



**NNU** · 南京师范大学  
NANJING NORMAL UNIVERSITY



# Decay Constant of Vector Bc Meson and its Experimental Feasibility

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2209.15521; 2303.02692

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5<sup>th</sup> Heavy Flavor and QCD Conf. @Wuhan

April 21, 2023

# Content



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- **Research Background**
- **Decay Constant of  $B_c^*$  in EFT**
- **Decay Width of  $B_c^*$  and its non-radiative decay modes**
- **Summary and Outlook**

# 1、Research Background

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➤ **Meson Puzzle:**  $B_c^+$  ( $\bar{b}c$ ) discovered in 1998, CDF @Fermi Lab)

Friday, March 20, 1998

Number 6

## CDF Corrals the Last of the Mesons

by Judy Jackson, Office of Public Affairs

By tradition, Fermilab scientists announce discoveries first at their own laboratory, and researchers from CDF, the Collider Detector at Fermilab, carried on the tradition with a March 5 seminar presenting the newest member of the meson family, the  $B_c$  meson.

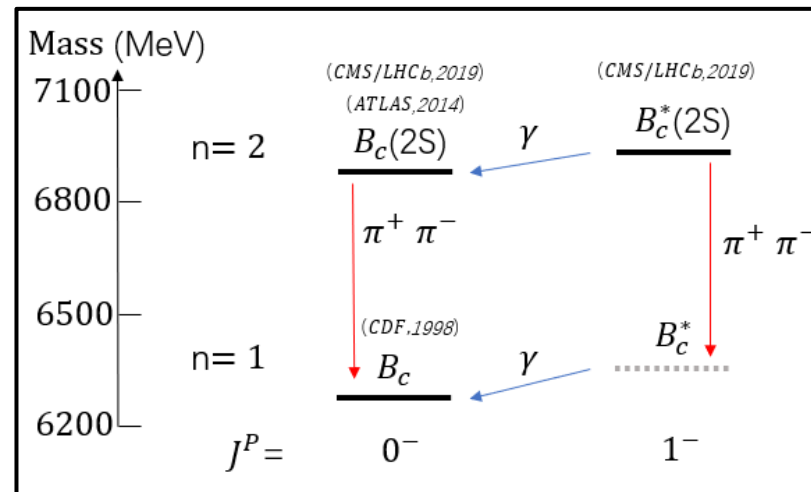
Fifty years ago, scientists discovered the first meson, the pion, in cosmic rays on a mountaintop. CDF's newly minted particle, the  $B_c$  ("Bee Sub See"), a combination of a charm quark and an antibottom quark, created in collisions at the Tevatron, is likely to be the last of the quark-antiquark pairs that constitute normal garden-variety mesons.

...at least the last of the normal ones

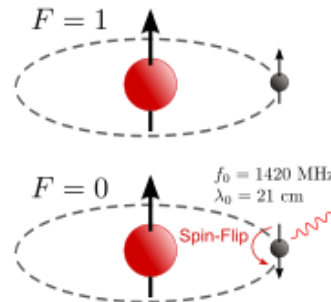
Physicist Shin-Hong Kim, of Tsukuba University in Japan, presented "the discovery of a non-elementary particle" to an all-Fermilab audience. Kim explained the CDF researchers' methods for identifying the  $B_c$  from its decay products and separating the particle's distinctive electronic signal from meson-mimicking

continued on page 2

$u\bar{u}$ $\pi^0, \eta, \eta'$	$u\bar{d}$ $\pi^+$	$u\bar{s}$ $K^+$	$u\bar{c}$ $D^0$	$u\bar{b}$ $B^+$
$d\bar{d}$ $\pi^0, \eta, \eta'$	$d\bar{s}$ $K^0$	$d\bar{c}$ $D^-$	$d\bar{b}$ $B^0$	
$s\bar{s}$ $\eta, \eta'$	$s\bar{c}$ $D_s^-$	$s\bar{b}$ $B_s^-$		
$c\bar{c}$ $J/\psi$	<b><math>c\bar{b}</math> <math>B_c^-</math></b>			
	$b\bar{b}$ $Y$			



$$E_0 \approx (6227 - m_b - m_c) \text{ MeV}, \quad \Delta E_0 \approx 60 \text{ MeV}$$



**Nonrelativistic system  
at femtoscale,  
similar to Hydrogen**

**1998,  $B_c(1S), 0^-$**

**51 years, ground meson**

# Ground Bc (0-) Decays

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➤ LHCb made great progress to detect Bc decay modes

Bottom decay ~20%, charm decay ~70%, both decay ~10%

## $B_c^+$ DECAY MODES $\times B(\bar{b} \rightarrow B_c)$

$B_c^-$  modes are charge conjugates of the modes below.

Mode	Fraction ( $\Gamma_i/\Gamma$ )	Confidence level
$\Gamma_1$ $J/\psi(1S)\ell^+\nu_\ell$ anything	seen	
$\Gamma_2$ $J/\psi(1S)\mu^+\nu_\mu$	seen	
$\Gamma_3$ $J/\psi(1S)\tau^+\nu_\tau$	seen	
$\Gamma_4$ $J/\psi(1S)\pi^+$	seen	
$\Gamma_5$ $J/\psi(1S)K^+$	seen	
$\Gamma_6$ $J/\psi(1S)\pi^+\pi^+\pi^-$	seen	
$\Gamma_7$ $J/\psi(1S)a_1(1260)$	not seen	
$\Gamma_8$ $J/\psi(1S)K^+K^-\pi^+$	seen	
$\Gamma_9$ $J/\psi(1S)\pi^+\pi^+\pi^+\pi^-\pi^-$	seen	
$\Gamma_{10}$ $\psi(2S)\pi^+$	seen	
$\Gamma_{11}$ $J/\psi(1S)D^0K^+$	seen	
$\Gamma_{12}$ $J/\psi(1S)D^*(2007)^0K^+$	seen	
$\Gamma_{13}$ $J/\psi(1S)D^*(2010)^+K^{*0}$	seen	
$\Gamma_{14}$ $J/\psi(1S)D^+K^{*0}$	seen	
$\Gamma_{15}$ $J/\psi(1S)D_s^+$	seen	
$\Gamma_{16}$ $J/\psi(1S)D_s^{*+}$	seen	
$\Gamma_{17}$ $J/\psi(1S)p\bar{p}\pi^+$	seen	

PDG-2022

## $B_c^+$ DECAY MODES $\times B(\bar{b} \rightarrow B_c)$

$B_c^-$  modes are charge conjugates of the modes below.

Mode Fraction ( $\Gamma_i/\Gamma$ ) Confidence level

Citation: J. Beringer et al. (Particle Data Group), PR **D86**, 010001 (2012) (URL: <http://pdg.lbl.gov>)

The following quantities are not pure branching ratios; rather the fraction  $\Gamma_i/\Gamma \times B(\bar{b} \rightarrow B_c)$ .

