

Subleading power corrections to double radiative B decays in SCET

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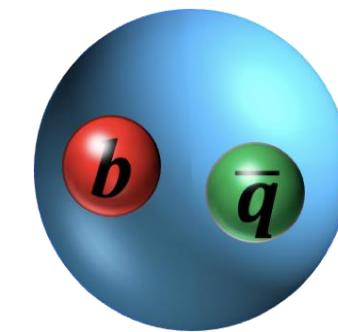
Based on

- YLS, Yu-Ming Wang and Yan-Bing Wei, JHEP12(2020) 169
- Dong-Hao Li, Lei-Yi Li, Cai-Dian Lu, YLS, PRD106, 094038(2022)
- Qin Qin, YLS, Chao Wang, Yu-Ming Wang, 2207.02691

第五届重味物理和量子色动力学研讨会@武汉东湖

What can one benefit from Heavy Quark Physics?

- ✓ Test of Standard Model (CPV)
- ✓ Search for “New Physics”
- ✓ Understanding QCD
- ✓ ...



High precision era of Heavy Quark Physics

- Experiment: LHCb+Belle-II+BES+*CEPC+STCF+...*
- ✓ Factorization+Multi-Loop calculation
- ✓ Power correction
- ✓ Lattice simulation

Role of B meson LCDA

- * Radiative leptonic/double radiative decays $\mathcal{A} \sim C(\mu)J(\omega, \mu) * \phi_B(\omega, \mu) + \mathcal{O}(1/m_b)$
(Lunghi et al, 2002, Bosch et al, 2003)

- * QCDF for nonleptonic decays (BBNS 1999)

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle = F^{BM_1} T_i^I \star \Phi_{M_2} + \Phi_B \star [H_i^{II} \star J^{II}] \star \Phi_{M_1} \star \Phi_{M_2} + \mathcal{O}(1/m_b)$$

- * PQCD approach (Li et al, Lu et al 2000): $\mathcal{M} \propto \int_0^1 dx_1 dx_3 \int d^2 \vec{b}_1 d^2 \vec{b}_3 \phi_B(x_1, \vec{b}_1, p_1, t)$
 $\times T_H(x_1, x_3, \vec{b}_1, \vec{b}_3, t) \phi_{M_3}(x_3, \vec{b}_3, p_3, t)$
 $\times S_t(x_3) \exp[-S_B(t) - S_3(t)].$

- * Radiative decays (Becher et al 2005)

$$\langle V\gamma | \mathcal{H}_{\text{eff}} | \bar{B} \rangle = 2m_B \left[C^A \zeta_{V\perp} + \frac{\sqrt{m_B} F(\mu) f_{V\perp}}{4} (C^B \otimes J_\perp) \otimes \phi_\perp^V \otimes \phi_+^B \right] + \mathcal{O}(1/m_b)$$

- * LCSR with B meson LCDA (Khodjamirian et al 2005)

How to determine B meson LCDA

- Constraint from evolution behavior
- Phenomenological model

Free parton model from WW approximation

Exponential/local duality model from QCD sum rules

Lattice simulation (benefit from LaMET)

- Fitted from experimental data:

radiative leptonic/double radiative decays.....

Double radiative decays in Belle-II

Observables	Belle 0.71 ab^{-1} (0.12 ab^{-1})	Belle II 5 ab^{-1}	Belle II 50 ab^{-1}
$\text{Br}(B_d \rightarrow \gamma\gamma)$	< 740%	30%	9.6%
$A_{CP}(B_d \rightarrow \gamma\gamma)$	—	78%	25%
$\text{Br}(B_s \rightarrow \gamma\gamma)$	< 250%	23%	—

(Belle Physics Book 2018)

Kinematics of double radiative decays

- * Parameterization of the amplitudes

$$\bar{\mathcal{A}}(\bar{B}_q \rightarrow \gamma\gamma) = -\frac{4G_F}{\sqrt{2}} \frac{\alpha_{\text{em}}}{4\pi} \epsilon_1^{*\alpha}(p) \epsilon_2^{*\beta}(q) \sum_{p=u,c} V_{pb} V_{pq}^* \sum_{i=1}^8 C_i T_{i,\alpha\beta}^{(p)}$$

