

CP violation in multi-body decays of heavy hadron

Zhen-Hua Zhang (张振华)

email: zhenhua_zhang@163.com

University of South China (南华大学)

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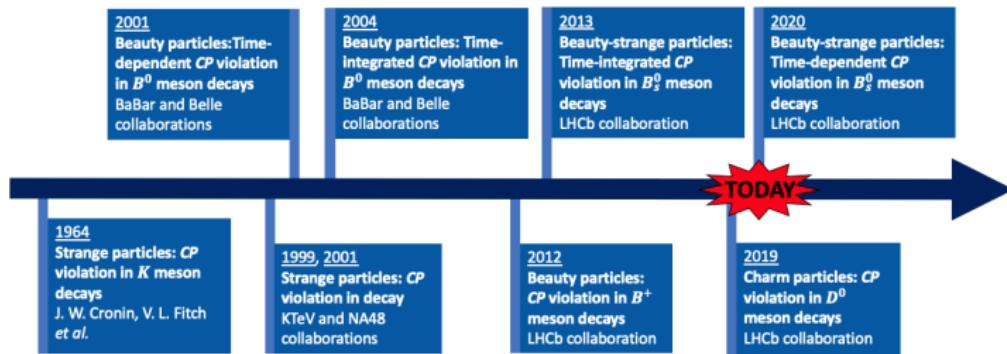
1 Motivations

2 Angular-correlation related CPA in four-body decays

3 Summary and Outlook

1 Motivations

current exp. status of CPV



Already observed CPV effects (PDG review)

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13.8 Observed CP violation effects

We conclude by listing the observables where CP violation has been observed at a level above 5σ [43, 61, 81]:

- Indirect CP violation in $K \rightarrow \pi\pi$ and $K \rightarrow \pi\ell\nu$ decays, and in the $K_L \rightarrow \pi^+\pi^-e^+e^-$ decay, is given by

$$|\epsilon| = (2.228 \pm 0.011) \times 10^{-3}. \quad (13.98)$$

- Direct CP violation in $K \rightarrow \pi\pi$ decays is given by

$$\Re(\epsilon'/\epsilon) = (1.65 \pm 0.26) \times 10^{-3}. \quad (13.99)$$

- CP violation in the interference of mixing and decay in the tree-dominated $b \rightarrow c\bar{s}$ transitions, such as $B^0 \rightarrow \psi K^0$, is given by (we use K^0 throughout to denote results that combine K_S and K_L modes, but use the sign appropriate to K_S):

$$\mathcal{S}_{\psi K^0} = +0.699 \pm 0.017. \quad (13.100)$$

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- CP violation in the interference of mixing and decay in modes governed by the tree-dominated $b \rightarrow c\bar{c}d$ transitions is given by

$$S_{D^{(*)}\bar{K}^0} = +0.71 \pm 0.09. \quad (13.101)$$

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$$\begin{aligned} S_{\psi\pi^0} &= -0.86 \pm 0.14, \\ S_{D^* D^-} &= -0.84 \pm 0.12, \\ S_{D^* \bar{D}^0} &= -0.81 \pm 0.06, \\ S_{D^+ D^-} &= -0.71 \pm 0.09. \end{aligned} \quad (13.102)$$

- CP violation in the interference of mixing and decay in various modes related to $b \rightarrow q\bar{q}s$ (penguin) transitions is given by

$$\begin{aligned} S_{\psi K^0} &= +0.74^{+0.11}_{-0.13}, \\ S_{q' K^0} &= +0.63 \pm 0.06, \\ S_{f_0 K^0} &= +0.69^{+0.10}_{-0.12}, \\ S_{K^+ K^-} &= +0.08^{+0.09}_{-0.10}. \end{aligned} \quad (13.103)$$

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- Direct CP violation in the $B_s^0 \rightarrow K^+K^-$ mode is given by

$$C_{K^+K^-} = 0.17 \pm 0.03. \quad (13.106)$$

- Direct CP violation in $B^+ \rightarrow D_s^{(*)}K^+$ decays ($D_s^{(*)}$ is the CP-even neutral $D^{(*)}$ state) are given by

$$A_{B^+ \rightarrow D_s K^+} = +0.139 \pm 0.009 \quad \text{and} \quad A_{B^+ \rightarrow D_s^* K^+} = -0.109 \pm 0.019, \quad (13.107)$$

while the corresponding quantity in the case that the neutral D meson is reconstructed in the suppressed $K^-\pi^+$ final state is

$$A_{B^+ \rightarrow D_{s0}^0 K^+} = -0.453 \pm 0.026, \quad (13.108)$$

- Direct CP violation has also been observed in $B^+ \rightarrow DK^+$ decays through differences between the Dalitz plot distributions of subsequent $D \rightarrow K_S\pi^+\pi^-$ decays.

- Direct CP violation in the $B^0 \rightarrow K^+\pi^-$ mode is given by

$$A_{B^0 \rightarrow K^+\pi^-} = -0.083 \pm 0.004. \quad (13.109)$$

- Direct CP violation in the $B_s^0 \rightarrow K^-\pi^+$ mode is given by

$$A_{B_s^0 \rightarrow K^-\pi^+} = +0.225 \pm 0.012. \quad (13.110)$$

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$$\mathcal{A}_{B^+ \rightarrow K^+K^-\pi^+} = -0.118 \pm 0.022. \quad (13.111)$$

- Large CP violation effects have been observed model-independently in certain regions of the phase space of $B^+ \rightarrow K^+K^-K^+$, $K^+K^-\pi^+$, $\pi^+\pi^-\pi^+$ and $\pi^+\pi^-\pi^+$ decays. An amplitude analysis has established a large CP violation effect associated with $\pi\pi \leftrightarrow KK$ S-wave rescattering in $B^+ \rightarrow K^+K^-\pi^+$ decays. In $B^+ \rightarrow \pi^+\pi^-\pi^+$ decays, amplitude analysis has established CP violation effects in the decay amplitude involving the $f_0(1270)$ resonance, in the $\pi^+\pi^-$ S-wave at low invariant mass, and in the interference between the $\pi^+\pi^-$ S-wave and the P-wave $B^+ \rightarrow p(770)\pi^+$ amplitude.

