



# Lattice QCD prediction of Pion & Kaon electromagnetic form factors at very large $Q^2$

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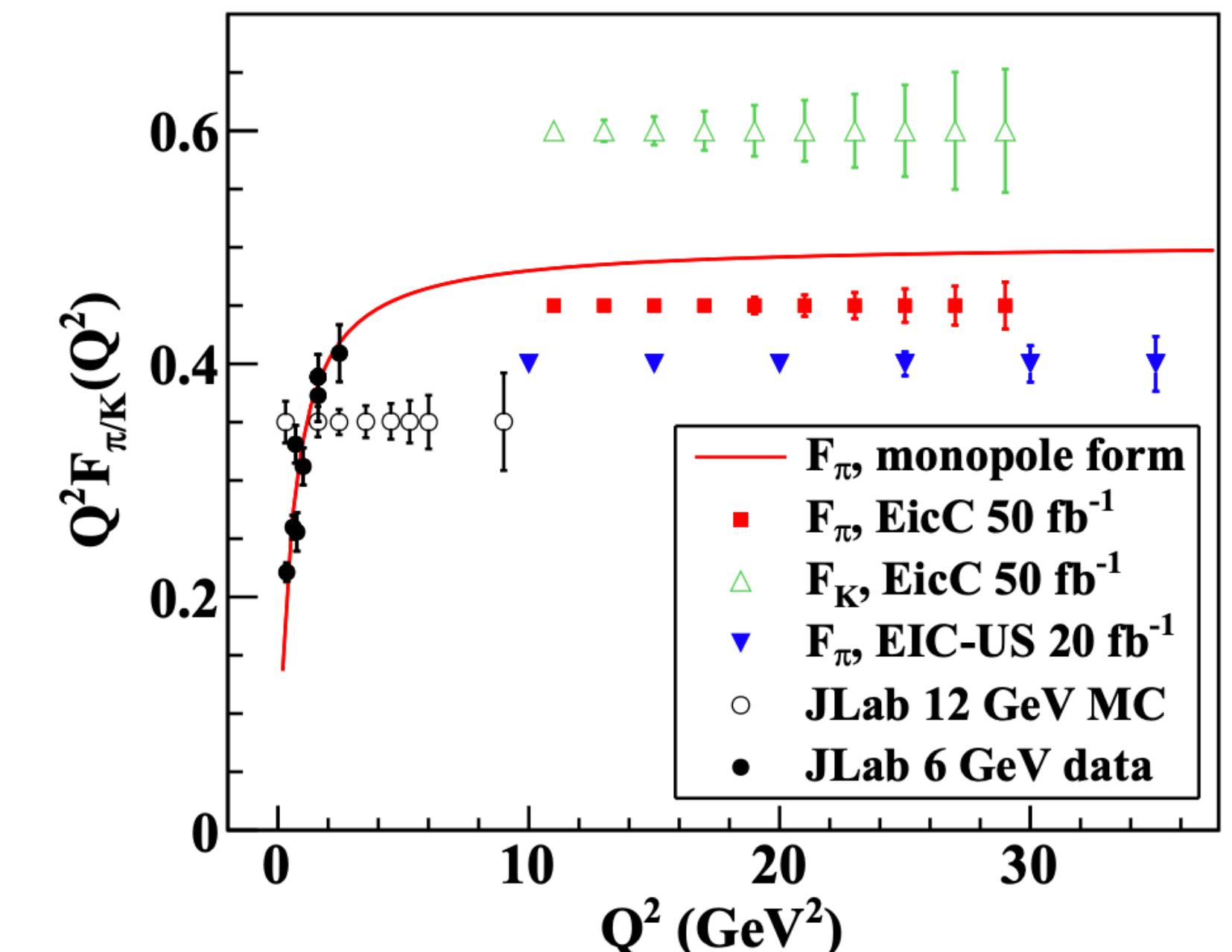
Work in progress & in collaboration with  
X. Gao(高翔), A.D. Hanlon, N. Karthik, S. Mukherjee, P. Petreczky,  
Q. Shi(施岐), P. Scior, S. Syritsyn and Y. Zhao

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# Hadron Electromagnetic (EM) form factor

$$\langle H(P_1) | J_\mu | H(P_2) \rangle = (P_1 + P_2)_\mu F_H(Q^2)$$

- ✿ Insights into hadron structure, i.e. on the charge distribution
- ✿ Together with PDF produces General Parton Distribution (GPD), i.e. a 3-d image of hadron
- ★ Experiments: Jlab, EiC, EicC
- ★ DSE, QCD sum rules, lattice QCD...



EicC white paper, arXiv:2102.09222

# Pion/kaon EM form factors

**Small  $Q^2$  limit:** hadronic picture

- Vector Meson Dominance  $\rightarrow$  Charge radius

$$r_{eff}^2(Q^2) = \frac{6(1/F_\pi(Q^2) - 1)}{Q^2}.$$

$$\langle r_\pi^2 \rangle = 0.42(2) \text{ fm}^2, \langle r_\pi^2 \rangle_{PDG} = 0.434(5) \text{ fm}^2$$

**Large  $Q^2$  limit:** partonic picture

$$Q^2 F_M(Q^2) \approx 16\pi \alpha_s(Q^2) f_M^2 \omega_M^2(Q^2), \quad \omega_M^2(Q^2) = e_{\bar{q}} \omega_{\bar{q}}^2(Q^2) + e_u \omega_u^2(Q^2)$$

Lepage & Brodsky, 79', 80'  
Efremov & Radyushkin 80'

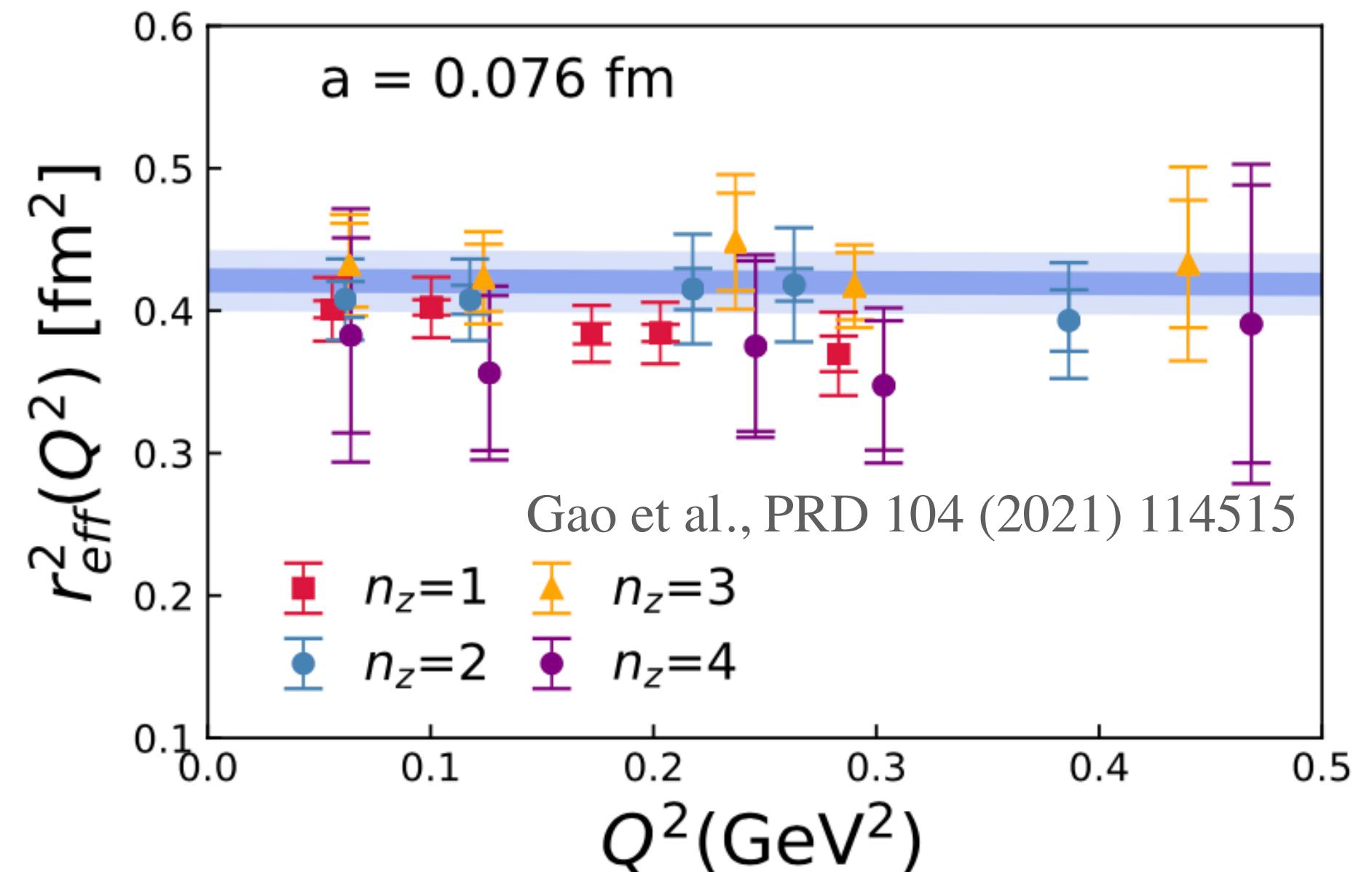
$$\omega_f = \frac{1}{3} \int_0^1 dx q_f(x) \phi_M(x, Q^2)$$

leading-twist parton distribution amplitude (DA)

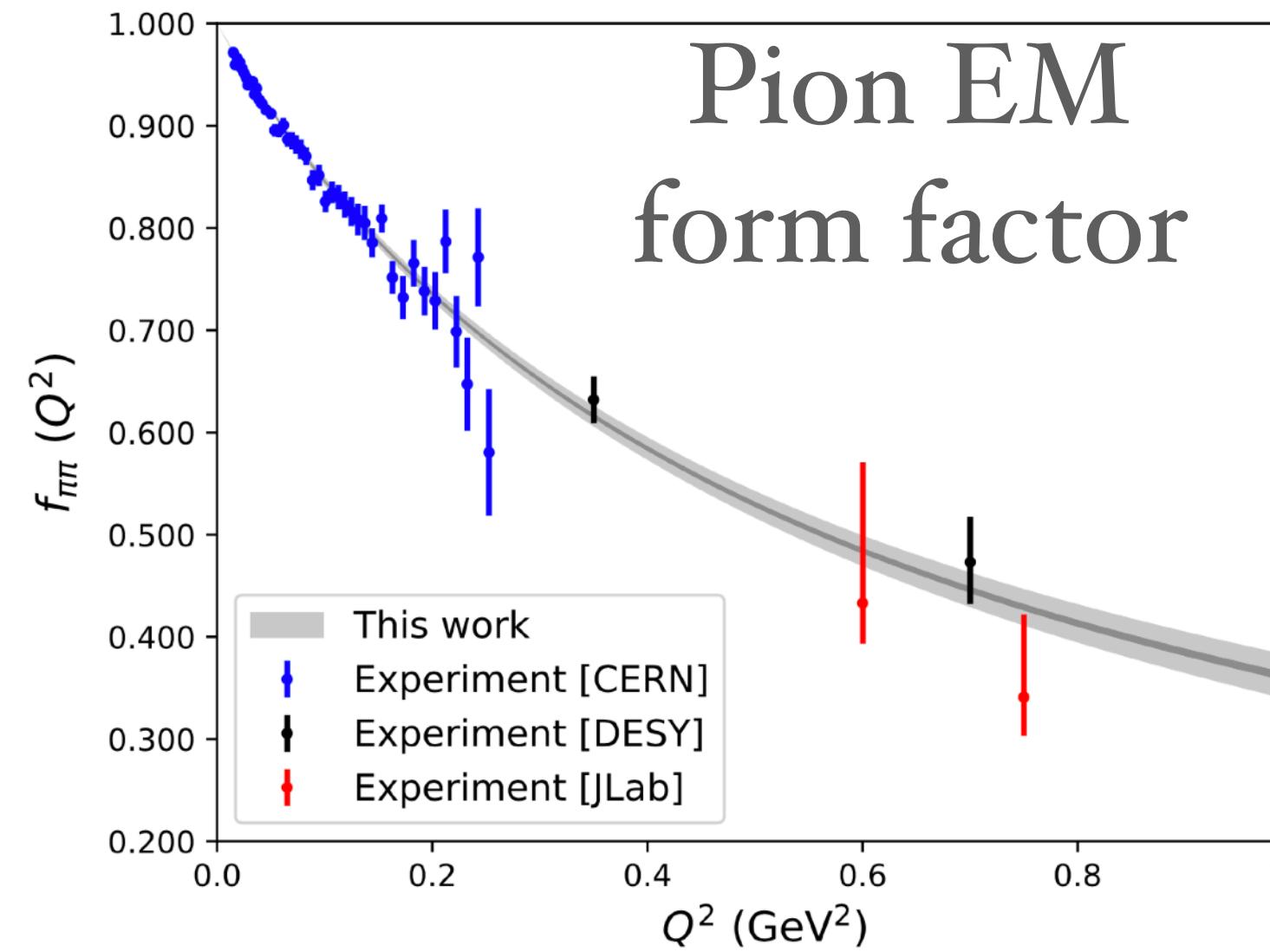
- Asymptotic DA:  $\phi_M(x, Q^2 \rightarrow \infty) = 6x(1-x)$

- DA from LQCD: pion & kaon etc. J. Hua et al. [LPC], Phys.Rev.Lett. 129 (2022) 13