$\Gamma_1$	$J/\psi(1S)\ell^+\nu_\ell$ anything
$\Gamma_2$	$J/\psi(1S)\pi^+$
$\Gamma_3$	$J/\psi(1S)\pi^+\pi^+\pi^-$
$\Gamma_4$	$J/\psi(1S)a_1(1260)$
$\Gamma_5$	$D^*(2010)^+\bar{D}^0$

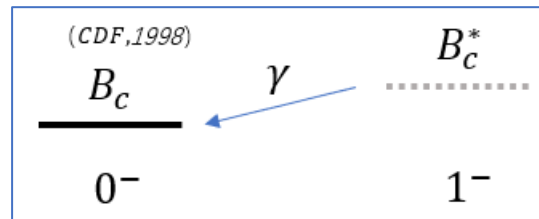
PDG-2012  
5 个衰变道

$\Gamma_{18}$	$\chi_c^0\pi^+$		
$\Gamma_{19}$	$p\bar{p}\pi^+$		
$\Gamma_{20}$	$D^0K^+$		
$\Gamma_{21}$	$D^0\pi^+$		
$\Gamma_{22}$	$D^{*0}\pi^+$		
$\Gamma_{23}$	$D^{*0}K^+$		
$\Gamma_{24}$	$D_s^+\bar{D}^0$		
$\Gamma_{25}$	$D_s^+D^0$		
$\Gamma_{26}$	$D_s^+\bar{D}^0$		
$\Gamma_{27}$	$D^+D^0$		
$\Gamma_{28}$	$D_s^{*+}\bar{D}^0$		
$\Gamma_{29}$	$D_s^+\bar{D}^*(2007)^0$		
$\Gamma_{30}$	$D_s^{*+}D^0$		
$\Gamma_{31}$	$D_s^+D^*(2007)^0$	$< 6.6 \times 10^{-4}$	90%
$\Gamma_{32}$	$D^*(2010)^+\bar{D}^0$	$< 3.8 \times 10^{-4}$	90%
$\Gamma_{33}$	$D^*(2010)^+\bar{D}^0, D^{*+} \rightarrow D^+\pi^0/\gamma$	not seen	
$\Gamma_{34}$	$D^+\bar{D}^*(2007)^0$	$< 6.5 \times 10^{-4}$	90%
$\Gamma_{35}$	$D^*(2007)^+D^0$	$< 2.0 \times 10^{-4}$	90%
$\Gamma_{36}$	$D^*(2010)^+D^0, D^{*+} \rightarrow D^+\pi^0/\gamma$	not seen	
$\Gamma_{37}$	$D^+D^*(2007)^0$	$< 3.7 \times 10^{-4}$	90%
$\Gamma_{38}$	$D_s^{*+}\bar{D}^*(2007)^0$	$< 1.3 \times 10^{-3}$	90%
$\Gamma_{39}$	$D_s^{*+}D^*(2007)^0$	$< 1.3 \times 10^{-3}$	90%
$\Gamma_{40}$	$D^*(2010)^+\bar{D}^*(2007)^0$	$< 1.0 \times 10^{-3}$	90%
$\Gamma_{41}$	$D^*(2010)^+D^*(2007)^0$	$< 7.7 \times 10^{-4}$	90%
$\Gamma_{42}$	$D^+K^{*0}$	not seen	
$\Gamma_{43}$	$D^+\bar{K}^{*0}$	not seen	
$\Gamma_{44}$	$D_s^+K^{*0}$	not seen	
$\Gamma_{45}$	$D_s^+\bar{K}^{*0}$	not seen	
$\Gamma_{46}$	$D_s^+\phi$	not seen	
$\Gamma_{47}$	$K^+K^0$	not seen	
$\Gamma_{48}$	$B_s^0\pi^+ / B(\bar{b} \rightarrow B_s)$	seen	

# Vector $B_c^*(1^-)$

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- Excited  $B_c$  states below BD threshold major decay to ground  $B_c$  state
- Hyperfine splitting between  $B_c^*(1S)$  and  $B_c(1S)$ : around 60MeV



- $B_c^*(1S)$  major (99.99%) electromagnetic decays to  $B_c(1S)$



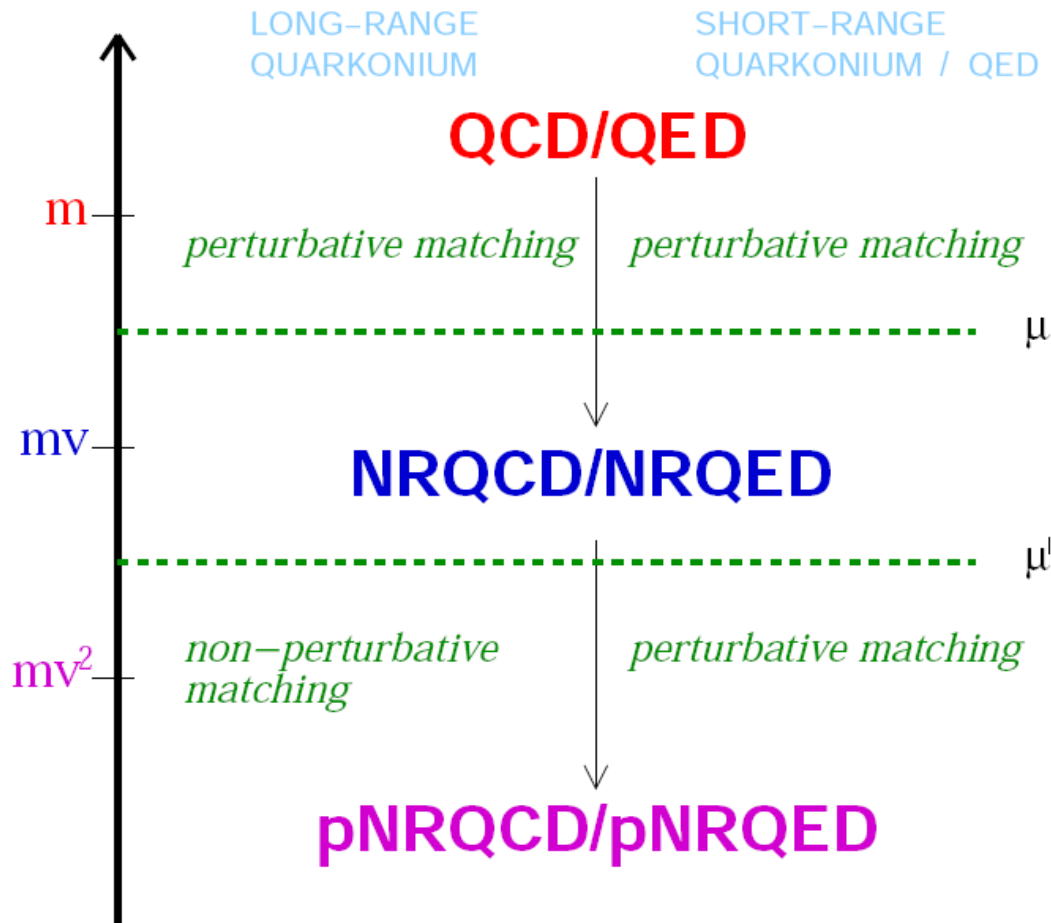
- However, 60MeV photon is hard to detect at LHC