- Direct CP violation has been established in the difference of asymmetries for $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$ decays

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overall CPA in three-body B^\pm decay (LHCb, 2206.07622)

$$A_{CP}(\pi\pi\pi) = +0.080 \pm 0.004 \pm 0.003 \pm 0.003$$

$$A_{CP}(KK\pi) = -0.114 \pm 0.007 \pm 0.003 \pm 0.003$$

$$A_{CP}(KKK) = -0.037 \pm 0.002 \pm 0.002 \pm 0.003$$

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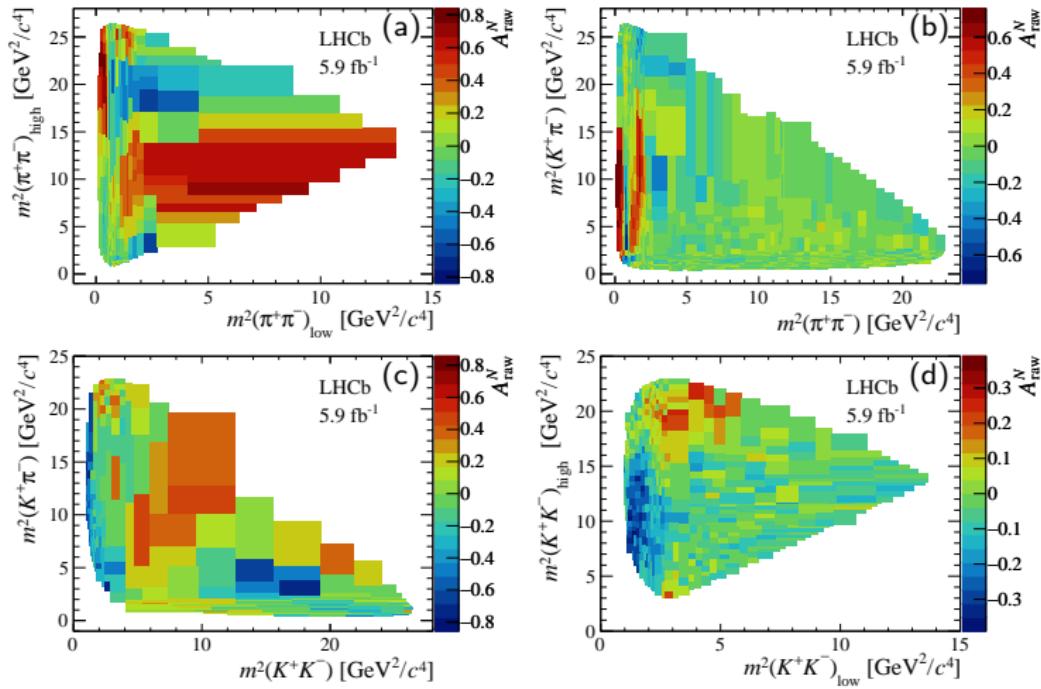
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CPV in baryon sector: not confirmed yet

- CPV has been observed in B , B_s , D meson decays.
- no confirmation of CPV in baryon sector from exp. side (LHCb, Belle(-II), BESIII)
 - CPA in two-body heavy baryons
 - regional/localised CPAs in multi-body decays of heavy baryons
 - Triple Product Asymmetry induced CPA in four-body decay
 - decay asymmetry parameters induced CPA
 - energy test method

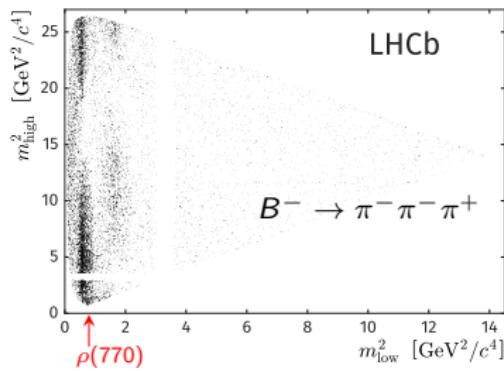
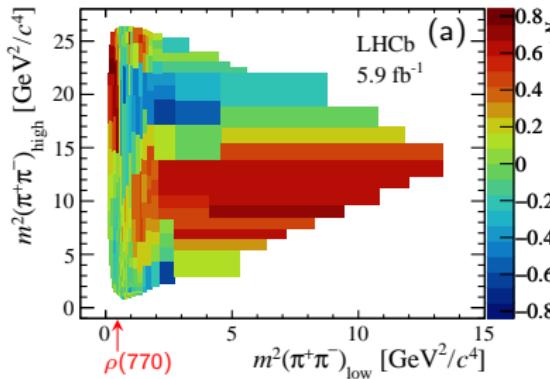
Large regional CPV in three-body decays of B^\pm

LHCb, 2206.07622



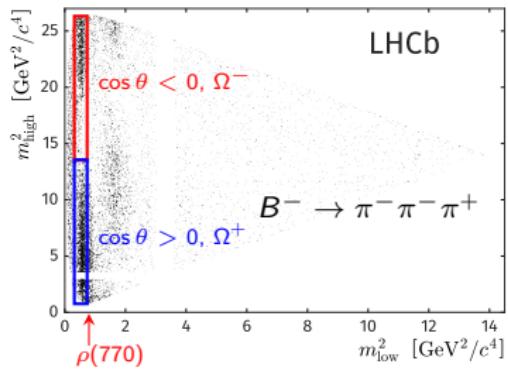
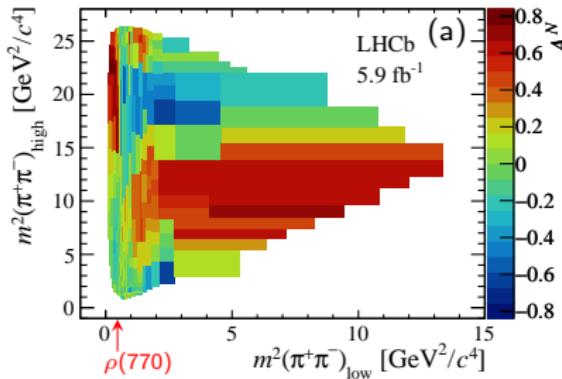
Forward-Backward Asymmetry induced CPA (FB-CPA)