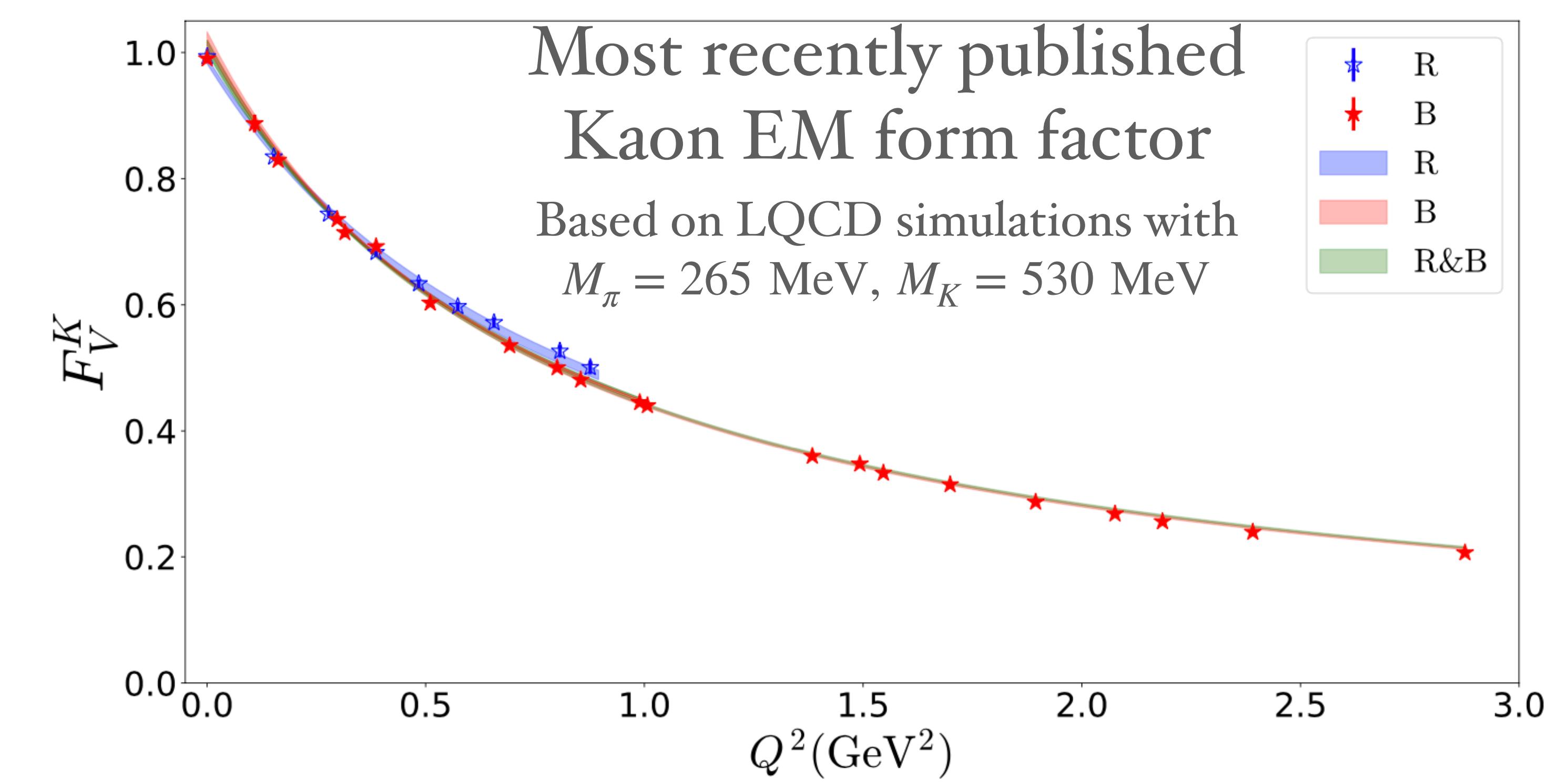
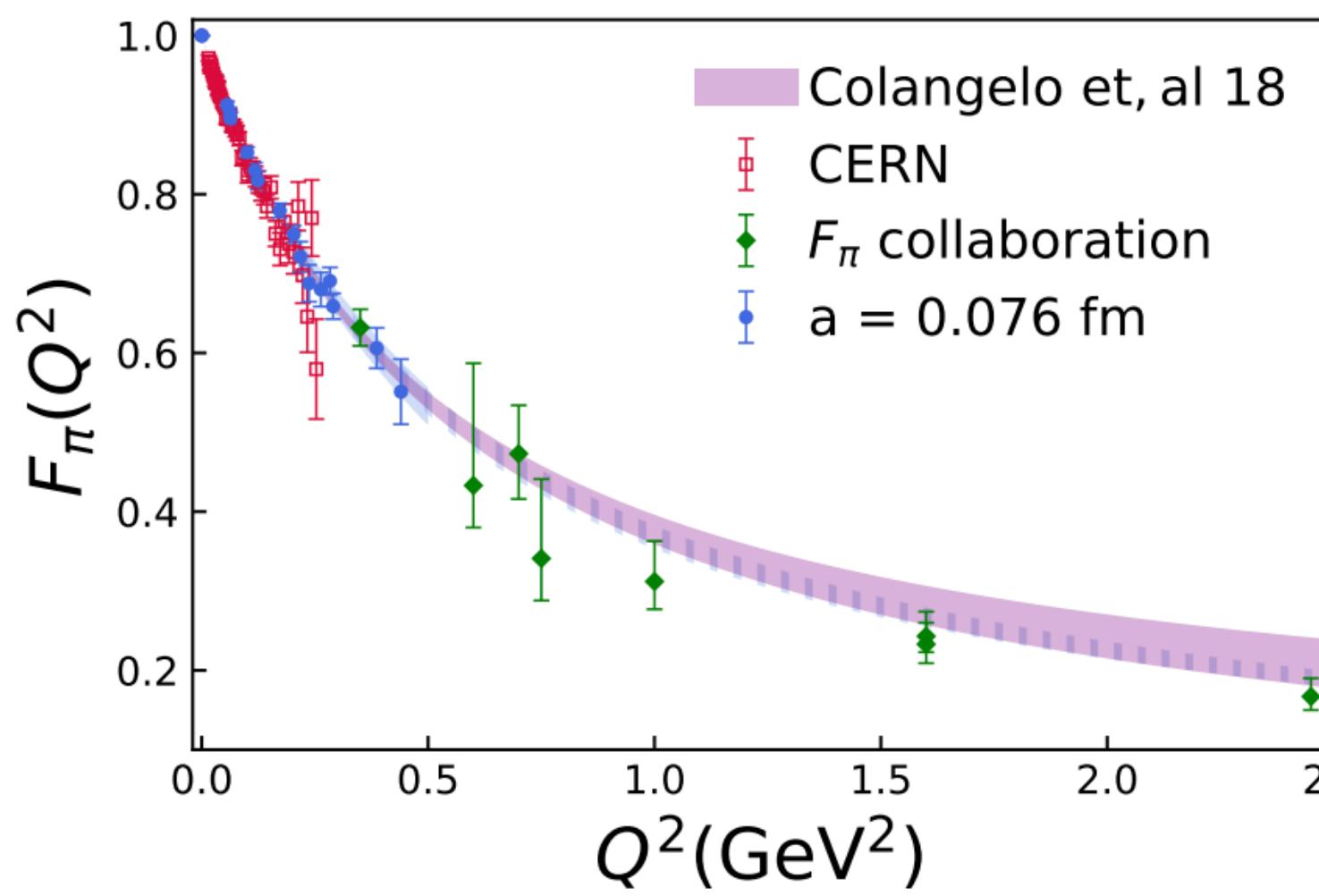
Gao et al., PRD 106 (2022) 074505, G. Bali et al., JHEP 08 (2019) 065, ...



# Current status: pion/kaon EM form factors from Lattice QCD



高翔 et al. Tsinghua-BNL-ANL, PRD 104 (2021) 114515

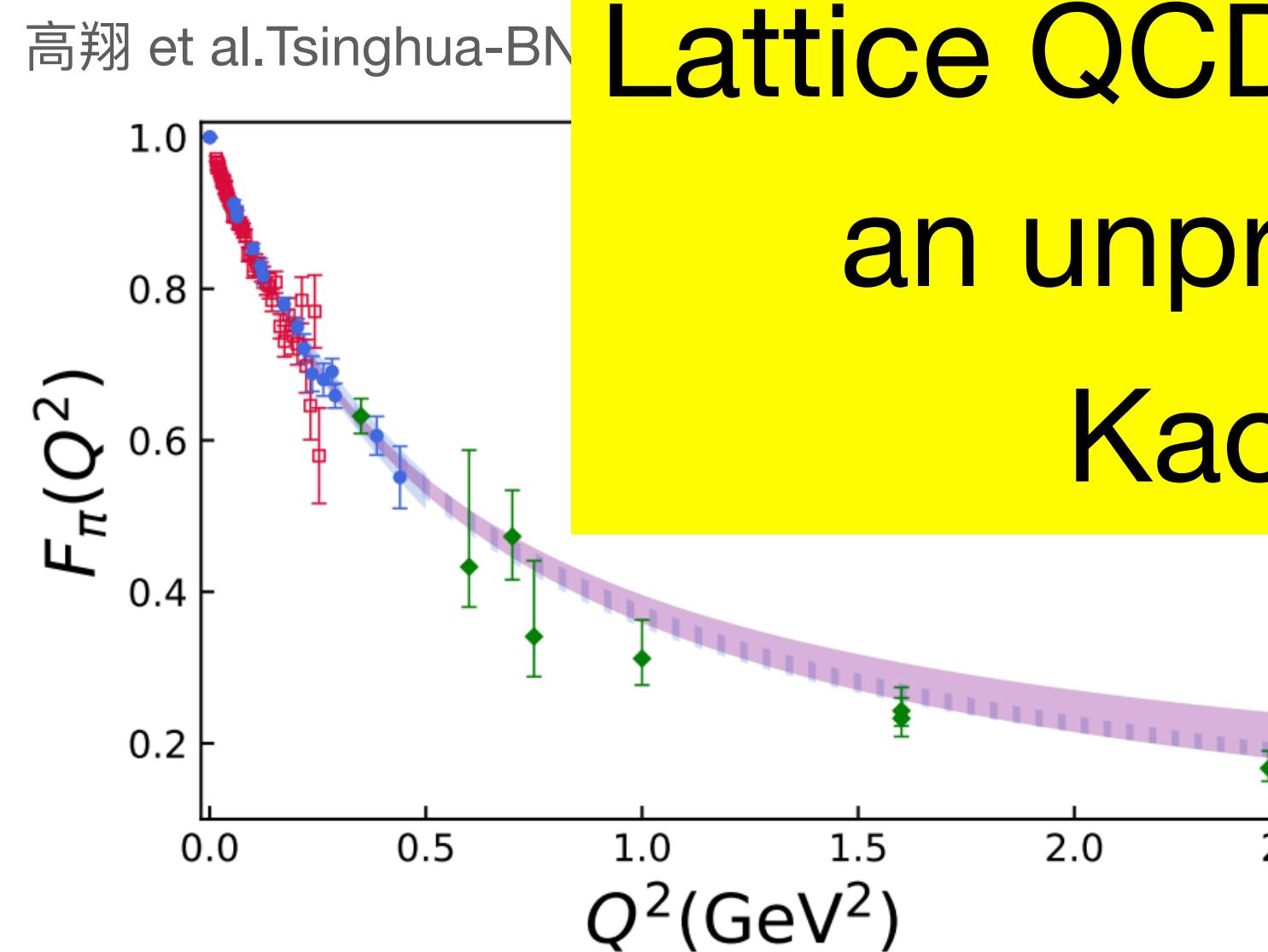
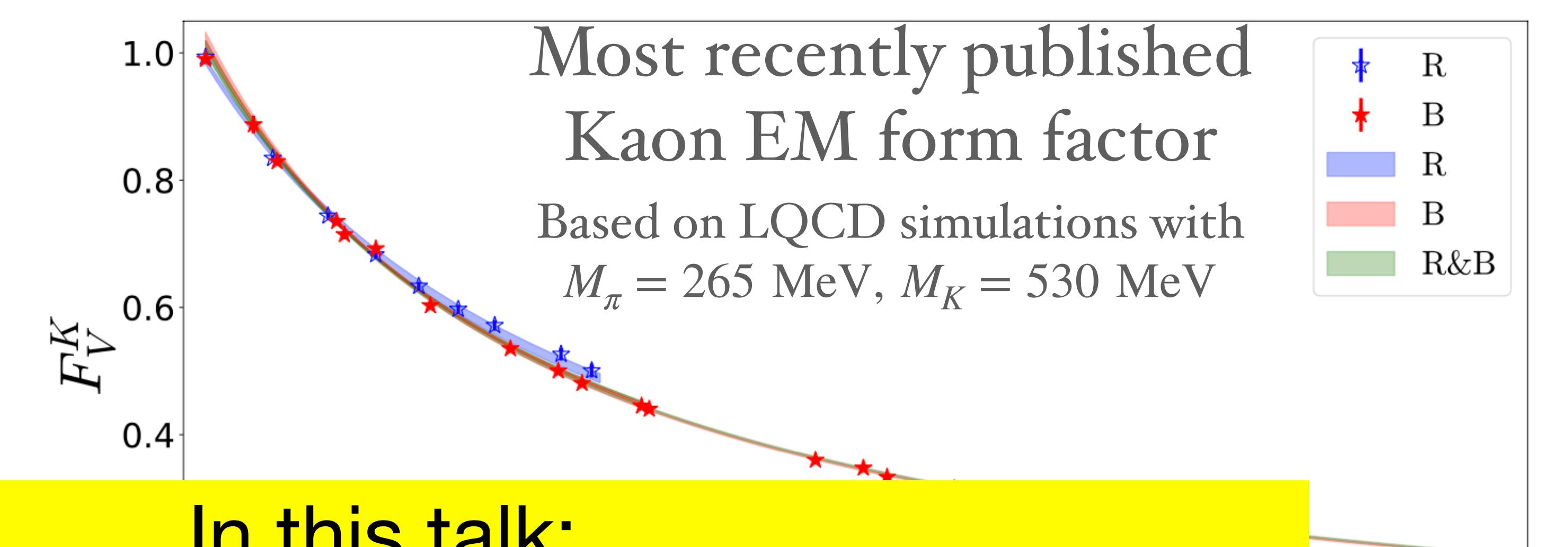
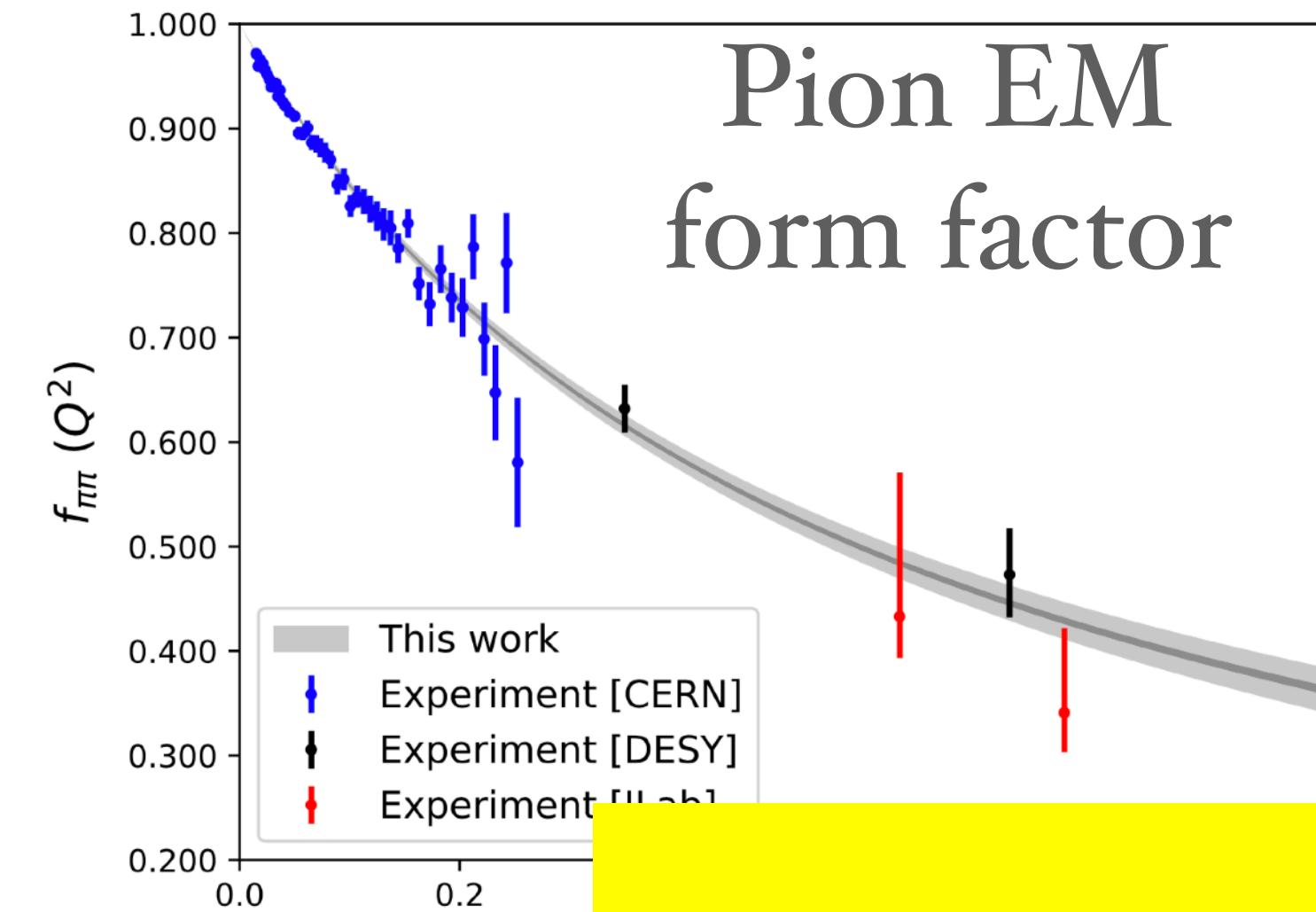