# 2、Decay Constant of $B_c^*$ in EFT

## ➤ Nonrelativistic EFT in QCD/QED

$$\alpha_s(mv) \sim v$$

$$v^2 \approx 0.1 \text{ for the } \Upsilon$$



Bodwin-Braaten-Lapage-1995

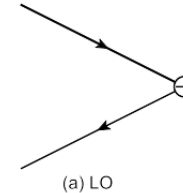
Pineda-Soto-Brambilla-Vairo-2000

# Decay Constant of $B_c^*$

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## ➤ $B_c^*$ decay constants in QCD

$$\langle 0 | \bar{b} \gamma^\mu c | B_c^*(P, \varepsilon) \rangle = f_{B_c^*}^\nu m_{B_c^*} \varepsilon^\mu,$$



## ➤ $B_c^*$ decay constants in NRQCD

$$f_{B_c^*}^\nu = \sqrt{\frac{2}{m_{B_c^*}}} \underbrace{C_v(m_b, m_c, \mu_f)}_{\text{matching coefficients}} \underbrace{\langle 0 | \chi_b^\dagger \sigma \cdot \varepsilon \psi_c | B_c^*(\mathbf{P}) \rangle}_{\text{NRQCD LDMEs}} (\mu_f) + O(v^2)$$

## ➤ Matching Formulae

$$Z_J Z_{2,b}^{\frac{1}{2}} Z_{2,c}^{\frac{1}{2}} \Gamma_J = C_J \tilde{Z}_J^{-1} \tilde{Z}_{2,b}^{\frac{1}{2}} \tilde{Z}_{2,c}^{\frac{1}{2}} \tilde{\Gamma}_J$$

$\tilde{Z}_J$ : NRQCD  $\overline{\text{MS}}$  current renormalization constants

# Calculation procedure

- **Feynman Diagrams & Amplitudes**  
(Packages: FeynArts / QGraf)
- **Feynman Amplitudes Simplification: Trace & Contraction**  
(Packages: FeynCalc / FormCalc / FormLink)
- **Feynman Integrals Reduction**  
(Packages: Apart(Feng) / FIRE/Kira /...)
- **Feynman Master Integrals Calculation:**  
(Packages: AMFlow(Ma et al) / FIESTA /...)

**A Systematic and Efficient Method to Compute Multi-loop Master Integrals** #1  
Xiao Liu (Peking U. and Peking U., SKLNPT), Yan-Qing Ma (Peking U. and Peking U., SKLNPT and Peking U., CHEP and CICQM, Beijing), Chen-Yu Wang (Peking U. and Peking U., SKLNPT) (Nov 27, 2017)  
Published in: *Phys.Lett.B* 779 (2018) 353-357 • e-Print: 1711.09572 [hep-ph]  
pdf DOI cite claim reference search 84 citations

**AMFlow: A Mathematica package for Feynman integrals computation via auxiliary mass flow** #2  
Xiao Liu (Peking U., SKLNPT and Oxford U., Theor. Phys.), Yan-Qing Ma (Peking U., SKLNPT and Peking U., CHEP and CICQM, Beijing) (Jan 27, 2022)  
Published in: *Comput.Phys.Commun.* 283 (2023) 108565 • e-Print: 2201.11669 [hep-ph]  
pdf DOI cite claim reference search 63 citations

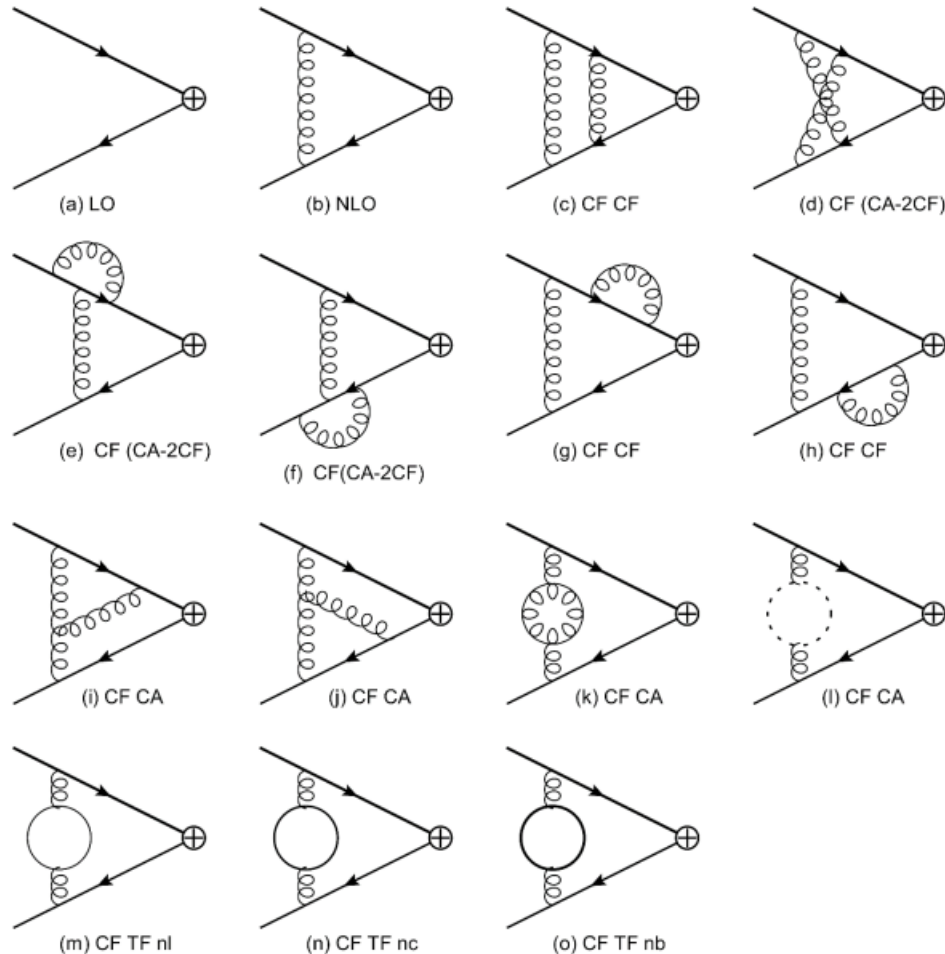
**See Prof. Ma Yanqing's talk tomorrow**