Corelation between reg. CPA and event distributions

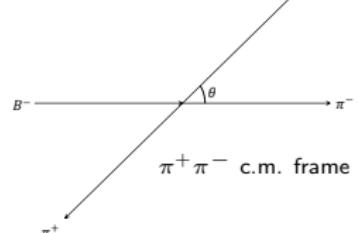


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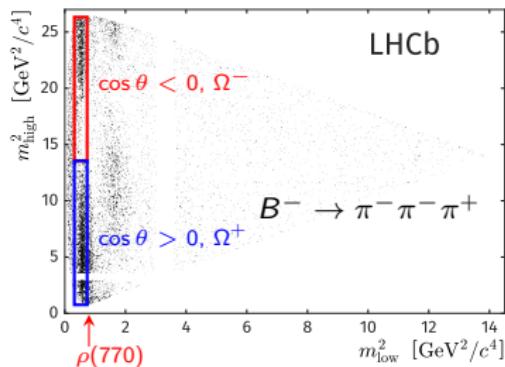
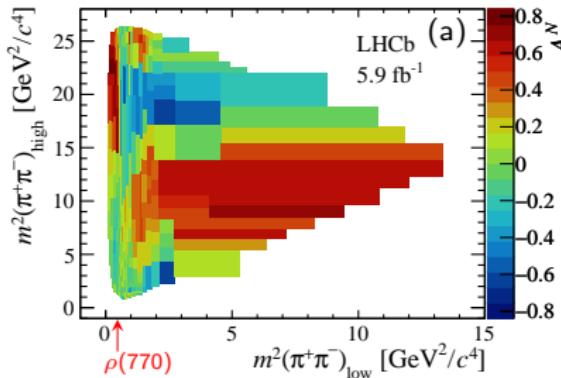


$$\cos \theta = \frac{m_{\text{high}}^2 - (m_{\text{high,max}}^2 + m_{\text{high,min}}^2)/2}{(m_{\text{high,max}}^2 - m_{\text{high,min}}^2)/2}$$



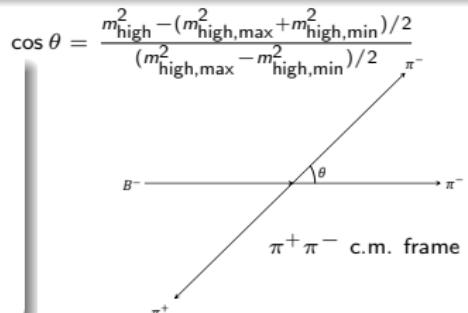
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FBA and FB-CPA

$$A_{B^-}^{FB} = \frac{N_{B^-}^{\Omega^+} - N_{B^-}^{\Omega^-}}{N_{B^-}^{\Omega^+} + N_{B^-}^{\Omega^-}}, \quad A_{CP}^{FB} = \frac{1}{2}(A_{B^-}^{FB} - A_{B^+}^{FB}).$$



(ZHZ, PLB820, 136537)

Interference of P ($\rho^0(770)$) and S ($f_0(500)$) wave resonances

$$\mathcal{A} = a_S + e^{i\delta} a_P \cos \theta, \quad a_{S(P)} = a_{S(P)}^{\text{tree}} + a_{S(P)}^{\text{penguin}}$$

$$\begin{aligned} A_{B-}^{FB} &= \frac{N_{B-}^{\Omega^+} - N_{B-}^{\Omega^-}}{N_{B-}^{\Omega^+} + N_{B-}^{\Omega^-}} = \frac{\left(-f_{-1}^0 + f_0^{+1}\right) |\mathcal{A}|^2 d \cos \theta}{f_{-1}^{+1} |\mathcal{A}|^2 d \cos \theta} \\ &= \frac{\Re(\langle a_S^* a_P e^{i\delta} \rangle)}{|\langle a_P \rangle|^2/3 + |\langle a_S \rangle|^2}. \end{aligned}$$

$$\begin{aligned} A_{CP}^{FB} &= \frac{1}{2} (A_{B-}^{FB} - A_{B+}^{FB}) \\ &= \frac{1}{2} \left(\frac{\Re(\langle a_S^* a_P e^{i\delta} \rangle)}{|\langle a_P \rangle|^2/3 + |\langle a_S \rangle|^2} - \frac{\Re(\langle \bar{a}_S^* \bar{a}_P e^{i\delta} \rangle)}{|\langle \bar{a}_P \rangle|^2/3 + |\langle \bar{a}_S \rangle|^2} \right) \end{aligned}$$



FBI-CPA can isolate the interference effect between S- and P-waves.

(ZHZ, PLB820, 136537)

A_{CP}^{FB}	\tilde{A}_{CP}^{FB}	$A_{CP}^{\Omega^+ + \Omega^-}$	$A_{CP}^{\Omega^+}$	$A_{CP}^{\Omega^-}$
0.224 ± 0.012	0.194 ± 0.013	0.099 ± 0.013	0.405 ± 0.020	-0.074 ± 0.017

(Y.-R. Wei, ZHZ, PRD 106(2022), 113002)

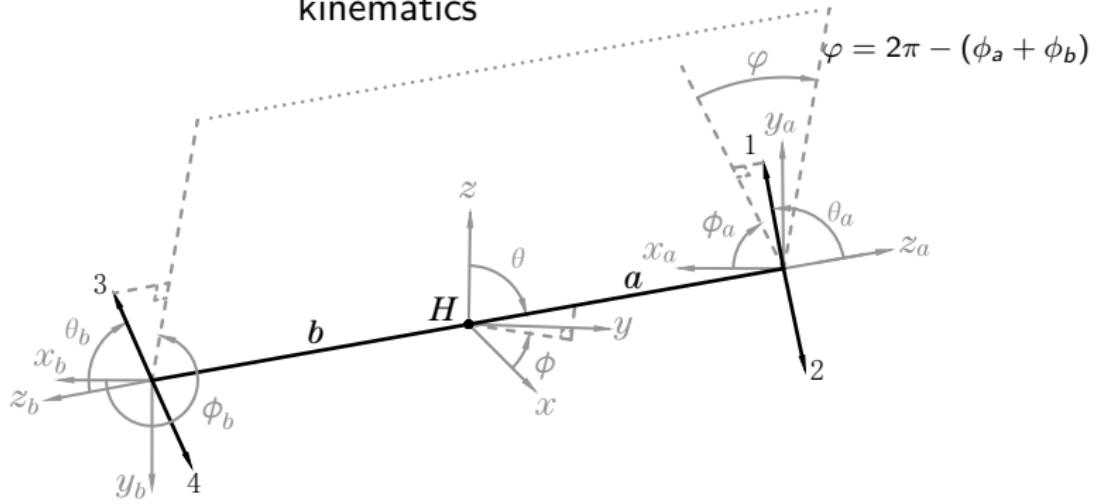
② Angular-correlation related CPA in four-body decays