Alexandrou et al., [ETMC], Phys.Rev.D 105 (2022) 5, 054502

Many computations on the pion form factor,  
but much less on kaon

Mostly restricted to  $Q^2 \lesssim 3 \text{ GeV}^2$

# Current status: pion/kaon EM form factors from Lattice QCD



In this talk:  
Lattice QCD prediction of EM form factors at up to  
an unprecedented large (**intermediate**)  $Q^2$ :  
Kaon  $\sim 28$  GeV $^2$ , Pion  $\sim 10$  GeV $^2$

Many computations on the pion form factor,  
but much less on kaon

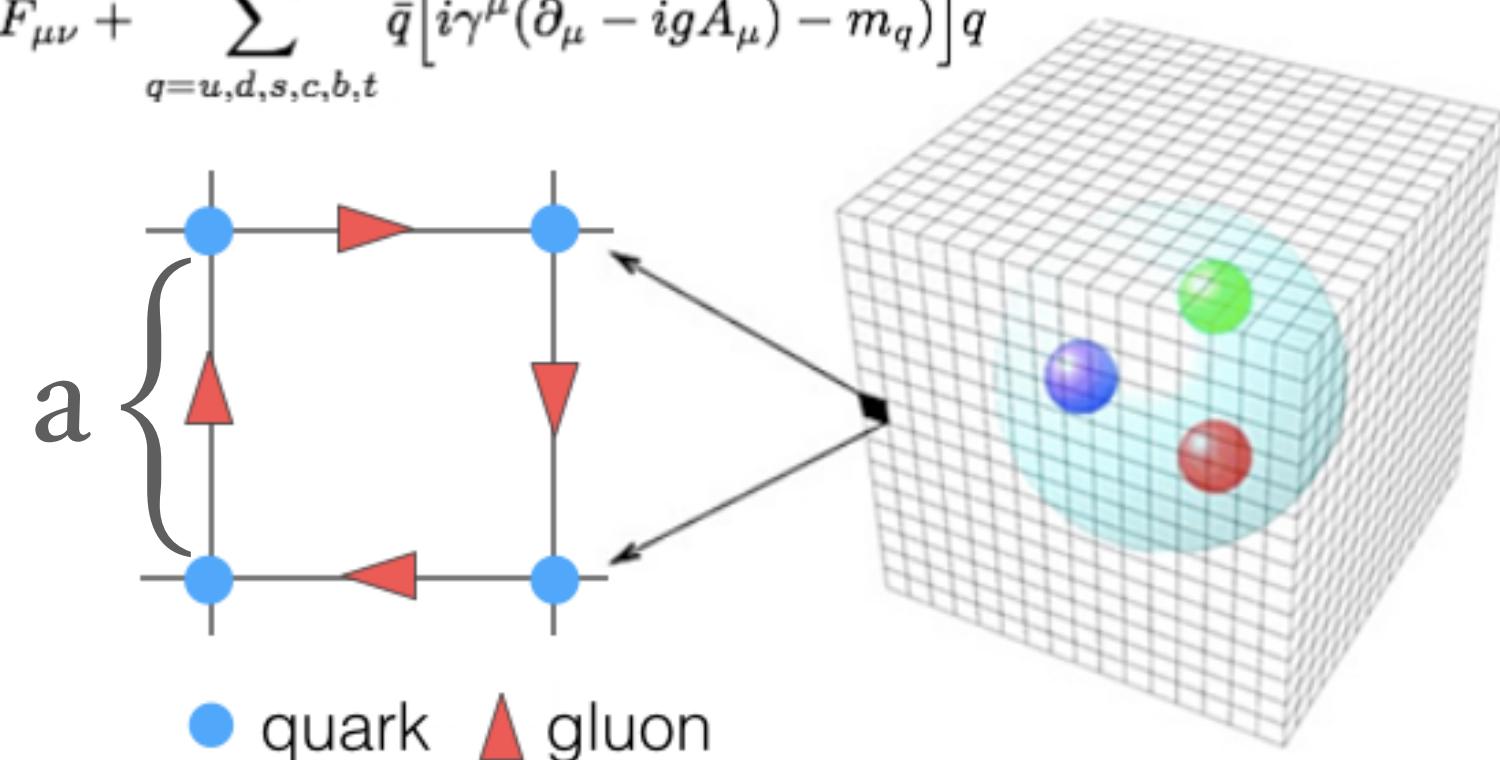
Mostly restricted to  $Q^2 \lesssim 3$  GeV $^2$

# The Lattice QCD

- Lattice simulations of QCD give first principle results

QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_{q=u,d,s,c,b,t} \bar{q}\left[i\gamma^\mu(\partial_\mu - igA_\mu) - m_q\right]q$$



- But need to have control of

- ♣ Thermodynamic limit  $V = 2 \sim 4 \text{ fm}$   $V \rightarrow \infty$
- ♣ Continuum limit  $a = 0.1 \sim 0.04 \text{ fm}$   $a \rightarrow 0$
- ♣ Chiral extrapolation  $M_\pi \sim 500 \rightarrow 200 \text{ MeV}$   $M_\pi = 140 \text{ MeV}$   
(Physical Point)
- ♣ Statistical errors  $N_{conf} \sim \mathcal{O}(1000)$   $N_{conf} \rightarrow \infty$

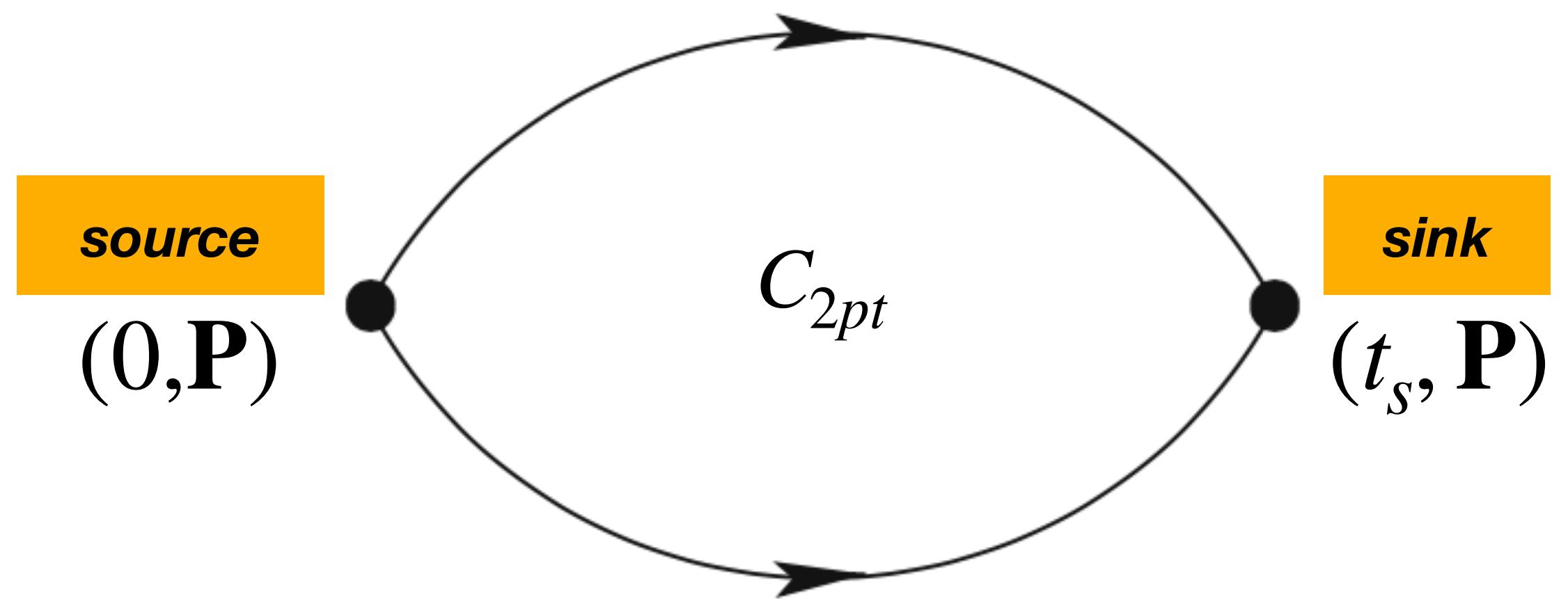
- Fast computers and algorithms are essential



EM form factor of Kaon:  $\langle K(P_1) | J_\mu | K(P_2) \rangle = (P_1 + P_2)_\mu F_K(Q^2)$

# Kaon at nonzero momentum

- Two point kaon correlation function



$$C_{2\text{pt}}(\mathbf{P}, t_s) = \langle [K(\mathbf{P}, t_s)][K(\mathbf{P}, 0)]^\dagger \rangle$$

$$K(\mathbf{P}, t) = \sum_{\mathbf{x}} \bar{s}(\mathbf{x}, t) \gamma_5 u(\mathbf{x}, t) e^{-i\mathbf{P}\cdot\mathbf{x}}$$

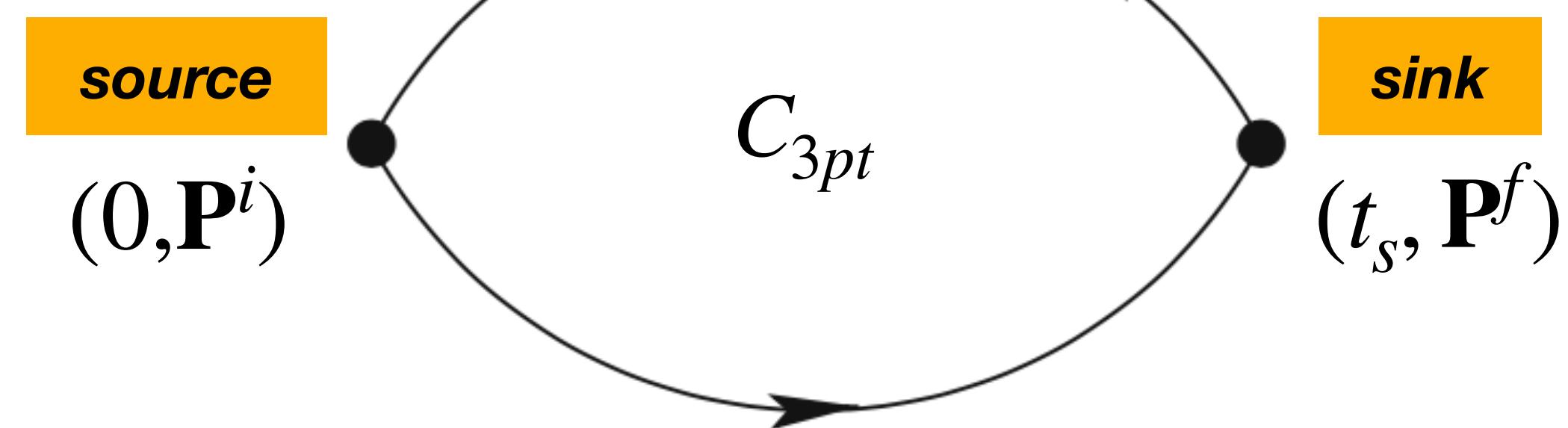
$$\mathbf{P} = \frac{2\pi}{N_\sigma} \mathbf{n} a^{-1}$$

- Determine energy of states from the energy decomposition:

$$C_{2\text{pt}}(\mathbf{P}, t_s) = \sum_{n=0}^{N_{\text{state}}-1} |\langle \Omega | K_S | n; \mathbf{P} \rangle|^2 (e^{-E_n t_s} + e^{-E_n (aL_t - t_s)})$$