- * The helicity amplitudes

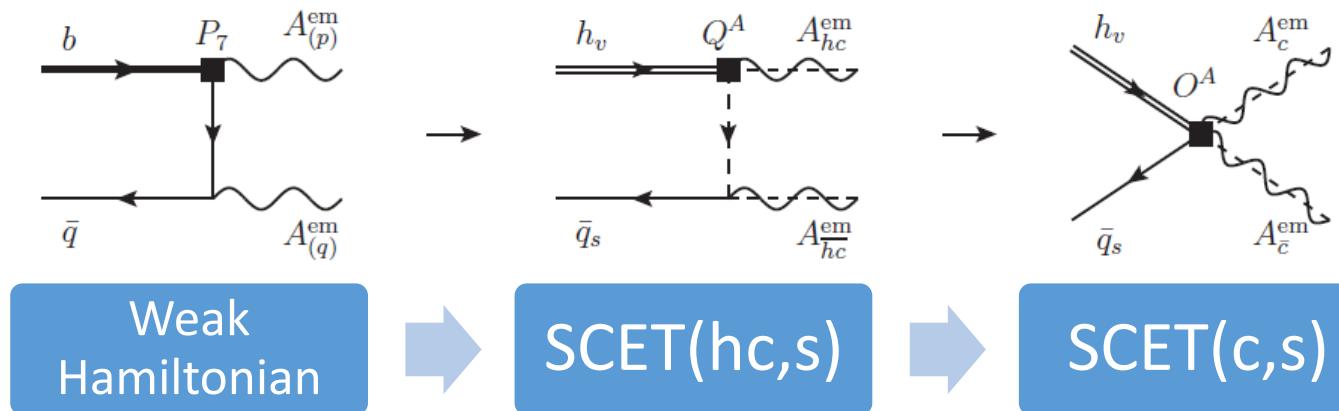
$$T_{i,\alpha\beta}^{(p)} = i m_{B_q}^3 \left[\left(g_{\alpha\beta}^\perp - i \varepsilon_{\alpha\beta}^\perp \right) F_{i,L}^{(p)} - \left(g_{\alpha\beta}^\perp + i \varepsilon_{\alpha\beta}^\perp \right) F_{i,R}^{(p)} \right]$$

- * The hierarchy structure

$$F_{i,L}^{(p)} : F_{i,R}^{(p)} = 1 : \left(\frac{\Lambda_{\text{QCD}}}{m_b} \right)$$

Factorization at leading power

- * The tree level matching in SCET



$$P_7 = -\frac{g_{\text{em}} \overline{m}_b(\nu)}{16 \pi^2} (\bar{q}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}$$

- * The power behavior of the matrix element of the operators can be obtained by the QCD analysis(Bosch et al 2002)

$$\mathcal{A}(B \rightarrow \gamma\gamma) \sim f_B \int_0^\infty \frac{\phi_B^+(\omega)}{\omega} d\omega = \frac{f_B}{\lambda_B} \sim \left(\frac{\Lambda}{m_b}\right)^{\frac{3}{2}} \frac{1}{\Lambda} \sim \lambda^{\frac{1}{2}}$$

