

# The size of $\psi \rightarrow \bar{B}B$ reaction

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Phys. Rev. D 104 (2021) 5, 054018

第五届重味物理和量子色动力学研讨会

2023. 04. 21.

武汉



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Question: How many partial waves does  $\psi$  decay need ?



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N(2220) 9/2+	$\pi N$ and $\gamma N$
N(2250) 9/2-	$\pi N$ and $\gamma N$
N(2300) 1/2+	$\psi(2S)$ decay
N(2570) 5/2-	$\psi(2S)$ decay
N(2600) 11/2-	$\pi N$
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$$E \sim 2 \text{ GeV} \rightarrow \mathbf{p} \sim 1 \text{ GeV}$$

$$r_{eff} \sim r_N \sim 1 \text{ fm}$$

$$\mathbf{L} \sim 1 \text{ fm} \times 1 \text{ GeV} \sim 5$$

$$J_{N^*} \sim \left[ 5 - \frac{3}{2}, 5 + \frac{3}{2} \right] \sim \left[ \frac{7}{2}, \frac{13}{2} \right]$$



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$\pi N$  and  $\gamma N$

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How to estimate the effective radius  
of the  $\psi$  decay ???

$$\psi \rightarrow \bar{B}(1/2-)B(1/2+) \longrightarrow r_{eff} \sim \frac{\bar{L}}{p}$$



# L-S scheme

$$\psi \rightarrow \bar{B}(1/2-)B(1/2+) \xrightarrow{L: \text{ S and D wave}} r_{eff} \sim \frac{\bar{L}}{p}$$



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This is a very useful PWA tool

PHYSICAL REVIEW C 67, 015204 (2003)

**Lorentz covariant orbital-spin scheme for the effective  $N^*NM$  couplings**

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$$J = S_\psi = S_{\bar{N}N^*} + L_{\bar{N}N^*}$$

$$S_{\bar{N}} = \frac{1}{2}, \quad S_{N^*} = \frac{2n+1}{2}$$
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$$\begin{aligned} \tilde{t}_{\mu_{i_1} \mu_{i_2} \dots \mu_{i_L}}^{(L)} &= \tilde{r}_{\mu_{i_1}} \tilde{r}_{\mu_{i_2}} \dots \tilde{r}_{\mu_{i_L}} + \sum_{l=1}^{\lfloor L/2 \rfloor} \sum_{i_1 < i_2 < \dots < i_{2l}=1}^L \frac{(-\tilde{r} \cdot \tilde{r})^l}{(2L-1)(2L-3)\dots(2L-2l+1)} \\ &\times \frac{1}{2l l!} (\tilde{g}_{\mu_{i_1} \mu_{i_2}} \tilde{g}_{\mu_{i_3} \mu_{i_4}} \dots \tilde{g}_{\mu_{i_{2l-1}} \mu_{i_{2l}}} + \text{permutation, } (2l)! \text{ term}) \\ &\times (\tilde{r}_{\mu_1} \tilde{r}_{\mu_2} \dots \tilde{r}_{\mu_{i_1-1}} \tilde{r}_{\mu_{i_1+1}} \dots \tilde{r}_{\mu_{i_2-1}} \tilde{r}_{\mu_{i_2+1}} \dots \tilde{r}_{\mu_{i_{2l-1}}} \tilde{r}_{\mu_{i_{2l}+1}} \dots \tilde{r}_{\mu_L}), \quad (12) \end{aligned}$$



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$$\text{If } S+L+J=\text{odd}, \quad M \sim \epsilon^{\alpha\beta\gamma\sigma} \psi_{\mu_i \mu_j, \alpha}^{(S)} \tilde{t}_{\mu'_i \mu_k, \beta}^{(L)} \varepsilon_{\mu'_j \mu'_k, \gamma} p_{\psi, \sigma} g^{\mu_i \mu'_i} g^{\mu_j \mu'_j} g^{\mu_k \mu'_k}$$

$$\text{If } S+L+J=\text{even}, \quad M \sim \psi_{\mu_i \mu_j}^{(S)} \tilde{t}_{\mu'_i \mu_k}^{(L)} \varepsilon_{\mu'_j \mu'_k} g^{\mu_i \mu'_i} g^{\mu_j \mu'_j} g^{\mu_k \mu'_k}$$

We also need to consider Parity conservation.



