Charmless two-body B decays in perturbative QCD factorization approach

Shan Cheng

Hunan University

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Jian Chai, **SC**, Yao-hui Ju, Da-cheng Yan, Cai-dian Lü and Zhen-jun Xiao

Overview

- In the post Higgs Era, the precise testing of SM and searching of NP are the core tasks of particle physics.
- HFP plays am important role in both two targets,
  B meson hadronic decays provides many processes with CPV.
- Timeline of B physics

  † 1973, *Kaboyashi* & *Maskawa* proposed a 3×3 unitary matrix (4 parameters) of quark mixing to accommodate CPV,
  † 1977, CFS-E288 at FermiLab discovered Υ meson ($b\bar{b}$), *Lederman*,
  † 1981, *Bigi* & *Santa* pointed out the expectation of large CPV in B^0 decay according to CKM theory,
  † 1987, *Oddone* proposed the construction of B factories to study CPV,
  † 1999, BABAR and Belle started running; 2001(04), $A_{CP}(t,f)(A_{CP})$ in B^0 decays,
  † 2009, LHCb played in to the game; 2013(20), $A_{CP}(A_{CP}(t,f))$ in B_s decays,
  2012, A_{CP} in B^+ decays; 2019, δA_{CP} in D decays,
  † Anomalies: $R_{K^{(*)}}$, R_D, P_5', $B_s \to \mu\mu$, $|V_{ub}|$, $|V_{cb}|$

Motivation: Experiment promotions

[borrowed from Alexander Lenz at FPCP2020]

- SuperKEKB(2018-2026) △ The first measurements of $B^+ \to \rho^+\rho^0$, $B^0 \to K^0\pi^0$ have been released [Belle-II, 2021]., △ A_{cp}, S_{cp} in $B^0 \to J/\psi K_s, \phi K_s, K_s\pi^0$ and $K\pi$ isospin sum rules [Belle-II, 2023]
- HL-LHC(2027-2033) △ $\mathcal{L} = 23(300)\mathrm{fb}^{-1}$ in phase 1(2), 2 order larger than LHC
- More precise study of B decays from the theoretical side is imperative

Motivation: Theoretical progresses

- High precision calculation of two-body charmless B decays
  - † NF: $\sim F_{B\to M_2} \otimes f_{M_1}$ [Bauer&Stech&Wirbel 1985,87]
    GF: pQCD corrections from $O_{i=1,2}$ and $O_{i=3,10}$ [Ali&Kramer&Lü 1998,99]
    QCDF: VC to $\mathcal{M}_{t,p}$ + correction to spectator scattering, full NNLO ($\mathcal{O}(\alpha_s^2)$)
    [Benele 2010, Bell 15, 20, Huber 16, Beneke 06,07, Jain 07]
  - † SCET: introduces different fields in different energy regions, simple kinematics but complicated dynamics [Bauer 2001, Chay 04, Becher 15], QCDF/SCET [Beneke 2015]
  - † $B \to \pi\pi$ decay is studied from LCSRs [Khodjamirian 2001,03,05]
    the high order & power corrections of $B \to P, V$ form factors LQCD [HPQCD 2013]
    [Bharucha 2016, Wang 15,16,20, Lü 19, Beneke 17, Gubernari 19, Cheng 17,19]
- To eliminate the end-point singularity emerged in collinear factorization, the PQCD approach is proposed by picking up the k_T of valence quarks.
  - † $B \to M$ FFs and the annihilation amp. are both calculable [Keum 2001, Lü 01]
  - † LO ($\mathcal{O}(\alpha_s)$) $B \to PP, PV, VV$ decays [Xiao 2007; Lü 02; Li 05, Li 06, Zou 15], [Hua 2021]
  - † partially NLO ($\mathcal{O}(\alpha_s^2)$): △ factorizable amplitudes [Cheng 2021], △ effective operators [Mishima 2003, Li 05], △ hard scattering [Li 2012, Cheng 14], [Li 13, Cheng 15,15, Hua 18], [Li 14, Liu 15,16], △ TMD wave function [Li-Wang 2014, 15]
- A timely update of two-body hadronic B decays is urgent.

New physics: $\mathcal{L}_{NP}$

$\downarrow$

Electroweak scale (m_W): $\mathcal{L}_{EW} + \mathcal{L}_{D>4}$

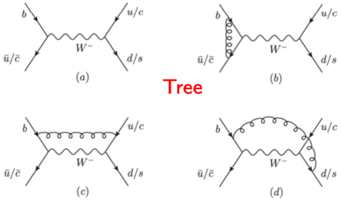
$\downarrow$

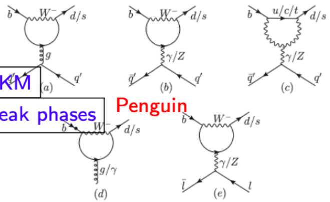
Heavy quark scale (m_b): $\mathcal{L}_{\rm eff} = -\frac{G_F}{\sqrt{2}} V_{\rm CKM} \sum_i C_i(\mu) O_i(\mu) + \mathcal{L}_{{\rm eff},D>6}$

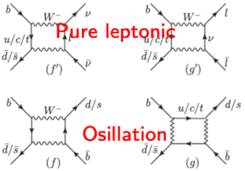
$\downarrow$

Hadron scale ($\Lambda_{\rm QCD}$): LCDAs, PDF, PDA

- Derive the effective Hamiltonian by integrating over m_W [Buchalla 1996]
  - † Product of two charged currents is expanded by a series of local operators O_i with the weighted coefficients C_i
- Dynamics at the scale $\mathcal{O}(m_W)$ is absorbed into Wilson coefficients $C_i(\mu)$
  - † C_i is obtained by matching the $\mathcal{L}_{\rm eff}$ with the full theory of weak decays [Ma 80, Inami&Lim 81, Clements 83]
- The rest go into the four fermion effective operators $O_i(\mu)$
- The key is to calculate the hadron matrix element $\langle M_1 M_2 | O_i | B \rangle$



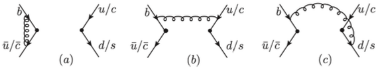




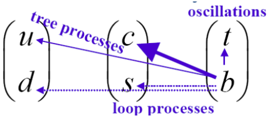
$\Downarrow$ Diagrams at scale $\mathcal{O}(m_W)$

\alpha_s

Effective tree, bullent denotes O_i



- Integrating over the m_W
- Weak phase difference between charged and FCNC of b decays

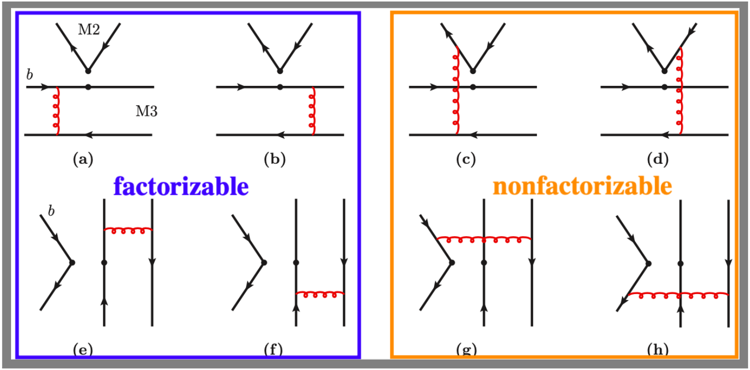


PQCD: Three scale factorization frame

- Diagrams at scales $\mathcal{O}(\Lambda_{\rm QCD}) - \mathcal{O}(m_b)$: Hadron matrix element $\langle M_1 M_2 | O_i | B \rangle$
- Factorization: detach the hard kernel $\mathcal{H}$ [O_i at scale $\mathcal{O}(m_b)$]

  from the hadron wave function Φ [B, M_1, M_2 mesons at scale $\mathcal{O}(\Lambda_{\rm QCD})$]
- Prediction power: $\mathcal{H}$ is calculated perturbatively order by order, Φs are universal

different strong phases

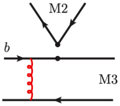


- End-point singularities appear in diagrams (a,b,e,f)

PQCD: Three scale factorization frame

- End-point singularities appear in diagrams (a,b,e,f)
  - † B rest frame, p_2 and p_3 are collinear with large momenta, $m_{2,3} \ll m_B$
  - † put on light cone: $p_2 = (\frac{m_B}{\sqrt{2}}, 0, \mathbf{0}_\mathrm{T})$, $p_3 = (0, \frac{m_B}{\sqrt{2}}, \mathbf{0}_\mathrm{T})$

    valence (anti-)quark: $k_2 = x_2 p_2$, $\bar{k}_2 = \bar{x}_2 p_2$



(a)

$$\mathcal{M}_a \propto \sum_{t=2,3} \int dx_1 dx_3 \kappa_t(x_i) \frac{\alpha_s(\mu)\phi_B(x_1)\phi_3^t(x_3)}{x_1(1-x_3)}$$

End-point singularity: $x_1 = 0$, $x_3 = 1$

- Picking up the transversal momentum, parton momentum is off-shell by k_T^2

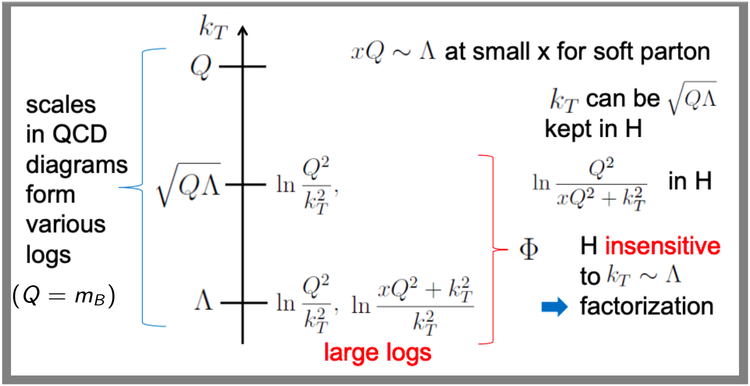
$$\mathcal{M}_a \propto \sum_{t=2,3} \int dx_1 dx_3 d\mathbf{k}_{1T} d\mathbf{k}_{3T} \kappa_t(x_i) \frac{\alpha_s(\mu)\phi_B(x_1, \mathbf{k}_{1T})\phi_3^t(x_3, \mathbf{k}_{3T})}{x_1 \bar{x}_3 m_B^2 - \mathbf{k}_T^2}$$

- End-point singularity at leading and subleading powers

$$\mathcal{H}_a \propto \frac{\alpha_s(\mu)}{x_1 \bar{x}_3 m_B^2 - \mathbf{k}_T^2} \sim \frac{\alpha_s(\mu)}{x_1 \bar{x}_3 m_B^2} - \frac{\alpha_s(\mu)\mathbf{k}_T^2}{(x_1 \bar{x}_3 m_B^2)^2} + \cdots$$