# Three point correlation function

$$O_\Gamma = \left( \frac{2}{3} \bar{u} \gamma_\mu u - \frac{1}{3} \bar{s} \gamma_\mu s \right)$$



$$C_{3pt}(\mathbf{P}^f, \mathbf{P}^i, \tau, t_s) = \langle [K_{S^f}(\mathbf{P}^f, t_s)] O_\Gamma(\mathbf{q}, \tau) [K_{S^i}^\dagger(\mathbf{P}^i, 0)]^\dagger \rangle$$

$$\mathbf{P}^i = \mathbf{P}^f - \mathbf{q}$$

$$Q^2 = -(\mathbf{P}^i - \mathbf{P}^f)^2$$

$$C_{3pt}(\mathbf{P}^f, \mathbf{P}^i; \tau, t_s) = \sum_{m,n} \langle \Omega | K_{S^f} | m; \mathbf{P}^f \rangle \langle m; \mathbf{P}^f | O_\Gamma | n; \mathbf{P}^i \rangle \langle n; \mathbf{P}^i | K_{S^i}^\dagger | \Omega \rangle \times e^{-(t_s - \tau) E_m^f} e^{-\tau E_n^i}$$

EM form factor:

Bare matrix element of kaon ground state  $F^B(Q^2) = \langle 0; \mathbf{P}^f | O_\Gamma | 0; \mathbf{P}^i \rangle$

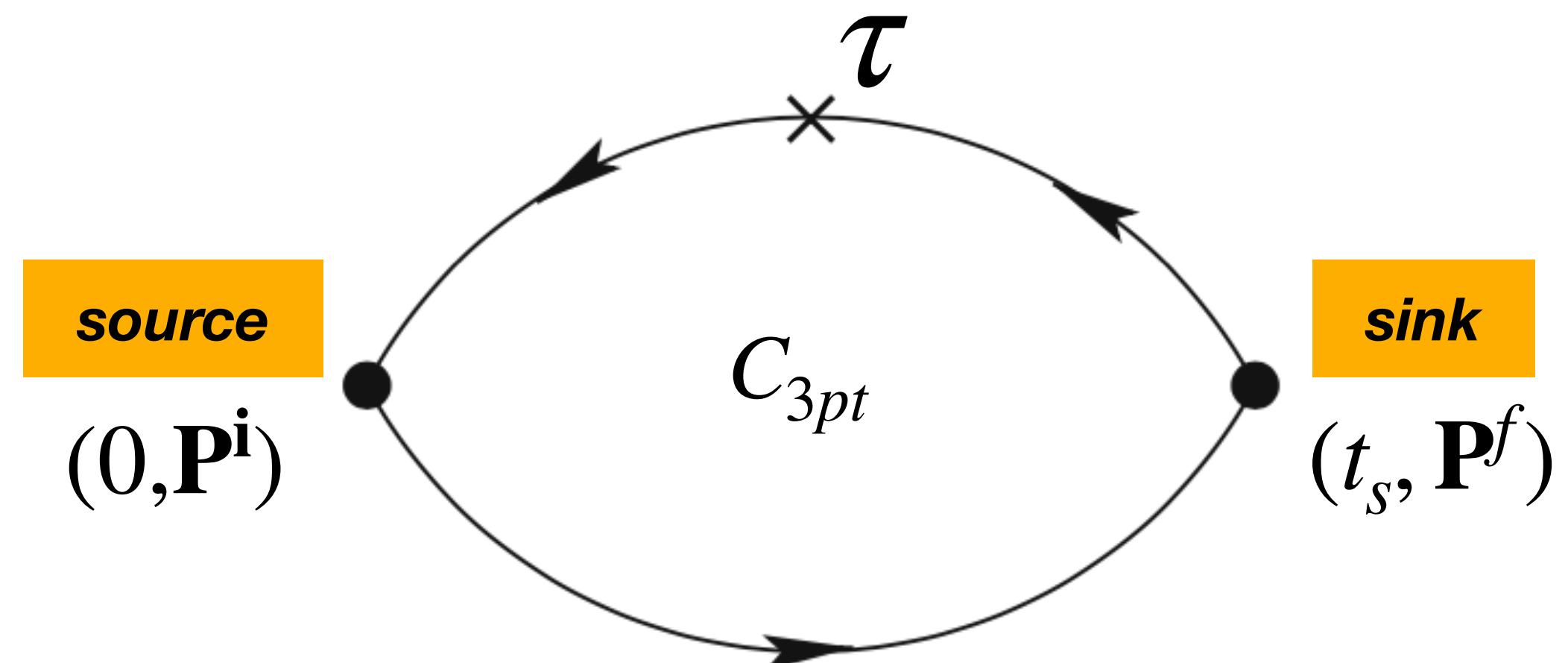
# Extraction of bare form factor

- Construct the ratio between 3 and 2-pt corr.:

$$R^{fi}(\mathbf{P}^f, \mathbf{P}^i; \tau, t_s) \equiv \frac{2\sqrt{E_0^f E_0^i}}{E_0^f + E_0^i} \frac{C_{3pt}(\mathbf{P}^f, \mathbf{P}^i; \tau, t_s)}{C_{2pt}(t_s, \mathbf{P}^f)} \times \left[ \frac{C_{2pt}(t_s - \tau, \mathbf{P}^i) C_{2pt}(\tau, \mathbf{P}^f) C_{2pt}(t_s, \mathbf{P}^f)}{C_{2pt}(t_s - \tau, \mathbf{P}^f) C_{2pt}(\tau, \mathbf{P}^i) C_{2pt}(t_s, \mathbf{P}^i)} \right]^{1/2}$$

- Bare form factor:

$$F^B(Q^2) = \lim_{\tau \rightarrow \infty, t_s \rightarrow \infty} R^{fi}(\mathbf{P}^f, \mathbf{P}^i, \tau, t_s)$$

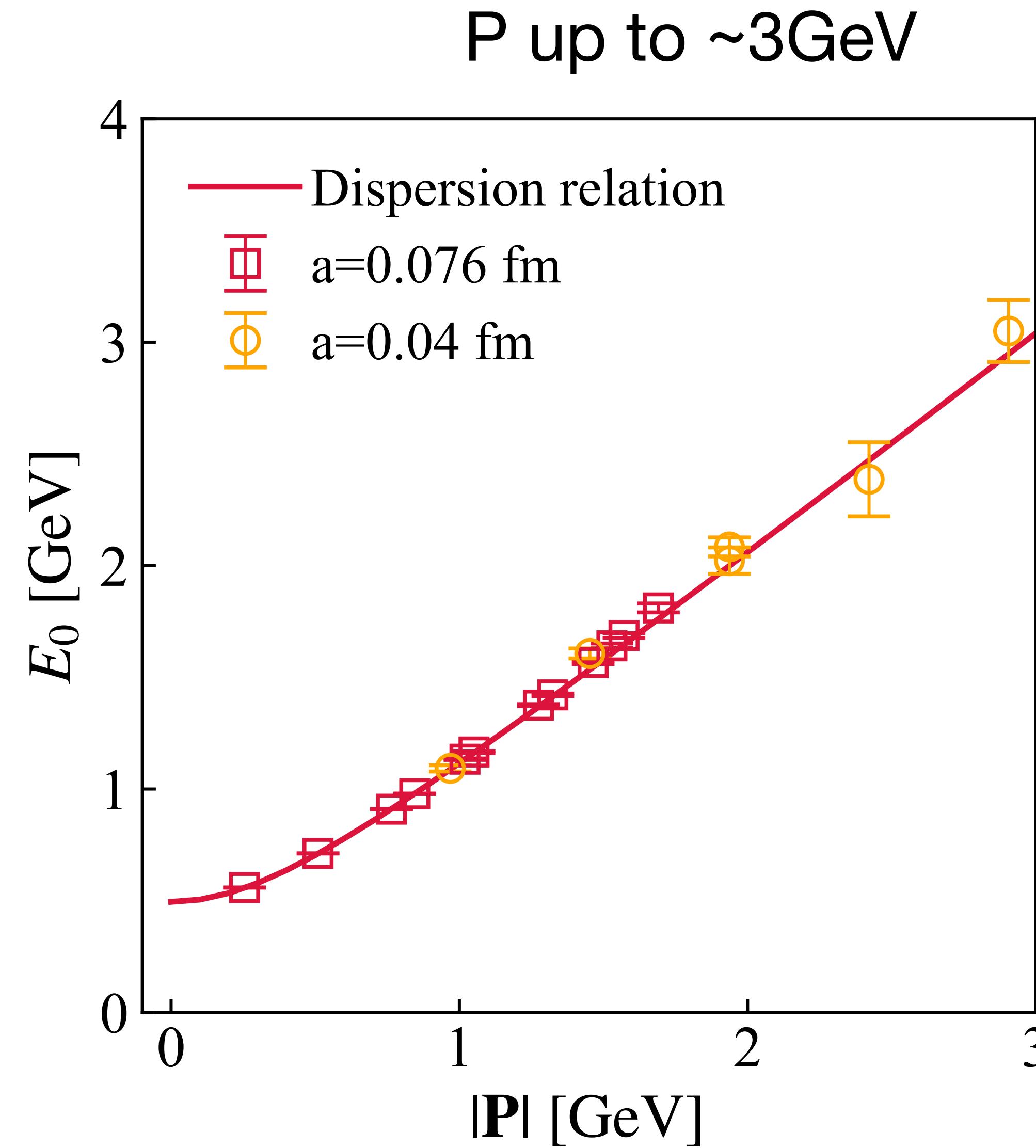
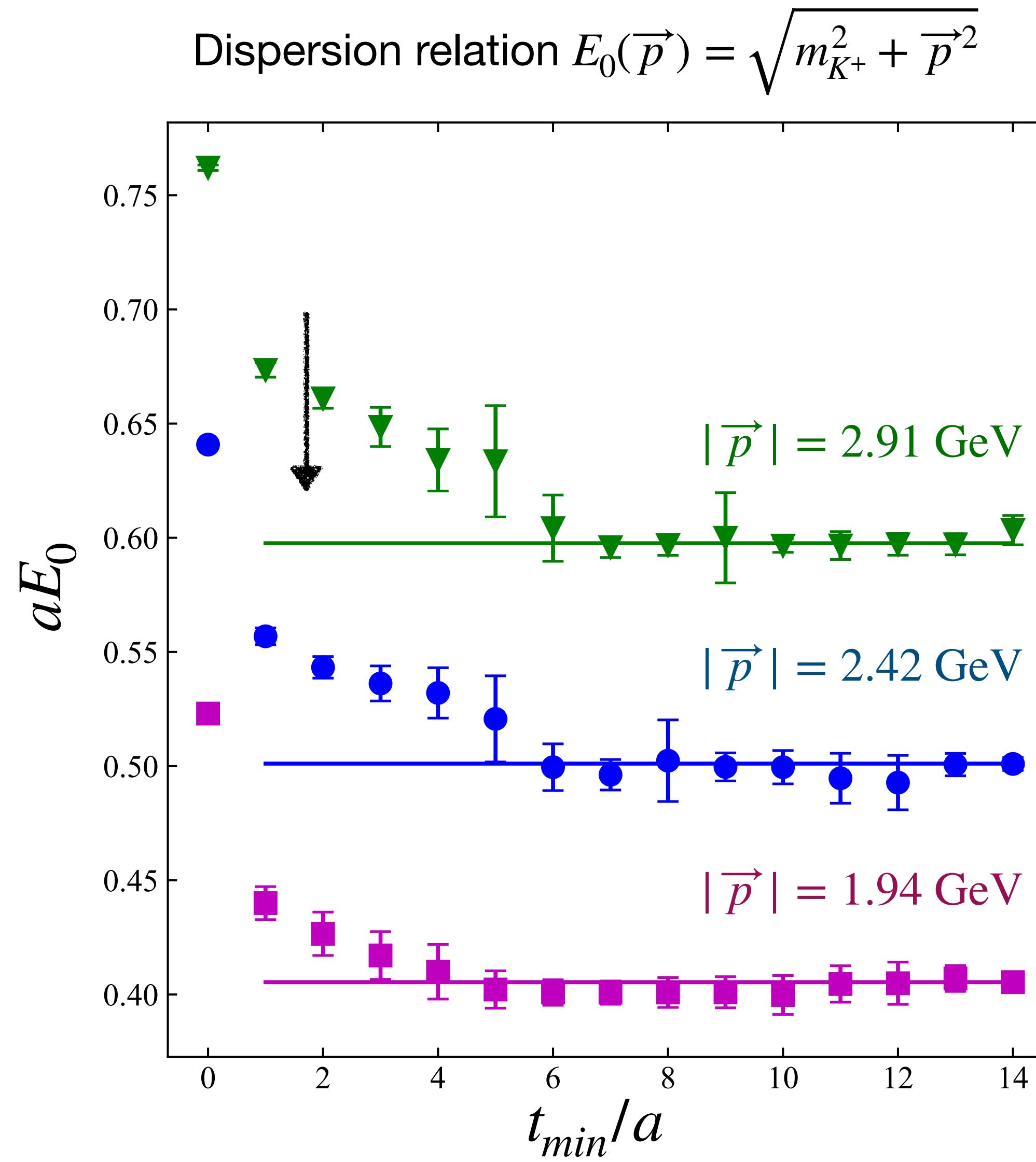


- Form factor:  $F(Q^2) = F^B \times Z_V$

# Lattice setup

- $N_f=2+1$  QCD on  $64^3 \times 64$  lattices with  $a=0.076$  &  $0.04$  fm ([HotQCD] configurations)
  - Sea quark: Highly Improved Staggered Quark (HISQ) action
  - Valence quark: Wilson-Clover action
- At the physical point:  $M_{\pi^+} = 140$  MeV,  $M_{K^+} = 497$  MeV
- Boost smearing with the corresponding signs of the quark momenta at source & sink
  - Pion: up to  $10$  GeV $^2$  with  $a = 0.076$  fm
  - Kaon: up to  $28$  GeV $^2$  with  $a = 0.076$  &  $0.04$  fm