Kinematics

“branching” four-body decay

$$H \rightarrow a^{(\prime)} (\rightarrow 12) b^{(\prime)} (\rightarrow 34)$$

kinematics

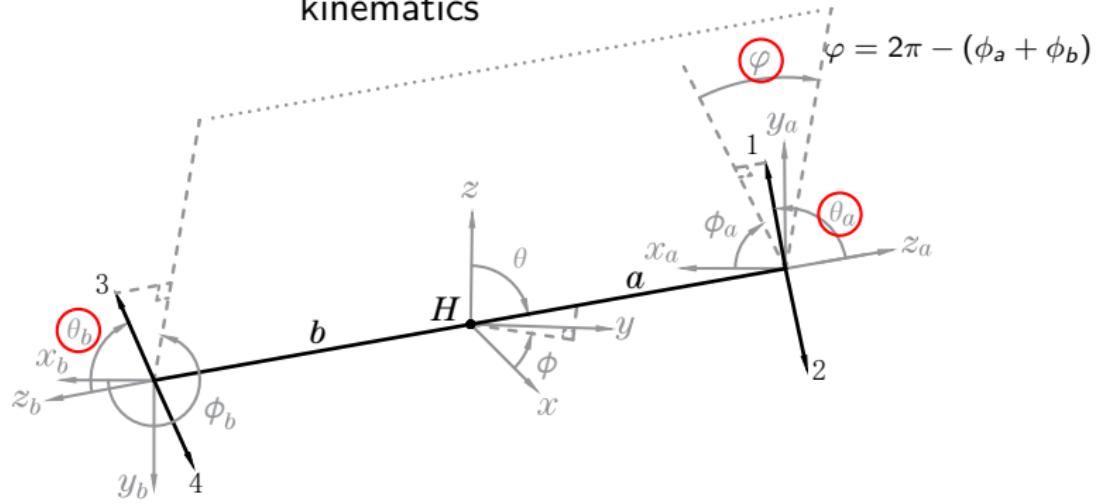


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kinematics



Dynamics

interferences

- $H \rightarrow a(\rightarrow 12)b(\rightarrow 34)$ and $H \rightarrow a'(\rightarrow 12)b(\rightarrow 34)$,
- $H \rightarrow a(\rightarrow 12)b(\rightarrow 34)$ and $H \rightarrow a(\rightarrow 12)b'(\rightarrow 34)$,

Dynamics

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- $H \rightarrow a(\rightarrow 12)b(\rightarrow 34)$ and $H \rightarrow a'(\rightarrow 12)b(\rightarrow 34)$, $H \rightarrow 12b$
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Genuine Interference (GI) to four-body decay.

Decay amplitude squared for unpolarized H :

$$\overline{|\mathcal{A}|^2} \propto \sum \gamma_{\sigma_{a'a}, \sigma_{b'b}}^{jl} d_{\sigma_{a'a}, 0}^j(\theta_a) d_{\sigma_{b'b}, 0}^l(\theta_b) e^{-i\sigma_{aa'}\varphi},$$

$$\gamma_{\sigma_{a'a}, \sigma_{b'b}}^{jl} = \frac{w_{jl}^{(ab, a'b')} \mathcal{G}_j^{(aa')} \mathcal{G}_l^{(bb')}}{\mathcal{I}_a \mathcal{I}_{a'}^* \mathcal{I}_b \mathcal{I}_{b'}^*},$$

$$\begin{aligned} w_{\sigma_{a'a}, \sigma_{b'b}}^{(ab, a'b')jl} &= \langle s_a - \sigma_a s_{a'} \sigma_{a'} | j \sigma_{a'a} \rangle \langle s_b - \sigma_b s_{b'} \sigma_{b'} | l \sigma_{b'b} \rangle \\ &\times (-)^{\sigma_a - s_a + \sigma_b - s_b} \mathcal{F}_{\sigma_a \sigma_b}^{H \rightarrow ab} \mathcal{F}_{\sigma_{a'} \sigma_{b'}}^{H \rightarrow a'b' *} \\ &\times \delta_{\sigma_{ab}, \sigma_{a'b'}} / 2, \end{aligned}$$

$$\mathcal{G}_j^{(aa')} = \sum_{\lambda_1 \lambda_2} (-)^{s_a - \lambda_{12}} \langle s_a - \lambda_{12} s_{a'}, \lambda_{12} | j 0 \rangle \mathcal{F}_{\lambda_1 \lambda_2}^{a \rightarrow 12} \mathcal{F}_{\lambda_1 \lambda_2}^{a' \rightarrow 12*},$$

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aim: find out the interference terms in γ^{jl} .

- Parity symmetry
- Properties of Clebsch-Gordan coefficients

Selection Rules

- Parity symmetry

- Properties of C-Gs

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Interf. terms show up in γ_{jl} when j and/or l :
 ("and" is for GI terms)

$$P_a P_{a'} (-)^j = 1, \quad P_b P_{b'} (-)^l = 1$$

$$|s_a - s_{a'}| \leq j \leq s_a + s_{a'}, \quad |s_b - s_{b'}| \leq l \leq s_b + s_{b'}$$

Selection Rules

- Parity symmetry

- Properties of C-Gs

$$\mathcal{G}_j^{(aa')} = \sum_{\lambda_1 \lambda_2} (-)^{s_a - \lambda_{12}} \langle s_a - \lambda_{12} s_{a'} \lambda_{12} | j0 \rangle \mathcal{F}_{\lambda_1 \lambda_2}^{a \rightarrow 12} \mathcal{F}_{\lambda_1 \lambda_2}^{a' \rightarrow 12*},$$

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Non-interf. terms show up in γ_{jl} when j and l :

$$j \text{ even, } l \text{ even}$$

$$0 \leq j \leq 2s_{a(l)}, \quad 0 \leq l \leq 2s_{b(l)}$$

Selection Rules

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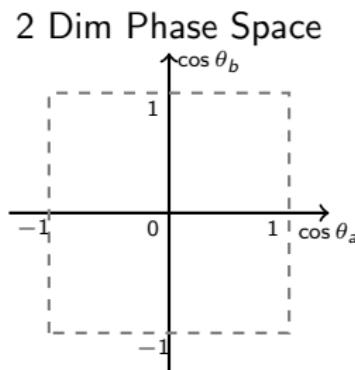
- The Spins and Parities of the intermediate resonances are the key factors.
- If a and a' as well as b and b' have opposite parities, interf. and non-interf. terms will be well separated, GI terms only contain in γ^{jl} when both j and l are odd.