# Two-loop Feynman diagrams



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➤ Feynman diagram up to two-loop order accuracy



# Two-loop results

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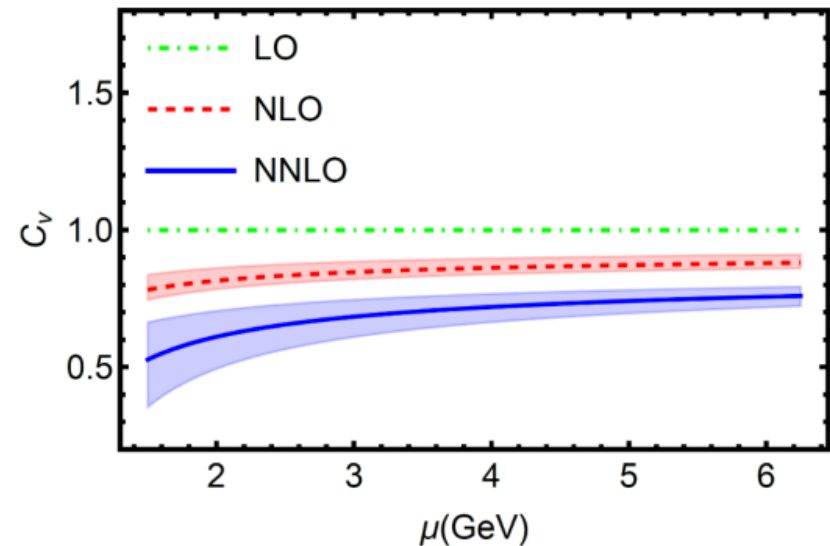
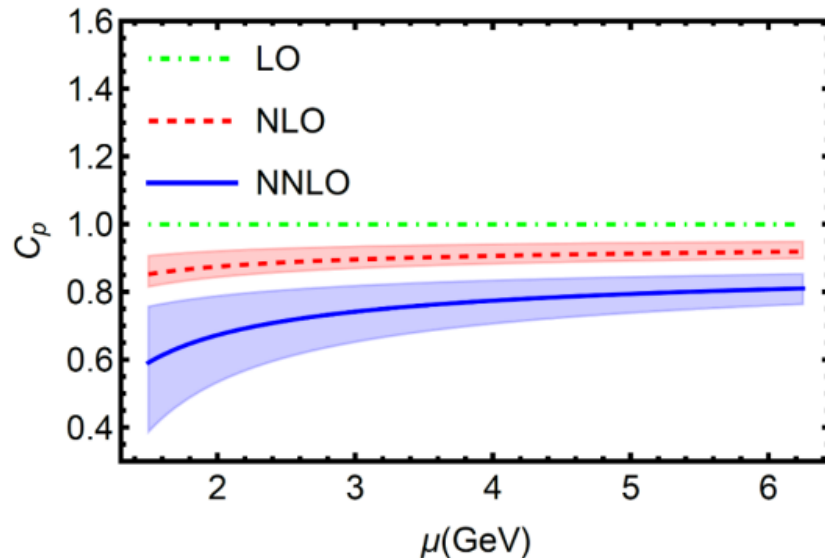
## ➤ Matching coefficients at two loop

Tao-Zhu-Xiao, 2209.15521

$\mu_f \in [1.5, 1.2, 1] \text{ GeV}$ ,  $\mu \in [6.25, 4.75, 3] \text{ GeV}$ ,  $m_b \in [5.25, 4.75, 4.25] \text{ GeV}$ ,  $m_c \in [2, 1.5, 1] \text{ GeV}$

	LO	NLO	NNLO
$C_p$	1	$0.9117^{+0.0072+0.0061-0.0156}_{+0-0.0160-0.0064+0.0263}$	$0.7897^{+0.0206+0.0119+0.0149}_{+0.0253-0.0482-0.0133-0.0141}$
$C_v$	1	$0.8697^{+0.0107+0.0061-0.0156}_{+0-0.0236-0.0064+0.0263}$	$0.7363^{+0.0230+0.0106+0.0117}_{+0.0191-0.0526-0.0117-0.0121}$

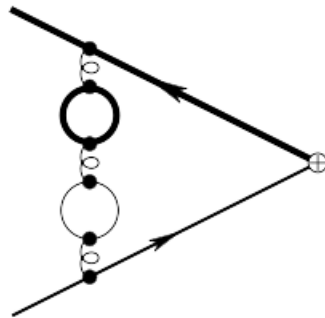
$\mu$  dependence for matching coefficients



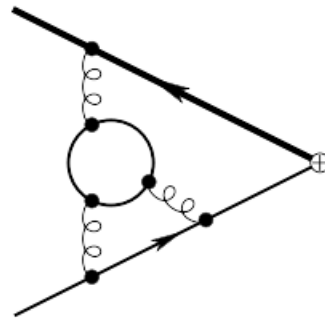
# Three-loop diagrams

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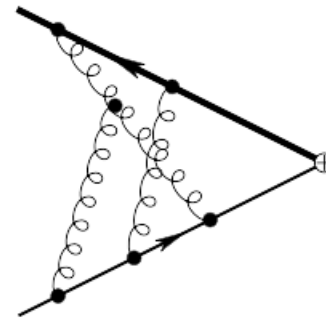
## ➤ Typical diagrams up to three-loop