# L-S scheme

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$\psi \rightarrow \bar{N}^* N^*$   
Such as  $\Sigma^* \bar{\Sigma}^*$

Here I mainly introduce it for  $\psi \rightarrow \bar{N}N^*$  case

Update: Recently, we propose a new method based on Group theory to write down **covariant** partial wave amplitudes within **L-S scheme** for three-particle vertex with **any spin**. And we also show that it is equivalent with usual helicity amplitude.

Covariant Orbital-Spin Scheme for Any Spin based on Irreducible Tensor

Hao-Jie Jing (Beijing, GUCAS), Di Ben (Beijing, Inst. Theor. Phys. and Beijing, GUCAS), Shu-Ming Wu (Beijing, GUCAS and Beijing, Inst. Theor. Phys.), Jia-Jun Wu (Beijing, GUCAS), Bing-Song Zou (Beijing, Inst. Theor. Phys. and Central South U., Changsha) (Jan 4, 2023)

e-Print: [2301.01575 \[hep-ph\]](https://arxiv.org/abs/2301.01575)



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# Two ways for the amplitude of $\psi \rightarrow \bar{B}B$

**Method 1**, using  $G_E^\psi$  and  $G_M^\psi$ , the amplitude,

$$M = -ie_g \bar{u}_B \left( G_E^\psi \gamma_\mu - \frac{2m_B}{r^2} (G_M^\psi - G_E^\psi) r_\mu \right) v_{\bar{B}} \epsilon^\mu$$
$$r^2 = q^2 = \frac{s}{4} - m_B^2$$

**G. Fäldt, EPJA 52 141(2016)**

**G. Fäldt and A. Kupsc, PLB 772 16(2017)**



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**Method 2**, using L-S scheme, the amplitude,

$$M = \psi_\mu^{(1)} (g_S g^{\mu\nu} + g_D e^{i\delta} \tilde{t}^{(2)\mu\nu}) \epsilon_\nu = \bar{u}_B \left( \gamma_\mu - \frac{(p_B - p_{\bar{B}})}{m_\psi + 2m_B} \right) v_{\bar{B}} (g_S g^{\mu\nu} + g_D e^{i\delta} \tilde{t}^{(2)\mu\nu}) \epsilon_\nu$$

$\psi(1^-) \rightarrow \bar{B}(1/2^-)B(1/2^+)$

$$S_\psi = S_{\bar{N}N^*} + L_{\bar{N}N^*}$$
$$1 = 1 + (0, 2)$$

Including S- and D-wave



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Including S- and D-wave

**Relationship**, there are three free parameters in both cases, we can define them as: an overall

coupling constant,  $\left| \frac{g_D}{g_S} \right| \left( \left| \frac{G_E^\psi}{G_M^\psi} \right| \right)$ , and  $\delta \left( \Delta\Phi \equiv \text{Arg} \left( \frac{G_E^\psi}{G_M^\psi} \right) \right)$ , typically,

$$e^{i\delta} \frac{g_D}{g_S} = -\frac{3}{2r^2} \frac{2m_B - r^2/(m_\psi + 2m_B) - 2m_B \left( G_E^\psi / G_M^\psi \right)}{2m_B - r^2/(m_\psi + 2m_B) + 2m_B \left( G_E^\psi / G_M^\psi \right)}$$



# Two ways for the amplitude of $\psi \rightarrow \bar{B}B$

**Method 1**, using  $G_E^\psi$  and  $G_M^\psi$ , the amplitude,  $M = -ie_g \bar{u}_B \left( G_E^\psi \gamma_\mu - \frac{2m_B}{r^2} (G_M^\psi - G_E^\psi) r_\mu \right) v_{\bar{B}} \epsilon^\mu$

**Method 2**, using PAW, the amplitude,  $M = \bar{u}_B \left( \gamma_\mu - \frac{(p_B - p_{\bar{B}})}{m_\psi + 2m_B} \right) v_{\bar{B}} (g_S g^{\mu\nu} + g_D e^{i\delta} \tilde{t}^{(2)\mu\nu}) \epsilon_\nu$

**Experimentally,**

(1) an overall coupling constant can be obtained from the partial width  $\Gamma$ .