- At the end-points, the power suppressed TMD terms are nonnegligible

- Introduce $\mathbf{k}_T$ to regularize the end-point singularity [Huang 1991]
- Enriches the study of hadron DAs, TMD definition with Wilson link, observables
- Scales of transversal momentum and the large logarithms [borrowed from H.N Li]



† Multiple scales and hence large single logarithms in $\mathcal{H}$ and Φ from QCD correction

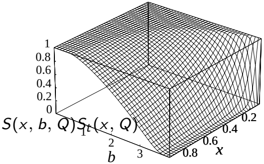
† Double logs in the soft-collinear regions $\alpha_s(\mu) \ln^2(\mathbf{k}_T^2/m_B^2)$

PQCD: Three scale factorization frame

- In order to repair the perturbative expansion, do resummation by using RGE
- k_T resummation for $\mathcal{H}$ and obtain $S(x_i, b_i, Q)$ [Botts 1989, Li 92]
  - † decreases the inverse power of the momentum transfer in the divergence amplitude
  - † exhibits high suppression for large transversal distances (small k_T) interactions
- Integrating over k_T, large log $\ln^2(x_i)$ when intermediate gluon is on shell
- threshold resummation for Φ and obtain $S_t(x_i, Q)$ [Li 1999]
  - † suppresses the small x_i regions
  - † repairs the self-consistency between $\alpha_s(t)$ and hard log $\ln(x_1 x_3 Q^2/t^2)$

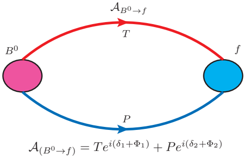
‡ dynamics with $k_T < \sqrt{Q\Lambda}$ is organized into $S(x, b, Q)$

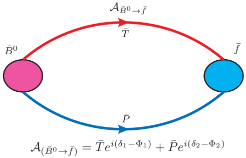
‡ dynamics in small x is suppressed by $S_t(x, Q)$



$$\mathcal{M}(B \to M_1 M_2) = \sum_i C_i(m_W, t) \otimes \mathcal{H}_i(t, b) \otimes \phi(x, b)\, \mathrm{Exp}\left[-s(p^+, b) - \int_{1/b}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_\phi(\alpha_s(\bar{\mu}))\right]$$

Sources of strong phase (differences) $\delta_{1,2}$ to generate $C\!\!\!/P$





- Different topologies: emission (real, $\delta_1 = 0$) and annihilation (plural, $\delta_2 \neq 0$)

$$\frac{1}{k_T^2 - xm_B^2 - i\epsilon} = \mathcal{P}\left(\frac{1}{k_T^2 - xm_B^2}\right) + i\delta(k_T^2 - xm_B^2).$$

- Sudakov expanent (NLO)
  - † center of mass scattering angle and angular distribution of scattering hadrons
  - † important in baryon decays but not in B meson decays
- NLO corrections to spectator emission amplitude from Glauber gluon
  - † only supplies a sizable phase to the pion final state
  - † modifies the interactions between different topological amplitudes
- on shell charm quark loop correction (NLO)

$B \to PP, PV, VV$ decays: Amplitudes

- General decomposition of Wilson coefficients for each certain effective weak vertex

spectator meson M_3
↑
$[q_1, q_2, q_3]$
↓
emission meson M_2

Weak vertex	Typical amplitudes	Wilson coefficients
$[s,s,s],[d,d,d]$	$\mathcal{E}^{\rm LL}/\mathcal{A}^{\rm LL}, \mathcal{E}^{\rm LL}_{NF}/\mathcal{A}^{\rm LL}_{NF}$	$a_3+a_4-\frac{a_9+a_{10}}{2}, \quad C_3+C_4-\frac{C_9+C_{10}}{2}$
	$\mathcal{E}^{\rm LR}/\mathcal{A}^{\rm LR}, \quad \mathcal{E}^{\rm LR}_{NF}/\mathcal{A}^{\rm LR}_{NF}$	$a_5-\frac{a_7}{2}, \quad C_5-\frac{C_7}{2}$
	$\mathcal{E}^{\rm SP}/\mathcal{A}^{\rm SP}, \quad \mathcal{E}^{\rm SP}_{NF}/\mathcal{A}^{\rm SP}_{NF}$	$a_6-\frac{a_8}{2}, \quad C_6-\frac{C_8}{2}$
$[d,s,s], \quad [s,d,d]$	$\mathcal{E}^{\rm LL}/\mathcal{A}^{\rm LL}, \quad \mathcal{E}^{\rm LL}_{NF}/\mathcal{A}^{\rm LL}_{NF}$	$a_4-\frac{a_{10}}{2}, \quad C_3-\frac{C_9}{2}$
	$\mathcal{E}^{\rm LR}/\mathcal{A}^{\rm LR}, \quad \mathcal{E}^{\rm LR}_{NF}/\mathcal{A}^{\rm LR}_{NF}$	$a_6-\frac{a_8}{2}, \quad C_5-\frac{C_7}{2}$
$[s,s,d], \quad [d,d,s]$	$\mathcal{E}^{\rm LL}/\mathcal{A}^{\rm LL}, \quad \mathcal{E}^{\rm LL}_{NF}/\mathcal{A}^{\rm LL}_{NF}$	$a_3-\frac{a_9}{2}, \quad C_4-\frac{C_{10}}{2}$
	$\mathcal{E}^{\rm LR}/\mathcal{A}^{\rm LR}, \quad \mathcal{E}^{\rm LR}_{NF}/\mathcal{A}^{\rm LR}_{NF}$	$a_5-\frac{a_7}{2}, \quad C_6-\frac{C_8}{2}$
$[u,u,s], \quad [u,u,d]$	$\mathcal{E}^{\rm LL}/\mathcal{A}^{\rm LL}, \quad \mathcal{E}^{\rm LL}_{NF}/\mathcal{A}^{\rm LL}_{NF}$	$a_2, \quad C_2$
	$\mathcal{E}^{\rm LR}/\mathcal{A}^{\rm LR}, \quad \mathcal{E}^{\rm LR}_{NF}/\mathcal{A}^{\rm LR}_{NF}$	$a_3+a_9, \quad C_4+C_{10}$
	$\mathcal{E}^{\rm SP}/\mathcal{A}^{\rm SP}, \quad \mathcal{E}^{\rm SP}_{NF}/\mathcal{A}^{\rm SP}_{NF}$	$a_5+a_7, \quad C_6+C_8$
$[s,u,u], \quad [d,u,u]$	$\mathcal{E}^{\rm LL}/\mathcal{A}^{\rm LL}, \quad \mathcal{E}^{\rm LL}_{NF}/\mathcal{A}^{\rm LL}_{NF}$	$a_1, \quad C_1$
	$\mathcal{E}^{\rm LR}/\mathcal{A}^{\rm LR}, \quad \mathcal{E}^{\rm LR}_{NF}/\mathcal{A}^{\rm LR}_{NF}$	$a_4+a_{10}, \quad C_3+C_9$
	$\mathcal{E}^{\rm SP}/\mathcal{A}^{\rm SP}, \quad \mathcal{E}^{\rm SP}_{NF}/\mathcal{A}^{\rm SP}_{NF}$	$a_6+a_8, \quad C_5+C_7$

- ie. Decay amplitude of $B^+ \to \pi^+ K^0$ at NLO

$[s, d, d]$

$$\mathcal{M}(B^+ \to \pi^+ K^0) = \frac{G_F}{\sqrt{2}} V^*_{ub} V_{us} \Big[a_1 \mathcal{A}^{\rm LL}_{\pi} + C_1 \mathcal{A}^{\rm LL}_{NF,\pi} + \mathcal{M}^{(\rm ql,u)}_{B\to K^0\pi^+} \Big] + \frac{G_F}{\sqrt{2}} V^*_{cb} V_{cs} \mathcal{M}^{(\rm ql,c)}_{B\to K^0\pi^+} - \frac{G_F}{\sqrt{2}} V^*_{tb} V_{ts} \Big[\Big(a_4 - \frac{a_{10}}{2}\Big) \mathcal{E}^{\rm LL}_{\pi} + \Big(a_6 - \frac{a_8}{2}\Big) \mathcal{E}^{\rm SP}_{\pi} + \Big(C_3 - \frac{C_9}{2}\Big) \mathcal{E}^{\rm LL}_{NF,\pi} + \Big(C_5 - \frac{C_7}{2}\Big) \mathcal{E}^{\rm LR}_{NF,\pi} + (a_4 + a_{10}) \mathcal{A}^{\rm LL}_{\pi} + (a_6 + a_8) \mathcal{A}^{\rm SP}_{\pi} + (C_3 + C_9) \mathcal{A}^{\rm LL}_{NF,\pi} + (C_5 + C_7) \mathcal{A}^{\rm LR}_{NF,\pi} + \mathcal{M}^{(\rm ql,t)}_{B\to K^0\pi^+} + \mathcal{M}^{(\rm mp)}_{B\to K^0\pi^+} \Big],$$

△ the glauber gluon corrections and TMD wave functions are not taken into account in this work

$B \to PP, PV, VV$ decays: Amplitudes

- Operator decomposition of $B \to PP$ decays

Tree/color-favoured tree emission

QCD **P**enguin

Color-suppressed tree emission

$\mathbf{P}_{\rm em}$: Electroweak penguin

E: tree annihilation amplitude

Topology	Channel
$\{\mathbf{P}, \mathrm{T}, \mathrm{C}, \mathrm{E}, \mathrm{P_{ew}}\}$	$\pi^0 K^+$, $\eta_q K^+$
$\{\mathbf{T}, \mathrm{P}, \mathrm{C}, \mathrm{E}, \mathrm{P_{ew}}\}$	$\pi^+ \eta_q$
$\{\mathbf{T}, \mathrm{C}, \mathrm{P_{ew}}\}$	$\pi^+ \pi^0$
$\{\mathbf{P}, \mathrm{E}, \mathrm{P_{ew}}\}$	$\pi^+ K^0$, $\eta_s K^+$, $K^+ \bar{K}^0$
$\{\mathbf{P}, \mathrm{P_{ew}}\}$	$\pi^+ \eta_s$
$\{\mathbf{T}, \mathrm{P}, \mathrm{E}, \mathrm{P_{ew}}\}$	$\pi^+ \pi^-$
$\{\mathbf{P}, \mathrm{T}, \mathrm{P_{ew}}\}$	$\pi^- K^+$
$\{\mathbf{C}, \mathbf{E}, \mathbf{P}, \mathrm{P_{ew}}\}$	$\pi^0 \pi^0, \pi^0 \eta_q$, $\eta_q \eta_q$
$\{\mathbf{P}, \mathrm{C}, \mathrm{P_{ew}}\}$	$\pi^0 K^0$, $\eta_q K^0$
$\{\mathbf{P}, \mathrm{P_{ew}}\}$	$\eta_s K^0$, $K^0 \bar{K}^0$, $\pi^0 \eta_s$, $\eta_s \eta_s$, $\eta_q \eta_s$
$\{\mathbf{E}, \mathrm{P}, \mathrm{P_{ew}}\}$	$K^+ K^-$

