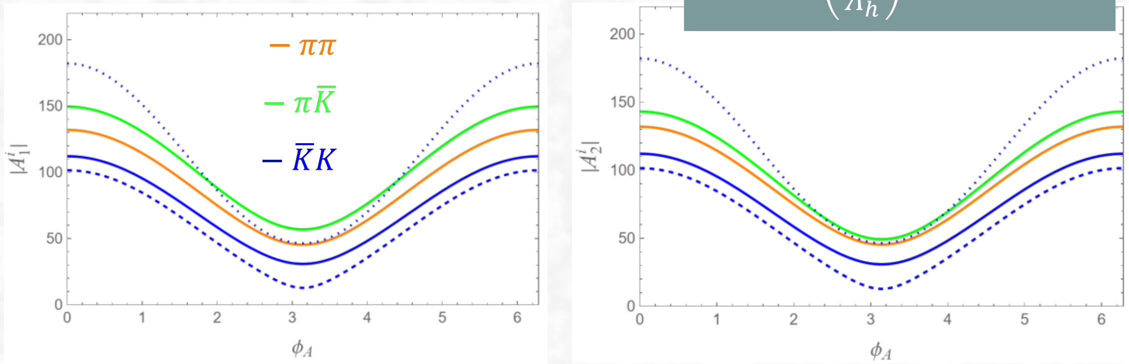
Ways to improve the modelling of annihilations

□ SU(3)-breaking effects in $A_{1,2}^i$: due to higher Gengengauber moments and quark masses

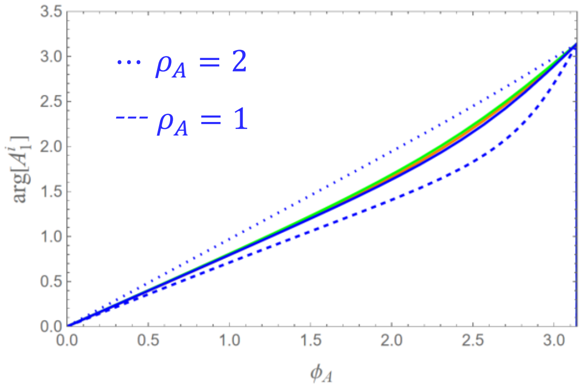
$$A_1^i(M_1M_2) = \pi\alpha_s \Big\{ 18(1 - a_1^{M_1} + a_2^{M_1})\Big[(1 + 3a_1^{M_2} + 6a_2^{M_2})X_A - (1 + 6a_1^{M_2} + 16a_2^{M_2})\Big] \quad +54(69 - 7\pi^2)a_1^{M_1}a_1^{M_2} - 36(385 - 39\pi^2)(a_1^{M_1}a_2^{M_2} - 2a_2^{M_1}a_1^{M_2})$$
$$-6(9 - \pi^2) - 18(10 - \pi^2)(3a_1^{M_1} - a_1^{M_2}) - 6(59 - 6\pi^2)(6a_2^{M_1} + a_2^{M_2}) \quad -18(9593 - 972\pi^2)a_2^{M_1}a_2^{M_2} + r_\chi^{M_1}r_\chi^{M_2}\left(2X_A^2\right)\Big\},$$

$$X_A = \ln\left(\frac{m_B}{\Lambda_h}\right)(1 + \rho_A e^{i\phi_A})$$

$$A_2^i(M_1M_2) = \pi\alpha_s \Big\{ 18(1 + a_1^{M_2} + a_2^{M_2})\Big[(1 - 3a_1^{M_1} + 6a_2^{M_1})X_A - (1 - 6a_1^{M_1} + 16a_2^{M_1})\Big]$$
$$-6(9 - \pi^2) - 18(10 - \pi^2)(a_1^{M_1} - 3a_1^{M_2}) - 6(59 - 6\pi^2)(a_2^{M_1} + 6a_2^{M_2})$$
$$+54(69 - 7\pi^2)a_1^{M_1}a_1^{M_2} - 36(385 - 39\pi^2)(2a_1^{M_1}a_2^{M_2} - a_2^{M_1}a_1^{M_2})$$
$$-18(9593 - 972\pi^2)a_2^{M_1}a_2^{M_2} + r_\chi^{M_1}r_\chi^{M_2}\left(2X_A^2\right)\Big\},$$



	$\pi\pi$	$\pi\bar{K}$	$\bar{K}K$
A_1^i	$31.7X_A - 51.5+6.2 + 1.5X_A^2$ $[18X_A - 18+5.2 + 1.5X_A^2]$	$37.6X_A - 63.4+6.5 + 1.6X_A^2$ $[18X_A - 18+5.2 + 1.6X_A^2]$	$23.4X_A - 36.0+5.2 + 1.7X_A^2$ $[18X_A - 18+5.2 + 1.7X_A^2]$
A_2^i	$31.7X_A - 51.5+6.2 + 1.5X_A^2$ $[18X_A - 18+5.2 + 1.5X_A^2]$	$34.6X_A - 56.2+6.9 + 1.6X_A^2$ $[18X_A - 18+5.2 + 1.6X_A^2]$	$23.4X_A - 36.0+5.2 + 1.7X_A^2$ $[18X_A - 18+5.2 + 1.7X_A^2]$



- ➢ $|A_{1,2}^i|$ can differ by more than **20%** in the **BBNS+ model**!
- ➢ The amplitude ratios $A_{1,2}^i(\pi\pi)/A_{1,2}^i(KK)$ get **enhanced** in the **BBNS+ model**! ➡ what we need!