(2)  $r = \frac{p_B}{m_\psi}$  and  $\delta$  can be determined by angular distribution,  $\frac{d\Gamma}{d\cos\theta} \sim (1 + \alpha \cos^2\theta)$ .

and  $\Delta\Phi = \text{Arg}\left(\frac{p_B}{m_\psi}\right)$

$$\begin{aligned} \alpha &= \frac{-3t^2r^2/3 - 9tr^2 \cos\delta}{9 + 3t^2r^2/3 - 3tr^2 \cos\delta} \\ \tan\delta &= \sqrt{\frac{3 + \alpha \left( \frac{2m_B}{m_\psi} - \frac{r^2}{m_\psi(m_\psi + 2m_B)} \right)}{3 - \alpha \left( \frac{2m_B}{m_\psi} - \frac{r^2}{m_\psi(m_\psi + 2m_B)} \right)}} \end{aligned}$$



# Two ways for the amplitude of $\psi \rightarrow \bar{B}B$

**Method 1**, using  $G_E^\psi$  and  $G_M^\psi$ , the amplitude,  $M = -ie_g \bar{u}_B \left( G_E^\psi \gamma_\mu - \frac{2m_B}{r^2} (G_M^\psi - G_E^\psi) r_\mu \right) v_{\bar{B}} \epsilon^\mu$

**Method 2**, using PAW, the amplitude,  $M = \bar{u}_B \left( \gamma_\mu - \frac{(p_B - p_{\bar{B}})}{m_\psi + 2m_B} \right) v_{\bar{B}} (g_S g^{\mu\nu} + g_D e^{i\delta} \tilde{t}^{(2)\mu\nu}) \epsilon_\nu$

**Experimentally,**

- (1) an overall coupling constant can be obtained from the partial width  $\Gamma$ .
- (2)  $t = \left| \frac{g_D}{g_S} \right|$  and  $\delta$  can be determined by angular distribution,  $\frac{d\Gamma}{d \cos \theta} \sim (1 + \alpha \cos^2 \theta)$ ,

and  $\Delta\Phi \equiv \text{Arg} \left( \frac{G_E^\psi}{G_M^\psi} \right)$

$$\begin{aligned} \frac{G_E^\psi}{G_M^\psi} &= \sqrt{\frac{-3t^2r^2/3 - 9rt^2 \cos \delta}{9 + 3t^2r^2/3 - 3rt^2 \cos \delta}} \\ &= \sqrt{\frac{3 + \alpha \left( \frac{2m_B}{m_B - m_\psi(m_\psi + 2m_B)} \right) r^2}{3 - r^2 \alpha^2}} \end{aligned}$$



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# The effective radius of the decay reaction

$$t = \left| \frac{g_D}{g_S} \right| \quad \alpha = \frac{-3t^2 r^2 / 8 - 9tr^2 \cos \delta}{9 + 5t^2 r^2 / 8 - 3tr^2 \cos \delta};$$

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Mode	$\alpha$	$\Delta\Phi$	$g_D/g_S$	$\delta$	$\Gamma_S/\Gamma_{\text{Total}}$	$r_{\text{eff}}$
$J/\psi \rightarrow \Lambda\bar{\Lambda}$	$0.461 \pm 0.013$ [23]	$0.74 \pm 0.019$ [23]	$0.180 \pm 0.005$	$-0.804 \pm 0.024$	$85.7 \pm 0.6\%$	$0.0488 \pm 0.0021$
$J/\psi \rightarrow \Sigma^+ \bar{\Sigma}^-$	$-0.508 \pm 0.010$ [24]	$-0.270 \pm 0.021$ [24]	$0.171 \pm 0.006$	$2.67 \pm 0.04$	$90.9 \pm 0.6\%$	$0.0362 \pm 0.0024$
$\psi(2S) \rightarrow \Sigma^+ \bar{\Sigma}^-$	$0.682 \pm 0.041$ [24]	$0.379 \pm 0.084$ [24]	$0.097 \pm 0.009$	$-0.33 \pm 0.10$	$88.3 \pm 2.0\%$	$0.033 \pm 0.006$



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Need further decay, such as  $\Lambda \rightarrow p \pi$

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It shows that the effective radius of  $\psi$  decay is very small. It leads to the small orbital angular momentum of the process of  $\psi \rightarrow \bar{N}N^*$ ,  $L = r_{\text{eff}} \times p$ , then it should be dominated by low spin  $N^*$ . Therefore, when dealing with  $N^*$  partial wave analysis in  $\psi$  decay, the highest partial wave can be taken as  $l = 5/2$ .