$B \to PP, PV, VV$ decays: Numerics

- Main uncertainties of PQCD calculation: high order QCD corrections & LCDAs

  $\downarrow$

  † characterized by the variation in the factorization scale
  † minimized by setting μ_t as the largest virtuality in hard scattering
  † two-loop expression for the strong coupling

- Input parameters of meson LCDAs

Meson	$\pi^\pm/\pi^0$	$K^\pm/K^0$	η_q	η_s
m/GeV [108]	0.140/0.135	0.494/0.498	0.104	0.705
f/GeV	0.130 [108]	0.156 [108]	0.125 [114]	0.177 [114]
m_0/GeV	1.400	1.892 [112]	1.087	1.990
a_1	0	0.076 ± 0.004 [113]	0	0
a_2	0.270 ± 0.047[14]	0.221 ± 0.082 [113]	0.250 ± 0.150 [115]	0.250 ± 0.150 [115]
Meson	$\rho^\pm/\rho^0$	$K^{*\pm}/K^{*0}$	ω	ϕ
m/GeV [108]	0.775	0.892	0.783	1.019
$f^\parallel$/GeV [9]	0.210/0.213	0.204	0.197	0.233
$f^\perp$/GeV	0.144/0.146 [116]	0.159 [9]	0.162 [9]	0.191 [9]
$a_1^\parallel$	0	0.060 ± 0.040 [117]	0	0
$a_1^\perp$	0	0.040 ± 0.030 [117]	0	0
$a_2^\parallel$	0.180 ± 0.037 [116]	0.160 ± 0.090 [117]	0.150 ± 0.120 [117]	0.230 ± 0.080 [117]
$a_2^\perp$	0.137 ± 0.030 [116]	0.100 ± 0.080 [117]	0.140 ± 0.120 [117]	0.140 ± 0.070 [117]

default scale $1\,\mathrm{GeV}$

$B \to PP, PV, VV$ decays: Numerics

- Anatomy of NLO corrections to $\mathcal{B}$ and $\mathcal{A}_{\rm CP}$ of $\pi\pi, \pi K$ modes

Mode	LO	+VC	+QL	+MP	$+\mathcal{F}^{\rm NLO}$	PDG [108]
$\mathcal{B}(B^+ \to \pi^+\pi^0)$	3.58	3.89	$\cdots$	$\cdots$	$4.18^{+1.32}_{-0.97}$	5.5 ± 0.4
$\mathcal{A}_{CP}$	−0.05	0.09	$\cdots$	$\cdots$	$0.08^{+0.09}_{-0.09}$	3 ± 4
$\mathcal{B}(B^0 \to \pi^+\pi^-)$	6.97	6.82	6.92	6.76	$7.31^{+2.38}_{-1.72}$	5.12 ± 0.19
$C_{\pi^+\pi^-}$	−23.4	−27.6	−13.8	−13.3	$-12.8^{+3.5}_{-3.3}$	-32 ± 4
$S_{\pi^+\pi^-}$	−31.1	−35.5	−46.4	−37.0	$-36.4^{+1.5}_{-1.5}$	-65 ± 4
$\mathcal{B}(B^0 \to \pi^0\pi^0)$	0.14	0.29	0.30	0.22	$0.23^{+0.07}_{-0.05}$	1.59 ± 0.26
$C_{\pi^0\pi^0}$	−3.1	60.1	73.6	77.6	$80.2^{+5.2}_{-6.7}$	33 ± 22
$\mathcal{B}(B^+ \to \pi^+K^0)$	17.0	20.8	28.0	19.4	$20.3^{+6.3}_{-4.4}$	23.7 ± 0.8
$\mathcal{A}_{CP}$	−1.19	−0.95	−0.06	−0.08	$-0.08^{+0.08}_{-0.09}$	-1.7 ± 1.6
$\mathcal{B}(B^+ \to \pi^0K^+)$	10.0	12.75	16.76	11.92	$12.3^{+3.8}_{-2.7}$	12.9 ± 0.5
$\mathcal{A}_{CP}$	−10.9	−5.20	2.26	2.48	$2.28^{+1.61}_{-1.74}$	3.7 ± 2.1
$\mathcal{B}(B^0 \to \pi^-K^+)$	14.3	18.0	23.9	16.4	$17.1^{+5.2}_{-3.7}$	19.6 ± 0.5
$\mathcal{A}_{CP}$	−15.2	−14.2	−4.16	−5.42	$-5.43^{+2.24}_{-2.34}$	-8.3 ± 0.4
$\mathcal{B}(B^0 \to \pi^0K^0)$	5.90	8.12	10.4	6.99	$7.38^{+2.11}_{-1.50}$	9.9 ± 0.5
$C_{\pi^0K^0}$	−2.62	−7.31	−6.57	−7.97	$-7.70^{+0.21}_{-0.13}$	0 ± 13
$S_{\pi^0K^0}$	70.1	73.5	71.6	71.9	$71.9^{+0.6}_{-0.6}$	58 ± 17

† $\mathcal{B}$: QL cancels with MP corrections, VC and NLO ffs do not have a significant effect

† NLO corrections change asymmetry parameters more significantly

† VC (QL) flips the sign of the direct CPV of $\pi^+\pi^0$ and $\pi^0\pi^0$ ($\pi^0 K^+$) modes
$\mathcal{A}_{CP}(B^+ \to K^+\pi^0) - \mathcal{A}_{CP}(B^+ \to K^+\pi^-) = 7.71^{+2.74}_{-2.92}(\rm PQCD)$ **VS** $12.0 \pm 2.4(\rm Data)$

† Color-suppressed modes ($\pi^0\pi^0, \pi^0 K^0$) are more sensitive to NLO corrections.

† PQCD shows a large direct CPV in $\pi^- K^+, \pi^+\pi^-$ modes in 2000 (LO), which are confirmed by BABRA and Belle afterward.

$B \to PP, PV, VV$ decays: Numerics

- Updated PQCD results for the branching ratios of $B \to PP$ decays (in units of 10^{-6})

η_q-η_s mixing

✓ η_q-η_s-η_g mixing [Fan 2012]

✓ Glauber gluon effect [Liu 2014]

Mode	PQCD	SCET1 [125]	SCET2 [125]	QCDF [127]	PDG [108]
$B^+ \to \pi^+ K^0$	$20.3^{+6.3+0.1}_{-4.4-0.1}$	$\cdots$	$\cdots$	$21.7^{+13.4}_{-9.1}$	23.7 ± 0.8
$B^+ \to \pi^0 K^+$	$12.3^{+3.8+0.1}_{-2.7-0.1}$	$\cdots$	$\cdots$	$12.5^{+6.8}_{-4.8}$	12.9 ± 0.5
$B^+ \to \eta' K^+$	$52.0^{+15.0+2.1}_{-10.8-0.7}$	69.5 ± 28.4	69.3 ± 27.7	$74.5^{+63.6}_{-31.6}$	70.4 ± 2.5
$B^+ \to \eta K^+$	$6.68^{+2.26+1.85}_{-1.60-0.96}$	2.7 ± 4.8	2.3 ± 4.5	$2.2^{+2.0}_{-1.3}$	2.4 ± 0.4
$B^+ \to K^+ \bar{K}^0$	$1.56^{+0.48+0.02}_{-0.34-0.02}$	$\cdots$	$\cdots$	$1.8^{+1.1}_{-0.7}$	1.31 ± 0.17
$B^+ \to \pi^0 \pi^+$	$4.18^{+1.30+0.22}_{-0.94-0.22}$ 4.45	$\cdots$	$\cdots$	$5.9^{+2.6}_{-1.6}$	5.5 ± 0.4
$B^+ \to \pi^+ \eta'$	$2.00^{+0.57+0.36}_{-0.42-0.31}$	2.4 ± 1.3	2.8 ± 1.3	$3.8^{+1.6}_{-0.8}$	2.7 ± 0.9
$B^+ \to \pi^+ \eta$	$2.62^{+0.78+0.45}_{-0.57-0.40}$	4.9 ± 2.0	5.0 ± 2.1	$5.0^{+1.5}_{-0.9}$	4.02 ± 0.27
$B^0 \to \pi^- K^+$	$17.1^{+5.2+0.1}_{-3.7-0.1}$	$\cdots$	$\cdots$	$19.3^{+11.4}_{-7.8}$	19.6 ± 0.5
$B^0 \to \pi^0 K^0$	$7.38^{+2.11+0.03}_{-1.50-0.04}$	$\cdots$	$\cdots$	$8.6^{+5.4}_{-3.6}$	9.9 ± 0.5
$B^0 \to \eta' K^0$	$52.3^{+14.9+2.1}_{-10.8-0.3}$	63.2 ± 26.3	62.2 ± 25.4	$70.9^{+59.1}_{-29.8}$	66 ± 4
$B^0 \to \eta K^0$	$4.63^{+1.57+1.51}_{-1.09-0.79}$	2.4 ± 4.4	2.3 ± 4.4	$1.5^{+1.7}_{-1.1}$	$1.23^{+0.27}_{-0.24}$
$B^0 \to K^0 \bar{K}^0$	$1.48^{+0.47+0.01}_{-0.33-0.00}$	$\cdots$	$\cdots$	$2.1^{+1.3}_{-0.8}$	1.21 ± 0.16
$B^0 \to K^+ K^-$	$0.046^{+0.058+0.009}_{-0.039-0.008}$	$\cdots$	$\cdots$	0.1 ± 0.04	0.078 ± 0.015
$B^0 \to \pi^+ \pi^-$	$7.31^{+2.35+0.38}_{-1.68-0.36}$ 5.35	$\cdots$	$\cdots$	$7.0^{+0.8}_{-1.0}$	5.12 ± 0.19
$B^0 \to \pi^0 \pi^0$	$0.23^{+0.07+0.01}_{-0.05-0.01}$ 0.61	$\cdots$	$\cdots$	$1.1^{+1.2}_{-0.5}$	1.59 ± 0.26
$B^0 \to \pi^0 \eta'$	$0.20^{+0.05+0.02}_{-0.03-0.01}$	2.3 ± 2.8	1.3 ± 0.6	$0.42^{+0.28}_{-0.15}$	1.2 ± 0.6
$B^0 \to \pi^0 \eta$	$0.20^{+0.06+0.02}_{-0.04-0.01}$	0.88 ± 0.68	0.68 ± 0.62	$0.36^{+0.13}_{-0.11}$	0.41 ± 0.17
$B^0 \to \eta\eta$	$0.37^{+0.09+0.08}_{-0.07-0.07}$	0.69 ± 0.71	1.0 ± 1.5	$0.32^{+0.15}_{-0.08}$	< 1
$B^0 \to \eta\eta'$	$0.29^{+0.07+0.06}_{-0.05-0.06}$	1.0 ± 1.6	2.2 ± 5.5	$0.36^{+0.27}_{-0.13}$	< 1.2
$B^0 \to \eta'\eta'$	$0.42^{+0.09+0.13}_{-0.07-0.11}$	0.57 ± 0.73	1.2 ± 3.7	$0.22^{+0.16}_{-0.08}$	< 1.7