Ways to improve the modelling of annihilations

☐ How to improve:

- Making the parameter X_A to be flavour dependent & depending on its origins;

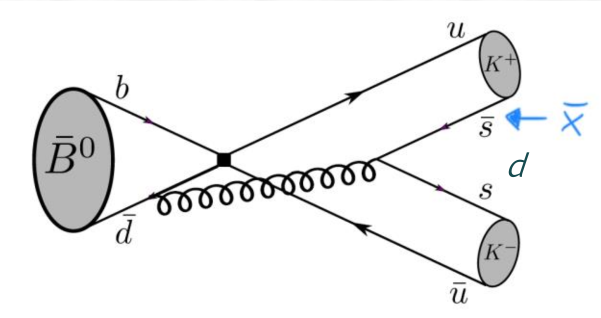
$$
\begin{aligned}
\int_0^1 dy\,\frac{\Phi_{M_1}(y)}{y^2} &= \Phi'_{M_1}(0)\int_0^1 dy\,\frac{1}{y} + \int_0^1 dy\,\frac{\Phi_{M_1}(y) - y\,\Phi'_{M_1}(0)}{y^2} &&\longrightarrow && 6X_0^{M_1} - 6\,,\\
\int_0^1 dx\,\frac{\Phi_{M_2}(x)}{\bar{x}^2} &= \Phi'_{M_2}(1)\int_0^1 dx\,\frac{1}{\bar{x}} + \int_0^1 dx\,\frac{\Phi_{M_2}(x) - \bar{x}\,\Phi'_{M_2}(1)}{\bar{x}^2} &&\longrightarrow && 6X_1^{M_2} - 6\,,\\
\int_0^1 dy\,\frac{\Phi_{m_1}(y)}{y} &= \Phi_{m_1}(0)\int_0^1 dy\,\frac{1}{y} + \int_0^1 dy\,\frac{\Phi_{m_1}(y) - \Phi_{m_1}(0)}{y} &&\longrightarrow && X_0^{m_1}\,,\\
\int_0^1 dx\,\frac{\Phi_{m_2}(x)}{\bar{x}} &= \Phi_{m_2}(1)\int_0^1 dx\,\frac{1}{\bar{x}} + \int_0^1 dx\,\frac{\Phi_{m_2}(x) - \Phi_{m_2}(1)}{\bar{x}} &&\longrightarrow && X_1^{m_2}\,,
\end{aligned}
$$

$$
\Rightarrow
$$

$$
\begin{aligned}
A_1^i(M_1M_2) &= \pi\alpha_s\left\{18X_1^{M_2} - 18 - 6(9-\pi^2) + r_\chi^{M_1} r_\chi^{M_2}\left(2X_0^{m_1}X_1^{m_2}\right)\right\},\\
A_2^i(M_1M_2) &= \pi\alpha_s\left\{18X_0^{M_1} - 18 - 6(9-\pi^2) + r_\chi^{M_1} r_\chi^{M_2}\left(2X_0^{m_1}X_1^{m_2}\right)\right\},
\end{aligned}
$$

$$
\Rightarrow \boxed{A_1^i(M_1M_2) \neq A_2^i(M_1M_2)}
$$

- To make it predictive, distinguish whether the endpoint configuration mediated by a soft strange quark (X_A^s) or a soft up or down quark (X_A^{ud}) .



☐ Advantages compared to original BBNS: two free parameters!

- **For $\pi\pi$ final states, only X_A^{ud} involved;**

easily to reproduce the data!

- **For KK final states, both X_A^{ud} (for $M_1M_2 = K^+K^-$) and X_A^s (for $M_1M_2 = K^-K^+$) involved;**

☐ Other interesting progress:

Lu, Shen, Wang, Wang, Wang 2202.08073; Boer talk @ SCET2023;

Neubert talk @ Neutrinos, Flavour and Beyond 2022

Summary

- ☐ **With dedicated LHCb & Belle II, precision era for B physics expected!**

- ☐ **NNLO calculation at LP in QCDF/SCET complete; but some puzzles still remain:**

  - ➢ **for class-I $B_q^0 \to D_q^{(*)-} L^+$ decays, $\mathcal{O}(4\text{–}5\sigma)$ discrepancies observed;**

  - ➢ **long-standing $\mathrm{Br}(\bar{B}^0 \to \pi^0\pi^0)$ and $\Delta A_{CP}(\pi K) = A_{CP}(\pi^0 K^-) - A_{CP}(\pi^+ K^-)$;**

  ➡ **sub-leading power corrections in QCDF/SCET need to be considered!**

- ☐ **Power-suppressed colour-octet matrix elements:**

  $\langle M_1 M_2 | [\bar{u}_c T^A h_v]_{\Gamma_1} [\bar{s}_{\bar{c}} T^A u_{\bar{c}}]_{\Gamma_2}(u) | \bar{B} \rangle = \hat{A}_{M_1 M_2} \mathfrak{F}_{M_2}^{BM_1}(u)$, with $\mathfrak{F}_{M_2}^{BM_1}(u)$ an arbitrary function

- ☐ **Improved treatments of annihilation amplitudes:** SU(3)-breaking effects & flavor-dependence of the building blocks $A_{1,2}^i$; ➡ **correct direction as expected!**

Thank You for your attention!