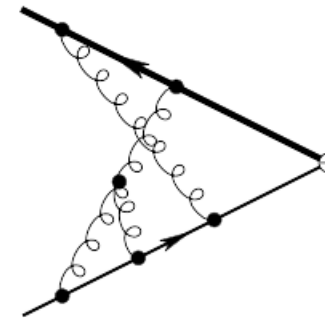
(a) TF nb TF n1 CF



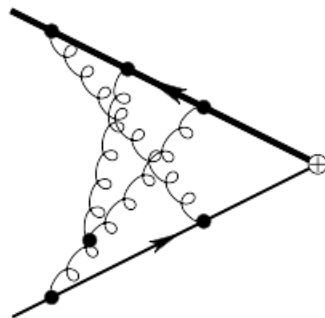
(b) TF nc (CA - 4CF) CF



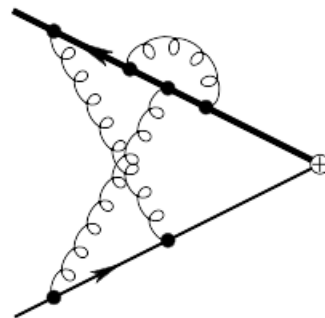
(c) CA (CA - 2CF) CF



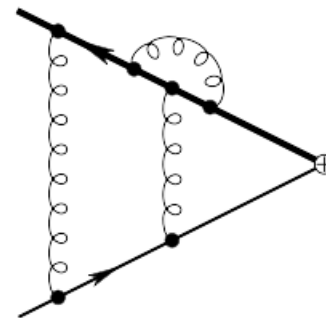
(d) CA (CA - 2CF) CF



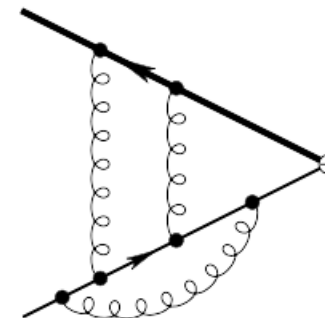
(e) CA (CA - 2CF) CF



(f) (CA - 2CF) (CA - 2CF) CF



(g) (CA - 2CF) CF CF



(h) (CA - 2CF) (CA - 2CF) CF

LO:1, NLO:1, NNLO:11, N<sup>3</sup>LO:268

# Results for vector and pseudoscalar

## ➤ Matching coefficients for vector current(state-of-art)

$$\mathcal{C} = 1 - 2.29 \left( \frac{\alpha_s^{(n_l)}}{\pi} \right) - 35.44 \left( \frac{\alpha_s^{(n_l)}}{\pi} \right)^2 - 1686.27 \left( \frac{\alpha_s^{(n_l)}}{\pi} \right)^3 + \mathcal{O}(\alpha_s^4),$$

for  $n_l = 3, n_c = 1, n_b = 0$ ,

Sang-Zhang-Zhou, arXiv:2210.02979

## ➤ Matching coefficients for pseudoscalar current(state-of-art)

$$\mathcal{C}(x_{\text{phys}}) = 1 - 1.62623 \left( \frac{\alpha_s^{(n_l)}(m_r)}{\pi} \right) - 6.51043 \left( \frac{\alpha_s^{(n_l)}(m_r)}{\pi} \right)^2 - 1520.59 \left( \frac{\alpha_s^{(n_l)}(m_r)}{\pi} \right)^3 + \mathcal{O}(\alpha_s^4)$$

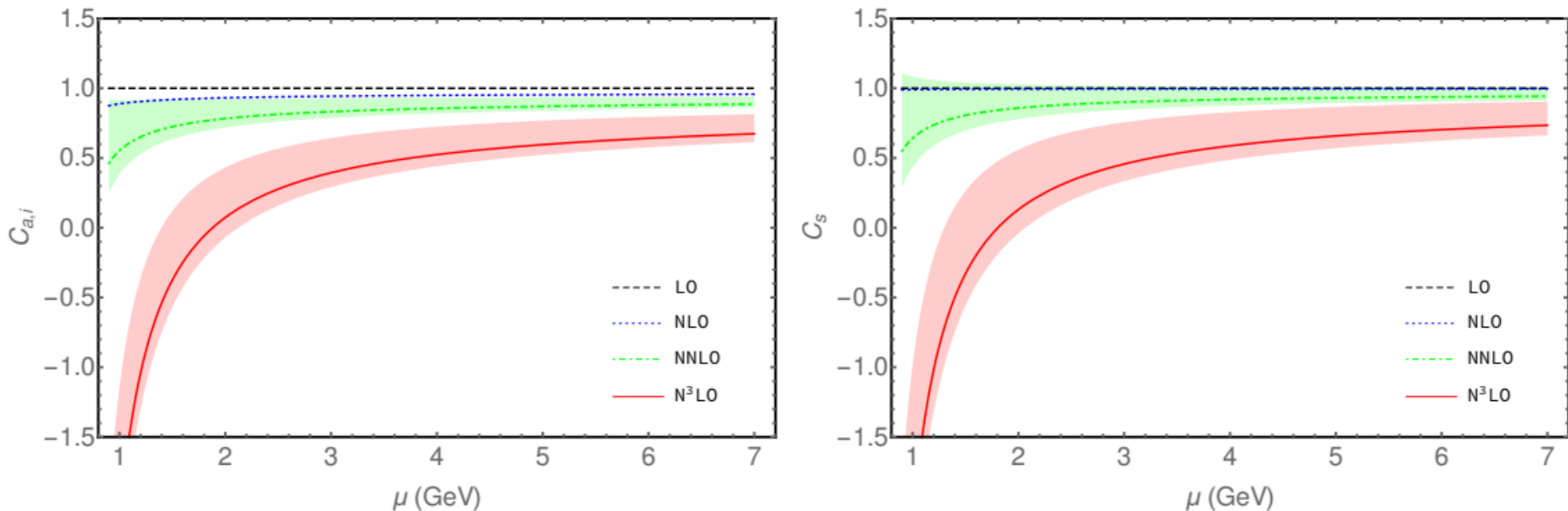
Feng-Jia-Mo-Pan-Sang-Zhang, arXiv:2208.04302

# Results for axial-vector and scalar



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## ➤ Matching coefficients for axial-vector and scalar up to three loop



Nonconvergence behaviors also in other two currents

Tao-Xiao-Zhu, arXiv: 2303.02692

# Sub-leading Contribution



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## ➤ Relativistic corrections

$$\begin{aligned} & \langle 0 | \overline{Q_1} \gamma^5 Q_2 | Q_2 \overline{Q_1} \rangle_{\text{QCD}} \\ &= \sqrt{2M_H} \left[ C_0^P \langle 0 | \chi_1^\dagger \psi_2 | Q_2 \overline{Q_1}(\mathbf{p}) \rangle_{\text{NRQCD}} + C_2^P \langle 0 | (\mathbf{D}\chi_1)^\dagger \cdot \mathbf{D}\psi_2 | Q_2 \overline{Q_1}(\mathbf{p}) \rangle_{\text{NRQCD}} + \dots \right] \end{aligned}$$

Employing EOM:  $\langle 0 | (\mathbf{D}\chi_1)^\dagger \mathbf{D}\psi_2 | Q_2 \overline{Q_1}(\mathbf{p}) \rangle = -2m_r E \langle 0 | \chi_1^\dagger \psi_2 | Q_2 \overline{Q_1}(\mathbf{p}) \rangle.$

$$f_{B_c^*} = 2 \sqrt{\frac{N_c}{m_{B_c^*}}} \left[ \mathcal{C}_v + \frac{d_v E_{B_c^*}}{12} \left( \frac{8}{M} - \frac{3}{m_r} \right) \right] |\Psi_{B_c^*}(0)|,$$

$$f_{B_c} = 2 \sqrt{\frac{N_c}{m_{B_c}}} \left[ \mathcal{C}_p - \frac{d_p E_{B_c}}{4m_r} \right] |\Psi_{B_c}(0)|,$$

# Wave function scale dependence

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## ➤ Wave function at origin

For Power-law potential  $V(r) = Ar^a + C$

Exact solution  $|\psi_\mu^n(0)|^2 = f(n, a)(A\mu)^{3/(2+a)}$

Scale relation  $|\Psi_{B_c^*}(0)| = |\Psi_{J/\psi}(0)|^{1-y} |\Psi_\Upsilon(0)|^y,$   
 $y = y_c = \ln((1 + m_c/m_b)/2) / \ln(m_c/m_b)$