† NLO corrections play an important role in penguin dominated models $\pi K, \eta' K$ and pure annihilation mode $K^0 K^0$

† PQCD predicted $\mathcal{B}(B_s \to \pi^+\pi^-) \sim 6 \times 10^{-6}$ in 2007 (LO), confirmed by CDF in 2011

$B \to PP, PV, VV$ decays: Numerics

- Updated PQCD results for the CPV of $B \to PP$ decays (in units of 10^{-2})

Mode	PQCD	SCET1 [125]	SCET2 [125]	QCDF [127]	PDG [108]
$B^+ \to \pi^+ K_S^0$	$-0.08^{+0.08+0.02}_{-0.09-0.02}$	$\cdots$	$\cdots$	$0.28^{+0.09}_{-0.10}$	-1.7 ± 1.6
$B^+ \to \pi^0 K^+$	$2.28^{+1.53+0.50}_{-1.65-0.57}$	$\cdots$	$\cdots$	$4.9^{+6.3}_{-5.8}$	3.0 ± 2.1
$B^+ \to \eta' K^+$	$-1.83^{+0.40+0.77}_{-0.40-1.03}$	-1 ± 1	7 ± 1	$0.45^{+1.4}_{-1.1}$	0.4 ± 1.1
$B^+ \to \eta K^+$	$-7.75^{+1.06+0.81}_{-0.99-0.43}$	33 ± 31	-33 ± 40	$-14.5^{+18.6}_{-28.1}$	-37 ± 8
$B^+ \to K^+ K_S^0$	$1.83^{+1.93+0.14}_{-1.87-0.18}$	$\cdots$	$\cdots$	-6.4 ± 2.0	14
$B^+ \to \pi^0 \pi^+$	$0.08^{+0.06+0.07}_{-0.06-0.04}$	$\cdots$	$\cdots$	$-0.11^{+0.06}_{-0.03}$	3 ± 4
$B^+ \to \pi^+ \eta'$	$68.9^{+2.4+1.0}_{-2.4-0.9}$	21 ± 21	2 ± 18	$1.6^{+10.6}_{-13.8}$	6 ± 16
$B^+ \to \pi^+ \eta$	$24.8^{+3.6+0.8}_{-3.3-0.7}$	5 ± 29	37 ± 29	$-5.0^{+8.7}_{-10.8}$	-14 ± 7
$B^0 \to \pi^- K^+$	$-5.43^{+1.86+1.26}_{-1.92-1.34}$	$\cdots$	$\cdots$	$-7.4^{+4.6}_{-5.0}$	-8.3 ± 0.4
$B^0 \to \pi^0 K_S^0$	$-7.70^{+0.17+0.12}_{-0.09-0.09}$	$\cdots$	$\cdots$	$-10.6^{+6.2}_{-5.7}$	$C_{\pi^0 K^0} = 0 \pm 13$
	$71.9^{+0.3+0.5}_{-0.3-0.5}$	$\cdots$	$\cdots$	$79.0^{+7.2}_{-5.7}$	$S_{\pi^0 K^0} = 58 \pm 17$
$B^0 \to \eta' K_S^0$	$-2.65^{+0.10+0.07}_{-0.10-0.11}$	1.1 ± 1.4	-2.7 ± 1.2	$3.0^{+1.0}_{-0.9}$	$C_{\eta' K^0} = -6 \pm 4$
	$69.8^{+0.1+0.1}_{-0.1-0.1}$	70.6	71.5	67.0 ± 1.4	$S_{\eta' K^0} = 63 \pm 6$
$B^0 \to \eta K_S^0$	$-7.88^{+0.14+0.06}_{-0.10-0.02}$	21 ± 21	-18 ± 23.2	$-23.6^{+16.0}_{-29.0}$	$\cdots$
	$70.0^{+0.2+0.2}_{-0.3-0.1}$	69	79	$79.0^{+8.9}_{-8.5}$	$\cdots$
$B^0 \to K_S^0 K_S^0$	$-17.3^{+0.6+0.4}_{-0.4-0.3}$	$\cdots$	$\cdots$	$-10.0^{+1.2}_{-2.0}$	$C_{K_S^0 K_S^0} = 0 \pm 40$
	$5.34^{+1.05+0.53}_{-1.06-0.49}$	$\cdots$	$\cdots$	$\cdots$	$S_{K_S^0 K_S^0} = -80 \pm 50$
$B^0 \to \pi^+ \pi^-$	$-12.8^{+3.3+1.1}_{-3.1-1.1}$	$\cdots$	$\cdots$	$17.0^{+4.5}_{-8.8}$	$C_{\pi^+\pi^-} = -32 \pm 4$
	$-36.4^{+0.5+1.4}_{-0.4-1.4}$	$\cdots$	$\cdots$	$-69^{+20.6}_{-13.5}$	$S_{\pi^+\pi^-} = -65 \pm 4$
$B^0 \to \pi^0 \pi^0$	$-80.2^{+5.2+0.4}_{-6.7-0.2}$	$\cdots$	$\cdots$	$57.2^{+33.7}_{-40.4}$	$C_{\pi^0\pi^0} = -33 \pm 22$
	$53.5^{+8.7+3.1}_{-8.4-3.0}$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

η_q-η_s mixing

✓ η_q-η_s-η_g mixing [Fan 2012]

$B \to PP, PV, VV$ decays: Numerics

- Updated PQCD results for the branching ratios of $B^+ \to PV$ decays (in units of 10^{-6})

η_q-η_s mixing

? η_q-η_s-η_g mixing

Mode	PQCD	SCET1 [128]	SCET2 [128]	QCDF [127]	PDG [108]
$B^+ \to \pi^+ K^{*0}$	$5.52^{+1.93+0.38}_{-1.36-0.41}$	$8.5^{+5.0}_{-3.9}$	$9.9^{+3.7}_{-3.2}$	$10.4^{+4.5}_{-4.2}$	10.1 ± 0.8
$B^+ \to \pi^0 K^{*+}$	$3.58^{+1.19+0.18}_{-0.82-0.15}$	$4.2^{+2.3}_{-1.8}$	$6.5^{+2.0}_{-1.8}$	$6.7^{+2.5}_{-2.3}$	6.8 ± 0.9
$B^+ \to \eta' K^{*+}$	$1.54^{+0.51+0.17}_{-0.34-0.08}$	$4.5^{+6.7}_{-4.0}$	$4.8^{+5.4}_{-3.7}$	$1.7^{+4.9}_{-1.6}$	$4.8^{+1.8}_{-1.6}$
$B^+ \to \eta K^{*+}$	$6.08^{+0.41+2.02}_{-0.30-1.45}$	$17.9^{+6.5}_{-6.1}$	$18.6^{+5.1}_{-5.3}$	$15.7^{+12.7}_{-8.3}$	19.3 ± 1.6
$B^+ \to K^+ \omega$	$6.17^{+1.25+1.59}_{-0.90-1.33}$	$5.1^{+2.6}_{-2.1}$	$5.9^{+2.2}_{-1.8}$	$4.8^{+5.6}_{-3.0}$	6.5 ± 0.4
$B^+ \to K^+ \phi$	$4.61^{+1.41+2.29}_{-0.82-0.63}$	$9.7^{+5.2}_{-4.2}$	$8.6^{+3.4}_{-2.9}$	$8.8^{+5.5}_{-4.5}$	$8.8^{+0.7}_{-0.6}$
$B^+ \to K^+ \rho^0$	$3.28^{+0.25+0.50}_{-0.21-0.48}$	$6.7^{+2.9}_{-2.4}$	$4.6^{+1.9}_{-1.6}$	$3.5^{+4.1}_{-4.5}$	3.7 ± 0.5
$B^+ \to K^0 \rho^+$	$6.11^{+0.45+0.96}_{-0.34-0.86}$	$9.3^{+5.0}_{-4.0}$	$10.1^{+4.3}_{-3.5}$	$7.8^{+9.6}_{-5.3}$	$7.3^{+1.0}_{-1.2}$
$B^+ \to K^+ \bar{K}^{*0}$	$0.47^{+0.15+0.02}_{-0.10-0.03}$	$0.49^{+0.28}_{-0.22}$	$0.51^{+0.2}_{-0.17}$	$0.80^{+0.36}_{-0.33}$	0.59 ± 0.08
$B^+ \to \bar{K}^0 K^{*+}$	$0.31^{+0.02+0.10}_{-0.02-0.09}$	$0.54^{+0.28}_{-0.22}$	$0.51^{+0.22}_{-0.18}$	$0.46^{+0.56}_{-0.31}$	$\cdots$
$B^+ \to \pi^+ \rho^0$	$4.96^{+1.34+0.13}_{-1.01-0.14}$	$10.7^{+1.2}_{-1.1}$	$7.9^{+0.8}_{-0.8}$	$8.7^{+3.2}_{-1.9}$	8.3 ± 1.2
$B^+ \to \pi^0 \rho^+$	$10.9^{+3.4+0.6}_{-2.4-0.6}$	$8.9^{+1.0}_{-1.0}$	$11.4^{+1.3}_{-1.1}$	$11.8^{+2.3}_{-1.8}$	10.9 ± 1.4
$B^+ \to \eta' \rho^+$	$4.06^{+1.22+0.84}_{-0.89-0.77}$	$0.37^{+2.5}_{-0.23}$	$0.44^{+3.2}_{-0.20}$	$5.6^{+1.2}_{-0.9}$	9.7 ± 2.2
$B^+ \to \eta \rho^+$	$5.59^{+1.68+1.17}_{-1.22-1.06}$	$3.9^{+2.0}_{-1.7}$	$3.3^{+1.9}_{-1.6}$	$8.3^{+1.3}_{-1.1}$	7.0 ± 2.9
$B^+ \to \pi^+ \omega$	$5.42^{+1.44+0.47}_{-1.10-0.45}$	$6.7^{+0.80}_{-0.70}$	$8.5^{+0.9}_{-0.9}$	$6.7^{+2.5}_{-1.5}$	6.9 ± 0.5
$B^+ \to \pi^+ \phi$	$0.042^{+0.014+0.002}_{-0.010-0.002}$	~ 0.003	~ 0.003	~ 0.043	0.032 ± 0.015

† NLO corrections play an important role in ϕ, ω involved modes, ω-ϕ mixing ?