Applications to $\Lambda_b^0 \rightarrow p\pi^-\pi^+\pi^-$: $N(1440) - N(1520)$ and $f_0(500) - \rho(770)$

Decay amplitudes squared, integrating out φ

$$\int \overline{|\mathcal{A}|^2} d\varphi \propto \sum_{jl} \Gamma_{jl}(s_{12}, s_{34}) P_j(c_{\theta_a}) P_l(c_{\theta_b})$$

$$\Gamma_{jl} = \sum_{a,a',b,b'} \frac{\mathcal{W}_{jl}^{(ab,a'b')} \mathcal{G}_j^{(aa')} \mathcal{G}_l^{(bb')}}{\mathcal{I}_a \mathcal{I}_{a'}^* \mathcal{I}_b \mathcal{I}_{b'}^*},$$



$$\mathcal{W}_{jl}^{(ab,a'b')} = \sum_{\sigma\rho} (-)^{\sigma-s_a+\rho-s_b} \langle s_a - \sigma s_{a'} | \sigma | j0 \rangle \langle s_b - \rho s_{b'} | \rho | l0 \rangle \mathcal{F}_{\sigma\rho}^{H \rightarrow ab} \mathcal{F}_{\sigma\rho}^{H \rightarrow a'b'*},$$

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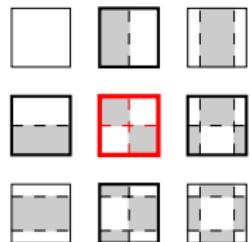
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Applications to $\Lambda_b^0 \rightarrow p\pi^-\pi^+\pi^-$: $N(1440) - N(1520)$ and $f_0(500) - \rho(770)$

c_{θ_a} and c_{θ_b} are correlated because of the GI terms

$$(\Gamma_{jl}) \sim \begin{pmatrix} \text{Non-int} & |N_{1440}N_{1520})|f|^2, \\ & |N_{1440}N_{1520})|\rho|^2 \\ \hline |(f\rho)|N_{1440}|^2, & (\textcolor{red}{N_{1440}N_{1520}f\rho}_{GI}) \\ |(f\rho)|N_{1520}|^2 & |(f\rho)|N_{1520}|^2 \\ \hline \text{Non-int} & |N_{1440}N_{1520})|\rho|^2 \\ & \text{Non-int} \end{pmatrix}.$$

- $(N_{1440}N_{1520}f\rho)_{GI}$: GI term.
- $j = l = 1$ for GI term, hence this GI term is proportional to $\cos \theta_a \cos \theta_b$.

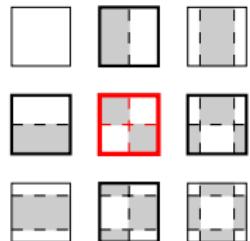


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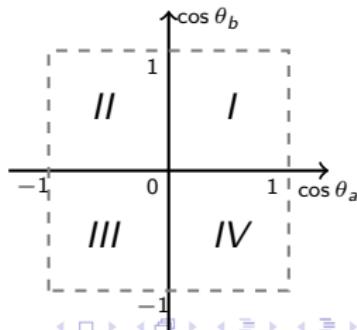


two-fold FBA (TFFBA): $j = l = 1$

$$\tilde{A}^{11} = \frac{(N_I - N_{II} + N_{III} - N_{IV})}{N}$$

TFFBA-CPA (ZHJ, PRD 107 (2023) 1, L011301)

$$A_{CP}^{11} = \frac{1}{2}(\tilde{A}^{11} - \bar{\tilde{A}}^{11})$$



Applications to $\Lambda_b^0 \rightarrow p\pi^-\pi^+\pi^-$: $N(1440) - N(1520)$ and $f_0(500) - \rho(770)$

φ : Up-down asymmetry and Left-right asymmetry

GI terms

- $d_{1,0}^1(\theta_a)d_{1,0}^1(\theta_b)\sin\varphi \sim \sin\theta_a \sin\theta_b \sin\varphi$
- $d_{1,0}^1(\theta_a)d_{1,0}^1(\theta_b)\cos\varphi \sim \sin\theta_a \sin\theta_b \cos\varphi$

$\sin\varphi$: TPA induced CPA (Up-Down Asymmetry induced CPA)

$$A_T \equiv \frac{N(C_T > 0) - N(C_T < 0)}{N(C_T > 0) + N(C_T < 0)} = \frac{N_U - N_D}{N_U + N_D},$$

$$N_{U/D} \equiv N(\sin\varphi > 0), \quad C_T \equiv (\mathbf{p}_1 \times \mathbf{p}_2) \cdot \mathbf{p}_3$$

$$A_{CP}^T \equiv \frac{1}{2}(A_T + \bar{A}_T),$$

$\cos\varphi$: Left-Right Asymmetry induced CPA

$$A^{LR} = \frac{N(C_q > 0) - N(C_q < 0)}{N(C_q > 0) + N(C_q < 0)} = \frac{N_L - N_R}{N_L + N_R},$$

$$N_{L/R} \equiv N(\cos\varphi \gtrless 0), \quad C_q \equiv (\mathbf{p}_1 \times \mathbf{p}_2) \cdot (\mathbf{p}_3 \times \mathbf{p}_4)$$

$$A_{CP}^{LR} \equiv \frac{1}{2}(A^{LR} - \bar{A}^{LR}).$$

③ Summary and Outlook

Summary and Outlook

- regional CPA and FB-CPA in three-body charged B meson decays
- interfer. of resonances with **opposite parities** generate FBA and **FB-CPA** in three-body decays of heavy hadrons
- Angular-correlated CPV observables in four-body decays, such as UD-CPA(TPA-CPA), LR-CPA, and 2-dim-FB-CPA, can help in the experimental CPV study.
- Looking forward to future experimental measurements.

Summary and Outlook

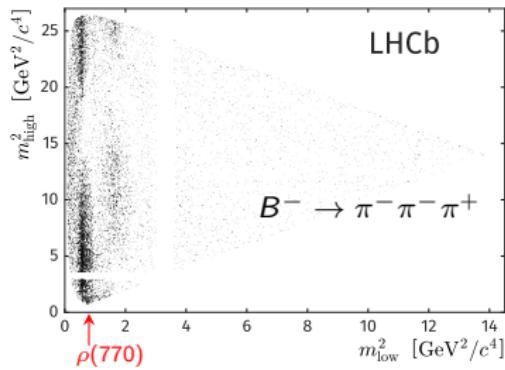
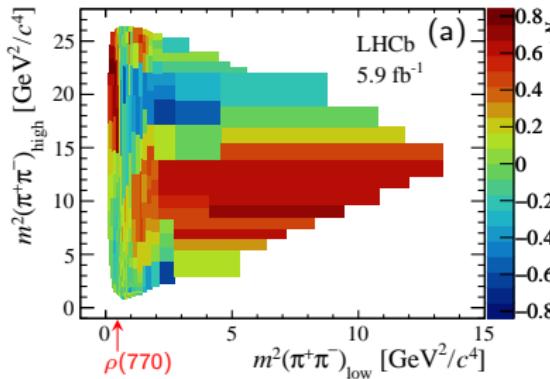
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Thank you for your attentions!