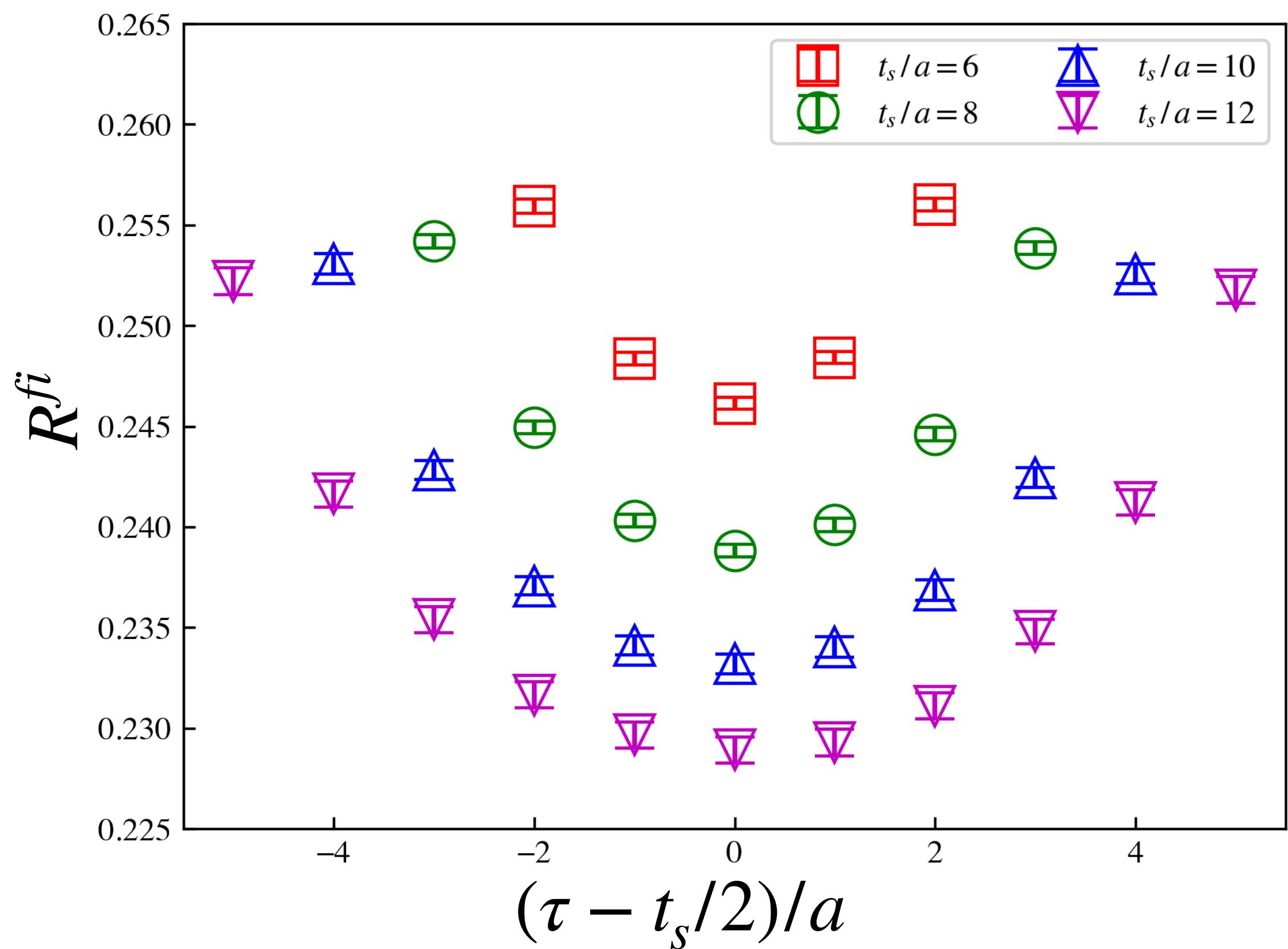
# Kaon at large momentum



# Lattice data of $R^{fi} \sim C_{3pt}/C_{2pt}$

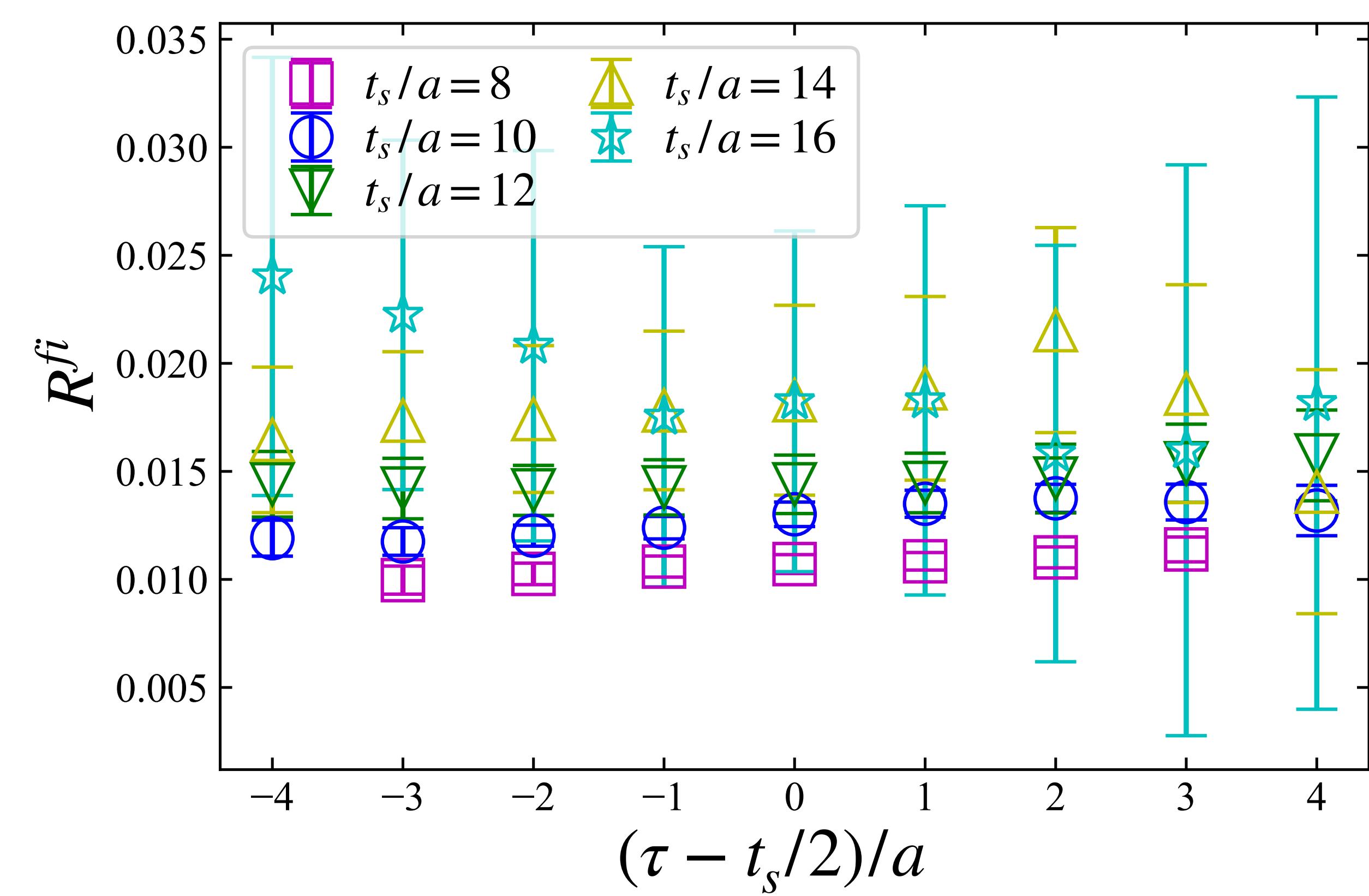
$$Q^2 = 2.34 \text{ GeV}^2$$

$$\mathbf{P}^i = (0, 0, 0.76) \text{ GeV}, \mathbf{P}^f = (0, 0, -0.76) \text{ GeV}$$



$$Q^2 = 28.37 \text{ GeV}^2$$

$$\mathbf{P}^i = (0, 0, 2.91) \text{ GeV}, \mathbf{P}^f = (0, 0, -2.42) \text{ GeV}$$

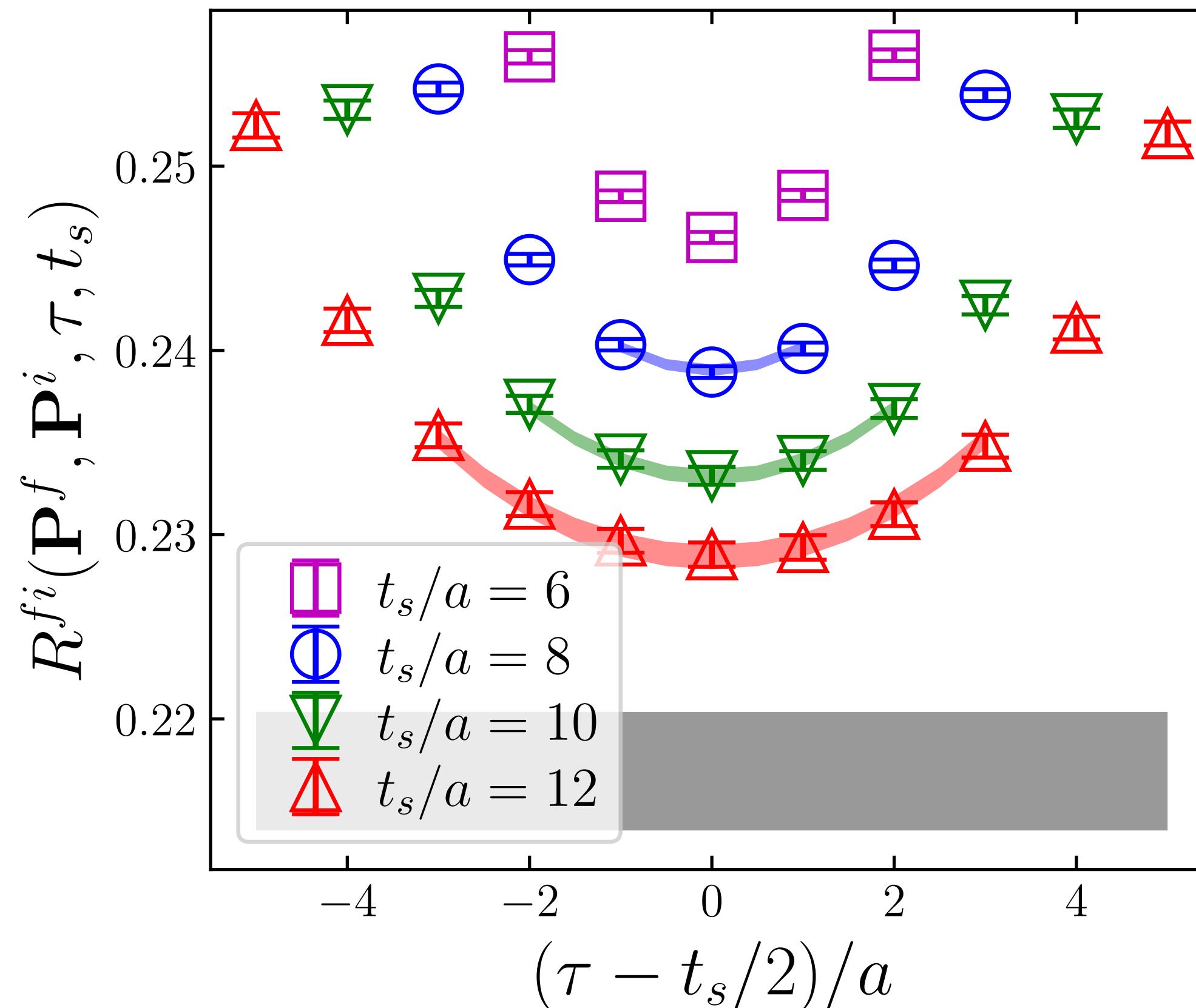


# Extraction of the form factor

$$N_{state} = 2: R^{fi}(\tau, t_s) = \left( \frac{\mathcal{O}_{00}}{F^B} + \frac{A_1}{A_0} \mathcal{O}_{11} e^{-t_s \Delta E} + \sqrt{\frac{A_1}{A_0}} \mathcal{O}_{01} e^{-\tau \Delta E} + \sqrt{\frac{A_1}{A_0}} \mathcal{O}_{10} e^{-(t_s - \tau) \Delta E} \right) / \left( 1 + \frac{A_1}{A_0} e^{-t_s \Delta E} \right), \Delta E = E_1 - E_0$$

$$Q^2 = 2.34 \text{ GeV}^2$$

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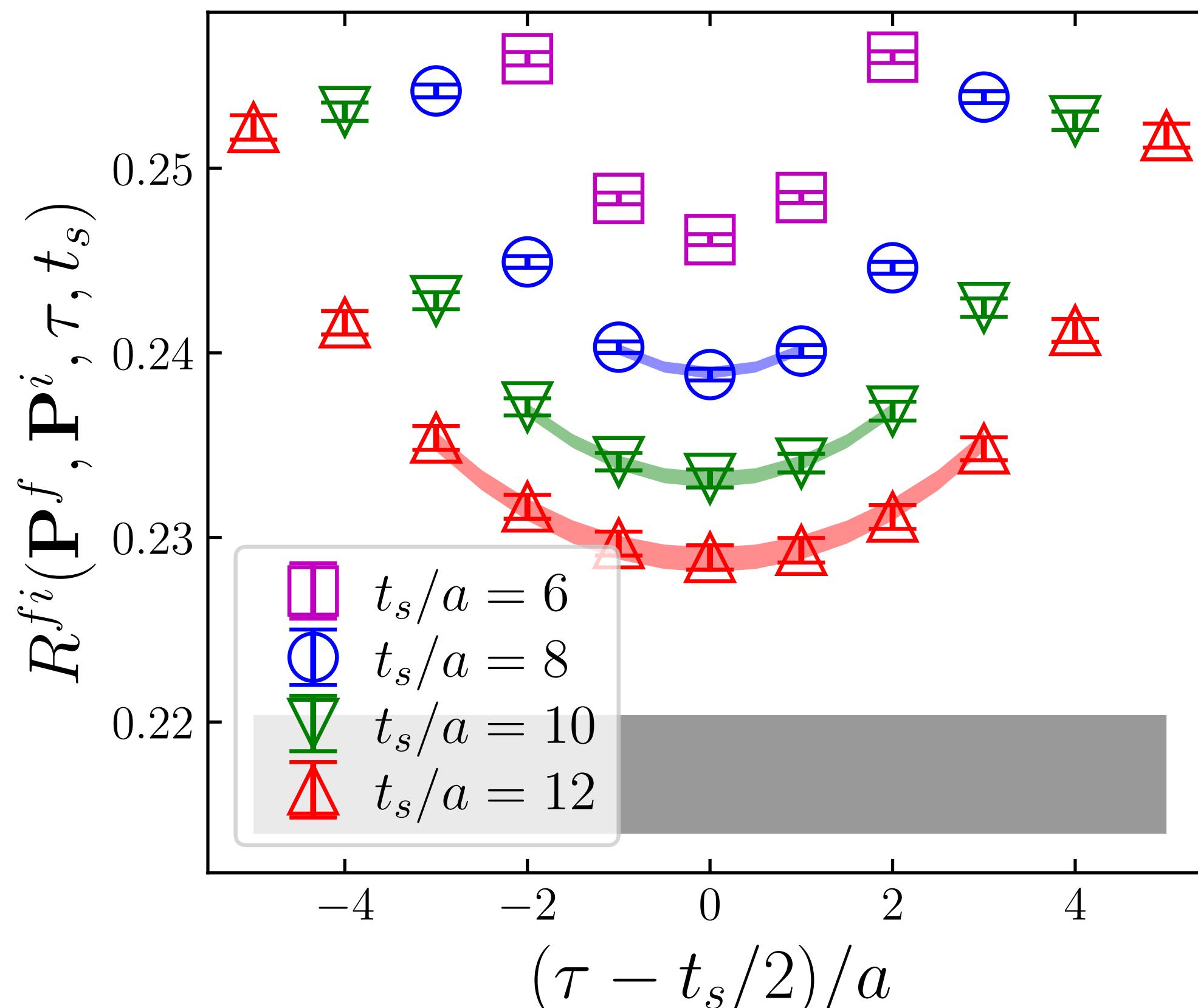
- Perform a 4-parameter fit to the ratio  $R^{fi}$  to extract  $F^B$
- Use the values of energy  $E_n$  and amplitude  $A_n$  extracted from  $C_{2pt}$