$B \to PP, PV, VV$ decays: Numerics

- Updated PQCD results for the CPV of $B^+ \to PV$ decays (in units of 10^{-2})

Mode	PQCD	SCET1 [128]	SCET2 [128]	QCDF [127]	PDG [108]
$B^+ \to \eta' K^{*+}$	$1.54^{+9.05+14.9}_{-8.16-9.74}$	$2.7^{+27.4}_{-19.5}$	$2.6^{+26.7}_{-32.9}$	$65.5^{+35.7}_{-63.9}$	-26 ± 27
$B^+ \to \eta K^{*+}$	$-34.5^{+2.5+0.9}_{-2.4-0.8}$	$-2.6^{+5.4}_{-5.5}$	$-1.9^{+3.4}_{-3.6}$	$-9.7^{+7.3}_{-8.0}$	2 ± 6
$B^+ \to K^+ \omega$	$31.5^{+0.6+0.1}_{-1.1-0.7}$	$11.6^{+18.2}_{-20.4}$	$12.3^{+16.6}_{-17.3}$	$22.1^{+19.6}_{-18.2}$	-2 ± 4
$B^+ \to \pi^+ K^{*0}$	$-0.94^{+0.26+0.04}_{-0.29-0.03}$	0	0	$0.4^{+4.5}_{-4.2}$	-4 ± 9
$B^+ \to \pi^0 K^{*+}$	$-0.01^{+4.40+1.12}_{-4.87-1.26}$	$-17.8^{+30.4}_{-24.7}$	$-12.9^{+12.0}_{-12.2}$	$1.6^{+11.5}_{-4.2}$	-39 ± 21
$B^+ \to K^+ \rho^0$	$58.7^{+4.3+3.2}_{-4.0-2.8}$	$9.2^{+15.2}_{-16.1}$	$16.0^{+20.5}_{-22.5}$	$45.4^{+36.1}_{-30.2}$	37 ± 10
$B^+ \to K^0 \rho^+$	$0.99^{+0.01+0.13}_{-0.01-0.18}$	0	0	$0.3^{+0.5}_{-0.3}$	-3 ± 15
$B^+ \to K^+ \bar{K}^{*0}$	$21.3^{+6.2+1.2}_{-5.7-1.4}$	$-3.6^{+6.1}_{-5.3}$	$-4.4^{+4.1}_{-4.1}$	$-8.9^{+3.0}_{-2.6}$	12 ± 10
$B^+ \to K^+ \phi$	$-1.93^{+0.66+0.66}_{-0.60-0.42}$	0	0	$0.6^{+0.1}_{-0.1}$	2.4 ± 2.8
$B^+ \to \pi^+ \phi$	0.0	$\cdots$	$\cdots$	0.0	1 ± 5
$B^+ \to \pi^+ \omega$	$-29.8^{+0.5+1.1}_{-0.4-0.8}$	$0.5^{+19.1}_{-19.6}$	$2.3^{+13.4}_{-13.2}$	$-13.2^{+12.4}_{-10.9}$	-4 ± 5
$B^+ \to \pi^+ \rho^0$	$14.9^{+0.4+0.5}_{-0.4-0.6}$	$-10.8^{+13.1}_{-12.7}$	$-19.2^{+15.6}_{-13.5}$	$-9.8^{+11.9}_{-10.5}$	0.9 ± 1.9
$B^+ \to \pi^0 \rho^+$	$-7.31^{+0.06+0.07}_{-0.02-0.03}$	$15.5^{+17.0}_{-19.0}$	$12.3^{+9.4}_{-10}$	$9.7^{+8.3}_{-10.8}$	2 ± 11
$B^+ \to \eta' \rho^+$	$29.0^{+0.4+0.0}_{-0.4-0.1}$	$-19.8^{+66.6}_{-37.6}$	$-21.7^{+135.9}_{-24.3}$	$1.4^{+14.0}_{-11.9}$	26 ± 17
$B^+ \to \eta \rho^+$	$-13.0^{+0.1+0.1}_{-0.1-1.5}$	$-6.6^{+21.5}_{-21.3}$	$-9.1^{+16.7}_{-15.8}$	$-8.5^{+6.5}_{-5.3}$	11 ± 11

large CPV predictions (boxed: $B^+ \to \eta' K^{*+}$, $B^+ \to \eta K^{*+}$, $B^+ \to K^+ \omega$)

large CPV in rare decay (boxed: $B^+ \to K^+ \bar{K}^{*0}$)

η_q-η_s mixing
? η_q-η_s-η_g mixing (boxed: $B^+ \to \eta' \rho^+$, $B^+ \to \eta \rho^+$)

† The measured direct CPV in $B \to PV$ is significantly larger than that in $B \to PP$

† It is hard to measure $B \to PV$ decays precisely $\Leftarrow$ vector meson is not stable

† Three-body B decays along with intermediate $B \to PV$ decays, but difficult to resolve

- Updated PQCD results for the branching ratios of $B^+ \to VV$ decays (in units of 10^{-6})

Mode	PQCD	SCET [130]	QCDF [127,131]	PDG [108]
$B^+ \to \rho^+ K^{*0}$	$9.40^{+1.43+1.05}_{-1.34-0.95}$	8.93 ± 3.18	$9.2^{+3.8}_{-5.5}$	9.2 ± 1.5
$B^+ \to \rho^0 K^{*+}$	$6.25^{+1.12+0.59}_{-0.84-0.53}$	4.64 ± 1.37	$5.5^{+1.4}_{-2.5}$	4.6 ± 1.1
$B^+ \to \omega K^{*+}$	$5.48^{+1.52+0.81}_{-1.36-0.66}$	5.56 ± 1.60	$3.0^{+2.5}_{-1.5}$	< 7.4
$B^+ \to \phi K^{*+}$	$12.3^{+1.7+1.5}_{-1.4-1.4}$	9.86 ± 3.39	$10.0^{+12.4}_{-3.5}$	10.2 ± 2.0
$B^+ \to K^{*+} \bar{K}^{*0}$	$0.66^{+0.12+0.09}_{-0.09-0.08}$	0.52 ± 0.18	$0.6^{+0.3}_{-0.3}$	0.91 ± 0.29
$B^+ \to \rho^0 \rho^+$	$14.0^{+4.1+0.4}_{-3.0-0.4}$	22.1 ± 3.7	$20.06^{+4.5}_{-2.1}$	24.0 ± 1.9
$B^+ \to \rho^+ \omega$	$10.9^{+2.8+1.0}_{-2.1-0.9}$	19.2 ± 3.1	$16.9^{+3.6}_{-1.8}$	15.9 ± 2.1
$B^+ \to \rho^+ \phi$	$0.042^{+0.011+0.004}_{-0.008-0.003}$	0.005 ± 0.001	⋯	< 3.0
$B^0 \to \rho^- K^{*+}$	$8.72^{+1.27+0.97}_{-0.96-0.87}$	10.6 ± 3.2	$8.9^{+4.9}_{-5.6}$	10.3 ± 2.6
$B^0 \to \rho^0 K^{*0}$	$3.37^{+0.38+0.43}_{-0.29-0.39}$	5.87 ± 1.87	$4.6^{+3.6}_{-3.6}$	3.9 ± 1.3
$B^0 \to \omega K^{*0}$	$5.93^{+0.89+1.74}_{-0.73-1.55}$	3.82 ± 1.39	$2.5^{+2.5}_{-1.5}$	2.0 ± 0.5
$B^0 \to \phi K^{*0}$	$11.8^{+1.6+1.5}_{-1.3-1.5}$	9.14 ± 3.14	10.0 ± 0.5	⋯
$B^0 \to K^{*0} \bar{K}^{*0}$	$0.38^{+0.09+0.02}_{-0.06-0.01}$	0.48 ± 0.16	$0.6^{+0.2}_{-0.3}$	0.83 ± 0.24
$B^0 \to K^{*+} K^{*-}$	$0.17^{+0.02+0.05}_{-0.02-0.03}$	⋯	$0.16^{+0.1}_{-0.1}$	< 2.0
$B^0 \to \rho^+ \rho^-$	$22.7^{+6.3+0.6}_{-4.8-0.6}$	27.7 ± 4.1	$25.5^{+2.8}_{-3.0}$	27.7 ± 1.9
$B^0 \to \rho^0 \rho^0$	$0.54^{+0.16+0.04}_{-0.11-0.04}$	1.00 ± 0.29	$0.9^{+1.9}_{-0.5}$	0.96 ± 0.15
$B^0 \to \rho^0 \omega$	0.76 [illegible] $_{-0.11-0.12}$	0.59 ± 0.19	0.08 [illegible] $_{-0.02}$	< 1.6
$B^0 \to \rho^0 \phi$	$0.019^{+0.005+0.002}_{-0.004-0.001}$	~ 0.002	⋯	< 3.3
$B^0 \to \omega\omega$	$1.21^{+0.24+0.31}_{-0.19-0.24}$	0.39 ± 0.13	$0.7^{+1.1}_{-0.4}$	1.2 ± 0.4
$B^0 \to \omega\phi$	$0.018^{+0.005+0.005}_{-0.004-0.005}$	~ 0.002	⋯	< 0.7
$B^0 \to \phi\phi$	$0.029^{+0.002+0.006}_{-0.002-0.006}$	⋯	⋯	< 0.027

isospin symmetry
smallness of $\mathcal{B}(\rho^0\rho^0)$
↓
$\mathcal{B}(\rho^+\rho^-) \sim 2\mathcal{B}(\rho^+\rho^0)$

VS (no new physics violates QCD isospin symmetry)

PQCD: $\sim$ 1.6
Data: $\sim$ 1

PQCD : $\mathcal{B}(\rho^0\rho^0) \sim 2\mathcal{B}(\pi^0\pi^0)$
Data : $\mathcal{B}(\rho^0\rho^0) \sim \mathcal{B}(\pi^0\pi^0)/2$

0.89 Glauber gluon

~~inconsistence between two B factories~~

† NLO corrections play an important role in rare modes $\rho^+\phi, \rho^0\rho^0(\omega, \rho), \omega\omega(\phi)$

$B \to PP, PV, VV$ decays: Numerics

$$\sqrt{2}\mathcal{M}(B^+ \to \pi^+\pi^0) = \mathcal{M}(B^0 \to \pi^+\pi^-) - \mathcal{M}(B^0 \to \pi^0\pi^0),$$
$$\sqrt{2}\mathcal{M}(B^+ \to \pi^+\rho^0 + \pi^0\rho^+) = \mathcal{M}(B^0 \to \pi^+\rho^- + \pi^-\rho^+) - 2\mathcal{M}(B^0 \to \pi^0\rho^0),$$
$$\sqrt{2}\mathcal{M}(B^+ \to \rho^+\rho^0) = \mathcal{M}(B^0 \to \rho^+\rho^-) - \mathcal{M}(B^0 \to \rho^0\rho^0).$$

- Updated PQCD results for the CPV of $B^+ \to VV$ decays (in units of 10^{-2})