Backup

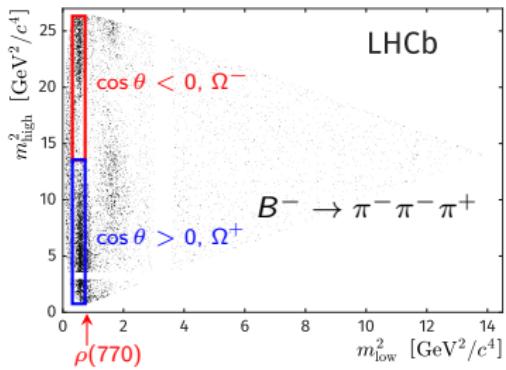
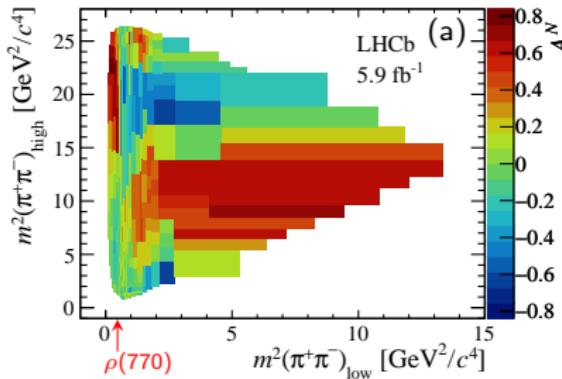
Forward-Backward Asymmetry induced CPA

Corelation between reg. CPA and event distributions

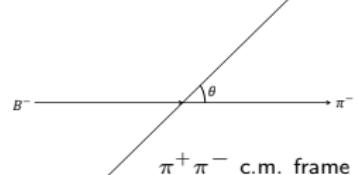


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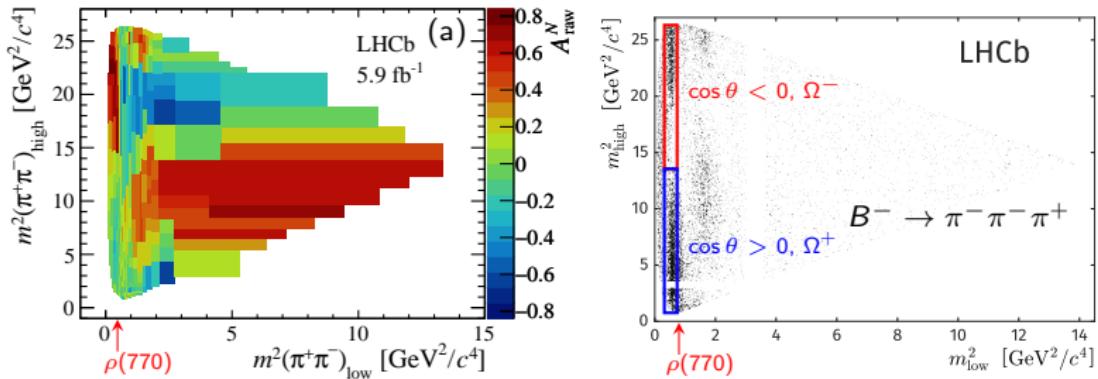


$$\cos \theta = \frac{m_{\text{high}}^2 - (m_{\text{high,max}}^2 + m_{\text{high,min}}^2)/2}{(m_{\text{high,max}}^2 - m_{\text{high,min}}^2)/2}$$



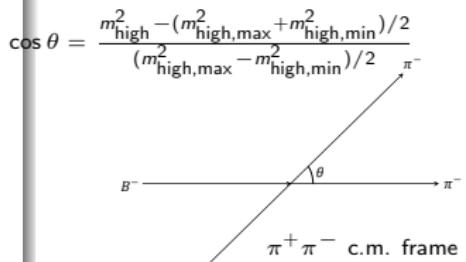
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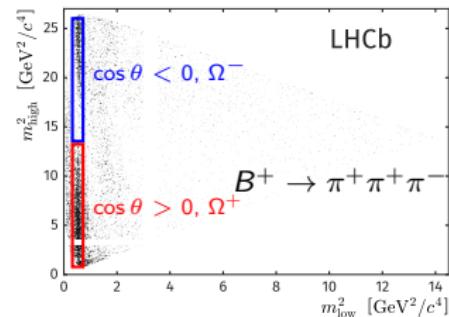
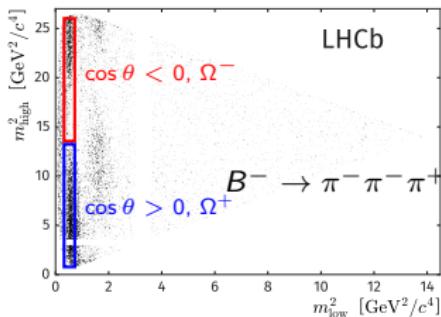


Forward-Backward Asymmetry (FBA)

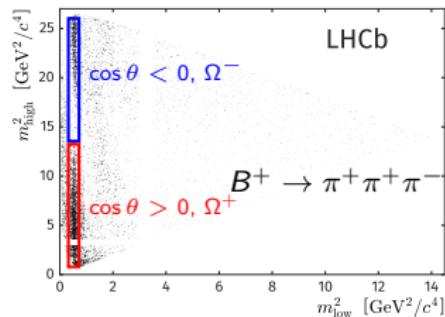
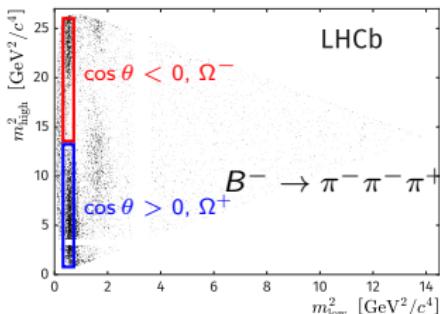
$$\begin{aligned} \mathcal{A} &= a_S + e^{i\delta} a_P \cos \theta, \quad a_{S(P)} = a_{S(P)}^{\text{tree}} + a_{S(P)}^{\text{penguin}} \\ A_{B^-}^{FB} &= \frac{N_{B^-}^{\Omega^+} - N_{B^-}^{\Omega^-}}{N_{B^-}^{\Omega^+} + N_{B^-}^{\Omega^-}} = \frac{\left(-f_{-1}^0 + f_0^+ \right) |\mathcal{A}|^2 d \cos \theta}{f_{-1}^+ |\mathcal{A}|^2 d \cos \theta} \end{aligned}$$