Leading power amplitude

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$$\begin{aligned}
 \bar{\mathcal{A}}_{\text{LP}}(\bar{B}_q \rightarrow \gamma\gamma) = & i \frac{4G_F}{\sqrt{2}} \frac{\alpha_{\text{em}}}{4\pi} \epsilon_1^{*\alpha}(p) \epsilon_2^{*\beta}(q) \left[g_{\alpha\beta}^\perp - i\varepsilon_{\alpha\beta}^\perp \right] e_q f_{B_q} m_{B_q}^2 \hat{U}_1(m_b, \mu_h, \mu) \hat{U}_2(m_b, \mu_h, \mu) \\
 & K^{-1}(m_b, \mu_h) \left[\sum_{p=u,c} V_{pb} V_{pq}^* \bar{m}_b(\nu) V_{7,\text{eff}}^{(p)}(m_b, \mu_h, \nu) \right] \\
 & \exp [S(\mu_0, \mu) + a_\gamma(\mu_0, \mu) + 2\gamma_E a_\Gamma(\mu_0, \mu)] \hat{\mathcal{J}} \left(\frac{\partial}{\partial \eta}, \mu \right) \left(\frac{m_b \bar{\omega}}{\mu^2} \right)^{-\eta} \\
 & \frac{\Gamma(1 + \eta + a_\Gamma(\mu_0, \mu)) \Gamma(1 - \eta)}{\Gamma(1 - \eta - a_\Gamma(\mu_0, \mu)) \Gamma(1 + \eta)} \exp \left[\int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} \frac{d\alpha}{\beta(\alpha)} \mathcal{G}(\eta + a_\Gamma(\mu_\alpha, \mu), \alpha) \right] \\
 & \left(\frac{\bar{\omega}}{\mu_0} \right)^{-a_\Gamma(\mu_0, \mu)} \tilde{\phi}_B^+(\eta + a_\Gamma(\mu_0, \mu), \mu_0) \Big|_{\eta \rightarrow 0},
 \end{aligned}$$

- Hard function **at two loop level** (CMM 1996).
- Jet function **at two loop level** (Liu, Neubert 2020)
- Complete NLL resummation (BJM2019, GN2020).

- * The tree level amplitude is proportional to $1/\lambda_B$
- * The QCD correction introduces the logarithmic moments

Power expansion in effective field theory

- * Heavy quark expansion

$$b(x) = e^{-i m_b v \cdot x} \left[1 + \frac{i \not{D}_\perp}{2 m_b} + \frac{(v \cdot \not{D}) \not{D}_\perp}{4 m_b^2} - \frac{\not{D}_\perp \not{D}_\perp}{8 m_b^2} + \mathcal{O}\left(\frac{1}{m_b^3}\right) \right] h_v(x)$$

$$\not{D}_\perp^\mu \equiv \not{D}^\mu - (v \cdot \not{D}) v^\mu.$$

- * Expand light quark field in SCET

$$\psi = \xi_c + \eta_c + \xi_{hc} + \eta_{hc} + q_s \equiv \psi^{(2)} + \psi^{(3)} + \psi^{(4)} + \psi^{(5)} + \dots$$

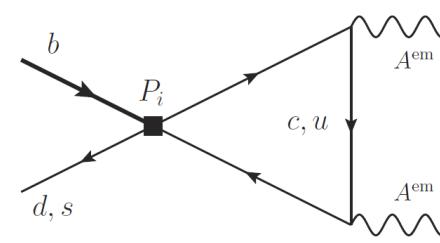
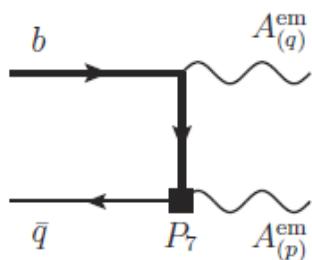
- * Building blocks of the operators (Beneke et al 2003)

$$n_-^\mu, n_+^\mu, g^{\mu\nu}, \epsilon^{\mu_\perp \nu_\perp \rho \sigma} n_{-\rho} n_{+\sigma}, \frac{1}{in_- \partial}, \frac{1}{in_- \partial},$$

$$\partial_\perp, A_{\perp c}, A_{\perp s}, n_+ \partial, n_+ A_s, n_- \partial, n_- A_c,$$