Mode	PQCD	SCET [130]	QCDF [127,131]	PDG [108]
$B^+ \to \rho^+ K^{*0}$	$0.58^{+0.13+0.16}_{-0.12-0.18}$	-0.56 ± 0.61	-0.3^{+2}_{-1}	-1 ± 16
$B^+ \to \rho^0 K^{*+}$	$30.6^{+0.5+0.1}_{-0.7-0.2}$	29.3 ± 31.0	43^{+13}_{-28}	31 ± 13
$B^+ \to \omega K^{*+}$	$43.0^{+1.7+3.8}_{-2.0-3.2}$	24.3 ± 27.1	29 ± 35	$\cdots$
$B^+ \to \phi K^{*+}$	$2.40^{+0.14+0.13}_{-0.14-0.10}$	-0.39 ± 0.44	0.05	-1 ± 8
$B^+ \to K^{*+} \bar{K}^{*0}$	$-26.8^{+2.3+1.0}_{-2.4-2.0}$	9.5 ± 10.6	$\cdots$	$\cdots$
$B^+ \to \rho^0 \rho^+$	$0.03^{+0.00+0.00}_{-0.01-0.00}$	0.0	0.06	-5 ± 5
$B^+ \to \rho^+ \omega$	$-25.9^{+1.8+1.3}_{-1.9-1.2}$	-13.6 ± 16.1	-8^{+3}_{-4}	-20 ± 9
$B^+ \to \rho^+ \phi$	0.0	0.0	$\cdots$	$\cdots$
$B^0 \to \rho^- K^{*+}$	$32.4^{+0.1+0.1}_{-0.1-0.2}$	20.6 ± 23.3	32^{+2}_{-14}	21 ± 15
$B^0 \to \rho^0 K^{*0}$	$-14.4^{+1.2+0.9}_{-1.4-1.0}$	-3.30 ± 3.91	-15 ± 16	-6 ± 9
$B^0 \to \omega K^{*0}$	$9.89^{+0.96+1.59}_{-0.80-1.12}$	3.66 ± 4.05	23^{+10}_{-18}	45 ± 25
$B^0 \to \phi K^{*0}$	$0.86^{+0.06+0.07}_{-0.06-0.06}$	-0.39 ± 0.44	$0.8^{+0.4}_{-0.5}$	0 ± 4
$B^0 \to \rho^+ \rho^-$	$-1.85^{+0.20+0.01}_{-0.11-0.00}$	-7.68 ± 9.19	11^{+11}_{-4}	$C_{\rho^+\rho^-} = 0 \pm 9$
	$-12.7^{+0.1+0.4}_{-0.1-0.3}$	$\cdots$	-19^{+9}_{-10}	$S_{\rho^+\rho^-} = -14 \pm 13$
$B^0 \to \rho^0 \rho^0$	$74.6^{+1.3+1.9}_{-1.9-2.3}$	19.5 ± 23.5	-53^{+26}_{-54}	$C_{\rho^0\rho^0} = 20 \pm 90$
	$1.38^{+0.74+2.15}_{-0.03-1.93}$	$\cdots$	16^{+50}_{-49}	$S_{\rho^0\rho^0} = 30 \pm 70$

- Updated PQCD results for the f_L of $B^+ \to VV$ decays (in units of 10^{-2})

Mode	PQCD$_{\rm LO}$ [51]	PQCD	SCET [130]	QCDF [127,131]	HFLAV [134]
$B^+ \to \rho^+ K^{*0}$	70.0 ± 5.0	$76.6^{+1.5}_{-1.4}$	45.0 ± 18.0	$48.0^{+52.0}_{-40.0}$	48 ± 8
$B^+ \to \rho^0 K^{*+}$	$75.0^{+4.0}_{-5.0}$	$80.0^{+1.5}_{-1.5}$	42.0 ± 14.0	$67.0^{+31.0}_{-48.0}$	78 ± 12
$B^+ \to \omega K^{*+}$	64.0 ± 7.0	$77.4^{+0.5}_{-0.9}$	53.0 ± 14.0	$67.0^{+32.0}_{-39.0}$	41 ± 19
$B^+ \to \phi K^{*+}$	$57.0^{+6.3}_{-5.9}$	$68.7^{+1.3}_{-1.5}$	51.0 ± 16.4	$49.0^{+51.0}_{-43.0}$	50 ± 5
$B^+ \to K^{*+} \bar{K}^{*0}$	74.0 ± 7.0	$82.4^{+1.1}_{-1.1}$	50.0 ± 16.0	$45.0^{+55.0}_{-38.0}$	82^{+15}_{-21}
$B^+ \to \rho^0 \rho^+$	98.0 ± 1.0	$96.9^{+0.1}_{-0.1}$	~ 100	96.0 ± 2.0	95 ± 1.6
$B^+ \to \rho^+ \omega$	97.0 ± 1.0	$96.3^{+0.3}_{-0.4}$	97.0 ± 1.0	$96.0^{+2.0}_{-3.0}$	90 ± 6
$B^+ \to \rho^+ \phi$	95.0 ± 1.0	$81.3^{+1.9}_{-1.8}$	~ 100	$\cdots$	$\cdots$
$B^0 \to \rho^- K^{*+}$	$68.0^{+5.0}_{-4.0}$	$75.7^{+1.5}_{-1.4}$	55 ± 14	$53.0^{+45.0}_{-32.0}$	38 ± 13
$B^0 \to \rho^0 K^{*0}$	$65.0^{+4.0}_{-5.0}$	$71.0^{+1.5}_{-1.3}$	61.0 ± 13.0	$39.0^{+60.0}_{-31.0}$	17.3 ± 2.6
$B^0 \to \omega K^{*0}$	65.0 ± 5.0	$77.7^{+0.4}_{-0.9}$	40.0 ± 20.0	$58.0^{+44.0}_{-17.0}$	69 ± 11
$B^0 \to \phi K^{*0}$	$56.5^{+5.8}_{-5.9}$	$69.5^{+1.2}_{-1.5}$	51.0 ± 16.4	$50.0^{+51.0}_{-44.0}$	49.7 ± 1.7
$B^0 \to K^{*0} \bar{K}^{*0}$	58.0 ± 8.0	$68.8^{+5.3}_{-5.3}$	50.0 ± 16.0	$52.0^{+48.0}_{-49.0}$	74 ± 5
$B^0 \to K^{*+} K^{*-}$	~ 100.0	~ 100.0	$\cdots$	~ 100.0	$\cdots$
$B^0 \to \rho^+ \rho^-$	95.0 ± 1.0	$93.8^{+0.1}_{-0.1}$	99.1 ± 0.3	$92.0^{+1.0}_{-3.0}$	$99.0^{+2.1}_{-1.9}$
$B^0 \to \rho^0 \rho^0$	$12.0^{+16.0}_{-2.0}$	$80.9^{+1.9}_{-1.9}$	87.0 ± 5.0	$92.0^{+7.0}_{-37.0}$	71^{+8}_{-9}
$B^0 \to \rho^0 \omega$	$67.0^{+8.0}_{-9.0}$	$74.2^{+0.1}_{-0.1}$	58.0 ± 14.0	$52.0^{+12.0}_{-44.0}$	$\cdots$
$B^0 \to \rho^0 \phi$	95.0 ± 1.0	$81.3^{+1.9}_{-1.8}$	~ 100	$\cdots$	$\cdots$
$B^0 \to \omega\omega$	$66.0^{+10.0}_{-11.0}$	$88.4^{+0.9}_{-0.8}$	64.0 ± 15.0	$94.0^{+4.0}_{-20.0}$	$\cdots$
$B^0 \to \omega\phi$	$94.0^{+2.0}_{-3.0}$	$80.8^{+0.8}_{-1.4}$	~ 100	$\cdots$	$\cdots$
$B^0 \to \phi\phi$	97.0 ± 1.0	$99.9^{+0.0}_{-0.0}$	$\cdots$	$\cdots$	$\cdots$

† PQCD showed f_L in penguin dominated $B \to VV$ channels down by annihilation mechanism in 2002 (LO), before the "polarization puzzle" appeared.

Conclusion

- The up-to-date PQCD predictions with including the current well-known NLO and sub-leading power corrections can explain most of the data.

† $K\pi, K\rho, K\omega, K\phi$ and $K^*\rho, K^*\omega, K^*\phi$ channels ✓✓ $K^*\pi, K^*K$ channels ✓

† f_L in $K^*\rho, K^*\omega, K^*\phi$ channels is still larger than the HFLAV result

LD effect in $B \to K^*$ transition ? NLO corrections to $B \to V$ form factors ?
width effect of the intermediate vector resonant (four-body decays) ?

† $\eta^{(\prime)}$ involved channels do not consist well with data

the large mixing mechanism η_q-η_s-η_g provides a possible solution

† The CPV of charged (neutral) B decays is (not) sensitive to the new added two power correction (heavy quark expansion), especially for the channels with at least one $\eta^{(\prime)}$ in the final state.

Opportunities and challenges of PQCD

- Corrections from 3 particle B meson DAs and high twist light meson DAs

† interaction between largely off-shell gluon with three-particle configurations $\mathcal{O}(\Lambda/m_B)$?

- Complete NLO calculation for two-body B meson decays

† vertex corrections, $B \to \rho$ type ff, tensor meson ffs, annihilation spectator amplitude $\cdots$

- Complete NLO calculation for the radiative and $P_{\rm EW}$ B meson decays

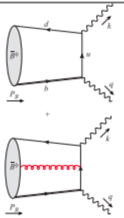
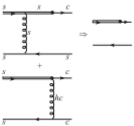
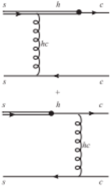
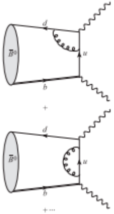
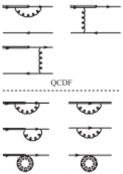
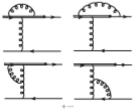
† B meson distribution amplitude

- TMD wave functions of B and B_c mesons, Λ_b baryon
- Systematic power counting with including k_T
- Sudakov factor of baryon and three particle configuration of meson
- Multibody B decay, more observables, CPV sources, factorization formula
- Input of meson and dimeson DAs, optimal choice of factorization scale

Thanks for your patience.