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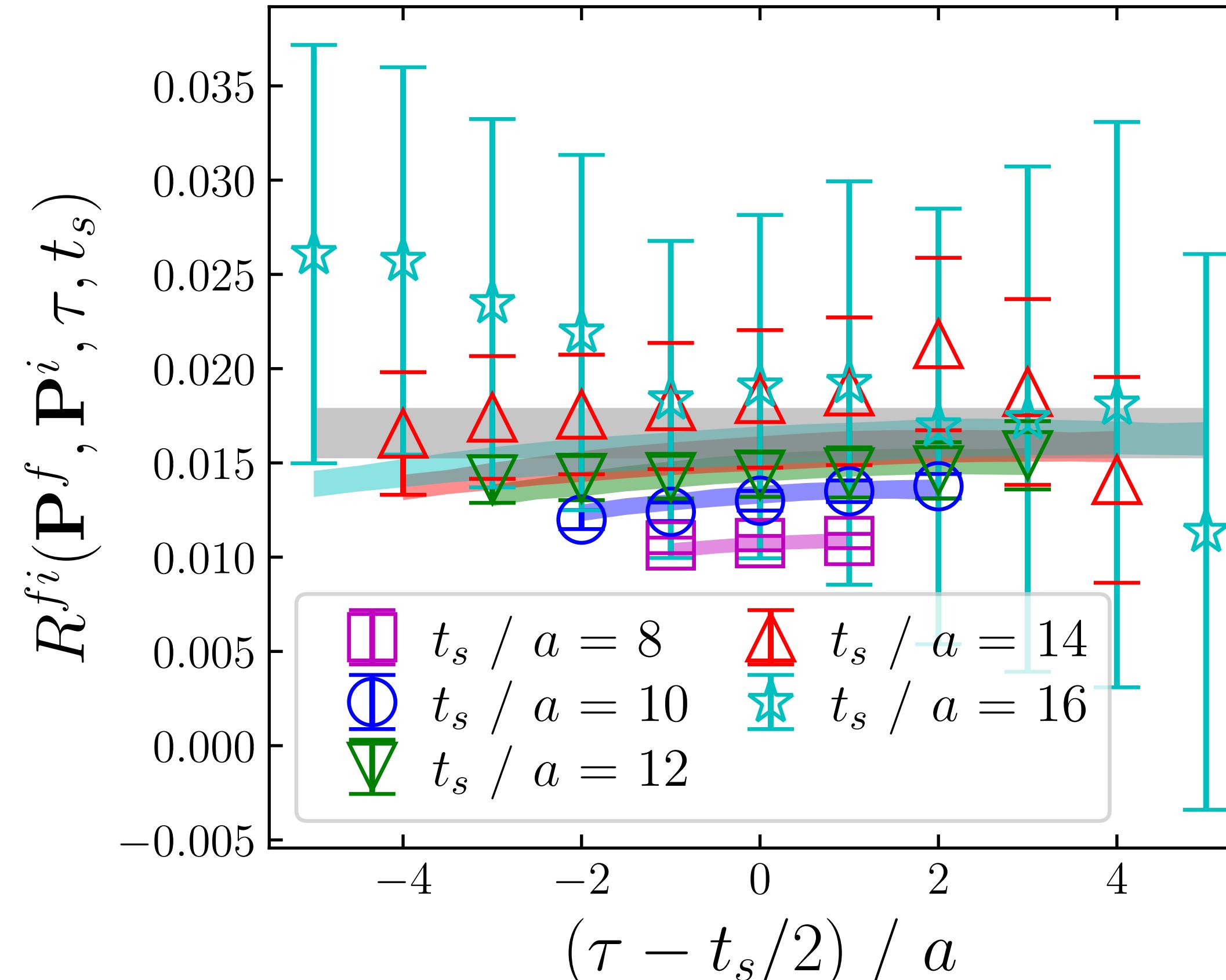
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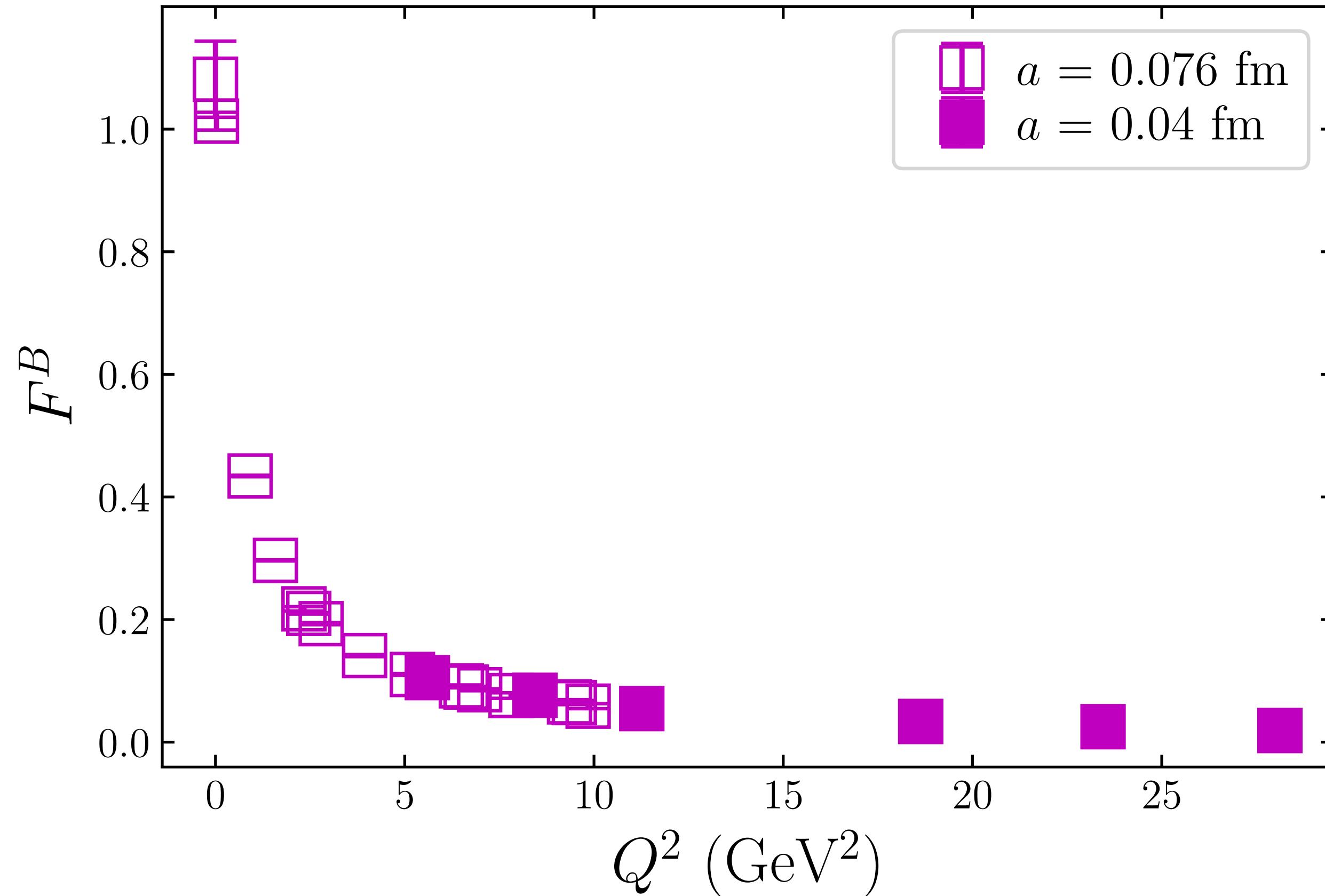


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# Bare form factor as a function of $Q^2$



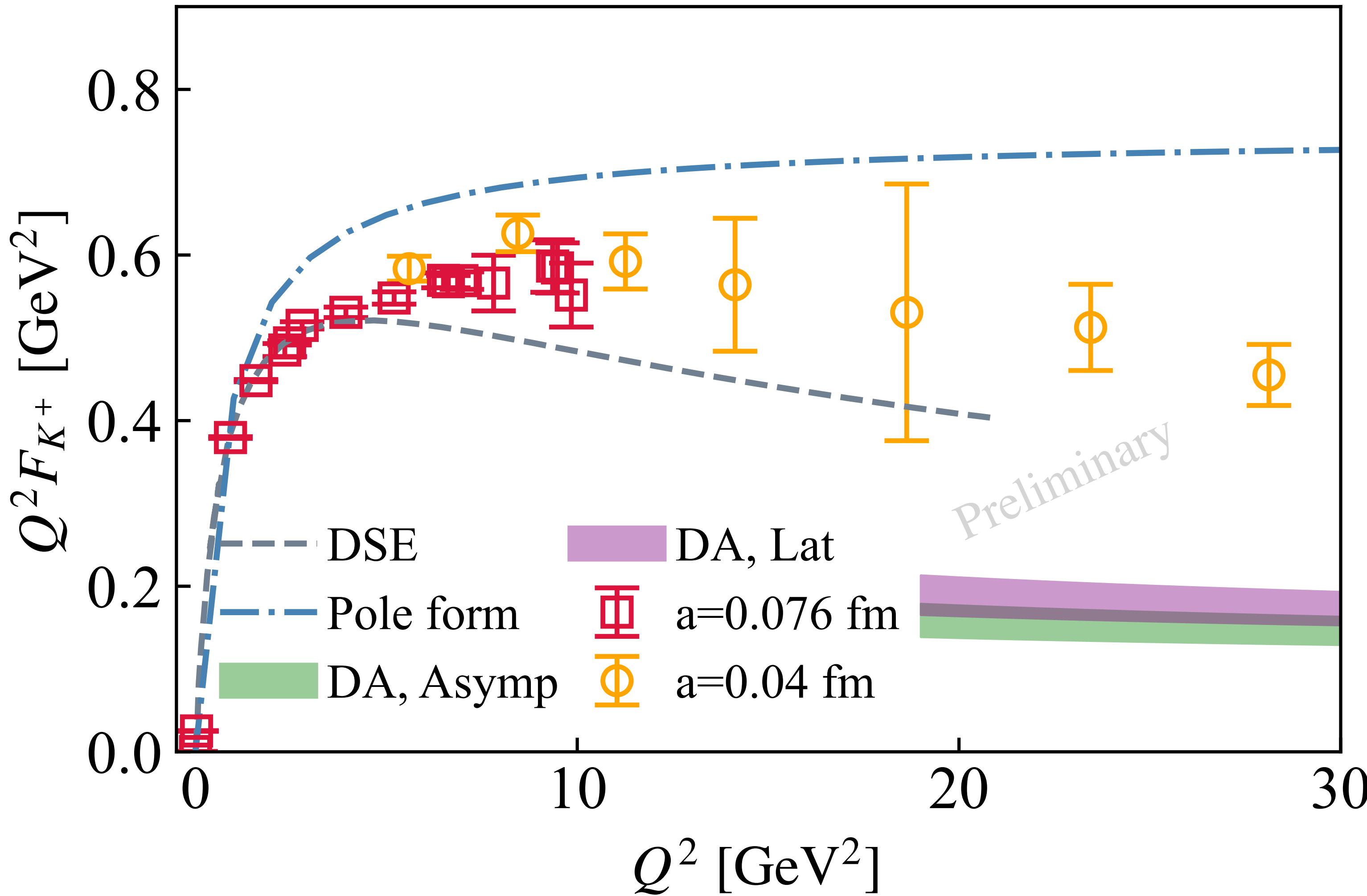
→  $F^B$  decreases as  $Q^2$  increases