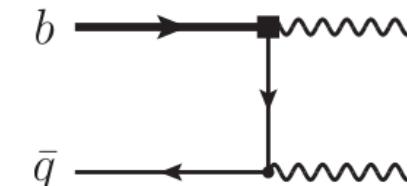
Subleading power contributions

- * Local contribution



Contribute to right-handed amplitude

- * Expansion of quark propagator

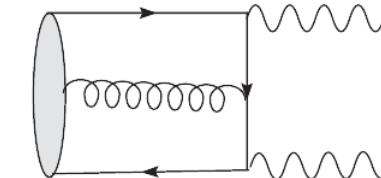


$$\frac{\not{p} - \not{k}}{(p - k)^2} = -\frac{1}{\omega} \frac{\not{\mu}}{2} + \boxed{\frac{\bar{n} \cdot k}{\bar{n} \cdot p \omega} \frac{\not{\mu}}{2} + \frac{\not{k}_\perp}{\bar{n} \cdot p \omega} + \frac{1}{\bar{n} \cdot p} \frac{\not{\mu}}{2}}$$

- * Heavy quark expansion

$$J^{(A2)} \supset (\bar{\xi}_{hc} W_{hc}) \gamma_\alpha^\perp P_L \left(\frac{i \not{D}_\perp}{2 m_b} \right) h_v + \dots$$

- * Higher twist LCDA



- * All the above contributions are factorizable

The strange quark mass term

- The mass term

$$T_{7,\alpha\beta}^{m_q, \text{NLP}} = [-i e_q \bar{m}_b(\nu) m_q f_{B_q} m_{B_q}] [g_{\alpha\beta}^\perp - i \varepsilon_{\alpha\beta}^\perp] \int_0^\infty d\omega \frac{\phi_B^-(\omega, \mu)}{\omega}$$

- End point behavior

$$\phi_B^+(\omega) \sim \omega, \quad \phi_B^-(\omega) \sim 1 \quad \text{as } \omega \rightarrow 0$$

- End point singularity and parameterization

$$\begin{aligned} \mathcal{I}_{\text{NLP}}^{m_q} &= \int_0^\infty d\omega \frac{\phi_B^-(\omega, \mu)}{\omega} = \left[\int_0^{\Lambda_{\text{UV}}} + \int_{\Lambda_{\text{UV}}}^\infty \right] d\omega \frac{\phi_B^-(\omega, \mu)}{\omega} \\ &= \left[\phi_B^-(0, \mu) X_{\text{NLP}} + \int_0^{\Lambda_{\text{UV}}} d\omega \frac{\phi_B^-(\omega, \mu) - \phi_B^-(0, \mu)}{\omega} \right] + \int_{\Lambda_{\text{UV}}}^\infty d\omega \frac{\phi_B^-(\omega, \mu)}{\omega} \end{aligned}$$

The quark mass term: dispersion approach

PRD106, 094038(2022)

- Start from the correlation function with Off-shell photon

$$\tilde{F}_{7,\text{NLP}}^{m_s} = -f_{B_s} \frac{m_s}{m_{B_s}} \frac{n_- q}{n_+ k} \int_0^\infty \frac{\phi_{B_s}^-(\omega)}{\omega - n_- k} = -f_{B_s} \frac{m_s}{m_{B_s}} \frac{n_- q}{n_+ k} \left[\int_0^{\omega_s} \frac{\phi_{B_s}^-(\omega)}{\omega - n_- k} + \int_{\omega_s}^\infty \frac{\phi_{B_s}^-(\omega)}{\omega - n_- k} \right]$$

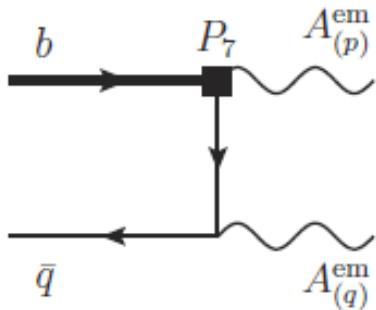
- Quark hadron duality: physical cutoff

$$F_{7,\text{LP}} + \tilde{F}_{7,\text{NLP}}^{m_s} + \tilde{F}_{7,\text{NLP}}^{\text{other}} = \frac{f_V m_V}{m_V^2 - k^2} \left[\frac{n_+ k}{m_{B_s}} T_1(q^2) + T_2(q^2) \right] + \frac{1}{\pi} \int_{\omega_s}^\infty d\omega' \frac{\rho_{\text{had}}(\omega')}{\omega' - n_- k - i0}$$