# The effective radius of the decay reaction

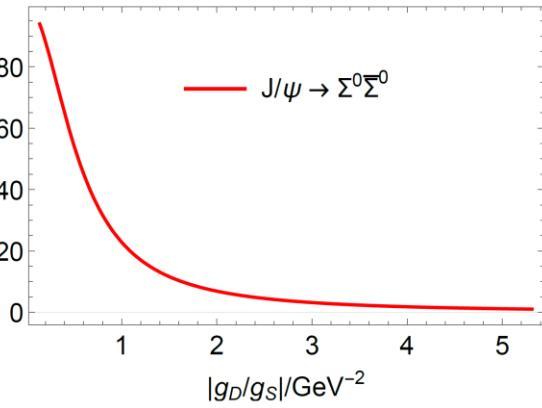
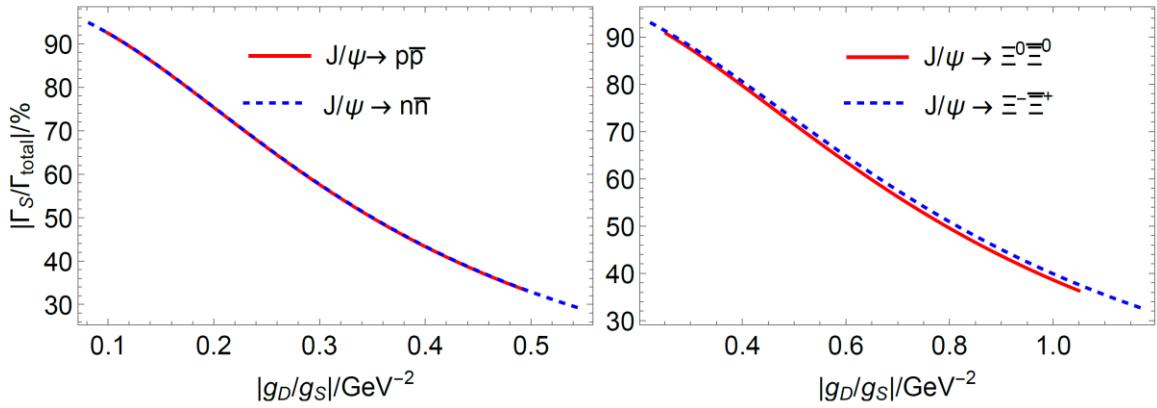
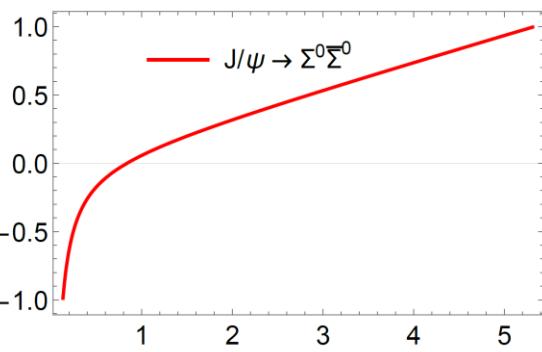
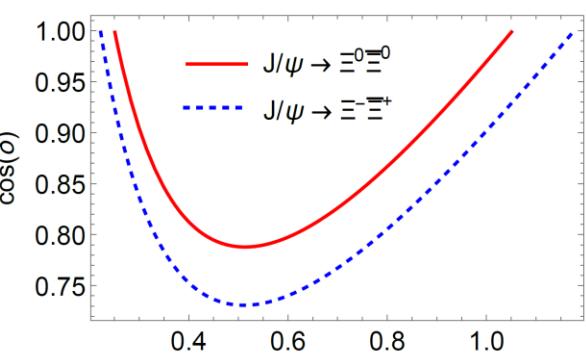
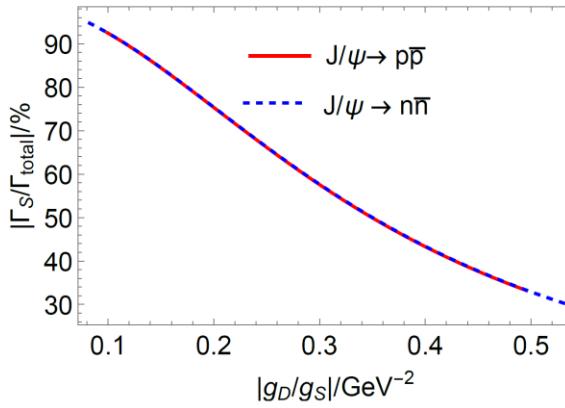
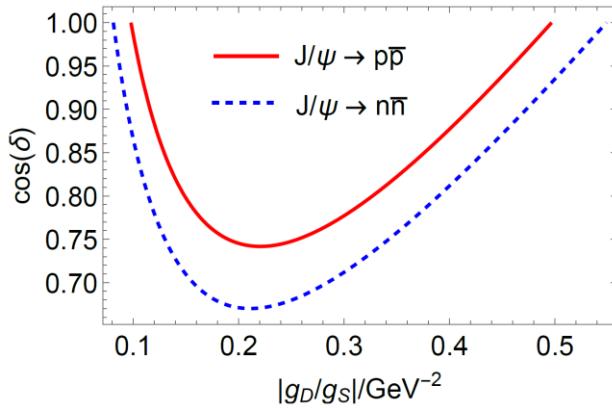
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However, we expect more data to test the effective radius of  $\psi$  decay.