Back Slides

Table 1 A diagrammic summary of different QCD-based approaches to study $B \to \pi$ form factor. [Cheng 2021]

Approach	LCSRs (B meson DAs)	QCDF/SCET	PQCD
Formula	$\Pi_i^{OPE}(Q^2, s_0^B, M^2)/(f_B m_B)$	$C_i^{A0}(Q^2)\,\xi_{B\pi}(Q^2) + C_i^{B1}(Q^2,\omega) \cdot \phi_B(\omega) \otimes J_i(Q^2,\omega,\nu) \otimes \phi_\pi(\nu)$	$\phi_B(\omega) \otimes H_i(Q^2,\omega,\nu) \otimes \phi_\pi(\nu)$
Diagrams (LO)	 [28, 29]	 [15, 16]	 [37, 38]
Diagrams (NLO)	 [30, 31]	 [17, 32]	 [39, 40]
Diagrams (NNLO)		 [33–36]	

PQCD: Progresses towards to NLO

$$\mathcal{M}(B \to M_1 M_2) = \sum_i C_i(m_W, t) \otimes \mathcal{H}_i(t, b) \otimes \phi(x, b)\, \mathrm{Exp}\left[-s(p^+, b) - \int_{1/b}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_\phi(\alpha_s(\bar{\mu}))\right]$$

- The NLO QCD/QED corrections to C_i has been finished [Buchalla, 1996, Rev. Mod. Phys]

$$C_1(m_W) = \frac{11}{2}\frac{\alpha_s(m_W)}{4\pi},$$

$$C_2(m_W) = 1 - \frac{11}{6}\frac{\alpha_s(m_W)}{4\pi} - \frac{35}{18}\frac{\alpha_{\rm em}}{4\pi},$$

$$C_3(m_W) = -\frac{\alpha_s(m_W)}{24\pi}\left[E_0(\frac{m_t^2}{m_W^2}) - \frac{2}{3}\right] + \frac{\alpha_{\rm em}}{6\pi}\frac{1}{\sin^2\theta_W}\left[2B_0(\frac{m_t^2}{m_W^2}) + C_0(\frac{m_t^2}{m_W^2})\right]$$

$$C_4(m_W) = -\frac{\alpha_s(m_W)}{8\pi}\left[E_0(\frac{m_t^2}{m_W^2}) - \frac{2}{3}\right],$$

...

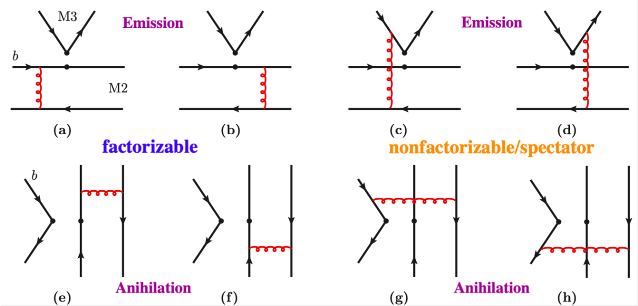
μ(GeV)	1.0	2.0	3.0	4.0	4.98
$\alpha_s(\mu)$	0.63, 0.47	0.39, 0.30	0.32, 0.25	0.29, 0.23	0.26, 0.21
$C_1(\mu)$	−0.27, −0.51	−0.61, −0.31	−0.85, −0.24	−1.05, −0.20	−0.83, −0.17
$C_2(\mu)$	1.12, 1.28	1.33, 1.15	1.50, 1.11	1.66, 1.09	1.48, 1.07
$C_3(\mu)$	0.01, 0.04	0.03, 0.02	0.05, 0.02	0.06, 0.01	0.05, 0.01
$C_4(\mu)$	−0.03, −0.09	−0.06, −0.05	−0.08, −0.04	−0.10, −0.04	−0.07, −0.03
$C_5(\mu)$	0.01, 0.02	0.02, 0.01	0.02, 0.01	0.02, 0.01	0.02, 0.01
$C_6(\mu)$	−0.03, −0.13	−0.09, −0.07	−0.15, −0.05	−0.20, −0.04	−0.14, −0.04
$C_7(\mu)$	0.00, −0.00	0.00, −0.00	0.00, −0.00	0.00, −0.00	0.00, −0.00
$C_8(\mu)$	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00
$C_9(\mu)$	−0.01, −0.01	−0.01, −0.01	−0.01, −0.01	−0.01, −0.01	−0.01, −0.01
$C_{10}(\mu)$	0.00, 0.01	0.01, 0.00	0.01, 0.00	0.01, 0.00	0.01, 0.00

LO NLO

† Inami-Lim functions B, C, D, E from box, Z, γ, g penguin diagrams, respectively

† Scale running from m_W to $\mathcal{O}(m_b)$ by evolution matrix: $C_i(\mu) = U(\mu, m_W) C_i(m_W)$

- The NLO corrections to ME $\langle M_1 M_2 | O_i | B \rangle$



† Vertex of effective operator in $\mathcal{M}_{a,b}$ Completed in collinear factorization

† $B \to P$ transition form factors in $\mathcal{M}_{a,b}$ Done up to twist three

† Electromagnetic form factors in $\mathcal{M}_{e,f}$ Done up to twist three for PP, PV

† Scalar form factor in $\mathcal{M}_{e,f}$ with helicity flip Done up to twist three

† Glauber gluon correction to $\mathcal{M}_{c,d}$ Done for $M = \pi$

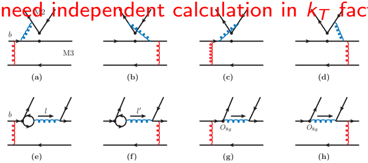
† NLO correction to spectator annihilation amplitude is still missing

PQCD: Progresses towards to NLO

- Vertex of effective operator in $\mathcal{M}_{a,b}$ [Beneke 2001, Mishima 03, Li 05]

Completed in collinear factorization

need independent calculation in k_T factorization



† vertex correction: does not involve the end-point singularity in collinear fact.

‡ absorbed into the effective Wilson coefficients according to the effective operators, ie.
$a_{1,2}(\mu) \to a_{1,2}(\mu) + \frac{\alpha_s(\mu)}{4\pi} \frac{C_{1,2}(\mu)}{N_C} V_{1,2}(M)$

‡ V_i has imaginary part, embodied into a_2 and $a_{3,10}$, and hence sensitive in the color-suppressed amplitudes

‡ ie., increase the Br of $\pi^0\pi^0$ channel by a factor 1.5 more important is to change the sign of CPV

† chromomagnetic penguin: does not involve the end-point singularity

‡ the same form as the QCDF

‡ another new invariant amplitude

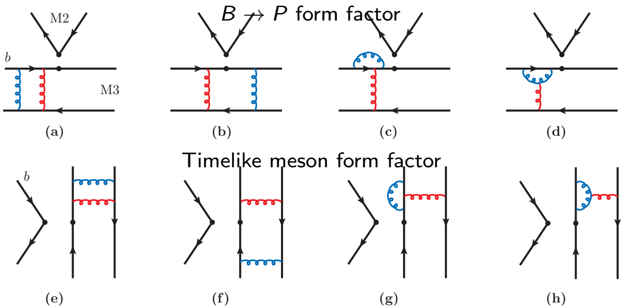
† quark loop: does not involve the end-point singularity/momentum redistribution in $\mathcal{H}$

‡ the same form as the QCDF, ie., $\mathcal{C}^{(u,c)}(\mu, l^2) = \left[\mathcal{G}^{(u,c)}(\mu, l^2) - \frac{2}{3}\right] C_2(\mu)$

‡ for the massive charm quark, $\mathcal{G}^c(\mu, l^2)$ has real and imaginary parts

‡ a new invariant amplitude depended on three meson wave functions, the correction is special to the operator O_5

- NLO Form factors in the factorizable amplitudes $\mathcal{M}_{a,b,e,f}$



† The IR safety of full amplitudes is one of the prerequisites for perturbative calculation

† Two types of IR singularities: soft ($l_\mu \sim (\Lambda, \lambda, \mathbf{\Lambda}_T^2)$) and collinear ($l_\mu \sim (Q, \Lambda^2/Q, \mathbf{\Lambda}_T^2)$)

gluon exchanged between two on-shell quark lines, gluon emission from a massless quark

† Factorize the QCD IR divergences in sequence of momentum, spin and color spaces

△ Eikonal approximation, detach the leading soft and collinear divs △ Fierz identity, spread out the fermion current into different twist △ Ward identity, sum over all color structures to guarantee gauge invariance

PQCD: Progresses towards to NLO

- $B \to P$ transition form factors in $\mathcal{M}_{a,b}$ Done up to twist three [Li 2012, Cheng 14]
  - IR divs cancel between the QCD quark diagrams and the effective diagrams of Φ

$$\Phi_B = \int \frac{dz^- d^2 z_T}{(2\pi)^3} e^{-ix_1' P_1^+ z^- + i\mathbf{k}_{1T}' \cdot \mathbf{z}_T} < 0 \mid \bar{q}(z) W_z(n_1)^\dagger I_{n_1;z,0} W_0(n_1) \not{n}_+ \Gamma h_\nu(0) \mid h_\nu \bar{d}(k_1) >,$$

$$\Phi_{\pi,P} = \int \frac{dy^+ d^2 y_T}{(2\pi)^3} e^{-ix_2' P_2^- y^+ + i\mathbf{k}_{2T}' \cdot \mathbf{y}_T} < 0 \mid \bar{q}(y) W_y(n_2)^\dagger I_{n_2;y,0} W_0(n_2) \gamma_5 q(0) \mid u(P_2 - k_2) \bar{d}(k_2)$$

$$W_z(n) = Pexp[-ig_s \int_0^\infty d\lambda n \cdot A(z + \lambda n)]$$

  - k_T dependent IR safety NLO hard kernel is obtained

$$H^{(1)}(x_1, \mathbf{k}_{1T}; x_2, \mathbf{k}_{2T}) = G^{(1)}(x_1, \mathbf{k}_{1T}; x_2, \mathbf{k}_{2T}) - \int dx_1' d^2\mathbf{k}_{1T}' \Phi_B^{(1)}(x_1, \mathbf{k}_{1T}; x_1', \mathbf{k}_{1T}') H^{(0)}(x_1', \mathbf{k}_{1T}'; x_2, \mathbf{k}_{2T})$$
$$- \int dx_2' d^2\mathbf{k}_{2T}' H^{(0)}(x_1, \mathbf{k}_{1T}; x_2', \mathbf{k}_{2T}') \Phi_{\pi,P}^{(1)}(x_2', \mathbf{k}_{2T}'; x_2, \mathbf{k}_{2T})$$

  - NLO correction gives $\sim 8\%$ enhancement to LO prediction of $B \to \pi$ form factors
  - NLO correction to $B \to \rho$ form factor is still missing

- Electromagnetic form factors Done up to twist three [Li 2010, Cheng 14], [Hua 18]
  - Soft divs cancel themselves in the quark diagrams
  - Collinear divs cancel between the QCD quark diagrams and the effective diagrams of Φ
  - NLO correction gives $\sim 20\%$ enhancement to LO prediction of pion EM form factors

PQCD: Progresses towards to NLO

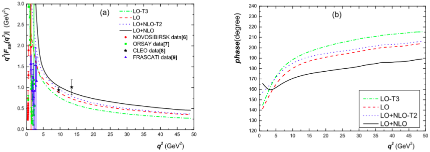
† Scalar form factors   Done up to twist three [Cheng 15]

- ‡ NLO correction gives $\sim -10\%$ enhancement to LO prediction

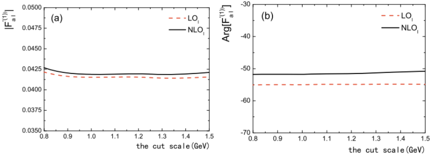
† Timelike form factors in $\mathcal{M}_{e,f}$ [Li 2012, Cheng 14,15]