→ Renormalization:  $F = F^B \times Z_V$

$$Z_V^{-1} = \langle 0 | \hat{\mathcal{O}} | 0 \rangle = 1.048, 1.024$$

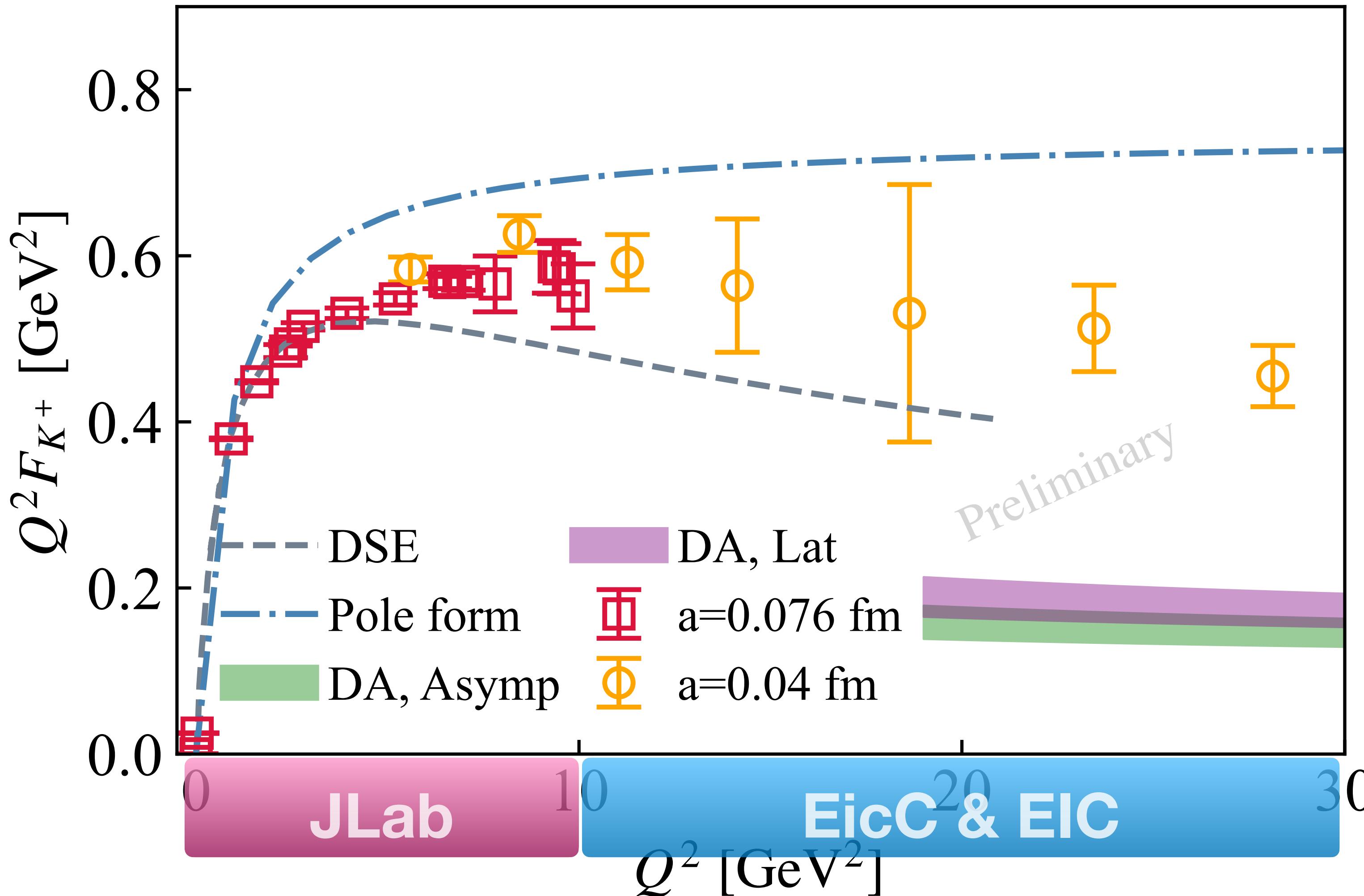
X. Gao et al., Phys.Rev.D 104 (2021) 11

# Kaon form factor up to $Q^2 \sim 28 \text{ GeV}^2$



DSE: Gao et al., PRD 96(2017)034024  
Pole form:  $Q^2/(1 + Q^2\langle r_K^2 \rangle/6)$   
DA, Lat: Bali et al., JHEP 08 (2019) 065

# Kaon form factor up to $Q^2 \sim 28 \text{ GeV}^2$

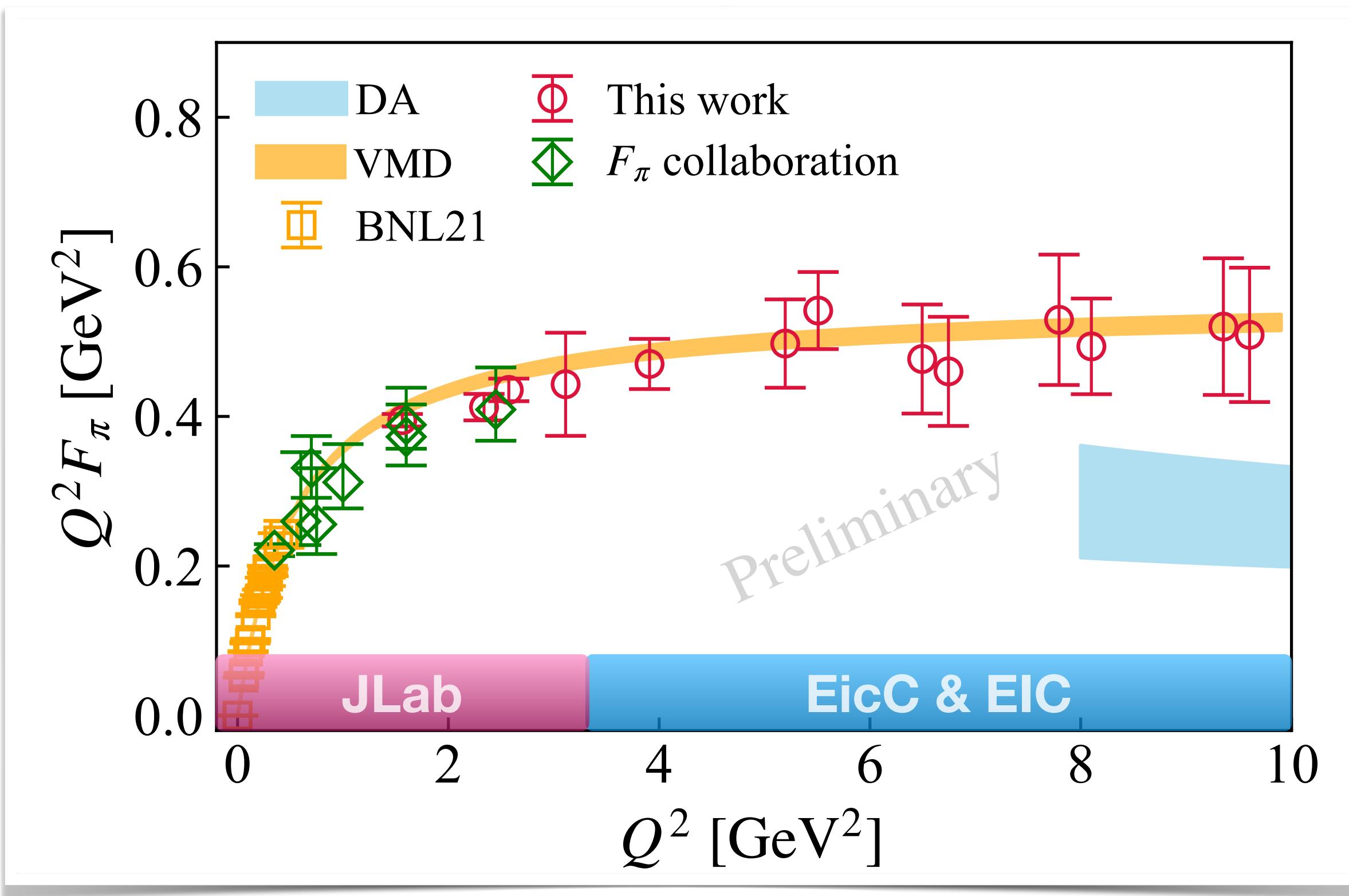


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# Pion form factor up to $Q^2 \sim 10 \text{ GeV}^2$



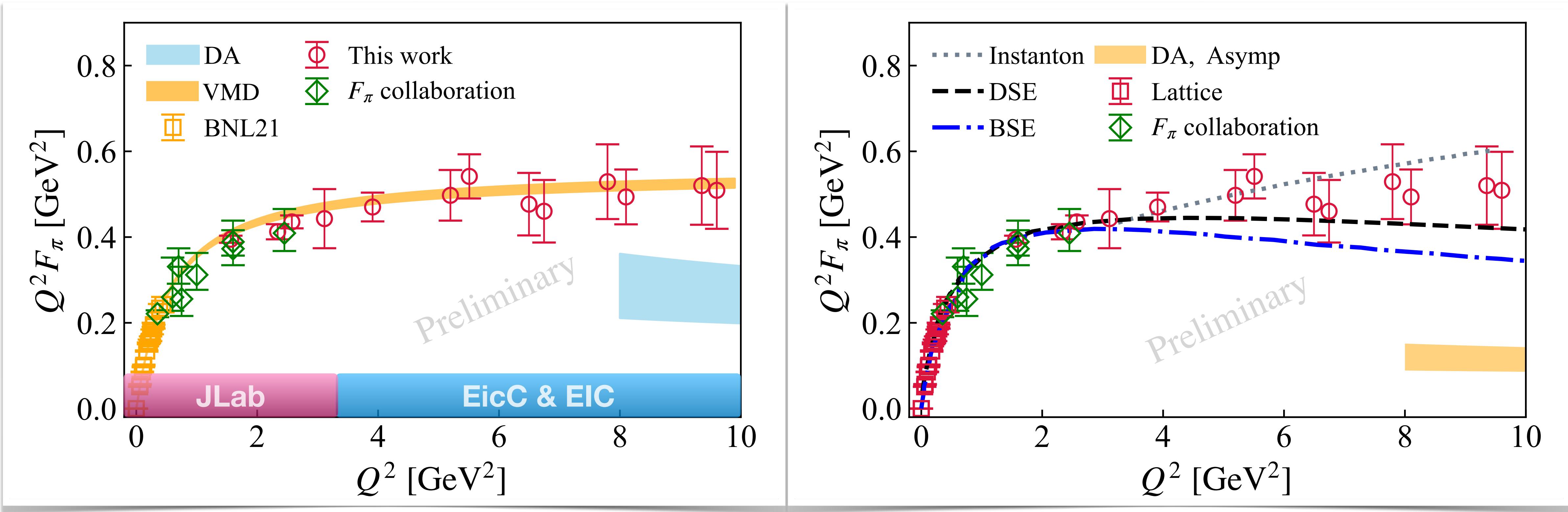
DA: Gao et al., arXiv:2206.04084

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BNL21: Gao et al., PRD 104 (2021) 114515

$F_\pi$  collaboration: Huber et al., PRC 78 (2008) 045203

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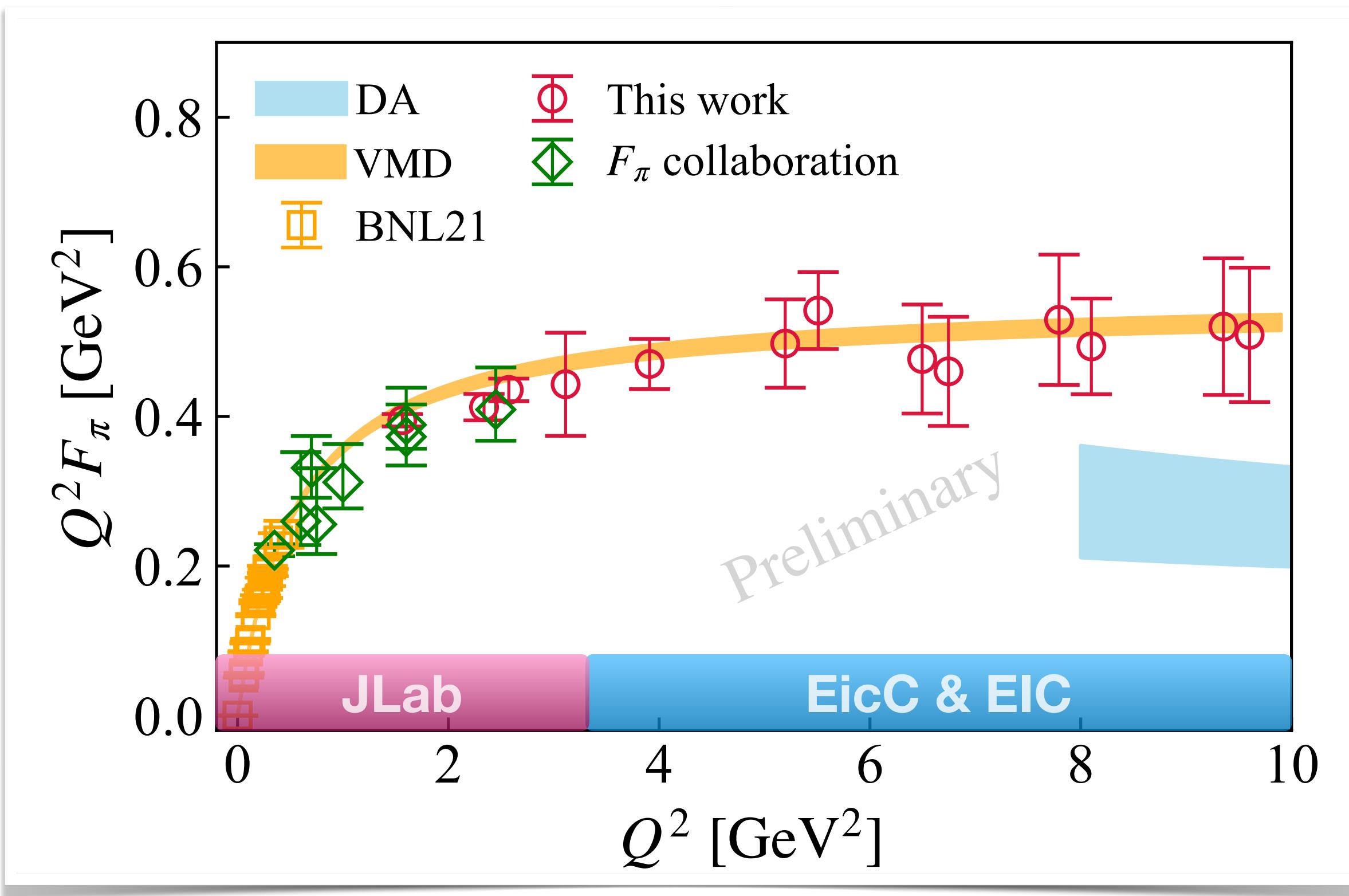
Instanton: Shuryak et al., PRD 103 (2021) 054028