- Below threshold: expressed in terms of form factors
above threshold: physical photon limit

$$F_{7,\text{NLP}}^{m_s} = -\frac{Q_s f_{B_s} \bar{m}_b m_s}{m_{B_s}^2} \left\{ \frac{m_{B_s}}{m_\phi^2} \int_0^{\omega_s} \exp\left(\frac{m_\phi^2 - m_{B_s} \omega'}{m_{B_s} \omega_M}\right) \phi_B^-(\omega') d\omega' + \int_{\omega_s}^\infty \frac{\phi_B^-(\omega')}{\omega'} d\omega' \right\}$$

The resolved photon contribution



Soft contribution (Khodjamirian 1997)

$$x^2 \sim 1/\Lambda^2$$

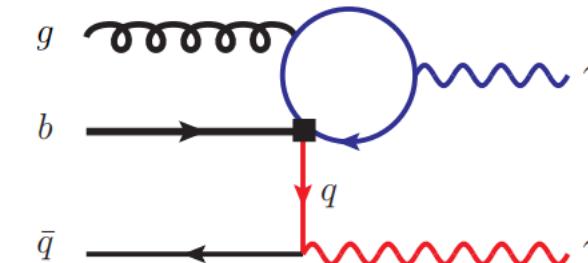
$$\begin{aligned} \tilde{T}_{7,\alpha\beta} = & 2 \overline{m}_b(\nu) \int d^4x e^{iq \cdot x} \langle 0 | T \{ j_\beta^{\text{em}}(x), \bar{q}_L(0) \sigma_{\mu\alpha} p^\mu b_R(0) \} | \bar{B}_q(p+q) \rangle \Big|_{q^2 < 0} \\ & + [p \leftrightarrow q, \alpha \leftrightarrow \beta] , \end{aligned}$$

$$F_{7,L}^{\text{soft, NLP}} = -\frac{e_q \overline{m}_b(\nu)}{m_{B_q}} \frac{1}{\pi} \int_0^{\omega_s} d\omega' \left[\frac{\bar{n} \cdot q}{m_V^2} \exp \left(\frac{m_V^2 - \bar{n} \cdot q \omega'}{\bar{n} \cdot q \omega_M} \right) - \frac{1}{\omega'} \right] \text{Im}_{\omega'} \tilde{F}_{7,L}(\bar{n} \cdot q, \omega')$$

The long distance penguin

Q. Qin etc., 2207.02691

- The soft photon emission from quark loop



- Power counting of charm quark mass: $\Lambda_{QCD} \ll m_c \sim m_b$ or $\Lambda_{QCD} \ll m_c \ll m_b$
 $m_c^2 \approx m_b \Lambda_{QCD}$: Anti-hard-collinear
- Integrate out the charm quark:

$$\begin{aligned}\mathcal{M}(g + b \rightarrow q + \gamma) = i \frac{4 G_F}{\sqrt{2}} \frac{g_{em} g_s}{4\pi^2} \sum_{p=u,c} V_{pb} V_{pq}^* & \left\{ \right. \\ & \left(C_2 - \frac{C_1}{2N_c} \right) Q_p [F(z_p) - 1] + 6 C_6 \sum_{q'} Q_{q'} [F(z_{q'}) - 1] \\ & + \left[\left(C_3 - \frac{C_4}{2N_c} \right) + 16 \left(C_5 - \frac{C_6}{2N_c} \right) \right] Q_q [F(z_q) - 1] \Big\} \\ & \times \left[\bar{q}(\tilde{q}) \gamma_\beta P_L G_{\mu\alpha} \tilde{F}^{\mu\beta} b(v) \right] \frac{p^\alpha}{(p - \ell)^2}\end{aligned}$$