PRL 124 (2020) 031801 [1909.05211]



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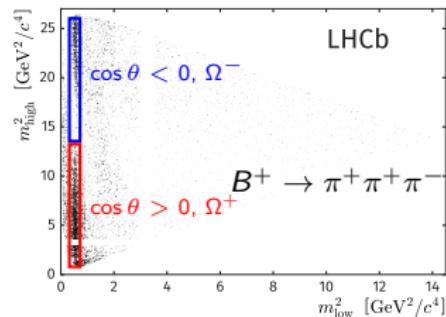
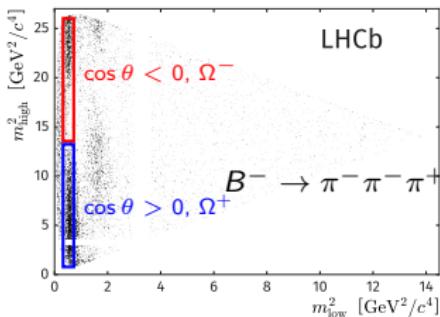
FBA Induced CP Asymmetry (FB-CPA)

$$\begin{aligned} A_{CP}^{FB} &= \frac{1}{2}(A_{B^-}^{FB} - A_{B^+}^{FB}) \\ &= \frac{1}{2} \left(\frac{\Re(\langle a_S^* a_P e^{i\delta} \rangle)}{|\langle a_P \rangle|^2/3 + |\langle a_S \rangle|^2} - \frac{\Re(\langle \bar{a}_S^* \bar{a}_P e^{i\delta} \rangle)}{|\langle \bar{a}_P \rangle|^2/3 + |\langle \bar{a}_S \rangle|^2} \right). \end{aligned}$$

FBI-CPA can isolate the interference effect between S- and P-waves.

(ZHZ, PLB820, 136537)

PRL 124 (2020) 031801 [1909.05211]



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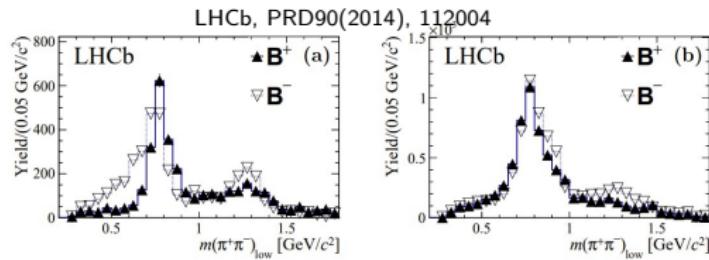
(ZHJ, PLB820, 136537)

direct-CPA-subtracted FB-CPA

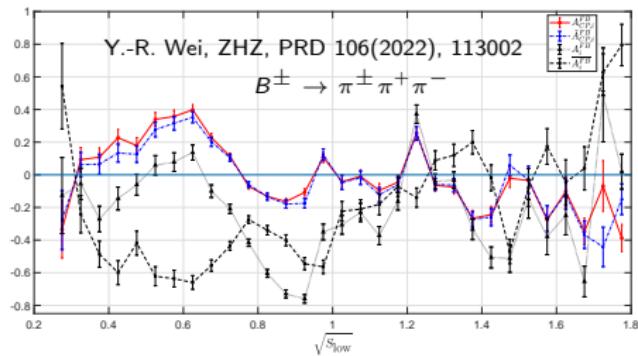
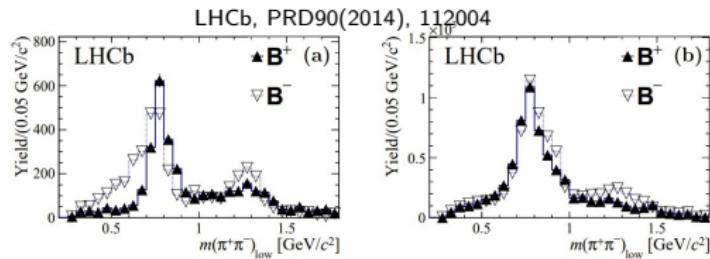
$$\begin{aligned} \tilde{A}_{CP}^{FB} &= \frac{N_{B^-}^{\Omega^+} - N_{B^-}^{\Omega^-} - N_{B^+}^{\Omega^+} + N_{B^+}^{\Omega^-}}{N_{B^-}^{\Omega^+} + N_{B^-}^{\Omega^-} + N_{B^+}^{\Omega^+} + N_{B^+}^{\Omega^-}} \\ &= \frac{\Re(\langle a_S^* a_P e^{i\delta} \rangle) - \Re(\langle \bar{a}_S^* \bar{a}_P e^{i\delta} \rangle)}{|\langle a_P \rangle|^2/3 + |\langle a_S \rangle|^2 + |\langle \bar{a}_P \rangle|^2/3 + |\langle \bar{a}_S \rangle|^2} \end{aligned}$$

(Y.-R. Wei, ZHZ, PRD 106(2022), 113002[2209.02348])

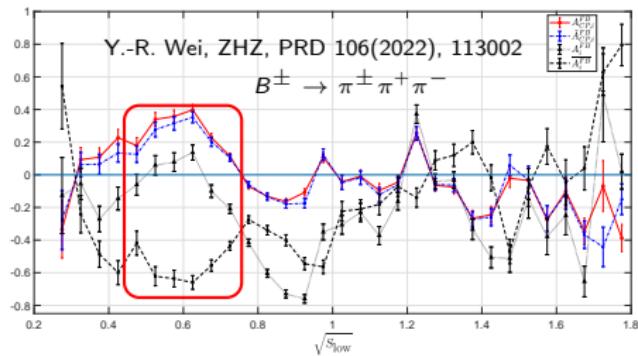
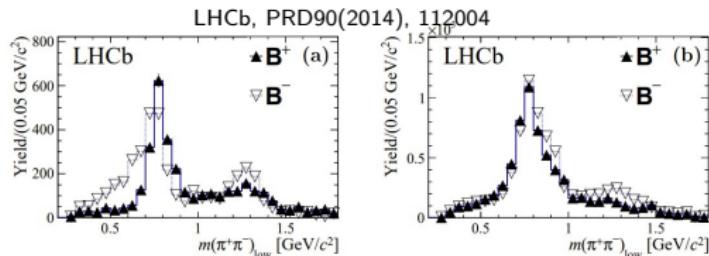
FB-CPA v.s. reg. CPA (based on LHCb data of PRD90(2014), 112004)