DSE: Gao et al., PRD 96 (2017) 034024

BSE: Ydrefors et al., PLB 820 (2021) 136494

DA, Asymp:  $\phi(x) = 6x(1 - x)$

# Pion form factor up to $Q^2 \sim 10 \text{ GeV}^2$

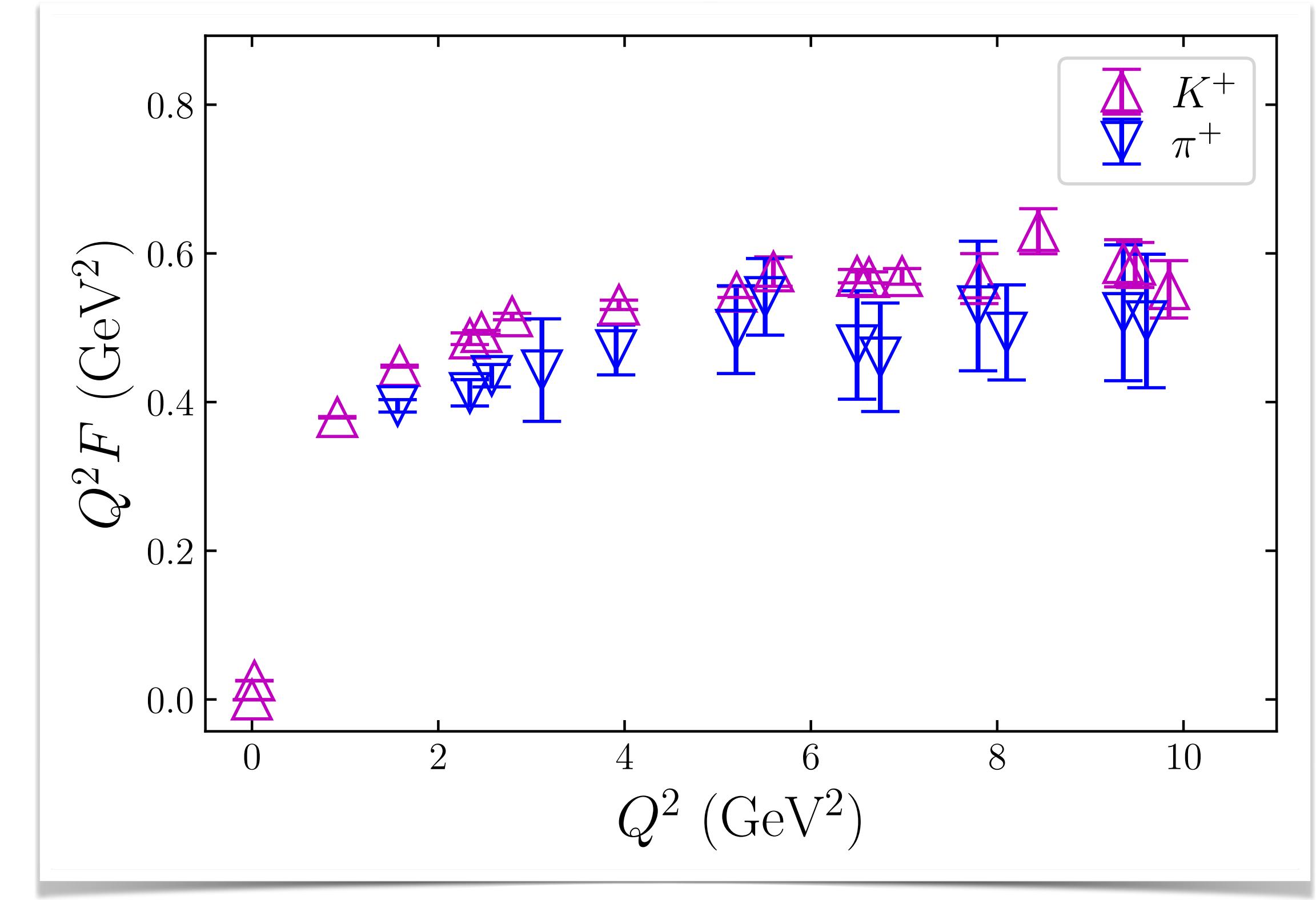


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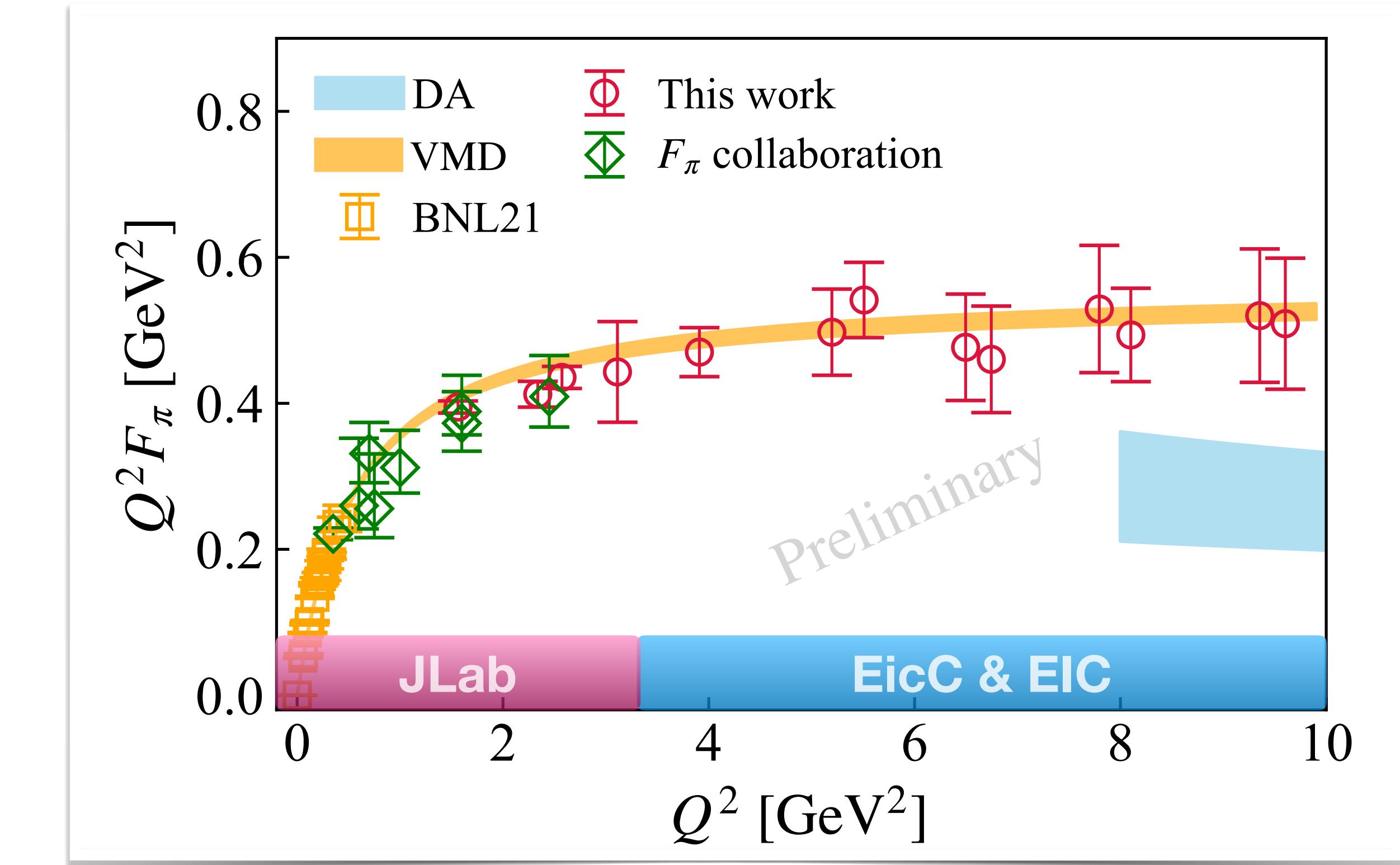
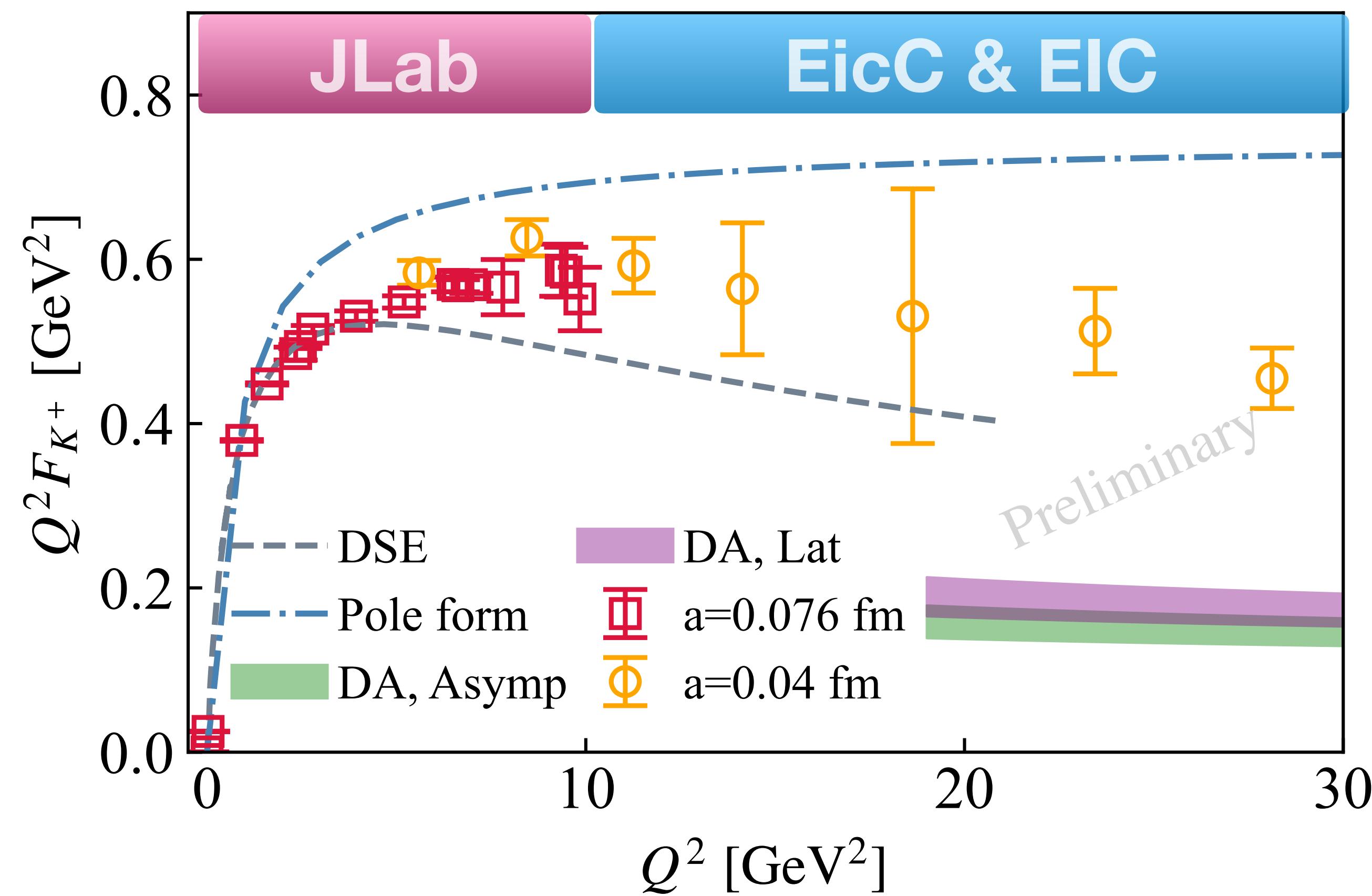
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# Summary

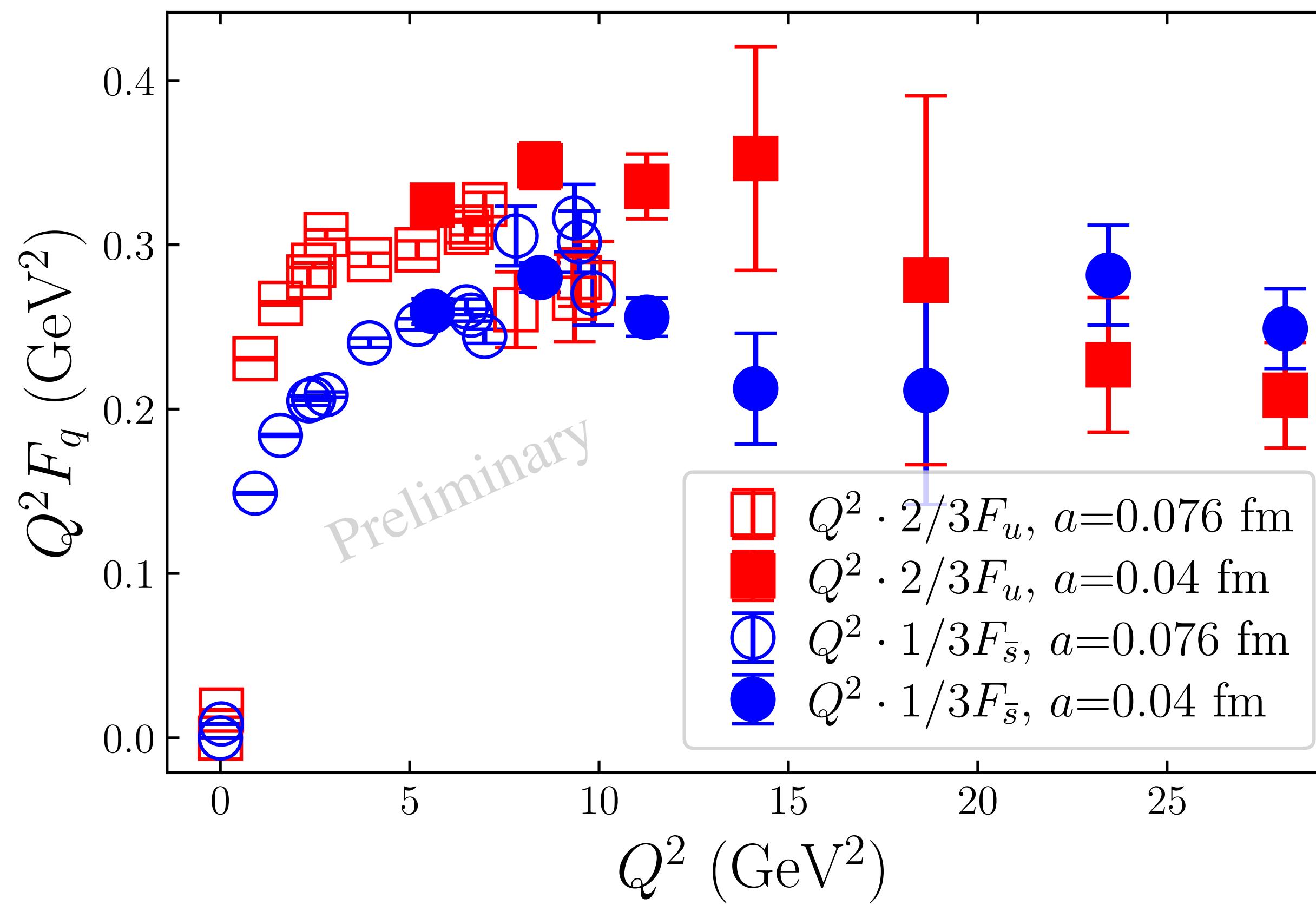
- ✓ A first LQCD prediction of Kaon and Pion electromagnetic form factors with  $Q^2$  up to  $\sim 28$  and  $10 \text{ GeV}^2$ , respectively



Q. Shi et al., work in progress

# Backup

# Up and strange quark components of kaon EM form factor



$$F_K = \frac{2}{3} F_u + \frac{1}{3} F_{\bar{s}}$$