Collins-Imbo-King-Martell, PLB 393 (1997) 155–160

$$|\psi_1(0)|^2 = |\psi_1^{(0)}(0)|^2 \left( 1 + \sum_{k=1}^n f_k a_s^k \right). \quad \begin{aligned} |\psi_1^{(0)}(0)|^2 &= \frac{(m_b C_F \alpha_s)^3}{8\pi}, \\ E_1^{(0)} &= -\frac{1}{4} m_b (C_F \alpha_s)^2, \end{aligned}$$

Beneke-Kiyo-Marquard-Penin-Piclum-Seide-Steinhauser, PRL. 112, 151801 (2014)

# Hyperfine splitting

➤ Hyperfine splitting for beauty-charm family

$$(\Delta M)_{i\bar{j}} = 32\pi\alpha_s(2\mu_{i\bar{j}})|\Psi_{i\bar{j}}(0)|^2/9m_i m_j ,$$

$$\Delta M_{c\bar{b}} = \alpha_s(2m_r)x^{1-2q} \left( \frac{\Delta M_{c\bar{c}}}{\alpha_s(m_c)} \right)^{1-q} \left( \frac{\Delta M_{b\bar{b}}}{\alpha_s(m_b)} \right)^q .$$

$$\Delta M_{c\bar{b}(1S)} = 63.8_{-8.4}^{+5.5}(q)_{-1.2}^{+1.2}(exp) \text{ MeV},$$

$$\Delta M_{c\bar{b}(2S)} = 26.4_{-3.3}^{+2.1}(q)_{-1.7}^{+1.5}(exp) \text{ MeV},$$

$$\Delta M_{b\bar{b}(1S)} = 62.3 \pm 3.2 \text{ MeV}$$

$$\Delta M_{b\bar{b}(2S)} = 24 \pm 4 \text{ MeV}$$

CMS 2019

$$\Delta M_{c\bar{b}(2S)} = 29.1 \pm 1.5(stat) \pm 0.7(syst) \text{ MeV}$$

LHCb 2019

$$\Delta M_{c\bar{b}(2S)} = 31.0 \pm 1.4(stat) \pm 0.0(syst) \text{ MeV}$$

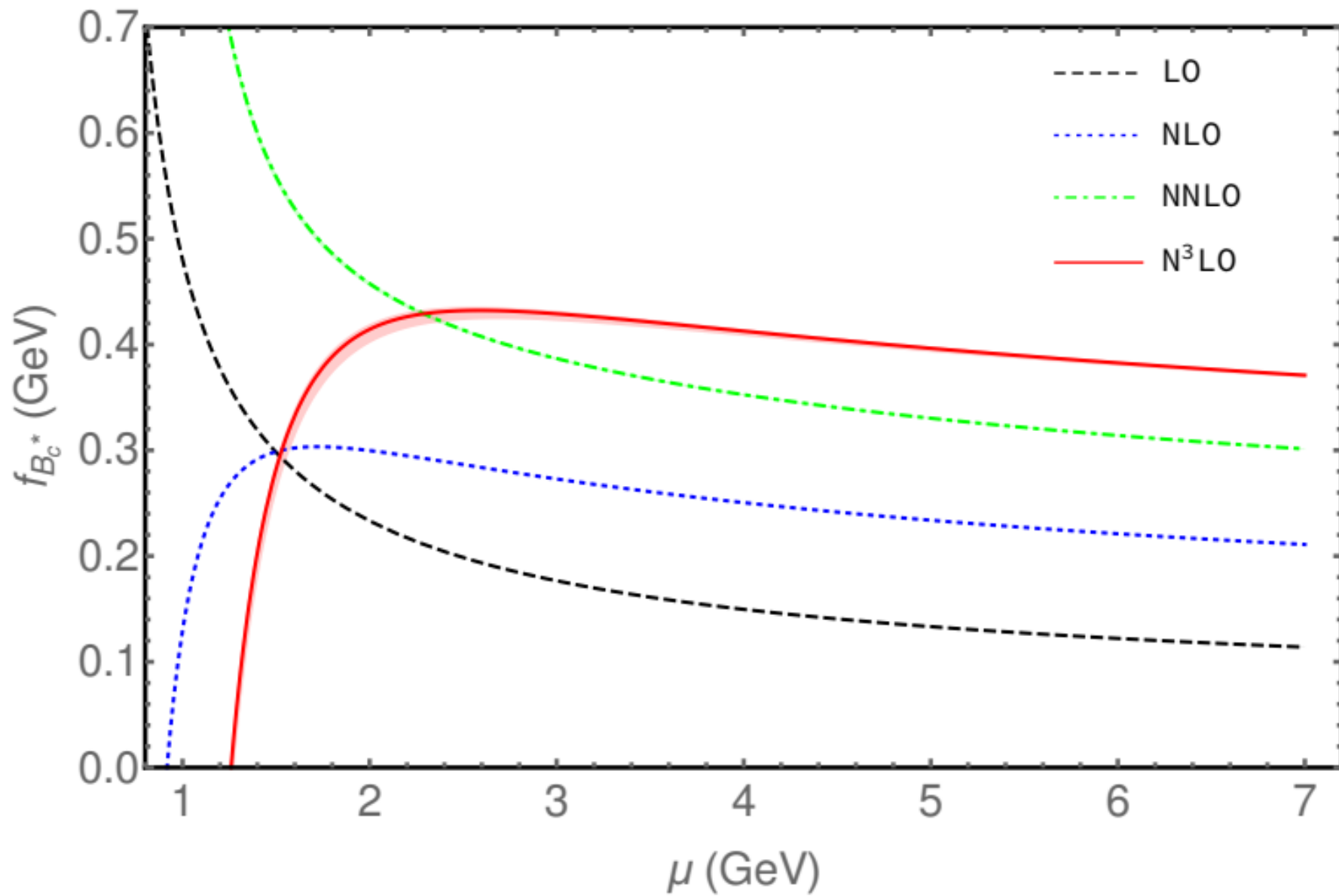
HPQCD lattice results

$$\Delta M_{c\bar{b}(1S)} = 54 \pm 4 \text{ MeV}$$

# Vector $B_c^*$ decay constant



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# Leptonic branching ratios

TABLE I: The  $N^3\text{LO}$  predictions for the pure leptonic branching ratios  $\mathcal{B}(B_c^{*+}(B_c^+) \rightarrow l^+ \nu_l)$  with  $l = (e, \mu, \tau)$ . The three uncertainties come from  $q \in [0.3, 0.4]$ ,  $\mu_f \in [0.4, 7]\text{GeV}$ ,  $\mu \in [2.2, 7]\text{GeV}$ , respectively.