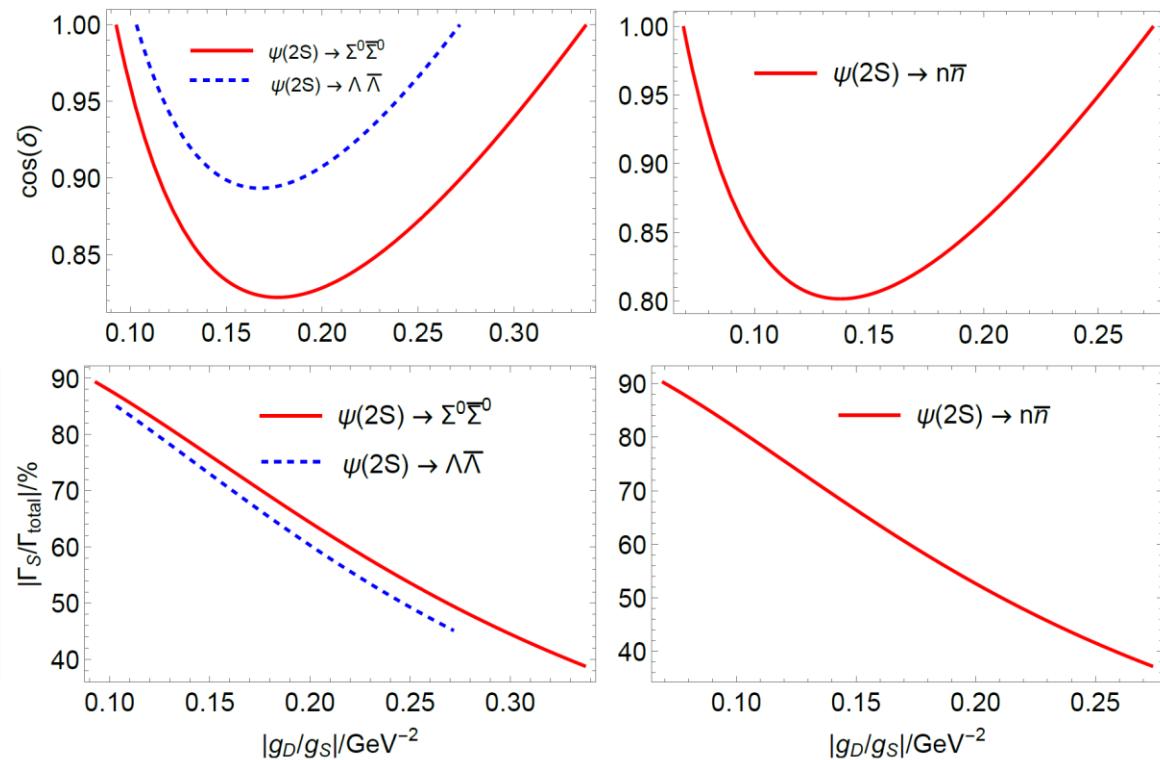
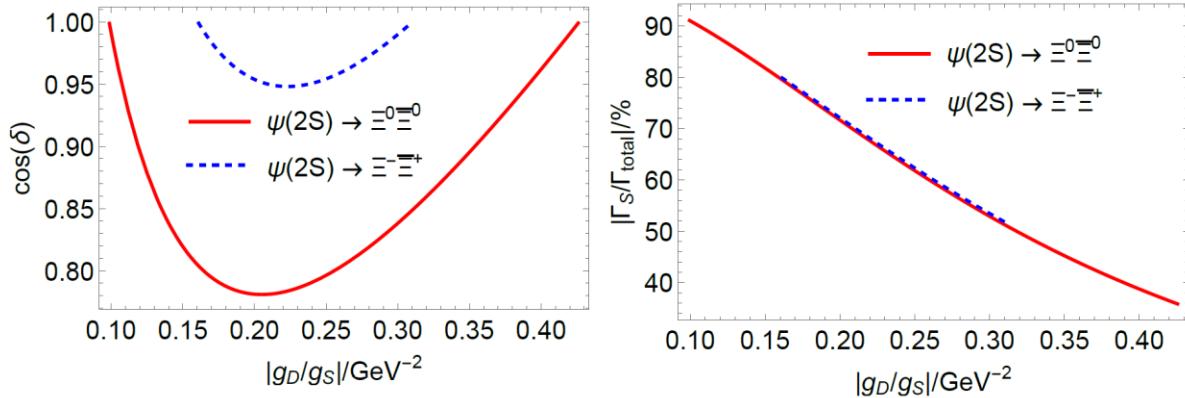
# The effective radius of the decay reaction