- ‡ Obtain timelike ffs from spacelike ones by analytical continuation from $-Q^2$ to Q^2, ie. $\ln(-Q^2 - i\epsilon) = \ln(Q^2) - i\pi$
- ‡ Timelike em ff contributes in $\mathcal{M}_{e,f}^{\mathbf{LL},\mathbf{LR}}$ when the final two mesons are not identical
- ‡ Enhance (reduce) the magnitude (phase) of the LO form factor by 20%-30% ($< 15^\circ$)
- ‡ Its correction to $B^0 \to \pi^0\eta^{(\prime)}$ can be expected as approximately 30% with $SU(3)$ flavor breaking

Timelike em form factor

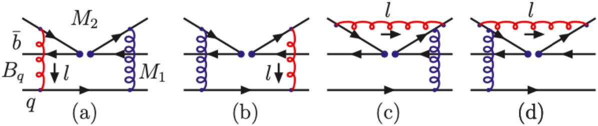


Timelike scalar form factor



- ‡ Timelike scalar ff becomes important in $\mathcal{M}_{e,f}^{\mathbf{SP}}$ when the final two mesons are identical (in this case $\mathcal{M}_{e,f}^{\mathbf{LL},\mathbf{LR}} = 0$)
- ‡ Its correction is very small in size with a large strong phase, main source of large CPV in $B \to \pi^0\pi^0$

- Gluon gluon effect in Spectator emission amplitude $\mathcal{M}_{c,d}$



† Glauber gluon $l \sim (\Lambda^2/m_B, \Lambda^2/m_B, \Lambda)$

† Glauber gluon from the pseudo-NambuGoldstone bosons brings significant effect

† Glauber gluon associated with the heavy B meson is not important and can be ignored

† Glauber effect formulates to an additional phase associated to π meson [Li 2014]

$$\bar{\phi}_M(\mathbf{b}', \mathbf{b}) = \frac{2\beta_M^2}{\pi}\phi_M(x)\exp\left[-2\beta_M^2 x b'^2 - 2\beta_M^2(1-x)b^2\right]. \qquad \text{phase parameter } \beta_M$$

‡ enhances the color-suppressed spectator tree amplitude

‡ changes the interference mode between it with other tree amplitudes, from destructive to instructive

‡ provides a possibility to understand the long-standing $\pi^0\pi^0$ puzzle [Liu 2015]

‡ and the $K\pi$ puzzle $\triangle A_{K\pi} = A_{\rm CP}^{\rm dir}(K^{\pm}\pi^0) - A_{\rm CP}^{\rm dir}(K^{\pm}\pi^{\mp})$ [Liu 2016]

- TMD wave functions

† Non-normalizable (unintegral) B meson DA $\phi_+(k^+, \mu)$ in the collinear factorization

△ divergence ($\sim 1/k^+$) does not break the collinear factoriation at LO, only $\lambda_B^{-1}(\mu) = \int dk^+ \phi_+(k^+, \mu)/k^+$ involved △ emerges at high orders with more moments interplaying △ an ambiguous renormalization of f_B [Li 2004]

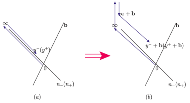
† TMD definition of B meson wave function under HQET

$$\langle 0|\, \bar{q}(y)\, W_y(n)^\dagger\, I_{n,y,0}(n)\, W_0(n)\, \Gamma h(0)\, |\bar{B}(v)\rangle$$
$$= \frac{-if_B m_B}{4} \mathrm{Tr}\Big[\frac{1+\not{v}}{2}\Big(2\phi_+(v^+y^-, y_T^2) + \frac{\phi_+(v^+y^-, y_T^2) - \phi_-(v^+y^-, y_T^2)}{v^+y^-}\not{y}\Big)\gamma_5\Gamma\Big]$$

△ $p_B = m_B v$, the coordinate of field $\bar{q}$ is $y = (0, y^-, \mathbf{b})$, moving along the light cone $n = n_- = (0, 1, \mathbf{0}_T)$ △ $W_y(n) = \mathcal{P}\mathrm{Exp}\left[-ig_s \int_0^\infty d\lambda n \cdot A(y + n\lambda)\right]$ △ $\phi_+(\phi_-)$ is the (sub-)leading twist DAs

† The light cone singularity in the TMD definition (b parameter) when $l \parallel n$

△ rotate the Wilson line away from the light cone
△ alleviate the factorization-scheme dependence by adhering it to a fixed off-shellness $n^2 \neq 0$
△ scheme dependence $\zeta_B^2 = 4(n \cdot p_B)^2/n^2$



△ evolution of DA $\phi_+(k^+, b, \mu) = S(k^+, b, \zeta_B)\, R(b, \mu, \zeta_B)\, \phi_+(k^+, b, 1/b)$ △△ IR $\alpha_s \ln^2(\zeta_B b)$ (resummation), UV $\ln(\mu b)$ (RGE), universal initial condition (soft divs regularized by m_g)

△ the normalization of f_B becomes realized in k_T factorization by $S(k^+, b, \zeta_B)$

$$\int_0^\infty dk^+ \lim_{b \to 1/k^+} \phi_+(k^+, b, \mu) = \int_0^\infty dk^+\, R(1/k^+, \mu, \zeta_B)\, \phi_+(k^+, 1/k^+, \mu)$$

PQCD: Progresses towards to NLO

† Rapidity singularity in the TMD definition with the lightlike Wilson line

$$\phi_\pi^{(1)} \otimes H^{(0)} \supset \int [dl] \frac{1}{[(k+l)^2+i0)][l_+ + i0][l^2+i0]} \times \left[H^{(0)}(x+l_+/p_+, \vec{k}_T + \vec{l}_T) - H^{(0)}(x, \vec{k}_T) \right].$$

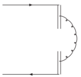
generated due to the Eikonal propagator

△ regularized by rotating the Wilson line away from light cone $n = (n^+, n^-, \mathbf{n}_T)$ and removed by the Collins soft subtraction [Collines 2003]

△ multiple non-light-like Wilson lines with the price of soft function and another scale parameter ρ [Ji 2004]

$$\phi_\pi(x, \vec{k}_T, y_u, \mu) \stackrel{?}{=} \int \frac{dz_-}{2\pi} \int \frac{d^2 z_T}{(2\pi)^2} e^{i(xp_+ z_- - \vec{k}_T \cdot \vec{z}_T)} \times \frac{\langle 0|\bar{q}(0) W_{n_-}^\dagger(+\infty, 0) \not{n}_- \gamma_5 [\text{tr. link}] W_{n_-}(+\infty, z) q(z)|\pi^+(p)\rangle}{\langle 0|W_{n_-}^\dagger(+\infty,0) W_u(+\infty,0) [\text{tr. link}] W_{n_-}(+\infty,z) W_u^\dagger(+\infty,z)|0\rangle}.$$

soft subtraction



Wilson line self energies in the TMD wave function

↓

$$\phi_\pi \supset \int [dl] \frac{u^2}{[l+i0)][u \cdot l + i0][u \cdot l - i0]} \times \delta(x' - x + l_+/p_+) \delta^{(2)}(\vec{k}'_T - \vec{k}_T + \vec{l}_T).$$

△ pinch singularity with $n^2 \neq 0$ [Bacchetta 2008]

△△ corresponds to the linear divergence in the length of the Wilson line in the coordinate space

$$\phi_\pi^C(x, \vec{k}_T, y_2, \mu) = \lim_{\substack{y_1 \to +\infty \\ y_u \to -\infty}} \int \frac{dz_-}{2\pi} \int \frac{d^2 z_T}{(2\pi)^2} e^{i(xp_+ z_- - \vec{k}_T \cdot \vec{z}_T)} \times \langle 0|\bar{q}(0) W_u^\dagger(+\infty,0) \not{n}_- \gamma_5 [\text{tr. link}] W_u(+\infty,z) q(z)|\pi^+(p)\rangle \times \sqrt{\frac{S(z_T; y_1, y_2)}{S(z_T; y_1, y_u) S(z_T; y_2, y_u)}}.$$

$$S(z_T; y_A, y_B) = \frac{1}{N_c} \langle 0|W_{n_B}^\dagger(\infty, \vec{z}_T)_{ca} W_{n_A}(\infty, \vec{z}_T)_{ad} W_{n_B}(\infty, 0)_{bc} W_{n_A}^\dagger(\infty, 0)_{db}|0\rangle.$$

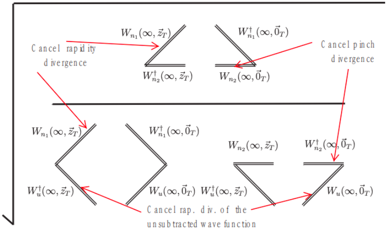
new soft function

△ Collins modification of TMD wave function with out pinch singularity [Collins 2011]

△△ rapidity of the gauge vector $n_2 = (e^{y_2}, e^{-y_2}, \mathbf{0}_T)$

△△ unsubtracted wave function only involves light cone Wilson lines △△ each soft factor has at most one off-light-cone Wilson line △△ rapidity safe and pinch safe [Collins 2014]

PQCD: Progresses towards to NLO



cancellation mechanism of the new Collins definition [borrowed from Y.M Wang]

△ Li-Wang definition with non-dipole Wilson line [Li 2015]

$$\phi_\pi^{\rm I}(x,\vec k_T,y_2,\mu) = \int\frac{dz_-}{2\pi}\int\frac{d^2z_T}{(2\pi)^2}\, e^{i(xp_+z_- - \vec k_T\cdot\vec z_T)}$$
$$\times\langle 0|\bar q(0)W^\dagger_{n_2}(+\infty,0)\not n_-\gamma_5\,[\text{links}@\infty]\,W_v(+\infty,z)\,q(z)|\pi^+(p)\rangle\,.$$

orthogonal Wilson lines $n_2\cdot v = 0$

△△ $n_2 = (e^{y_2}, e^{-y_2}, \mathbf{0}_T)$ and $v = (-e^{y_2}, e^{-y_2}, \mathbf{0}_T)$
△△ the Wilson-line self energies vanishes an hence no pinch singularity in the Feynman gauge

△△ reproduces the collinear logarithm of QCD diagrams for $\phi^I_\pi\otimes H^{(0)}$

‡ Exclusive B decays at NLO: △ $\ln^2(\zeta_B^2/m_B^2)$, $\ln x\ln(\zeta_B^2/m_B^2)$ △ resummed to all order by resolving the evolution equation of Φ_B on ζ_B^2 △ suppresses the shape of $\phi_+(k^+, b, \mu)$ near the end point $k^+\to 0$ [Li 2013]

† Joint singularity in the pion-photon form factor [Li 2014]

△ $\ln x\ln(\zeta_\pi^2/k_T^2)$ △ joint resummation △ strong suppression for small x and large b △ joint resummation improved pion wave function does not bring sizable corrections