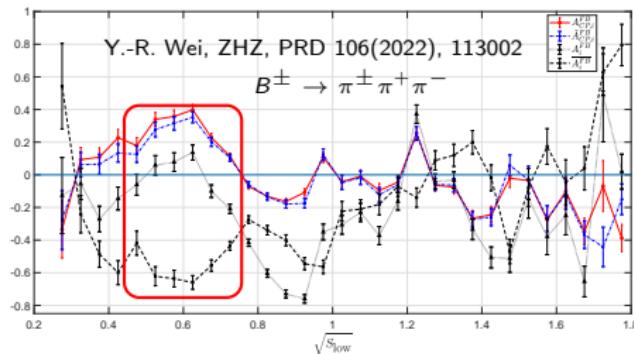
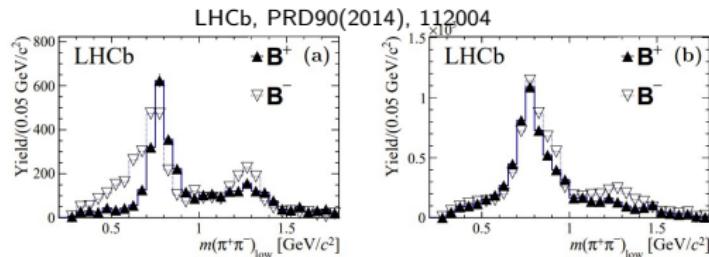
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FB-CPA v.s. reg. CPA (based on LHCb data of PRD90(2014), 112004)



FB-CPA v.s. reg. CPA (based on LHCb data of PRD90(2014), 112004)



A_{CP}^{FB}	\tilde{A}_{CP}^{FB}	A_{CP}^{Ω}	$A_{CP}^{\Omega^+}$	$A_{CP}^{\Omega^-}$
0.224 ± 0.012	0.194 ± 0.013	0.099 ± 0.013	0.405 ± 0.020	-0.074 ± 0.017

Decay amplitude squared

Without integrating out φ , without the assumption of unpolarized H :

$$\overline{|\mathcal{A}|^2} \propto \sum \gamma_{\sigma_{a'a}, \sigma_{b'b}}^{jl} d_{\sigma_{a'a}, 0}^j(\theta_a) d_{\sigma_{b'b}, 0}^l(\theta_b) e^{i\sigma_{aa'}\phi_a} e^{i\sigma_{bb'}\phi_b},$$

$$\gamma_{\sigma_{a'a}, \sigma_{b'b}}^{jl} = \frac{w_{jl}^{(ab, a'b')} \mathcal{G}_j^{(aa')} \mathcal{G}_l^{(bb')}}{\mathcal{I}_a \mathcal{I}_{a'}^* \mathcal{I}_b \mathcal{I}_{b'}^*},$$

$$P(\theta) = \frac{1}{2} \begin{pmatrix} \cos \theta & -\sin \theta \\ -\sin \theta & -\cos \theta \end{pmatrix} P_z,$$

$$\begin{aligned} w_{\sigma_{a'a}, \sigma_{b'b}}^{(ab, a'b')jl} &= \langle s_a - \sigma_a s_{a'} | \sigma_{a'} | j \sigma_{a'a} \rangle \langle s_b - \sigma_b s_{b'} | \sigma_{b'} | l \sigma_{b'b} \rangle \\ &\times (-)^{\sigma_a - s_a + \sigma_b - s_b} \mathcal{F}_{\sigma_a \sigma_b}^{H \rightarrow ab} \mathcal{F}_{\sigma_{a'} \sigma_{b'}}^{H \rightarrow a'b' *} \\ &\times P_{\sigma_{ab}, \sigma_{a'b'}}(\theta), \end{aligned}$$

For unpolarized H , $P_z = 0$:

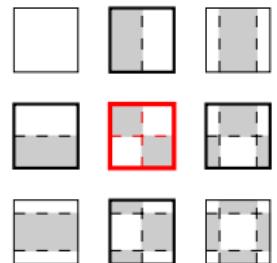
$$\begin{aligned} \sigma_{aa'} &= \sigma_{bb'}, \text{ and} \\ e^{i\sigma_{aa'}\phi_a} e^{i\sigma_{bb'}\phi_b} &\rightarrow e^{-i\sigma_{aa'}\varphi}. \end{aligned}$$

Applications to $\Lambda_b^0 \rightarrow p\pi^-\pi^+\pi^-$: $N(1440) - N(1520)$ and $f_0(500) - \rho(770)$

$\Lambda_b^0 \rightarrow N(\rightarrow p\pi^-)f/\rho(\rightarrow pi^+\pi^-)$: c_{θ_a} and c_{θ_b} are correlated.

$$(\Gamma_{jl}) \sim \begin{pmatrix} \text{Non-int} & |(N_{1440}N_{1520})|f|^2, & \text{Non-int} \\ |(N_{1440}N_{1520})|\rho|^2 & \color{red}{(N_{1440}N_{1520}f\rho)_{GI}} & |(f\rho)|N_{1520}|^2 \\ \frac{|(f\rho)|N_{1440}|^2,}{|(f\rho)|N_{1520}|^2} & & \text{Non-int} \\ \text{Non-int} & |(N_{1440}N_{1520})|\rho|^2 & \text{Non-int} \end{pmatrix}.$$

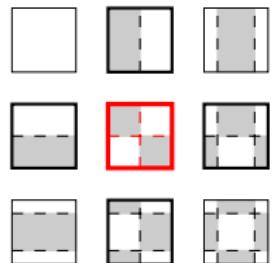
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two-fold FBA (TFFBA): $j = 1 = l$

$$\tilde{A}^{11} = \frac{(N_I - N_{\bar{I}} + N_{\bar{II}} - N_{\bar{N}})}{N}$$

TFFBA-CPA

$$A_{CP}^{11} = \frac{1}{2}(\tilde{A}^{11} - \overline{\tilde{A}^{11}})$$

dir-CPV-subtracted TFFBA-CPA

$$\hat{A}_{CP}^{11} = \frac{(N_I - N_{\bar{I}} + N_{\bar{II}} - N_{\bar{N}}) - (\bar{N}_I - \bar{N}_{\bar{I}} + \bar{N}_{\bar{II}} - \bar{N}_{\bar{N}})}{N + \bar{N}}$$