Mode	$\alpha$	$r_{\text{eff}}(\text{fm})$
$J/\psi \rightarrow p\bar{p}$	$0.595 \pm 0.027[26]$	[0.023 – 0.214]
$J/\psi \rightarrow n\bar{n}$	$0.50 \pm 0.25[26]$	[0.016 – 0.228]
$J/\psi \rightarrow \Sigma^0 \bar{\Sigma}^0$	$-0.449 \pm 0.028[25]$	[0.022 – 0.396]
$J/\psi \rightarrow \Xi^0 \bar{\Xi}^0$	$0.66 \pm 0.08[28]$	[0.044 – 0.308]
$J/\psi \rightarrow \Xi^- \bar{\Xi}^+$	$0.58 \pm 0.12[29]$	[0.034 – 0.331]



# The effective radius of the decay reaction

Mode	$\alpha$	$r_{\text{eff}}(\text{fm})$
$\psi_{2S} \rightarrow \Lambda \bar{\Lambda}$	$0.82 \pm 0.10[25]$	$[0.040 - 0.148]$
$\psi_{2S} \rightarrow p \bar{p}$	$1.03 \pm 0.09[27]$	???
$\psi_{2S} \rightarrow n \bar{n}$	$0.68 \pm 0.23[27]$	$[0.024 - 0.156]$
$\psi_{2S} \rightarrow \Sigma^0 \bar{\Sigma}^0$	$0.71 \pm 0.15[25]$	$[0.030 - 0.172]$
$\psi_{2S} \rightarrow \Xi^0 \bar{\Xi}^0$	$0.65 \pm 0.23[28]$	$[0.027 - 0.196]$
$\psi_{2S} \rightarrow \Xi^- \bar{\Xi}^+$	$0.91 \pm 0.27[29]$	$[0.061 - 0.148]$