The long distance penguin

- Integrate out hard-collinear light quark field: nonlocal operator with two light-cone direction
- A novel soft function

$$\begin{aligned} & \langle 0 | (\bar{q}_s S_n)(\tau_1 n) (S_n^\dagger S_{\bar{n}})(0) (S_{\bar{n}}^\dagger g_s G_{\mu\nu} S_{\bar{n}})(\tau_2 \bar{n}) \bar{n}^\nu \not{n} \gamma_\perp^\mu \gamma_5 (S_{\bar{n}}^\dagger h_v)(0) | \bar{B}_v \rangle \\ &= 2 \tilde{f}_B(\mu) m_B \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \exp [-i(\omega_1 \tau_1 + \omega_2 \tau_2)] \Phi_G(\omega_1, \omega_2, \mu), \end{aligned}$$

- Normalization

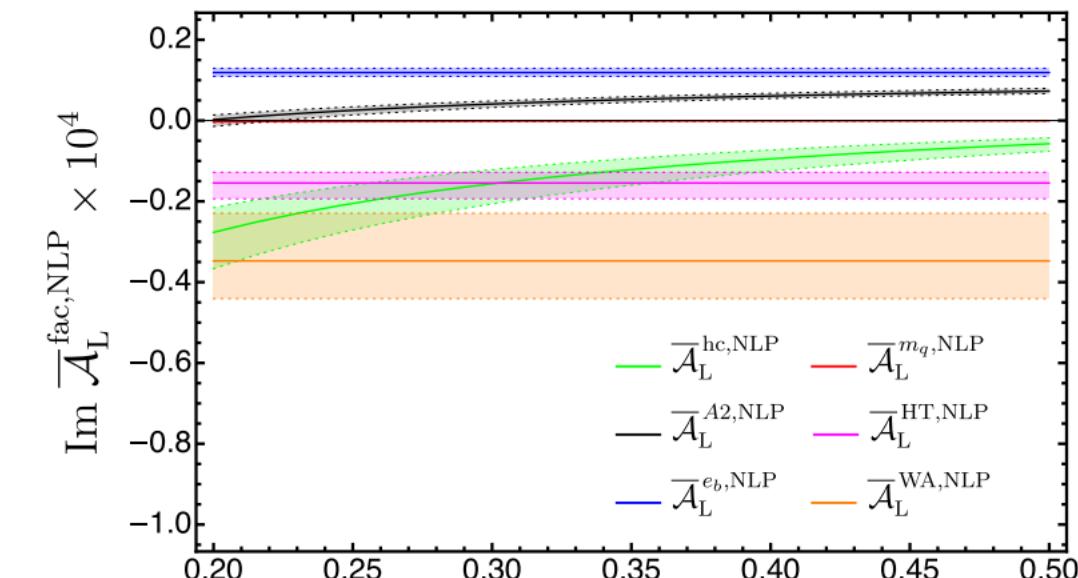
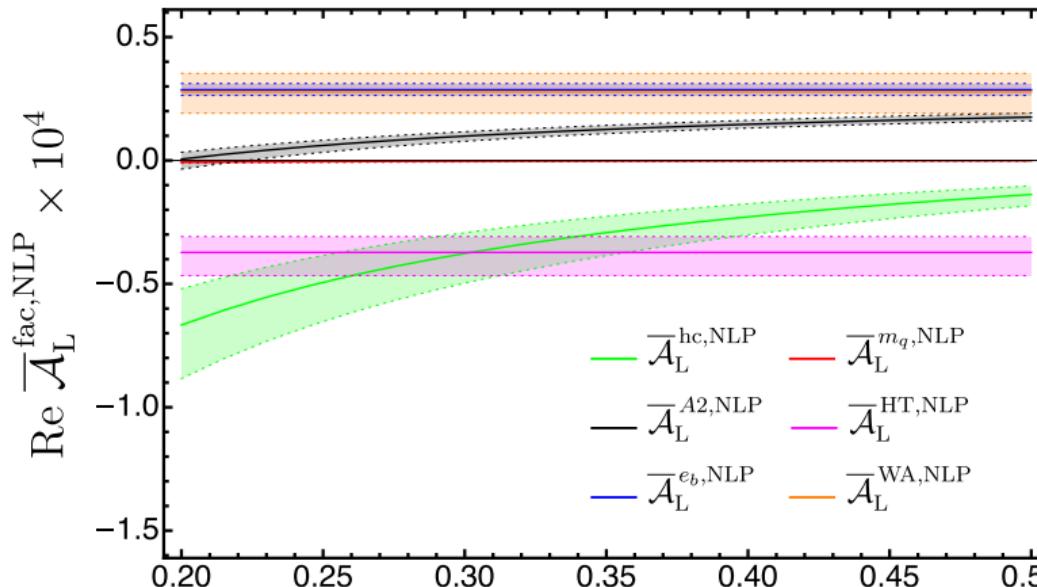
$$\begin{aligned} \int_0^\infty d\omega_1 \Phi_G(\omega_1, \omega_2, \mu) &= \int_0^\infty d\omega_1 \Phi_4(\omega_1, \omega_2, \mu) \\ \int_0^\infty d\omega_2 \Phi_G(\omega_1, \omega_2, \mu) &= \int_0^\infty d\omega_2 \Phi_5(\omega_1, \omega_2, \mu) \\ \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \Phi_G(\omega_1, \omega_2, \mu) &= \frac{\lambda_E^2 + \lambda_H^2}{3}, \end{aligned}$$