Branching ratios	$N^3\text{LO}$
$\mathcal{B}(B_c^{*+} \rightarrow e^+ \nu_e)$	$(3.85_{-0.46+0.03+0.37}^{+0.29-0.07-1.35}) \times 10^{-6}$
$\mathcal{B}(B_c^{*+} \rightarrow \mu^+ \nu_\mu)$	$(3.85_{-0.46+0.03+0.37}^{+0.29-0.07-1.35}) \times 10^{-6}$
$\mathcal{B}(B_c^{*+} \rightarrow \tau^+ \nu_\tau)$	$(3.40_{-0.41+0.03+0.33}^{+0.25-0.06-1.19}) \times 10^{-6}$
$\mathcal{B}(B_c^+ \rightarrow e^+ \nu_e)$	$(1.91_{-0.23+0.12+0.22}^{+0.15-0.19-0.70}) \times 10^{-9}$
$\mathcal{B}(B_c^+ \rightarrow \mu^+ \nu_\mu)$	$(8.18_{-1.00+0.52+0.94}^{+0.63-0.83-2.99}) \times 10^{-5}$
$\mathcal{B}(B_c^+ \rightarrow \tau^+ \nu_\tau)$	$(1.96_{-0.24+0.12+0.23}^{+0.15-0.20-0.72}) \times 10^{-2}$

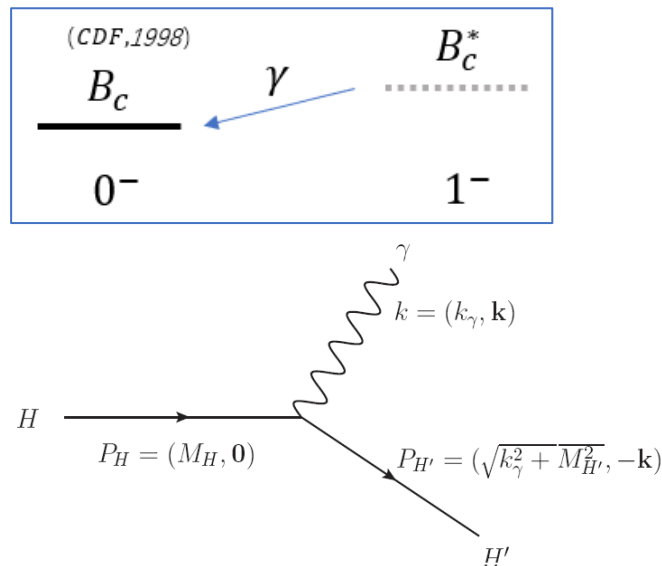
$$\Gamma_{\text{tot}}(\tilde{B}_c^*) = 60\text{eV}$$

$$\tau_{B_c} = 0.51\text{ps.}$$

# 3、Decay width of $B_c^*$ and its non-radiative decay modes

arXiv: 2304.XXXXX

- $B_c^*$  (1S) major (99.99%) electromagnetic decays to  $B_c(1S)$ : M1 transition



$$\begin{aligned} \mathcal{L}_\varphi = & \varphi^\dagger \left( iD_0 + \frac{\mathbf{D}^2}{2m} + \frac{\mathbf{D}^4}{8m^3} \right) \varphi \\ & + g\varphi^\dagger \left( \frac{c_F}{2m} \boldsymbol{\sigma} \cdot \mathbf{B} + i \frac{c_s}{8m^2} \boldsymbol{\sigma} \cdot [\mathbf{D} \times, \mathbf{E}] + \frac{c_D}{8m^2} [\mathbf{D} \cdot, \mathbf{E}] \right) \varphi \\ & + ee_Q \varphi^\dagger \left( \frac{c_F^{em}}{2m} \boldsymbol{\sigma} \cdot \mathbf{B}^{em} + i \frac{c_s^{em}}{8m^2} \boldsymbol{\sigma} \cdot [\mathbf{D} \times, \mathbf{E}^{em}] + \frac{c_D^{em}}{8m^2} [\mathbf{D} \cdot, \mathbf{E}^{em}] \right) \varphi \\ & + ee_Q \varphi^\dagger \left( \frac{c_{W1}^{em}}{8m^3} \{ \mathbf{D}^2, \boldsymbol{\sigma} \cdot \mathbf{B}^{em} \} - \frac{c_{W2}^{em}}{4m^3} \mathbf{D}^i \boldsymbol{\sigma} \cdot \mathbf{B}^{em} \mathbf{D}^i \right) \varphi \\ & + ee_Q \varphi^\dagger \left( \frac{c_{p'p}^{em}}{8m^3} [(\boldsymbol{\sigma} \cdot \mathbf{D})(\mathbf{B}^{em} \cdot \mathbf{D}) + (\mathbf{D} \cdot \mathbf{B}^{em})(\boldsymbol{\sigma} \cdot \mathbf{D})] \right) \varphi \\ & + ee_Q \varphi^\dagger \left( i \frac{c_M^{em}}{8m^3} \{ \mathbf{D} \cdot, \{ \mathbf{D} \times, \mathbf{B}^{em} \} \} \right) \varphi, \end{aligned}$$

$$|H(\mathbf{P}, \lambda)\rangle^{(0)} = \int d^3R \int d^3r e^{i\mathbf{P} \cdot \mathbf{R}} \text{Tr} \left\{ \phi_{H(\lambda)}^{(0)}(\mathbf{r}) S^\dagger(\mathbf{r}, \mathbf{R}) |US\rangle \right\},$$