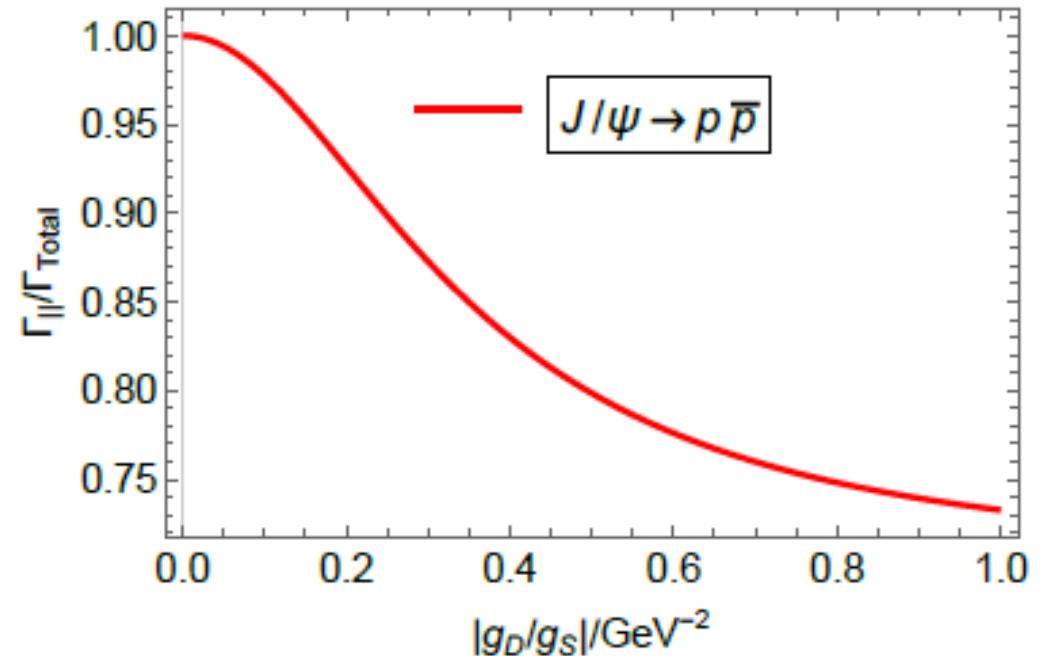


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Since  $p(n)$  can not further decay, thus  $\Delta\Phi$  cannot be measured. We need new observable, such as  $\Gamma_{//}$  to stand for the decay width of the process where  $p(n)$  and  $\bar{p}(\bar{n})$  have the same polarization,

$$\Gamma_{//} = \frac{|\vec{p}_B|}{32\pi^2 m_\psi^2} \int \frac{1}{2} \sum_{S_\psi, S_B = S_{\bar{B}}} |M(S_\psi, S_B, S_{\bar{B}})|^2 d\Omega = \Gamma_S + 0.7\Gamma_D$$



# Summary

- Introduction of L-S scheme
- Through  $\psi \rightarrow \bar{B}(1/2-)B(1/2+)$  to estimate the effective radius of  $\psi$  decay, around 0.05fm.
- It leads to low spin  $N^*$  dominate in the  $\psi \rightarrow \bar{N}N^*$ , thus, highest partial wave analysis require the total J of  $N^*$  with  $l = 5/2$ .
- It provides a unique window to search low spin baryon resonances with masses above 2 GeV in  $\psi$  decay.
- We need more experimental data to confirm it.



# Thanks Very Much !

Advertisement Time

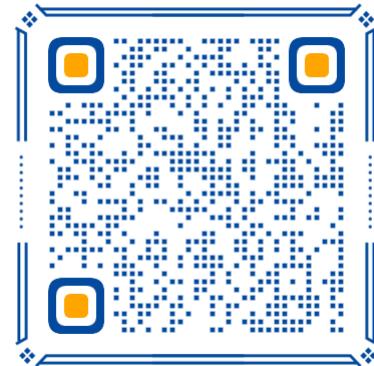


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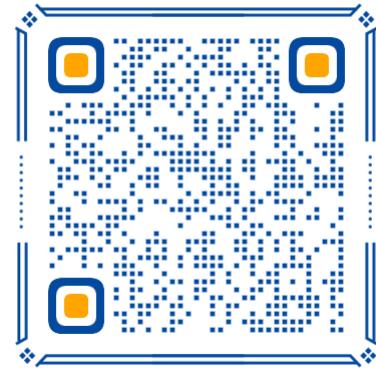
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