NOT LCDA!

The RG evolution can change
the sign of the argument

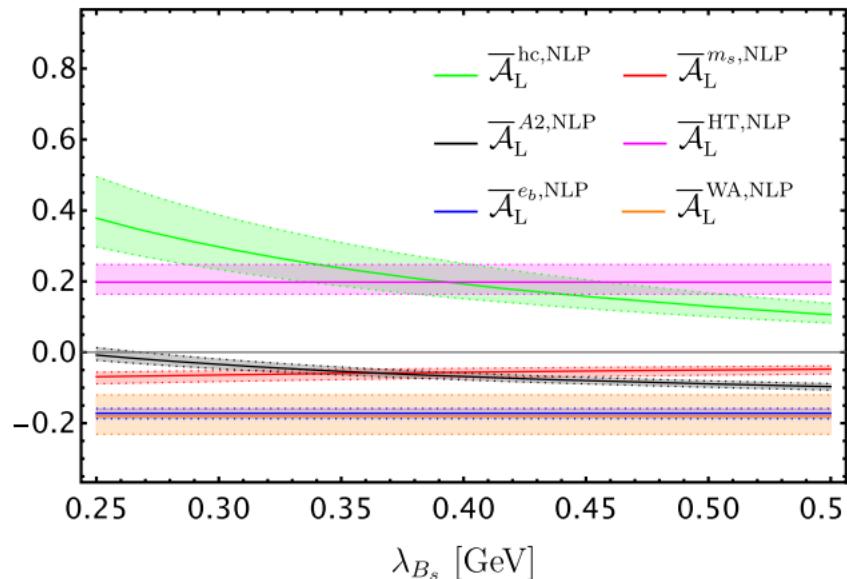
- The amplitude is factorizable
- Can to be generalized to $B \rightarrow K(K^*) l^+ l^-, K^* \gamma$

Numerical result for $B \rightarrow \gamma\gamma$



- There exists cancellation between different type of NLP contribution, the overall correction is about 10%-20%
- The λ_B dependence of the NLP contribution is quite different from LP one.