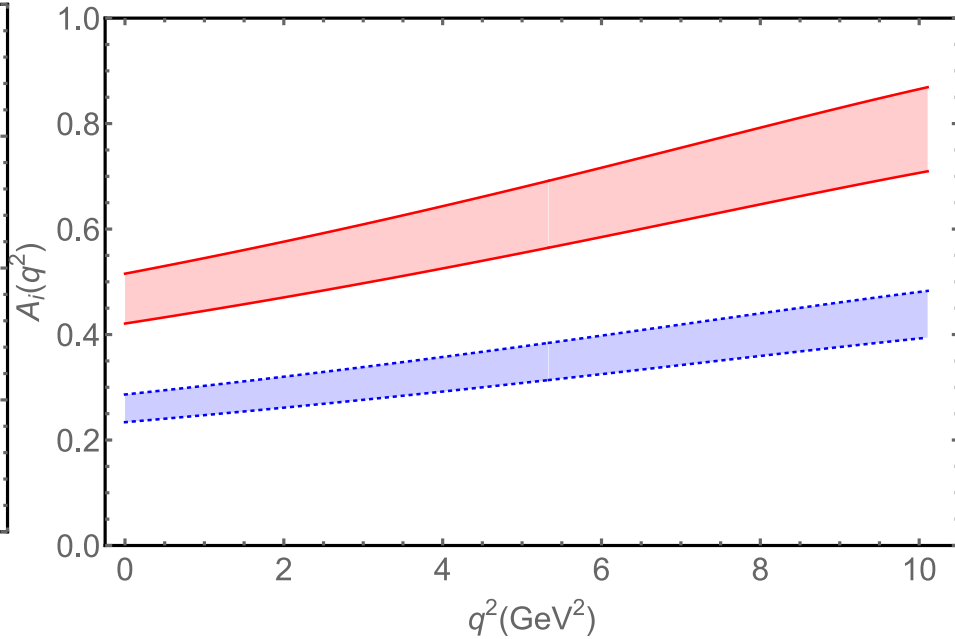
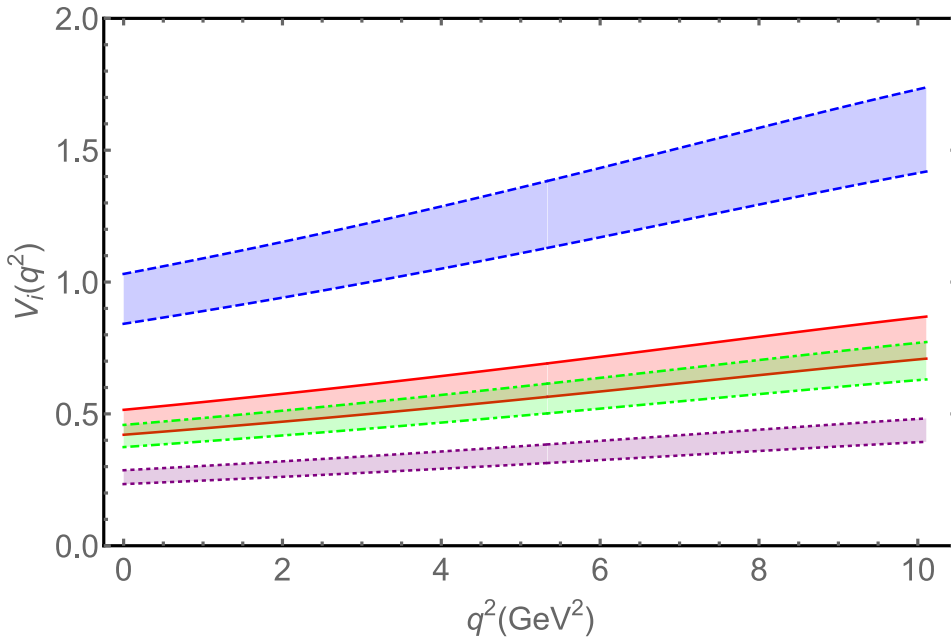
$$\Gamma(B_c^*(1S)) \approx \Gamma(B_c^*(1S) \rightarrow B_c(1S) + \gamma) \sim 54 \text{ eV}$$

- However, around 60MeV photon is hard to detect at LHC

# Bc\* decays to J/psi form factors

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$$\begin{aligned}
 \langle J/\psi (\epsilon', p') | \bar{b} \gamma_\mu c | B_c^* (\epsilon, p) \rangle &= - (\epsilon \cdot \epsilon'^*) [P_\mu V_1 (q^2) - q_\mu V_2 (q^2)] - (\epsilon \cdot q) \epsilon'^*_\mu V_3 (q^2) + (\epsilon'^* \cdot q) \epsilon_\mu V_4 (q^2) \\
 &\quad + \frac{(\epsilon \cdot q) (\epsilon'^* \cdot q)}{M^2 - M'^2} \left[ \left( P^\mu - \frac{M^2 - M'^2}{q^2} q^\mu \right) V_5 (q^2) + \frac{M^2 - M'^2}{q^2} q^\mu V_6 (q^2) \right], \\
 \langle J/\psi (\epsilon', p') | \bar{b} \gamma_\mu \gamma_5 c | B_c^* (\epsilon, p) \rangle &= i \varepsilon_{\mu\nu\alpha\beta} \epsilon^\alpha \epsilon'^{* \beta} \left[ \left( P^\nu - \frac{M^2 - M'^2}{q^2} q^\nu \right) A_1 (q^2) + \frac{M^2 - M'^2}{q^2} q^\nu A_2 (q^2) \right] \\
 &\quad + \frac{i \varepsilon_{\mu\nu\alpha\beta} P^\alpha q^\beta}{M^2 - M'^2} [\epsilon'^* \cdot q \epsilon^\nu A_3 (q^2) - \epsilon \cdot q \epsilon'^{* \nu} A_4 (q^2)] \\
 &\quad + i \varepsilon_{\rho\nu\alpha\beta} \epsilon^\rho \epsilon'^{* \nu} P^\alpha q^\beta [P^\mu A_5 (q^2) - q^\mu A_6 (q^2)],
 \end{aligned}$$



# Bc\* non-radiative modes (selected)

$\ell = e, \mu$	This work	Chang-Wang-Zhu-Li(2020)	Wang et al(2018)	Dai-Zhang-Oset(2018)
$B_c^{*+} \rightarrow J/\psi + \ell^+ + \nu_\ell$	$(2.10 - 3.15) \times 10^{-7}$	$(5.44^{+4.46}_{-2.49}) \times 10^{-7}$	$5.37 \times 10^{-7}$	$2.91 \times 10^{-7}$

	This work	Yang-Wang-Huang-Chang-Sun(2022)	Liu-Wan(2022)
$B_c^{*+} \rightarrow J/\psi + \pi^+$	$(1.86 - 2.79) \times 10^{-8}$	$5.8 \times 10^{-8}$	$2.4 \times 10^{-8}$

$$\mathcal{B}(B_c^{*+} \rightarrow \mu^+ \nu_\mu) \quad (3.85^{+0.29-0.07-1.35}_{-0.46+0.03+0.37}) \times 10^{-6}$$

➤ **Cross section ~100nb at LHC from single parton scattering**

Chang-Wu, EPJC 38,267(2004)

**increased up to ~ μb if consider the double parton scattering**

➤ **around hundreds of Jpsi+pi and thousands of Jpsi+mu+nv events for 10fb<sup>-1</sup> data.**

**Future LHCb 300fb<sup>-1</sup>**

➤ ...

**See Prof. He Jibo's talk**

# Summary and Outlook

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- ✓ Convergent  $B_c^*$  decay constant up to three-loop accuracy is obtained;
- ✓  $B_c^*$  decay width is studied ( $\sim 50\text{-}60\text{eV}$ ) from its radiative decay;  $B_c^{*+}(1S) \rightarrow B_c^+(1S) + \gamma$
- ✓ Hunting for vector  $B_c^*$  meson possibly by non-radiative decay modes

## Outlook

$$B_c^{*+}(1S) \rightarrow \mu^+ + \nu_\mu \quad B_c^{*+}(1S) \rightarrow J/\psi + X$$

- Nontrivial high-order calculation of wave function in un-equal mass cases
- Rare Decays for  $B_c$  meson family at LHC/CEPC/Super-Z

**Thank you a lot!**