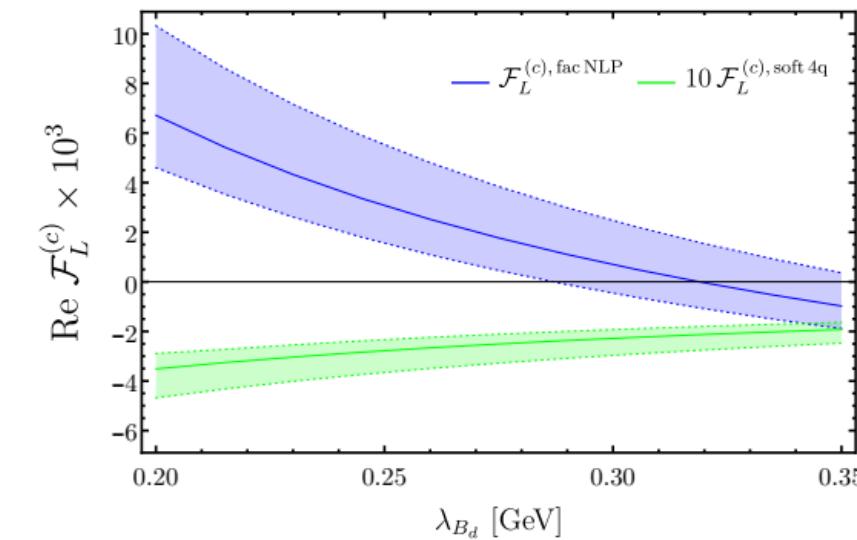
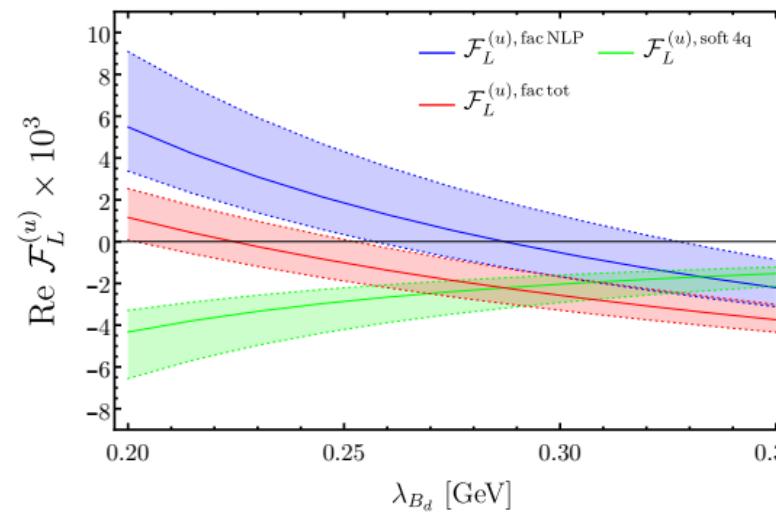
Numerical result for strange quark mass effect



Contributions	Central value	Total error	Error from λ_{B_s}
Leading power(LP)	3.87	+5.69 -1.85	+5.66 -1.77
LP + NLP	3.25	+2.09 -1.73	+1.54 -0.80
LP + NLP + quark mass	3.49	+2.21 -1.80	+1.67 -0.87

- The quark mass contribution can enhance the branching ratio over 6%

The long distance quark loop contribution



- The charm loop contribution is highly suppressed
- This effect enhances mixing induced CP violation about 30%, but has negligible impact on other observables

The ratio of inverse moment

$$\begin{aligned}\frac{\mathcal{BR}(B_s \rightarrow \gamma\gamma)}{\mathcal{BR}(B_d \rightarrow \gamma\gamma)} &= \frac{\tau_{B_s}}{\tau_{B_d}} \left| \frac{V_{ts}}{V_{td}} \right|^2 \left(\frac{m_{B_s}}{m_{B_d}} \right)^3 \left(\frac{f_{B_s}}{f_{B_d}} \right)^2 \left(\frac{\lambda_{B_d}}{\lambda_{B_s}} \right)^2 \left(\frac{1 - y_d^2}{1 - y_s^2} \right) + \mathcal{O} \left(\frac{\Lambda}{m_b}, \alpha_s \right) \\ &= 33.80 \left(\frac{\lambda_{B_d}}{\lambda_{B_s}} \right)^2 + \mathcal{O} \left(\frac{\Lambda}{m_b}, \alpha_s \right).\end{aligned}$$

- The theoretical uncertainty is less than 5%-10%

Thanks for your attention.